

# The demand for currency and the welfare cost of inflation in a 21<sup>st</sup> century economy\*

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## Abstract

The interest elasticity of money demand is key in the evaluation of the welfare cost of inflation. Yet not much is known about it in the typical environment of a modern economy, characterized by low inflation and a developed financial system.

This paper uses a unique panel of Italian household data (on currency holdings, banking services, interest rates, consumption, etc.) that allows a precise estimation of the demand for currency. The data span the 1989-2002 period, characterized by a widespread diffusion of bank branches, automated teller machines (ATMs) and, in recent years, low inflation rates. The estimates show that the interest elasticity is large and significant over the first part of the nineties, but essentially nil afterwards.

We present a currency inventory model that introduces uncertain withdrawal options between costly and non-costly withdrawals. Financial innovation is described as an increase in the availability of the latter, to capture the diffusion of e.g. bank branches and ATMs. The model predictions are consistent with several features of the data and inform us on the welfare costs of (low) inflation in a financially-developed economy.

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# 1 Introduction

In a monetary economy inflation requires individuals to spend resources to economize on cash balances. Reducing inflation diminishes the resources that each citizen dedicates to such an unproductive activity, and therefore increases welfare. This simple and powerful insight, made explicit by the seminal contributions of Bailey (1956) and Friedman (1969), is still the core of the theory on the costs of inflation.

Measuring the welfare costs of inflation in accordance to this view requires information on the interest elasticity of money demand, which allows one to quantify the magnitude of the distortion induced upon the holdings of non-interest bearing assets by a higher rate of inflation. Following this line of reasoning Lucas (2000) estimates that the welfare gain of reducing inflation from about 14 to 3 per cent is comparable to a permanent increase of one per cent of GDP. Moreover, he shows that while this quantification is relatively robust to the money demand specification adopted to fit the historical U.S. aggregate data over the twentieth century, the gain delivered by a further inflation reduction crucially hinges on the interest elasticity of money demand at low interest rates. Depending on the money demand specification, the gain of reducing inflation from 3 per cent to zero ranges from about 1 per cent of GDP to basically nil. As Lucas put it “money holding behavior at low interest rates is central for estimating welfare costs”. Yet not much is known about the interest elasticity of the demand for money in the typical environment of a modern economy, characterized by low inflation and a developed financial system.

This paper uses a unique panel of Italian household data (on currency holdings, interest rates, consumption, territorial diffusion of banking services, etc.) to quantify the interest elasticity of the demand for currency. Micro data are essential for a precise estimation because, as shown by Attanasio, Guiso and Jappelli (2002), different degrees of financial sophistication across households may bias the inference on money-holdings behavior drawn from aggregate data. The data span the 1989-2002 period, characterized by strong financial innovation and, in recent years, low inflation rates. This provides an ideal natural laboratory to study money demand at low interest rates.

The estimates show that the interest elasticity is large and significant over the first part of the sample, but essentially nil in more recent years. We present a currency “inventory” model that introduces uncertain withdrawal options between costly and non-costly withdrawals. Financial innovation is described as an increase in the availability of the latter, which is consistent with, e.g., the diffusion of bank branches and ATMs. The model predictions offer an interpretation of the time-varying elasticity detected by our estimates and contribute to understanding the welfare costs of (low) inflation in a financially-developed economy.

The paper is organized as follows. The next section describes the database and portrays the main developments in the financial variables of interest over the past 13 years. Estimates of the demand for currency, which extend the results of Attanasio et al. (2002), are presented in Section 3. Section 4 proposes a simple modification of the textbook Baumol-Tobin inventory model which allows us to interpret our empirical

findings and to discuss the welfare costs of inflation. A final section concludes.

## 2 Currency and financial innovation in the last 15 years: data sources and descriptive statistics

### 2.1 Sources

The main data source is the *Survey of Household Income and Wealth* (SHIW), a periodic survey conducted by the Bank of Italy since 1965<sup>1</sup> on a rotating sample of Italian households. The survey collects information on several social and economic characteristics of the household members, such as age, gender, education, employment, income, real and financial wealth, consumption and saving behavior. Each survey is conducted on a sample of about 8000 households.<sup>2</sup>

We focus on the surveys conducted from 1989 to 2002 because they include a section that is dedicated to the household cash management. This contains data on the average monthly number of withdrawals (from ATM and bank tellers), the average amount of cash held by the household, the minimum level of currency holdings that triggers a new withdrawal.

Two additional data sources are the *Italian Central Credit Register* and the *Supervisory Reports to the Bank of Italy*. The former includes information on the interest rate paid by banks on checking accounts disaggregated by year and province.<sup>3</sup> The latter collects the reports that Italian banks file to the Bank of Italy for supervisory reasons and contains information on the supply of various financial services, such as the diffusion of bank branches and of ATMs, at provincial level.<sup>4</sup>

### 2.2 A first look at the data

Descriptive sample statistics on the household currency management for the 1989 - 2002 period are reported in Table 1.

The first line shows that about 15 per cent of the households did not hold a checking account in 2002. This fraction does not vary much from 1989 to 2002. During the same period, the fraction of households who possess an ATM card increases sharply, from 15 to 55 per cent. This diffusion is accompanied by a marked increase in the supply of ATMs and bank branches (see Table 2).

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<sup>1</sup>The frequency of the survey has been annual from 1965 to 1987 and bi-annual since 1989.

<sup>2</sup>The use of micro data drawn from surveys may pose problem of intertemporal comparability and of misreporting. Concerning the former, most of the questions related to the data used in this paper are virtually unchanged. With respect to the latter problem, available investigations suggest that misreporting mainly affects information on financial wealth and, to a lesser degree, on real wealth and incomes; See Battipaglia and D'Alessio (1997).

<sup>3</sup>See Miller (2000) for a detailed description of this database.

<sup>4</sup>Italian provinces were 95 until 1995 and became 103 afterwards. The size of provinces is broadly comparable to that of U.S. counties.

Table 1: Household currency management

Variable	1989	1991	1993	1995	1998	2000	2002
Fraction with a checking account	0.88	0.87	0.85	0.85	0.86	0.85	0.85
Fraction using ATMs	0.15	0.29	0.34	0.40	0.49	0.52	0.55
Average currency holdings	672	554	396	445	388	374	376
No bank account	660	582	359	441	376	396	494
With bank account	674	549	403	445	390	370	355
No ATM card	665	572	426	482	423	427	405
With ATM card	719	505	368	403	364	335	329
Minimum currency <sup>a</sup>	157	133	141	108	133	137	123
No ATM card	151	131	147	112	157	154	143
With ATM card	184	137	133	103	115	126	113
Average withdrawal at a bank	...	487	655	554	520	488	482
No ATM card	...	478	660	547	515	505	484
With ATM card	...	511	645	563	526	474	481
Average withdrawal at an ATM	...	259	243	235	231	232	217
Total number of trips <sup>b</sup>	...	29.0	27.4	30.3	41.9	44.4	43.0
To the bank (no ATM card)	...	18.4	12.5	13.1	19.9	16.6	17.5
To the bank (with ATM card)	...	14.9	12.7	10.7	11.9	13.0	10.5
To the ATM	...	35.5	36.7	39.2	46.8	48.9	46.2
Total number of observations	8,262	8,135	7,979	8,134	7,122	8,001	8,010

Source: Bank of Italy - *Survey of Household Income and Wealth* ; entries computed using sample weights. - Currency data are deflated and expressed in euro (base: 2002). - Variables' definitions are available in the appendix. - Notes: <sup>a</sup>Reported level that triggers a withdrawal. - <sup>b</sup>Per year.

Looking at currency, the average amount held by the household (at constant prices) almost halved during the past 15 years. The reduction, which is common across households with different withdrawal technologies, is largest for those who own an ATM card. The average size of the ATM withdrawal decreased over the past 15 years. Note, moreover, that the average withdrawal at an ATM is substantially smaller than the typical withdrawal at a bank desk. Finally, the minimum amount of currency that triggers a withdrawal is substantially larger than zero (and greater for agents without the ATM), suggesting that a precautionary motive may play a role in cash-holding decisions. This variable too displays a downward trend over the period, which is strongest for the households who possess an ATM card.

Table 2 reports summary statistics on the supply of financial services (bank branches and ATMs diffusion) and two measures of the interest rate on deposits. The first measure is drawn from the “Central Credit Register” and is based on interest rates disaggregated by year and province. The second one, labeled “Individual”, is obtained from a specific SHIW question available since 1995.<sup>5</sup> Two remarks are due. First, interest rates paid on deposits record a substantial reduction since 1989. The decline is particularly significant in the second half of the nineties. Second, the cross-sectional distribution of the interest rate maintains a relatively large coefficient

<sup>5</sup>Households were asked to report the gross interest rate earned on deposits. Information on individual interest rates is available only for a small subset of our sample.

of variation even in the more recent years when the average interest rate is very low. This is an extremely important feature of the data because variability in interest rates is key for the inference on the interest elasticity of the demand for currency.

Table 2: Financial development, interest rates and inflation

Variable	1989	1991	1993	1995	1998	2000	2002
Bank branches <sup>a</sup>	288	341	384	421	470	501	532
ATM points <sup>a</sup>	101	221	315	393	505	570	649
Interest rate <sup>b</sup>	6.96 (0.49)	6.74 (0.52)	6.10 (0.42)	5.23 (0.32)	2.15 (0.23)	1.16 (0.22)	0.77 (0.15)
Interest rate (Individual)	...	...	...	5.47 (1.56)	2.09 (1.59)	2.07 (1.50)	1.60 (1.31)
Inflation <sup>c</sup>	6.3	6.3	4.6	5.2	2.0	2.5	2.5

Note: Standard errors in parenthesis. - <sup>a</sup> Per million residents. - <sup>b</sup> Source: Central credit register. Arithmetic mean of interest rates disaggregated by province and year. - <sup>c</sup> Consumer price index. All items.

### 3 Empirical evidence on the demand for currency

The descriptive evidence of the previous section highlights that currency holding behavior is highly heterogeneous across households, especially concerning the deposit and withdrawal technology.<sup>6</sup> As shown by Attanasio et al. (2002) (AGJ henceforth) accounting for this heterogeneity is important to obtain precise measures of the interest elasticity, due to the presence of an endogenous sample selection problem.<sup>7</sup>

We thus follow their methodological lead and estimate the demand for currency integrating their sample, which ended in 1995, with the data made available by the three following surveys (1998, 2000, 2002) using Heckman’s two-step procedure. The

<sup>6</sup>Moreover, while part of this heterogeneity has been reduced, e.g. by the increasing diffusion of ATM cards, other important differences persist (e.g. the percentage of households without a checking account is roughly constant).

<sup>7</sup>The selection problem arises because the sample used for the estimation of equation (3) is not randomly selected: the choice of opening a bank account (or to have an ATM card) is likely affected by unobserved individual characteristics that also influence currency demand. In such a case, estimates obtained through OLS are biased and inconsistent. The procedure developed by Heckman (1979) (the so called “Heckit” estimator) allows to solve for this problem. The methodology consists of two steps. First a probit is estimated to obtain a measure of these unobserved characteristics (the “inverse Mills ratio”). Then, the inclusion of this measure as an auxiliary variable in the baseline OLS regression controls for unobserved characteristics, yielding consistent estimates. As in AGJ, the currency equation is estimated separately for the sample of households with bank accounts and ATM cards and for those with bank accounts and no ATM card. This implies the estimation of two probits. First a probit for having a bank account is estimated on all the observations. Then a probit for having the ATM card conditional on having a bank account. This allows us to obtain the two variables that are necessary to correct OLS estimates.

latest survey data, moreover, seem ideal to investigate the property of the demand for currency in a low-interest rates and financially developed economy see Table 2).<sup>8</sup>

As in AGJ, the baseline specification is derived from the McCallum and Goodfriend (1987) extension of the Baumol-Tobin inventory model. The consumer chooses currency balances,  $m$ , to minimize the sum of the cost of transaction time  $\tau w$  (the product of transaction time  $\tau$  and the time of cost transaction  $w$ ) and foregone interest rates  $Rm$ , subject to a transaction technology:

$$\begin{aligned} \min_{\tau, m} \tau w + Rm & \quad (1) \\ \text{subject to } \tau = Ac^\gamma \left(\frac{c}{m}\right)^\beta & \end{aligned}$$

where  $A$  is a technology parameter and  $c$  denotes consumption. The internal solution of the minimization problem in (1) implies that the average holdings of real balances are equal to:

$$m = \left(\frac{wA\beta}{R}\right)^{\frac{1}{(1+\beta)}} c^{(\beta+\gamma)/(1+\beta)} \quad (2)$$

which yields the baseline specification for the money demand equation:

$$\log m_{i,t} = \frac{1}{1+\beta} \log \beta + \frac{1}{1+\beta} \log w_{i,t} A_{i,t} - \frac{1}{1+\beta} \log R_{i,t} + \frac{\beta+\gamma}{1+\beta} \log c_{i,t} \quad (3)$$

The estimates of equation (3) are reported in Table 3. For comparability, the control variables included in the regression replicate the set of AGJ, with the exception that we use time dummies instead of a quadratic trend specification to describe the time-series developments in the “cost of time” ( $\log w_{i,t} A_{i,t}$ ) and that we augment the baseline equation with regional dummies.<sup>9</sup>

The currency demand estimates for households who own a checking account (without ATM card) and for those who own a checking account and an ATM card are presented in Table 3. The first and the fourth columns of the Table report the full sample estimates of equation (3), for each group of households. The magnitude of consumption elasticities is comparable to that found by AGJ. In particular, as was the case for AGJ, the elasticity for the households with checking account and no ATM card is greater than that detected for the more financially developed households.

The findings concerning the interest elasticity, however, differ markedly from those of AGJ. When the equation is estimated over the whole 1989-2002 sample, the interest elasticity is not statistically different from zero, which compares to negative and statistically significant estimates found by AGJ over the 1989-1995 period. This suggests that the value of the interest elasticity might not be stable over the whole sample period.

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<sup>8</sup>As noted above, it is important for the estimation that notwithstanding the reduction of the average rate, the cross-sectional variance of the interest rate paid on deposits remains high.

<sup>9</sup>Neither choice affects the main results of the regression.

Table 3: The Demand for Currency

	Checking account			ATM card		
	1989-2002	1989-1995	1998-2002	1989-2002	1989-1995	1998-2002
log(consumption)	0.330 (0.016)	0.296 (0.017)	0.408 (0.056)	0.265 (0.023)	0.259 (0.031)	0.294 (0.039)
log(interest rate)	-0.016 (0.053)	-0.288 (0.129)	0.119 (0.080)	-0.077 (0.048)	-0.815 (0.203)	0.082 (0.059)
Dummies:						
Less than elementary school	0.008 (0.033)	0.030 (0.035)	-0.004 (0.104)	0.045 (0.070)	0.070 (0.093)	-0.007 (0.111)
Elementary schooling	-0.005 (0.027)	0.037 (0.029)	-0.055 (0.086)	0.068 (0.034)	0.075 (0.047)	0.036 (0.056)
Junior high school	-0.008 (0.023)	0.021 (0.024)	-0.061 (0.059)	0.068 (0.019)	0.045 (0.028)	0.078 (0.028)
High school	-0.042 (0.021)	-0.038 (0.023)	-0.050 (0.049)	-0.019 (0.016)	-0.034 (0.023)	-0.005 (0.022)
Male head	0.080 (0.011)	0.085 (0.013)	0.073 (0.020)	0.026 (0.013)	0.042 (0.022)	0.020 (0.016)
Mills ratios:						
Checking account	-0.589 (0.016)	-0.662 (0.019)	-0.477 (0.029)	-0.680 (0.031)	-0.888 (0.047)	-0.551 (0.041)
ATM card	0.016 (0.033)	-0.024 (0.038)	0.017 (0.102)	0.075 (0.053)	0.088 (0.063)	0.108 (0.096)
Constant	2.520 (0.180)	3.383 (0.291)	1.291 (0.442)	3.007 (0.297)	4.335 (0.537)	1.924 (0.414)
Regional dummies	yes	yes	yes	yes	yes	yes
Year dummies	yes	yes	yes	yes	yes	yes
Sample size	25285	17827	7458	21535	9385	12150
R <sup>2</sup>	0.246	0.245	0.179	0.196	0.213	0.132

Note: Note: Standard errors in parenthesis. - The equation is estimated using Heckman's two-step procedure. - The dependent variable in the probit regression for the households with a Checking account (ATM card) equals one if the household has at least one account (ATM card), zero otherwise. The probit regressors also include dummies for education (4 levels), gender and area of residence (city center, semi center, outskirts), as well as a measure of financial wealth and ATM per capita in province. - The dependent variable in the currency equation is the logarithm of real currency. The currency equation also includes the number of children, adults and income earners in the household and the age and the occupational status of the household head.

In fact, when estimated over the AGJ subsample (second and fifth columns of Table 3) a negative and significant interest elasticity emerges for both groups, while the regressions over the more recent sample period (1998-2002) indicate an elasticity that is not significantly different from zero (third and sixth columns). No instability in the consumption elasticity emerges across the two subsamples.<sup>10</sup>

<sup>10</sup>Two additional differences with respect to AGJ estimates concern the significance of the other regressors. In our case, neither the schooling dummies nor the correction term (Mills Ratio) obtained from the probit on ATM owners seem to have a significant effect on currency demand.

The reduction of the interest elasticity illustrated above is a rather robust feature of the data.<sup>11</sup> The inventory model that underlies the classical inventory theory, however, posits a constant elasticity. Neither an interest rate reduction nor financial innovation affect the interest elasticity. In the next section we therefore propose a modification to the classical inventory model with the aim to provide an interpretation for reduction in the interest elasticity discussed above.

## 4 Interpreting the evidence

This section seeks to interpret the time variation in the interest elasticity detected above and discuss its implications for the welfare costs of inflation. That requires us to depart from the McCallum and Goodfriend inventory model, that implies a *constant* interest rate elasticity of the demand for currency. Other aspects of that model, moreover, are at odds with our data. In particular, the average balance before withdrawal is much larger than zero, which suggests a “precautionary component” in the demand for currency that cannot be generated by a deterministic specification.

The distinguishing feature of our model is to explicitly account for the fact that, in a modern economy, agents often have an opportunity to withdraw at basically no costs, e.g. every time they find a withdrawal point next to the shopping point. This contrasts with the McCallum and Goodfriend assumption that all cash withdrawals are equally costly.

The spread of ATMs provides one instance of financial development which has significantly affected the cost and the availability of withdrawals. Evidence on the increasing importance of the ATM as withdrawal devices is provided in Tables 1 and 2. During the nineties both the territorial diffusion of ATMs and the population fraction who possesses an ATM card have increased significantly (respectively, from 221 to 649 ATM points per million residents and from 15 to 55 per cent).

Altogether, we want to ask whether explicitly accounting for the diffusion of cheaper withdrawal technologies helps interpreting the evidence presented above.

### 4.1 A search model of the demand for currency

Consider an agent who, in each period, finances an exogenous stream of consumption expenditures equal to  $c$  (time is discrete and the agent has an infinite horizon). The agent needs cash to buy goods (i.e. we rule out credit goods). Shopping takes place in one of several locations of this economy, which are may or may not be endowed with a cash dispenser (think of it as either an ATM or a bank branch).

Our model captures the existence of both costly and non-costly withdrawals by assuming that if the agent happens to be shopping at a location that has a cash dispenser, then a withdrawal can take place at no extra cost. When the agent ends

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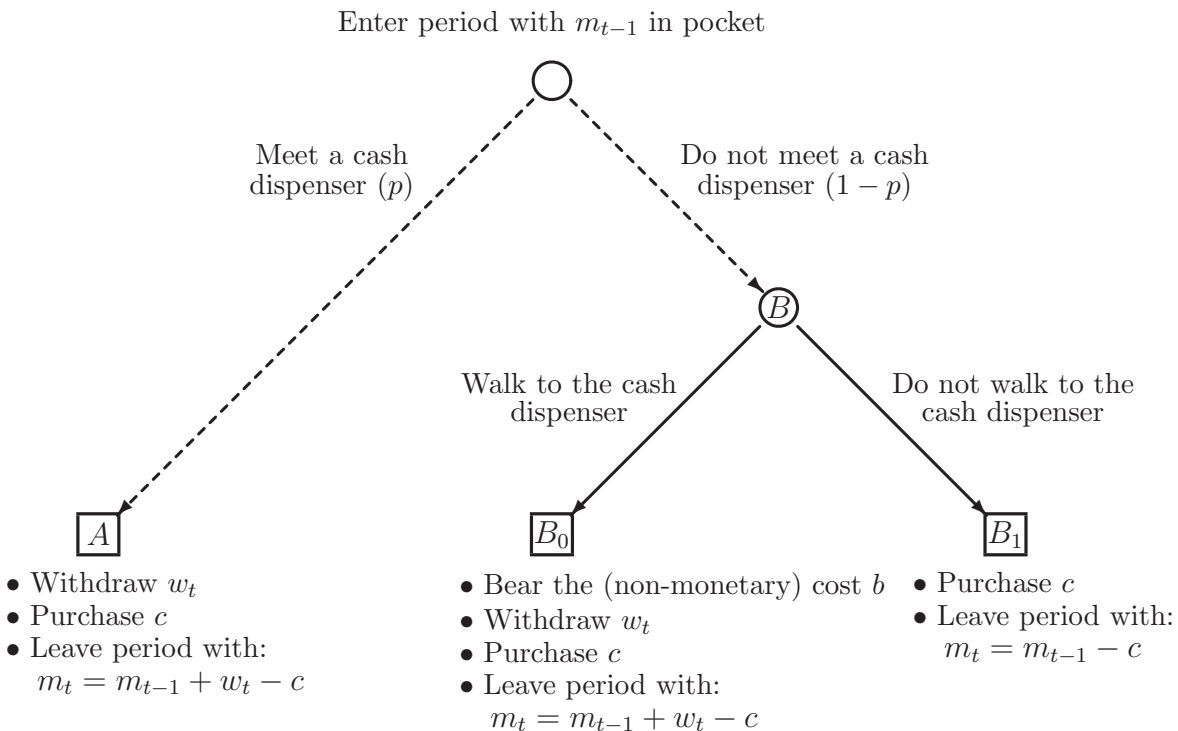
This result is likely related to the fact that, unlike AGJ, our equation includes temporal and regional dummies.

<sup>11</sup>SANDRO: - report that similar results on the variation of the  $i$ -elasticity emerge from a host of specifications, allowing for different functional forms.

up shopping in a location without cash dispenser, a withdrawal requires her to waste time searching for a cash dispenser, this entails a non monetary cost equal to  $b$  (this withdrawal technology is completely analogous to the one in the inventory model of Section 3). We measure the diffusion of the costly withdrawal technology by the probability  $(1 - p)$  that the agent ends up shopping in a location without a cash dispenser.

When every shopping point is endowed with a cash dispenser ( $p = 1$ ), the shopper can reduce cash holdings to zero. However, as long as the probability  $p$  is less than one, the optimal quantity of currency demanded by the agent will be greater than zero. This quantity weights the costs of forgone interest on deposit ( $Rm$ , where  $R$  denotes the net interest rate) against the (expected) costs of ending up without cash at a shopping location, in which case the agent must search for a dispenser and bear the non-monetary cost  $b$ .

Figure 1: Timing of actions within each period



The sequence of events in a typical period of this economy is depicted in Figure 1. The agent goes shopping carrying  $m_{t-1}$  currency from the previous period. With probability  $p$ , the agent ends up shopping in a location which has a cash dispenser. At this point (node  $A$  in the figure), the agent makes a withdrawal  $w_t$ , she then consumes  $c$  and leaves the period with  $m_t = m_{t-1} + w_t - c$  currency. With probability  $(1 - p)$ , the agent ends up in a location without cash dispenser (node  $B$  in the figure). She may decide to search for it (bearing the non-monetary cost  $b$ ), withdraw  $w_t$  and consume  $c$  (node  $B_0$  in Figure 1). Or she may decide not to search and pay for

current consumption using the available currency holdings. In this case the end of period currency is  $m_t = m_{t-1} - c$  (node  $B_1$  in Figure 1).

Formally, the agent's problem can be written recursively as follows. Let  $V_A(m)$  be the expected cost of financing the consumption flow for an agent who holds an amount  $m \geq 0$  of currency and is matched with a cash dispenser. Let  $V_B(m)$  be the corresponding expected cost for an agent who is unmatched. The value function  $V_A(m)$  is given by :

$$V_A(m) = \min_w \{R(m+w) + \delta[pV_A(m+w-c) + (1-p)V_B(m+w-c)]\} \quad (4)$$

subject to  $m+w \geq c$ . The Bellman equation states that the optimal currency choice weights the current costs of a greater foregone interest with the future ( $\delta$  denotes the discount factor) expected costs implied by the current withdrawal. For an agent who is not matched with a cash dispenser:

$$V_B(m) = \begin{cases} \min\{b + V_A(m); Rm + \delta[pV_A(m-c) + (1-p)V_B(m-c)]\} & \text{for } m \geq c \\ b + V_A(m) & \text{otherwise} \end{cases} \quad (5)$$

where the second line indicates that the agent must search for a cash dispenser and bear the cost  $b$  whenever currency holdings are not sufficient to pay for the current period consumption.

The policy function for this problem  $w(s, m)$  is a map that, given the state of the world ( $s = A$  or  $s = B$ ) and the amount of money in hand, prescribes the agent whether and how much to withdraw.

**Proposition 1.** *The optimal policy function is:*

$$w^*(s, m) = \begin{cases} m^* - m & \text{if } s = A \\ 0 & \text{if } s = B \text{ and } m \geq c \\ m^* - m & \text{if } s = B \text{ and } m < c \end{cases} \quad (6)$$

*Proof.* Let  $z = m + w$ , and define

$$m^* = \arg \min_z \{Rz + \delta(1-p)V_B(z-c)\}$$

Note that:

$$V_A = \frac{Rm^* + \delta(1-p)V_B(m^* - c)}{1 - \delta p}$$

solves the Bellman equation (4). Hence the value function  $V_A$  is constant and the optimal withdrawal for the agent matched with a cash dispenser is  $w^* = m^* - m$ .

Define the function  $f(m) \equiv R(m) + \delta[pV_A + (1-p)V_B(m-c)]$ , over the domain  $m = nc$  with  $n = 1, 2, 3, \dots$

Note from (4) that  $V_A = \min_n f(nc) = f(n^*c)$ , which implies that  $V_B \geq V_A$ . The integer  $n^* \equiv \frac{m^*}{c}$  corresponds to the number of consumption periods financed by the optimal withdrawal.

Let us now show that it is not optimal to walk to a cash dispenser when  $c \leq m \leq n^*c$ .

Compute the difference between the cost of walking to a dispenser,  $C_W(nc)$ , and the cost of not-walking,  $C_N(nc)$ , when when currency holding is  $nc$ , this yields:

$$C_W(nc) - C_N(nc) = b(1 - \delta(1 - p)) + R(n^* - n)c + \delta(1 - p)$$

$(b + Rm^* + \delta[pV_A + (1 - p)V_B(m^* - c)])$  with the cost of not walking:  $(b + Rc + \delta[pV_A + (1 - p)(V_A + b)])$ . Some algebra shows that the difference between these two is always greater than zero. Therefore, an agent at node  $B$  only chooses to withdraw when her cash balances are insufficient to finance the current consumption.

Assume (which we verify below) that  $V_B(nc) = f(nc)$  for  $1 \leq n \leq n^*$ . By recursive substitution into 5 we can express the value of  $V_B$  as a function of the currency stock  $nc$  as follows:

$$V_B(nc) = \frac{1 - \varphi^n}{1 - \varphi} \delta p V_A + cR \sum_{j=1}^n \frac{(1 - \varphi^j)}{1 - \varphi} + \varphi^n (V_A + b)$$

This shows that  $V_B(nc)$  is a convex function of  $n$ , hence that it is decreasing for  $1 \leq n \leq n^*$ , i.e. over the  $[c, m^*]$  range. Hence, if it is not optimal to walk to the dispenser when  $m = c$ , it is never optimal to walk to a dispenser for  $m \leq m^*$ .  $\square$

The above result shows that there is an optimal level of currency balances,  $m^*$ , which is the same for all agents who decide to withdraw, irrespective of current holdings. We can then define the integer value  $n^* \equiv \frac{m^*}{c}$  which corresponds to the number of consumption periods financed by the optimal withdrawal.

The policy rule thus prescribes the following: replenish currency balances till the target level  $m^*$  whenever a cash dispenser is available. Otherwise, the rule prescribes not to walk to a dispenser, provided currency holdings are sufficient to finance consumption. The only instance in which the fixed cost  $b$  is paid is when the agent is not matched with a dispenser *and* currency holdings are insufficient to pay for consumption.<sup>12</sup>

## 4.2 The demand for currency and interest elasticity

The model shows that the typical agent in this economy will be holding different amount of currency in different period, depending on whether she was matched with

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<sup>12</sup>Intuitively, for an agent at node  $B$  with  $m \geq c$  it is not optimal to walk to the dispenser, since this choice entails paying the fixed cost  $b$  with certainty while the choice of not walking to the dispenser allows her to postpone paying this cost.

cash a dispenser this period, one period ago, and so on. The model thus generates a whole distribution for currency holdings.

Computing this distribution is necessary to study how the average currency holdings of a representative agent respond to changes in the nominal rate of interest and to the diffusion of cash dispensers (as measured by the probability  $p$ ). Let the function  $\Theta(m)$  denote the density of the end-of-period currency balances  $m$ .

Only a fraction  $\pi$  of agents replenish their balances in each period: after consuming they are left with currency holdings equal to  $m^* - c$ . The fraction  $\pi$  includes  $p$  individuals who are matched with a cash-dispenser (node  $A$  in Figure 1) and  $(1 - p)^{n^*} \pi$  agents who pay the cost  $b$  and undertake a costly withdrawal. The latter group is given by those agents who remain unmatched with a dispenser for  $n^*$  consecutive periods (node  $B_0$  in Figure 1). The value for  $\pi$  thus satisfies:  $\pi = p + (1 - p)^{n^*} \pi$ , which yields  $\pi = \frac{p}{1 - (1 - p)^{n^*}}$ .

Table 4: Currency holdings distribution

Currency Holdings( $m_i$ )	0	$c$	...	$(n^* - \kappa) c$	...	$(n^* - 1) c$
Withdrawal when matched( $w_i$ )	$m^*$	$m^* - c$	...	$\kappa c$	...	$c$
Population Fraction $\Theta(m_i)$	$(1 - p)^{n^* - 1} \pi$	$(1 - p)^{n^* - 2} \pi$	...	$(1 - p)^{\kappa - 1} \pi$	...	$\pi$

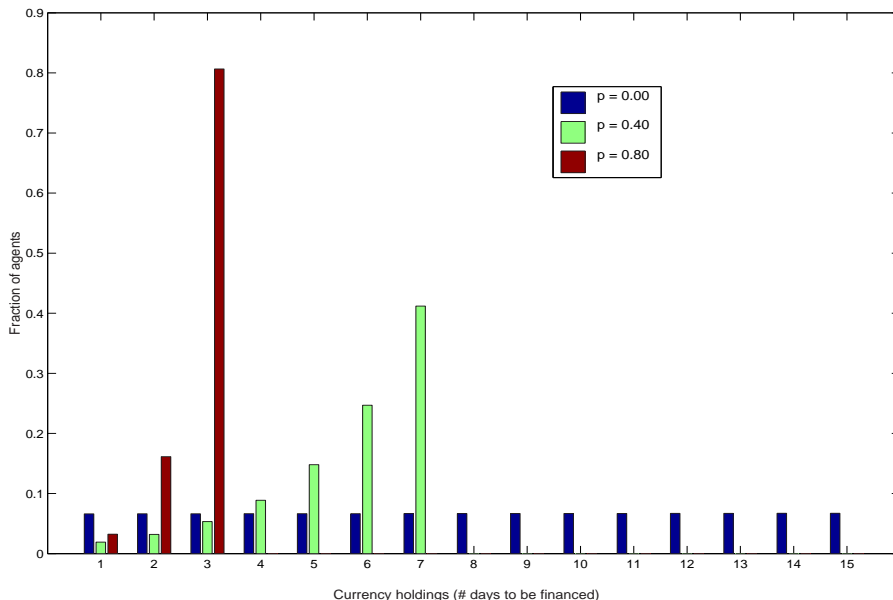
Similarly, one can compute the fraction of agents with end-of-period balances equal to zero,  $\Theta(0)$ . This is given by those agents who, after a withdrawal, remain un-matched with a cash dispenser for  $n^* - 1$  periods (hence  $\Theta(0) = (1 - p)^{n^* - 1} \pi$ ). Following this line of reasoning, we compute the population fraction of agents with end-of-period balances equal to  $m^* - \kappa c$  (where  $\kappa = 1, 2, 3, \dots, n^*$ ), which is given by  $\Theta(m^* - \kappa c) = (1 - p)^{\kappa - 1} \pi$ . Table 4 summarizes the distribution of currency holdings in this economy.

A few numeric examples are used to illustrate the workings of the model for three alternative levels of the probability of a cheap withdrawal  $p$ .<sup>13</sup> The basic parametrization assumes the unit length of a period is a day, a daily consumption flow  $c = 1$  and that the utility loss of a costly withdrawal is  $b = 1/24$ , i.e. about the value of one hour of time. We consider values of the nominal interest rate between 0.5 and 20 per cent and an intertemporal discount rate  $\delta = 0.96$  (both variables are expressed in annual rates).

The three distributions of currency holdings originated by this parametrization are shown in Figure 2 for a given net nominal annual interest rate of 10 per cent ( $R = 0.1$ ). When all withdrawals are costly ( $p = 0$ ), the distribution of currency is uniform (the blue bars in the figure), as in the Baumol-Tobin case. As the probability of free withdrawals becomes positive, the distribution becomes unimodal, with the largest mass of agents located at the maximum currency holdings ( $m^* - c$ ), and the remaining mass distributed to the left of this value in a decreasing fashion. It is also

<sup>13</sup>Alvarez and Lippi (2004) provide an analytical characterization of these results.

Figure 2: Currency distribution for different values of  $p$  when  $R = 0.10$



apparent that the size of the optimal balance  $m^*$  is decreasing in  $p$ , as one could intuitively expect. This reflects the fact that the demand for currency in this model has a precautionary component. The reason that motivates an agent to withdraw more currency than what is strictly necessary to pay for today's consumption is to reduce the chances of having to pay the cost  $b$  at some point in the future. A greater value of  $p$ , by reducing the odds of incurring in a costly withdrawal, reduces the agent's desired currency holdings.

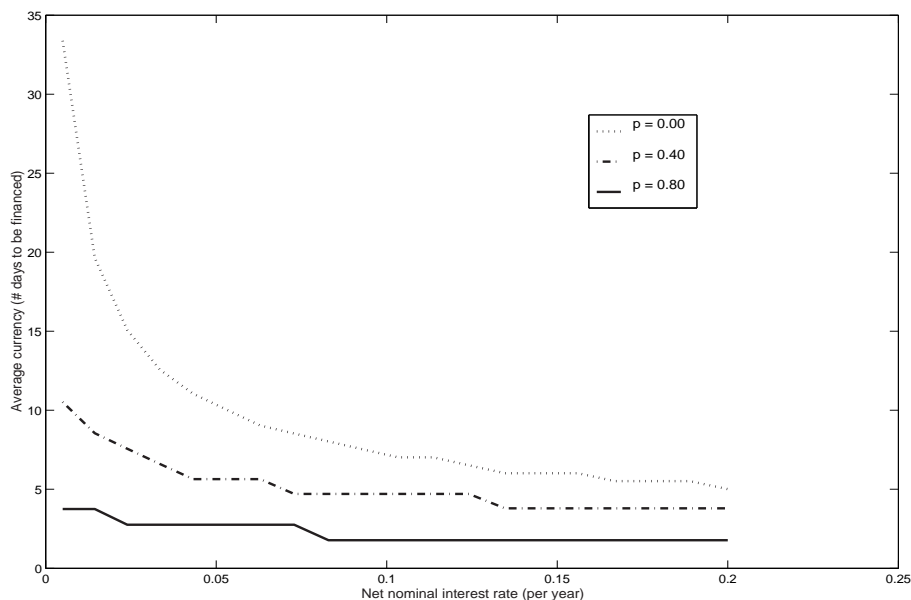
The average currency balance held by each agent in this economy can be readily calculated as:

$$\bar{m} = \sum_i [m_i \cdot \Theta(m_i)] \quad (7)$$

Figure 3 depicts three schedules of the demand for currency corresponding to each of the  $p$  values (low, medium, high) described above. Each curve describes the average currency balance of an agent as a function of the nominal interest rate on deposits,  $R$ . For low values of  $p$ , the curve is downward sloping, with an interest elasticity that is approximately constant at around 0.5. The figure clearly shows that, as the probability of a costly withdrawal decreases, the interest elasticity of the demand for currency gets smaller. The model thus reproduces a qualitative feature of the data, namely that the interest rate elasticity of the demand for currency may decrease with time as the diffusion of ATMs and bank branches allows the agents to avoid the costly withdrawals implied by the inventory model.

Finally, the model can be used to provide a partial equilibrium assessment of the welfare costs of inflation. The natural measure of welfare costs of our model is,

Figure 3: Currency Demand for different values of  $p$



analogously to the Baumol-Tobin approach, the deadweight loss associated to the costly withdrawal:  $b$ . The per-period welfare loss can thus be calculated from the proportion of individuals who bear the cost  $b$  on average in each period:<sup>14</sup>

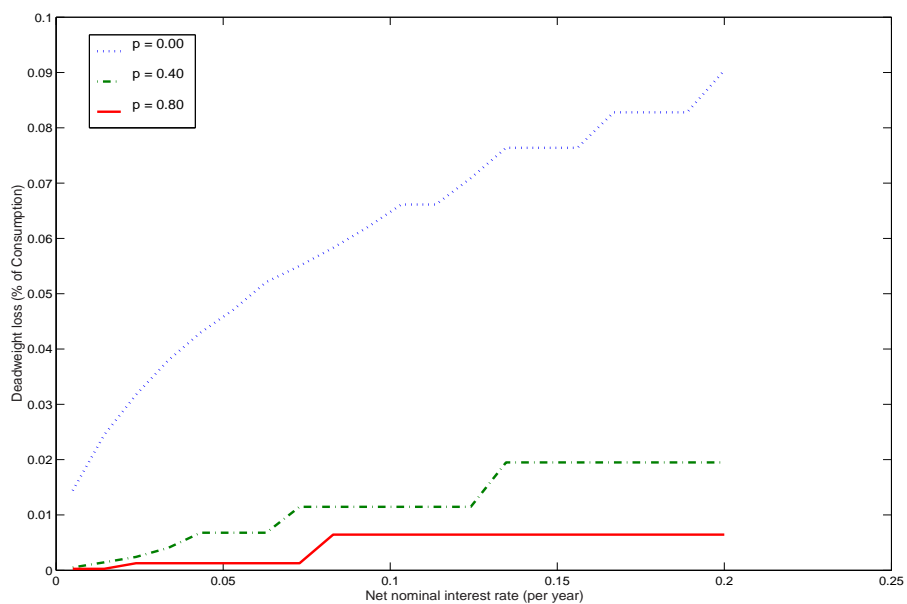
$$l = b(1 - p)^{n^*} \pi \quad (8)$$

Figure 4 shows three profiles for welfare losses as a function of the net nominal interest rate  $R$  corresponding to the three levels of  $p$  considered above. When all withdrawals are costly ( $p = 0$ ), the welfare loss is steeply increasing in  $R$ . The loss associated with a nominal interest of 5 per cent has a magnitude of about 5 per cent of consumption, quite a large number. The figure shows that the welfare costs of inflation (i.e. nominal interest rate) decrease sharply as the cheap withdrawal technology becomes widely available (greater  $p$ ). The welfare costs of a 5 per cent nominal interest rate when  $p = 0.8$  is about 0.1 per cent of consumption.

Finally it is important to notice that in our model the welfare gain achievable from a reduction in inflation crucially depends on the parameter  $p$ . In particular, in a regime characterized by a low level of  $p$  (the dotted line of Figure 4), the reduction in the welfare loss resulting from a fall in inflation is systematically larger than that obtained in a regime characterized by an higher level of  $p$  (e.g. the solid line of Figure 4).

<sup>14</sup>This value can also be interpreted as the expected welfare loss for an individual who lives in this economy.

Figure 4: Welfare Losses for different values of  $p$



## 5 Concluding remarks

[TBW]

## A Data description

This section draws from Attanasio et al. (2002). Information on sample design and response rates of the Survey of Household Income and Wealth can be found in Brandolini and Cannari (1994). In the empirical estimates, all demographic variables are referred to the head of the household. All monetary variables are deflated using the consumer price index, expressed in 1995 lire and then converted to euros.

*ATM ownership.* In each year, respondents report ownership of an ATM card. The surveys also contain information about the use of ATMs. In practice, virtually all those reporting having an ATM card also report using the ATM card.

*Currency.* The following question was asked to household heads in each of the surveys: *What is the average amount of currency usually held in your family?*

*Minimum amount of currency.* The following question was asked to household heads in each of the surveys: *Usually, what is the amount of currency that you have at home before you choose to make a currency withdrawal?*

*Number of withdrawals and average withdrawal.* The following questions were asked to household heads in each of the surveys: *Think about a normal month. How many currency withdrawals are made by you or members of your household? What is the average currency withdrawn?* These questions are asked separately for withdrawals at a bank or a post office, and at an ATM point.

*Consumption.* Consumption is the sum of the expenditure on food consumption, entertainment, education, clothes, medical expenses, housing repairs and additions, and imputed rents. Expenditures on durable goods (vehicles, furniture and appliances, or art objects) are therefore not included in the definition of consumption.

*Deposits.* Include checking accounts, savings accounts, and postal deposits.

*Education of the household head.* This variable is originally coded as follows: no education (zero), completed elementary school (five years), completed junior high school (eight years), completed high school (13 years), completed college (18 years), and graduate education (more than 20 years). The variables actually used in regression are dummies for each of the classes described above.

*Financial wealth.* Sum of checking accounts, savings accounts, postal deposits, government paper, corporate bonds, mutual funds and other managed accounts, and stocks. Data have been corrected to guarantee comparability across time.

*Interest rate on deposits.* We have data on the average nominal interest rate on checking accounts by year (1989, 1991, 1993, 1995, 1998, 2000 and 2002) and provinces (95 up to 1995 and 103 afterwards). The source is the Central Credit Register. This data set is then merged with the 1989-02 SHIW.

*Number of ATM points per province.* Data on the number of ATM points in each year/province are provided by the Supervisory Reports to the Bank of Italy. This data set is then merged with the 1989-02 SHIW.

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