Is There a Role for Small Caps in International Equity Portfolios? The Effects of Variance Risk.*

Massimo GUIDOLIN†
Federal Reserve Bank of St. Louis

Giovanna NICODANO‡
Center for Research on Pensions and Welfare Policies
and University of Turin

January 2005

Abstract

Small capitalization stocks are known to have asymmetric risk across bull and bear markets. This paper investigates how variance risk affects international equity diversification by examining the portfolio choice of a power utility investor confronted with an asset menu that includes (but is not limited to) European and North American small equity portfolios. Stock returns are generated by a multivariate, multi-state regime switching process that is able to account for both non-normality and predictability of stock returns. Non-normality matters for portfolio choice because the investor has a power utility function, implying a preference for positively skewed returns and aversion to kurtosis. We find that small cap portfolios command large optimal weights only when regime switching (and hence variance risk) is ignored. Otherwise a rational investor ought to hold a well-diversified portfolio. However, the availability of small caps substantially increases expected utility, in the order of riskless annualized gains of 3 percent and higher. These findings are robust to a number of modifications concerning the coefficient of relative risk aversion, the investment horizon, short-sale possibilities, and the exact structure of the asset menu.

Keywords: strategic asset allocation; markov-switching; size effects; variance risk.

1. **Introduction**

A number of recent papers have focussed on the asset pricing of small capitalization firms.[1] For instance, Fama and French (1993) report that, over a recent sample, a portfolio comprising small firms paid a return of 0.74 percent per annum in excess of the return on a portfolio composed of large firms. There is also strong

*We thank seminar participants at CERP, University of Turin. Financial support from CeRP and the Italian Research Ministry is gratefully acknowledged. All errors remain our own.

†Research Division, P.O. Box 442, St. Louis, MO 63166, United States.

‡University of Turin, Faculty of Economics, Corso Unione Sovietica, 218bis - 10134 Turin, ITALY. Tel: +39-011.6706073, Fax: +39-011.6706062

[1] The size effect in the US markets is studied by Banz (1981), Reinganum (1981), and Keim (1983) among others. More recently, Pastor (2000) estimates an average monthly premium of 0.17% per month from 1927 to 1996. However, the premiun turns out to be unstable in that a negative premium persists between 1953 and 1965 and then again throughout the 1990s. Moreover, his estimate is -0.13% per month for the period 1982-1996.

international evidence of size effects (Fama and French, 1998).[2] Since these patterns in returns appear to let investors build zero net investment portfolios with positive expected returns, they are commonly held as being incompatible with asset pricing models such as the CAPM and often labeled as 'anomalies'.

At the same time, several papers have recently focused on international optimal equity portfolio allocation under a variety of assumptions concerning the width of the asset menu and/or the salient features for the underlying process generating asset returns, e.g. Ang and Bekaert (2002). To our knowledge, no specific attention has been given to rational portfolio choices involving small capitalization firms, despite the well established finding that small caps yield a higher risk premium than large stocks both in the US and in Europe, i.e. in all major world equity markets. Our paper brings together these two literatures and studies the contribution of small caps to the international diversification of stock portfolios.

Such an effort appears not only to be warranted but also to make a crucial point in the light of the recent developments of the asset pricing literature that struggles to explain the rational foundations of size effects. For instance, the size premium has been recently interpreted as a reward for the lower liquidity of small caps (see (Amihud and Mendelsohn, 1986; Brennan and Subrahmanyam, 1999). If this is the case, then investors with longer horizons (hence unlikely to actively trade the stocks) ought to consider small caps as an attractive diversification vehicle, since they would earn the small cap premium without incurring into large illiquidity costs. However the results in papers like Gompers and Metrick (2001) imply that in practice it is precisely institutional investors such as pension funds and university endowments − which often have longer horizons than individuals and could therefore benefit from the illiquidity of small caps − that have lower ownership shares in both small and low turnover companies. So it appears that there must be something else about small caps that does repel long-horizon investor from buying large chunks of their capital, seizing the corresponding premium. In fact, there is evidence that small caps are highly sensitive to systemic illiquidity and volatility (Amihud, 2002) which are priced risk factors (Pastor and Stambaugh, 2003; Ang, Hodrick, Xing, and Zhang, 2003). In other words, investors may discount small caps because their return is low when aggregate volatility is high, and/or because their volatility is high when aggregate return is low (Acharya and Pedersen, 2004). Our paper is a quantitative exploration of the effects of these properties of US and European small cap stocks for optimal asset allocation choices under realistic specifications for both investors' preferences and the joint stochastic process driving asset returns.

Our paper investigates how *variance risk* − the tendency of small cap returns to be low when aggregate volatility is high and of small cap volatility to be high when 'market' returns are below average − affects investors' portfolio demands by analyzing the composition of international stock portfolios for a power utility (constant relative risk aversion) investor with varying investment horizon. In particular, we document the importance of small caps for optimal portfolios and proceed to calculate the welfare costs of restricting the asset menu to large North American, European and Pacific stocks vs. the unrestricted case in which portfolios are also allowed to include small caps. Importantly, both exercises are separately performed with reference to both the case in which the asset menu is expanded to include US and European small caps, as well a framework in which European small caps are considered in isolation. The case of European small caps is especially important: First, the European small size effect has been almost neglected by the asset pricing literature (with the exception of Annaert et al., 2002) that has instead focussed principally on US

[2] Annaert et al. (2002) argue that the premium on European small caps between 1974-2000 is equal to 16.8 per annum after accounting for transaction costs.

data. It is therefore important to prevent our quantitative estimates of the importance of small caps for portfolio choice to depend entirely on some well-known features of the North American data. Second, as a matter of fact, US small caps experienced an unprecedented performance in the first part of our sample period (between January 1999 and June 2003). Since a concern has been expressed that the size premium may contain long and persistent swings (see e.g. Pástor, 1999 and Guidolin and Timmermann, 2004a), we believe to be of primary importance to obtain broader evidence involving major stock markets, such as the British, German, and French ones (all covered by our MSCI EU-small series).

Traditionally, portfolio choice problems have been studied assuming joint normality of the distribution of asset returns (e.g. Elton, Gruber, Brown, and Goetzmann, 2003), often in a mean-variance framework. However, it is now well known that stock portfolios exhibit non-normal features, such as asymmetric distributions with fat tails and the tendency for returns to be more highly correlated when below the mean (i.e. in bear markets) than when above the mean (in bull markets), see Longin and Solnik (2001).[3] Asymmetries are especially relevant for small caps which suffer more from credit constraints in cyclical downturns due to their lower collateral (Perez-Quiros and Timmerman, 2000; Ang and Chen, 2002). Furthermore, there has long been evidence of predictable returns (Campbell, 1987; Keim and Stambaugh, 1986; Fama and French, 1998; Pesaran and Timmerman, 1995). This is why we represent stock returns through a Markov switching process, that is able to account for both non-normality and predictability.[4] Differently from previous papers, we characterize endogenously the number of regimes,[5] the number of lags and the distribution of the error terms. As recently discussed by Ang and Bekaert (2002), Guidolin and Timmermann (2004b), and Jondeau and Rockinger (2004), possible departures of excess stock returns from joint multivariate normality may be of first-order importance for long-run optimal asset allocation when investors are characterized by power utility, implying a preference for a positively skewed final wealth process (besides for a higher mean) and aversion to the kurtosis (besides the variance) of final wealth.[6]

Using a 1999-2003 weekly MSCI data set for four major equity portfolios (Europe large and small, North America, and Pacific), we find that the joint distribution of international excess stock returns is well captured by a three-state multivariate regime switching model. The three states required to characterize the data are easy to interpret and can be ordered by increasing mean returns (risk premia). In the intermediate regime − that we label *normal* because of its high persistence (average duration) − European small caps returns exhibit an extremely low variance which makes their Sharpe ratio relatively high. Thus a risk averse investor, who is assumed to believe to be in this regime at the time the optimal weights are computed, would invest 100% of her stock portfolio in European small caps for horizons up to two years.

[3]Butler and Joaquin (2002) characterize the consequence of asymmetric correlations in bear and bull markets in an international portfolio diversification framework and show that risk averse investor may want to tilt portfolio weights away from stock markets characterized by the highest correlations during downturns.

[4]Ang and Chen (2002) report that regime switching models may replicate the asymmetries in correlations observed in stock returns data better than GARCH-M and Poisson jump processes. There is now a large body of empirical evidence suggesting that returns on stocks and other financial assets can be captured by this class of models. While a single Gaussian distribution generally does not provide an accurate description of stock returns, the regime switching models that we consider have far better ability to approximate the return distribution and can capture outliers, fat tails and skew. See Guidolin and Timmermann (2005), Turner, Startz and Nelson (1989), and Whitelaw (2001).

[5]Butler and Joaquin (2002) simply define their three regimes (bear, normal, and bull) according to the level of domestic returns. Each regime is exogeneously constrained to collect exactly one-third of the sample.

[6]Non increasing absolute risk aversion is suggested by Arrow (1971) as a desirable property of utility functions. See also Harvey and Siddique (2000).

On the other hand, the possible change in (regime-specific) variance is the highest just for European small caps: in particular, excess returns variance almost doubles when the regime shifts from normal to bear. The high variance 'excursion' across regimes for European small caps is compounded by the presence of high and a negative co-skewness of with other asset returns, which means that potentially the European small variance is high when other excess returns are negative, and European small returns small when the 'market' is highly volatile. similarly, the co-kurtosis of European small excess returns with other excess returns series is rather high − i.e. the variance of the European small class tends to correlate with the variance of other assets. We relate both these features to a general tendency of European small caps to display a disproportionate variance risk. The striking implication is then that a rational investor ought to give European small caps a rather limited weight (as low as 10% only) when her beliefs are initialized in a state of 'ignorance' on the nature of the current regime, i.e. with state probabilities matching their long-run, unconditional counterparts. This shows that higher moments of the return distribution considerably reduce the desirability of small caps for portfolio diversification purposes.[7] These results provide the portfolio choice counterparts of the asset pricing features uncovered in Acharya and Pedersen (2004) and potentially explains the empirical portfolio choice data documented by Gompers and Metrick (2001).

Our results change quantitatively but are *qualitatively* robust when *both* European and North American small caps are introduced in the analysis. In this case, even initializing the experiment to the set of ergodic state probabilities implied by a similar three-state regime switching model for excess stock returns, we obtain that small caps − both North American (with approximate weight of 40-50%) and European (with approximate weight of 20%) − enter optimal long-run portfolios with a weight exceeding 50% for all investment horizons. Moreover, the demand for small caps appears much more stable across regimes (assuming they are initially known), which is easily explained by the finding that both North American small caps and Pacific stocks represent good hedges for European small caps that help stabilize and improve overall portfolio properties outside the normal regime. However, the fact remains that equity portfolios with excellent Sharpe ratio properties may command an optimal weight substantially below 100% because of their bad variance risk properties.

The implication of our paper is that the scarce interest of some important classes of investors − mainly those with long horizons and that are unlikely to incur in high transaction costs − for small capitalization firms may be a rational outcome of the statistical properties of the returns on small caps, in particular of high variance risk (along with illiquidity). Whether and why this represents an equilibrium is beyond the scope of our paper. The claim that it may be rational to limit one's committment to small caps does not imply on the other hand that small caps should be considered irrelevant in international portfolio diversification terms. Even when their weight is moderate, we find that the welfare loss from imposing restrictions on the asset menu that bar the access to small caps may lead to first-order magnitude costs (e.g. 3 percent for long horizons). On the contrary such estimates of the welfare improvements from their use in asset allocation should provide a powerful incentive to policy makers to keep sustaining the development of more liquid and accessible markets where small caps should be traded.

Our work is closely related to Ang and Bekaert (2002), and Guidolin and Timmermann (2004a) who investigate the effects on portfolio diversification of time-varying correlations across markets when regime

[7] On the contrary, it emerges that North American large caps, which have relatively low variance in the bear state, represent the quality stocks that investors look for to maximized their expected utility in the case of downturns.

shifts are accounted for.[8] As is customary in this literature and similarly to these papers, we overlook the analysis of inflation risk, informational differences, and currency hedging costs that – while generally important – may not radically affect rational choices of a large investor who can perfectly hedge currency risk and fails to have a precise reference basket for consumption purposes. Ang and Bekaert work with US, German and UK excess stock returns. They fail to reject the hypothesis that correlations are constant across regimes, and test whether the US portfolio weight in each regime is different from 100%, conditional on assuming – as we do – that regimes are perfectly correlated across countries. Differently from Ang and Bekaert, we focus here on issues of international diversification across small and large capitalization firms. Guidolin and Timmermann (2004a) find strong evidence of time-variation in the joint distribution of US returns on a stock market portfolio and portfolios reflecting size- and value effects. Mean returns, volatilities and correlations between these equity portfolios are found to be driven by underlying regimes that introduce short-run market timing opportunities for investors. However, their asset allocation exercises are limited to menus including Fama and French's (1993) value and size-tracking (HML and SMB, respectively) zero-investment portfolios, while in our paper we are interested in a more standard optimal portfolio exercise in which positive net investments in large and small cap equity portfolios are allowed.

Das and Uppal (2004) study the effects of infrequent price changes on international equity portfolio choice. Equity returns are generated by a multivariate jump diffusion processe where jumps are simultaneous and perfectly correlated across assets. We also assume that regimes are perfectly correlated across stock portfolio returns, but allow for persistence of market regimes. While this prevents us from obtaining their simple analytic results, it allows to compute portfolio allocations conditional on a given regime when the investor anticipates the probability of a regime shift next period. While the ex-ante cost of overlooking shifts is small both in Das and Uppal (2004) and in our paper, it is high when a normal state is prevailing. This observation can be especially important for shorter-term investors, who tailor their portfolio allocation to the state.

This paper is organized as follows. Section 2 presents the portfolio choice problem and gives details on the multivariate regime switching model used in this paper to describe the process of asset returns. Section 3 describes the data, while Section 4 reports our econometric estimates and provides an assessment of their economic implications for optimal portfolio choices. This section reports the most meaningful results of the paper and is organized around three sub-sections, each describing different experiments and reporting homogeneous sets of results. Section 5 performs a number of robustness checks. Section 6 concludes.

2. The Model

2.1. *Regimes in International Equity Returns*

Many papers have found evidence of regimes in the distribution of returns on individual stock portfolios or pairs of these (e.g., Ang and Bekaert (2002), Perez-Quiros and Timmermann (2000), Ramchand and Susmel (1998), Turner, Startz and Nelson (1989), Whitelaw (2001)). Similarly to Guidolin and Timmermann (2004a,c), in this paper we extend this literature to model the joint distribution of a vector of n portfolio returns, $\mathbf{r}_t = (r_{1t}, r_{2t}, ..., r_{nt})'$ over some sample period $t = 1, ..., T$. Suppose that $\mathbf{r}_t$ follows a multivariate regime switching process driven by a common discrete state variable, S_t, which takes integer values between

[8] See also Ramchand and Susmel (1998) and Butler and Joaquin (2002).

1 and k:

$$\mathbf{r}_t = \boldsymbol{\mu}_{s_t} + \sum_{j=1}^{p} \mathbf{A}_{j,s_t} \mathbf{r}_{t-j} + \boldsymbol{\varepsilon}_t. \tag{1}$$

$\boldsymbol{\mu}_{s_t} = (\mu_{1s_t}, ..., \mu_{ns_t})'$ is a vector of mean returns in state s_t, $\mathbf{A}_{j,s_t}$ is an $n \times n$ matrix of autoregressive coefficients at lag j in state s_t and $\boldsymbol{\varepsilon}_t = (\varepsilon_{1t}, ..., \varepsilon_{nt})' \sim N(0, \boldsymbol{\Sigma}_{s_t})$ is the vector of return innovations which are assumed to be jointly normally distributed with zero mean and state-specific covariance matrix $\boldsymbol{\Sigma}_{s_t}$. Innovations to returns are thus drawn from a Gaussian mixture distribution. As pointed out by Marron and Wand (1992), mixtures of normal distributions provide a very flexible family that can be used to approximate numerous other distributions.[9] They can capture skew and kurtosis in a way that is easily characterized as a function of the mean, variance and persistence parameters of the underlying states. They can also accommodate predictability and serial correlation in returns and volatility clustering since they allow the first and second moments to follow a step function driven by shifts in the underlying regime process, c.f. Timmermann (2000).

Moves between states are assumed to be governed by the $k \times k$ transition probability matrix, $\mathbf{P}$, with generic element p_{ji} defined as

$$\Pr(s_t = i | s_{t-1} = j) = p_{ji}, \quad i, j = 1, .., k. \tag{2}$$

Each regime is hence the realization of a first-order Markov chain. We allow S_t to be unobserved and treat it as a latent variable. This captures the idea that investors do not know the true state with certainty even though the time-series of returns $\{\mathbf{r}_j\}_{j=1}^{t}$ provides partial information about the identity of the current state, s_t.

Importantly, the model (1) - (**??**) nests several popular models from the literature as special cases. In the case of a single state, $k = 1$, we obtain a linear VAR model with predictable mean returns provided that there is at least one lag for which $\mathbf{A}_j \neq 0$. Such VAR models have become popular in the literature on optimal asset allocation under predictable risk premia, see e.g. Campbell and Viceira (1999), Kandel and Stambaugh (1996), and Lynch (2001). Absent significant autoregressive terms, the discrete-time equivalent of the i.i.d. Gaussian model adopted as a benchmark by most of the literature on portfolio choices (see e.g. Barberis, 2000 and Brennan and Xia, 2001) is obtained. In the following we conduct a thorough specification process, i.e. we let asset returns data endogeneously determine the number of regimes k, the VAR order p, and possibly the presence of heteroskedasticity in the form of a regime-switching covariance matrix for returns.

In the presence of multiple regimes ($k \geq 2$) our model generally implies various types of predictability in the return distribution. When regimes are persistent and mean returns, $\boldsymbol{\mu}_{s_t}$, differ across states, expected returns vary over time. Similarly, when the covariance matrices, $\boldsymbol{\Sigma}_{s_t}$, differ across states there will be predictability in higher order moments such as volatilities, correlations, skews and tail thickness. Predictability is therefore not confined to mean returns but carries over to the entire return distribution. Effectively the return distribution is calculated as a weighted average of the individual, state-specific distributions using weights that are updated as new return data arrive.

[9] Mixtures of normals can also be viewed as a nonparametric approach if the number of states, k, is allowed to grow with the sample size.

2.2. *The Portfolio Choice Problem*

Consider the 'pure' asset allocation problem for an investor with power utility defined over terminal wealth, W_{t+T}, coefficient of relative risk aversion $\gamma > 0$, and a time horizon T:

$$u(W_{t+T}) = \frac{W_{t+T}^{1-\gamma}}{1-\gamma}, \tag{3}$$

The investor is assumed to maximize expected utility by choosing at time t a portfolio allocation to large stocks, small stocks and bonds, $\boldsymbol{\omega}_t \equiv [\omega_t^l \; \omega_t^s \; \omega_t^b]'$, while $1 - \boldsymbol{\omega}_t' \boldsymbol{\iota}_3$ is invested in riskless, one-month T-bills.[10] For simplicity we assume the investor has unit initial wealth. Portfolio weights are adjusted every $\varphi = \frac{T}{B}$ months at B equally spaced points t, $t + \frac{T}{B}$, $t + 2\frac{T}{B}$, ..., $t + (B-1)\frac{T}{B}$. When $B = 1$, $\varphi = T$ and the investor simply implements a buy-and-hold strategy.

Let $\boldsymbol{\omega}_b$ ($b = 0, 1, ..., B-1$) be the portfolio weights on the risky assets at these rebalancing times. Then $1 - \boldsymbol{\omega}_b' \boldsymbol{\iota}_3$ is the weight on T-bills at time $t + b\frac{T}{B}$ and

$$u(W_B) = \frac{W_{t+T}^{1-\gamma}}{1-\gamma} = \frac{W_B^{1-\gamma}}{1-\gamma}.$$

With regular rebalancing the investor's optimization problem is therefore

$$\begin{gathered} \max_{\{\boldsymbol{\omega}_j\}_{j=0}^{B-1}} \quad E_t \left[\frac{W_B^{1-\gamma}}{1-\gamma} \right] \\ s.t. \; W_{b+1} = W_b \boldsymbol{\omega}_b' \exp(\mathbf{R}_{b+1}) \\ \mathbf{R}_{b+1} \equiv \mathbf{r}_{t_b+1} + \mathbf{r}_{t_b+2} + ... + \mathbf{r}_{t_{b+1}} \end{gathered} \tag{4}$$

for $b = 0$, 1, ..., $B-1$. Here $\exp(\mathbf{R}_{b+1})$ denotes a $n \times 1$ vector of exponentiated, cumulative returns. The associated time (step) b derived utility of wealth function is then:

$$J(W_b, \mathbf{r}_b, \boldsymbol{\theta}_b, \boldsymbol{\pi}_b, t_b) \equiv \max_{\{\boldsymbol{\omega}_j\}_{j=b}^{B-1}} E_{t_b} \left[\frac{W_B^{1-\gamma}}{1-\gamma} \right]. \tag{5}$$

Here $\boldsymbol{\theta}_b = \left(\left\{ \boldsymbol{\mu}_{i,b}, \{\mathbf{A}_{j,i,b}\}_{j=1}^{p}, \boldsymbol{\Sigma}_{i,b} \right\}_{i=1}^{k}, \mathbf{P}_b \right)$ is a vector that collects the parameters of the regime switching model (here b is the time index while i is the regime index) and $\boldsymbol{\pi}_b$ is the (column) vector of probabilities for each of the k possible states, conditional on information at time t_b.

We consider two distinct investment problems. The first rules out short-selling so that $\mathbf{e}_j' \boldsymbol{\omega}_b \in [0,1]$ ($j = 1, ..., n$) and $\boldsymbol{\omega}_b' \boldsymbol{\iota}_3 = 1$. Here $\mathbf{e}_j$ is a $n \times 1$ vector of zeros with a 1 in the jth place and $\boldsymbol{\iota}_3$ is a $n \times 1$ vector of ones. In the second problem short selling is allowed and there are no constraints on $\boldsymbol{\omega}_b$.

Under power utility the Bellman equation conveniently simplifies to[11]

$$J(W_b, \mathbf{r}_b, \boldsymbol{\theta}_b, \boldsymbol{\pi}_b, t_b) = \frac{W_b^{1-\gamma}}{1-\gamma} Q(\mathbf{r}_b, \boldsymbol{\theta}_b, \boldsymbol{\pi}_b, t_b). \tag{6}$$

[10] Following the literature we assume that the return on T-bills is known and constant each period.

[11] Assuming $\gamma \neq 1$. Under logarithmic utility we have $J(W_b, \mathbf{r}_b, \boldsymbol{\theta}_b, \boldsymbol{\pi}_b, t_b) \equiv \ln W_b + Q(\mathbf{r}_b, \boldsymbol{\theta}_b, \boldsymbol{\pi}_b, t_b)$.

c.f. Ingersoll (1987, pp. 240-242). Investors' learning is incorporated in this setup by letting them optimally update their beliefs about the underlying state at each point in time using the formula (c.f. Hamilton (1994, pp. 692-693)):

$$\boldsymbol{\pi}_{b+1}(\hat{\boldsymbol{\theta}}_t) = \frac{\left(\boldsymbol{\pi}_b'(\hat{\boldsymbol{\theta}}_b)\hat{\mathbf{P}}_b^{\varphi}\right)' \odot \boldsymbol{\eta}(\mathbf{r}_{b+1}; \hat{\boldsymbol{\theta}}_b)}{[(\boldsymbol{\pi}_b'(\hat{\boldsymbol{\theta}}_b)\hat{\mathbf{P}}_b^{\varphi})' \odot \boldsymbol{\eta}(\mathbf{r}_{b+1}; \hat{\boldsymbol{\theta}}_b))]'\boldsymbol{\iota}_k}. \tag{7}$$

Here $\odot$ denotes the element-by-element product, $\hat{\mathbf{P}}_t^{\varphi} \equiv \prod_{i=1}^{\varphi} \hat{\mathbf{P}}_t$, and $\boldsymbol{\eta}(\mathbf{y}_{b+1})$ is the $k \times 1$ vector whose jth element gives the density of observation $\mathbf{r}_{b+1}$ in the jth state at time t_{b+1} conditional on $\hat{\boldsymbol{\theta}}_b$:

$$\boldsymbol{\eta}(\mathbf{r}_{b+1}; \hat{\boldsymbol{\theta}}_b) \equiv \begin{bmatrix} f(\mathbf{r}_{b+1}|s_{b+1} = 1, \{\mathbf{r}_{t_b-j}\}_{j=0}^{p-1}; \hat{\boldsymbol{\theta}}_b) \\ f(\mathbf{r}_{b+1}|s_{b+1} = 2, \{\mathbf{r}_{t_b-j}\}_{j=0}^{p-1}; \hat{\boldsymbol{\theta}}_b) \\ \vdots \\ f(\mathbf{r}_{b+1}|s_{b+1} = k, \{\mathbf{r}_{t_b-j}\}_{j=0}^{p-1}; \hat{\boldsymbol{\theta}}_b) \end{bmatrix}$$

$$= \begin{bmatrix} (2\pi)^{-\frac{N}{2}}|\hat{\boldsymbol{\Sigma}}_1^{-1}|^{\frac{1}{2}} \exp\left[-\frac{1}{2}\left(\mathbf{r}_b - \hat{\boldsymbol{\mu}}_1 - \sum_{j=0}^{p-1}\hat{\mathbf{A}}_{1j}\mathbf{r}_{t_b-j}\right)' \hat{\boldsymbol{\Sigma}}_1^{-1}\left(\mathbf{r}_b - \hat{\boldsymbol{\mu}}_1 - \sum_{j=0}^{p-1}\hat{\mathbf{A}}_{1j}\mathbf{r}_{t_b-j}\right)\right] \\ (2\pi)^{-\frac{N}{2}}|\hat{\boldsymbol{\Sigma}}_2^{-1}|^{\frac{1}{2}} \exp\left[-\frac{1}{2}\left(\mathbf{r}_b - \hat{\boldsymbol{\mu}}_2 - \sum_{j=0}^{p-1}\hat{\mathbf{A}}_{2j}\mathbf{r}_{t_b-j}\right)' \hat{\boldsymbol{\Sigma}}_1^{-1}\left(\mathbf{r}_b - \hat{\boldsymbol{\mu}}_2 - \sum_{j=0}^{p-1}\hat{\mathbf{A}}_{2j}\mathbf{r}_{t_b-j}\right)\right] \\ \vdots \\ (2\pi)^{-\frac{N}{2}}|\hat{\boldsymbol{\Sigma}}_k^{-1}|^{\frac{1}{2}} \exp\left[-\frac{1}{2}\left(\mathbf{r}_b - \hat{\boldsymbol{\mu}}_k - \sum_{j=0}^{p-1}\hat{\mathbf{A}}_{kj}\mathbf{r}_{t_b-j}\right)' \hat{\boldsymbol{\Sigma}}_1^{-1}\left(\mathbf{r}_b - \hat{\boldsymbol{\mu}}_k - \sum_{j=0}^{p-1}\hat{\mathbf{A}}_{kj}\mathbf{r}_{t_b-j}\right)\right] \end{bmatrix} \tag{8}$$

Such learning effects can be important since optimal portfolio choices obviously depend not only on future values of asset returns, but also on future perceptions of the likelihood of being in each of the unobservable regimes ($\boldsymbol{\pi}_{t_b+j}$). In practice, the state probabilities are updated in calendar time and not at the frequency of the portfolio rebalancing.

Solving (4) by standard backward induction techniques is, unfortunately, a formidable task (see e.g. the discussion in Barberis, 2000, pp. 256-260) so our approach assumes that investors condition on their parameter estimates, $\hat{\boldsymbol{\theta}}_b$. Under standard discretization techniques the investor first needs to use a sufficiently dense grid of size G, $\{\boldsymbol{\theta}_b^j, \boldsymbol{\pi}_b^j\}_{j=1}^G$ to update $\boldsymbol{\theta}_{b+1}$ and $\boldsymbol{\pi}_{b+1}$ from $\boldsymbol{\theta}_b$ and $\boldsymbol{\pi}_b$. The structure of the model in (1) and the implied regime-dependence of most of the parameters it implies, make it obvious that dozens of parameters are likely to be equired to adequately capture the joint distribution a relatively large vector of international stock portfolios returns. Standard numerical techniques are not feasible for this problem or would at best force us to use a very rough discretization grid, introducing large approximation errors.[12]

Since W_b is known at time t_b, $Q(.)$ simplifies to

$$Q(\mathbf{r}_b, \boldsymbol{\pi}_b, t_b) = \max_{\boldsymbol{\omega}_b} E_{t_b}\left[\left(\frac{W_{b+1}}{W_b}\right)^{1-\gamma} Q\left(\mathbf{r}_{b+1}, \boldsymbol{\pi}_{b+1}, t_{b+1}\right)\right]. \tag{9}$$

Optimal portfolio choices will reflect not only hedging demands for assets due to stochastic shifts in investment opportunities but also a hedging motive caused by changes in investors' beliefs concerning future

[12] In the context of a simpler single state model, Barberis (2000) also assumes that the investor is endowed with limited recursive updating capabilies. Barberis' single-state setup allows him to adopt a Bayesian approach to account for parameter estimation uncertainty. Clearly there is a trade-off between allowing for unobservable regimes and the extent to which Bayesian learning and parameter estimation uncertainty can be considered.

state probabilities $\boldsymbol{\pi}_{t_b+j}$. In fact, the investor's perception of the regime probabilities, $\boldsymbol{\pi}_b$, is the only state variable and the basic recursions can be written as

$$\begin{aligned} Q(\boldsymbol{\pi}_b, t_b) &= \max_{\boldsymbol{\omega}_b} E_{t_b}\left[\left(\frac{W_{b+1}}{W_b}\right)^{1-\gamma} Q\left(\boldsymbol{\pi}_{b+1}, t_{b+1}\right)\right], \\ \boldsymbol{\pi}_{b+1}(\hat{\boldsymbol{\theta}}_t) &= \frac{\left(\boldsymbol{\pi}_b'(\hat{\boldsymbol{\theta}}_b)\hat{\mathbf{P}}_b^{\varphi}\right)' \odot \boldsymbol{\eta}(\mathbf{r}_{b+1}; \hat{\boldsymbol{\theta}}_b)}{[(\boldsymbol{\pi}_b'(\hat{\boldsymbol{\theta}}_b)\hat{\mathbf{P}}_b^{\varphi})' \odot \boldsymbol{\eta}(\mathbf{r}_{b+1}; \hat{\boldsymbol{\theta}}_b))]'\boldsymbol{\iota}_k}. \end{aligned} \tag{10}$$

Therefore we calculate asset allocations under optimal filtering, allowing for unobservable states. In our model investors have to account for revisions in future beliefs $\boldsymbol{\pi}_{b+q}$ $(q \geq 1)$ when determining the optimal asset allocation.

2.3. *Solution Method*

Various approaches have been followed in the literature on portfolio allocation under predictable returns. Barberis (2000) employs simulation methods to study a 'pure' allocation problem without interim consumption. Ang and Bekaert (2002) and Lynch (2001) solve for the optimal asset allocation using Gaussian quadrature methods. Campbell and Viceira (1999) and Campbell, Chan and Viceira (2003) derive approximate analytical solutions for an infinitely lived investor. Finally, some papers have derived closed-form solutions by working in continuous-time, e.g. Kim and Omberg (1996) for the case without interim consumption and Wachter (2002) for the case with interim consumption and complete markets.

Ang and Bekaert (2002) were the first to study asset allocation under regime switching. They consider pairs of international stock market portfolios under regime switching with observable states, so the state variable simplifies to a set of dummy indicators. This setup allows them to apply quadrature methods based on a discretization scheme. Our framework is quite different since we treat the state as unobservable and calculate asset allocations under optimal filtering (7).[13]

To deal with the latent state we use Monte-Carlo methods for integral (expected utility) approximation. For example, for a buy-and-hold investor with $\varphi = T$, we follow Barberis (2000) and approximate the integral in the expected utility functional as follows:

$$\max_{\boldsymbol{\omega}_t} N^{-1} \sum_{n=1}^{N} \left\{ \frac{\left[\boldsymbol{\omega}_t' \exp\left(\sum_{i=1}^{T} \mathbf{r}_{t+i,n}\right)\right]^{1-\gamma}}{1-\gamma} \right\}. \tag{11}$$

Here $\boldsymbol{\omega}_t' \exp\left(\sum_{i=1}^{T} \mathbf{r}_{t+i,n}\right)$ is the portfolio return in the n-th Monte Carlo simulation. Each simulated path of portfolio returns is generated using draws from the model (1)-(**??**) that allow regimes to shift randomly as governed by the transition matrix, $\mathbf{P}$. We use $N = 30,000$ simulations.[14] The appendix provides further details on the numerical techniques employed in the solutions.

[13] Ang and Bekaert (2002) conjecture that when regimes are unobservable, the problem becomes considerably more difficult since - as they correctly point out - all possible sample paths must be considered.

[14] Experiments with similar problems in Guidolin and Timmermann (2004b) indicated that for $n = 4$, a number of simulations N between 20,000 and 40,000 trials delivers satisfactory results in terms of accuracy and minimization of simulation errors.

2.4. *Dynamic Rebalancing*

We next consider the asset allocation of an investor who adjusts portfolio weights every φ months. Once again we numerically solve the Bellman equation. We discretize the compact $[0,1]$ interval that defines the domain of each of the state variables on G equi-distant points. Backward induction methods can then be used as follows.[15] Suppose that $Q(\boldsymbol{\pi}_{b+1}, t_{b+1})$ is known at all points $\boldsymbol{\pi}_{b+1} = \boldsymbol{\pi}^j_{b+1}$, $j = 1, 2, ..., G^{k-1}$. This will be true at time $t_B \equiv t+T$ as $Q(\boldsymbol{\pi}^j_B, t_B) = 1$ for all values of $\boldsymbol{\pi}^j_B$ on the grid. Then we can solve equation (4) to obtain $Q(\boldsymbol{\pi}_b, t_b)$ by choosing ω_b to maximize

$$E_{t_b}\left[\left\{\boldsymbol{\omega}_b' \exp(\mathbf{R}_{b+1,n}(s_b))\right\}^{1-\gamma} Q(\boldsymbol{\pi}^j_{b+1}, t_{b+1})\right].$$

The multiple integral defining the conditional expectation is calculated by Monte Carlo methods. For each $\boldsymbol{\pi}_b = \boldsymbol{\pi}^j_b$, $j = 1, 2, ..., G^{k-1}$ on the grid we draw in calendar time N samples of $\varphi-$period returns $\{\mathbf{R}_{b+1,n}(s_b)\}_{n=1}^N$ from the regime switching model, where $\mathbf{R}_{b+1,n}(s_b) \equiv \sum_{i=1}^{\varphi} \mathbf{r}_{t_b+i,n}(s_b)$. The expectation is then approximated as

$$N^{-1}\sum_{n=1}^{N}\left[\left\{\boldsymbol{\omega}_b' \exp(\mathbf{R}_{b+1,n}(s_b))\right\}^{1-\gamma} Q(\boldsymbol{\pi}^{(j,n)}_{b+1}, t_{b+1})\right],$$

where $\boldsymbol{\pi}^{(j,n)}_{b+1}$ denotes the element $\boldsymbol{\pi}^j_{b+1}$ on the grid used to discretize the state space that is closest to

$$\boldsymbol{\pi}_{b+1,n} = \frac{\left(\boldsymbol{\pi}_b'\hat{\mathbf{P}}_b^{\varphi}\right)' \odot \boldsymbol{\eta}(\mathbf{r}_{b+1}; \hat{\boldsymbol{\theta}}_t)}{[(\boldsymbol{\pi}_b'\hat{\mathbf{P}}_b^{\varphi})' \odot \boldsymbol{\eta}(\mathbf{r}_{b+1}; \hat{\boldsymbol{\theta}}_t))]'\boldsymbol{\iota}_k},$$

using the distance measure $\sum_{i=1}^{k-1} |(\boldsymbol{\pi}^j_{b+1})'\mathbf{e}_i - (\boldsymbol{\pi}_{b+1,n})'\mathbf{e}_i|$. Starting from time t_{B-1}, we work backwards through the B rebalancing points until $\hat{\boldsymbol{\omega}}_t \equiv \hat{\boldsymbol{\omega}}_0$ is derived. The appendix provides further details on the iterative backward solution to the asset allocation problem.

2.5. *Welfare Cost Measures*

In the following we will need at several points one unified way to quantify the utility costs of somehow restricting the investor's asset allocation problem. In these situations, we follow Ang and Bekaert (2002), Ang and Chen (2002), Guidolin and Timmermann (2004a,b), and Lynch (2001), and obtain estimates of the compensatory variation. Call $\hat{\boldsymbol{\omega}}^R_t$ the vector of portfolio weights obtained imposing restrictions on the portfolio choice problem. For instance, $\hat{\boldsymbol{\omega}}^R_t$ may be the vector of optimal asset demands when the investor is forced to ignore the existence of regime shifts. We aim at comparing the investor's expected utility under the unrestricted model – leading to some optimal set of controls $\hat{\boldsymbol{\omega}}_t$ – to that derived assuming the investor is constrained to choose at time t the restricted optimum, $\hat{\boldsymbol{\omega}}^R_t$. Define now $V(W_t, \mathbf{r}_t; \hat{\boldsymbol{\omega}}_t)$ the optimal value function of the unconstrained problem, and $V(W_t, \mathbf{r}_t; \hat{\boldsymbol{\omega}}^R_t)$ the 'constrained' optimal value function. Since a restricted model is by construction a special case of a more general, unrestricted model, the following holds:

$$V(W_t, \mathbf{r}_t; \hat{\boldsymbol{\omega}}^R_t) \leq V(W_t, \mathbf{r}_t; \hat{\boldsymbol{\omega}}_t).$$

[15] For instance, when $G = 11$ the points are defined as 0, 0.1, 0.2, ..., 1 and a $(k-1)$-dimensional grid on a maximum of G^{k-1} points is built. The grid has fewer than G^{k-1} points since each of the points satisfies the constraint $\boldsymbol{\pi}^j_b \boldsymbol{\iota}_k = 1$, $j = 1, 2, ..., G^{k-1}$.

We compute the compensatory premium, π_t^R, that an investor with relative risk aversion coefficient γ is willing to pay to obtain the same expected utility from the constrained and unconstrained asset allocation problems as:

$$\pi_t^R = \left[\frac{V(W_t, \mathbf{r}_t; \hat{\boldsymbol{\omega}}_t)}{V(W_t, \mathbf{r}_t; \hat{\boldsymbol{\omega}}_t^R)}\right]^{\frac{1}{1-\gamma}} - 1. \tag{12}$$

The interpretation is that an investor, if endowed with an initial wealth of $(1 + \pi_t^R)$, would tolerate to be constrained to solve some kind of restricted asset allocation problem leading to $V(W_t, \mathbf{r}_t; \hat{\boldsymbol{\omega}}_t^R) \leq V(W_t, \mathbf{r}_t; \hat{\boldsymbol{\omega}}_t)$ only. Several types of restrictions are analyzed in what follows. For simplicity, we limit ourselves to consider buy-and-hold strategies.[16]

3. Data

We use weekly data from the MSCI total return indices data base for Pacific, North American, European Small Caps and European Large Caps (MSCI Europe Benchmark). Returns on North American Large Caps are computed as a weighted average of the MSCI US Large Cap 300 Index and the D.R.I. Toronto Stock Exchange 300, using as weights the total relative capitalizations of the US and Canada on the world total.[17] In practice, the US large caps index receives a weight of 94.4% vs. a 5.6% for the Canadian index.

All data are total returns series, inclusive of dividends, adjusted for stock splits, etc. Returns are expressed in the local currencies (or weighted averages thereof) as provided by MSCI. This implies a − in fact rather common, see e.g. Ang and Bekaert (2002) and Butler and Joaquin (2002) − assumption that our investor has high enough size and sophistication to be able to fully hedge currency positions, so that her wealth/consumption patterns are unrelated to the dynamics of the exchange rate between the national and foreign currencies.

The sample period is January 1, 1999 - June 30, 2003. A Jan. 1, 1999 starting date for our study is justified by the widespread evidence of substantial portfolio reallocations induced by the disappearing currency risk in the European Monetary Union (Galati and Tsatsaronis, 2001; European Central Bank, 2001). Given the relatively short sample period enforced by the 'Euro structural break' in an asset menu that includes European stock returns, we employ data at a monthly frequency, which anyway guarantee the availability of 234 observations for each of the series. Furthermore, notice that in any event our sample does straddle one complete stock market cycle, capturing both the last months of the stock market rally of 1998-1999, its fall in March 2000, the crash of September 11 2001, and the subsequent, timid recovery. This cycle clearly appears in the time series plots of cumulative total returns in figure 1.

Tables 1 and 2 report basic summary statistics for asset returns. Since about half of our sample is characterized by bear markets, average mean returns are low or even negative for all portfolios under consideration. However − as discussed in the Introduction − small caps represent an exception. In particular, European small caps are characterized by a non-negligible annualized 14.4% positive median return, followed by North American small caps with 12.8% per year.[18] The resulting (median-based)

[16] These clearly give lower bounds to the implied welfare costs, see e.g. Guidolin and Timmermann (2004b). As a matter of fact, under dynamic rebalancing predictability gives an investor a chance to aggressively act upon the information on the state; therefore ignoring predictability when rebalancing is possible implies even higher (sometimes enormous) utility costs.

[17] It is well known that while the MSCI Europe Benchmark index targets mainly large capitalization firms, no equivalent for North America (i.e. US and Canada) is available from MSCI.

[18] The size premium in Europe has been documented by Annaert et al. (2002) using monthly data drawn from large samples

Sharpe ratios for small capitalization firms make them highly appealing in a (static) portfolio perspective that only stresses the first two moments: North American small caps display a 0.59 Sharpe ratio, while European small caps score a stunning 0.89.

On the other hand, table 1 leads immediately to question the validity of an approach that relies only on the sign and/or ratio of the first two moments. First, while small caps have good Sharpe ratios and give positive mean returns, their full-sample estimates of higher order moments may turn disappointing for an investor with standard (and not necessarily mean-variance) prefereces: their skewness is negative, indicating that there are asymmetries in the marginal density of returns that make negative draws more likely than positive ones; their kurtosis is rather high, in excess of a Gaussian benchmark, indicating that extreme realizations are also more likely than in a simple Gaussian i.i.d. framework. Second, opposite remarks apply in fact to other stock portfolios, in particular the North Americal (overall) and Pacific ones: their skewness is positive, which may be seen as an expected utility-enhancing feature by many investors; their kurtosis is rather moderate, essentially close to what a Gaussian i.i.d. framework would imply. These remarks obviously make it pressing to give a quantitative assessment to the main question of this paper: When and how much do higher order moment properties and variance risk (or the lack thereof) matter for optimal asset allocation?

The last two columns of the table reveal that while the evidence for serial correlation in level is limited to European and small caps portfolios, the evidence for volatility clustering – i.e. for the tendency of squared returns to dominate over certain periods – is widespread, which points to the possible need of models that capture heteroskedastic patterns.

Finally, table 2 reports the correlation coefficients of portfolio returns. It is interesting to see that Pacific stock returns have structurally lower correlations (around 0.4 - 0.6 only) with other portfolios than all other pairs covered by the table. In principle, this feature makes of Pacific stocks an excellent hedging tool. All other pairs display correlations in the order of 0.7 - 0.8, which is fairly high but also expected in the light of the evidence in the literature that all major international stock markets are become increasingly prone to synchroneous comovements (e.g. Longin and Solnik, 1995). Unsurprisingly the correlation between North American and North American large caps returns is as high as 0.997, which is easy to understand in the light of the disproporniate market share giant multinationals count for in the US and Canada.

4. International Portfolio Diversification

In this section, we present the main results of the paper on the importance of small caps for international diversification purposes. The section is organized around ?? sub-sections, each devoted to a distinct portfolio choice exercise. In each case, we start by presenting and discussing econometric estimates of multivariate regime switching model and proceed then to calculate and interpret optimal portfolio weights. In particular, the sequence is as follows: first, we set up a benchmark by studying a simpler (and traditional) portfolio problem in which the asset menu is restricted to Pacific, North American large caps and European large caps equity portfolios ($n = 3$). The idea is to familiarize with the econometric framework and with

dating back to 1976. Notice that we use the median of returns as estimators of location: for variables characterized by substantial asymmetries (negative skewness), it seems that the median may be more meaningful than the mean. As a matter of fact, the median European small return is only 1.2% per year, although this must be interpreted in the light of the -0.78 skewness coefficient characterizing this asset class.

the type of asset allocation results it can provide. Next, we allow our investor to buy European small caps ($n = 4$). The choice to expand the asset menu leveraging on European small caps first is justified by their high ratio between median returns and their standard deviation. However, European small caps are also the stock portfolio exhibiting the worst third and fourth moment properties. Hence it is natural to start our investigation of the role of small caps from this portfolio. Finally, we further expand the asset menu and proceed to add to our North American large stocks equity portfolio the MSCI North American small portfolio ($n = 5$). Obviously, this third exercise maximizes the chances for small stocks to play an important role in diversification terms, especially because North American and European small caps appear imperfectly correlated (0.73 from table 2). For the time being we impose no-short sale restrictions; this assumption is removed in Section 5. Similarly, we focus initially on the simpler buy-and-hold case (see e.g. Barberis, 2000) but proceed then to analyze dynamic results in Section 5.

4.1. *Benchmark Results: Restricted Asset Menu*

4.1.1. **Model selection and estimates**

Table 3 reports the results of a model specification search concerning the case in which the asset menu consists of European large caps, North American large, and the Pacific equity portfolios ($n = 3$). In practice we estimate a variety of multivariate regime switching models, including the special cases in which $k = 1$ (no regimes), $p = 0$ (no VAR), and the model has a regime-independent, time-invariant covariance matrix (homoskedasticity).[19] Clearly, $k = 1$ and $p = 0$ result in a IID multivariate Gaussian model that implies the absence of predictability. Otherwise, our model search allows for $k = 1, 2, 3$, and 4, for $p = 0, 1, 2$, and entertains both homoskedastic and heteroskedastic models.

In table 3, three different statistics helpful for model specification purposes are reported. The fourth column shows for all estimated model the likelihood ratio (LR) statistic for the test of $k = 1$ (linearity, in the sense that the model reduces to a homoskedastic Gaussian VAR(p) model). Importantly, in this case there a number of parameters that are not identified (estimable) under the null hypothesis of $k = 1$, so that the even asymptotically the LR test has a non-standard distribution. Similarly to Guidolin and Timmermann (2004b,c) we therefore report corrected, Davis (1977)-type upper bound for the associated p-values − i.e. we adopt a conservative approach that escapes nuisance parameter problems. The table shows that most regime switching models ($k \geq 2$) one can propose do a better job than simpler linear models at capturing the salient features of the joint density of the stock returns data. We conclude that the null of linearity − i.e. the absence of non-linearities in the form of regime switching − in international stock returns data is resoundingly rejected. In the light of previous results (e.g. Ramchand and Susmel, 1998, Ang and Bekaert, 2002, Butler and Joaquin, 2002), this is hardly surprizing.

The fifth and sixth columns of table 3 present two information criteria, the Bayesian (BIC) and Hannan-Quinn (H-Q) statistics. Their purpose in econometric approaches to model specification is to trade-off in-sample fit (in terms of maximized log-likelihood) against parsimony (in terms of number of parameters to be estimated, i.e. residual degrees of freedom) and hence out-of-sample forecasting accuracy. By construction, the best performing model ought to minimize such criteria. Importantly, in this case we obtain that the

[19] Estimation of the model is relatively straightforward and proceeds by optimizing the likelihood function associated with our model. Since the underlying state variable, s_t, is unobserved we treat it as a latent variable and use the EM algorithm to update our parameter estimates, c.f. Hamilton (1989).

same model minimizes both the BIC and the H-Q criteria. This happens in correspondence of a relatively simple and parsimonious (characterized by 20 parameters, in the face of a total of 702 observations) model with $k = 2$, $p = 0$, and regime-dependent covariance matrix. Although it is not customary or trivial to put standard errors around information criteria, one notices that in practice no other model comes even close to our best pick, a MSIH(2,0), in terms of implied BIC and H-Q values.[20]

Table 4 details the MLE parameter estimates in panel B.[21] Looking at the sign and size of the estimated means, we can label the two regimes as 'normal' and 'bear', in the sense that mean returns are negative and relatively large in the second state (in the order of from -0.002 to -0.005 per week, i.e. from -10% to -25% on an annualized basis). However, estimated means are never significant, which is not a new finding in the regime switching class (Ang and Bekaert, 2001, report similar findings). On the opposite, second moment are precisely estimated. This suggests that the two regimes are more accurately characterized by the second moments than by the first ones. The normal/stable state is then a very persistent regime (average duration exceeds 6 months) implying moderate volatilities (roughly 17-18% on annualized basis) and high correlation across pairs of stock indices. The bear/volatile state is less persistent (its average duration is only 9 weeks) and implies much higher volatilities (as high as 40% a year in the case of European large caps).

Figure 2 deepens our understanding of the nonlinearities implied by the estimated model by plotting the full-sample, smoothed state probabilities of the two states over the sample period. In particular, the bear/volatile probabilities peak in correspondence to turbulent periods (e.g. the Winter of 2000, at the peak of the tech bubble of the late 1990s) and to situations of rapidly declining prices (e.g. September 11, 2001 and the drop in prices of the first part of 2002. Overall, the figure gives the impression of international equity markets smoothly cycling between extended (the associated ergodic, long-run probability is 73%) periods of normal, stable markets and shorter (their ergodic frequency is 27%) bursts of volatile and declining prices.

4.1.2. **Implied portfolio weights**

We describe and discuss two sets of portfolio weights estimates. A first exercise computes optimal asset allocation at the end of June 2003 for an investor who has available all the data used so far for estimation purposes and who has obtained the ML parameter estimates we have shown in table 4. Therefore this is a simulation exercise in which the unknown model parameters are calibrated to coincide with the full-sample estimates previously obtained. Clearly, such type of an exercise may prove disappoint to some Readers, as the resulting assessment of the role played by small caps in international diversification may then turn to dramatically depend on the peculiar set of parameter estimates one obtains on the available data. As a result, we supplement the first type of exercise with calculation of real time optimal portfolio weights, each vector being based on a recursively updated set of parameter estimates. Further details are provided in the following.

Figure 3 shows optimal portfolio shares as a function of the investment horizon for a buy-and hold investor who employs parameter estimates obtained as of the end of June 2003. Results for two alternative

[20] Further specification tests − based on implied, unconditional high-order co-moments − are discussed in Section 4.2.2.

[21] Panel A reports as a benchmark the corresponding $k = 1$ model, a simple trivariate IID Gaussian framework in which both means and covariances are time-invariant.

levels of the coefficient of relative risk aversion are reported, $\gamma = 5$ and 10.[22] Each plot concerns one of the available equity portfolio and reports five alternative portfolio schedules: two of them condition on knowledge of the current, initial state of the markets (normal or bear); two other schedules imply the existence of uncertainty on the nature of the regime, by assuming that either all regimes are identically weighted or that their probabilities match their long run, ergodic frequencies (in this case 0.73 and 0.27, for normal and bear states, respectively); one last schedule depicts the optimal choice by a myopic investor who (incorrectly, see table 3) believes that international stock returns are adequately described by a multivariate Gaussian IID model with time invariant means and covariance matrix.[23] Oddly, European large caps would be completely ignored by investor with mild risk aversion for all investment horizons. The only demand for European large stocks is generated for $\gamma = 10$ and starting from the normal state, when the variance of European large stocks is particularly small. Investors should otherwise demand North American large and Pacific stocks in proportions depending both on the assumption on the initial state and on their investment horizon. North American large stocks are more attractive in the short-run and when starting from the bear state (regime 2) when their mean returns are negative but also much higher than all other stock portfolios. However, as the horizon T grows, the weight in North American large stocks generally declines (with the exception of regime 1). Opposite considerations apply to Pacific stocks. As a benchmark, the optimal weight to North American large stocks is 33% for $T = 1$ week and declines to 16% for $T = 2$ years; the difference between 100% and these weights is invested in Pacific equities.

In the normal state, the slopes are reversed: the North American schedule becomes upward sloping while the Pacific one is downward sloping. This is a result of the fact that Pacific stocks have the highest Sharpe ratio in the normal state, although even starting from the normal state the probability of a switch to the bear regime increases over time thus justifying increased caution towards this stocks. Importantly, there are importance differences between the regime-switching portfolio weights and the IID benchmark that ignores predictability, especially for the case of the normal regime when $\gamma = 5$: while the IID weights are 38% in North American large stocks and 62% in Pacific stocks, the regime- and horizon-dependent optimal choices assign much less weight to the former portfolio (the difference is almost 20% at long horizons when the comparison is performed with the steady-state schedule).[24] Both these effects, the presence of well-defined slopes in optimal investment schedules and structural differences between the implications for portfolio choices across the regime switching and IID cases, have been described and interpreted in a different context by Guidolin and Timmermann (2004b).

Figure 4 shows the welfare costs of ignoring regimes and adopting instead a simpler, IID no-predictability benchmark. These estimates are important for two reasons. First, they provide us with a quantitative assessment of the *economic* important of regimes in our asset allocation problem. Second, they attach an 'economic' price to the differences in optimal portfolio weights between regime-switching and IID case we have noticed in Figure 3. Clearly, the welfare costs strongly depend on the assumed initial state as well as

[22]These values span the typical coefficients employed in the literature.

[23]These schedules are completely flat, i.e. they imply that the investment horizon is irrelevant for asset allocation purposes. From the seminal papers by Samuelson (1969) and Merton (1969) it is well-known that this is the case for multivariate IID processes.

[24]Notice that there is no reason to think that the IID schedule ought to be an average of the regime-specific ones: the unconditional (long-run) joint distribution implied by a Gaussian IID and a multivariate regime switching model need not be the same; on the opposite, our specification tests offer evidence that the null of a Gaussian IID model may be rejected, an indication that the unconditional density of the data differs from the one implied by a switching model.

risk-aversion, being higher under moderate values for γ and assuming the investor correctly knows she is initially in regime 1 (normal). However what matters the most is the order of magnitude: while the bear state does not seem to imply particularly high welfare loss,[25] an investor who ignores the initial regime and purely conditions on long-run ergodic probabilities would 'feel' a long-run (for $T = 2$ years) welfare loss of almost 20% of her initial wealth. This estimate is large and stresses that regimes should not be ignored when approaching international diversification problems.

Unfortunately, figure 3 does not easily reveal how sensitive portfolio choice is to the arrival of new information on the prevailing regime. In order to shed light on this issue, we recursively estimate the parameters of the regime switching model with data covering the expanding samples Jan. 1999 - Dec. 2001, Jan. 1999 - first week of Jan. 2002, etc. up to the full sample Jan. 1999 - June 2003 previously employed. For simplicity, we stick to the MSIH(2,0) as the selected regime switching model. Figure 5 plots recursive optimal portfolio weights for $\gamma = 5$ and 10 and for five alternative investment horizons spanning the range 1 week - 2 years previously employed. As a benchmark, we also plot as a solid bold line the IID, myopic asset allocation.[26] The plot clearly shows that our previous remarks are not an artifact of the particular sample period we have selected: The demand for Pacific stocks is relatively stable, both over time and over investment horizons. Even though European large caps have become less and less attractive over time, as the incidence of the bear state has increased, it is clear that their demand is very thin and mostly concentrated on the long-horizon segment. Additionally, we notice considerable variation of optimal weights over time, although most of the changes do appear for short investment horizon, which is consistent with the agent paying attention to the regime-specific density characterizing asset returns. In fact, notice that the two columns of plots are rather similar, although movements are more accentuated for $T = 1$ and 4 weeks and for $\gamma = 5$, when the investor is more sensitive to revisions in estimated means. Finally, optimal regime switching weights are substantially different and more volatile than the myopic ones, especially at short horizons, when rational behavior ought to strongly depend on the perception of the regime.[27]

These results nicely set up the background against which we will proceed to measure the variance and higher moment risk characterizing small caps. When the asset menu is restricted to European and North American large caps only (besides an overall Pacific portfolio), international diversification is substantial both in end-of-sample simulations and in real time experiments, although the highest proportions go to North American large and Pacific equities. The welfare costs of ignoring regime switching (in favor of a non-predictability model) are non-negligible and support our claim that shifts in the first two moments of the joint distribution of returns play a crucial role in portfolio decisions. Next, we proceed to the main question of this paper: should small caps play a major role in international portfolio decisions?

[25] Indeed the bear regime implies portfolio weights that fall very close to the IID ones.

[26] In this case time variation of portfolio weights is simply imputable to the recursive updating of the ML parameter estimates of means, variances, and covariances. On the contrary, in the regime-switching case the time variation is caused both by (regime-specific) parameter updating and by recursive revision of the probability of the prevailing state.

[27] We also compute recursive estimates of the utility costs of ignoring regimes and observe that for long enough horizons the loss oscillates between 1 and 3 percent in annualized terms over most of the sample. Consistently with results in figure 4, peaks of 5 percent (in annual terms) and higher are reached in correspondence to periods characterized as a bear state (e.g. the Summer of 2002).

4.2. *Diversifying with European Small Caps*

4.2.1. **Model selection and estimates**

Table 5 repeats our model specification search with reference to a model with four equity portfolios: European large and small stocks, North American large, and Pacific. Since $n = 4$, even models identical to those estimated in table 3 imply a different number of parameters, as the dimension of the relevant vectors and matrices changes. Also in this case, the evidence against the null of a linear, IID Gaussian model is overwhelming in terms of likelihood ratio tests, even when p-values are adjusted in the manner explained in Section 4.1.1. In this case the information criteria provide contrasting indications: while the H-Q sides for a rather 'expensive' (in terms of number of parameters, 52) two-regime model with a VAR(1) structure, the BIC is undecided between a homoskedastic three-regime model and a heteroskedastic one (in both cases $p = 0$). Given the pervasive evidence of in table 1 (from the Ljung-Box statistic for squared returns) of volatility clustering − which is unsurprising in weekly data − we select the latter MSIH(3,0) model, even though the BIC would favor a homoskedastic one.[28]

Table 6 (panel B) reports ML parameter estimates (as well as standard multivariate IID benchmarks, panel A). The model implies a very intuitive characterization of the three regimes. Interestingly, in this case most of the estimated mean returns are highly significant and their lack of significance does help to identify one of the regimes. So, differently from the model in table 4, the three-state model has now tight implications for both means and covariance matrices in the different regimes. The dominant state, at least in terms of long-run ergodic probabilities, is the second, which we label *normal* regime. In this state, mean returns are essentially zero, volatilities are moderate (around 15% a year for all portfolios), correlations are fairly high. This regime is highly persistent with an average duration in excess of 7 months. Undoubtedly, between 1999 and 2003 markets have spent most of the time in this type of regime, with negligible trends and waiting for news of one sign or the other. In fact, the ergodic long-run probability of the normal regime is 72%.

When the international equity markets are not in a normal state, there are two possibilities. With an (ergodic) probability of 13%, they are in the first, *bear* regime, when mean returns are negative and significant across all portfolios. European large caps seem to be particularly prone to large downturns, as their bear state mean is -5% per week. The bear regime is also a high-volatility state: the variance of all portfolio drastically increases when markets switch from normal to bear states, with peaks of volatility in excess of 21% per year (for European stocks). Interestingly, some of the implied correlations strongly decline when going from regime 2 to 1, with Pacific stocks being almost uncorrelated with both North American and European large caps. However, the persistence of regime 1 is low: starting from a bear state there is only a 22% probability of staying in such a state. As a result, the average duration of such a state is less than 2 weeks. This fits the common wisdom that sharp market declines happen suddenly a tend to span only a few consecutive trading days. Figure helps us once to more to visualize the nature of regime 1 as a bear state: it occurs relatively frequently in our sample (e.g. the week of September 11, 2001 is picked up by this state) but it rarely lasts more than 3 weeks.

The rest of the time (15%), the international equity markets find themselves in a bull regime in which

[28] The MSIH(3,0) model implies the estimation of 48 parameters, although with 936 observations this still amount to a reasonable saturation ratio of $936/48 = 19.5$, i.e. roughly 20 observations per parameter.

mean returns are positive, high, and significant. Also in this case, European large caps are characterized by the highest mean, 3.7% in a week. Once more, volatility is high in the bull regime: this is true for all markets, although the wedge vs. the normal volatilities are extreme for both large caps portfolios, for which the bull volatility is almost twice the normal one (e.g. it is 27% in annualized terms for European large caps). Also in this case, correlations decline when compared to the normal regime. Those implying Pacific stocks become systematically negative, which obviously makes of Pacific equities a great hedge in this regime. Also the bull regime has low persistence, with a 'stayer' probability of 0.29 only and an average duration of less than 2 weeks. Figure 6 confirms these statistical facts but also raise an intruguing suspicion: bull states tend to cluster in the same periods in which bear states appear. The fourth plot in figure 6 make this claim clair: for each period in the sample, we proceed to sum the smoothed probability of regimes 1 and 3. This gives an ex-post estimate of the total probability of being in a high volatility state. We find that although bull and bear regimes are non-persistent, the overall 'high volatility' regime is. It captures periods which are now known as extremely volatile, e.g. early 2001 with the accounting scandals in the US or the Fall of 2001, after the terror attacks to New York City and the market holiday. This is confirmed by the special structure of the estimated transition matrix in table 5: notice that although the 'stayer' probabilities of regimes 1 and 3 are small, they both have rather high probabilities (0.78 and 0.54, respectively) of switching from bear to bull and from bull to bear. This means that extended periods of time may come to be characterized by highly volatile returns, although the signs of the underlying means may be quickly switching back and forth.[29,30]

4.2.2. **Implied portfolio weights**

At face value, it seems that the role of European small caps (henceforth EUSC) in optimal portfolio choices may strongly depend on the regime: EUSC have the best and second-best Sharpe ratios in the normal and bull states (a non-negligible 0.21 and a stellar 0.77, respectively), and display the worst possible combination (negative mean and high variance) in the bear state. However, it is not clear how these contrasting information may influence the choices of some investor who does not observe the state. Furthermore, notice that speculating on the Sharpe ratio to trace back portfolio implication may be grossly incorrect when portfolios have adverse higher-moment properties and/or feature high variance risk.

Figure 7 shows the end-of-sample results of our portfolio calculations as a function of the usual parameters, i.e. risk aversion, investment horizon, and assumptions on the initial regime. The demand for EUSC is essentially 100% independently of the horizon and of γ when the initial state is normal one. Independence of the initial regime is easily justified by the fact that the normal state is highly persistent. The schedule for the bull state provides on the other hand evidence that using the Sharpe ratio may be grossly misleading: in regime 3 EUSC are never demanded as all the weight is given to North American large and Pacific stocks

[29] In practice, table 5 implies that it is rather unlikely that a bear state be followed by normal conditions. Normally markets will 'rebound' by going through 1-3 weeks of bear conditions. The bear state is then more likely to be followed by calmer, highly correlated markets, since the probability of a switch from 3 to 2 is 0.17. Guidolin and Timmermann (2005) report a similar pattern for US equity and bond data and a longer sample period.

[30] Readers may be concerned for the equilibrium justification of the existence of a state with negative stock returns. However – unless all investor have 1-week investment horizons – this does not imply a zero or negligible demand for stocks, as for longer horizons switching to better (or even wonderful) states with zero or positive mean returns is not only possible, but almost sure provided the horizon is long enough.

(plus European large caps for horizons between 1 and 3 weeks). Even though European large stocks do have the best Sharpe ratio in the bull state, the intuition behind the finding that their demand does not survive the test of longer (and more plausible) horizons T, is that while North American large caps still provide a respectable 0.62 Sharpe ratio, Pacific stocks provide their perfect hedge. Unsurprisingly, EUSC fail to enter the optimal portfolio in the bear state (North American and Pacific stocks still dominate the rational decision).

Even more interesting is the result concerning the 'steady-state' allocation to EUSC, when the investor does not know the regime and merely assumes that all regimes are possible, with a probability equal to their long-run unconditional measure. In this case – one may argue the most realistic in applications, since regimes are in fact not observable – EUSC play a very limited role. Their weight is actually zero for short horizons ($T = 1, 2$ weeks) and grows to an unimpressive 10% for longer horizon. Once more, the steady-state portfolio puts almost identical weights on North American and Pacific equities.[31] On the opposite, the IID myopic portfolio would be grossly incorrect, when compared to the steady-state regime switching weights, as it would place high weights on EUSC (87%) and Pacific stocks (13%). Finally, European large caps keep playing a modest role, which is in fact consistent with what we have found in Section 4.1.2. and with the idea that the definition of the regimes implied by the model is not structurally 'perturbed' by the addition of a fourth asset to the menu.

Figure 8 shows also for this case our estimates of the welfare costs of ignoring the regimes implied by asset returns data. Since figure 7 stresses the existence of important differences between regime-switching and IID myopic weights, it is less than surprising to see that the utility loss from ignoring predictability is once more of a first-order magnitude: for instance, a relatively risk-averse ($\gamma = 10$), long-horizon ($T = 2$ years) investor who ignores the nature of the current regime would be still ready to ignore the regime shift if compensated by a lump-sum, riskless increase equal to roughly 4% of her initial wealth. These estimates are of course much larger should we endow the investor with precise information on the nature of the current state (and assuming the information is highly useful, as it is in the bear and bull regimes), as the welfare loss climbs in this case to 15-20% of wealth. Once more, even when the asset menu is enlarged to include EUSC, there seems to be no good reason for ignoring regime shifts in the (conditional) distribution of asset returns.

Once more these results do not seem to entirely depend on the point in time in which they have been performed. In figure 9 we recursively estimate our MSIH(3,0) model and proceed to compute optimal portfolio weights similarly to what we have done in Section 4.1.2. Also in this case, the average (over time) weight assigned to EUSC remains only approximately 39%, while also European large caps acquire substantial importance (26%), followed by North American large and Pacific stocks (23 and 12%).[32] Also in this case, ignoring regime switching would assign way too high a weight on EUSC, in excess of 80% on average (the rest goes to Pacific stocks). As a result, our recursive estimates of the welfare loss of ignoring regime switching (not reported) become huge in certain parts of the sample, exceeding annualized compensatory variation of 5-10% even under the most adverse parameters and investment horizons.

[31] Notice that this conclusion are sensible to the way in which the investor's uncertainty on the current regime is defined. If equal probabilies are assigned to all regimes, we find that the EUSC weight remains around 90%. Therefore it is the shifting of weight away from the bull and bear states that is responsible for our findings.

[32] These are also averaged across investment horizons, although slopes tend to be moderate, consistently with the shapes reported in figure 7. These figures are for the $\gamma = 5$ case. Under high risk aversion they are 36, 23, 26, and 15 percent.

4.2.3. **Making sense of the results: variance risk**

Our simulations find that under realistic assumptions concerning knowledge of the initial state, a rational investor should invest rather a limited proportion of her wealth in EUSC. Tables 7 and 8 report several statistical findings that help us put this result into perspective. Several recent papers (Das and Uppal, 2004; Jondeau and Rockinger, 2003: Guidolin and Timmerman, 2005c) have in fact stressed that investor with power utility functions are not only averse to variance and high correlations between pairs of asset returns − as normally recognized − but also averse to negative skewness and to high co-kurtosis, i.e. to properties of the co-higher order moments of the joint distribution of asset returns.[33] For instance, rational investors will dislike assets the returns of which tend to become highly volatile at times in which the price of most of the other assets (or some reference portfolio of other assets) declines: in this situations, the expected utility of the investor is hurt both by the low expected mean returns as well as by the high variance contributed by the asset under scrutiny.[34] Similarly, investors ought to be suspicious of assets the price of which declines when the volatility of most other assets increases. Investors will also dislike assets the volatility of which increases when most other assets are also volatile. Unless of special configurations of the correlation matrix, this means that overall portfolio returns are bound to become increasingly uncertain. We say that an asset that suffers from this bad co-higher moment properties is subject to high *variance risk*.

Table 7 and 8 clearly pin down these undesirable properties of EUSC. In table 7 we calculate the co-skewness coefficients,

$$S_{i,j,l} \equiv \frac{E[(r_i - E[r_i])(r_j - E[r_j])(r_l - E[r_l])]}{\{E[(r_i - E[r_i])^2]E[(r_j - E[r_j])^2]E[(r_l - E[r_l])^2]\}^{1/2}},$$

between all possible triplets of portfolio returns i, j, l. We do that both with reference to the data (for which simple moments estimates of numerator and denominator can be found) as well for the three-state regime switching model estimated in Section 4.2.1. In the latter case, since closed-form solutions for higher order moments are hard to come by in the multivariate regime switching case, we employ simulations to have a Monte Carlo estimate of the (unconditional) co-moments under regime switching. Based on our discussion, variance risk relates to the cases in which the triplet boils down to a pair, i.e. either $i = j$, or $i = l$, or $j = l$.[35] When $i = j = l$ we will be looking at the standard own skewness coefficient of some portfolio return. In table 7, bold coefficients highlight large estimates in absolute value. Clearly, there is an amazing correspondence between signs and magnitudes of co-skewness coefficients in the data and the unconditional estimates under our estimated regime switching model. Similarly to Das and Uppal (2004) we interpret this result as a sign of correct specification of the model.[36] Furthermore, notice that the co-skewness coefficients $S_{EUSC,EUSC,j}$ are all negative and large in absolute value for all possible js: the volatility of EUSC is indeed higher when each of the other portfolios performs poorly. On the opposite,

[33]Of course, as a special case of this we have also that power utility investors will dislike the own negative skewness and the own (excess) kurtosis of univariate asset returns series (see Guidolin and Timmermann, 2004c and references therein).

[34]This is the case in the model of Vayanos (2004), where fund managers are subject to uncertain withdrawals in bear markets.

[35]We do report also coefficients for genuine triplets, although those are harder to interpret. However, they can still be very useful for assessing potential misspecification problems with a given model.

[36]These findings confirm Ang and Chen's (2002) claim that markov switching models are fit to capture non-normalities in stock returns.

the similar co-skewness coefficients for most other indices (e.g. $S_{EU\_large,EU\_large,j}$ for varying js) are close to zero and sometimes even positive. Worse, a few of the $S_{EUSC,j,j}$ coefficients are also large and negative (when j = Pacific), sign that EUSC may be losing ground exactly when some of the other assets become volatile. Therefore EUSC does display considerable variance risk. On the top of variance risk, from tables 1 and 7 it emerges that EUSC also show high and negative own-skewness (i.e. left asymmetries in the marginal distribution which imply higher probability of below-mean returns), another feature a rational risk-averse investor ought to dislike.

Of course, it might be hard to balance off co-skewness coefficients involving EUSC with different magnitudes or signs. In these cases, it is sensible to calculate quantities similar to those appearing in table 7 for portfolio returns vs. some aggregate portfolio benchmark. For our purposes we use a plain equally weighted portfolio (EW_ptf, 25% in each stock index), although results proved fairly robust to other notions of benchmark portfolio. Once more the match between data- and model-implied coefficients is striking. In particular, in panel A of table 8 we obtain model estimates $S_{EUSC,EUSC,EW\_ptf} = -0.60$ and $S_{EUSC,EW\_ptf,EW\_ptf} = -0.44$, i.e. the variance of EUSC is high when equally weighted returns are below average, and EUSC returns are below average when the variance of the equally weighted portfolio is high. This is another powerful indication of the presence of variance risk plaguing EUSC. For comparison purposes, panel B of table 8 we repeat calculations for European large stocks and obtain negligible (or even positive) coefficients.[37] Therefore while the demand for European large caps is modest because of their low Sharpe ratios (with the exception of the bull state and $T = 1, 2$ weeks), the demand for EUSC is essentially limited by their poor third-moment properties, in particular by their asymmetric marginal density and variance risk. The co-skewness $S_{EUSC,EUSC,EW\_ptf}$ is reminiscent of the covariance between EUSC illiquidity and market return in Acharya and Pedersen (2004). S_{EUSC,EW_ptf,EW_ptf} is akin to the covariance between EUSC return and market illiquidity. Thus, these moments are reminiscent of the risks that are potentially priced in the liquidity CAPM of Acharya and Pedersen (2004). In a sense, we can claim to be providing a portfolio choice rationale for their pricing formula, without resorting to exogenous illiquidity costs that are necessary in a mean-variance framework.

Table 9 performs an operation similar in spirit to table 7, but with reference to the fourth co-moments of equity returns.[38] Once more − although some discrepancies now appear (as expected, as the order of moments grows their accurate estimation becomes more troublesome) − we find a striking correspondence between large co-kurtosis coefficients measured on the data and unconditional coefficients implied by our regime switching model (estimated by simulation). Generally speaking, EUSC tends to have dradful co-kurtosis properties: for instance $K_{EUSC,EUSC,j,j}$ exceeds 2.2 for all js and tends to be higher than all other similar coefficients involving other portfolios, which means that the volatility of EUSC tends to be high exactly when the volatility of all other portolios is high. As already revealed by table 1, also the own-kurtosis of EUSC substantially exceeds a Gaussian reference point of 3. Table 8 confirms that also the model-implied $K_{EUSC,EUSC,EW\_ptf,EW\_pft}$ is 3.3, which is one of the highest among these types of coefficients (see for instance the 3.0 estimate for large caps). $K_{EUSC,EUSC,EW\_ptf,EW\_pft}$ is reminiscent of an indicator of covariance between EUSC illiquidity and market illiquidity. All in all, we have also some evidence that the extreme tails of the marginal density of EUSC tends to be fatter than what found

[37]Results are similar for North American and Pacific portfolios and are available upon request.

[38]Co-kurtosis coefficients are formally defined in the legend to table 9.

for other portfolios and that their volatility might be dangerously co-moving with that of other assets (or the 'market', if well proxied by an equally-weighted benchmark). These higher-moment properties all contribute to make small caps a much less attractive asset class than what one might conjecture based on their sample means (medians) or even their (unconditional) Sharpe ratios.

4.2.4. **Welfare Costs of Ignoring European Small Caps**

Gompers and Metrick (2002) observe that institutions do not usually invest in small caps, because they prefer liquid assets. This is surprising for long-horizon investors, such as pension funds and university endowments, that could profit from their higher Sharpe ratios and diversification potential. Our evidence concerning the high variance risk (and poor higher-order moments) of EUSC may in principle be able explain their neglect as higher moments of their return distribution increase skewness and kurtosis of wealth and hence of expected utility (under a non-linear, power transformation). However: Does this mean that there is no utility loss from restricting the available asset menu to exclude small caps?

We provide a preliminary answer by considering the stark case of EUSC. We consider this exercise extremely informative because we have found that: (1) EUSC ought to have a limited role in optimal portfolio choices despite their seemingly promising full-sample (unconditional) Sharpe ratios; (2) we have indeed discovered that EUSC display bad co-higher moment properties, which we have synthetized writing that *their variance risk is high.* This is case in which we may have a legitimate suspicion that completely eradicating European small caps from the problem will make a tiny damage to the welfare of our investor.

In practice, we proceed to perform compensatory variation calculations similar to those in Sections 4.1.2 and 4.2.2. In this case we identify $V(W_t, \mathbf{r}_t; \hat{\boldsymbol{\omega}}_t^R)$ with the value function under a restricted asset menu that rules our EUSC; on the other hand, $V(W_t, \mathbf{r}_t; \hat{\boldsymbol{\omega}}_t)$ is the value function of the portfolio problem entertained in this Section 4.2.[39] Table 10 reports a number of results. The conclusion is that $-$ in spite of their drawbacks and their limited optimal weight $-$ the loss from constraining the choice to disreagard EUSC would be of a first-order magnitude. Even a moderate 10%, highly-regime dependent weight assigned to an asset may substantially increase the expected utility from a portfolio choice problem. Therefore there is no direct mapping between Gompers and Metrick's remark that small caps seem to be unimportant and the conclusion that their market and marketability are irrelevant. In particular, end-of-sample calculations (panel A, no short sales) show that the annualized utility loss of ignoring EUSC declines with the investment horizons, starts at exceptionally high levels (e.g. 60% a year in the ergodic probability case) to diminish to approximately 3 percent when $T = 2$ years. While for short horizon the assumed coefficient of relative risk aversion seems to be important, as T grows this is not the case. Panel B documents real time results, distinguishing between three different sample (the last two break down Jan. 2002 - June 2003 into two shorter, 9-month periods to have a sense of the stability of the results over time). Interestingly, mean compensatory variations are now even higher, reaching levels in excess of 10 percent per year even at long horizons and in the worst real time sub-samples.[40]

[39] Notice that we cannot simply expect $V(W_t, \mathbf{r}_t; \hat{\boldsymbol{\omega}}_t^R) \leq V(W_t, \mathbf{r}_t; \hat{\boldsymbol{\omega}}_t)$ as the two value functions concern problems solved under different data, statistical models, and parameter estimates. This means that the introduction of EUSC may in principle even hurt an investor! In practice however, we anticipate this will not be the case.

[40] Panel B of table 10 also displays standard deviations for welfare loss estimations. In only ones case the pseudo t-statistic is not significant at a standard 5 percent size. This means that our conclusion that omitting EUSC in real time implies high utility loss does not purely depend on some isolated peaks.

Our conclusion is that − if we interpret these utility loss estimates as upper bounds for the transaction costs that might be consistent with the irrelevance of EUSC − when faced with compensatory variation in excess of 3 percent per year (easily as large as 10 percent per year), it is difficult to think that small caps are not important for international diversification purposes. Although it is well-known that the effective costs paid when transacting on small caps strongly depend on the nature of the trader (e.g. because some of the costs are fixed and can be diluted by transacting relatively large blocks), on tax considerations, and on the frequency of trading, it is unlikely that any sensible estimate of the costs implied by long-run buy-and-hold positions (i.e. revised only every one or two years) may systematically exceed the spectrum of welfare loss estimates we have found. So, modest and strongly regime-dependent optimal weights and high doses of variance risk are still compatible with a claim that small caps are key to a correct and truly expected utility enhancing international portfolio diversification.

4.3. *The Role of Small Caps in an Extended Asset Menu*

We anticipate at this point that at least one basic objection may be standing: Why and when draw general inferences on the issue of the portfolio role of small caps from the EUSC case. Even though we have presented our reasons to start the exercise by augmenting at first the asset menu using EUSC, in this Section we proceed to further expand the menu to also include North American small caps (NASC), besides the North American large portfolio, i.e. $n = 5$. We repeat the usual process of reporting estimation and portfolio outputs separately, even though the general logic and approach remains unchanged vs. Sections 4.1 and 4.2 and therefore many details are now omitted to save space.

4.3.1. **Model selection and estimates**

We perform once more our model selection search using information criteria. An unreported table similar to tables 3 and 5 shows that both the BIC and H-Q criteria keep selecting a three-state heteroskedastic regime switching model with $p = 0$ (MSIH(3,0)), i.e. in which regime switching is responsible of most of the autoregressive structure in levels noticed in table 1. Such a model implies estimation of as many as 66 parameters, although with (5×234) 1,170 observations this gives once more an acceptable saturation ratio of 18.[41]

Table 11 shows ML estimates for both the regime switching (panel B) and a benchmark IID Gaussian models (panel A). The characterization of the three regimes remains essentially identical to Section 4.2.1: the second regime is a normal state in which mean returns are small or even nil (with the exception of NASC, that give a significant mean return of 24 percent per year) and in which volatilities are small (the highest annualized estimate is 17%); correlations are all fairly high, including those involving Pacific stocks. The normal state is highly persistent. The first regime is a bear state in which mean returns are significantly negative and large (e.g. -4% per week for European large caps), volatilities are high (between 25 and 50% higher than in the normal state), and correlations moderate. The third regime is a bull state implying high and significant means, high volatilities and rather modest correlations. Notice that once more all correlations involving Pacific stocks turn negative and some of them are now even significantly

[41]The increase in the number of parameter is essentially cause by the fact that while with $n = 4$ each covariance matrix has 10 free parameters, with $n = 5$ such a number is 15 as 5 additional covariances must be estimated.

so. The bear and bull states are non-persistent; however the structure of the estimated transition matrix is such that the world equity markets may easily enter a high volatility meta-state in which they cycle between regimes 1 and 3 with sustained fluctuations but relatively small chances to settle down to the normal state of affairs. A comparison of tables 11 and 5 shows that the characterization of the states implied by the model is essentially unchanged when adding NASC to the asset menu: this is an important finding that corroborates the validity of our three-state regime switching model. In fact, we omit plots of the smoothed state probabilities that would look almost indistinguishable from those in figure 6 already. The ergodic probabilities of the three regimes remain almost unchanged, 0.17, 0.65, and 0.18, respectively.

4.3.2. **Implied portfolio weights**

Although Section 4.2 has provided abudant examples of how looking at both unconditional and regime-specific Sharpe ratios may be misleading when the investor has power utility preferences and asset returns follow more complicated, nonlinear stochastic processes, we anyway start by stressing how in this metric NASC dominate EUSC and – for that matter – all other equity portfolios. Panel A of table 11 shows that NASC have a Sharpe ratio of 0.06 vs. 0.01 for EUSC and negative ratios for all other portfolios. Figure 10 plots optimal portfolio schedules as a function of risk aversion and once more also reports IID myopic schedules obtained under the estimated model in panel A of table 11. As a reflection of the difference in Sharpe ratios, an investor would indeed invest most of her wealth (58%) in NASC, another important proportion in EUSC (29%), and the remainder (13%) in Pacific stocks, essentially for hedging reasons given the low correlations between Pacific and other portfolio returns. Incredibly, this means that 87% of the overall wealth ought to be invested in small caps, North American and European.

Once more, these portfolio advice would be grossly incorrect, both because it ignores the existence of predictability patterns induced by the structure of the transition matrix, and because it does not take into account variance risk. In fact, the regime switching portfolio schedules in figure 10 contain dramatic departures from the solid, bold lines flattened by the IID myopic assumption: focussing on the case of $\gamma = 5$ and assuming the investor ignores the current regime, her committment to NASC would remain large (and increasing in T) but would be located in the 40-50% range; once more, EUSC imply large amounts of variance risk and poor third- and fourth-order moment properties, which brings their weights down to 15-20%. This means that there is then the opportunity to invest between 30 and 45 percent in other portfolios, mainly the Pacific one. Of course, optimal allocations also turn out to be strongly regime-dependent: for instance, the bear state 1 is highly favorable to NASC investments as these stocks do have the highest Sharpe ratio in this regime, while Pacific stocks provide a relatively good hedge; however as T grows it is clear that the probability of leaving the bear state grows, so that investment schedules revert to their ergodic counterparts. Finally, North American large caps appear with moderate weights only in the extreme regimes 1 and 3, i.e. they should optimally be included in the portfolio only 35% of the time, which quite a modest assessment of their overall importance.

Also in this case, we use a logic similar to table 9 to rationalize these findings and measure variance risk and the effects of higher order moments on portfolio choices. Table 12 performs computations of co-skewness and co-kurtosis coefficients vs. an equally weighted portfolio, both under the available data and under the three-state regime switching model of table 11. In the latter case, simulations are employed to measure unconditional co-moments. We find estimates $S_{NASC,EW\_ptf,EW\_ptf} = -0.29$ and $S_{NASC,NASC,EW\_ptf} =$

-0.25 that approximately fit the sample moments; moreover, $K_{NASC,NASC,EW\_ptf,EW\_ptf} = 2.20$, close to the sample estimate of 2.75.[42] This means that both small cap portfolios we have evidence that their variance increases when the variance of the market is high, that their variance is high when the market is bear, and that their returns are below average when the market is unstable. These properties (along with own kurtosis and skewness) explain why our portfolio results do not completely reflect simple Sharpe ratio-based arguments and why both portfolios receive a much higher weight under the myopic IID calculations than in the plots in figure 10. The estimates in table 12 also make it clear that NASC imply substantially less variance risk than EUSC − hence their higher weights in figure 10.[43]

Figure 11 reports real time results (for $\gamma = 5$) and confirms that our conclusion are far from an artifact of the end-of-sample estimates in table 11: small caps play a substantial role in international diversification although − despite their excellent Sharpe ratio − their variance risk and higher order moment properties reduce somewhat their relevance, for instance from an average 90% myopic IID weight to less than 60% under regime switching, when their complex statistical features are taken into account (see the sixth plot at the bottom of the figure).[44] This wedge of roughly 30 percent in portfolio weight is a prima facie measure of the importance of variance risk, co-skewness and co-kurtosis in international diversification.

We conclude by performing the usual two types of welfare cost calculations. While the utility loss of ignoring predictability remains large (especially when the investor is given knowledge of the current state), the most important result concerns the utility loss of ruling out diversification through small caps, similarly to table 10. Specifically, we identify $V(W_t, \mathbf{r}_t; \hat{\boldsymbol{\omega}}_t^R)$ with the value function under a restricted asset menu that rules our both NASC and EUSC, while $V(W_t, \mathbf{r}_t; \hat{\boldsymbol{\omega}}_t)$ is the value function of the portfolio problem entertained in this Section. Assuming $\gamma = 5$, we find that the utility loss of restricting the asset menu is enormous (in annualized terms) over the short horizon (e.g. 39% for $T = 1$ week) and remains of the same order of magnitude as in Section 4.2.4 over long horizons (e.g. 4.7% for $T = 1$ year and 3.7% for $T = 2$ years). Results are only slightly smaller when risk aversion is set to higher levels (e.g. under $\gamma = 10$ we have 2.4% for $T = 1$ year and 1.5% for $T = 2$ years). Even a welfare loss of 'only' 150 basis points (!) on annualized, riskless basis appears enormous in the light of similar experiments performed in the literature. Once more, we find that the availability of small caps significantly increases expected utility (especially at short and intermediate horizons) through better risk diversification opportunities; moreover, in this Section NASC also entered portfolios with non-negligible and clearly interpretable weights.

[42]The evidence of variance remains strong for EUSC: the regime switching estimates are $S_{EUSC,EW\_ptf,EW\_ptf} = -0.31$, $S_{EUSC,EUSC,EW\_ptf} = -0.28$, and $K_{EUSC,EUSC,EW\_ptf,EW\_ptf} = 3.06$. Notice that these values are different from those in table 9 as they are obtained for a different asset menu and statistical model.

[43]Figure 10 also stresses that in this exercise the coefficient of relative risk aversion has first-order effects. Mainly, we observe a shift of weights from NASC to EUSC, although the overall effect is to make small caps less important (e.g. from 65 to 60% in steady state and for $T = 2$ years). As shown by Guidolin and Timmermann (2004c), as γ increases, progressively more weight is given to higher order moments when making optimal portfolio choices. In this sense, our remarks on the effects of variance risk may then represent a lower bound as principally based on the case $\gamma = 5$.

[44]Averaging across periods and investment horizons, the exact numbers for regime switching weights are: 43% NASC, 13% EUSC, 31% Pacific, 11% European large caps, and 2% North American large caps. Notice the negligible real time weight to be assigned to North American large stocks, a much stronger evidence of their modest role than what obtained by Guidolin and Timmermann (2004b) with reference to US domestic portfolio diversification only.

5. **Robustness Checks**

5.1. *Dynamic Rebalancing*

Section 4 has focussed entirely on the buy-and-hold case, when $\varphi = T$. However $-$ especially given that we have at times entertained long investment horizons up to 2 years $-$ buy-and-hold is inconstitent with the very idea that international equity returns are predictable, in the sense that the predicted probabilities of future states are: a rational investor should change the structure of her portfolio as new information is acquired and beliefs on current and future regimes are recursively revised. This means that dynamic portfolio strategies in which $\varphi < T$ are much more plausible than simpler, buy-and-hold ones. We therefore repeat calculations of portfolio weights from Section 4.2 ($n = 4$, including EUSC) for $\gamma = 5$ and a few alternative assumptions on the rebalancing frequency, $\varphi = 1$, 4, 16, 26 (bi-annual rebalancing), and 52 (i.e. annual rebalancing). In the light of the average durations of regimes 1 and 3 (less than 2 weeks), the cases $\varphi = 1$ and 4 do seem the most plausible ones, although transaction costs and other frictions (unmodeled here) may suggest in practice using higher values of φ.

Table 13 reports optimal portfolio weights.[45] As previously observed by Guidolin and Timmermann (2004a, b), rebalancing hardly changes the main implications found under simpler, buy-and-hold strategies, although it normally makes portfolio weights: (i) much more reactive to the initial state, and (ii) much less sensitive to the investment horizon. This is also the case in our set up: dynamic strategies imply positive and high weights on EUSC only when the investor knows the state is the normal one. In this case the optimal weight is actually extreme, 100%. This makes sense as EUSC have excellent Sharpe ratio in regime 2. Since EUSC's Sharpe ratio is also fairly good in the bull state, a positive demand exists also in this case, even though the proportions are small and limited to very high rebalancing frequencies. The demand for EUSC in the steady-state case is instead rather limited, zero for short horizons up to between 7 and 20% for $T = 2$ years. Clearly, rebalancing possibilities fail to overturn our previous finding that $-$ because of their high variance risk and poor skewness and kurtosis properties $-$ small caps may in practice result much less attractive than what their high Sharpe ratios may lead us to conjecture (as reflected by their 87% IID myopic weight).

5.2. *Long Horizons*

Another sensible objection is that the type of institutional investor studied by Gompers and Metrick (2001) may in fact have horizons much longer than the 2 years ceiling we have used in Section 4. However, we are hesitant before assigning our rational investor with horizons (and hence prediction intervals) that are close or even exceed the length of the data set (Jan. 1999 - June 2003, four and half years long) we have employed for estimation purposes. In any event, figure 12 shows optimal portfolio schedules for the case $\gamma = 5$ and when the investment horizon is extended up to $T = 5$ years. For simplicity, we report results for buy-and-hold portfolio directly comparable to Section 4.2.2, i.e. $n = 4$ and the asset menu includes EUSC. Figure 12 reports and interesting and very intuitive phenomenon already notice by Guidolin and Timmermann (2004b) in other applications: even though short- to medium-term horizon weights may strongly depend on the regime, as T grows all optimal investment schedules tend to converge towards their steady-state counterparts. This makes sense, as the best long-run forecast an agent may form about the

[45] Results are also available for the restricted asset menu case $n = 3$ but are not reported to save space.

future state is simply that all regimes are possible with probabilities identical to their ergodic frequencies. More importantly for our application, figure 12 shows evidence that even for very long horizons compatible with the objectives with large-size institutional investors, the optimal weight assigned to EUSC appears rather limited as a result of their high variance risk. Furthermore, even assuming a strong initial belief in the normal regime 2, for $T = 5$ years we have that the EUSC weight will be at most 55%, since over long periods markets are bound to transition out of the normal state and spend a fair share of time in both bull and bear states (where North Americal large stocks dominate).

5.3. *Short Sales*

Although selling short complex equity indices appears to be much more problematic than shorting individual stocks, the optimal asset allocation literature has developed a tradition of also computing and reporting unconstrained weights, in the sense that both negative positions and positions exceeding 100% of the initial wealth be allowed. We therefore perfom afresh portfolio calculations for the case in which $\mathbf{e}_j'\boldsymbol{\omega}_b \in [-4, 4]$ $(j = 1, ..., n)$ and $\boldsymbol{\omega}_b'\boldsymbol{\iota}_3 = 1$, i.e. weights are allowed to vary between -400 and +400%.[46] Once more, we limit the experiments to the cases of $n = 3$ and $n = 4$, although it would be only tedious to perform unconstrained portfolio calculations also for the $n = 5$ framework.

Figure 13 shows a sample of the resulting optimal portfolio weights. Removing the no-short sale constraint hardly changes our conclusion concerning the desirability of EUSC in international portfolio diversification: while a myopic investor who operates under a (false) IID framework would in fact invest in excess of 130% of her initial wealth in EUSC to exploit their high Sharpe ratio (and would finance this choice by essentially shorting European large stocks), in a regime switching framework the demand for EUSC essentially depend on the initial state. It is still very high under the second, normal regime (in excess of 250%!), but in the most plausible case of unknown regime, the weight is only 20%, not very different from the results of Section 4.2.2. Risk aversion increases this proportion to almost 40%, but it remains true that the highest regime switching weights still keep involving all other assets as well (with the exception of European large caps).[47]

Table 10 actually contains compensatory variation estimates that extend to the case of short sales. In particular the ergodic panel of the table highlights that admitting short sales enhances our estimate of the welfare gains from using small caps in international portfolio diversification, as most estimates (for both $\gamma = 5$ and 10) do increase when short sales are admitted. The worst-case estimate remains a long-run annualized riskless 3 percent, obtained assuming $\gamma = 10$. Therefore also in this experiment, small caps command only moderate portfolio weight but also imply rather large welfare improvements.

[46] As discussed by Barberis (2000) and Kandel and Stambaugh (1996) allowing short-sales (or even $\mathbf{e}_j'\boldsymbol{\omega}_b = 1$) creates problems when returns come from an unbounded density, in the sense that bankruptcy becomes possible and expected utility is not (or ill-) defined for non positive terminal wealth. As discussed in Guidolin and Timmermann (2004a), when Monte Carlo methods are used, this forces the researcher to truncate the distribution from which returns are simulated to avoid instances of bankruptcy. This means that returns are not simulated from the econometric models estimated in Section 4, but from suitably truncated distribution in which the probability mass is redistributed to sum to one. We accomplish the truncation by applying rejection methods that essentially re-draw wealth paths when they imply bankrupcy.

[47] Since differences between IID and regime switching weights widen when short sales are admitted, we generally find that in this case the welfare costs of ignoring regimes are much higher than what reported in Sections 4.1.2 and 4.2.2.

6. **Concluding Comments**

It is well known from the econometrics and finance literature that recurrent regime shifts are often required in order to correctly model the unconditional and conditional multivariate density of asset returns. In this paper we have found further evidence that such a statistical characterization can also be very helpful when modeling international equity returns. Since multivariate regime switching models imply that equity returns may display rich and interesting co-higher moment properties (co-skewness and co-kurtosis), in this paper we have asked whether such properties – that we have collectively labeled *variance risk* – may offer an explanation for a puzzling empirical fact: otherwise sophisticated institutional investors that are likely to possess long investment horizons seem to avoid investing in small capitalization stocks (e.g. Gompers and Metrick, 2002). As these stocks generally offer interesting mean returns and high Sharpe ratios, this observations has recently spurred many interesting attempt of explanation.

In fact we have found in this paper an excellent example of a class of small caps stocks, European small caps, which display an average premium over large caps which is not purely justified by their variance and which fail to enter in massive proportions the optimal portfolio of a rational investor who: (i) has power utility, and (ii) takes into account the existence of predictability of the regimes characterizing the joint distribution of the available data. A powerful display of the existence of variance risk in EUSC is our result that while their optimal weight in a myopic (based on a counterfactual IID Gaussian assumption) portfolio ought to be close to 90%, their optimal weight under regime switching and when the state is unobservable is always less than 20%. However, such a finding does not make small caps irrelevant for portfolio diversification: for instance our estimates of the welfare loss associated with dropping them from the asset menu were often in excess of 5% in a riskless, annualized metric. Even if our paper has ignored transaction costs and other frictions, it is difficult to think that – even when trading on rather illiquid small caps – a large-scale institutional investor might face costs of trading exceeding 500 basis points or more.

These results stand when the asset menu is extended to include a North American small capitalization portfolio, in the sense the in spite of the exceptional average premia and Sharpe ratio that NASC have yielded, we find that under realistic assumptions the combined weights of European and North American small caps fails to exceed 50% and remains at least 30 percent below what we would have obtained assuming a simple IID framework that ignores variance risk and higher-moment properties.

There are several natural extensions and/or completions of our paper. First, our result support an emerging view in the asset pricing literature that the so-called size premium (see Fama and French, 1993) may be not an anomaly but instead just a rational premium associated with the illiquidity and the high variance risk of small caps. As a matter of fact, we have found that the demand for small caps might be severely limited by their variance risk, thus explaining low equilibrium prices and high returns. However, it is clear that our model with regime shifts and power utility preferences is not yet an equilibrium model, while extensions in this direction would be interesting. XXXYYY is a first example in this direction, although only in a mean-variance set up. Second, we have computed estimates of the welfare losses caused by imposing restrictions on the asset menu and concluded that although their optimal proportions are much less than exceptional, small capitalization stocks may still be extremely helpful in international diversification programs. Needless to say, small caps are known to be traded on illiquid and expensive markets. It would be interesting to explicitly introduce transaction costs in our asset allocation exercise and explicitly check the robustness of our results. Balduzzi and Lynch (1999) and Lynch and Balduzzi (2000) show how this could be accomplished in discrete time frameworks akin to ours.

Finally, our results from Section 4.3 have rich implications for the general issue of the limits and benefits of international equity portfolio diversification. For instance, since Tesar and Werner (1995) it has been observed that investor in many countries and particularly in the US tend to grossly under-diversify their equity portfolios. Our paper has shown that regime shifts (especially as they affect the covariance matrices of returns) deeply affect the composition of optimal stock portfolios. North American large caps are observed to be the least volatile asset in bear markets. Following Vayanos (2004), they can easily construed as the the quality asset the investors should flight to in market downturns. Indeed, their portfolio share grows from zero in the normal state to 30% in bear markets. However, flight to quality is not complete in our setting. Other equity portfolios remain in high demand: Pacific stocks allow to dampen portfolio volatility changes since they have low correlation with both North American large stocks in bear states and with both NASC and EUSC in bull states. Thus, the desire to hedge both potential losses and potential increases in portfolio variance preserves the diversification of international portfolios, contrary to results in Ang and Bekaert (2002) where the optimal portfolio may be entirely composed of US stocks. In this sense, our paper seems to call the attention on the dependence of previous claims in the literature on details concerning sample periods or the asset menu. The overall impression is that the results on the high benefits of international diversification may be more robust than recently claimed.

References

[1] Acharya V., and L., Pedersen, 2004, "Asset Pricing with Liquidity Risk", *Journal of Financial Economics,* forthcoming.

[2] Ang A., and G., Bekaert, 2001, International Asset Allocation with Regime Shifts, *Review of Financial Studies*, 15, 1137-1187.

[3] Ang A. and J., Chen, 2002, "Asymmetric Correlations of Equity Portfolios", *Journal of Financial Economics, 63, 443-494.*

[4] Ang, Hodrick, Xing, and Zhang ????

[5] Annaert, J., J., Crombez, B., Spinel, and F., Van Holle, 2002, "Value and Size Effects: Now You See It, Now You Don't", Working Paper 146, Ghent University.

[6] Amihud Y., and H., Mendelson, 1986, "Asset Pricing and the Bid-Ask Spread", *Journal of Financial Economics,* 17, 223-249.

[7] Arrow, K., 1971, *Essays in the Theory of Risk Bearing*, Chicago: Markham Publishing Co.

[8] Balduzzi, P., and A., Lynch, 1999, Transaction Costs and Predictability: Some Utility Cost Calculations, *Journal of Financial Economics,* 52, 47-78.

[9] Banz, R., 1981, "The Relationship Between Returns and Market Value of Common Stocks", *Journal of Financial Economics,* 9, 3-18.

[10] Barberis, N., 2000, "Investing for the Long Run When Returns Are Predictable", *Journal of Finance*, 55, 225-264.

[11] Brennan, M., and Y., Xia, 2001, Assessing Asset Pricing Anomalies, *Review of Financial Studies,* 14, 905–942.

[12] Butler K., and D., Joaquin, 2002, "Are the Gains from International Equity Portfolio Diversification Exaggerated? The Influence of Downside Risk in Bear Markets", *Journal of International Money and Finance,* 21, 981-1011.

[13] Campbell, J., 1987, "Stock Returns and the Term Structure", *Journal of Financial Economics, 18, 373-399.*

[14] Campbell, J., Chan, and L., Viceira, 2003, A Multivariate Model of Strategic Asset Allocation, *Journal of Financial Economics*, 67, 41-80.

[15] Campbell, J., and L., Viceira, 1999, Consumption and Portfolio Decisions when Expected Returns Are Time Varying, *Quarterly Journal of Economics*, 114, 433-495.

[16] Das, S., and R., Uppal, 2004, Systemic Risk and International Portfolio Choice, *Journal of Finance*, forthcoming.

[17] Davies, R., 1977, "Hypothesis Testing When a Nuisance Parameter Is Present Only Under the Alternative", *Biometrika*, 64, 247-254.

[18] Elton E., M., Gruber, S., Brown, and W., Gotzmann, 2003, *Modern Portfolio Theory and Investment Analysis*, John Wiley & Sons.

[19] European Central Bank, 2001, *The Euro Equity Markets.*

[20] Fama, E., and K. French, 1993, "Common Risk Factors in the Returns of Stocks and Bonds", *Journal of Financial Economics,* 33, 3-56.

[21] Fama, E., and K. French, 1998, "Value versus Growth: The International Evidence", *Journal of Finance,* 53, 1975-1999.

[22] Galati G., and K., Tsatsaronis, 2001, "The Impact of the Euro on Europe's Financial Markets", Bank of International Settlements, working paper No. 100.

[23] Gompers P., and A., Metrick, 2002, "Institutional Investors and Equity Prices", *Quarterly Journal of Economics*, 116, 229-260.

[24] Guidolin, M. and A., Timmermann, 2004a, Size and Value Anomalies under Regime Switching, mimeo, Federal Reserve Bank of St. Louis and UCSD.

[25] Guidolin, M. and A., Timmermann, 2004b, Strategic Asset Allocation and Consumption Decisions under Multivariate Regime Switching, mimeo, Federal Reserve Bank of St. Louis and UCSD.

[26] Guidolin, M. and A., Timmermann, 2004c, International Asset Allocation under Regime Switching, Skew and Kurtosis Preferences, mimeo, Federal Reserve Bank of St. Louis and UCSD.

[27] Guidolin, M. and A., Timmermann, 2005, An Econometric Model of Nonlinear Dynamics in the Joint Distribution of Stock and Bond Returns. *Journal of Applied Econometrics,* forthcoming.

[28] Hamilton, J., 1989, A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle, *Econometrica*, 57, 357-384.

[29] Hamilton, J., 1994, *Time Series Analysis.* Princeton University Press.

[30] Harvey, C., and A., Siddique, 2000, "Conditional Skewness in Asset Pricing Tests", *Journal of Finance*, 55, 1263-1295.

[31] Ingersoll, J., 1987, *Theory of Financial Decision Making.* Rowland and Littlefield.

[32] Jondeau, E., and M., Rockinger, 2003, "Optimal Portfolio Allocation Under Higher Moments", mimeo, HEC Lausanne.

[33] Kandel, S., and R., Stambaugh, 1996, On the Predictability of Stock Returns: An Asset Allocation Perspective, *Journal of Finance*, 51, 385-424.

[34] Keim, D., 1983, "Size-Related Anomalies and Stock Return Seasonality: Further Empirical Evidence", *Journal of Financial Economics,* 12, 13-32.

[35] Keim, D., and R., Stambaugh, 1986, Predicting Returns in the Stock and Bond Markets, *Journal of Financial Economics,* 17, 357-390.

[36] Kim, T.,S., and E., Omberg, 1996, "Dynamic Nonmyopic Portfolio Behavior", *Review of Financial Studies,* 9, 141-161.

[37] Longin, F., and B., Solnik, 1995, "Is the Correlation in International Equity Returns Constant:1960-1990?", *Journal of International Money and Finance*, 14, 3-26.

[38] Longin, F., and B., Solnik, 2001, "Correlation Structure of International Equity Markets During Extremely Volatile Periods", *Journal of Finance*, 56, 649-676.

[39] Lynch, A., 2001, "Portfolio Choice and Equity Characteristics: Characterizing the Hedging Demands Induced by Return Predictability", *Journal of Financial Economics*, 62, 67-130.

[40] Lynch, A., and P., Balduzzi, 2000, Transaction Costs and Predictability: The Impact on Portfolio Choice, *Journal of Finance,* 55, 2285-2310.

[41] Keim, D., and R., Stambaugh, 1986, "Predicting Returns in Stock and Bond Markets", *Journal of Financial Economics, 17, 357-390.*

[42] Marron, J., and M., Wand, 1992, Exact Mean Integrated Squared Error, *Annals of Statistics*, 20, 712-736.

[43] Merton, R., 1969, "Lifetime Portfolio Selection: the Continuous-Time Case", *Review of Economics and Statistics*, 51, 247-257.

[44] Pastor, L., 2000, "Portfolio Selection and Asset Pricing Models", *Journal of Finance*, 55, 179-224.

[45] Pastor L., and R., Stambaugh, 2003, "Liquidity Risk and Expected Stock Returns", *Journal of Political Economy*, 111, 643-684

[46] Peres-Quiros G., and A., Timmermann, 2000, "Firm Size and Cyclical Variations in Stock Returns", *Journal of Finance,* 55, 1229-1262.

[47] Pesaran H., and A., Timmerman, 1995, "Predictability of Stock Returns: Robustness and Economic Significance", *Journal of Finance,* 50,1201-1228.

[48] Ramchand, L., and R., Susmel, 1998, "Volatility and Cross Correlation Across Major Stock Markets", *Journal of Empirical Finance*, 5, 397-416.

[49] Reinganum, M., 1981, "Misspecification of Capital Asset Pricing: Empirical Anomalies Based on Earnings Yields and Market Values", *Journal of Financial Economics,* 9, 19-46.

[50] Samuelson, P., 1969, Lifetime Portfolio Selection by Dynamic Stochastic Programming, *Review of Economics and Statistics*, 51, 239-246.

[51] Tesar, L. and Werner, I., 1995, "Home Bias and High Turnover", *Journal of International Money and Finance,* 14, 467-492.

[52] Timmermann, A., 2000, Moments of Markov Switching Models, *Journal of Econometrics*, 96, 75-111.

[53] Turner, C., R., Startz, and C., Nelson, 1989, A Markov Model of Heteroskedasticity, Risk, and Learning in the Stock Market, *Journal of Financial Economics*, 25, 3-22.

[54] Vayanos, D., "Flight to Quality, Flight to Liquidity and the Pricing of Risk", mimeo, Sloan School of Management, M.I.T., 2004.

[55] Wachter, J., 2002, "Portfolio and Consumption Decisions Under Mean-Reverting Returns: An Exact Solution for Complete Markets", *Journal of Financial and Quantitative Analysis*, 37, 63-91.

[56] Whitelaw, R., 2001, "Stock Market Risk and Return: An Equilibrium Approach", *Review of Financial Studies,* 13, 521-548.

Appendix - Backward Solution of the Asset Allocation Problem under Regime Switching

Suppose the optimization problem has been solved backwards at the rebalancing points t_{B-1}, ..., t_{b+1} so that $Q(\boldsymbol{\pi}_{b+1}^{j}, t_{b+1})$ is known for all values $j = 1, 2, ..., G$ on the discretization grid. For each $\boldsymbol{\pi}_b = \boldsymbol{\pi}_b^j$, it is then possible to find $Q(\boldsymbol{\pi}_b^j, t_b)$ at time t_b. For concreteness, consider the case of $p = 0$, i.e. the conditional mean function does not imply any autoregressive structure. Approximating the expectation in the objective function

$$E_{t_b}\left[\left\{\boldsymbol{\omega}_b' \exp\left(\mathbf{R}_{b+1}^{p}\right)\right\}^{1-\gamma} Q(\boldsymbol{\pi}_{b+1}^{j}, t_{b+1})\right]$$

by Monte Carlo methods requires drawing N samples of asset returns $\{\mathbf{R}_{b+1,n}(\boldsymbol{\pi}_b^j)\}_{n=1}^{N}$ from the $(b+1)\varphi$-step-ahead joint density of asset returns conditional on period-t parameter estimates, $\widehat{\boldsymbol{\theta}}_t = (\{\hat{\boldsymbol{\mu}}_{t,i}, \widehat{\boldsymbol{\Sigma}}_{t,i}\}_{i=1}^{k}$, $\hat{\mathbf{P}}_t)$ assuming that $\boldsymbol{\pi}_b^j$ is optimally updated to $\boldsymbol{\pi}_{b+1}(\boldsymbol{\pi}_b^j)$. The algorithm consists of the following steps:

1. For a given $\boldsymbol{\pi}_b^j$ and for each possible future regime $s_{b+1} = j$ calculate the $(b+1)\varphi$-step ahead probability of being in each of the four regimes as $\boldsymbol{\pi}_{b+1|b} = (\boldsymbol{\pi}_b^j)'\hat{\mathbf{P}}_t^{\varphi}$, using that $\hat{\mathbf{P}}_t^{\varphi} \equiv \prod_{j=1}^{\varphi} \hat{\mathbf{P}}_t$ is the φ-step ahead transition matrix.

2. For each possible future regime, s_b, simulate N $\varphi-$period returns $\{\mathbf{R}_{b+1,s}(s_b)\}_{n=1}^{N}$ in calendar time from the regime switching model

$$\mathbf{r}_{t_b+i,n}(s_b) = \hat{\boldsymbol{\mu}}_{s_{t_b+i}} + \boldsymbol{\varepsilon}_{t_b+i,n},$$

   where $\mathbf{R}_{b+1,n}(s_b) \equiv \sum_{i=1}^{\varphi} \mathbf{r}_{t_b+i,n}(s_b)$ and $\boldsymbol{\varepsilon}_{t_b+i,n} \sim N(\mathbf{0}, \widehat{\boldsymbol{\Sigma}}_{s_{t_b+i}})$. At all rebalancing points this simulation allows for stochastic regime switching as governed by the transition matrix $\hat{\mathbf{P}}_t$. For example, if we start in regime 1, between t_b+1 and t_b+2 there is a probability $\hat{p}_{12} \equiv \mathbf{e}_1'\hat{\mathbf{P}}_t\mathbf{e}_2$ of switching to regime 2, and a probability $\hat{p}_{11} \equiv \mathbf{e}_1'\hat{\mathbf{P}}_t\mathbf{e}_1$ of staying in regime 1.

3. Combine the simulated $\varphi-$period asset returns $\{\mathbf{R}_{b+1,n}\}_{n=1}^{N}$ into a random sample of size N, using the probability weights contained in the vector $\boldsymbol{\pi}_b^j$:

$$\mathbf{R}_{b+1,n}(\boldsymbol{\pi}_b^j) = \sum_{i=1}^{4} (\boldsymbol{\pi}_b^j)'\mathbf{e}_i \mathbf{R}_{b+1,n}(s_b = i).$$

4. Update the future regime probabilities perceived by the investor using the formula

$$\boldsymbol{\pi}_{b+1,n}(\boldsymbol{\pi}_b^j) = \frac{\left(\boldsymbol{\pi}_b'(\hat{\boldsymbol{\theta}}_b)\hat{\mathbf{P}}_b^{\varphi}\right)' \odot \boldsymbol{\eta}(\mathbf{r}_{b+1}; \hat{\boldsymbol{\theta}}_b)}{[(\boldsymbol{\pi}_b'(\hat{\boldsymbol{\theta}}_b)\hat{\mathbf{P}}_b^{\varphi})' \odot \boldsymbol{\eta}(\mathbf{r}_{b+1}; \hat{\boldsymbol{\theta}}_b))]'\boldsymbol{\iota}_k}$$

obtaining an $N \times 4$ matrix $\{\boldsymbol{\pi}_{b+1,n}(\boldsymbol{\pi}_b^j)\}_{n=1}^{N}$, each row of which corresponds to a simulated row vector of perceived regime probabilities at time t_{b+1}.

5. For all $n = 1, 2, ..., N$, calculate the value $\tilde{\boldsymbol{\pi}}_{b+1,n}^j$ on the discretization grid ($j = 1, 2, ..., G$) that is closest to $\boldsymbol{\pi}_{b+1,n}(\boldsymbol{\pi}_b^j)$ according to the metric $\sum_{i=1}^{3} |(\boldsymbol{\pi}_{b+1}^j)'\mathbf{e}_i - \boldsymbol{\pi}_{b+1,n}'\mathbf{e}_i|$, i.e.

$$\tilde{\boldsymbol{\pi}}_{b+1,n}^j(\boldsymbol{\pi}_b^j) \equiv \arg\min_{\mathbf{x} \in \boldsymbol{\pi}_{b+1}^j} \sum_{i=1}^{3} |\mathbf{x}'\mathbf{e}_i - \boldsymbol{\pi}_{b+1,n}'\mathbf{e}_i|.$$

Knowledge of the vector $\{\tilde{\boldsymbol{\pi}}_{b+1,n}^j(\boldsymbol{\pi}_b^j)\}_{n=1}^{N}$ allows us to build $\{Q(\boldsymbol{\pi}_{b+1}^{(j,n)}, t_{b+1})\}_{n=1}^{N}$, where $\boldsymbol{\pi}_{b+1}^{(j,n)} \equiv \tilde{\boldsymbol{\pi}}_{b+1,n}^j(\boldsymbol{\pi}_b^j)$ is a function of the assumed vector of regime probabilities $\boldsymbol{\pi}_b^j$.[48]

6. Solve the program

$$\max_{\boldsymbol{\omega}_b(\boldsymbol{\pi}_b^j)} N^{-1} \sum_{n=1}^{N} \left[\left\{ \boldsymbol{\omega}_b' \exp\left(\mathbf{R}_{b+1,n}(\boldsymbol{\pi}_b^j) \right) \right\}^{1-\gamma} Q(\boldsymbol{\pi}_{b+1}^{(j,n)}, t_{b+1}) \right],$$

which for large values of N provides an arbitrarily precise Monte-Carlo approximation of the expectation $E\left[\left\{ \boldsymbol{\omega}_b' \exp\left(\mathbf{R}_{b+1,n}(\boldsymbol{\pi}_b^j) \right) \right\}^{1-\gamma} Q(\boldsymbol{\pi}_{b+1}^j, t_{b+1}) \right]$. The optimal value function corresponding to the optimal portfolio weights $\hat{\boldsymbol{\omega}}_b(\boldsymbol{\pi}_b^j)$ defines $Q(\boldsymbol{\pi}_b^j, t_b)$ for the jth point on the initial grid.

The algorithm is applied to all possible values $\boldsymbol{\pi}_b^j$ on the discretization grid until all values of $Q(\boldsymbol{\pi}_b^j, t_b)$ are obtained for $j = 1, 2, ..., G$. It is then iterated backwards until $t_{b+1} = t + \varphi$. At that stage the algorithm is applied one last time, taking $Q(\boldsymbol{\pi}_{t+\varphi}^j, t + \varphi)$ as given and using one row vector of perceived regime probabilities $\boldsymbol{\pi}_t$, the vector of smoothed probabilities estimated at time t. The resulting vector of optimal portfolio weights $\hat{\boldsymbol{\omega}}_t$ is the desired optimal portfolio allocation at time t, while $Q(\boldsymbol{\pi}_t, t)$ is the optimal value function.

In the simpler buy-and-hold case ($\varphi = T - t$) step 2 is replaced with a simulation routine that for each possible future regime, s_b, simulates N asset returns of length T, $\{\mathbf{R}_{T,s}(s_b)\}_{n=1}^{N}$ from the Markov switching model

$$\mathbf{r}_{t+i,s}(s_b) = \hat{\boldsymbol{\mu}}_{s_{t+i}} + \hat{\boldsymbol{\Sigma}}_{s_{t+i}} \boldsymbol{\varepsilon}_{t+i,n},$$

[48] This step may be avoided when $Q(\boldsymbol{\pi}_{b+1}^j, t_{b+1})$ is constant for all values on the discretization grid. This happens when $t_{b+1} = T$ and implies that the vector of state-dependent optimal portfolio weights determined at step $b+1$ $\{\hat{\boldsymbol{\omega}}_{b+1}(\boldsymbol{\pi}_{b+1}^j)\}_j$ is invariant to changes in $\boldsymbol{\pi}_{b+1}^j$ on the discretization grid.

where $\mathbf{R}_{T,s}(s_b) \equiv \sum_{i=1}^{T} \mathbf{r}_{t+i,n}(s_b)$. In other words, a matrix of monthly returns $\{\mathbf{r}_{t+i,s}(s_b)\}_{n=1}^{S}{}_{i=1}^{T}$ is first generated and then aggregated by summation into N long-term asset returns $\{\mathbf{R}_{T,s}(s_b)\}_{n=1}^{N}$. Steps 1 and 4-6 are, on the other hand, irrelevant in the buy-and-hold case since the objective simplifies to:

$$\max_{\boldsymbol{\omega}_t} N^{-1} \sum_{n=1}^{N} \left\{ \frac{\left[\boldsymbol{\omega}_t' \exp(\mathbf{R}_{T,n})\right]^{1-\gamma}}{1-\gamma} \right\},$$

where $\mathbf{R}_{T,n} = \sum_{i=1}^{4} (\boldsymbol{\pi}_t \mathbf{e}_i) \mathbf{R}_{T,n}(s_b = i)$. The absence of steps that were otherwise needed to solve a fully specified dynamic asset allocation problem makes computations feasible.

Table 1

Summary Statistics for International Stock Returns

The table reports basic moments for weekly equity total return series (including dividends, adjusted for stock splits, etc.) for a few international portfolios. The sample period is January 1999 – June 2003. All returns are expressed in local currencies. Means, medians, and standard deviations are annualized by multiplying weekly moments by 52 and $\sqrt{52}$, respectively. LB(j) denotes the j-th order Ljung-Box statistic.

Portfolio	Mean	Median	St. Dev.	Skewness	Kurtosis	LB(4)	LB(4)-squares
Europe – Large Caps	-0.079	-0.081	0.267	0.186	4.975	20.031**	32.329**
Europe – Small Caps	0.012	0.144	0.161	-0.778	4.815	16.202**	29.975**
North America	-0.041	-0.125	0.206	0.237	3.720	7.120	13.248*
North America – Large Caps	-0.012	-0.114	0.206	0.277	3.673	6.981	12.396*
North America – Small Caps	0.101	0.128	0.218	-0.181	3.384	15.849**	11.374*
Pacific	-0.035	0.006	0.187	-0.086	3.395	3.138	2.667

* denotes 5% significance, ** significance at 1%.

Table 2

Correlation Matrix of International Stock Returns

The table reports linear correlation coefficients for weekly equity total return series (including dividends, adjusted for stock splits, etc.) for a few international portfolios. The sample period is January 1999 – June 2003. All returns are expressed in local currencies.

	EU – Large	EU – Small	North America	North Am. – Large	North Am. – Small	Pacific
EU – Large Caps	1	0.782	0.747	0.754	0.695	0.509
EU – Small Caps		1	0.668	0.672	0.727	0.540
North America			1	0.997	0.795	0.484
North Am. – Large Caps				1	0.795	0.484
North Am. – Small Caps					1	0.427
Pacific						1

Table 3

Model Selection for Returns on European Large Caps, North American Large Caps, and Pacific Equity Portfolios

The table reports estimates for the multivariate Markov switching conditionally heteroskedastic VAR model:

$$\mathbf{r}_t = \boldsymbol{\mu}_{s_t} + \sum_{j=1}^{p} A_{js_t} \mathbf{r}_{t-j} + \boldsymbol{\varepsilon}_t$$

where $\boldsymbol{\mu}_{s_t}$ is the intercept vector in state s_t, A_{js_t} is the matrix of autoregressive coefficients associated with lag $j \geq 1$ in state s_t and $\boldsymbol{\varepsilon}_t = [\varepsilon_{1t}\ \varepsilon_{2t}\ \varepsilon_{3t}]' \sim N(\mathbf{0}, \Sigma_{s_t})$. The unobserved state variable s_t is governed by a first-order Markov chain that can assume *k* distinct values. *p* autoregressive terms are considered. The sample period is January 1999 – June 2003. MSIAH(k,p) stands for Markov Switching Intercept Autoregressive Heteroskedasticity model with k states and p autoregressive lags.

Model (k,p)	Number of parameters	Log-likelihood	LR test for linearity	BIC	Hannan-Quinn
Base model: MSIA(1,0)					
MSIA(1,0)	9	1597.00	NA	-13.4398	-13.5191
MSIA(1,1)	18	1607.08	NA	-13.3736	-13.5327
MSIA(1,2)	27	1610.42	NA	-13.2490	-13.4884
Base model: MSIA(2,0)					
MSIA(2,0)	14	1599.35	4.6972 (0.971)	-13.3433	-13.4666
MSIH(2,0)	20	1639.69	85.3730 (0.000)	**-13.5482**	**-13.7244**
MSIA(2,1)	32	1639.42	64.6713 (0.000)	-13.3236	-13.6064
MSIAH(2,1)	38	1642.85	71.5345 (0.000)	-13.2127	-13.5486
MSIA(2,2)	50	1663.94	107.0428 (0.000)	-13.1705	-13.6137
Base model: MSIA(3,0)					
MSIA(3,0)	21	1628.50	63.0003 (0.000)	-13.4292	-13.6143
MSIH(3,0)	33	1656.26	118.5173 (0.000)	-13.3867	-13.6775
MSIA(3,1)	48	1659.77	105.3812 (0.000)	-13.1240	-13.5483
MSIAH(3,1)	60	1681.08	147.9954 (0.000)	-13.0261	-13.5565
Base model: MSIA(4,0)					
MSIA(4,0)	30	1633.58	73.1593 (0.000)	-13.2628	-13.5272
MSIA(4,1)	66	1684.87	155.5868 (0.000)	-12.9184	-13.5017
MSIH(4,0)	48	1667.89	141.7696 (0.000)	-13.1364	-13.5594
MSIAH(4,1)	84	1703.65	193.1344 (0.000)	-12.6584	-13.4009

Table 4

Estimates of a Two-State Regime Switching Model for Large European, North American Large Caps, and Pacific Equity Portfolios

The table shows estimation results for the regime switching model:

$$r_t = \mu_{s_t} + \varepsilon_t$$

where $\mathbf{r}_t$ is a 3×1 vector collecting weekly total return series, $\boldsymbol{\mu}_{s_t}$ is the intercept vector in state s_t, and $\boldsymbol{\varepsilon}_t = [\varepsilon_{1t}\ \varepsilon_{2t}\ \varepsilon_{3t}]' \sim N(\mathbf{0}, \Sigma_{s_t})$. The sample period is January 1999 – June 2003. The unobservable state s_t is governed by a first-order Markov chain that can assume two values. The first panel refers to the single-state case k = 1. Asterisks attached to correlation coefficients refer to covariance estimates. For mean coefficients and transition probabilities, standard errors are reported in parenthesis.

	Panel A – Single State Model		
	Europe – Large caps	North America Large	Pacific
1. Mean excess return	-0.0015	-0.0008	-0.0007
2. Correlations/Volatilities			
Europe – Large caps	0.0370***		
North America - Large caps	0.7470***	0.0285***	
Pacific	0.5086***	0.4843***	0.0259***
	Panel B – Two State Model		
	Europe – Large caps	North America Large	Pacific
1. Mean excess return			
Normal State	-0.0002	-0.0003	0.0010
Bear State	-0.0046	-0.0020	-0.0048
2. Correlations/Volatilities			
Normal state:			
Europe – Large caps	0.0253***		
North America - Large caps	0.7318***	0.0231***	
Pacific	0.5845***	0.6077***	0.0227***
Bear state:			
Europe – Large caps	0.0559***		
North America - Large caps	0.7681***	0.0387***	
Pacific	0.4675**	0.3607*	0.0321***
3. Transition probabilities	Normal State	Bear State	
Normal State	0.9605***	0.0395	
Bear State	0.1084**	0.8916	

* denotes 10% significance, ** significance at 5%, *** significance at 1%.

Table 5

Selection of Regime Switching Model for Returns on European, North American, and Pacific Equity Portfolios – Effects of Adding European Small Caps

The table reports estimates for the multivariate Markov switching conditionally heteroskedastic VAR model:

$$r_t = \mu_{s_t} + \sum_{j=1}^{p} A_{js_t} r_{t-j} + \varepsilon_t$$

where $\boldsymbol{\mu}_{s_t}$ is the intercept vector in state s_t, A_{js_t} is the matrix of autoregressive coefficients associated with lag $j \geq 1$ in state s_t and $\varepsilon_t = [\varepsilon_{1t}\ \varepsilon_{2t}\ \varepsilon_{3t}\varepsilon_{4t}]' \sim N(0, \Omega_{s_t})$. The unobserved state variable s_t is governed by a first-order Markov chain that can assume *k* distinct values. *p* autoregressive terms are considered. The sample period is January 1999 – June 2003. MSIAH(k,p) stands for Markov Switching Intercept Autoregressive Heteroskedasticity model with k states and p autoregressive lags.

Model (k,p)	Number of parameters	Log-likelihood	LR test for linearity	BIC	Hannan-Quinn
Base model: MSIA(1,0)					
MSIA(1,0)	14	2277.84	NA	-19.1423	-19.2657
MSIA(1,1)	30	2321.25	NA	-19.2230	-19.4882
MSIA(1,2)	46	2325.78	NA	-18.9699	-19.3777
Base model: MSIA(2,0)					
MSIA(2,0)	20	2293.17	30.6600 (0.000)	-19.1335	-19.3097
MSIH(2,0)	30	2309.30	62.9205 (0.000)	-19.0382	-19.3026
MSIA(2,1)	52	2377.18	111.8710 (0.000)	-19.1885	**-19.6281**
MSIAH(2,1)	62	2377.86	99.2137 (0.000)	-18.9002	-19.4482
MSIA(2,2)	84	2379.88	94.2066 (0.000)	-18.4838	-19.2285
Base model: MSIA(3,0)					
MSIA(3,0)	28	2328.06	100.4450 (0.000)	**-19.2452**	-19.4919
MSIH(3,0)	48	2373.25	190.8288 (0.000)	**-19.2252**	-19.5882
MSIA(3,1)	76	2384.26	126.0169 (0.000)	-18.6877	-19.3594
MSIAH(3,1)	96	2432.60	222.6945 (0.000)	-18.6347	-19.4832
Base model: MSIA(4,0)					
MSIA(4,0)	38	2330.84	106.0120 (0.000)	-19.0358	-19.3707
MSIA(4,1)	102	2429.12	215.7464 (0.000)	-18.4645	-19.3661
MSIH(4,0)	68	2393.42	231.1690 (0.000)	-18.8713	-19.4706

Table 6

Estimates of a Three-State Regime Switching Model for European, North American, and Pacific Equity Portfolios – Effects of Adding European Small Caps

The table shows estimation results for the regime switching model:

$$r_t = \boldsymbol{\mu}_{s_t} + \boldsymbol{\varepsilon}_t$$

where $\mathbf{r}_t$ is a 4×1 vector collecting weekly total return series, $\boldsymbol{\mu}_{s_t}$ is the intercept vector in state s_t, and $\boldsymbol{\varepsilon}_t = [\varepsilon_{1t}\ \varepsilon_{2t}\ \varepsilon_{3t}\varepsilon_{4t}]' \sim N(\mathbf{0}, \Sigma_{s_t})$. The unobservable state s_t is governed by a first-order Markov chain that can assume three values. The first panel refers to the single-state case k = 1. Asterisks attached to correlation coefficients refer to covariance estimates. For mean coefficients and transition probabilities, standard errors are reported in parenthesis.

	Europe – Large caps	North America Large	Pacific	Europe – Small caps
Panel A – Single State Model				
1. Mean excess return	-0.0015	-0.0008	-0.0007	0.0002
2. Correlations/Volatilities				
Europe – Large caps	0.0370***			
North America - Large caps	0.7470***	0.0285***		
Pacific	0.5086***	0.4843***	0.0259***	
Europe – Small caps	0.7816***	0.6680***	0.5403***	0.0222***
Panel B – Three State Model				
	Europe – Large caps	North America Large	Pacific	Europe – Small caps
1. Mean excess return				
Bear State	-0.0501***	-0.0268***	-0.0256***	-0.0288***
Normal State	-0.0005	-0.0006	0.0007	0.0032**
Bull State	0.0374***	0.0214***	0.0157***	0.0136***
2. Correlations/Volatilities				
Bear state:				
Europe – Large caps	0.0300***			
North America - Large caps	0.6181***	0.0247***		
Pacific	0.1000	0.0544	0.0277***	
Europe – Small caps	0.7028***	0.5843***	0.5045**	0.0290***
Normal state:				
Europe – Large caps	0.0246***			
North America - Large caps	0.7182***	0.0226***		
Pacific	0.5694***	0.6022***	0.0219***	
Europe – Small caps	0.7062***	0.6369***	0.5759***	0.0153***
Bull state:				
Europe – Large caps	0.0370***			
North America - Large caps	0.5739***	0.0343***		
Pacific	-0.1242	-0.0515	0.0241***	
Europe – Small caps	0.7114***	0.5137***	-0.3581**	0.0177***
3. Transition probabilities	Bear State	Normal State	Bull State	
Bear State	0.2190*	0.0012	0.7798	
Normal State	0.0349	0.9650***	0.0001	
Bull State	0.5416***	0.1698**	0.2886	

* denotes 10% significance, ** significance at 5%, *** significance at 1%.

Table 7

Sample and Implied Co-Skewness Coefficients

The table reports the sample co-skewness coefficients,

$$S_{i,j,l} \equiv \frac{E[(r_i - E[r_i])(r_j - E[r_j])(r_l - E[r_l])]}{\{E[(r_i - E[r_i])^2]E[(r_j - E[r_j])^2]E[(r_l - E[r_l])^2]\}^{1/2}}$$

(*i, j, l* = Europe large, North America large, Pacific, Europe small) and compares them with the co-skewness coefficients implied by a three-state regime switching model:

$$\mathbf{r}_t = \boldsymbol{\mu}_{s_t} + \Sigma_{s_t}\boldsymbol{\varepsilon}_t,$$

where $\boldsymbol{\varepsilon}_t \sim I.I.D.\ N(\mathbf{0}, \mathrm{I}_4)$ is an unpredictable return innovation. Coefficients under multivariate regime switching are calculated employing simulations (50,000 trials) and averaging across simulated samples of length equal to the available data (January 1999 – June 2003). In the table NA stands for 'North American small caps', and Pac for 'Pacific' equity portfolios.

Coeff.	Sample	MS – ergodic
$S_{EU\_large,NA,Pac}$	-0.052	-0.077
S_{EU_large,NA,EU_small}	-0.150	-0.151
$S_{EU\_large,Pac,EU\_small}$	**-0.315**	**-0.308**
S_{NA,Pac,EU_small}	-0.226	-0.202
$S_{EU\_large,EU\_large,NA}$	0.110	0.025
$S_{EU\_large,EU\_large,Pac}$	-0.126	-0.131
$S_{EU\_large,EU\_large,EU\_small}$	-0.167	-0.228
$S_{NA,NA,Pac}$	0.005	-0.007
S_{NA,NA,EU_small}	-0.111	-0.070
S_{NA,NA,EU_large}	0.149	0.095
S_{Pac,Pac,EU_small}	**-0.493**	**-0.341**
S_{Pac,Pac,EU_large}	-0.203	-0.151
$S_{Pac,Pac,NA}$	-0.140	-0.086
$S_{EU\_small,EU\_small,EU\_large}$	**-0.467**	**-0.460**
$S_{EU\_small,EU\_small,NA}$	**-0.367**	**-0.323**
$S_{EU\_small,EU\_small,Pac}$	**-0.525**	**-0.487**
$S_{EU\_large, EU\_large, EU\_large}$	0.186	0.110
$S_{NA,NA,NA}$	0.237	0.170
$S_{Pac,Pac,Pac}$	-0.086	-0.169
$S_{EU\_small, EU\_small, EU\_small}$	**-0.711**	**-0.722**

Table 8

Sample and Implied Co-Skewness and C-Kurtosis Coefficients of European Small Caps vs. an Equally Weighted International Equity Portfolio

The table reports average sample co-skewness coefficients,

$$S_{i,j,l} \equiv \frac{E[(r_i - E[r_i])(r_j - E[r_j])(r_l - E[r_l])]}{\{E[(r_i - E[r_i])^2]E[(r_j - E[r_j])^2]E[(r_l - E[r_l])^2]\}^{1/2}}$$

$$K_{i,j,l,b} \equiv \frac{E[(r_i - E[r_i])(r_j - E[r_j])(r_l - E[r_l])(r_b - E[r_b])]}{\{E[(r_i - E[r_i])^2]E[(r_j - E[r_j])^2]E[(r_l - E[r_l])^2]E[(r_b - E[r_b])^2\}^{1/2}}$$

(*i, j, l* = Europe large, North America large, Pacific, Europe small, Equally weighted portfolio) and compares them with the co-kurtosis coefficients implied by a three-state regime switching model. Coefficients under multivariate regime switching are calculated employing simulations.

	Co-Skewness		Co-Kurtosis	
	Sample	MS - ergodic	Sample	MS - ergodic
	European Small Caps			
$S_{EU\_small,EU\_large,EW\_ptf}$	**-0.300**	**-0.318**	–	–
S_{EU_small,NA,EW_ptf}	-0.235	-0.205	–	–
S_{EU_small,Pac,EW_ptf}	**-0.439**	**-0.381**	–	–
$S_{EU\_small,EU\_small,EW\_ptf}$	**-0.604**	**-0.566**	–	–
$S_{EU\_small,EW\_ptf,EW\_ptf}$	**-0.440**	**-0.412**	–	–
$S_{EU\_small,NA,Pac,EW\_ptf}$	–	–	1.330	1.402
$S_{EU\_small,NA,EU\_large,EW\_ptf}$	–	–	**2.596**	**2.192**
$S_{EU\_small,Pac,EU\_large,EW\_ptf}$	–	–	1.478	1.622
$S_{EU\_small,EU\_small,Pac,EW\_ptf}$	–	–	**2.094**	**2.133**
$S_{EU\_small,EU\_small,NA,EW\_ptf}$	–	–	**2.623**	**2.460**
$S_{EU\_small,EU\_small,EU\_large,EW\_ptf}$	–	–	**3.220**	**2.927**
$S_{EW\_ptf,EW\_ptf,EU\_small,Pac}$	–	–	1.945	**2.133**
$S_{EW\_ptf,EW\_ptf,EU\_small,NA}$	–	–	**2.680**	**2.428**
$S_{EW\_ptf,EW\_ptf,EU\_small,EU\_large}$	–	–	**3.168**	**2.790**
$S_{EW\_ptf,EW\_ptf,EU\_small,EU\_small}$	–	–	**3.460**	**3.262**
$S_{EW\_ptf,EW\_ptf,EU\_ptf,EU\_small}$	–	–	**3.903**	**3.713**
$S_{EU\_small,EU\_small,EU\_small,EU\_ptf}$	–	–	**3.315**	**3.071**
	European Large Caps			
$S_{EU\_large,EU\_small,EW\_ptf}$	**-0.300**	**-0.318**	–	–
S_{EU_large,NA,EW_ptf}	0.037	0.016	–	–
S_{EU_large,Pac,EW_ptf}	-0.190	-0.184	–	–
$S_{EU\_large,EU\_large,EW\_ptf}$	0.031	-0.074	–	–
$S_{EU\_large,EW\_ptf,EW\_ptf}$	-0.097	-0.154	–	–
$S_{EU\_large,NA,Pac,EW\_ptf}$	–	–	1.327	1.335
$S_{EU\_large,NA,EU\_small,EW\_ptf}$	–	–	**2.596**	**2.192**
$S_{EU\_large,Pac,EU\_small,EW\_ptf}$	–	–	1.478	1.622
$S_{EU\_large,EU\_large,NA,EW\_ptf}$	–	–	**3.128**	**2.483**
$S_{EU\_large,EU\_large,Pac,EW\_ptf}$	–	–	1.465	1.616
$S_{EU\_large,EU\_large,EU\_small,EW\_ptf}$	–	–	**3.320**	**2.730**
$S_{EW\_ptf,EW\_ptf,EU\_large,Pac}$	–	–	1.691	1.841
$S_{EW\_ptf,EW\_ptf,EU\_large,NA}$	–	–	**2.997**	**2.521**
$S_{EW\_ptf,EW\_ptf,EU\_large,EU\_small}$	–	–	**3.168**	**2.790**
$S_{EW\_ptf,EW\_ptf,EU\_large,EU\_large}$	–	–	**3.650**	**3.005**
$S_{EW\_ptf,EW\_ptf,EU\_ptf,EU\_large}$	–	–	**3.458**	**3.021**

$S_{EU\_large,EU\_large,EU\_large,EU\_ptf}$	–	–	**4.119**	**3.190**

Table 9

Sample and Implied Co-Kurtosis Coefficients

The table reports the sample co-kurtosis coefficients,

$$K_{i,j,l,b} \equiv \frac{E[(r_i - E[r_i])(r_j - E[r_j])(r_l - E[r_l])(r_b - E[r_b])]}{\{E[(r_i - E[r_i])^2]E[(r_j - E[r_j])^2]E[(r_l - E[r_l])^2]E[(r_b - E[r_b])^2\}^{1/2}}$$

(*i, j, l* = Europe large, North America large, Pacific, Europe small) and compares them with the co-kurtosis coefficients implied by a three-state regime switching model:

$$\mathbf{r}_t = \boldsymbol{\mu}_{s_t} + \Sigma_{s_t}\boldsymbol{\varepsilon}_t,$$

where $\boldsymbol{\varepsilon}_t \sim I.I.D.\ N(\mathbf{0}, I_4)$ is an unpredictable return innovation. Coefficients under multivariate regime switching are calculated employing simulations (50,000 trials) and averaging across simulated samples of length equal to the available data (January 1999 – June 2003). In the table NA stands for 'North American small caps', and Pac for 'Pacific' equity portfolios.

Coeff.	Sample	MS – erg.	Coeff.	Sample	MS – erg.
$K_{EU\_large,NA,Pac,EU\_small}$	1.025	1.093	$K_{Pac,Pac,EU\_small,EU\_small}$	**2.193**	**2.080**
$K_{EU\_large, EU\_large,NA, EU\_small}$	**2.725**	**2.125**	$K_{EU\_large,EU\_large,EU\_large,NA}$	**3.450**	**2.586**
$K_{EU\_large, EU\_large,NA, Pac}$	1.137	1.123	$K_{EU\_large,EU\_large,EU\_large,Pac}$	1.354	1.457
$K_{EU\_large, EU\_large,Pac, EU\_small}$	1.234	1.377	$K_{EU\_large,EU\_large,EU\_large,EU\_small}$	**3.727**	**2.847**
$K_{NA,NA,EU\_large,Pac}$	1.215	1.131	$K_{NA,NA,NA,Pac}$	1.549	1.381
$K_{NA,NA,EU\_large,EU\_small}$	**2.395**	**2.002**	K_{NA,NA,NA,EU_small}	**2.463**	**2.212**
K_{NA,NA,Pac,EU_small}	1.086	1.129	$K_{Pac,EU\_small,EU\_small,EU\_small}$	1.922	1.852
$K_{Pac,Pac,EU\_large,EU\_small}$	1.330	1.496	K_{NA,NA,NA,EU_large}	**2.955**	**2.536**
$K_{Pac,Pac,EU\_large,NA}$	1.243	1.273	K_{Pac,Pac,Pac,EU_large}	1.469	1.606
$K_{Pac,Pac,EU\_large,NA}$	1.117	1.221	$K_{EU\_small,EU\_small,EU\_small,EU\_large}$	**3.508**	**3.290**
$K_{EU\_small,EU\_small,EU\_large,NA}$	**2.505**	**2.191**	$K_{Pac,Pac,Pac,NA}$	1.394	1.455
$K_{EU\_small,EU\_small,EU\_large,Pac}$	1.517	1.655	$K_{EU\_small,EU\_small,EU\_small,NA}$	**2.760**	**2.665**
$K_{EU\_small,EU\_small,,NA,Pac}$	1.246	1.376	$K_{EU\_small,EU\_small,EU\_small,Pac}$	**2.437**	**2.363**
$K_{EU\_large,EU\_large,NA,NA}$	**2.985**	**2.412**	$K_{EU\_large,EU\_large,EU\_large,EU\_large}$	**4.975**	3.646
$K_{EU\_large,EU\_large,Pac,Pac}$	1.229	1.562	$K_{NA,NA,NA,NA}$	3.689	3.434
$K_{EU\_large,EU\_large,EU\_small,EU\_small}$	**3.324**	**2.856**	$K_{Pac,Pac,Pac,Pac}$	3.395	3.258
$K_{NA,NA,Pac,Pac}$	1.510	1.495	$K_{EU\_small,EU\_small,EU\_small,EU\_small}$	**4.815**	**4.758**
$K_{NA,NA,EU\_small,EU\_small}$	**2.369**	**2.198**			

Table 10

Annualized Percentage Welfare Costs from Ignoring European Small Caps

The table reports the (annualized, percentage) compensatory variation from restricting the asset menu to exclude European small caps. The table shows welfare costs as a function of the investment horizon; calculations were performed under a variety of assumptions concerning the coefficient of relative risk aversion and the possibility to short-sell. The investor is assumed to have a simple buy-and-hold objective. Panel A and B present results for end-of-sample simulations (when assumptions are imposed on the regime probabilities) and for real-time portfolios, respectively.

	Investment Horizon T (in weeks)					
	T=1	**T=4**	**T=12**	**T=24**	**T=52**	**T=104**
Panel A – Simulations (based on end-of-sample parameter estimates)						
	Equal probabilities					
γ =5	34.94	11.87	5.92	4.38	4.33	2.96
γ =10	3.57	1.86	1.24	1.06	1.03	0.74
γ =5, short sales allowed	42.42	19.42	12.55	11.77	11.97	7.77
γ =10, short sales allowed	3.53	1.43	0.79	0.61	0.53	0.41
	Ergodic Probabilities					
γ =5	60.11	10.55	5.79	4.63	4.62	3.17
γ =10	8.40	2.19	1.18	0.97	0.88	0.69
γ =5, short sales allowed	77.90	9.95	5.68	4.95	5.02	3.51
γ =10, short sales allowed	41.81	9.86	5.21	4.26	3.89	3.00
Panel B – Real time recursive results						
	Full sample (Jan. 2002 – June 2003)					
Mean	40.31	21.21	22.11	22.86	23.79	16.26
Median	39.98	26.43	24.39	22.71	22.82	15.41
Standard deviation	23.16	8.44	6.23	8.49	14.58	15.76
t-stat	1.80	5.62	13.92	15.27	14.41	13.94
	First sub-sample (Jan. 2002 – Sept. 2003)					
Mean	21.27	24.63	27.71	29.12	30.36	20.47
Median	59.35	37.47	32.66	32.92	33.17	21.69
Standard deviation	22.14	8.91	6.42	8.34	14.47	15.92
t-stat	0.76	4.32	11.75	13.79	13.10	12.52
	Second sub-sample (Oct. 2002 – June 2003)					
Mean	62.28	17.88	16.70	16.76	17.22	11.88
Median	32.16	23.72	21.11	20.35	20.00	13.63
Standard deviation	24.26	7.99	5.18	6.88	11.52	12.14
t-stat	1.74	3.60	9.10	9.91	9.34	9.16

Table 11

Selection of Regime Switching Model for Returns on Equity Portfolios – Effects of Adding European and North American Small Caps

The table shows estimation results for the regime switching model:

$$r_t = \boldsymbol{\mu}_{s_t} + \boldsymbol{\varepsilon}_t$$

where $\mathbf{r}_t$ is a 4×1 vector collecting weekly total return series, $\boldsymbol{\mu}_{s_t}$ is the intercept vector in state s_t, and $\boldsymbol{\varepsilon}_t = [\varepsilon_{1t}\ \varepsilon_{2t}\ \varepsilon_{3t}\varepsilon_{4t}\varepsilon_{5t}]' \sim N(\mathbf{0}, \Sigma_{s_t})$. The unobservable state s_t is governed by a first-order Markov chain that can assume three values. The first panel refers to the single-state case k = 1. Asterisks attached to correlation coefficients refer to covariance estimates.

	Panel A – Single State Model				
	Europe – Large caps	North America – Large caps	Pacific	Europe – Small caps	North America – Small caps
1. Mean excess return	-0.0015	-0.0010	-0.0007	0.0002	0.0019
2. Correlations/Volatilities					
Europe – Large caps	0.0370***				
North America – Large caps	0.7537***	0.0285***			
Pacific	0.5086**	0.4822**	0.0259***		
Europe – Small caps	0.7816***	0.6718***	0.5403**	0.0222***	
North America – Small caps	0.6948***	0.7992***	0.4267**	0.7275***	0.0301

	Panel B – Three State Model				
	Europe – Large caps	North America – Large caps	Pacific	Europe – Small caps	North America – Small caps
1. Mean excess return					
Bear State	-0.0403***	-0.0248***	-0.0218***	-0.0214***	-0.0216**
Normal State	-0.0015	-0.0009	0.0004	0.0024*	0.0046**
Bull State	0.0337***	0.0204***	0.0153***	0.0131***	0.0134**
2. Correlations/Volatilities					
Bear state:					
Europe – Large caps	0.0365***				
North America – Large caps	0.6850***	0.0256***			
Pacific	0.3579**	0.2229*	0.0285***		
Europe – Small caps	0.8049***	0.6547***	0.6004***	0.0324***	
North America – Small caps	0.7759***	0.6757***	0.3714**	0.7092***	0.0378***
Normal state:					
Europe – Large caps	0.0242***				
North America – Large caps	0.7443***	0.0216***			
Pacific	0.5445**	0.6008***	0.0212***		
Europe – Small caps	0.7096***	0.6616***	0.6046***	0.0146***	
North America – Small caps	0.6869***	0.8410***	0.5779**	0.7370***	0.0234***
Bull state:					
Europe – Large caps	0.0359***				
North America – Large caps	0.5386***	0.0330***			
Pacific	-0.0551	-0.0067	0.0245***		
Europe – Small caps	0.6581***	0.4863**	-0.3451*	0.0167***	
North America – Small caps	0.4895*	0.7983***	-0.2535*	0.5554***	0.0314***

3. Transition probabilities	Bear State	Normal State	Bull State
Bear State	0.2450**	0.0005	0.7545
Normal State	0.0457*	0.9542***	0.0001
Bull State	0.5351**	0.1656*	0.2993*

* denotes 10% significance, ** significance at 5%, *** significance at 1%.

Table 12

Co-Skewness and C-Kurtosis Coefficients for Small Caps vs. an Equally Weighted Portfolio

Coefficients under multivariate regime switching are calculated employing simulations (50,000 trials) and averaging across simulated samples of length equal to the available data (January 1999 – June 2003).

	Co-Skewness		Co-Kurtosis	
	Sample	MS - ergodic	Sample	MS - ergodic
	European Small Caps			
$S_{EU\_small,EU\_large,EW\_ptf}$	**-0.303**	-0.279		
$S_{EU\_small,NA\_large,EW\_ptf}$	-0.225	-0.114		
$S_{EU\_small,NA\_small,EW\_ptf}$	**-0.327**	**-0.309**		
$S_{EU\_small,Pacifc,EW\_ptf}$	**-0.415**	**-0.410**		
$S_{EU\_small,EW\_ptf,EW\_ptf}$	**-0.422**	**-0.314**		
$S_{EU\_small,EU\_small,EW\_ptf}$	**-0.591**	-0.275		
$S_{EU\_small,NA\_large,NA\_small,EW\_ptf}$			**2.225**	1.414
$S_{EU\_small,NA\_large,Pac,EW\_ptf}$			1.288	**2.307**
$S_{EU\_small,NA\_large,EU\_large,EW\_ptf}$			**2.594**	**2.308**
$S_{EU\_small,NA\_small,Pac,EW\_ptf}$			1.357	1.725
$S_{EU\_small,NA\_small,EU\_large,EW\_ptf}$			**2.474**	1.485
$S_{EU\_small,EU\_large,Pac,EW\_ptf}$			1.431	**2.545**
$S_{EU\_small,EU\_small,NA\_large,EW\_ptf}$			**2.627**	**2.619**
$S_{EU\_small,EU\_small,NA\_small,EW\_ptf}$			**2.709**	1.700
$S_{EU\_small,EU\_small,Pac,EW\_ptf}$			**2.007**	**2.782**
$S_{EU\_small,EU\_small,EU\_large,EW\_ptf}$			**3.178**	**2.629**
$S_{EW\_ptf,EW\_ptf,EU\_small,NA\_large}$			**2.670**	**2.663**
$S_{EW\_ptf,EW\_ptf,EU\_small,NA\_small}$			**2.646**	1.872
$S_{EW\_ptf,EW\_ptf,EU\_small,Pac}$			1.827	**2.907**
$S_{EW\_ptf,EW\_ptf,EU\_small,EU\_large}$			**3.094**	**2.751**
$S_{EW\_ptf,EW\_ptf,EU\_small,EU\_small}$			**3.377**	**3.058**
$S_{EW\_ptf,EW\_ptf,EW\_ptf,EU\_small}$			**3.222**	**3.136**
$S_{EU\_small,EU\_small,EU\_small,EW\_ptf}$			**3.845**	**3.173**
	North American Small Caps			
$S_{NA\_small,EU\_large,EW\_ptf}$	-0.146	-0.270		
$S_{NA\_small,NA\_large,EW\_ptf}$	-0.040	-0.156		
$S_{NA\_small,EU\_small,EW\_ptf}$	**-0.327**	**-0.309**		
$S_{NA\_small,Pacifc,EW\_ptf}$	-0.206	**-0.457**		
$S_{NA\_small,EW\_ptf,EW\_ptf}$	-0.200	-0.286		
$S_{NA\_small,NA\_small,EW\_ptf}$	-0.174	-0.252		
$S_{NA\_small,NA\_large,EU\_small,EW\_ptf}$			**2.225**	1.414
$S_{NA\_small,NA\_large,Pac,EW\_ptf}$			1.260	1.518
$S_{NA\_small,NA\_large,EU\_large,EW\_ptf}$			**2.421**	1.435
$S_{NA\_small,EU\_small,Pac,EW\_ptf}$			1.357	1.725
$S_{NA\_small,EU\_small,EU\_large,EW\_ptf}$			1.474	1.485
$S_{NA\_small,EU\_large,Pac,EW\_ptf}$			1.220	1.785
$S_{NA\_small,NA\_small,NA\_large,EW\_ptf}$			**2.422**	1.655
$S_{NA\_small,NA\_small,EU\_small,EW\_ptf}$			1.869	1.827
$S_{NA\_small,NA\_small,Pac,EW\_ptf}$			1.431	1.991
$S_{NA\_small,NA\_small,EU\_large,EW\_ptf}$			**2.442**	1.793
$S_{EW\_ptf,EW\_ptf,NA\_small,NA\_large}$			**2.617**	1.767
$S_{EW\_ptf,EW\_ptf,NA\_small,EU\_small}$			**2.646**	1.872
$S_{EW\_ptf,EW\_ptf,NA\_small,Pac}$			1.576	**2.162**
$S_{EW\_ptf,EW\_ptf,NA\_small,EU\_large}$			**2.725**	1.930
$S_{EW\_ptf,EW\_ptf,NA\_small,NA\_small}$			**2.747**	**2.199**
$S_{EW\_ptf,EW\_ptf,EW\_ptf,NA\_small}$			**2.936**	**2.318**
$S_{NA\_small,NA\_small,NA\_small,EW\_ptf}$			**2.825**	**2.263**

Table 13

Effects of the Rebalancing Frequency

This table reports the optimal weight to be invested in the various equity portfolios as a function of the rebalancing frequency for an investor with power utility and a constant relative risk aversion coefficient of 5. Nominal returns are assumed to be generated by the three-state regime switching model:

$$r_t = \mu_{s_t} + \varepsilon_t$$

Rebalancing Frequency	Investment Horizon T (in months)					
	T=1	T=4	T=12	T=24	T=52	T=104
Panel A - Optimal Allocation to European Small Cap Stocks						
IID (no predictability)	0.87	0.87	0.87	0.87	0.87	0.87
Bear state 1						
Buy-and-hold	0.00	0.00	0.00	0.00	0.00	0.00
Bi-annually	0.00	0.00	0.00	0.00	0.00	0.00
Quarterly	0.00	0.00	0.00	0.00	0.00	0.00
Monthly	0.00	0.00	0.00	0.00	0.01	0.03
Weekly	0.00	0.05	0.01	0.02	0.03	0.04
Normal state 2						
Buy-and-hold	1.00	1.00	1.00	1.00	1.00	1.00
Bi-annually	1.00	1.00	1.00	1.00	1.00	1.00
Quarterly	1.00	1.00	1.00	1.00	1.00	1.00
Monthly	1.00	1.00	1.00	1.00	1.00	1.00
Weekly	1.00	1.00	1.00	1.00	1.00	1.00
Bull state 3						
Buy-and-hold	0.00	0.00	0.00	0.00	0.00	0.00
Bi-annually	0.00	0.00	0.00	0.00	0.03	0.04
Quarterly	0.00	0.00	0.00	0.00	0.04	0.05
Monthly	0.00	0.00	0.00	0.01	0.01	0.02
Weekly	0.00	0.00	0.00	0.01	0.01	0.01
Steady-state probabilities						
Buy-and-hold	0.00	0.00	0.05	0.11	0.10	0.10
Bi-annually	0.00	0.00	0.05	0.11	0.18	0.18
Quarterly	0.00	0.00	0.05	0.11	0.18	0.19
Monthly	0.00	0.00	0.08	0.13	0.20	0.20
Weekly	0.00	0.00	0.00	0.02	0.06	0.07
Panel B - Optimal Allocation to European Large Cap Stocks						
IID (no predictability)	0.00	0.00	0.00	0.00	0.00	0.00
Bear state 1						
Buy-and-hold	0.00	0.00	0.00	0.00	0.04	0.05
Bi-annually	0.00	0.00	0.00	0.00	0.08	0.09
Quarterly	0.00	0.00	0.00	0.00	0.09	0.10
Monthly	0.00	0.00	0.09	0.08	0.07	0.06
Weekly	0.00	0.00	0.04	0.02	0.01	0.00
Normal state 2						
Buy-and-hold	0.00	0.00	0.00	0.00	0.00	0.00
Bi-annually	0.00	0.00	0.00	0.00	0.00	0.00
Quarterly	0.00	0.00	0.00	0.00	0.00	0.00
Monthly	0.00	0.00	0.00	0.00	0.00	0.00
Weekly	0.00	0.00	0.00	0.00	0.00	0.00
Bull state 3						
Buy-and-hold	1.00	0.37	0.03	0.00	0.00	0.00
Bi-annually	1.00	0.37	0.03	0.00	0.00	0.00
Quarterly	1.00	0.37	0.03	0.00	0.00	0.00
Monthly	1.00	0.37	0.18	0.09	0.08	0.08
Weekly	1.00	1.00	0.97	0.90	0.88	0.87
Steady-state probabilities						
Buy-and-hold	0.00	0.00	0.00	0.00	0.00	0.00
Bi-annually	0.00	0.00	0.00	0.00	0.00	0.00
Quarterly	0.00	0.00	0.00	0.00	0.00	0.00
Monthly	0.00	0.00	0.00	0.00	0.00	0.00
Weekly	0.00	0.00	0.00	0.00	0.00	0.00

Table 13 (continued)

Effects of the Rebalancing Frequency

Rebalancing Frequency	Investment Horizon T (in months)					
Panel C - Optimal Allocation to North American Large Cap Stocks						
	T=1	**T=4**	**T=12**	**T=24**	**T=52**	**T=104**
IID (no predictability)	0.00	0.00	0.00	0.00	0.00	0.00
Bear state 1						
Buy-and-hold	0.44	0.59	0.60	0.60	0.57	0.57
Bi-annually	0.44	0.59	0.60	0.60	0.51	0.50
Quarterly	0.44	0.59	0.60	0.60	0.50	0.49
Monthly	0.44	0.59	0.49	0.50	0.50	0.49
Weekly	0.44	0.46	0.48	0.49	0.50	0.50
Normal state 2						
Buy-and-hold	0.00	0.00	0.00	0.00	0.00	0.00
Bi-annually	0.00	0.00	0.00	0.00	0.00	0.00
Quarterly	0.00	0.00	0.00	0.00	0.00	0.00
Monthly	0.00	0.00	0.00	0.00	0.00	0.00
Weekly	0.00	0.00	0.00	0.00	0.00	0.00
Bull state 3						
Buy-and-hold	0.00	0.30	0.56	0.57	0.57	0.56
Bi-annually	0.00	0.30	0.56	0.57	0.59	0.59
Quarterly	0.00	0.30	0.56	0.57	0.58	0.58
Monthly	0.00	0.30	0.42	0.50	0.54	0.53
Weekly	0.00	0.00	0.00	0.00	0.02	0.02
Steady-state probabilities						
Buy-and-hold	0.55	0.53	0.51	0.46	0.46	0.46
Bi-annually	0.55	0.53	0.51	0.46	0.40	0.40
Quarterly	0.55	0.53	0.51	0.46	0.40	0.39
Monthly	0.55	0.53	0.47	0.45	0.39	0.38
Weekly	0.55	0.51	0.46	0.43	0.38	0.36
Panel D - Optimal Allocation to Pacific Stocks						
	T=1	**T=4**	**T=12**	**T=24**	**T=52**	**T=104**
IID (no predictability)	0.13	0.13	0.13	0.13	0.13	0.13
Bear state 1						
Buy-and-hold	0.56	0.41	0.40	0.40	0.39	0.38
Bi-annually	0.56	0.41	0.40	0.40	0.41	0.41
Quarterly	0.56	0.41	0.40	0.40	0.41	0.41
Monthly	0.56	0.41	0.42	0.42	0.42	0.42
Weekly	0.56	0.49	0.47	0.47	0.46	0.46
Normal state 2						
Buy-and-hold	0.00	0.00	0.00	0.00	0.00	0.00
Bi-annually	0.00	0.00	0.00	0.00	0.00	0.00
Quarterly	0.00	0.00	0.00	0.00	0.00	0.00
Monthly	0.00	0.00	0.00	0.00	0.00	0.00
Weekly	0.00	0.00	0.00	0.00	0.00	0.00
Bull state 3						
Buy-and-hold	0.00	0.33	0.41	0.43	0.43	0.44
Bi-annually	0.00	0.33	0.41	0.43	0.38	0.37
Quarterly	0.00	0.33	0.41	0.43	0.38	0.37
Monthly	0.00	0.33	0.40	0.40	0.37	0.37
Weekly	0.00	0.00	0.03	0.09	0.09	0.10
Steady-state probabilities						
Buy-and-hold	0.45	0.47	0.44	0.43	0.44	0.44
Bi-annually	0.45	0.47	0.44	0.43	0.42	0.42
Quarterly	0.45	0.47	0.44	0.43	0.42	0.42
Monthly	0.45	0.47	0.45	0.42	0.41	0.42
Weekly	0.45	0.49	0.54	0.55	0.56	0.57

Figure 1

Time Series Plots of International Cumulated Stock Returns

The graphs plot weekly equity gross total return series (including dividends, adjusted for stock splits, etc.) for a few international portfolios. The sample period is January 1999 – June 2003. All returns are expressed in local currencies. Each graph plots the dynamics of the value of unit of local currency invested in the first week of 1999.

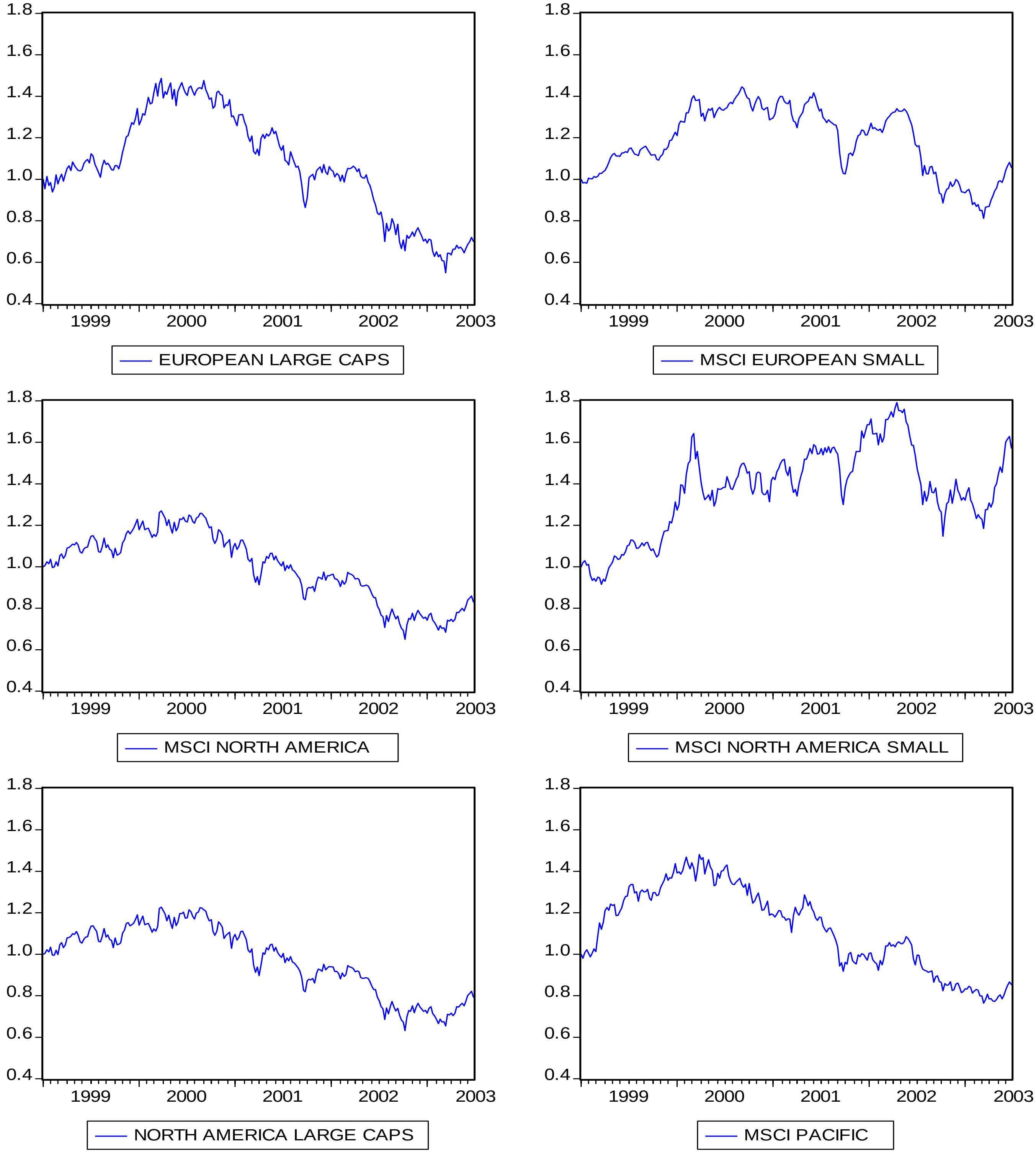


Figure 2

Smoothed State Probabilities: Two-State Model for European, North American, and Pacific Equity Portfolios

The graphs plot the smoothed probabilities of regimes 1-2 for the multivariate Markov Switching model comprising weekly total return series for North American, Pacific, and a European large caps portfolio (Dow Jones Stoxx 50).

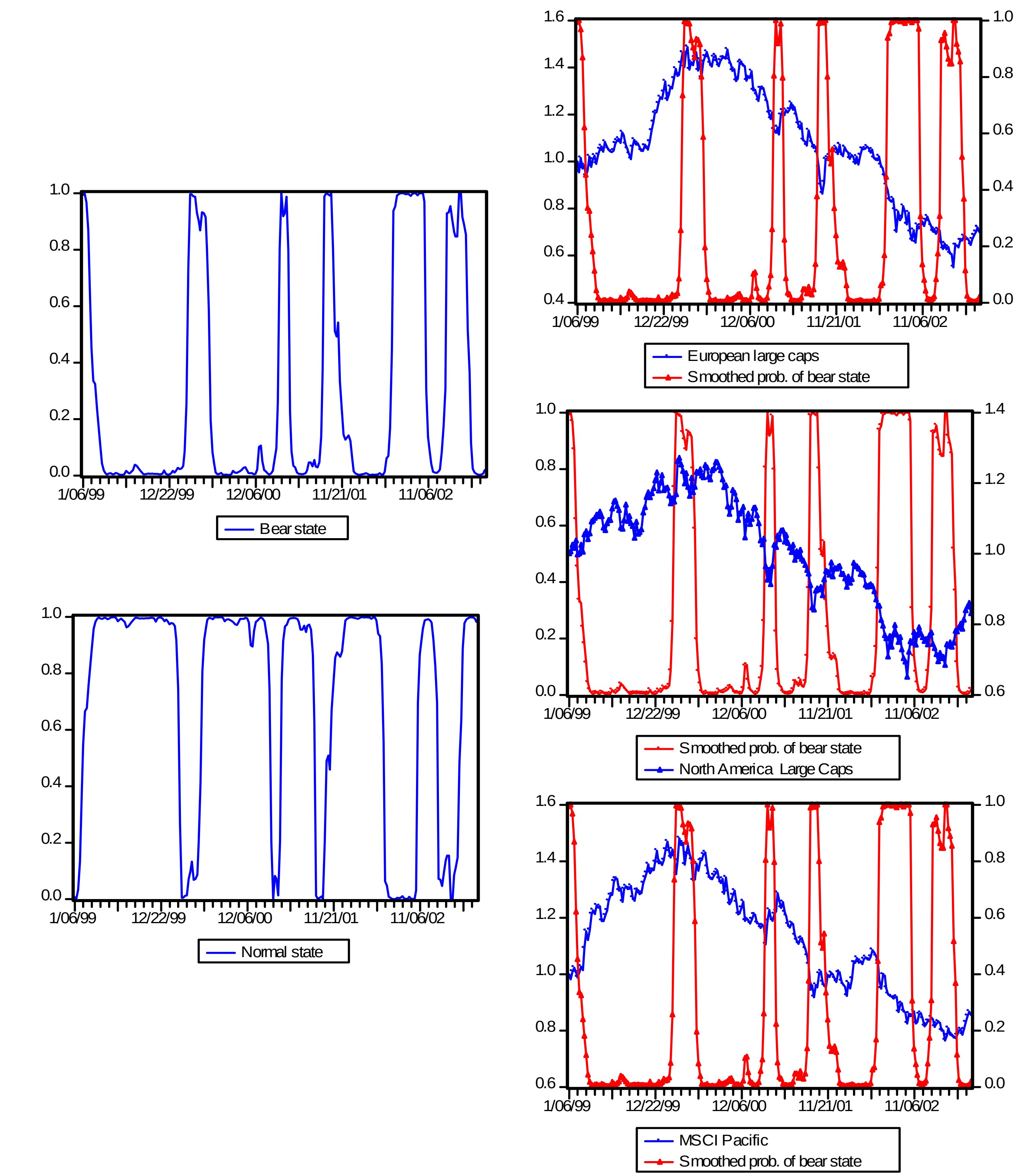


Figure 3

Buy-and-Hold Optimal Allocation – Restricted Asset Menu

The graphs plot the optimal international equity portfolio weights when returns follow a two-state Markov Switching model as a function of: (i) the coefficient of relative risk aversion; (ii) the investment horizon. As a benchmark (bold horizontal lines) we also report the IID/Myopic allocation that obtains when returns have an IID multivariate Gaussian distribution.

$\gamma = 5$ $\gamma = 10$

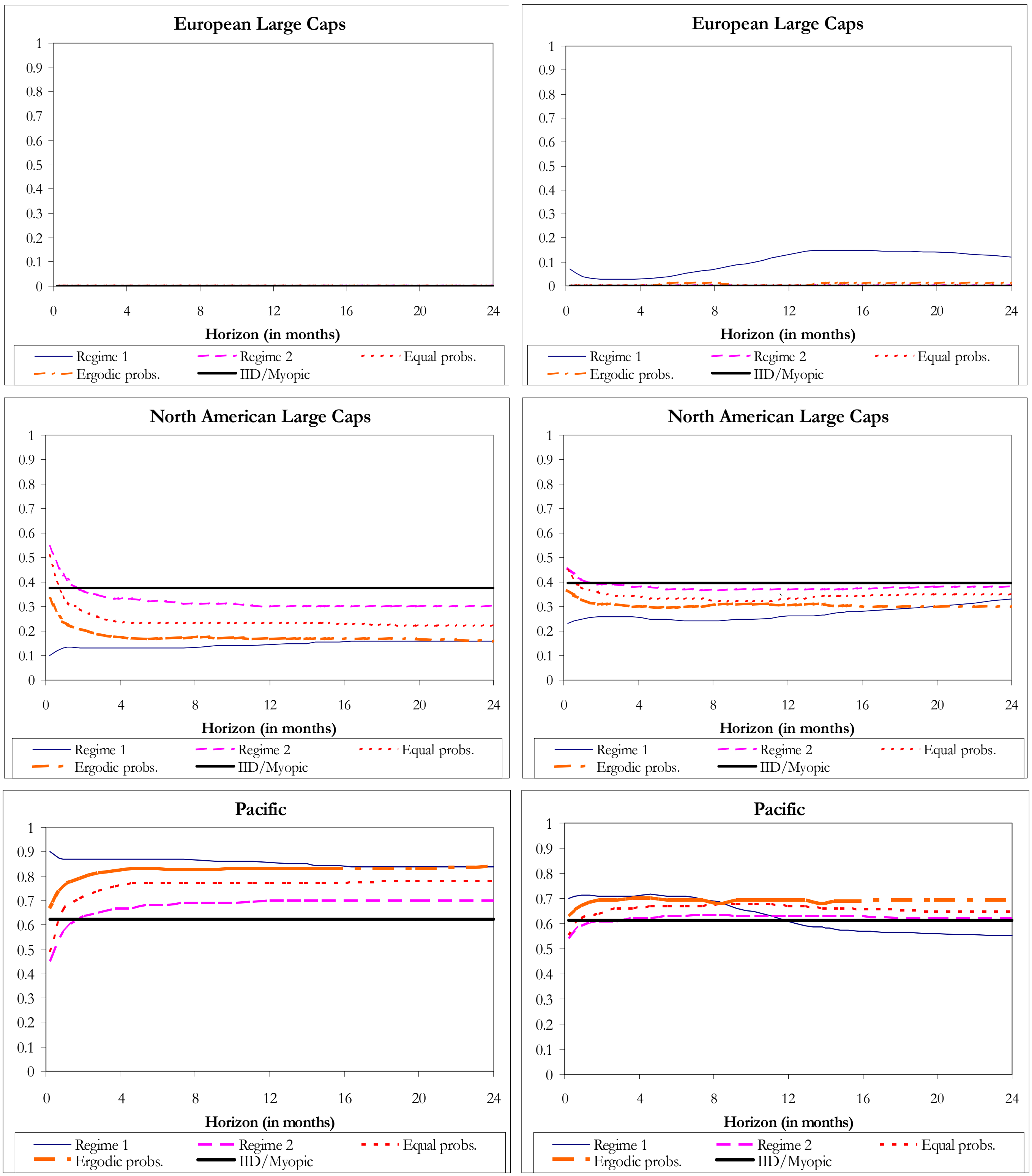
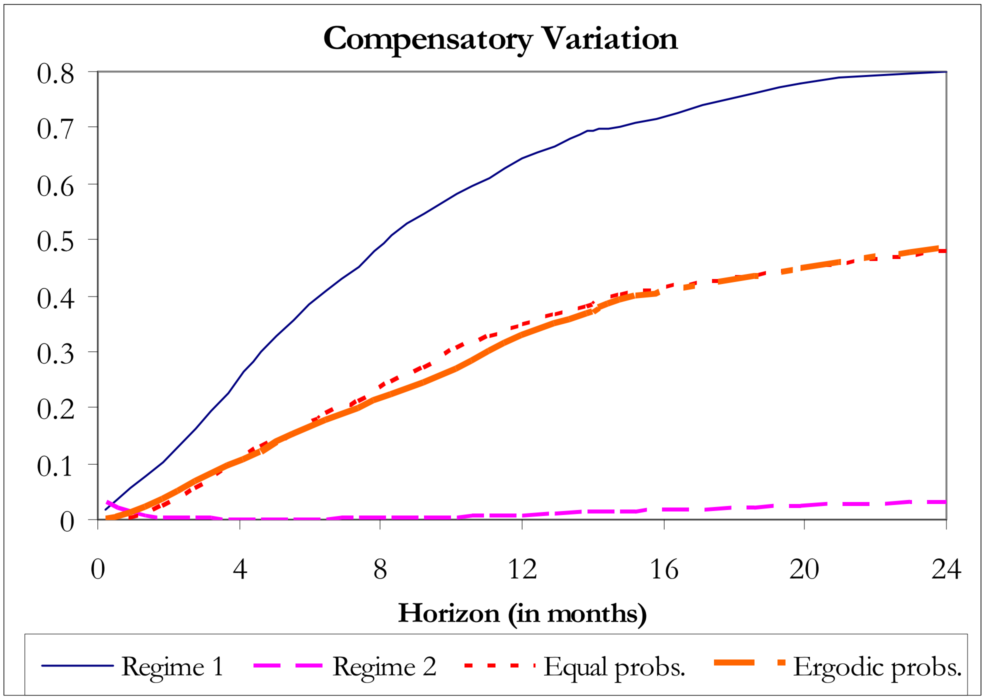


Figure 4

Welfare Costs of Ignoring Regime Switching – Restricted Asset Menu

The graphs plot the compensatory variation (as a fraction of initial wealth) from ignoring the presence of regime switches in the multivariate process of asset returns. The graphs plot the welfare costs as a function of the investment horizon; calculations were performed for two alternative levels of the coefficient of relative risk aversion. The investor is assumed to have a simple buy-and-hold objective.

$\gamma = 5$



$\gamma = 10$

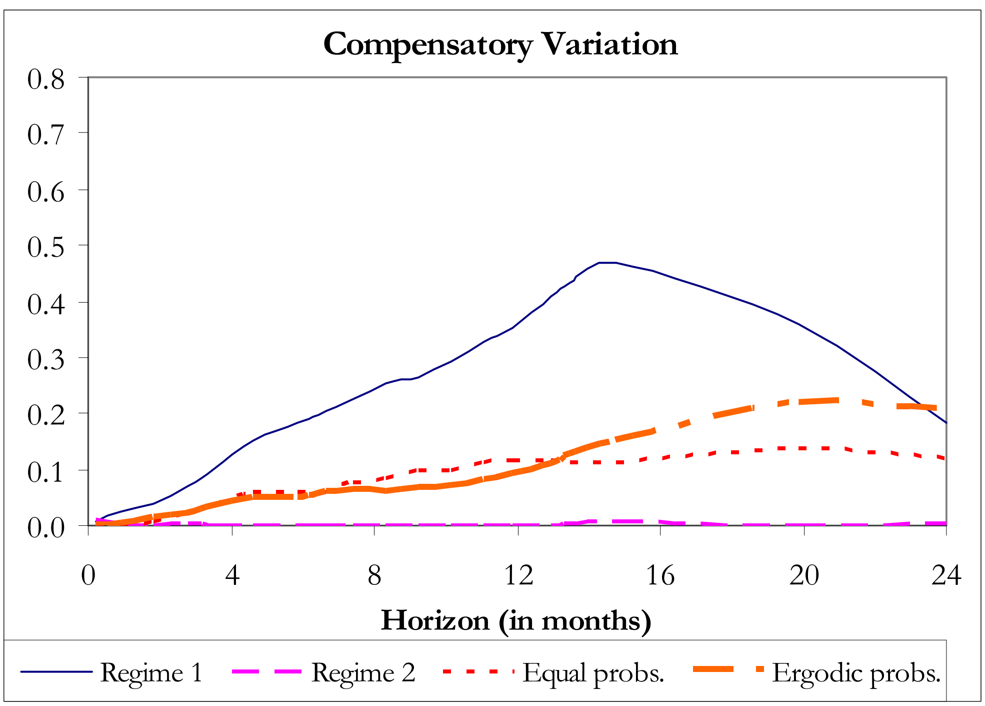


Figure 5

Buy-and-Hold Real Time Optimal Allocation – Restricted Asset Menu

The graphs plot the optimal international equity portfolio weights when returns follow a two-state Markov Switching model as a function of the coefficient of relative risk aversion for a few alternative investment horizons. The optimizing portfolio choice is recursively computed at the end of all weeks in the sample January 2002 – June 2003. In correspondence of each week, the models' parameters are re-estimated on an expanding window of data. As a benchmark (bold lines) we also report the IID/Myopic allocation that obtains when returns have an IID multivariate Gaussian distribution.

$\gamma = 5$ $\gamma = 10$

Figure 6

Smoothed State Probabilities: Three-State Model for European, North American, and Pacific Equity Portfolios – Effects of Adding European Small Caps

The graphs plot the smoothed probabilities of regimes 1-3 for the multivariate Markov Switching model comprising weekly total return series for North American large, Pacific, and a European small (MSCI) and large caps portfolios. The bottom right panel shows the sum of the smoothed probabilities of states 1 and 3, characterized by high volatility.

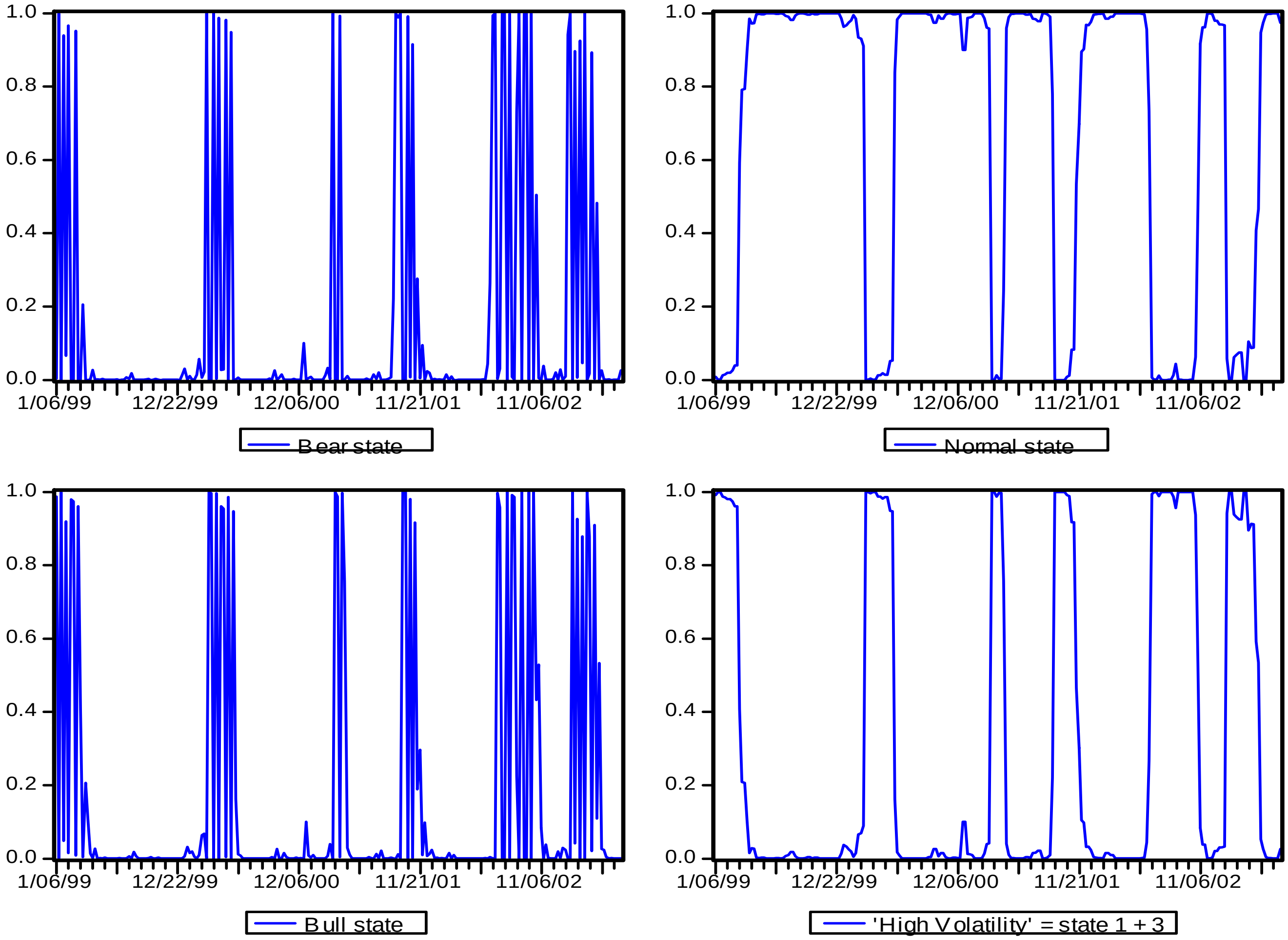


Figure 7

Buy-and-Hold Optimal Allocation

The graphs plot the optimal international equity portfolio weights when returns follow a three-state Markov Switching model as a function of: (i) the coefficient of relative risk aversion; (ii) the investment horizon. As a benchmark (bold horizontal lines) we also report the IID/Myopic allocation. The asset menu includes European small caps.

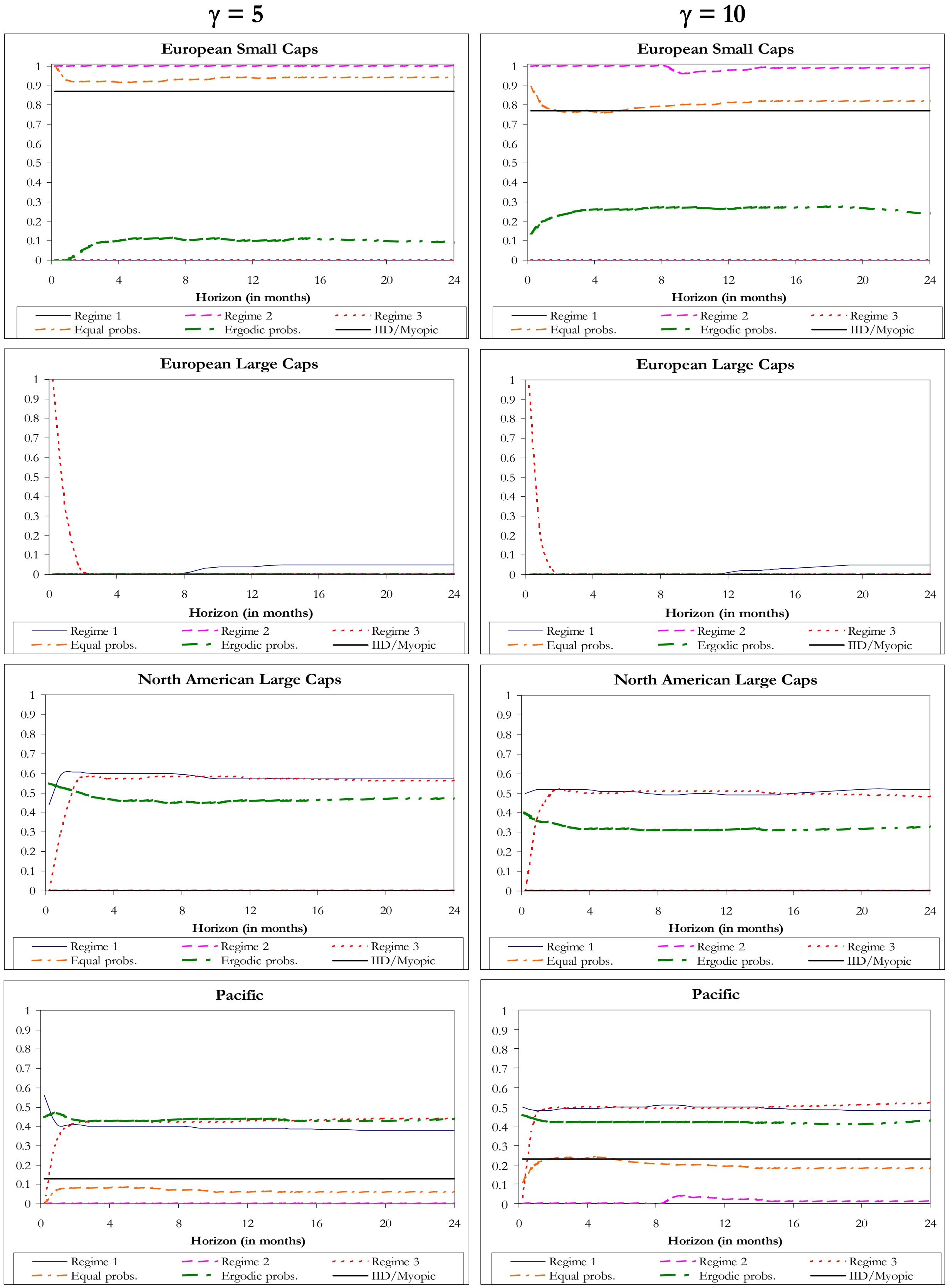
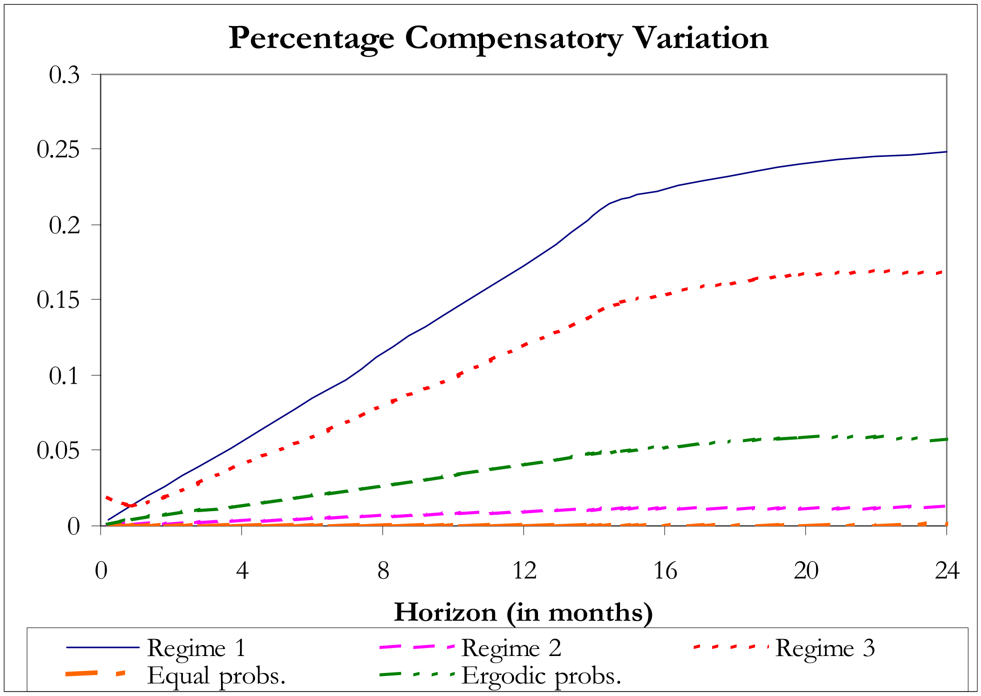


Figure 8

Welfare Costs of Ignoring Regime Switching

The graphs plot the percentage compensatory variation from ignoring the presence of regime switches in the multivariate process of asset returns. The graphs plot the welfare costs as a function of the investment horizon; calculations were performed for two alternative levels of the coefficient of relative risk aversion. The investor is assumed to have a simple buy-and-hold objective.

$\gamma = 5$



$\gamma = 10$

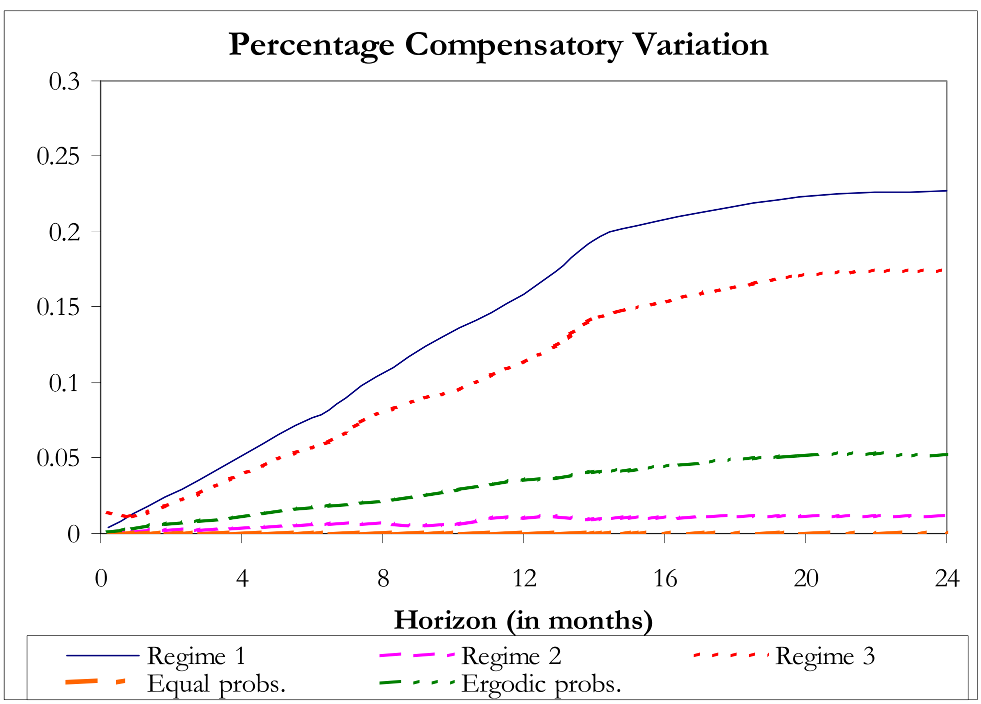


Figure 9

Buy-and-Hold Real Time Optimal Allocation

The graphs plot the optimal international equity portfolio weights when returns follow a three-state Markov Switching model. The optimizing portfolio choice is recursively computed at the end of all weeks in the sample January 2002 – June 2003. As a benchmark (bold lines) we also report the IID/Myopic allocation.

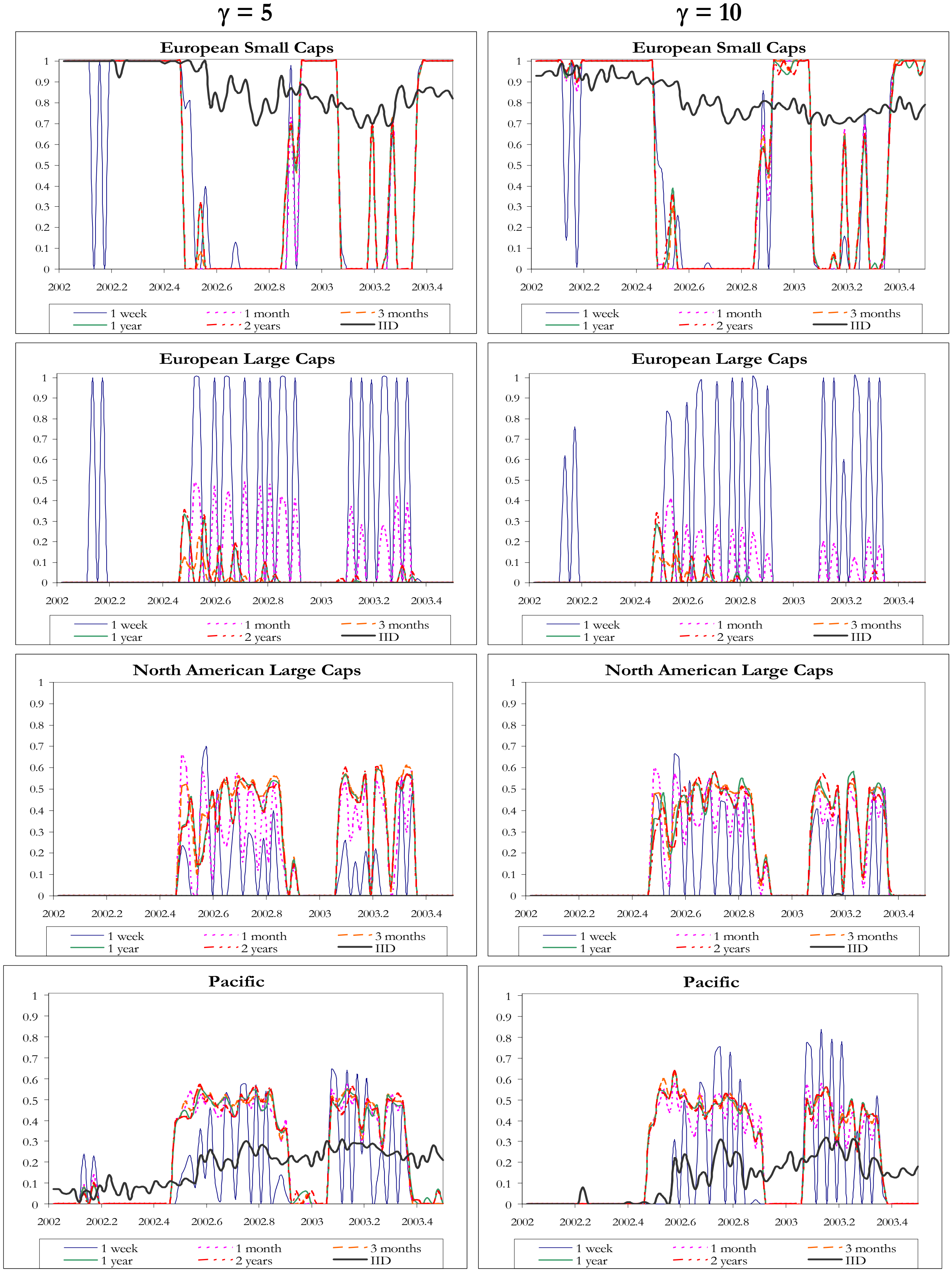


Figure 10

Buy-and-Hold Optimal Allocation – Asset Menu Expanded to North American Small Caps

The graphs plot the optimal international equity portfolio weights when returns follow a three-state Markov Switching model as a function of: (i) the coefficient of relative risk aversion; (ii) the investment horizon.

$\gamma = 5$ $\gamma = 10$

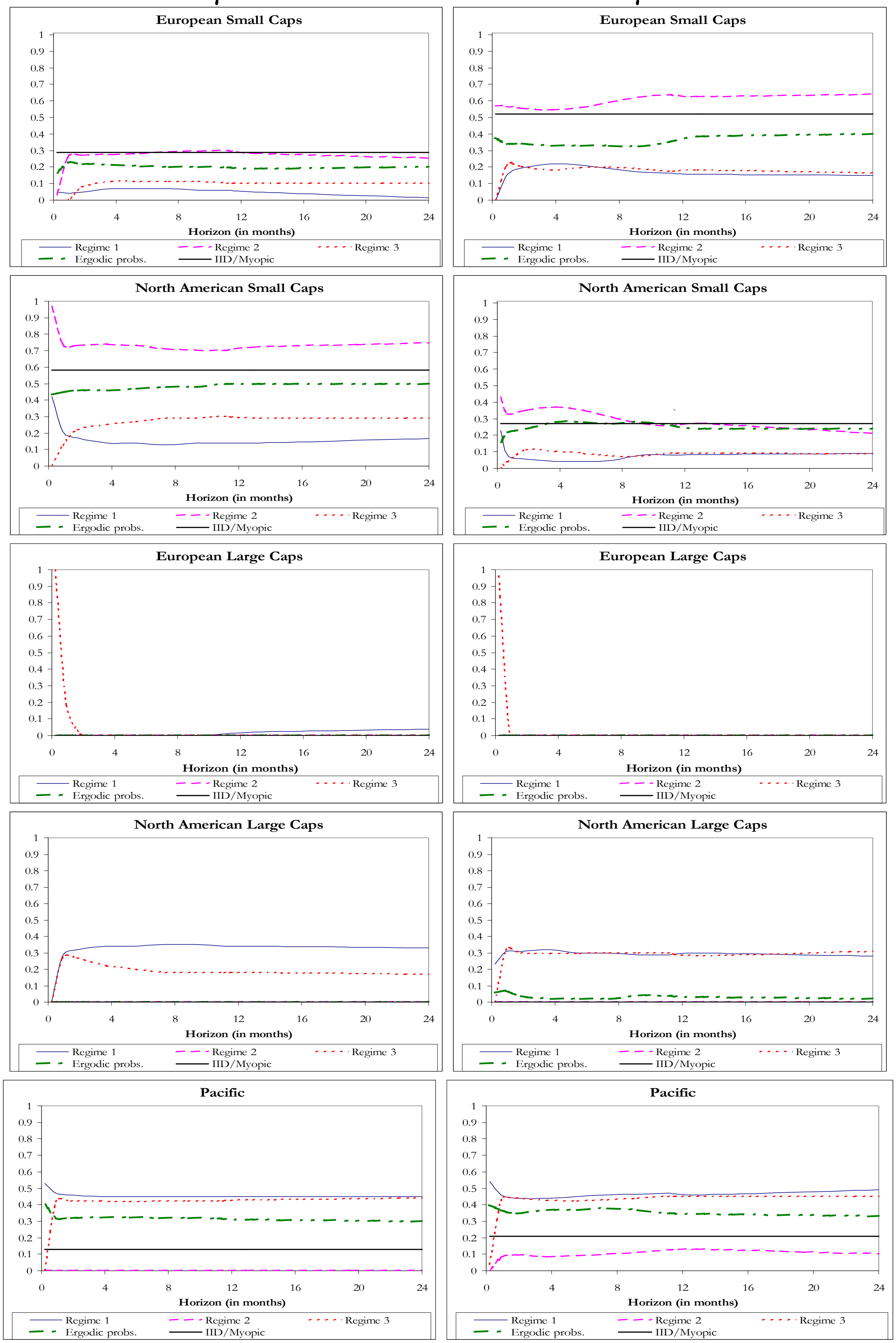


Figure 11

Buy-and-Hold Real Time Optimal Allocation – Asset Menu Expanded to North American Small Caps

The graphs plot the optimal international equity portfolio weights when returns follow a three-state Markov Switching model. The optimizing portfolio choice is recursively computed at the end of all weeks in the sample January 2002 – June 2003. As a benchmark (bold lines) we also report the IID/Myopic allocation. The coefficient of relative risk aversion is set to 5.

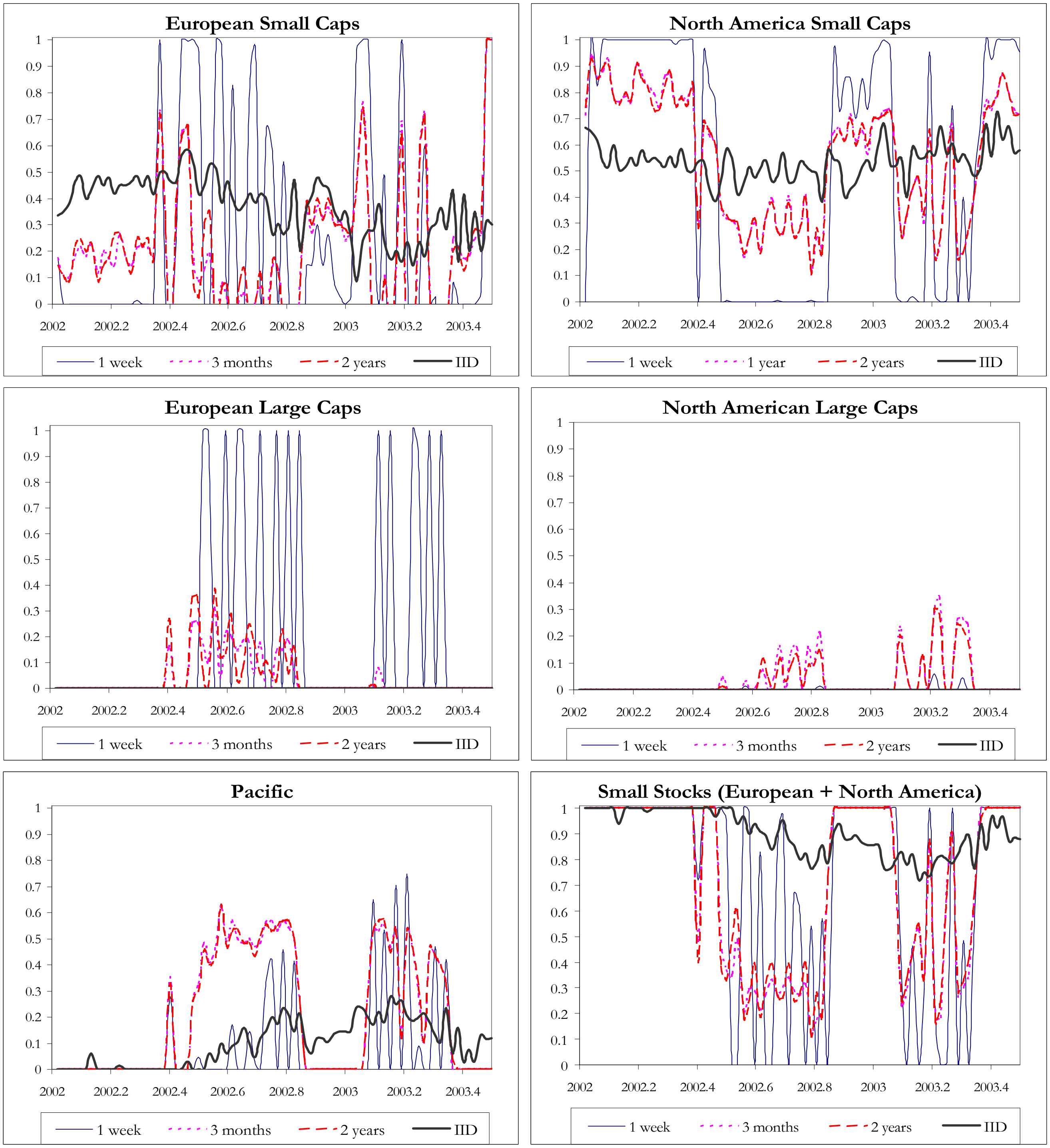


Figure 12

Buy-and-Hold Optimal Allocation – Long Horizon

The graphs plot the optimal international equity portfolio weights when returns follow a three-state Markov Switching model and the coefficient of relative risk aversion is set at 5, as a function of the investment horizon. As a benchmark (bold horizontal lines) we also report the IID/Myopic allocation. The asset menu includes European small caps.

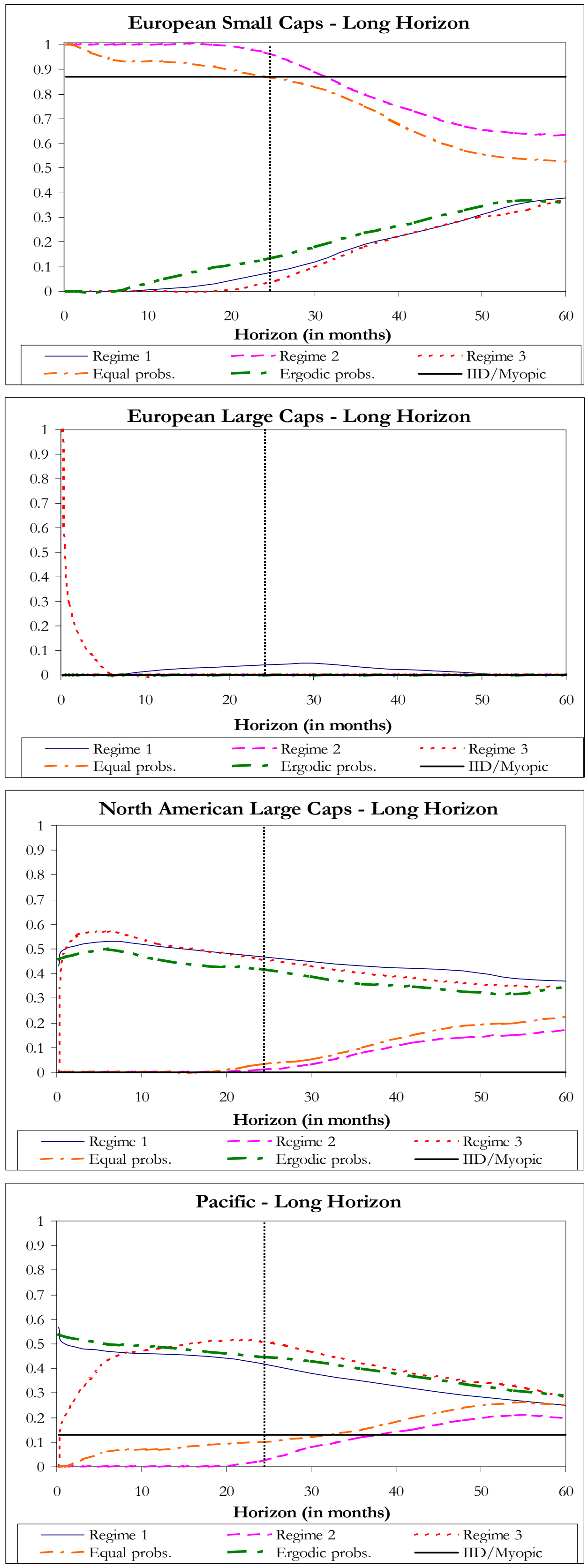


Figure 13

Buy-and-Hold Optimal Allocation – Short Sales Allowed

The graphs plot the optimal international equity portfolio weights when returns follow a three-state Markov Switching model as a function of: (i) the coefficient of relative risk aversion; (ii) the investment horizon. As a benchmark (bold horizontal lines) we also report the IID/Myopic allocation. The asset menu includes European small caps.