Competition with endogenous health risks*

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Abstract

We study a general equilibrium model where workers' preferences and productivity depend on their health status, and occupational choices affect health risks. We show that efficiency typically requires agents of the same type to obtain different expected utility when assigned to different occupations. If health mainly affects production capabilities, workers with riskier jobs must get higher expected utilities under mild conditions. The same holds when health mainly affects preferences if health and consumption are sufficiently good substitutes, while the converse obtains when health and consumption goods are complements. As a corollary, compensating wage differentials which equalize the utilities of workers in different jobs are generally incompatible with efficiency. Competitive equilibria are first best if lottery contracts are enforceable, but typically not when agents can trade only assets with deterministic payoffs. Finally, we show that that, absent asymmetric information, there exist deterministic transfers among workers which allow to achieve first best efficiency.

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1 Introduction

This paper studies a simple general equilibrium model where the aggregate distribution of health is determined jointly with the allocation of labor and consumption goods. Our model has three key features. First, the distribution of workers' health risks depends on their occupational choices. Second, health affects agents' productivity, and their preferences, namely their production and consumption capabilities. Third, occupational choices are indivisible, that is each occupation is defined by an indivisible set of tasks, and each worker may choose at most one occupation and the associated health distribution[1]. These assumptions capture some of the most significant real-life determinants and effects of individual health status. Indeed, occupational choices have both direct and indirect effects on health risks. They directly affect the probability distribution of future health states by influencing the likelihood of work-related injuries and diseases. Moreover, they may change workers' health risks indirectly by affecting their location choices, for instance by inducing them to locate in an area that is less safe (e.g., more crime-ridden or with poorer health facilities). Generally, health status also affects workers' productivity, labor endowment and preferences, as largely documented by the empirical literature. Workers choosing their occupations must then also take into account that their health status will influence their future income, as well as the utility obtainable through future consumption and labor activities. Finally, an important real-world feature of most health risks associated to production activities is that, because of human capital and occupational indivisibilities, they are typically only partially diversifiable. Agents cannot diversify health risks because of the non-convexities due to specialization, which indeed lead most workers to choose a single occupation.

Our setting encompasses both the direct and the indirect effects of jobs on health risks. We study the properties of efficient and equilibrium allocations of an economy with labor indivisibilities, where different distributions of health are associated to different occupations. In our model, agents offer labor in a competitive labor market, produce several goods, and use financial (insurance) markets to transfer income across individual health states.

Our first objective is to illustrate how the economic trade-offs involving health, consumption and production activities shape the Pareto frontier of the economy. In this respect, the crucial feature of our model is that agents' production and consumption capabilities are endogenous[2]. An important consequence of this endogeneity is that the separation between production and consumption decisions, which is standard in welfare

[1] The important restriction here is that a worker cannot choose an arbitrarily large number of jobs and offer a small amount of labor in each of it. The assumption of complete indivisibilty of occupations is made for simplicity.

[2] The efficiency properties of economies where health risks are exogenous haev been quite well understood in the literature, (see Arrow (1952), Evans-Viscusi (1990) and Grossman 1979 among others). Much less work exists on the efficiency properties of economies where health risks are affected by agents' production and consumption choices.

analysis (see Mas Colell et al.(1995)), does not hold in our context. The lack of separation has two fundamental effects on the Pareto frontier of the economy. First, ex-ante efficiency typically requires agents of the same type to get different expected utility levels when assigned to different occupations, this way precluding interim efficiency. Second, efficient consumption vectors of each type of agent are typically such that their shadow value (calculated at the Pareto-optimal shadow prices) differs from the shadow value of their initial endowment of consumption goods and production factors, preventing *budget balancing* within each occupation. Both results rely on a basic optimality argument. Precisely because health risks are specific to occupations, and both preferences and productivity are state-dependent, any pair of ex-ante identical workers with different occupations will generally feature different preferences and expected utility functions. Indeed, although the utilities of these workers are the same in each individual health state, their probabilities of facing each health state will differ since health distributions vary across occupations. For this reason, agents' expected utility functions will be occupation-specific; and the equalization of marginal utilities of contingent goods across agents, which is a standard ex ante efficiency condition, will typically prevent either interim efficiency (i.e. the equalization of the expected utility of agents of the same type assigned to different jobs) or budget balancing.

These results are new at the best of our knowledge and crucially depend on the endogeneity of health risks. Indeed, were optimal consumption and labor decision not influenced by health status and occupational choices, non convexities due occupational indivisibilities alone would become irrelevant in large economies.

Moreover, the inconsistency between ex ante and interim optimality opens a number of theoretical and policy issues that we investigate in the paper. To begin, both ex ante efficient utility's wedges across occupations and optimal cross-jobs transfers must now be characterized in order to understand the relevant efficiency trade-offs between health and the other consumption goods. Moreover, the sub-optimality of interim efficient allocations also raises the question of whether (and what class of) assets with random payoffs, can be used to implement Pareto efficient cross-transfers in a competitive equilibrium. Finally, by looking at the other face of the coin, i.e. taking a policy point of view, the need for cross-transfers across occupations leads also to study what policy interventions in the health market may possibly result welfare beneficial when contracts with random payoffs are unenforceable[3].

Our characterization of Pareto optima, which is performed under some simplifying assumptions on agents' preferences, show that the properties of *ex ante efficient utility's wedges across occupations and cross-jobs transfers* crucially depend both on the riskiness of health distributions and on the relative extent to which health affects agents' consumption and production capabilities. Precisely, ordering the health risk of different occupations

[3] This appear an important issue to address as real life health insurance market are often heavily regulated and many health insurance policies are provided at regulated prices.

according to first-order stochastic dominance, under mild assumptions on the elasticity of labor supply we find that if health mainly affects production capabilities (i.e. the disutility of labor or the productivity or the labor endowment), then agents assigned to riskier occupations must get a higher expected utility and a value of consumption higher than shadow value of their initial endowment of consumption goods and production factors, . The same result holds if health mainly affect preferences, provided the degree of substitutability between health and consumption goods is sufficiently large; while the converse obtains when health and consumption goods are complements; in this case, indeed, agents using safer technologies obtains an higher expected utility and a value of consumption larger than the value of their endowment.

All these results relate closely with some important contributions of the literature on compensating wage differentials. This literature focuses on the determination of the equilibrium wage premia commanded by risky, or otherwise unpleasant, jobs, and also analyzes the economic determinants of the wage versus health and safety risks trade-off (see Rosen (1986) Evans and Viscusi (1993), and Lucas (1972) among other). In particular, wage premia are characterized and estimated by imposing the condition that agents of the same type get the same utility in all occupation (i.e. by imposing interim efficiency) which is informally justified as a result of competitive labor market interactions. Our paper is at the best of our knowledge the first to analyze the determinants of welfare-maximizing compensating wage differentials, and to show that wage differentials satisfying interim efficiency typically prevent ex-ante efficiency.

In the second part of the paper we turn to the competitive analysis. We consider two alternative contractual regimes, labeled the "high-transaction-costs regime" and the "low-transaction-costs regime", respectively. In the former, there exist competitive insurance markets to cope with all idiosyncratic risks as in Malinvaud (1973) and Cass, Chichilnisky and Wu (1996), but only financial contracts with deterministic returns are enforceable. In the latter regime, agents can "trade" also lottery contracts, i.e. contracts with random payoffs. The first regime turns out to be the natural benchmark for understanding both the properties of the Pareto frontier and the welfare properties of competitive markets. The analysis of the high-transaction-costs regime is warranted by several reasons. First, as we already pointed out, all the theoretical and empirical literature on non-pecuniary job attributes and compensation wage differentials has only considered contracts with deterministic payoffs. Since this literature is a natural reference point for the problem at hand, we wish to know under what conditions competition with deterministic contracts leads to efficiency. Second, on the empirical side, the use of lottery contracts (or the use of other financial instruments that may replicate allocations obtainable through random contracts) does not appear to be very widespread in real markets[4].Finally, on a theoretical ground,

[4]Kehoe, Levine and Prescott (2001) show that, if there exists a sufficient number of assets paying units of numeraire in sunspot states of the world, competitive equilibria are first best efficient. At least in our setting, however, efficient trades of financial instruments leading to random allocation are typically such that workers must take possibly big short positions in the asset markets. This is often impossible in real

the use of "optimal" random contracts may sometimes result severely restricted either by limited liability constraints, or by the costs of verifying the characteristics and outcomes of the random devices that are needed to implement them.

Standard arguments imply that a competitive equilibrium exists and is typically locally unique both in the high and in the low-transaction-cost regimes. In both cases "insurance" is traded at fair prices, consumption allocations differ across workers of the same type with different occupations, and equilibrium wage differentials provide a premium for health risks.

However, expected wage differentials, and more generally the efficiency properties of competitive equilibria, differ markedly in our contractual environments. If lottery contracts are enforceable, competition leads to efficiency and both welfare theorems hold. Lottery contracts ensure ex-ante optimality precisely by allowing agents to make the cross-job transfers that are needed for the equalization of marginal utilities.

Conversely, if lottery contracts are unenforceable, competitive equilibria satisfy a specific interim efficiency condition, guaranteeing the equal treatment of the agents of the same type assigned to different occupations; but they are typically not ex ante efficient. Indeed, by equalizing the expected utilities of workers employed in different sectors, competition creates a wedge between their marginal utilities of expected income.

An important corollary is that the welfare properties of competitive equilibria with exogenous and endogenous individual risks dramatically differ. Indeed, under exogenous individual risks the first welfare theorem holds provided that competitive markets exist for insuring all individual risks through assets with deterministic payoffs (see Malinvaud (1973) and Cass, Chichilniski and Wu (1996)). The results of our welfare analysis are also related with the literature on indivisibilities (see Garrett (1995)[5] and the general equilibrium literature with asymmetric information initiated by Prescott-Townsend (1984a,b) (see also Allen and Gale (2003), Arnott and Stiglitz (1986), Bennardo-Chiappori (2003), Cole (1990), Rustichini and Siconolfi (2003), and Bennardo (2004) among others). This literature shows that random contracts may be welfare beneficial as they allow to convexify asymmetric information environment, where incentive constraints typically introduce natural non convexities[6]. In particular, it provides several examples where random contracts are Pareto beneficial; in our environment we prove the stronger result that random contracts are *almost always* necessary to achieve efficiency through the market. Moreover, differently from most of the literature on asymmetric information, our paper focus on the

life markets also because of incentive problems.

[5] Garrett (1995) studies lottery equilibria in economies with indivisibilities, but mainly focuses on existence issues in finite economies, and does not characterize Pareto optimal allocations.

[6] Our paper has in particular some close connections with that of Rustichini and Siconolfi. These authors, while mainly interested in economies with asymmetric information, also show that in economies with symmetric information, state-dependent preferences and endogenous individual risks, competitive equilibria are Pareto optimal if lottery contracts are enforceable. However, they neither characterize Pareto-optimal allocations with state-dependent preferences nor investigate under what conditions assets (contracts) with non-degenerate random payoffs are effectively welfare beneficial.

role that lottery contracts may play as market devices designed to implement cross-jobs transfers[7].

Finally, in the last part of the paper we show that Pareto optima can be implemented through deterministic transfers' policies that do not hinge on mechanisms with random payoffs. These policies impose cross-subsidies among insurance policies designed for workers choosing different occupations. The sign and the magnitude of the transfer received by each worker is determined by the difference between the (shadow) value of his consumption and that of his production and his endowment, both calculated at the optimal shadow prices. Under a natural non-manipulability requirement of the policy scheme, Pareto improving transfers' policies also require minimum wages[8].

Notewhorty, two contributions of the existing literature by Rogerson (1988) and Hansen (1985) provide examples where policy interventions (i.e. provision of unemployment insurance) based on deterministic transfer are Pareto improving in a setting where only deterministic contracts can be signed. Specifically, in the models analyzed by Rogerson and Hansen unemployment insurance may be welfare beneficial if labor supply choices are indivisible (workers may choose either to work or not but not how much to work) and a positive fraction of workers are unemployed at equilibrium [9]. Differently from Rogerson and Hansen, in our model cross-jobs transfers policies are typically welfare beneficial in the absence of lottery contracts, even if we assume that labor supply choices within each occupation are not indivisible and may depend on health state, and independently from the presence of equilibrium unemployment.

2 The economy

Demography, consumption goods and preferences. A continuum of measure 1 of consumers-workers produce C consumption goods. There exists a finite set , $I = \{1, ..., I\}$, of agents' types; i and μ_i is the total fraction (measure) of type-i agents. All agents face health risks that may affect their preferences, endowments and productivity. The set $\Theta = \{\theta_1, ..., \theta_N\}$ of possible health states is assumed to be finite, and $\theta \in \Theta$ represents a generic health state. Type-i agent has an endowment $e_i \in \Re^C_+$ which is the same in all individual states, and an amount L of time which is allocated between work, l, and leisure x_L. The maximal fraction of his total time that each agent can devote to work may depend on his health state, and is denoted $L(\theta)$; $L(\theta)$ is weakly increasing in θ, i.e., agents with better health states can work more.

Agents preferences are (health) state dependent and represented by the utility function

[7] See however Bennardo (2004) for a similar result on a multicommodity production economy with moral hazard.

[8] Imposing minimal wages eliminates firms' incentives to decrease wages in the sector where workers get higher expected utilities.

[9] This may result from labor indivisibilities.

$U_i(x,\theta)$, where $x = (x_1, .., x_C, x_L) \in \Re^C_+ \times [0, L]$. $U_i(x,\theta)$ is twice continuously differentiable, strictly increasing and strictly concave, and has indifference surfaces with closure in $\Re^C_{++}$.

Technologies and uncertainty. Competitive firms produce goods by employing workers and labor is the only production factor. Each firm can hire a positive measure of agents, while each worker can work for at most one firm; indivisibilities due to specialization prevents workers from performing different jobs[10]. There are $T = C$ production sectors and only one type of occupation is offered in each sector. Each worker's contribution to production is measurable and may depend on his health state. Precisely, and a type-i worker who is employed in sector t, and supplies l^t_i units of labor, produces $y^t_i(\theta) = a^t_i(\theta)l^t_i$ units of commodity t when his health state is θ. The state contingent productivity of type-i workers employed in sector-t, $a^t_i()$, is weakly increasing in θ. Moreover, each (type-i) worker's distribution of health states, $\langle p^t_i, \Theta \rangle$, with $p^t_i = (p^t_i(\theta_0), ..., p^t_i(\theta_N))$, depends on his occupation. Finally, health shocks are identically and independently distributed among workers with the same occupation, and independently distributed across sectors. The endogeneity of the health distribution can be due to the direct effects of labor activities on prospective workers' health. It can also be interpreted as the consequence of localization choices, which are determined by occupational choices.

Timing. The economy lasts two periods, $\tau = 0, 1$; at $\tau = 0$, agents trade in financial and labor markets. At $\tau = 1$, health shocks are realized; subsequently agents supply labor in production, and consumption goods are traded and consumed.

The contracting space (the set of enforceable contracts) will be defined in section 4.

Throughout, we will use the following notation: $x^t_i(\theta)$ is a generic state contingent consumption vector for a type-i agent occupied in sector t, with $x^t_i = (x^t_i(\theta))_{\theta\in\Theta}$ and $x = (x^t_1(\theta), ..., x^t_I(\theta))^{t\in T}_{\theta\in\Theta}$; $l^t_i(\theta)$ is a state contingent of labor for a type-i agent occupied in sector t. Finally $\alpha_i = (\alpha^1_i, ..., \alpha^T_i)$ represents an assignment of type-i workers to production sectors.

3 Ex ante and interim Pareto optimality

Ex ante Pareto optimality. Let $u^t_i(x^t_i) = \sum_{\theta\in\Theta} p^t_i(\theta) U(x^t_i(\theta), \theta)$ be the Von Neumann Morgenstern expected utility of a type-i agent employed in sector t, and let $\bar{x}^t_{ic} = \sum_{\theta\in\Theta} x^t_{ic}(\theta)p^t_i(\theta)$ be the expected consumption of commodity c of type-i worker employed in sector t.

By the law of large numbers, a *feasible allocation* of consumption goods, labor, and workers, $\langle x, \alpha\rangle$ must satisfy the following constraints:

[10] This extreme assumption, tough realistic in many circumstances, is made for simplicity. Assuming that each agent can perform a finite subset of jobs at the same time would only complicate the notation.

$$\sum_{i\in I}\mu_i \sum_{t\in T}\alpha_i^t \bar{x}_{ic}^t \leq \sum_{i\in I}\mu_i (e_i^c + \alpha_i^c y_i^c) \qquad \forall\ c = 1, ..., C \tag{1}$$

$$y_i^t = \sum_{\theta\in\Theta} p_i^t(\theta)\, a_i^t(\theta) l_i^t(\theta) \qquad \forall\ t = 1, .., T \tag{2}$$

$$l_i^t(\theta) + x_{iL}^t(\theta) \leq L \quad \forall\ \theta;\ \sum_{t\in T}\alpha_i^t = 1 \qquad \forall\ i = 1, .., I \tag{3}$$

Let F the set of feasible allocations, and let:

$$U = \left\{ \bar{u} = (\bar{u}_2, ..., \bar{u}_I) : \exists\ \langle x, \alpha\rangle \in F,\ \text{such that } \sum_{t\in T}\alpha_i^t u_i^t(x_i^t) \geq \bar{u}_i \text{ for } i = 2, .., I \right\}$$

Each allocation in the set of Pareto optima solves $\max_{(x,\alpha)\in U}\sum_{t\in T}\alpha_1^t u_1^t(x_1^t)$ *s.t.* $\sum_{t\in T}\alpha_i^t u_i^t(x_i^t) \geq \bar{u}_i$ for $i = 2, .., I$, for some $\bar{u} \in U$.

Note that we are not imposing that all agents of the same type must get the same expected utility independently from their occupation. While such a condition is typically satisfied by the optima of a convex economy, there is no reason to impose it as part of the definition of first best allocations. Moreover, by maximizing $\sum_{t\in T}\alpha_1^t u_1^t(x_1^t)$, one implicitly assumes that efficient mechanisms can randomly assign agents to occupations.

On the other hand, we are not taking into account the possibility that agents obtain random consumption allocations in the optimum. In our setting, this is without loss of generality since a standard risk aversion argument imply that random consumption allocations are always suboptimal.

Interim Pareto optimality. The following definition of interim Pareto optimality will play a crucial role in the welfare analysis of competitive equilibria with contractual incompleteness and unenforceable lottery contracts.

An *interim optimal allocation with equal treatment* maximizes $\sum_{t\in T}\alpha_1^t u_1^t(x_1^t)$ under (1)-(3) and the additional constraints $u_i^t(x_i^t) = u_i^{t'}(x_i^{t'})$ for each $t \neq t'$ such that $\alpha_i^t > 0, \alpha_i^{t'} > 0$.

4 Competitive equilibria

Throughout we shall assume that there exist competitive spot markets for all goods, as well as markets for insuring *all* risks through assets with *deterministic* payoffs. We shall study either a setting where lottery contracts (assets with random payoffs) are *enforceable* or one in which they are *unenforceable*. The two settings will be referred to as *perfect competition with low transaction costs* and *with high transaction costs*, respectively. Studying both these cases is useful to fully understand either the beneficial role that random contracts may play in our economy, or the effects of a somewhat natural market friction, that, as we explained in the introduction, may make costly or prevent their use.

4.1 Perfect competition with *high* transaction costs

Following the approach taken in several contributions of the general equilibrium literature on individual risks, we assume that there exist competing, risk neutral[11] intermediaries who offer securities with payoffs contingent on individual states. Specifically, let $h_{i\theta}^t$ a security paying one unit of numeraire in the individual health state θ, to type-i agents employed in sector t. A security market for all individual risks, types and occupations is assumed to exist[12]. Let $z_{i\theta}^t$ and $\hat{z}_{i\theta}^t$ be the units of $h_{i\theta}^t$ purchased by type-i agents employed in sector t, and the per capita units of this security offered in the market, respectively. And denote $\phi_i^t(\theta)$ the price of $h_{i\theta}^t$.

Production firms are also price-taking agents, which trade state contingent labor services and consumption goods at linear prices.

Finally let denote $w_i^t(\theta)$ the state contingent wage of type-i worker in the $t - th$ occupation (sector), with $w_i^t = (w_i^t(\theta))_{\theta \in \Theta}$ [13]. Finally, $q \in \Re_+^C$ denotes a vector of *spot prices* and q_c the $c - th$ component of this vector[14].

Because of the presence of indivisibilities in labor supply, it is expositionally convenient[15] to consider the possibility that workers choose their occupation by using mixed strategies. Denote $\varphi_i = (\varphi_i^1, .., \varphi^t, ...\varphi^T) \in \Delta^T$, the T dimensional simplex, a generic probability vector according to which a type-i worker chooses his occupation. Notice that the law of large number allows to interpret φ_i^t as the fraction of type-i agents who ex post get an occupation in sector t.

A competitive (Walrasian) equilibrium with high transaction costs is then an allocation $(x_i^{t*}, \varphi_i^{t*})_{i \in I}^{t \in T}$, a per capita vector of securities' offers and purchases $(\widehat{z}_i^{t*}, z_i^{t*})_{i \in I}^{t \in T}$ and a vector of state contingent prices $(q, (\phi_i^t, w_i^t))_{i \in I}^{t \in T}$ satisfying the following conditions.

(I) Type-i agents maximize their utility by choosing

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[11] Intermediaries' risk neutrality is, as usual, justified by the assumption of large numbers.

[12] Hence, the total number of open security markets is $\#\Theta \times T \times I$

[13] The introduction of individual risks in a competitive settings requires that assets' payoffs which are contingent on individual shocks must also be contingent on agents' types. This has been clarified in Malinvaud (1973) and Rustichini Siconolfi (2003),

[14] In the absence of aggregate uncertainty, spot market prices are independent from the realizations of individual shocks that wash-out at the aggregate level.

[15] In our continuum setting, equilibrium mixed strategies on occupations can be interpreted as different fractions of agents choosing a pure strategy in equilibrium. In other words, given any mixed strategy equilibrium strategy profile there exists a payoff equivalent profile of pure strategies satisfying all feasibility conditions. Using mixed strategies is, however, convenient for expositional reasons.

$$\left(x_i^{t*}, \varphi_i^{t*}, z_i^{t*}\right) \in \arg\max \sum_{t\in T} u_i^t(x_i^t)\varphi_i^t \tag{4}$$

$$s.t. \sum_{c\in C} q_c(x_{ic}^t(\theta) - e_i^c) = w_i^t(\theta)\left(L - x_{iL}^t(\theta)\right) + z_i^t(\theta) \quad \forall\, (\theta, t) \tag{5}$$

$$\sum_{\theta\in\Theta} z_i^t(\theta)\, \phi_i^t(\theta) \leq 0 \quad \forall\, t \tag{6}$$

where (5)are the spot market budget constraints and (6) is the initial period budget constraint.

(II) Production firms' and intermediaries, respectively, solve the following programs:

$$l_i^{t*} \in \arg\max \sum_{\theta\in\Theta} p_i^t(\theta)\left(q_t y_i^t(\theta) - w_i^t(\theta)\, l_i^t(\theta)\right) \quad \text{s.t.} \quad y_i^t(\theta) \leq a_i^t(\theta)\, l_i^t(\theta) \quad \forall\, \theta \tag{7}$$

$$\widehat{z}_i^{t*} \in \arg\max \sum_{\theta\in\Theta} (\phi_i^t(\theta) - p_i^t(\theta))\widehat{z}_i^t(\theta) \quad \text{s.t.} \quad \sum_{\theta\in\Theta, i\in I} \mu_i p_i^t(\theta)\, \widehat{z}_i^t(\theta) \geq 0 \;\; \forall\, t \tag{8}$$

(III) Consumption, labor and financial markets clear:

$$\sum_{i\in I}\mu_i \sum_{t\in T} \varphi_i^t \bar{x}_{ic}^{t*} \leq \sum_{i\in I}\mu_i(e_i^c + \varphi_i^c y_i^c) \quad \forall\, c = 1,..,C \tag{9}$$

$$L - x_{iL}^{t*}(\theta) = l_i^{t*}(\theta), \text{ and } z_i^t(\theta) = \widehat{z}_i^{t*}(\theta) \text{ for all } i,\ t \text{ and } \theta. \tag{10}$$

4.2 Perfect competition with *low* transaction costs.

We will now introduce lottery contracts in our competitive setting. Specifically, we will assume that, *before making any market trade*, agents can buy lotteries from financial intermediaries. These lotteries allow to obtain a vector of prizes (a payoff vector in units of numeraire) with positive probabilities. Following Arnott-Stiglitz (1987), in the literature such randomizations have been referred to as ex ante random contracts.

Formally, we define a lottery contract, $\mathcal{C} = ((\gamma, G), \rho(\gamma, G))$, as: (i) a finite distribution (γ, G) with probabilities $\gamma = (\gamma^1, ..., \gamma^M) \in \Delta^M$ and payoffs $g = (g^1, ..., g^M) \in \Re^M$, with M finite, and, (ii) a price $\rho(\gamma, G) \in \Re$. The interpretation of $\mathcal{C}$ is as follows: an agent signing $\mathcal{C}$ with a financial intermediary pays him the price $\rho(\gamma, G)$, while the financial intermediary commits to deliver to the agent the payoff g^m with probability γ_m. A random devise, whose characteristics are publicly verifiable, is then used by the contracting parties. Such a device chooses an artificial state of the world by selecting a positive integer $m \in \{1, ..., M\}$ with probability γ^m. Subsequently, the intermediary pays g^m to the agent whenever the integer

m turns out to be selected. The expected profit that a generic intermediary makes from signing $((\gamma, G), \rho)$ is $\rho(\gamma, G) - \sum_{m \in M} \gamma^m g^m$.

A very general formulation of the competitive equilibrium in the space of random allocations would require all possible lottery contracts (an infinite set) to be priced in equilibrium (as in Rustichini-Siconolfi (2003)) and would allow agents to possibly sign many lottery contracts. In order to avoid either the technicalities arising with infinite dimensional commodity space, or a complex notation, we will make the following simplifying and unrestrictive assumptions on the functioning of markets for lottery contracts. First, all fair lottery contracts with a payoff support of dimension $M \leq T+1$ are offered in the market. Second, an agent will sign at most one lottery contract, and all contracts signed have a support of dimension $M = T+1$. Third, an agent will offer labor in sector t if and only if he receives the *t-th* payoff of the lottery contract he has signed.

The first assumption is justified by an arbitrage argument. Assuming that each agent signs at most one lottery contract is also unrestrictive. Indeed it is straightforward to verify that any finite distribution of net payoffs that can be obtained by means of N fair lottery contracts can also be obtained through a single contract[16] contained in the set of fair lotteries. Finally, it is individually optimal for all agents to choose a lottery contract with at most $M = T+1$ payoffs different from zero. Indeed, a risk averse agent will never find it optimal to choose a lottery contract such that: (i) he receives the payoffs g^m and $g^{m'}$, with $g^m \neq g^{m'}$, with positive probabilities γ^m and $\gamma^{m'}$ respectively, and (ii) he chooses to work in sector t either when he receives g^m or $g^{m'}$. This is true as by convexity there always exists another fair contract, say $\mathcal{C}'$, which pays $\gamma^m g^m + \gamma^{m'} g^{m'}$ with probability $\gamma^m + \gamma^{m'}$ which is strictly preferred to $\mathcal{C}$. Hence if a fair contract is selected by an agent, it must necessarily have at most $M = T+1$[17] positive payoffs. Given this fact, the hypothesis that an agent will offer labor in sector t if and only if he receives the $t-th$ payoff of his lottery contract amounts to be a convenient notational convention.

A competitive (Walrasian) equilibrium with low transaction costs is then an allocation $(\tilde{x}_i^t)_{i \in I}^{t \in T}$, a per capita vector of assets offers and purchases $(\tilde{\hat{z}}_i^t, \tilde{z}_i^t)_{i \in I}^{t \in T}$, a vector of lottery contracts $(\mathcal{C}_i)_{i \in I}$, and a vector of prices $(\tilde{q}, \tilde{\phi}_i^t, \tilde{w}_i^t)_{i \in I}^{t \in T}$ satisfying the following conditions.

(I) Type-i agents maximize their utility by choosing

$$(\tilde{x}_i, \tilde{z}_i^t, \mathcal{C}) \in \underset{\mathcal{C} \in \Gamma}{\arg\max} \sum_{t \in T} \gamma^t u_i^t(x_i^t) \tag{11}$$

[16] Such a contract is defined by probabilities and payoffs which are linear combinations of the probabilities and the payoffs of the the N fair lottery contracts

[17] The dimension is T+1 and not T because an agent may also decide not to supply labor conditionally on receiving one of the possible payoffs of the contract.

$$s.t.\ \ \sum_{c\in C} q_c(x^t_{ic}(\theta) - e^c_i) = w^t_i(\theta)\left(L - x^t_{iL}(\theta)\right) + z^t_i(\theta) + g^t - \rho(\gamma, G) \qquad \forall\ (\theta, t) \tag{12}$$

$$\sum_{\theta\in\Theta} z^t_i(\theta)\,\phi^t_i(\theta) \le 0 \quad \forall\ t \tag{13}$$

where (12)-(13) are the budget constraints and $\Gamma = \left\{(\gamma, G), \rho(\gamma, G)) : \rho(\gamma, G) = \sum_{t\in T}\gamma^t g^t\right\}$.

(II) Production firms' and intermediaries, solve the same programs (i.e., 7 -8) as in the competitive equilibrium with deterministic contracts :

(III) Consumption financial and labor markets clear:

$$\sum_{i\in I}\mu_i \sum_{\theta\in\Theta, t\in T} \widetilde{\gamma}^t \widetilde{x}^t_{ic}(\theta) p^t_i(\theta) = \sum_{i\in I}\mu_i(e^c_i + \widetilde{\gamma}^c \sum_{\theta\in\Theta} p^c_i(\theta)\,\widetilde{l}^c_i(\theta)\,a^c_i(\theta)),\ \forall\ c = 1, .., C \tag{14}$$

$$L - \tilde{x}^t_{iL}(\theta) = \tilde{l}^t_i(\theta),\ \text{ and } \tilde{z}^t_i(\theta) = \widehat{z}^t_i(\theta) \text{ for all } i,\ t \text{ and } \theta. \tag{15}$$

5 Pareto optimal allocations

In this section we characterize Pareto optimal allocations. When it turns out to be convenient we will assume the Inada conditions, $D_cU_i(x, \theta) \to +\infty$ as $x_c \to 0$ for all θ, and i, and $D_cU_i(x, \theta) \to 0$ as $x_c \to \infty$ for all θ and i, which imply internal solutions. Let denote λ the vector of $(I - 1)$ Lagrange multipliers associated to the utility constraints, $\sum_{t\in T}\alpha^t_i u_i(x^t_i) \ge \overline{u}_i$ for all i, and assume by convention $\lambda_1 = 1$. Moreover, let η define the vector of the C Lagrange multipliers associated the feasibility constraints.

The first order conditions with respect to $(x^t_i(\theta), x^t_{iL}(\theta), \alpha^t_i)$ of the (ex ante) Pareto program are respectively:

$$\lambda_i D_cU_i(x^t_i, \theta) = \eta_c\mu_i \text{ for all } c, t \text{ and } i \tag{16}$$

$$\lambda_i U_{ix_L}(x^t_i, \theta) = \eta_t a^t_i(\theta)\mu_i \text{ for all } t \text{ and } i \tag{17}$$

$$\lambda_i(u^t_i(x^t_i) - u^{t'}_i(x^{t'}_i)) = \mu_i(Z^t_i - Z^{t'}_i) \text{ for all } t \ne t' \text{ and } i \tag{18}$$

where for all $t = 1, .., T$,

$$Z^t_i = \sum_{c\in C, \theta\in\Theta}(\eta_c(p^t_i(\theta)x^t_{ic}(\theta) - e^c_i) - \eta_t \sum_{c\in C, \theta\in\Theta} p^t_i(\theta)a^t_i(\theta)\left(L - x^t_{iL}(\theta)\right)$$

represents the difference between the value of the consumption of a type-i workers employed in sector t, *as measured by the vector of shadow prices* η, and the sum of the value of its *endowments* and its production.

As usual, (16) and (17) imply that marginal rates of substitution between all pairs of state contingent commodities are the same for all types.

The first order conditions with respect to α are less standard and play a crucial role in our analysis. Precisely, they say that the difference between the expected utilities of type-i workers employed in sector t and sector t' is proportional to the difference $Z_i^t - Z_i^{t'}$. Only if $Z_i^t - Z_i^{t'} = 0$, the type-i workers assigned to the occupations $t - th$ and $t' - th$, respectively, will get the same utility, and ex ante and interim optima coincide. This is, indeed, the distinguishing feature of the setting we are studying.

In order to state next proposition we shall use the following notation. Denote $t_i = \left(\left\langle p_i^t, \Theta \right\rangle, A_i^t\right)_{t \in T}$, with $A_i^t = \left\{a_i^t(\theta_1), ..., a_i^t(\theta_N)\right\}$, the sector t production process used by type-i worker. Let $\varepsilon = \langle e, T, U \rangle$ represent an economy with aggregate endowment $e \in \Re_{++}^C$, a vector $\mathbf{t} = (\mathbf{t}_1, ..., \mathbf{t}_I)$ of production technologies and profile $U = (U_1, ..., U_I)$ of utility functions . The set of possible economies is then defined as $\mathcal{E} = E \times T \times \mathcal{U}$, where $E = \Re_{++}^C$ is the endowment space, $T = \Re_{++}^{I \times C \times \# \Theta} \times \Delta^\Theta$ is the technology space, and $\mathcal{U} = \prod_{i=1}^I \mathcal{U}^i$ is the set of admissible profiles of utility functions (one for each type) which will be precisely defined in the proof of the next proposition.

Proposition 1 *(i) For each vector of reservation utilities $\bar{u}$, the Pareto optimum is unique; (ii) Moreover, if the number of produced goods is larger than the number of agents' types, then there exists a generic subset $\mathcal{S} \subset \mathcal{E}$ in which ex ante and interim Pareto optima are different.*

The proof is provided in the appendix and requires the use of some differential topology arguments. Its basic idea, however, relies on a simple argument. As there exists an unique optimum for each vector of reservation utilities, this optimum is uniquely defined by the system of first order conditions of the Pareto program, that we denote $\mathcal{F}() = 0$. Moreover, by definition interim efficiency requires the equalization of expected utilities of workers of the same type assigned to different occupations. Then either the solution of $\mathcal{F}() = 0$ satisfies the system of additional conditions, $u_i^t(x_i^t) = u_i^{t'}(x_i^{t'})$ for all α^{it} and $\alpha^{it'}$ strictly positive, which define interim efficiency, or such conditions do not hold and ex ante and interim efficiency are incompatible. The proof demonstrates that the former case is exceptional[18]. Finally, an immediate corollary of this proposition is that first best efficiency typically requires a random allocation of workers across occupations and transfers of resources across workers assigned to different occupations.

[18]It is worthwhile to stress that under standard regularity conditions ex ante and interim efficient allocations typically coincide in economies with exogenous distributions of individual risks. This remains true also in the presence of occupational indivisibilities.

The intuition for this result is that agents expected utilities are "occupation dependent" as occupational choices determine workers health distributions. And for this reason, the equalization of the marginal utilities of consumption goods across occupations, required by the first order conditions, is generally not consistent with the equalization of expectted utilities, i.e. with $u_i^t(x_i^t) = u_i^{t'}(x_i^{t'})$.

6 Characterization of Pareto optimal allocations in simple economies

In light of the results of the previous section, it becomes natural to investigate how the properties of Pareto efficient allocations, and the Pareto optimal transfers among workers of the same type assigned to different occupations, are affected by the *degree* of *health riskiness* associated to these occupations.

For simplicity only, we shall analyze these issues within a simplified setting where two goods are produced by a representative agent. These assumptions on the number of goods and agents' types are completely unrestrictive. Whereas it turns out to be useful we will also make some mild assumptions on agents' preferences that will allow us to obtain stronger characterization results by neglecting certain types of income effects.

According to the notation used before, now $x = (x_1,\ x_2,\ x_L)$ and $\hat{x} = (x_1,\ x_2)$ will denote a generic vector of goods and a vector of produced goods, respectively. We continue to use the same notation for endowments and technologies.

In order to interpret the results derived in this section it will be important to distinguish the effects of health status on the utility of consumption from those on the disutility of labor. For this purpose, from now on agents' certainty utility function will be written as:

$$U(x,\theta) = f\left(x,\theta\right) - \psi\left(L - x_L, \hat{x}, \theta\right)$$

where $U(x,\theta)$ satisfies all the assumptions we have previously stated, and where f and ψ represent the utility of consumption commodities and the disutility of labor, $l = L - x_L$, respectively. According to this representation, both f and ψ may possibly depend on θ. By introducing $\hat{x}$ among the arguments of ψ, we take into account the possibility that consumption activities affect workers' disutility of labor[19].

In order to evaluate how the *degree* of health riskiness associated to different occupations contributes to determine the properties of efficient allocations and transfers across workers, throughout all this section we assume that $\left\langle p^1, \Theta\right\rangle$ first order stochastically dominates (FOSD) $\left\langle p^2, \Theta\right\rangle$. Let $P^i(\theta_n) = \sum_{\theta \leq \theta_n} p^i(\theta)$, formally we impose $P^1\left(\theta_n\right) \leq P^2\left(\theta_n\right), \forall\, \theta_n \in \Theta$, with at least one strict inequality.

[19] In the real world, one can easily found either instances where a larger consumption of consumption goods (such as food and housing, drugs, kindergarden services, etc...) reduces the disutility of labor, and instances where a larger consumption of certain goods (alcohol etc.) increases the disutility of labor.

Within this setting, we shall show that the properties of Pareto optimal allocations and the direction of transfers among workers using technologies with different degree of riskiness crucially depends either on the *direct effect of health status on agents well being*, or on the effects of health on: (i) the utility of consumption commodities; (ii) the disutility of labor; (iii) the labor endowment and labor productivity. We will refer to (i) as the health effect on *consumption capabilities* and to (ii) and (iii) as the health effects on *production capabilities*. For the sake of clarity, we will consider each of these effects in isolation.

6.1 Health effects on consumption capabilities

We begin by considering how the effects of health status on the utility of consumption influence the properties of Pareto optima. For this purpose, we assume here that workers supply labor inelastically ($l^t(\theta) = L$ for $t = 1, 2$ and for all θ), and that both workers' productivity and their labor endowment are independent from health status. Precisely, we will set here $a(\theta) = a$ and $L(\theta) = L$ for all θ.

Preliminarily to the characterization of Pareto optima, we state the following well know result which turns out to be a useful tool for the analysis.

Lemma 2 *For any map $g : \Theta \to \Re^+,\ \theta \to g(\theta)$, with $dg(\theta_{n+1}) = g(\theta_{n+1}) - g(\theta_n)$, the following identity holds:*

$$\sum_{\theta\in\Theta} \left(p^1(\theta) - p^2(\theta)\right) g(\theta) := \sum_{n=0}^{N} \left(P^2(\theta_n) - P^1(\theta_n)\right) dg(\theta_{n+1})\ \forall\, n = 0, ...N.$$

For completeness, the proof of the lemma is provided in the appendix.

Next proposition explains how the direct health effect on well being, measured by $U_\theta(x, \theta)$, and the effect of health on the (marginal) utility of consumption, measured by $U_{c\theta}(x, \theta)$, contribute to determine the optimal allocation, and hence the difference, $\Delta u^P = u^1(x^{1P}) - u^2(x^{2P})$, between the expected utilities obtained at the optimum by the workers assigned to the two occupations. Precisely, it shows that when consumption goods and health are complements, or at least not very good substitutes, i.e. when the vector $(U_{c\theta}(x, \theta)/U_\theta(x, \theta))_{c\in C}$ is not *too negative*, efficiency imposes that workers using safer technologies obtain a larger expected utility, while the opposite is true for $U_{c\theta}$ negative and sufficiently large.

Proposition 3 *The Pareto optimal allocation satisfies the following properties: (i) assume $U(x, \theta)$ is supermodular in x for any given θ and it has increasing differences in (x, θ), then $u^1(x^{1P}) > u^2(x^{2P})$. (ii) Assume $U(x, \theta)$ is supermodular in x for any given θ and it has decreasing differences in (x, θ), then there exist two positive functions $g_1(x, \theta)$ and $g_2(x, \theta)$ such that $U_{c\theta}(x, \theta) / U_\theta(x, \theta) \gtreqless -g_c(x, \theta)$ for all (x, θ) where $x_L = 0$ then $u^1(x^{1P}) \gtreqless u^2(x^{2P})$.*

Proof. See the Appendix. ■

Intuitively, Pareto optimality requires risk-averse workers assigned to different occupations to get the same consumption in each individual health state (i.e., $x^1(\theta) = x^2(\theta)$ for all θ). Now, consider first the case where consumption goods and health are complements, i.e., $U_{c\theta}(x, \theta) > 0$, in this case optimality requires agents' consumption is larger in better health states. As $U_\theta(x, \theta) > 0$, the Pareto optimal level of utility, $U_\theta\left(x^P(\theta), \theta\right)$ increases in θ. Moreover, since worker using safer technologies (in the sense of first order stochastic dominance) experience better health states with larger probabilities they will also obtain larger utility levels with higher probabilities. As a consequence, the expected utility of workers using safer technologies will be larger at the Pareto optimum. Conversely, if consumption goods and health are substitutes, i.e., if $U_{c\theta}(x, \theta) < 0$, optimal consumption will be smaller in better health states. Moreover, when these substitution effects are sufficiently large to compensate the direct effect of health on utility, the Pareto optimal level of agents' utility is decreasing in θ. So that, using the same line of reasoning as above, one can conclude that the expected utility of workers using riskier technologies is larger at the Pareto optimum.

It is also worthy noting that the previous proposition only considers the cases where both goods are complements or substitutes with health and where all produced goods are complements. All these assumptions can be easily weakened to some extent. More importantly, the result of proposition (3) can be naturally interpreted in terms of the properties of the indirect utility function, $V(q, qe, \theta)$, associated to $U(x, \theta)$. Indeed, the basic facts used in the proves of both propositions is that Δu^P has the same sign as $\Delta U = U_\theta\left(x^P(\theta), \theta\right) d\theta + \sum_{c=1,2} \eta_c (x_c^P(\theta_{n+1}) - x_c^P(\theta_n))$. Now, the first term of this sum is positive as U is increasing in health. While the value of the second term of the sum is exactly the value of the second mixed derivative of the indirect utility, $V_{I\theta}(q, I, \theta)$[20], which measure the degree of complementarity between income and health (at each given prices' vector). It follows that the sign of Δu^P is positive whenever $\partial^2 V(q, I, \theta)/\partial I \partial\theta$ is positive (i.e., income and health are complements), while it is negative if health and income are substitutes and their degree of substitutability is sufficiently large.

Finally, let $I^{tP} = \sum_c \eta_c^P e_c + \sum_{\theta\in\Theta} \eta_t^P p^t(\theta) y^t(\theta))$, with $t = 1, 2$ be the sum of the expected endowment and production of sector-t workers calculated at the Pareto optimal shadow prices, η_t^P. Also, let $E\eta^P x^{Pt} = \sum_{\theta\in\Theta} \eta_c^P p^t(\theta) x_c^{tP}(\theta))$ be the value of the expected consumption of sector-t workers. Proposition (3) together with the optimality condition on workers' assignment on α^t have a simple but important corollary: they imply that the sign and magnitude of health effects on the marginal utility of consumption (i.e. the sign and the magnitude of $U_{c\theta}$) determines the sign of the transfers between workers using different technologies. This corollary is formally stated below.

[20] As $V(q, I, \theta) \equiv \max_{x\in\Re^+} U(x, \theta)$ s.t. $qx \leq I + qw$, $\partial V(q, I, \theta)/\partial I$ equals λ, the lagrange multiplier associated to the budget constraint, and $\partial^2 V(q, I, \theta)/\partial I \partial\theta = \partial\lambda/\partial\theta$. Straightforward comparative statics then yields: $\partial\lambda/\partial\theta = U_1 \frac{U_{1\theta}|U_{22}| + U_{2\theta} U_{12}}{\Lambda} + U_2 \frac{U_{2\theta}|U_{11}| + U_{1\theta} U_{12}}{\Lambda}$.

Corollary 4 *If* $U_{c\theta}(x,\theta) \gtreqless -U_{\theta}(x,\theta)\, g_c(x,\theta)$ *then* $Z^{1P} \gtreqless 0$ *and* $Z^{2P} \lesseqgtr 0$.

Finally, as we showed that the sign and the magnitude of $U_{c\theta}$ are crucial in determining either Δu^P or the direction of the transfers across jobs, in concluding this section it is useful to briefly discuss the possible real life determinants of $U_{c\theta}$. Whenever θ can be considered as an "input" that agents use in their consumption activities the it is natural to assume $f_{c\theta}$ positive. However, there also exist specific class of goods for which it is natural to assume $f_{c\theta} < 0$. An important example is surely that of the consumption of drugs; it is undisputable that the marginal utility of drugs is decreasing with health. On the other hand, it is less obvious to assess what assumptions one should impose on $\psi_{c\theta}$. Even so, if one limit attention to *low income* agents who spend a relatively large fraction of their income in satisfying nutritional and housing needs, it seems mostly sensible to assume that: (i) higher levels of consumption decrease the disutility of labor; and that (ii) this consumption effect gets stronger for lower health states, as additional consumption and additional health are substitute in reducing the disutility of labor. This is equivalent to impose $\psi_{c\theta} < 0$[21]. Hence at least in this case it is reasonable to expect bot h $f_{c\theta} < 0$ and $\psi_{c\theta} < 0$; the sign of $U_{c\theta} = f_{c\theta} - \psi_{c\theta}$ will then depend on whether health effects on production capabilities are more or less important than those on consumption capabilities.

6.2 Health effects on the disutility of labor

In this section we analyze how the effects of health risks on the disutility of labor affects the difference between the expected utilities that agents assigned to the risky and the safe sector get in equilibrium. For this reason, we will neglect the effect of health on preferences for consumption goods, productivity and labor endowment, by assuming either $f_{x\theta}(x,\theta) = 0$ for all $x \in X$ and $\theta \in \Theta$, $a(\theta) = a$ and $L(\theta) = L$ for all $\theta \in \Theta$. As we want to focus on the effects of health on labor supply here, differently from the previous section, we *will not* anymore assume that labor supply is completely inelastic. Finally, we will assume that the marginal disutility of labor decreases with health.

Next proposition shows that if the direct health effect on agents' utility is relatively "more important" than its effect on the marginal disutility of labor (i.e., on production capabilities) then agents working in the safer sector will get an higher expected utility in equilibrium. On the contrary, if the effect of health on the marginal disutility of labor (i.e., on production capabilities) is relatively more important and agents' marginal disutility of

[21] As an example, consider the case of an unskilled worker, who live in a low-income African country with higher diffusion rates of contagious diseases (such as malaria or AIDS), and spend a large fraction of his income in buying food and housing services. Contracting the disease generally increases the worker's disutility of labor (i.e $\psi_{\theta} < 0$) by impairing his phisical working aptitudes. In addition, it is completely sensible that the more adequately this worker can satisfy his basic consumption needs the smaller will be the effects of the disease on his labor disutility.Making this assumption is equivalent to impose $\psi_{c\theta} < 0$.

labor is “sufficiently increasing”, then agents working in the riskier sector will get an higher expected utility in equilibrium.

Proposition 5 *(i) If $\psi_\theta(l,\theta)$ sufficiently small, then $\Delta u^P = u^1(x^{1P}) - u^2(x^{2P}) > 0$, $Z^{1P} > 0$ and $Z^{2P} < 0$; (ii) if $\psi_\theta(l,\theta)$ is sufficiently large and $|\psi_{l\theta}(l,\theta)/\sigma_\psi(l,\theta)|$ has strictly positive upper and lower bounds, there exists a positive number k such that if $\sigma_\psi(l,\theta) > k$, $\Delta u^P < 0$ and $Z^{1P} < 0$ and $Z^{2P} > 0$; (iii) assume $\psi_{lll}(l,\theta) \geq 0$ for all (l,θ), if $\psi_{l\theta}(l,\theta)/\psi_\theta(l,\theta) \geq 2\sigma(l,\theta)$ then $\Delta u^P \leq 0$, $Z^{1P} < 0$, and $Z^{2P} > 0$.*

Proof. See the Appendix. ■

The intuition for the results just stated is as follows. Other things (wages) being equal, the maximization of social surplus requires agents to work more in good health states, as in these states the disutility of labor is lower. Workers using riskier technologies will enjoy *less often* good health status, and this effect reduces $\Delta u^P = u^1(x^{2P}) - u^2(x^{1P})$. On the other hand, the direct effect of health on utility, i.e. $U_\theta(x,\theta) < 0$ goes in the direction of increasing Δu^P. Moreover Pareto optimality imposes compensating (shadow) wage differentials in favor of riskier occupation. Since the individual labor supply is increasing in wages, the compensating wages differential effect leads workers assigned to the riskier occupation to supply more labor in each individual health state, and hence increases Δu^P. Consistently, proposition (5) shows that workers assigned to riskier occupations must indeed get a higher expected utility in the optimum if the health effect on the disutility of labor, $\psi_{l\theta}(l,\theta)$ is sufficiently larger than the direct effect of health on utility, $U_\theta(l,\theta)$, and if mild assumptions on the curvature of the utility function are satisfied. These conditions insure that the disutility of labor grows at sufficiently high rates. On the other hand a very large direct effect of health, measured by $U_\theta(l,\theta)$, implies $\Delta u^P > 0$.

6.3 Health effects on labor endowment and productivity

6.3.1 State dependent labor endowment

We shall now focus on the effects of health risks on labor endowment. For this reason, we will neglect the effect of health on preferences and productivity, by assuming either $f_\theta(x,\theta) = 0$ and $\psi_\theta(x,\theta) = 0$ for all $x \in X$ and $\theta \in \Theta$, or $a(\theta) = a$ for all $\theta \in \Theta$. Here again it is convenient to assume that workers supply labor inelastically, i.e. $l^t(\theta) = L(\theta)$ for all θ and $t = 1, 2$.(in this case, indeed, the shocks on labor endowment have the maximal impact on workers’ optimal consumption decisions). In fact, the assumption of an inelastic labor supply, does not change qualitatively the result of the analysis of this section, provided that the agents’ labor supply is maximal in at least one health state. The case where an agent experiencing a very bad health status cannot work (has no labor endowment) is only the simplest real life example where labor supply is maximal in some health states. Indeed, if our static setting is interpreted as a reduced form of a dynamic model where agents supply labor in several periods, the assumption that for certain health status workers supply all their (positive) labor endowment may often result quite realistic. For instance, a driver who typically suffer of overuse syndromes (and is in a relatively bad health state) cannot work more than a certain number of hours in a year if he wants to continue to perform his job and to avoid more important future health problem. Whenever the maximal amount of time that this worker can devote to labor in each interval of time is relatively low, he will plausibly wish to supply all his labor endowment.

Next proposition will show that under mild conditions on preferences the effects of health on labor endowment generate a positive utility differential that agents employed in risky sectors obtain at the optimum.

Proposition 6 *Assume* $\langle p^1, \Theta \rangle$ *FOSD* $\langle p^2, \Theta \rangle$. *There exists two strictly positive real number* b *and* δ *such that* $U_{cL} > -\delta$ *for* $c = 1, 2$ *and* $U_{12} > -b$ *imply* $u^2(x^{2P}) > u^1(x^{1P})$, $Z^{1P} < 0$ *and* $Z^{2P} > 0$.

The proof of proposition 7 follows the same logic as the proof of proposition 3 and is left to the reader, while its intuition relies on a concavity argument. Since the utility function is concave, it is efficient that agents with the same health state consume the same vector of state contingent goods, independently from which technology they are operating (i.e., $x^{1P}(\theta) = x^{2P}(\theta)$ for all θ). Moreover, workers will enjoy more leisure in worse health states where the maximal amount of time they can employ in working activities is lower. This, together with our assumption that the substitutability between leisure and consumption is limited (i.e., $U_{cl} > -b$ for all c) imply that workers gets a larger utility when they experience a worse health status and, for this reason, must work less. As workers using safer technologies experience better health states with larger probabilities, they will also obtain lower utility levels with larger probabilities. For this reason, the expected utility of workers using riskier technologies will be larger at the Pareto optimum.

Once more, in this case the first order conditions for optimality imply that the sign of the transfer, $(E\eta^P x^{1P} - I^{1P})$ going from workers using the riskier technologies to the ones using the safer technology is the same as the sign of $\Delta u^P = u^1(x^{2P}) - u^2(x^{2P})$, and workers using the riskier technology must get a positive subsidy at the optimum under the assumptions of proposition (6).

6.4 State dependent productivity

In this section we consider the case in which health risks only affect individual production. Precisely, we will now assume that agents' individual productivity, $a(\theta)$, is increasing in the health status, θ. and neglect the effects of health on preferences and labor endowment by assuming: $U_\theta(x, \theta) = 0$ for all x and θ.We begin by stating the following proposition, whose interpretation is analogous to that provided for proposition (5)

Proposition 7 *Assume $\sigma_\psi(l)$ has strictly positive upper and lower bounds, then if $\sigma_\psi(l)$ and $\partial a(\theta)/\partial\theta$ are sufficiently large for all θ, then $\Delta u^P = u^1(x^{1P}) - u^1(x^{2P}) < 0$, $Z^{1P} < 0$ and $Z^{2P} > 0$.*

The proof of this claim uses exactly the same argument developed in the proof of proposition (5), thus it will be omitted.

Differently from proposition (5), however, proposition (7) does not present results for the case of low values of $\sigma_\psi(l)$. Indeed, in the setting we are studying in this section, it can be showed that first order stochastic dominance does not provide in general sufficient restriction to sign Δu^P. Nevertheless, below we characterize optima for the specific but important case where one technology is assumed to be safe (i.e., it allows to obtain the highest health state with probability 1). For this case, we show that efficiency requires agents using riskier technologies to get higher utility at the optimum, whenever the elasticity of labor supply, $\zeta_{l,w}$, is non decreasing with respect to the shadow wage. This assumption appears in line with the empirical findings; its interpretation is that agents who are already "working a lot" are less reactive to wage increases. In order to state next proposition, denote $h(l) = \psi'(l)l$. Consistently, define $h^t(l^t(\theta)) = \sum_{\theta\in\Theta} p^t(\theta)h(l^t(\theta))$ and $\Delta h(l^t(\theta)) = h^2(l^2(\theta)) - h^1(l^2(\theta))$. Moreover let $\sigma_\psi(l) = \psi''(l)/\psi'(l)$ and $\sigma_h(l) = h''(l)/h'(l)$ respectively the absolute measures of convexity for the functions ψ and h.

Let $l(\eta a(\theta), \theta)$ be the contingent labor supply implicitly defined by the first order conditions of the Pareto program. Then $\zeta = dl(w_\theta, \theta)/dw_\theta)/(l(w_\theta, \theta)/w_\theta)$, with $w_\theta = \eta a(\theta)$ represents a measure of sensitivity of the equilibrium labor supply with respect to the the shadow wage w_θ. The next lemma will turn out to be useful in characterizing the optimum.

Lemma 8 *$\partial\zeta_{l,w}/\partial w \gtreqless 0$ for all (t, θ) if and only if $\sigma_\psi(l) \gtreqless \sigma_h(l)$.*

The proof follows immediately after straightforward algebraic manipulations, and it is omitted.

Next proposition characterizes optimal utility wedges and cross transfers across jobs.

Proposition 9 *Assume $p^1(\theta) = 1$ for $\theta = \theta_{\bar{N}}$. If $\partial\zeta_{l,w}/\partial w \gtreqless 0$ then $u^1(x^{1P}) \gtreqless u^2(x^{2P})$, $Z^{1P} \lesseqgtr 0$ and $Z^{2P} \gtreqless 0$.*

Proof. See the Appendix. ■

7 Characterization of competitive equilibria

In this section we characterize competitive equilibria for economies with either high or low transaction costs. We begin by proving the existence of a competitive equilibrium. The proof exploits the convexifying effect of large numbers.

Proposition 10 *An equilibrium always exists either in economies with high or low transaction costs.*

Proof. See the Appendix. ■

Next proposition states the first welfare theorem for an economy with low transaction costs.

Proposition 11 *Competitive equilibria in economies with low transaction costs are first best allocations.*

The argument of the proof is standard and it is omitted.

The logic of the first welfare theorem can also be used to prove that competitive equilibria in economies with high transaction costs are interim efficient allocations with fair treatment.

Proposition 12 *Competitive equilibria in economies with high transaction costs are interim efficient allocation with fair treatment .*

Proof. See the appendix. ■

Given the results proved in characterizing of Pareto optimal allocations, proposition 12 has the following immediate but important corollaries

Corollary 13 *Competitive equilibria with high transaction costs are typically not first best allocations.*

This simply follows by the results that competitive equilibria with high transaction costs are interim efficient and that ex-ante and interim efficient allocations are typically different.

We conclude the equilibrium analysis by looking at the properties of equilibrium prices. Next proposition shows that in both classes of economies we are studying agents trade individual securities at fair prices, and that state contingent wages are equal to the value of state contingent labor productivity for each type of worker. Moreover, occupations which are riskier for health in the sense of first order stochastic dominance command relatively higher expected wage. Finally, under low transaction costs, equilibrium prices (wages) are such that value of consumption of agents of the same type assigned to different occupations typically differs from the sum of the values of its endowment and its production. By using lottery contracts agents transfer wealth across occupations in such a way that agents with the higher (respectively lower utility) expected utility consume get a positive (negative) transfer. Or, saying it differently, lottery contracts allow to make transfers across occupations. Let $\tilde{Z}_i^t = \sum_{c \in C, \theta \in \Theta} (q_c (p_i^t(\theta) x_{ic}^t(\theta) - e_i^c) - q_t p_i^t(\theta) a_i^t(\theta) (L - x_{iL}^t(\theta))$.

Proposition 14 *In all competitive equilibria with either high or low transaction costs the following properties hold: (i) securities prices are fair, i.e.* $\phi_i^t(\theta) = g_i^t p_i^t(\theta)$ *for some* $g_i^t \in \Re_+$ *for all* $i \in I$, $t \in T$ *and* $\theta \in \Theta$*; (ii)* $w_i^t(\theta) = b_i^t a_i^t(\theta)$ *for some* $b_i^t \in \Re_+$ *for all* $i \in I, t \in T$ *and* $\theta \in \Theta$*; (iii) ; if* $\langle p_i^t, \Theta \rangle$ *first order stochastically dominates* $\left\langle p_i^{t'}, \Theta \right\rangle$ *and strictly positive measures of type-i agents are assigned to both sector* t *and* t' *then* $\sum_{\theta\in\Theta} p_i^t(\theta) w_i^t(\theta) > \sum_{\theta\in\Theta} p_i^{t'}(\theta) w_i^{t'}(\theta)$*; (iv) In any equilibrium with low transaction costs such that positive measures of type-i agents are employed in sector t and in sector* t'*, then* $u_i^t(x_i^t) - u_i^{t'}(x_i^{t'}) \gtreqless 0$ *if and only if* $\tilde{Z}_i^t - \tilde{Z}_i^{t'} \gtreqless 0$

Proof. *(i)* and *(ii)* result from a standard application of the separating hyperplane theorem**;** *(iii)*. as $a_i^t(\theta)$ is weakly increasing for all θ, the proof follows immediately by *(ii)* and lemma (2); *(iv)* immediately follows from the first order condition with respect to γ_i^t of the agents maximization program and the ex ante budget constraint. ■

Part (iii) of the previous proposition expresses in our general equilibrium setting the compensating wage differentials principle which is prominent in the literature on compensating wage differentials. Part (iv) of the same proposition however, together with the results of characterization of equilibria and Pareto optima previously stated, has the following important implication. Pareto optimal wage differentials do not satisfy the fair treatment conditions under which wage differentials have been commonly derived in the literature on non pecuniary job characteristics. The result of that literature are for this reason based on an implicit assumption of market suboptimality.

8 Second Welfare Theorem and decentralization

In this section, we will show that if agents' types are public information, the Pareto optima can be implemented as competitive equilibria with deterministic transfers even when lottery contracts are unenforceable. To prove this result, we introduce a class of policy schemes based on of deterministic transfers across agents and minimal wages. Minimal wages allow the Pareto-planner to ration labor supply in some sectors of the economy so as to decentralize through transfers allocations that do not satisfy the requirement of interim efficiency.

In order to simplify the formal description of the policy instruments, in this section we assume that each agent makes all his assets' trades with only one intermediary. This somewhat realistic and unrestrictive assumption allows to reinterpret each agent's vector of assets' trades as an insurance contract. Let s_i^t the monetary transfer[22] received by a type-i agent who signs a health insurance policy designed for sector-t workers, also denote f_i^t the (possibly negative) monetary transfer received by a sector-t firm for each type-i

[22] We will use monetary transfer as a synonimus of "transfer in units of numeraire".

worker employed. Finally let $\hat{w}_i^t(\theta)$ the minimal state contingent wage for type-i workers employed in sector t under the health state θ.

A transfers' policy, $\wp = (s, f, \hat{w})$, is then defined as: (i) a vector, $s = (s_i^t)_{i\in I}^{t\in T}$, of subsidies to the workers (ii) a vector $f = (f_i^t)_i^{t\in T}$of transfers to the firms and a vector $w = (\hat{w}_i^t(\theta))_{i\in I,\,\theta\in\Theta}^{t\in T}$ of state contingent minimal wages. Feasible policies must be budget-balancing. Formally, a budget balancing regulatory policy is an element of the set: $P = \left\{\wp : \sum_{t\in T, i\in I} \alpha_i^t s_i^t + \alpha_i^t f_i^t = 0\right\}$ where α_i^t is the measure of type-i workers who are effectively assigned to sector t in an equilibrium with transfers[23].

As minimal wages may induce rationing, market clearing rules must be carefully specified. We assume that in any equilibrium with transfers all commodity as well as asset markets clear without rationing at "walrasian" prices (i.e., exactly as in the absence of transfers), and that firms labor demand is not rationed. Differently, as transfers and minimal wages may well make some occupations more attractive than others, we shall assume that whenever in an equilibrium with transfers $sector - t$ workers receive a higher utility than $sector - t'$ workers, for some $t' \neq t$, the probability that a worker will be assigned to sector t is equal to α_i^t, i.e. the measure of type-i workers assigned to sector t.

The motivation for the clearing rules of consumption and assets markets is the same as that often used to justify walrasian market clearing in the definition of competitive equilibrium: at any non walrasian prices vector rationed firms and/or agents would have an incentive to manipulate prevailing prices[24]. The same argument justifies the assumption that labor demand is never rationed in equilibrium. Finally, our workers' assignment rule to firms can be tought as the result of a *decentralized* job search process where workers simultaneously apply for several occupations in a first stage, then applications are randomly selected whenever the number of workers applying for a job is larger than the number of vacancies posted; while in a final stage workers, whose applications may have been selected by several firms, choose an occupation within the set of offers. Notewhorty, while this type of assignment mechanism introduces a randomization on agents' labor supply, the transfers policies we consider are completely deterministic, and hence do not need any random device for their implementation.

A *rational expectation equilibrium with transfers and minimal wages* $\left\{\varphi_i^t, x, \alpha, z, p, w, \phi, \wp\right\}$ is now formally defined by the following conditions: (i) consumers' choose $\left(x_i^t, \alpha_i^t, z_i^t(\theta)\right)$ by maximizing $\sum_t u_i^t(x_i^t)\varphi_i^t$ subject to the budget constraints

$$\sum_{c\neq L} q_c(x_{ic}^t(\theta) - e_i^c) = w_i^t(\theta)\left(L(\theta) - x_{iL}^t(\theta)\right) + z_i^t(\theta) + s_i^t \quad \forall\ (\theta, t)$$

[23] As the focus of this section is mostly on the decentralizability of Pareto optimal allocations and workers'assignments, we will not introduce further notation to indicate the workers' assignment of a generic equilibrium with trransfers. For simplicity we prefer to denote by α a workers' assignment, as in the definition of efficient allocations.

[24] See for instance Mas Colell and others (p.315).

$$\sum_{\theta\in\Theta} z_i^t(\theta)\,\phi_i^t(\theta) \le 0 \quad \forall\, t; \qquad \varphi_i^t \le \alpha_i^t$$

and to a set of constraint of the type $\varphi_i^t \le \alpha_i^t$, which state that a type-i agent who offers labor in sector t will be assigned to that sector with a probability α_i^t equal to the measure of type-i workers who are effectively assigned to sector t in equilibrium; (ii) production firms' labor demand, l_i^t, and intermediaries assets' supply, $\widehat{z}_i^t$, satisfy the same conditions as in the competitive equilibrium with deterministic contracts (i.e. conditions (7) and (8)) except that, because of the presence of transfers, the firms' objective function is now $\sum_{\theta\in\Theta} p_i^t(\theta)\left(q_t y_i^t(\theta) - (w_i^t(\theta) - f_i^t)(l_i^t(\theta)\right)$; (iii) the minimal wages' constraints, $w_i^t(\theta) \ge \hat{w}_i^t(\theta)$, are satisfied; and (iv) all feasibility conditions hold.

Next proposition shows that Pareto optimal allocations can be implemented as equilibria with transfers. We show that optimal policy schemes generally hinge on state and sector contingent minimal wages. We also prove that, in the case of inelastic labor supply, uniform minimal wages suffice to implement Pareto optima. A continuity argument then implies that, whenever the elasticity of labor supply is sufficiently small, there exists Pareto improving regulatory policies imposing uniform minimal wages [25].

Proposition 15 *Any Pareto optimal allocation can be implemented as an equilibrium with transfers and state contingent minimal wages. Moreover if workers' labor supply is completely inelastic for any positive wage, all Pareto optima are implementable through policies imposing uniform minimal wages.*

Proof. See Appendix. ■

9 Conclusive remarks and extensions

The endogeneity of the individual health distributions' generates specific "cost-benefit trade-offs" involving agents' marginal utilities of health and consumption goods and their occupational choices. We have studied how these trade-offs determine the shape of the Pareto frontier of the economy, and the agents' competitive behavior. The relative magnitude of health effects on production and consumption capabilities determines either the sign of Pareto optimal compensating wage differentials or the optimal differences in the expected utility levels obtained by workers who use different technologies. Ex ante Pareto optimality also requires cross-jobs transfers of resources. Competitive equilibria are ex ante efficient if lottery contracts are enforceable, but not otherwise. The specific form of contract incompleteness, associated to the enforceability costs of lotteries, may justify policy schemes implementing cross-transfers across occupations.

[25] Uniform minimal wages and sector dependent minimal wages are both observed in developed countries.

All these results have been derived under many simplifying assumptions on the production side of the economy, and assuming away all asymmetric information problems which may affect either the amount of labor supplied by workers or their insurance schemes.

In a recent literature several contributions (Cole-Prescott (1997), Ellickson-Grodal-Scothcmer-Zame (1999,2001), Makowski-Ostroy (2003)) have developed general equilibrium analyses focusing on the basic and complex issues of the pricing of agents' contributions to clubs and production firms, and of the internal organization of these istitutions. Our conjecture based on the analysis of this paper is that the result of generic inconsistency between ex ante and interim optimality continue to hold in most of the setting studied in these literature, once it is assumed that the organization of production within a club or a firm preferences affects workers'utility. A result in this spirit is also obtained by Bennardo (2004) in a moral hazard economy where health effects are not considered but occupational choices affect the agents' indirect utility via incentive constraints[26]. Finally, in future work we wish to extend our analysis and consider a more developed dynamic setting in order to study how health effects modify saving decisions either under symmetric or asymmetric information.

[26] The presence of moral hazard, however, significantly affects the properties of Pareto optimal cross-job transfers.

10 Bibliography

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11 Appendix

Proof of proposition (1)

Part (i). The proof is based on a standard concavity argument and it is left to the reader.

Part (ii). In order to prove the genericity result, we need to formally define $\mathcal{U}^i$. Following the literature[27] assume that, beyond satisfying the assumptions previously stated, agents' preferences satisfy the following property: a sequence $U_{ik}(x_i,\theta)$ in $\mathcal{U}^i$ converges to $U_i(x_i,\theta) \in \mathcal{U}^i$ if and only if $U_{ik}(x_i,\theta)$, $DU_{ik}(x_i,\theta)$ and $D^2U_{ik}(x_i,\theta)$ uniformly converge to $U_i(x_i,\theta)$, $DU_i(x_i,\theta)$ and $D^2U_i(x_i,\theta)$, respectively, for all θ, on any compact subset of $\Re^{C+1}_{++}$[28].

Let $\xi = (x,\alpha,\eta,\lambda)$ define the vector of variables in the Pareto program, where η and λ are the vectors of Lagrange multipliers associated to the feasibility constraints for produced goods, and to the reservation utility constraints, $\sum_t \alpha_i^t u_i^t(x_i^t) \geq \overline{u}_i$ for all $i \neq 1$, respectively.

We consider first the case where the solution of the Pareto program is internal. A Pareto optimum then solves:

$$\mathcal{F}(\xi,\varepsilon,\bar{u}) = \begin{pmatrix} \lambda_i D_c U_i(x_i^t,\theta) - \eta\mu_i \\ -\lambda_i U_{ix_L}(x_i^t,\theta) + \eta_t a_i^t(\theta)\mu_i \\ \lambda_i(u_i^t(x_i^t) - u_i^T(x_i^T)) - \mu_i(Z_i^t - Z_i^T) \text{ for all } t \neq T \\ \sum_{i\in I}\mu_i(\overline{x}_i - e_i) - \sum_{i\in I}\mu_i y_i \\ \sum_{t\in T}\alpha_i^t u_i^t(x_i^t) - \overline{u}_i \ \ for\ all\ i \neq 1 \end{pmatrix}_{i\in I, t\in T, \theta\in\Theta} = \mathbf{0}$$

for some vector of weights, $\bar{u} = (\bar{u}_i)_{i\neq 1}$, where $\lambda_1 = 1$ and $Z_i^t = \sum_{c\in C,\theta\in\Theta}(\eta_c(p_i^t(\theta)x_{ic}^t(\theta) - e_i^c) - \eta_t p_i^t(\theta)a_i^t(\theta)(L - x_{iL}^t(\theta))$ for all t. Interim efficiency implies $u_i^t(x_i^t) = u_i^{t'}(x_i^{t'})$ for all t,t' such that $\alpha_i^t > 0$ and $\alpha_i^{t'} > 0$. Given an arbitrary $\varepsilon \in \mathcal{E}$, assume without loss of generality that $\alpha_1^t \in (0,1)$ for $t=1$ and $t=T$ and define the following extended system of equations:

$$\mathcal{G}(\xi,\varepsilon,\bar{u}) = \begin{pmatrix} \mathcal{F}(\xi,\varepsilon,\bar{u}) \\ u_1^1(x_1^1) - u_1^T(x_1^T) \end{pmatrix} = \mathbf{0}$$

[27] See Geneakoplos-Polemarchakis (1986) and A. Citanna, A. Kajii and A. Villanacci (1994) for a detailed discussion of this technique.

[28] In other words, we assume that $\mathcal{U}_i$ is endowed with the subspace topology of the C^2 uniform convergence topology on compact sets. Notice also that $\mathcal{U} = \prod_{i=1}^{I}\mathcal{U}_i$ is endowed with product topology.

Finally let $\mathcal{S}_{\bar{u}} = \{\varepsilon \in \mathcal{E} : \mathcal{G}(\xi, \varepsilon, \bar{u}) = 0\}$ be the subset of economies where a solution, $\xi(\varepsilon, \bar{u})$ of $\mathcal{G}()$ exists for any $\bar{u}$. We will show that ex ante and interim Pareto optima are generically different, by proving the equivalent statement that the complement of $\mathcal{S}_{\bar{u}}$ is open and dense.

(i) *Density*

The space $\mathcal{E}$ of economies on which density must be proved is an infinite dimensional space. However, as *density* is a local property, in accomplishing this task one may restrict attention to a properly defined finite subset of $\mathcal{E}$. Specifically, we will restrict attention to a linear subspace of $\mathcal{U}$ which will be defined as follows. Fix arbitrarily an economy $\bar{\varepsilon} \in \mathcal{E}$ and a vector $\bar{u}$, and let $x_{\bar{\varepsilon}}^{\bar{u}P}$ the Pareto optimal allocation associated to $\bar{\varepsilon}$, and to the reservation constraints defined by the vector of Pareto weights $\bar{u}$. Given an utility profile $\hat{U} \in \mathcal{U}$ of $\bar{\varepsilon}$, consider the perturbed utility functions of the type:

$$U_i(x_i, \theta) = \hat{U}_i(x_i, \theta) + \kappa_i(\theta) + \beta_i(\theta)\left(x_i - x_{i\bar{\varepsilon}}^{\bar{u}P}(\theta)\right)$$

where $\kappa_i(\theta)$ is a scalar and $\beta_i(\theta)$ denotes a $(C+1)$ dimensional vector for all (θ, i).Assume $|(\kappa_i(\theta)|$ and $\|\beta_i(\theta)\|$ sufficiently small for all (θ, i). The set of certainty utility functions, $\widehat{\mathcal{U}}$, defined by all possible perturbations obtained in this way is a finite linear subspace of $\mathcal{U}$. Let $\widehat{\mathcal{E}} = E \times T \times \widehat{\mathcal{U}}$. Define $\widehat{\mathcal{S}}_{\bar{u}} = \left\{\varepsilon \in \widehat{\mathcal{E}} : \mathcal{G}(\xi, \varepsilon, \bar{u}) = 0\right\}$ and let $(\xi^{\bar{u}P}, \varepsilon^{\bar{u}P})$ a generic point such that $\mathcal{G}() = 0$. We now show that the complement of $\widehat{\mathcal{S}}_{\bar{u}}$ is dense by proving that $D_{(\xi,\varepsilon)}\mathcal{G}(\xi^{\bar{u}P}, \varepsilon^{\bar{u}P})$, has full row rank, i.e., that $\mathcal{G}()$ is transversal to zero. To simplify, let $\kappa_i^t = \sum_{\theta \in \Theta} p_i^t(\theta)\kappa_i(\theta)$ for all pairs (t, i), $e^c = \sum_{i \in I} \mu_i e_i^c$ for all c and assume, without loss of generality, that $\alpha_i^T \in (0, 1)$ for all i. Moreover, let $e \in \Re^C$, $a = (a_1(\theta), ..., a_I(\theta))$ with $a \in \Re^{(T-1)\times I}$ for some $\theta \in \Theta$ and $a_i(\theta) = (a_i^1(\theta), .., a_i^{T-1}(\theta)) \in \Re^{T-1}$, $\kappa^T = (\kappa_2^T, .., \kappa_I^T) \in \Re^{I-1}$, and $\beta = (\beta_{1L}(\theta), .., \beta_{IL}(\theta)) \in \Re^I$. Straightforward elementary operations then imply[29] that the rank of $D_{(\xi,\varepsilon)}\mathcal{G}(\xi^{\bar{u}P}, \varepsilon^{\bar{u}P})$ is equal to the rank of the matrix:

$$\mathbf{A} = \begin{pmatrix} \text{Equ./Var.} & x & e & a & \kappa^T & \kappa_1^1 & \beta \\ \text{FOCs}(x) & \mathbf{H} & \mathbf{0} & \mathbf{B} & \mathbf{0} & \mathbf{0} & \mathbf{E} \\ \text{FEAs} & \mathbf{0} & -\mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \text{FOCs}(\alpha) & * & \mathbf{0} & \mathbf{C} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \text{UT.CON.} & * & \mathbf{0} & \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \text{INT.} & * & \mathbf{0} & \mathbf{0} & \mathbf{0} & 1 & \mathbf{0} \end{pmatrix}$$

where the matrix $\mathbf{H}$, the submatrix of the Hessians, has full rank since preferences are strictly convex; $\mathbf{I}$ denotes a $(C+1)$ dimensional identity matrix; $\mathbf{B}$ is a matrix with a zero in all entries except for the ones in correspondence of the first order conditions with respect to $x_{iL}^t(\theta)$ which are of the form $\eta_t\mu_i$ for all (i, t); $\mathbf{C}$ is a $(T-1) \times I$ dimensional square

[29] This can be easily verified by using the condition $u_1^1(x_1^1) - u_1^T(x_1^T) = 0$ to rewrite the FOC with respect to α_1^1 in $\mathcal{G}$ in the form $Z_1^1 - Z_1^T = 0$.

matrix with a zero in all entries except for the ones of the principal diagonal which take the form $p_i^t(\theta)l_i^t(\theta)$ for all (i,t); and $\mathbf{E}$ is a matrix with a zero in all entries except for the elements in correspondence of the first order conditions with respect to $x_{iL}^t(\theta)$ having a 1 for all (i,t).

Consider the matrix

$$\mathbf{M}_i = \begin{pmatrix} \text{Equ./Var.} & x_i & a_i(\theta) & \beta_i \\ \text{FOCs}(x_i) & \hat{\mathbf{H}}_i & \mathbf{0} & \mathbf{E}_i \\ \text{FOCs}(\alpha_i) & * & \mathbf{I}_i & \mathbf{0} \end{pmatrix} \quad where \;\; \hat{\mathbf{H}}_i = \begin{pmatrix} \mathbf{I}.. & ..\mathbf{0}.. & ..\mathbf{0}.. & ..\mathbf{0} \\ \vdots & \vdots & \vdots & \vdots \\ *.. & ..\mathbf{I}.. & ..\mathbf{0}.. & ..\mathbf{0} \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{0}.. & ..*.. & ..\mathbf{I}.. & ..\mathbf{0} \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{0}.. & ..\mathbf{0}.. & ..*.. & ..\mathbf{I} \end{pmatrix}$$

where the "$*$" indicates submatrices having some non zero entries. Straightforward elementary operations allow to show that $\mathbf{A}$ has full row rank if $\mathbf{M}_i$ has full row rank for all $i = 1,..,I$. It is immediate to verify that $\mathbf{M}_i$,and hence $\mathbf{A}$, have full rank. Thus $\mathcal{G}()$ is transversal to zero and $\widehat{\mathcal{S}}_{\bar{u}}$ is dense when the Pareto program has an internal solution. Finally, the proof extends to the case where $\mathcal{F}()$ has a corner solution such that $\alpha_i^t \in \{0,1\}$, say $\alpha_i^t = 0$, for some pairs (i,t). Indeed, in this case it suffices to replace $\mathcal{G}() = 0$ by an equivalent system $\mathcal{G}'()$ which is the same as $\mathcal{G}() = 0$ except that α_i^t is fixed to zero in all the equations and the first order condition with respect α_i^t does not appear anymore. □

(ii) *Openness*

Let $\mathcal{P}_{\bar{u}} = \{(\xi,\varepsilon) : \mathcal{F}(\xi,\varepsilon,\bar{u}) = 0\}$ denote the Pareto optimal manifold, and consider the natural projection of $\mathcal{P}_{\bar{u}}$: $\pi : \mathcal{P}_{\bar{u}} \rightarrow \mathcal{E}$, $\pi(\xi,\varepsilon,\bar{u}) = \varepsilon$. As proper mappings take closed sets into closed set, $\mathcal{S}_{\bar{u}}$ is open if the natural projection is *proper.* Hence we need to prove that for any sequence $(\xi_k^{\bar{u}P},\varepsilon_k)_{k=1}^{\infty}$ such that $F(\xi_k^{\bar{u}P},\varepsilon_k,\bar{u}) = 0$ for all k, and $\varepsilon_k \rightarrow \varepsilon$, as $k \rightarrow \infty$, there exists a convergent subsequence of $(\xi_k^{\bar{u}P})_{k=1}^{\infty}$ with limit $\xi^{\bar{u}P}$ such that $F(\xi^{\bar{u}P},\varepsilon,\bar{u}) = 0$.

In order to do this, note first that $\left\{\alpha_k^{\bar{u}P}\right\}_{k=1}^{\infty}$ must converge, say to α, as it belongs to the compact set $[0,1]^{T\times I}$. Moreover boundary conditions imply $\left\{x_k^{i\bar{u}P}\right\}_{k=1}^{\infty} \gg 0$ for all i, while strict convexity of preferences imply there exists a positive vector G such $x_k^{i\bar{u}P} < G$, hence $\left\{x_k^{\bar{u}P}\right\}_{k=1}^{\infty}$ must converge, say to x. Given the assumptions we made on preference $U_k^i(x^i,\theta) \rightarrow U^i(x^i,\theta)$ implies $DU_k^i(x^i,\theta) \rightarrow DU^i(x^i,\theta)$ uniformly on compact sets for all (x^i,θ), then $DU_k^i(x_k^{i\bar{u}P},\theta) \rightarrow DU^i(x^{i\bar{u}P},\theta)$ for all (i,θ). Finally, using the first order conditions with respect to x and α one gets $(\lambda_k^{\bar{u}P},\eta_k^{\bar{u}P}) \rightarrow (\lambda^{\bar{u}P},\eta^{\bar{u}P})$. This completes the proof. □

Proof of lemma (2)

Consider $-\sum_{n=0}^{N} dg(\theta_{n+1})\Delta P(\theta_n)$ with $\Delta P(\theta_n) = P^1(\theta_n) - P^2(\theta_n)$. Developing this expression one gets:

$$\sum_{n=0}^{N} \Delta P(\theta_n) dg(\theta_{n+1}) = -dg(\theta_1)\left(p^1(\theta_0) - p^2(\theta_0)\right) - ... - dg(\theta_N)\left(\sum_{n=0}^{N-1} p^1(\theta_n) - \sum_{n=0}^{N-1} p^2(\theta_n)\right)$$

this simplifies to $\sum_{\theta\in\Theta} p^1(\theta) g(\theta) - \sum_{\theta\in\Theta} p^2(\theta) g(\theta)$. $\square$

Proof of proposition (3)

The first order conditions with respect to x^{tP} of the Pareto program, which take the form $U_c(x^t(\theta), 0, \theta) = \eta_c$ with $c = 1, 2$, together with strict concavity of $U(x, \theta)$ imply $x^{1P}(\theta) = x^{2P}(\theta) = x^P(\theta)$ for all θ. Let $x^P : \Theta \to \Re^2_+$, $\theta \to x^P(\theta)$, be the map which associates the optimal consumption vector $x^P(\theta)$ to each $\theta \in \Theta$. Define $d\theta = \theta_{n+1} - \theta_n$ for all n, and assume $d\theta$ sufficiently small, then one has:

$$dU\left(x^P(\theta), \theta\right) = U\left(x^P(\theta_{n+1}), \theta_{n+1}\right) - U\left(x^P(\theta_n), \theta_n\right) \approx \sum_{c=1,2} dx^P_c(\theta)\eta^P_c + U_\theta\left(x^P(\theta), \theta\right) d\theta \tag{19}$$

By lemma (2), $u^1(x^{1P}) \gtreqless u^2(x^{2P})$ if $dU\left(x^P(\theta), \theta\right) \gtreqless 0$. Moreover, from the above approximation it follows that $\sum_{c=1,2} dx^P(\theta)\eta^P_c + U_\theta\left(x^P(\theta), \theta\right) d\theta \gtreqless 0$ implies $u^1(x^{1P}) \gtreqless u^2(x^{2P})$. For $d\theta$ small $dx^P_c(\theta)$ can be approximated as: $dx^P_1(\theta) \approx (U_{1\theta}|U_{22}| + U_{2\theta}U_{21})/\Lambda)d\theta$ and $dx^P_2(\theta) \approx (U_{2\theta}|U_{11}| + U_{1\theta}U_{12})/\Lambda)d\theta$ where $\Lambda = U_{11}U_{22} - (U_{12})^2 > 0$. Summing up, we obtain:

$$\sum_{c=1,2} dx^P_c(\theta)\eta_c \approx U_1 \frac{U_{1\theta}|U_{22}| + U_{2\theta}U_{12}}{\Lambda} d\theta + U_2 \frac{U_{2\theta}|U_{11}| + U_{1\theta}U_{12}}{\Lambda} d\theta \tag{20}$$

(19) and (20) then imply that $dU\left(x^P(\theta), \theta\right) \gtreqless 0$ if

$$(U_{1\theta}(U_1|U_{22}| + U_2U_{12}) + \frac{\Lambda}{2}U_\theta) + (U_{2\theta}(U_2|U_{11}| + U_1U_{12}) + \frac{\Lambda}{2}U_\theta) \gtreqless 0$$

Part (i). Increasing differences in (x, θ) imply $U_{c\theta} \geq 0$ for all (x, θ) and $c = 1, 2$, supermodularity in x for any given θ implies $U_{12} \geq 0$ for all (x, θ). Since by assumption $U_\theta \geq 0$, the result immediately follows by the above equation.

Part (ii). Decreasing differences in (x, θ) imply $U_{c\theta} \leq 0$ for all (x, θ) and $c = 1, 2$, supermodularity in x for any given θ implies $U_{12} \geq 0$ for all (x, θ). Now set $g_1(x, \theta) = \Lambda/2\,(U_1|U_{22}| + U_2U_{12})$, and $g_2(x, \theta) = \Lambda/2\,(U_2|U_{11}| + U_1U_{12})$, the above expression implies that if $U_{c\theta}(x, \theta)/U_\theta(x, \theta) \gtreqless -g_c(x, \theta) \Leftrightarrow u^1(x^{1P}) \gtreqless u^2(x^{2P})$ for all c. $\square$

Proof of proposition (5)

As a preliminary result we prove the following lemma.

Lemma 16 *Assume $\langle p^1, \Theta\rangle$ FOSD $\langle p^2, \Theta\rangle$, then the Pareto optimal allocation satisfies the following property: $\eta^P_2 > \eta^P_1$.*

Proof. The proof is by contradiction. Let $h(l, \theta) \equiv \psi_l(l, \theta)l$ and $\Psi(l, \theta) \equiv \psi(l, \theta) - h(l, \theta)$. By using the first order conditions with respect to $l^t(\theta)$ of the Pareto program, for all t and θ, the first order condition with respect to α rewrites as: $\Delta\Psi \equiv \sum_{\theta\in\Theta} p^2(\theta)\Psi(l^2(\theta), \theta) -$

$\sum_{\theta\in\Theta} p^1(\theta)\Psi(l^1(\theta),\theta) = 0$.Now assume $\eta_1^P > \eta_2^P$, $\psi_l(l^t(\theta),\theta) = \eta_t$ together with $\psi_{ll}() > 0$ imply $l^1(\theta) > l^2(\theta)$ for all θ. From $\Psi_l() = -\psi_{ll}() < 0$ for all θ it follows $\Delta\Psi \geq \sum_{\theta\in\Theta}(p^1(\theta) - p^2(\theta))(-\Psi(l^2(\theta),\theta))$. Moreover, by differentiating $\psi_l(l^t(\theta),\theta) = \eta_t$ and $\Psi(l,\theta)$ one gets $d\Psi(l^t(\theta),\theta)/d\theta = \psi_\theta(l^t(\theta),\theta) < 0$ for all θ. As $\langle p^1, \Theta\rangle$ FOSD $\langle p^2, \Theta\rangle$, lemma (2), then implies $\Delta\Psi > 0$ which contradicts Pareto optimality. □

We turn now to the proof of the propostion.

Part (i). Let $\sigma_\psi(l,\theta) \equiv \psi_{ll}(l,\theta)/\psi_l(l,\theta)$ for all (l,θ). Summing by parts, Δu^P can be written as

$$\Delta u^P = -(\sum_{\theta\in\Theta}(p_1(\theta) - p_2(\theta))\psi(l^{1P}(\theta),\theta) + \sum_{\theta\in\Theta} p_2(\theta)(\psi(l^{2P}(\theta),\theta) - \psi(l^{1P}(\theta),\theta))$$

Now, denote η_t^P the lagrange multiplier associated to the feasibility condition with respect to good t $(t = 1, 2)$. Let $l_1(\theta)$ be the implicit (continuous) function defined by the first order condition $\psi_l(l(\theta),\theta) = \eta_1^P$; and let $l(\theta,\eta)$ be the implicit function defined by the equation $\psi_l(l(\theta),\theta) = \eta$. We then have,

$$\Delta u^P = \sum_{n\in N} \Delta P(\theta_n) \int_{\theta_n}^{\theta_n+\Delta\theta} (d\psi(l_1(\theta),\theta)/d\theta)d\theta + \sum_{\theta\in\Theta} p^2(\theta) \int_{\eta_1^P}^{\eta_2^P} 1/\sigma_\psi(l(\theta,\eta),\theta)d\eta$$

where, for each $\theta_n \in \Theta$, $\Delta P(\theta_n) = (P^1(\theta_n) - P^2(\theta_n))$, and $P^t(\theta_n) = \sum_{\theta\leq\theta_n} p^t(\theta_n)$. Finally, by using the definitions of $l_1(\theta)$ and $l(\theta,\eta)$ one obtains:

$$\Delta u^P = \sum_{n\in N} \Delta P(\theta_n) \int_{\theta_n}^{\theta_n+\Delta\theta} \left(-\frac{\psi_{l\theta}(l_1(\theta),\theta)}{\sigma_\psi(l_1(\theta),\theta)} + \psi_\theta(l_1(\theta),\theta)\right) d\theta + \sum_{\theta\in\Theta} p^2(\theta) \int_{\eta_1^P}^{\eta_2^P} 1/\sigma_\psi(l(\theta,\eta),\theta)d\eta. \tag{21}$$

Since $\eta_2^P > \eta_1^P$ by lemma (16) and $\sigma_\psi(l,\theta) \geq 0$ for all (l,θ), the second addendum is positive. By first order stochastic dominance it is immediate to see that the first addendum of the above equation is also positive when $\psi_\theta(l,\theta)$ is sufficiently small. Hence Δu^P is positive. □

Part (ii). To begin, observe that $|\psi_{l\theta}(l,\theta)/\sigma_\psi(l,\theta)| > 0$ implies that $\sigma_\psi() < \infty$, which in turn, entails $\psi_l() > g > 0$ for all $(l^P(\theta),\theta)$. This together with lemma (2), and the continuity of the utility function, imply the existence of a strictly positive number d such that if $dl(\theta) = l_2^P(\theta) - l_1^P(\theta) \leq d$ for all θ then $\Delta u^P < 0$. Thus, it remains to prove that $\Delta u^P < 0$ whenever $l_2^P(\theta) - l_1^P(\theta) > d$ for all θ. Consider equation (21), as $\Delta P(\theta_n) < 0$ for all $n \in N$, and $-\psi_{l\theta}(l,\theta)/\sigma_\psi(l,\theta)$ positive for all θ, we have $\sum_{n\in N} \Delta P(\theta) \int_{\theta_n}^{\theta_n+\Delta\theta}(d\psi(l_1(\theta),\theta)/d\theta)d\theta < 0$ for $\psi_\theta(l,\theta)$ sufficiently small. Then, $\Delta u^P < 0$ if $\sigma_\psi(l,\theta)$ is sufficiently large, and there exists a finite number h such that $\Delta\eta^P = \eta_2^P - \eta_1^P < h$. To prove the existence of this upper bound we use an optimality argument. As (x^P, α^P) solves the Pareto program, one must have $EU(x^P, \alpha^P) \geq EU(x', \alpha')$

for all feasible $(x', \alpha') \neq (x^P, \alpha^P)$. In particular, consider the consumption allocation $\hat{x}$ such that $\hat{x}_c^t(\theta) = x_c^{tP}(\theta)$ for $c = 1, 2$; $\hat{l}^t = L - \hat{x}_L^{tP} = \bar{l} = \beta l^{1P} + (1-\beta) l^{2P}$, with $\beta \in (0,1)$. Since $l^{1P} < l^{2P}$, a continuity argument implies that for any β sufficiently small there exists a real number k such that $\hat{\alpha} = \alpha^P + k\beta$, and $(\hat{x}, \hat{\alpha})$ satisfy the feasibility constraints (possibly as inequality). Thus take $(\hat{x}, \hat{\alpha})$ which satisfy the above properties; we must have $\Delta EU = EU(x^P, \alpha^P) - EU(\hat{x}, \hat{\alpha}) \geq 0$. By adding and subtracting $EU(\hat{x}, \alpha^P)$ to ΔEU,and then using the first order conditions of the Pareto program and rearranging, one gets $\Delta EU = A^P + B^P$ where $A^P = \sum_{t \in T, \theta \in \Theta} \alpha_t^P p^t(\theta) \Delta\psi(l^t(\theta))$, with $\Delta\psi(l^t(\theta)) = (\psi(\bar{l}(\theta), \theta) - \psi(l^{tP}(\theta), \theta))$, and where:

$$B^P = (\hat{\alpha}_1 - \alpha_1^P) \sum_{n \in N} \Delta P(\theta_n) \int_{\theta_n}^{\theta_n + \Delta\theta} \left(\alpha_1^P \left(-\frac{\psi_{l\theta}(l_1(\theta), \theta)}{\sigma(l_1(\theta), \theta)} \right) + \alpha_2^P \left(-\frac{\psi_{l\theta}(l_2(\theta), \theta)}{\sigma(l_2(\theta), \theta)} \right) \right) d\theta$$

Moreover, from the first order conditions of the Pareto program and the convexity of ψ it follows that $A^P < A' = \sum_{t \in T} \alpha_t^P \eta_t^P \sum_{\theta \in \Theta} p^t(\theta) \Delta l^t(\theta) > 0$ where $\Delta l^t(\theta) = (\bar{l}(\theta) - l^{tP}(\theta))$. Using the definition of $\bar{l}(\theta)$ we then get:

$$A' = \alpha_1^P \eta_1^P (1-\beta) \sum_{\theta \in \Theta} p^1(\theta)(l^{2P}(\theta) - l^{1P}(\theta)) - \alpha_2^P \eta_2^P \beta \sum_{\theta \in \Theta} p^2(\theta)(l^{2P}(\theta) - l^{1P}(\theta))$$

As we are considering the case where $(l^{2P}(\theta) - l^{1P}(\theta)) > d$, the above expression implies $A^P \to -\infty$ as $\eta_2^P - \eta_1^P \to \infty$. In turn, since $|\psi_{l\theta}(l, \theta)/\sigma_\psi(l, \theta)| < k$, there must exist a real number D such that for all θ, $\psi_{l\theta}(l_t(\theta), \theta) < D$. This implies that $B^P < (\hat{\alpha}_1 - \alpha_1^P) D$. We can conclude that $\Delta EU = A^P + B^P > 0$ implies $\eta_2^P - \eta_1^P < h$ for some positive h. This allows to conclude that $\Delta u^P < 0$ for $\sigma_\psi(l, \theta)$ sufficiently large. Indeed $\sum_{n \in N} \Delta P(\theta_n) \int_{\theta_n}^{\theta_n + \Delta\theta} (d\psi(l_1(\theta), \theta)/d\theta) d\theta$ is strictly positive and bounded below; while $\sum_{\theta \in \Theta} p^2(\theta) \int_{\eta_1^P}^{\eta_2^P} 1/\sigma_\psi(l(\theta, \eta), \theta) d\eta$ becomes arbitrarily small for $\sigma_\psi(l, \theta) \to \infty$, since $\eta_2^P - \eta_1^P$ is bounded. □

Part (iii). Define the function $T(\eta, \theta) \equiv \sigma_\psi^{-1}(l(\eta, \theta), \theta) - l(\eta, \theta)$, Inada conditions imply $T(0, \theta) = 0$ for all θ. Moreover one can readily verify that $\psi_{lll}(l, \theta) \geq 0$ for all (l, θ) implies $\partial T(\eta, \theta)/\partial\eta \leq 0$ for all (η, θ). It immediately follows that $l(\eta, \theta) \geq \sigma_\psi^{-1}(l(\eta, \theta), \theta)$ for all (η, θ). This inequality together with (21) imply

$$\Delta u^P \leq \sum_{n \in N} \Delta P(\theta_n) \int_{\theta_n}^{\theta_{n+1}} \left(-\frac{\psi_{l\theta}(l_2(\theta), \theta)}{\sigma_\psi(l_2(\theta), \theta)} + \psi_\theta(l_2(\theta), \theta) \right) d\theta + \sum_{\theta \in \Theta} p^1(\theta) \left(\int_{\eta_1^P}^{\eta_2^P} l(\eta, \theta) d\eta \right) \tag{22}$$

Let the function $h(l, \theta) \equiv \psi_l(l, \theta) l$ for all (l, θ). Consider the first order condition with respect to α of the Pareto program:

$$\sum_{\theta \in \Theta} p^2(\theta) \psi(l^{2P}(\theta), \theta) - \sum_{\theta \in \Theta} p^1(\theta) \psi(l^{1P}(\theta), \theta) = \eta_2^P \sum_{\theta \in \Theta} p^2(\theta) l^{2P}(\theta) - \eta_1^P \sum_{\theta \in \Theta} p^1(\theta) l^{1P}(\theta)$$

Substituting the optimality conditions $\psi_l(l^t(\theta),\theta) = \eta_t^P$ for all θ, $t = 1, 2$, and adding and substracting $\sum_{\theta\in\Theta} p^1(\theta)\psi(l^2(\theta),\theta)$ to the left hand side of the above equation, and $\sum_{\theta\in\Theta} p^1(\theta)h(l^2(\theta),\theta)$ to the right hand side, one gets:

$$\sum_{n\in N} \Delta P(\theta_n) \int_{\theta_n}^{\theta_{n+1}} \psi_\theta(l_2(\theta),\theta)d\theta = \sum_{\theta\in\Theta} p^1(\theta) \left(\int_{\eta_1^P}^{\eta_2^P} l(\eta,\theta)d\eta \right)$$

From this equality together with (22) it follows:

$$\Delta u^P \leq \sum_{n\in N} \Delta P(\theta_n) \int_{\theta_n}^{\theta_{n+1}} \left(-\frac{\psi_{l\theta}(l_2(\theta),\theta)}{\sigma(l_2(\theta),\theta)} + 2\psi_\theta(l_2(\theta),\theta) \right) d\theta$$

Then $\Delta u^P \leq 0$ since first order stochastic dominance implies $\Delta P(\theta) < 0$ for all θ and by assumption the integrand function is positive. $\Box$

Proof of proposition (9)

As a preliminar result we prove the following lemma:

Lemma 17 *Assume $p^1(\theta) = 1$ for $\theta = \theta_N$, if $\sigma_\psi(l) \gtreqless \sigma_h(l)$ then $\Delta\psi = 0$ implies $\Delta h \lesseqgtr 0$.*

Proof. Setting $\Delta\psi = 0$, one gets $\psi\left(l^1(\theta_N)\right) = \sum_{\theta\in\Theta} p^2(\theta)\psi\left(l^2(\theta)\right)$. $l^1(\theta_N)$ then defines the certainty equivalent of $\sum p^2(\theta)\psi\left(l^2(\theta)\right)$. Let $l^2(h)$ denote the certainty equivalent for $\sum_{\theta\in\Theta} p^2(\theta)h\left(l^2(\theta)\right)$, we have $\Delta h = h(l^2(h)) - h(l^1(\theta_N))$. Since $h(l)$ is an increasing function and $\sigma_\psi(l) \gtreqless \sigma_h(l)$ implies $l^2(\psi) \gtreqless l^2(h)$, it follows that $\Delta h \lesseqgtr 0$ if $\sigma_\psi(l) \gtreqless \sigma_h(l)$. $\Box$

We now turn to the proof of the proposition. Consider first the extreme case where $p^1(\theta) = 1$ for $\theta = \theta_{\bar{N}}$.

(Part 1): $\sigma_\psi(l) < \sigma_h(l)$ implies $u^1(x^{1P}) < u^2(x^{2P})$.

To prove the claim we use introduce an auxiliary maximization program, this program maximizes the expected utility of the representative agent under the feasibility constraints and the additional constraint

$$\Delta\psi = \sum_{\theta\in\Theta} p^2(\theta)\psi(l^2(\theta)) - \sum_{\theta\in\Theta} p^1(\theta)\psi(l^1(\theta)) \leq 0 \tag{23}$$

The FOCs of the auxiliary program with respect to $l^t(\theta)$, $t = 1, 2$, and α are:

$$\psi'(l^1(\theta_N)) = \eta_1 a(\theta_N) + \frac{\varkappa}{\alpha}\psi'(l^1(\theta_N)) \tag{24}$$

$$\psi'(l^2(\theta)) = \eta_2 a(\theta) - \frac{\varkappa}{1-\alpha}\psi'(l^2(\theta)) \ \ \forall\, \theta \in \Theta \tag{25}$$

$$\sum p^2(\theta)\psi(l^2(\theta)) - \psi(l^1(\theta_N)) = \eta_2 \sum p^2(\theta)a(\theta)l^2(\theta) - \eta_1 a(\theta_N)l^1(\theta_N) \tag{26}$$

where $\varkappa \geq 0$ is the multiplier associated with the constraint $\Delta\psi \leq 0$. Substituting (24) and (25) into (26) one gets:

$$\Delta\psi = \Delta h + \varkappa\left(\frac{h^2}{1-\alpha} + \frac{h^1}{\alpha}\right) \tag{27}$$

We begin by showing that if $\sigma_\psi(l) < \sigma_h(l)$ the solution of the auxiliary program solves the Pareto program. This is equivalent to prove that $\sigma_\psi(l) < \sigma_h(l)$, implies either that $\varkappa = 0$ or that $\Delta\psi = \sum_{\theta\in\Theta} p^2(\theta)\psi(l^2(\theta)) - \psi(l^1(\theta_N)) \leq 0$ at the Pareto optimum. Suppose, on the contrary, that $\varkappa > 0$, then $\Delta\psi = 0$, and by equation (27) it follows $\Delta h = -\varkappa\left(\frac{h^2}{1-\alpha} + \frac{h^1}{\alpha}\right) < 0$. But this contradicts lemma (17) since $\sigma_\psi(l) < \sigma_h(l)$ implies $\Delta h > 0$ whenever $\Delta\psi = 0$.

Now note that $x^{1P}(\theta) = x^{2P}(\theta) = x^P$ for all θ implies $u^1(x^{1P}) - u^2(x^{2P}) = \Delta\psi$. It then remains to show that $\sigma_\psi(l) < \sigma_h(l)$ also implies $\Delta\psi < 0$ (i.e. that the inequality in (23) is satisfied strictly). This is true as $\Delta\psi = 0$ implies $\Delta h > 0$ whenever $\sigma_\psi(l) < \sigma_h(l)$ so that (27) cannot be satisfied.

(Part 2): $\sigma_\psi(l) > \sigma_h(l)$ implies $u^1(x^{1P}) > u^2(x^{2P})$.

Proving this claim requires the same type of argument developed in part 2. Precisely, one first introduces an auxiliary program where the expected utility of the representative agent is maximized subject to the feasibility conditions and the additional constraint $-\Delta\psi \leq 0$, and then use the FOCs of this program to prove the result. The details of the proof are left to the reader.

(Part 3): $\sigma_\psi(l) = \sigma_h(l)$ implies $u^1(x^{1P}) = u^2(x^{2P})$.

Consider the auxiliary maximization program which maximizes the expected utility of the representative agent under the feasibility constraints and the additional constraint $\Delta\psi = 0$. Notice that this program is the same as the auxiliary program used in part 1 except that now the additional constraint on $\Delta\psi$ is imposed as equality. Notice that the FOCs of the program with respect to $l^t(\theta)$, $t = 1, 2$, and α are exactly conditions (24), (25) and (26). Substituting (24) and (25) into (26) one gets again equation (27) where $\varkappa \gtreqless 0$ is the Lagrange multiplier associated to the constraint $\Delta\psi = 0$. The claim is now proved by contradiction. Assume $\varkappa \neq 0$, the constraint $\Delta\psi = 0$ is then binding, and equation (27) implies then that $\Delta h = -\varkappa\left(\frac{h^2}{1-\alpha} + \frac{h^1}{\alpha}\right) \neq 0$ which by lemma (17) is impossible since $\sigma(l,\psi) = \sigma(l,h)$ and $\Delta\psi = 0$ imply $\Delta h = 0$. Therefore, $\sigma_\psi(l) = \sigma_h(l)$ must imply that $\varkappa = 0$, the solution of the auxiliary problem solves the pareto program so that $\Delta\psi = 0$ and $u^1(x^{1P}) = u^2(x^{2P})$ at the solution of that program. □

Proof of proposition (10)

We begin with the case of low transaction costs economies in which only deterministic contracts are enforceable.

Consider the auxiliary program which maximizes $\sum_{t\in T} u_i^t(x_i^t)\gamma_i^t$ within the compact set of allocations defined by the agents' budget constraints and the additional constraints $\gamma_i^t \geq \varepsilon$ for all t and i, and $x_i^t \in \bar{X}$,with $\bar{X}$ finite. Under Inada conditions , the constraint $\gamma_i^t \geq \varepsilon$ is not binding for ε sufficiently small. Moreover, since the per capita endowment of the economy is finite, the constraint $x_i^t \in \bar{X}$ is not binding for $\bar{X}$ sufficiently large if $\gamma_i^t \geq \varepsilon$. And hence the set of solutions of the competitive equilibrium agents' maximization program and of the auxiliary program coincide for ε sufficiently small and $\bar{X}$ sufficiently large. As both production and intermediation technologies are linear, equilibrium prices satisfy the following properties: $\phi_i^t(\theta) = g_i^t p_i^t(\theta)$ for some $g_i^t \in \Re_+$ for all $i \in I$ and $t \in T$ and $\theta \in \Theta$; (ii) $w_i^t(\theta) = p_i^t(\theta) b_i^t a_i^t(\theta)$, and all firms make zero profits. Using the zero profit conditions and normalizing assets and spot prices, the budget correspondence can be rewritten as:

$$B_i^t(q) = \sum_{\theta\in\Theta} p_i^t(\theta) \, (\sum_{c\in C} q_c(x_{ic}^t(\theta) - e_i^c - a_i^t(\theta) q_t(L - x_{iL}^t(\theta)) \leq 0 \quad \forall \, t$$

This correspondence is continuous for all $q \gg 0$. As a consequence the correspondence

$$(\zeta_i^t(p), \varphi_i^t(q)) = \left\{ (x_i^t, \varphi_i^t) : (x_i^t, \varphi_i^t) \in \arg\max \sum_{t\in T} u_i^t(x_i^t)\varphi_i^t \text{ s.t. } (x_i^t, \varphi_i^t) \in B^i(q) \right\}$$

is upper hemicontinuous. While $\gamma_i^t(q)$ is also convex valued, however, $\zeta_i^t(q)$ has not a convex graph. By construction, however, the per capita demand correspondence $\xi_i^t(p) = \sum_{t\in T} \gamma_i^t(q)\zeta_i^t(q)$ is upperhemicontinuous and convex valued. Hence, a standard application of the Kakutani fixed point theorem, in the space $\bar{X} \times \Delta^{C-1}$ implies the existence result.

The existence proof of an equilibrium for the high transaction costs case follows exactly the same lines.

Proof of proposition 12

We start by demonstrating that competitive equilibria satisfy the fair treatment condition. Suppose, this is not true; if $u_i^t(x_i^t) > u_i^{t'}(x_i^{t'})$ then it is individually optimal for all type i agents to choose $\varphi_i^t = 0$ which contradicts $(\varphi_i^t, \varphi_i^{t'}) > 0$. This argument is valid for any t hence the fair treatment requirement must necessarily be satisfied. Now let $\langle x^*, \varphi^*, q^*, z^*, \phi^* \rangle$ a competitive equilibrium and assume it is not interim efficient. Assume that there exists a feasible allocation $\langle \widehat{x}, \widehat{\varphi}, \rangle \neq \langle x^*, \varphi^* \rangle$ such that $u_i^t(\widehat{x}_i^t) = u_i^{t'}(\widehat{x}_i^{t'})$ for all i, t and t' such that $(\widehat{\varphi}_i^t, \widehat{\varphi}_i^{t'}) > 0$ and $\langle \widehat{x}_i, \widehat{\varphi}_i \rangle \succeq_i \langle x_i^*, \varphi_i^* \rangle$ with at least one type strictly preferring $\langle \widehat{x}_i, \widehat{\varphi}_i \rangle$. Hence, it must be true that:

$$\sum_{t\in T} \widehat{\varphi}_i^t \sum_{\theta\in\Theta} p_i^t(\theta) \sum_{c\in C} q_c^*(\widehat{x}_{ic}^t(\theta) - e_i^c) \geq \sum_{t\in T} \varphi_i^t \sum_{\theta\in\Theta} p_i^t(\theta) q_t^* a_i^t(\theta)(L - \widehat{x}_{Li}^t(\theta)) \quad \text{for all } i$$

where the inequality must be strict for at least one i (i.e. $\langle \widehat{x}_i, \widehat{\varphi}_i \rangle$ must violates at least one budget constraint). Multiplying both sides of each type's budget constraint by the measure of agents of that type and adding up one obtains:

$$\sum_{c\in C} q_c^* \left(\sum_{i\in I} \mu_i \left(\sum_{t\in T,\theta\in\Theta} \widehat{\varphi}_i^t p_i^t(\theta) \sum_{c\in C} \widehat{x}_{ic}^t(\theta) - e_i^c - \sum_{t\in T,\theta\in\Theta} \varphi_i^t p_i^t(\theta) a_i^t(\theta)(L - \widehat{x}_{Li}^t(\theta)) \right) \right) > 0$$

which implies that $\langle \widehat{x}_i, \widehat{\varphi}_i \rangle$ violates feasibility, and provides the required contradiction.

Proof of proposition (15)

Consider a solution, $(\hat{\alpha}, \hat{x})$, of the Pareto program associated to a feasible vector of reservation utilities, $\overline{u}$. There exists an equilibrium with transfers such that

$$\wp = (s_i^t = \sum_{c\in C,\theta\in\Theta} p_i^t(\theta)(\eta_c(\hat{x}_{ic}^t(\theta) - e_i^c) - \eta_t a_i^t(\theta)(L(\theta) - \hat{x}_{iL}^t(\theta))); \ \hat{w}_i^t(\theta) = \eta_t a_i^t(\theta); \ f_i^t(\theta) = 0)$$

$\varphi_i^t = \alpha_i^t = \hat{\alpha}_t^i$, $x = \hat{x}$; and $q_j/q_1 = \eta_j/\eta_1; \phi_i^t(\theta) = p_i^t(\theta); w_i^t(\theta) = \eta_t a_i^t(\theta)$.

Indeed, for $\phi_i^t(\theta) = p_i^t(\theta)$, $q_j/q_1 = \eta_j/\eta_1$, by using the budget constraints defining the agent program one obtains:

$$\sum_{\theta\in\Theta} p_i^t(\theta) \left(\sum_{c\neq L} \eta_c(x_{ic}^t(\theta) - e_i^c) - \eta_t a_i^t(\theta) \hat{l}_i^t(\theta) - s_i^t \right) \leq 0$$

where $\hat{l}_i^t(\theta) = (L - \hat{x}_{iL}^t(\theta))$. The vector $(x^i = \hat{x}^i, \ \varphi_i^t = \hat{\alpha}_t^i)$ then solves the type-i agents maximization program for $q_j/q_1 = \eta_j/\eta_1$ and $s_i^t = \sum_{c\in C,\theta\in\Theta} p_i^t(\theta)(\eta_c(\hat{x}_{ic}^t(\theta) - e_i^c) - \eta_t a_i^t(\theta)(L(\theta) - \hat{x}_{iL}^t(\theta)))$, the budget constraints and all the first order conditions of the program calculated in the point $\varphi_i^t = \alpha_i^t$.

Moreover, by construction the transfers' policy is budget balancing. Finally, all the market clearing conditions are satisfied at $\phi_i^t(\theta) = p_i^t(\theta)$ and $w_i^t(\theta) = \hat{w}_i^t(\theta) = \eta_t a_i^t(\theta)$. Indeed, at these prices $\hat{l}_i^t(\theta)$ is part of the indeterminate labor demand schedule of sector t firms, and the supply of all state contingent assets is also indeterminate and the feasibility conditions for consumption commodities are satisfied.

Assume now that workers' labor supply is completely inelastic for any positive wage, and take a generic Pareto optimal allocation $(\hat{\alpha}, \hat{x})$. It must necessarily be such that $\hat{x}_{iL}^t(\theta) = L$ if $w_i^t(\theta) = 0$ and $\hat{x}_{iL}^t(\theta) = L - L(\theta)$ if $w_i^t(\theta) > 0$. Consider now a transfers' policy $\wp$ such that

$$s_i^t = \sum_{c\in C,\theta\in\Theta} p_i^t(\theta)(\eta_c(\hat{x}_{ic}^t(\theta) - e_i^c) - \eta_t a_i^t(\theta)(L(\theta) - \hat{x}_{iL}^t(\theta)));$$

$$\hat{w}_i^t(\theta) = \hat{w} = \max_t \eta_t \sum_{\theta\in\Theta} p_i^t(\theta) a_i^t(\theta); \ f_i^t = \eta_t \sum_{\theta\in\Theta} p_i^t(\theta) a_i^t(\theta) - \max_t \eta_t \sum_{\theta\in\Theta} p_i^t(\theta) a_i^t(\theta)$$

By using the same arguments as in the first part of the proof one can easily check that given this policy scheme $\varphi_i^t = \alpha_i^t = \hat{\alpha}_t^i$, $x = \hat{x}$, if the price vector is such that $q_j/q_1 = \eta_j/\eta_1; \phi_i^t(\theta) = p_i^t(\theta); w_i^t(\theta) = \hat{w}$.