The Strategic Value of Incomplete Contracts for Competing Hierarchies*

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Abstract

We provide a rationale for contractual incompleteness in a setting with competing manufacturer-
retailer hierarchies. In a class of simple manufacturers-retailers economies, characterized by adverse
selection and moral hazard, we show that principals dealing with (exclusive) agents who compete on a
downstream market, may prefer to leave contracts silent on some (potentially) verifiable performance
measures whenever certain other aspects of agents’ activity remain noncontractible. Two effects are
at play once one moves from a complete to an incomplete contracting. First, reducing the number of
screening instruments has a detrimental effect on principals’ profits as it makes information revelation
more costly and drives up the information rents associated to informational asymmetry. Second, it may
provide principals with strategic power in that it allows to force competing hierarchies to behave in a more
friendly manner on the downstream market. In contrast with previous literature, conjecturing a positive
relationship between transaction costs and contracting incompleteness, it is demonstrated that equilibrium
contracts display forms of incompleteness whenever agency costs are small enough. Implications on the
use of vertical arrangements based on resale price maintenance in imperfectly competitive industries are
also discussed.

1 Introduction

The basic principal-agent paradigm predicts that an optimal contract must limit as much as possible agents’
discretion within agency relationships. According to this view, contracts have to be complete in the sense that
principals should profitably exploit all available screening and monitoring instruments in order to prevent
agents’ misbehavior. Strikingly, however, seldom in real life contracts display such high degree of complexity.
Contracts appear rather simple and, more importantly, quite often fail to specify verifiable obligations of
contracting counterparts. Such examples of arm’s length relationships are widespread in business practices.
Manufacturers often delegate marketing and advertising activities to retailers; shareholders typically delegate

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to managers firms’ competitive and organizational strategies; lenders usually leave entrepreneurs free to perform certain tasks that affect the profitability of their ventures; insurance companies monitor only to a limited extent the behavior of insurees, etc..

So far a few rationales behind this incompleteness puzzle have been provided by the contracting literature. First, high transaction costs can limit individuals’ capability to enforce complete contracts because many relevant aspects of agents’ performances are not verifiable at the time of enforcing contracting obligations. Second, bounded rationality may prevent complete contracting as not all possible future contingencies can be foreseen at the time contracts are designed. Nevertheless, these views clearly fail to explain why there are circumstances under which sophisticated agents deliberately choose to write incomplete contracts even though transaction costs are low.

In the current paper we address this issue by taking a traditional agency perspective. Specifically, we argue that once one moves from the isolated principal-agent set-up to games played by competing hierarchies, the design of incomplete contracts can be rationalized by the interaction between agency costs associated to alternative incentive schemes and contracting externalities. In a setting with competing hierarchies, encompassing both adverse selection and moral hazard, it is demonstrated that when agents impose externalities on each other, and some aspects of the individuals’ actions are nonverifiable, leaving contracts incomplete or silent on certain (potentially) verifiable agents’ performance measures may have a strategic value. Incomplete contracts emerge at equilibrium even when more complex arrangements can be enforced without additional transaction costs, i.e., further enforcement and monitoring outflows.

More precisely, we show that the equilibrium determinants of contracting incompleteness hinge on three somewhat natural aspects of information asymmetries: (i) the way different screening instruments shape agency costs; (ii) the type of externalities that bilateral negotiations between principal-agent pairs impose on competing organizations; (iii) the effectiveness of agency constraints on firms’ technological frontier.

To take an archetypical example of incomplete contracting, we consider manufacturers-retailers relationships. Particular emphasis has indeed been stressed by the IO literature on the very incomplete nature of the arrangements regulating the terms of trade between vertically related firms. Not only manufacturers often delegate marketing activities to retailers, but they also frequently give up vertical control by refusing to impose contractual restraints that would reduce agency costs and, typically, would allow to improve upon allocative efficiency. We assume that two retailers (agents or dealers) selling differentiated products compete on a downstream market by setting quantities. Production of final outputs requires an essential raw input which is supplied by a pair of exclusive, upstream suppliers (principals or manufacturers). Retailers

1The applied literature on this topic (Lafontaine and Slade, 1997, among many others) has recently argued that the empirical evidence, fairly consistent across industries and firms, quite often appears to be inconsistent with some aspects of the theoretical predictions based on agency theory.

2Clearly, our analysis can be also extended to many other environments where competition takes place between vertical hierarchies or where agents, dealing with exclusive principals, impose externalities on each other. For instance it is easy to verify that our results would also hold in a multiple regulators set-up where different authorities regulate economic sectors characterized by technological spill-overs among firms; in the case of lenders financing competing entrepreneurs; or in an insurance economy where risk-neutral principals share risk with individuals imposing consumption externalities on each other.
are selected from a very large population of agents, so that upstream firms dictate the terms of trade in
the wholesale-retail relationship. Downstream demands are uncertain and only agents observe a pay-off
relevant signal which realizes before contracts are designed: an adverse selection problem. Moreover, an
unverifiable demand-enhancing activity (promotional expenditures and/or production of indivisible services)
is carried out by downstream agents: a moral hazard issue. Principals and agents are risk-neutral. Each
upstream firm hires an agent before production occurs, but after uncertainty about demand is realized. Two
alternative wholesale-retail trade regimes are analyzed. Specifically, each principal can either commit to an
incomplete contract, referred to as a quantity fixing scheme (QF), or to a more sophisticated arrangement,
comparable to resale price maintenance (RPM). A QF arrangement is incomplete relative RPM in the sense
that, beyond fixing the quantity supplied to final consumers, it leaves the downstream firm free to choose its
most preferred level of demand-enhancing activity. Instead, a RPM mechanism, besides fixing the quantity
supplied in the final market, also restrains the retail price charged to final consumers to indirectly control
the retailer’s non-market activity.3 A three-stage game is examined. In the first stage, before setting their
competitive strategies, upstream suppliers (simultaneously) make a public announcement on the type of
mechanism that they will enforce at the contracting stage. In the second stage, allocations are secretly ne-
gotiated after all players have observed the announcement made in the first stage. Finally, in the third stage,
product market competition takes place and payments are made according to the prespecified contracting
rules.

Within this framework, we prove that the degree to which agents’ (unverifiable) effort affects competing
organizations plays a crucial role in shaping the features of equilibrium contracts. Two opposite effects
are at play once one moves from a complete (RPM) to an incomplete contract (QF) in this framework.
On the one hand, an incomplete contract leaves more information rents simply because it limits the set of
screening instruments. On the other hand, introducing a source of contracting incompleteness might provide
principals with strategic power in that it allows them to induce a more friendly behavior by rivals at the
market stage. Noteworthy, while the former effect has been widely discussed in previous contributions, the
second effect is novel in the principal-agent literature and fully driven by our focus on competition between
vertical hierarchies.

We provide simple conditions, related to the severity of asymmetric information problem, under which
incomplete contracting may endogenously emerge when agents impose either positive or negative externali-
ties on each other. The key argument is as follows, when demands for each good are independent, i.e., there
are no externalities between agents and so retailers are monopolists in their own markets, the enforcement
of a complete contract is beneficial to principals because it reaches the best trade-off between efficiency and

3Of course, in real life suppliers can credibly commit to use retail price restrictions in several ways. First, one can imagine
that investments in specific monitoring technologies and/or delegation to third parties (such as intermediaries or experts) the
task of monitoring retail prices, have the goal of signaling in a credible way the use of retail price restrictions. Second, in a
dynamic perspective, suppliers can credibly induce their competitors to believe that retail price restrictions will be used through
reputation.
rent extraction. With competing hierarchies a new and opposite channel through which contracting incompleteness can affect principals’ profits comes at play. By leaving downstream agents free to set some aspects of their performance, each principal can influence in her own interest the market behavior of competing organizations under certain circumstances.

After having proved that incomplete contracts can be signed at equilibrium, we relate contracting incompleteness to the nature of the externality between downstream retailers. In particular, when agents impose negative (resp. positive) externalities on each other, that is goods are substitutes (resp. complements) and effort has a selfish (resp. cooperative) value, the game of contractual choices has multiple equilibria. In a free-riding environment, that emerges when goods are substitutes and effort has a selfish value, multiple equilibria may emerge only if retail competition is not very intense, when this condition is not satisfied only complete contracts are signed at the equilibrium. We also show that when there are multiple equilibria incomplete contracts are always Pareto superior (resp. inferior) relative to complete ones whenever effort has a cooperative (resp. selfish) value. A welfare analysis aimed at assessing the social desirability of retail price restriction is also performed.

Section 2 relates our work to the literature on incomplete contracting. In Section 3, by introducing a simple example, we motivate our approach arguing that when all aspects of agents’ performance are verifiable contracting incompleteness will never emerge at equilibrium. Section 4 presents an illustrative example showing why, and under what conditions, contracting incompleteness may have a strategic value whenever some aspects of agents’ performances are nonverifiable. In Section 5, the model is extended to a fully symmetric competing hierarchies framework. Finally, Section 6 discusses the extent to which our analysis applies to more general environments. Section 7 concludes. All proofs are relegated to an Appendix.

2 Related Literature

The common wisdom identifies the major sources of contracting incompleteness in the presence of transaction costs and bounded rationality. As argued by Mookerjee (2005), apart from the usual noncooperative incentive and participation constraints, these approaches search for additional constraints on contracts that rationalize simple real-world mechanisms. On the one hand, as suggested by the property rights approach (Grossman and Hart, 1986, and Hart and Moore, 1990, among others), agents’ inability to describe future (uncertain) events might severely prevent them from signing complete contracts. On the other hand, even in the presence of very sophisticated agents, enforcement and monitoring costs could limit the possibility of writing complete contracts as well (see Maskin, 2002, Maskin and Tirole, 1999, and Segal, 1999, among others). Although based on different presumptions, both approaches support the view that the standard principal-agent analysis is unsatisfactory in building a convincing theoretical foundation for contracting incompleteness. The present paper questions this view and it argues that, once one moves from the isolated principal-agent set-up to games

\footnote{Basically, each incentive feasible allocation implementable under incomplete contracting can always be replicated by a complete contract.}
played by competing hierarchies, contracting incompleteness can be more easily rationalized by looking at how agency costs change with the nature of the contractual externalities between agents. Few previous studies have addressed the issue of contracting incompleteness by taking a perspective somewhat more in line with the principal-agent paradigm.

**Information Asymmetries, Dynamics and Renegotiation:** Dewatripont and Maskin (1990, 1995) investigate the set of optimal contingencies on which an incentive contract should depend whenever renegotiation is possible. They show that, within a repeated principal-agent set-up, where renegotiation issues produce dynamic externalities, it might happen that a principal voluntarily limits the set of publicly observable screening devices in order to relax future renegotiation constraints. Similarly, Crémer (1995) shows that in a repeated moral hazard setting, principals may voluntarily weaken the effectiveness of their (ex post) monitoring technology in order to make less likely renegotiation threats. Martimort (1999) and Olsen and Torsvik (1993) show also how moving away from a centralized regulation by introducing competing regulators, another form of incompleteness, may improve commitment in an intertemporal context. Rather than looking at the effects of incomplete contracts on renegotiation constraints with the same principal, we focus on their strategic value and analyze the extent to which those arrangements weaken the market behavior of agents’ dealing with competing principals.

Bernheim and Whinston (1998) show that when some aspects of individuals’ behavior are observable but not verifiable, before playing a dynamic game, two individuals may profitably agree on designing contracts which are silent on some potentially contractible aspects of the relationship. Bernheim and Whinston analyze a class of complete information games where the possibility for players to sign (ex ante) binding contracts allows to limit in a mutually beneficial way the space of actions in their subsequent interactions. Two major aspects differentiate our work by this paper. First, we analyze an incomplete information set-up and incentive constraints arise here from the need for inducing information revelation whereas incentive constraints are deduced from the self-enforceability of nonverifiable actions in their framework. Second, differently form them we examine a competing organizations set-up.

**Information Asymmetries and Signaling:** Aghion and Hermalin, (1990), Allen and Gale (1992) and Spier (1992) present a principal-agent model in which asymmetric information leads to contractual incompleteness. With an informed principal, incomplete contracts may profitably be used to signal the principal’s type when transactions costs are sufficiently high.

**Ignorance in Vertical Contracting:** Caillaud and Rey (1995) analyze the optimal information structure of producers with respect to their retailers. They show that ignorance on the retailer’s cost function might create a strategic advantage that could outweigh the associated agency costs. While they rather focus on the value of ignorance in environments where principals can chose whether to acquire (at no additional costs) the relevant market information or, alternatively, to be (strategically) uninformed, in our set-up full extraction is prevented by the moral hazard component. As we shall discuss, this assumption is key for

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5In this respect, their analysis seems much more related to Dewatripont and Maskin (1990, 1995) and Crémer (1995).
equilibria to display incomplete contracting.\textsuperscript{6} Finally, Kessler (1999) examines an agency model where the agent’s information structure is endogenous and the possibility of remaining uninformed for the agent has a positive strategic value. The crucial difference between our paper and Kessler’s analysis is that, while she analyzes the strategic value of ignorance from the agents’ perspective, we rather focus on principals’ incentives to create contracting incompleteness.

**Miscellaneous:** Our analysis is also closely related to the literature on vertical restraints (D’Amato et al., 2005, Gal-Or, 1991, 1992, 1999, Martimort, 1996, Rey and Stiglitz, 1995 and Rey and Tirole, 1986). The fundamental difference between us and this literature is that while previous contributions have adopted a complete contracting approach and mainly taken the set of control instruments as given, we endogenize this set.\textsuperscript{7} From an organizational design viewpoint our results also provide a simple explanation for the evidence showing that often, in real life, upstream manufacturers delegate non-market decisions, such as advertising and marketing activities, to downstream retailers (see for instance Laffontaine and Slade, 1997, and Sheppard, 1993). In this respect, besides providing a theory of delegation based on information asymmetries, we also contribute to assess the welfare properties of vertical arrangements based on vertical price control (RPM) in imperfectly competitive industries. Specifically, normative implications advocating for the lawfulness of vertical arrangements based on retail price control are drawn.

3 **The Model**

**Environment:** Consider a downstream industry where two retailers, $R_1$ and $R_2$, producing two symmetrically differentiated products compete by setting quantities. The production of final output requires an essential raw input which is supplied by a pair of exclusive upstream suppliers, $S_1$ and $S_2$. The inverse market demand function facing the good produced by the retailer-$i$ is uncertain and is defined by:\textsuperscript{8}

$$p_i(\tilde{\theta}, e_i, e_{-i}, q_i, q_{-i}) = \tilde{\theta} + e_i + \sigma e_{-i} - q_i + \rho q_{-i},$$

where $q_i$ is the quantity produced of good-$i$, $p_i$ is the retail price level charged for this product, and $\tilde{\theta}$ is a common shock affecting both demands.

\textsuperscript{6}Relatedly, Kastl and Piccolo (2005) examine a dynamic game where vertical separation can be supported as an equilibrium strategy in a infinitely repeated game, provided that contracting externalities are intense enough and information asymmetries are not particularly severe.

\textsuperscript{7}An exception is Martimort and Piccolo (2006).

\textsuperscript{8}This demand system is generated by a representative consumer whosew preferences are:

$$V(q_1, q_2, I, \theta) = \sum_{i=1,2} e_i(q_i + \sigma q_{-i}) + \theta \left( \sum_{i=1,2} q_i \right) - \frac{1}{2} \left( \sum_{i=1,2} q_i^2 \right) + \rho q_1 q_2 + I.$$

The sign of $\rho$ determines whether produced goods are complements, independents or substitutes. Whereas the sign of $\sigma$ determines whether retailers impose positive ($\sigma \geq 0$) or negative externalities ($\sigma < 0$) on each other through their unverifiable activities.
This parameter is drawn on the compact support $\Theta \equiv [\bar{\theta}, \theta]$ according to the cumulative distribution function $F(\theta)$ having a (strictly) positive density, $f(\theta)$, with $|f'(\theta)|$ being bounded. We assume also that the hazard rate $h(\theta) = (1 - F(\theta))/f(\theta)$ is monotonically decreasing, $h(\theta) < 0$ for all $\theta \in \Theta$. The realization of $\hat{\theta}$ is private information of retailers at the time contracts are signed, and $e_i$ denotes an unverifiable activity (effort) performed by each retailer-$i$.

The effort variable has two effects on the (inverse) demands system. First, it enhances own consumers’ willingness to pay.$^9$ Second, it may also influence the competitor’s inverse demand.

Following Che and Hausch (1999) we say that efforts have a cooperative (resp. selfish) nature if $\sigma \geq 0$ (resp. $< 0$). Throughout we shall assume that $|\sigma| \leq 1$ in order to guarantee that own-effort effects are larger than cross ones, i.e., $\partial p_i(.)/\partial e_i \geq \partial p_i(.)/\partial e_{-i}$.\(^{10}\) Furthermore, $\rho$ denotes the measure of products’ differentiation and it also satisfies $|\rho| \leq 1$ in order to guarantee that own-price effects are larger than cross-price ones in the direct demands’ system, $\partial q_i(.)/\partial p_i \geq \partial q_i(.)/\partial p_{-i}$.

We shall focus either on the case where $\sigma \rho \geq 0$ or in that where $\sigma > 0$ and $\rho < 0$ which captures a free-riding set-up.$^{11}$

Providing effort is costly and we denote by $\psi(e_i)$ the disutility function satisfying $\psi'(e_i) \geq 0$, the Inada conditions $\psi'(0) = 0$, $\lim_{e_i \rightarrow +\infty} \psi'(e_i) = +\infty$, and $\psi''(e_i) > 1/2$ for all $e_i \in \mathbb{R}_+$ in order to guarantee well behaved optimization programs.

Finally, as for production technologies, we assume that both upstream and downstream firms produce at constant marginal costs normalized to zero.

Most of our results will be derived by using Taylor expansions, hence they hold for $\Delta \theta = \bar{\theta} - \theta$ small enough. Alternatively, they also hold more generally whatever $\Delta \theta$ when $\psi(e)$ is quadratic since Taylor expansions are then exact.

**Mechanisms:** We assume that the mechanisms ruling each hierarchy are private, i.e., cannot have any commitment power. However, the choice of a contractual mode, i.e., whether quantity forcing QF or resale price maintenance RPM is chosen, is itself observable.

Within this framework, we follow Myerson (1982) and Martimort (1996) to characterize the set of incentive feasible allocations in each hierarchy. Indeed, for any output choice made by retailer-$i$, there is no loss of generality in looking for $S_i$’s best response to $S_{-i}$’s contractual offer within the class of direct and truthful mechanisms.

If a QF arrangement has been chosen, such mechanism is of the form $\{t_i(\hat{\theta}_i), q_i(\hat{\theta}_i)\}_{\hat{\theta}_i \in \Theta}$ where $\hat{\theta}_i$ is retailer-$i$’s report on the demand parameter, and $t_i(\hat{\theta}_i)$ (resp. $q_i(\hat{\theta}_i)$) the corresponding fixed-fee (resp.

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$^9$This assumption seems completely natural if one interprets effort as being production of indivisible goods and/or investment in advertising.

$^{10}$One could reasonably interpret the effort as being production of indivisible services and/or advertising investments. When goods are substitutes $e_i$ affects negatively $p_j$ as it may have a business stealing effect, i.e., $\sigma < 0$. Whereas, when goods display forms of complementarities or network effects it is natural to assume that $e_i$ has also a positive impact on $p_j$, i.e., $\sigma > 0$.

$^{11}$In fact, assuming that produced goods are complements, $\rho > 0$, and that efforts create negative externalities, $\sigma < 0$, seems unreasonable, thus it can be ruled out.
output) paid (resp. supplied) to (resp. from) the supplier $S_i$. If RPM is preferred, an incentive mechanism is of form \( \{t_i(\hat{\theta}_i), q_i(\hat{\theta}_i), p_i(\hat{\theta}_i)\} \) where $p_i(\hat{\theta}_i)$ denotes the retail price of good-$i$ following report $\hat{\theta}_i$.

A RPM arrangement is more complete relative to QF because this latter contract leaves unspecified the retail market price. Therefore, it is reasonable to think of a QF arrangement as being equivalent to a \textit{vertically decentralized} organizational structure or to an \textit{arm’s length} relationship. Under this contractual scheme, the upstream manufacturer does not have enough instruments to monitor the promotional effort level exerted by the retailer. Instead, RPM will be seen to replicate the constrained \textit{vertical integration} outcome, since by dictating the retail price and the quantity sold to the retailer, the upstream manufacturer is able to \textit{control directly} the retailer’s effort level, although there is no possibility of full extraction.

\textbf{Timing, Strategies and Equilibrium Concept:} Firms play a three-stage game whose sequence of events is as follows:

1. Each supplier $S_i$ either publicly commits to verify the (ex post) realization of the retail price in market-$i$, together with $R_i$’s sales level or, alternatively, she might give up price control and use only sales as a screening device.\footnote{In practice suppliers can credibly commit to use retail price restrictions in several ways. First, one can imagine that (observable) investments in specific monitoring technologies may induce competitors to believe that retail price restrictions will be enforced. Second, in a dynamic perspective, suppliers may well achieve the same goal through reputation effects.}

2. Uncertainty about demand realizes and only $R_1$ and $R_2$ observe it.

3. Each supplier $S_i$ secretly offers a menu of contracts to his own retailer $R_i$. If the contract is accepted, $R_i$ reports a message $\hat{\theta}_i \in \Theta$ to $S_i$ about the realized demand state. Effort is exerted, product market competition takes place and, finally, payments are made after verifiable actions have been observed. If the offer is turned down, $S_i$ and $R_i$ enjoy their outside options which are normalized to zero, and $R_{-i}$ acts as a monopolist in the downstream market.

The equilibrium concept we use is \textit{Perfect Bayesian Equilibrium} with the added refinement that, provided $R_i$ receives any unexpected offer from $S_i$ he still believes that $R_{-i}$ will produce the same quantity. We denote by $\mathcal{G}$ the three stages game of contractual choices cum mechanisms offers and market interactions.

\textbf{Complete Information Benchmark:} Under complete information the two kinds of contracts QF and RPM implement the same allocation. Both mechanism can emerge in an equilibrium of $\mathcal{G}$. Let us denote by \( \{p^*(\theta), q^*(\theta), e^*(\theta)\} \) the solution to the following equation:

\begin{equation}
\theta + (1 + \sigma)\phi(q^*(\theta)) - (2 - \rho)q^*(\theta) = 0,
\end{equation}

with $\phi(.) = \psi(\cdot)^{-1}$, $q^*(\theta) = \psi'(e^*(\theta))$ and $p^*(\theta) = q^*(\theta)$.

\textbf{Proposition 1} Assume $\Delta \theta$ is small enough and $\psi''(e) > \max \left\{ \frac{1}{2}, \frac{1+\rho}{2-\rho} \right\}$ for all $e \in \mathbb{R}_+$. Then the following properties are satisfied:
• Efforts, outputs and fixed fees are the same under both contractual regimes: \( q^P(\theta) = q^Q(\theta) = q^*(\theta), e^Q(\theta) = e^P(\theta) = e^*(\theta), \) and \( t^P(\theta) = t^Q(\theta) \) for all \( \theta; \)

• Efforts and outputs are increasing in \( \theta, \) i.e., \( \dot{e}^*(\theta) \geq 0 \) and \( \dot{q}^*(\theta) \geq 0 \) for all \( \theta. \)

This result is similar to Katz (1991) arguing that the use of agents in games of competing hierarchies does not affect the equilibrium outcomes if there exists a contract which perfectly internalizes the vertical externality between the principals and their agents. We shall see below that this result drastically changes under asymmetric information. Furthermore, it should be noticed that, under complete information, both types of contracts produce the same external effect on third parties, i.e., they generate the same consumers’ surplus and total welfare. As we prove, this property no longer holds once asymmetric information is introduced in the model, this result will thus motivate our welfare analysis.

4 Preliminary Remarks

Before presenting the formal analysis it is worthwhile explaining why the issue of contracting incompleteness must be addressed in an asymmetric information framework. To this end we shall use a simple example of vertical contracting under asymmetric information. The centerpiece of the argument rests on the idea that, when some aspects of agents’ performance are nonverifiable, the interaction between agency costs, second-best outcomes and the presence of non-market externalities that competing agents impose on each other may provide principals with strategic power.

Consider the simple case where while \( R_1 \) must deal with \( S_1 \) to receive inputs, whereas \( R_2 \) and \( S_2 \) are vertically integrated and produce as a unique entity, labeled \( S_2-R_2. \) Furthermore, assume that providing effort is too costly for both retailers, i.e., \( \psi(e) = +\infty \) for all \( e \in \mathbb{R}_+, \) so that each (inverse) demand function is defined by \( p_i(\bar{\theta}, q_i, q_j) = \bar{\theta} - q_i + \rho q_{-i} \) for \( i = 1, 2. \)

To begin with, observe that when \( S_1 \) monitors both retail price and sales he can fully extract the information rent of his retailer and infer (correctly at the equilibrium) the production of \( S_2-R_2. \)

Lemma 2 When \( S_1 \) monitors both sales and retail price no rents are left to \( R_1. \)

At an equilibrium of the game outcomes must solve the following type-contingent system of first-order conditions equalizing marginal revenues to marginal costs within each pair:

\[
\theta - 2q_i^P(\theta) + \rho q_i^P(\theta) = 0 \quad \text{for } i = 1, 2.
\]

\[^{13}\text{Since demands are increasing in } \theta \text{ it is completely natural to focus on cases where output is also increasing in } \theta. \]

\[^{14}\text{This mechanism is similar to Gal-Or (1991).} \]
This yields a symmetric equilibrium output:

\[ q_i^P(\theta) = \frac{\theta}{2 - \rho} \text{ for } i = 1, 2. \]

Consider now the optimal contract when \( S_1 \) no longer controls the retail price of his retailer. The upstream supplier does not have enough instruments to achieve full extraction. The underlying idea is standard, high-demand types have an incentive to mimic low-demand ones, and \( S_1 \) has to grant information rents in order to induce information revelation. To trade-off optimally allocative efficiency and rent extraction, the supplier must also distort downward the output relative to the complete information level. In this context, as we have argued above, the Revelation Principle applies in \( S_1-R_1 \) hierarchy (keeping as fixed the production of \( R_2 \)). Denoting by \( U_1(\theta) \) retailer \( R_1 \)'s information rent, we have:

\[
U_1(\theta) = p_1(\theta, q_1(\theta), q_2(\theta))q_1(\theta) - t_1(\theta) = \max_{\hat{\theta} \in \Theta} \left\{ p_1(\theta, q_1(\hat{\theta}), q_2(\hat{\theta}))q_1(\hat{\theta}) - t_1(\hat{\theta}) \right\}.
\]

Using standard techniques developed in Martimort (1996), we obtain the following first- and second-order local conditions for incentive compatibility:

\[
\hat{U}_1(\theta) = (1 + \rho q_2(\theta))q_1(\theta), \quad (IC_1)
\]

\[
(1 + \rho q_2(\theta))q_1(\theta) \geq 0, \quad (IC_2)
\]

which, together with the agent’s participation constraint

\[
U_1(\theta) \geq 0 \forall \theta \in \Theta, \quad (PC)
\]

define the set of incentive feasible allocations in \( S_1-R_1 \) hierarchy.

Expressing the fixed-fee as a function of \( U_1(\theta) \) and \( q_1(\theta) \), \( S_1 \)'s problem, denoted thereafter by \((\mathcal{P})\), can be rewritten as:

\[
(\mathcal{P}) : \max_{(q(\cdot), U_1(\cdot))} \int_{\Theta} \left\{ p_1(\theta, q(\theta), q_2(\theta))q_1(\theta) - U_1(\theta) \right\} f(\theta)d\theta
\]

subject to \((IC_1)\), \((IC_2)\) and \((PC)\).

A contractual externality similar to the competing-contracts effect studied in Martimort (1996) is at play in this environment. Specifically, information rents have now a different structure relative to those arising in a standard isolated principal-agent framework. In fact, the revelation of a good demand state provides two pieces of information to \( S_1 \). First, it reveals that consumers’ willingness to pay is large. Second,
since $\theta$ affects symmetrically demands, it also provides information on the market behavior of the opponent hierarchy, which (in equilibrium) will produce more in better demand states. In particular, information rents now are also affected by a measure of the competitiveness, $\rho q_2(\theta)$, indicating how aggressively the integrated structure $S_2-R_2$ behaves at the market stage. The steeper $q_2(\theta)$ is the lower is its level at any given $\theta$. This implies in turn that, when $\dot{q}_2(\theta)$ increases, the integrated pair $S_2-R_2$ behaves less (resp. more) aggressively at the market stage so as to make $S_1$ less (resp. more) willing to grant information rents if goods are substitutes (resp. complements).

By using standard techniques, and assuming that information rents are increasing in $\theta$ so that $(PC)$ binds only at $\theta$, we solve a relaxed program $(P')$, neglecting $(IC_2)$ in $(P)$,

$$(P') : \max_{\{q_1(\cdot)\}} \int_\theta \{p_1(\theta, q_1(\theta), q_2(\theta))q_1(\theta) - h(\theta)(1 + \rho \dot{q}_2(\theta))q_1(\theta)\} dF(\theta).$$

Optimizing pointwise yields the following first-order condition which is both necessary and sufficient (given our assumptions ensuring concavity of the objective) for optimality:

$$(3) \quad \theta - 2q_1^Q(\theta) + \rho q_2^Q(\theta) - h(\theta)(1 + \rho q_2^Q(\theta)) = 0.$$

At a best-response to what the integrated structure produces, $S_1$ equalizes her virtual marginal revenues to zero at each $\theta$. Under asymmetric information, everything happens as if the demand parameter $\theta$ is now replaced by a lower virtual demand parameter, namely $\theta - h(\theta)(1 + \rho q_2^Q(\theta))$, which captures also the extent of competitive pressure on the downstream market.

On the other hand, $S_2-R_2$ chooses its output level so as to maximize profit under complete information, i.e., $\theta - 2q_2^Q(\theta) + \rho q_1^Q(\theta) = 0$. From equation (3) one can show that in a Nash equilibrium of the market subgame $R_1$’s output, $q_1^Q(\theta)$, must be a solution to the following differential equation:

$$(4) \quad \dot{q}_1^Q(\theta) = \frac{(2 + \rho)(\theta - (2 - \rho)q_1^Q(\theta) - h(\theta))}{\rho^2 h(\theta)},$$

with the boundary condition $q_1^Q(\theta) = q_1^F(\theta)$.

Let $\Pi_1^Q(\theta)$ be $S_1$’s state contingent profits under a given mechanism $\omega \in \{QF, RPM\}$, and denote by $\Pi'_1 = E[\Pi_1^Q(\theta)]$ its expectation. Formally we have:

$$\Pi_1^Q(\theta) = q_1^Q(\theta)(\theta + \sigma q_2^Q(\theta) - \dot{q}_1^Q(\theta) - h(\theta)(1 + \sigma q_2^Q(\theta)))$$

and,

$$\Pi_1^R(\theta) = q_1^R(\theta)(\theta + \sigma q_2^R(\theta) - \dot{q}_1^R(\theta))$$

The next Proposition summarizes the results for this simple case.
Proposition 3 The following properties are satisfied at each $\rho \in [-1, 1]$:

- $q_1^Q(\theta) \leq q_1^P(\theta)$ for all $\theta$ (with equality only at $\bar{\theta}$);
- $S_1$ strictly prefers to control the retail price; i.e., $\Pi_1^P > \Pi_1^Q$.

When the retail price is not monitored, the upstream manufacturer chooses the quantity schedule so as to equalize her virtual marginal revenues to zero. By doing so, output is reduced below the complete information level for rent extraction reasons. On the contrary, under vertical price control, $S_1$ can extract $R_1$’s private information at no costs, so $S_1$ can always replicate any allocation implemented when sales are the only screening instrument by means of vertical price control, but of course this is never optimal.

The lesson of this simple example is that restricting the set of screening instruments used to control the agents’ behavior, i.e., not controlling the retail price in the present setting, is never worthwhile when the supplier can achieve the vertically integrated structure first-best profit under complete contracting.

Of course, when, besides produced output, the agent does not perform any other unverifiable task affecting both his own principal’s payoff and the competing organization objective function, a complete contract can always replicate every allocation obtained with an incomplete one, hence contracting incompleteness has no commitment value. To properly address the question one should thus consider environments where information asymmetries prevent upstream suppliers to achieve full extraction even under complete contracting. The rest of the analysis goes in this direction and, as we shall see in the next sections, in such a framework the interaction between contracting externalities created by (extra) unverifiable tasks performed by agents, information rents and second-best outcomes associated to information asymmetries may give a commitment value to an incomplete contract.

5 An Illustrative Example

We start our analysis with a simple example considering again a fully integrated firm competing with a vertically separated supplier-retailer hierarchy. Once the argument highlighted by this simple example will be clear, it will be easier to understand the properties of our complete model, which entails symmetric competing hierarchies. To further simplify the analysis, we also assume that $S_2$-$R_2$ does not exert any effort (formally this amounts to assume $\psi_2(e_2) = +\infty$ for all $e_2 \in \mathbb{R}_+$).

The system of (inverse) market demands is thus defined by:

$$p_1(\tilde{\theta}, e_1, q_1, q_2) = \tilde{\theta} + e_1 - q_1 + \rho q_2 \quad \text{and} \quad p_2(\tilde{\theta}, e_1, q_2, q_1) = \tilde{\theta} + \sigma e_1 - q_2 + \rho q_1.$$

Observe that none of the contractual regimes allows $S_1$ to fully extract $R_1$’s information rents since even when the retail price can be contracted upon, $S_1$ cannot disentangle the impact of the demand parameter $\theta$ and the retailer’s effort on the residual demand the latter faces. The underlying idea is as follows: the
possibility for the retailer to claim that large sales are due to a high effort level, whereas they result instead from a high demand, induces the upstream manufacturer to give up some information rent to the high demand retailer (high $\theta$) in order to induce truth-telling. As a result, the second-best allocation will be characterized by a downward distortion of both quantity and effort supplied by the retailer when he faces low demand states. This information rent, of course, depends on the chosen contractual mode.

As we shall clearly point out below, the choice of the contractual mode has two opposite effects on $S_1$'s profits whenever $R_1$'s effort and output have the same impact on $S_2$-$R_2$'s (inverse) demand, i.e., $\text{sign}(\rho) = \text{sign}(\sigma)$. On the one hand, committing to an incomplete contract may lead the upstream supplier to grant more information rents relative to RPM because a screening instrument is given up: an agency effect. On the other hand, an incomplete contract may have a strategic value relative to a complete one in that it enables $S_1$ to force $S_2$-$R_2$ to behave in a more friendly manner at the market stage: a strategic effect. Indeed, besides creating a vertical externality between $S_1$ and $R_1$, an incomplete contract also generates an horizontal externality affecting the competitive behavior of the rival pair $S_2$-$R_2$. The tension between these two effects will depend upon the severity of the agency problem. More precisely, when information rents are small enough, the agency effect will be dominated by the strategic one. In a free riding set-up, i.e., $\sigma > 0$ and $\rho < 0$, however, the oversupply of effort provided by $R_1$ under QF makes its competitor more aggressive at the market stage so as to make the strategic effect reinforcing the agency one. An incomplete contract is thus always dominated by the complete one.

Below we solve the game in two steps. First, we characterize the market allocation under both contractual regimes. Then, the equilibrium contract will be derived by using a backward induction argument.

$S_2$-$R_2$'s Program: To begin with, let us briefly analyze the program solved by $S_2$-$R_2$. For each realization of $\hat{\theta}$ and any incentive feasible allocation $\{e_1(\theta), q_1(\theta)\}_{\theta \in \Theta}$ implemented by $S_1$, the vertically integrated structure $S_2$-$R_2$ solves:

\[
\max_{q_2 \in \mathbb{R}^+} p_2(\theta, e_1(\theta), q_2, q_1(\theta)) q_2.
\]

The corresponding necessary and sufficient condition yields the following reaction function at each $\theta$:

\[
q_2(\theta) = \frac{\theta + \rho q_1(\theta) + \sigma e_1(\theta)}{2},
\]

From (6) one can infer that $S_1$ has an incentive to choose strategically the contractual mode to soften $S_2$-$R_2$'s behavior at the competitive stage. In fact, the contractual regime maximizing the term $\{\sigma e_1(\theta) + \rho q_1(\theta)\}$ in equation (6) will be the most preferred one either when goods are substitutes or when they are complements, everything else being kept constant.\footnote{Obviously, this effect must be traded off with the effect of contracting incompleteness on information rents, which may well go in the opposite direction of decreasing $S_1$’s profits.}

Complete Contracting: Let us first consider a RPM arrangement. In this case $S_1$ can contract also the
retail market price besides the quantity supplied by the downstream firm to final consumers. The effort level is then indirectly fixed as a function of $\theta$ through the inverse demand, i.e., $e_1 = p_1 + q_1 - \rho q_2 - \theta$. RPM is less flexible than QF just because when the retailer faces a retail price target, she is indirectly forced to choose the effort level.\footnote{Indeed, under retail price restrictions the upstream producer has full control of all available instruments. See also Blair and Lewis (1994) and Martimort and Piccolo (2006).}

Let us define retailer $R_1$'s information rent as:

$$U_1(\theta) = p_1(\theta)q_1(\theta) - \psi(p_1(\theta) + q_1(\theta) - \rho q_2(\theta) - \theta) - t_1(\theta).$$

By definition of incentive compatibility, we have:

$$U_1(\theta) = \max_{\hat{\theta} \in \Theta} \left\{ p_1(\hat{\theta})q_1(\hat{\theta}) - \psi(p_1(\hat{\theta}) + q_1(\hat{\theta}) - \rho q_2(\hat{\theta}) - \theta) - t_1(\hat{\theta}) \right\}.$$

This yields the following first- and second-order local conditions for incentive compatibility:

$$\dot{U}_1(\theta) = (1 + \rho \dot{q}_2(\theta))\psi'(e_1(\theta)), \quad (IC_1)$$

$$(1 + \rho \dot{q}_2(\theta))(1 + \dot{e}_1(\theta) + \rho \dot{q}_2(\theta)) \geq 0, \quad (IC_2).$$

Then, $(IC_1)$ and $(IC_2)$ together with the retailer’s participation constraint

$$U_1(\theta) \geq 0 \; \forall \; \theta \in \Theta, \quad (PC)$$

define the set of incentive feasible allocations in $S_1$-$R_1$ hierarchy for a fixed output schedule $q_2(.)$ chosen by the rival pair $S_2$-$R_2$. $S_1$'s problem, denoted thereafter by $(P^F)$, is to design a menu of contracts to maximize the expected franchise fee he receives from $R_1$ subject to the participation and incentive compatibility constraints, together with the additional restriction required by the retail price target.

Expressing the fixed fee as a function of the retailer’s revenue and information rent we can thus write $(P^F)$ as:

$$(P^F): \max_{\{U_1(.), q_1(.), e_1(.)\}} \int_{\Theta} \{ (\theta + e_1(\theta) - \rho q_2(\theta) - q_1(\theta))q_1(\theta) - \psi(e_1(\theta)) - U_1(\theta) \} f(\theta) d\theta,$$

subject to $(IC_1)$, $(IC_2)$ and $(PC)$.

We will first assume and check ex post that $(1 + \rho \dot{q}_2(\theta)) \geq 0$ for all $\theta$. Then $U_1(\theta)$ is increasing and
(PC) binds only at $\theta$, i.e., the lowest demand parameter. We obtain immediately:

$$U_1(\theta) = \int_{\theta}^{\bar{\theta}} (1 + \rho q_2(x))\psi'(e_1(x))dx.$$ 

Integrating by parts into the expression of the maximand of $(P^P)$ and neglecting $(IC_2)$ (that will be checked ex post also) yields a simplified optimization problem $(P^{P*})$:

$$(P^{P*}) : \max_{\{q_1(\cdot), e_1(\cdot)\}} \int_{\theta}^{\bar{\theta}} \left\{ (\theta + e_1(\theta) - q_1(\theta) + \rho q_2(\theta))q_1(\theta) - \psi(e_1(\theta)) - h(\theta)(1 + \rho q_2(\theta))\psi'(e_1(\theta)) \right\} f(\theta)d\theta.$$ 

At a best response to the schedule $q_2(\cdot)$ implemented by the competing pair $S_2-R_2$, the production and effort in $S_1-R_1$ hierarchy are respectively given by the following first-order conditions:

$$q_1(\theta) = p_1(\theta) = \theta + e_1(\theta) + \rho q_2(\theta) - q_1(\theta),$$

$$q_1(\theta) = \psi'(e_1(\theta)) + h(\theta)(1 + \rho q_2(\theta))\psi''(e_1(\theta)).$$

Notice that the dichotomy result (Laffont and Tirole, 1993, Ch. 3) holds in this setting, that is since under RPM the only screening variable is effort, which is downward distorted relative to the complete information level, output is produced according to the efficient rule that would prevail under complete information.

By using (6) together with (7) and (8), one can also check that the allocation $\{e_1^P(\theta), q_1^P(\theta)\}_{\theta \in \Theta}$ solves the following system of differential equations:

$$\dot{q}_1^P(\theta) = \frac{2(q_1^P(\theta) - \psi'(e_1^P(\theta))) + h(\theta)\psi''(e_1^P(\theta))(2 + \rho(1 + \sigma e_1^P(\theta)))}{\rho^2 h(\theta)\psi''(e_1^P(\theta))},$$

and,

$$\dot{e}_1^P(\theta) = \frac{(4 - \rho^2)q_1^P(\theta) - (2 + \rho)}{2 + \rho \sigma},$$

with the boundary conditions $q_1^P(\bar{\theta}) = q_1(\bar{\theta})$ and $e_1^P(\bar{\theta}) = e_1(\bar{\theta})$.

Simple algebra shows that at an equilibrium the complete information allocation, $\{e_1^*(\theta), q_1^*(\theta)\}_{\theta \in \Theta}$, is now defined by the following equations:

$$(2 + \rho)\theta + (2 + \rho \sigma)\phi(q_1^*(\theta)) - (4 - \rho^2)q_1^*(\theta) = 0,$$

17Given concavity of the objective, these conditions are also sufficient.
and
\[ q_1^*(\theta) = \psi'(e_1^*(\theta)). \]

Next Proposition characterizes some useful features of the equilibrium allocation under RPM.

**Proposition 4** Assume \( \Delta \theta \) is small enough and \( \psi''(e) > \max \left\{ \frac{1}{2}, \frac{2+\sigma\rho}{1-2\rho(\sigma+\rho)} \right\} \) for all \( e \in \mathbb{R}_+ \). Then the following properties hold:

- \( e_1^P(\theta) \leq e_1^*(\theta) \) and \( q_1^P(\theta) \leq q_1^*(\theta) \) for all \( \theta \) (with equality holding only at \( \bar{\theta} \))
- \( (IC_2) \) always holds.

These are standard properties of adverse selection models, so we shall not discuss them (see Laffont and Martimort, 2002).

**Incomplete Contracting:** Under QF, \( S_1 \) does not observe the retail price, but she still observes the quantity supplied by \( R_1 \) on the retail market. A QF mechanism can thus be viewed as an incomplete contract relative to RPM since the upstream producer voluntarily gives up one of the possible screening instruments.

Let us now redefine retailer \( R_1 \)’s information rent under a QF regime as:

\[ U_1(\theta) = -t_1(\theta) + \max_{e_1 \in \mathbb{R}_+} \left\{ (\theta + e_1 - q_1(\theta) + \rho q_2(\theta))q_1(\theta) - \psi(e_1) \right\}. \]

By definition of incentive compatibility, we have:

\[ U(\theta) = \max_{\hat{\theta} \in \Theta} \left\{ -t_1(\hat{\theta}) + \max_{e_1 \in \mathbb{R}_+} \left\{ (\theta + e_1 - q_1(\hat{\theta}) + \rho q_2(\theta))q_1(\hat{\theta}) - \psi(e_1) \right\} \right\}. \]

From which we obtain the following first- and second-order local conditions for incentive compatibility:

(11) \[ \dot{U}_1(\theta) = (1 + \rho q_2(\theta))q_1(\theta), \]

(12) \[ (1 + \rho q_2(\theta))q_2(\theta) \geq 0, \]

to which we must also add the participation constraint:

(13) \[ U(\theta) \geq 0 \forall \theta \in \Theta. \]

\[ 18 \text{One can easily show that under some regularity conditions these properties also hold when } \Delta \theta \text{ is arbitrarily chosen.} \]
This leads to state $S_1$’s contracting problem, denoted thereafter as $(P^Q)$, as:

$$(P^Q) : \max_{\{U_1(.)q_1(.)\}} \int_{\theta}^{\theta'} \{(\theta + \phi(q_1(\theta)) - q_1(\theta) + \rho q_2(\theta)q_1(\theta) - U_1(\theta)) f(\theta)d\theta.$$

subject to (11), (12) and (13).

For any given quantity schedule specified by the direct revelation mechanism QF, $R_1$ gains flexibility under a quantity-fixing arrangement in the sense that the effort level is chosen to command more information rents than it would be efficient from the manufacturer’s viewpoint. More specifically, while choosing the optimal effort level, the retailer does not internalize the impact of his effort on the information rent given up by the upstream manufacturer. QF introduces a vertical externality between the manufacturer and his retailer. As rents and effort are positively related via quantity, it will be thus profitable to oversupply effort relative to RPM everything else being kept equal.

We will again first assume and check ex post that $$(1 + \rho e_2(\theta)) \geq 0$$ for all $\theta$. Then $U_1(\theta)$ is increasing and (13) binds at $\theta$ only. We obtain immediately:

$$U_1(\theta) = \int_{\theta}^{\theta'} (1 + \rho q_2(x))q_1(x)dx.$$

Integrating by parts the above expression and inserting into the maximand of $(P^Q)$ yields the expression of the relaxed program $(P^{Q'})$:

$$(P^{Q'}) : \max_{\{q_1(.)\}} \int_{\theta}^{\theta'} \{(\theta + \phi(q_1(\theta)) - q_1(\theta) + \rho q_2(\theta)q_1(\theta) - \psi(\phi(q_1(\theta))) - h(\theta)(1 + \rho q_2(\theta)q_1(\theta))\} f(\theta)d\theta.$$

At a best-response to the schedule $q_2(.)$ implemented by the $S_2$-$R_2$ pair we get:

$$(14) \quad \theta + \phi(q_1^Q(\theta)) - 2q_1^Q(\theta) + \rho q_2^Q(\theta) - h(\theta)(1 + \rho q_2^Q(\theta)) = 0.$$

Differentiating equation (6) yields $2q_2^Q(\theta) = 1 + \rho q_1^Q(\theta) + \sigma e_1^Q(\theta)$, using this condition together with $e_1^Q(\theta) = \phi'(q_1^Q(\theta))q_1^Q(\theta)$, one can immediately show that $q_1^Q(\theta)$ solves the the following differential equation:

$$(15) \quad q_1^Q(\theta) = \frac{(2 + \rho)(\theta - h(\theta)) + (2 + \rho \sigma)\phi(q_1^Q(\theta)) - q_1^Q(\theta)(4 - \rho^2)}{\rho h(\theta)(\sigma \phi'(q_1^Q(\theta)) + \rho)};$$

with the boundary condition $q_1^Q(\theta) = q_1^*$. Under the QF regime the dichotomy result no longer holds. In fact, since $S_1$ gives up a screening instrument by not controlling the retail price, the only screening device is output which must be downward distorted relative to the complete information level for rent extraction reasons. This point will be key to
prove that an incomplete contract might be preferred to a complete one.

Next Proposition characterizes some useful features of the equilibrium allocation under QF.

**Proposition 5** Assume \( \Delta \theta \) is small enough and \( \psi''(e) > \max \left\{ \frac{1}{2}, \frac{1+\sigma}{1-\rho^2} \right\} \) for all \( e \in \mathbb{R}_+ \). Then the following properties hold:

- \( e_1^Q(\theta) \leq e_1^P(\theta) \) and \( q_1^Q(\theta) \leq q_1^P(\theta) \) for all \( \theta \) (with equality only at \( \overline{\theta} \));
- \( 1 + \rho q_2^Q(\theta) \geq 0 \) for all \( \theta \in \Theta \).

**Choice of the Contractual mode:** We now turn to determine whether RPM or QF is chosen at equilibrium. As a preliminary result we state the next Proposition which provides a useful description of how outputs and efforts are ordered under both regimes. This result will be key for our equilibrium characterization.

**Proposition 6** Assume \( \Delta \theta \) is small enough and \( \psi''(e) > \max \left\{ \frac{1}{2}, \frac{2+\sigma}{4-2\rho(\sigma+\rho)}, \frac{1+\sigma\rho}{2-\rho^2} \right\} \) for all \( e \in \mathbb{R}_+ \). Then the following properties hold:

- \( e_1^Q(\theta) \geq e_1^P(\theta) \) for all \( \theta \) with equality holding only at \( \overline{\theta} \);
- \( q_2^Q(\theta) \geq q_2^P(\theta) \) (resp. \( < \)) if and only if \( \sigma \geq 0 \) (resp. \( < \)) for all \( \theta \) with equality only at \( \overline{\theta} \) when \( \sigma \neq 0 \);
- \( q_1^Q(\theta) \geq q_1^P(\theta) \) (resp. \( < \)) if and only if \( \rho \sigma \geq 0 \) (resp. \( < \)) for all \( \theta \) with equality only at \( \overline{\theta} \) when \( \rho \sigma \neq 0 \).

The economic interpretation of the above result is simple. When a QF contract is enforced, the upstream manufacturer gives up the control of the promotional activities (effort) supplied by the downstream retailer. This decentralized organizational mode, crucially, allows the downstream firm to increase his information rent by playing on his effort choice. More specifically, as effort affects positively quantity, \( R_1 \) finds it profitable to supply more effort (relative to RPM) in order to enjoy more rents.

From equation (6), the output produced by the integrated structure \( S_2-R_2 \) is directly affected by the effort level. Since \( e_1^Q(\theta) \geq e_1^P(\theta) \) for all \( \theta \), an incomplete contract provides \( S_1 \) with strategic power in that it shifts its \( S_2-R_2 \)'s reaction function. That is \( q_2(.) \) increases (resp. diminishes) when one moves from RPM to QF whenever \( \sigma \geq 0 \) (resp. \( < \)). Besides introducing a vertical externality between \( S_1 \) and \( R_1 \), an incomplete contract also creates an horizontal externality between the competing hierarchies.

By moving from RPM to QF, three different effects are at play simultaneously. First, for any given output level, \( R_1 \) will exert more effort under QF relative to RPM. Under QF, the agent is residual claimant for the full impact of his effort on enhancing demand. This effect raises effort and thus \( R_1 \)'s output: a demand-enhancing effect. Second, since sales is the only screening instrument under QF, one needs to

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\(^{19}\)Remember that under QF the downstream retailer chooses his effort according to the efficient rule \( q_1 = \psi'(e_1) \).
distort it downward for rent extraction reasons: a *rent extraction effect*. Third, because of the horizontal externality, the output of the $S_2-R_2$ pair is shifted upward (resp. downward) when goods are substitutes (resp. complements): a *strategic effect*.

When the agents’ effort choices do not create externalities, $\sigma = 0$, or goods are independent, $\rho = 0$, the strategic effect is absent. Therefore the present set-up displays the same features as the sequential monopolies model studied in Martimort and Piccolo (2006). In particular, the demand enhancing and the rent extraction effects exactly compensate each other in the cutting-edge case where $\psi''(e_1) = 0$, so $q_1^Q(\theta) = q_1^P(\theta)$ for all $\theta$.

As we have argued above, though, when products are differentiated and the effort choice creates contracting externalities, the strategic effect reinforces when $\rho > 0$. In this case QF increases effort and so it moves $q_2(.)$ in a more suitable manner relative to RPM. In fact, since effort influences negatively (resp. positively) the output produced by the competing firm when goods are substitutes (resp. complements), and QF commands an effort larger than that produced under RPM, the result follows immediately since a QF contract always forces $S_2-R_2$ to behave in a more friendly manner (relative to RPM) on the retail market.

When instead $\rho < 0$ and $\sigma > 0$, the strategic effect makes $S_2-R_2$ more aggressive at the market stage since the consumers’ willingness to pay increases and $q_2(.)$ increases, which in turn lowers $q_1(.)$ as quantities are strategic substitutes.

Now, let $\Pi_1^P = E_\theta[\Pi_1^P(\theta)]$ and $\Pi_1^Q = E_\theta[\Pi_1^Q(\theta)]$ define $S_1$’s profits under RPM and QF, respectively. Next proposition shows that incomplete contracts may have a commitment value under certain conditions related to the severity of the agency problem and to the presence of externalities between agents.

**Proposition 7** Assume $\Delta \theta$ is small enough and $\psi''(e) > \max\left\{ \frac{1}{2}, \frac{1+\sigma\rho}{2-\rho^2} \right\}$ for all $e \in \mathbb{R}_+$. Then the following properties hold:

- $\Pi_1^Q > \Pi_1^P$ whenever $\rho \sigma > 0$;
- $\Pi_1^Q \leq \Pi_1^P$ whenever $\rho \sigma \leq 0$.

Of course, once one moves from a complete to an incomplete contract, information rents increase because a screening instrument is given up and information revelation becomes more costly, an *agency cost effect*. When goods are independent or effort does not create externalities, this effect drives the upstream supplier to always prefer RPM. When goods are differentiated and there are contracting externalities such that both effort and output by $R_1$ affect in the same direction $S_2-R_2$’s demand and make it behave in a more friendly manner, the strategic effect may outweigh excessive agency costs. In fact, as an incomplete contract allows $S_1$ to force $S_2-R_2$ to behave in a more friendly manner at the market stage, it raises the supplier’s profits by increasing effort and so the expected transfer that can be extracted from $R_1$. 
6 Symmetric Hierarchies

We now extend the example presented in the previous section to a more symmetric model where each downstream retailer deals with an exclusive upstream supplier who remains asymmetrically informed on market demand. So the retailer \( R_i \) faces the following inverse demand function:

\[
p_i(\hat{\theta}, e_i, e_{-i}, q_i, q_{-i}) = \hat{\theta} + e_i + \sigma e_{-i} - q_i + \rho q_{-i}
\]

on the downstream market. Again, two effects are key in determining the equilibrium of the game \( G \). First, an incomplete contract commands more information rents. Second, for any given mechanism ruling the competing hierarchies, contractual incompleteness may create a strategic effect influencing rivals’ market behavior. Specifically, the vertical externality that a QF mechanism creates within each vertical hierarchy is translated horizontally on the competing organization. Increasing retailers’ effort, indeed, provides a beneficial effect on a supplier’s profits to the extent that it weakens the competitive stance of the opposing hierarchy on the downstream market. Of course, since we now examine a full model of competing organizations, multiple equilibria of the game of contracting mode choices can emerge.

Furthermore, as our analysis is also related to the practice of vertical restraints, which have been for a long time at heart of antitrust and regulation concerns, we shall also discuss some welfare properties of the equilibrium contracts and relate them to the use of retail price restrictions in vertical contracting.

Complete Contracting: With this hierarchy framework it is straightforward to describe the set of incentive feasible allocations within \( S_i-R_i \) pair with the following first- and second-order local conditions for incentive compatibility:

\[
(16) \quad \hat{U}_i(\theta) = (1 + \sigma \hat{e}_{-i}(\theta) + \rho \hat{q}_{-i}(\theta))\psi'(e_i(\theta)),
\]

\[
(17) \quad (1 + \sigma \hat{e}_{-i}(\theta) + \rho \hat{q}_{-i}(\theta))(1 + \hat{e}_i(\theta) + \sigma \hat{e}_{-i}(\theta) + \rho \hat{q}_{-i}) \geq 0,
\]

and the participation constraint:

\[
(18) \quad U_i(\theta) \geq 0, \ \forall \theta \in \Theta.
\]

Equipped with this characterization, we may define \( S_i \)’s problem as:

\[
(P_i^P) : \max_{\{U_i(\cdot), e_i(\cdot), q_i(\cdot)\}} \int_{\theta} \{ (\theta + e_i(\theta) + \sigma e_{-i}(\theta) - q_i(\theta) + \rho q_{-i}(\theta))q_i(\theta) - \psi(e_i(\theta)) - U_i(\theta) \} f(\theta) d\theta,
\]

subject to (16), (17) and (18).
Assume Proposition 8 characterizes some useful features of this allocation. Then the following properties hold:

\[ U_i(\theta) = \int_0^\theta (1 + \sigma \dot{e}_{-i}(\theta) + \rho \dot{q}_{-i}(\theta)) \psi'(e_i(\theta)) \, dx. \]

Inserting \( U_i(\theta) \) into the maximand of \( (P_i^P) \) and integrating by parts yields the following relaxed program \( (P_i^{P_i}) \):

\[
(P_i^{P_i}) : \max_{\{q_i(\cdot), e_i(\cdot)\}} \int_\Theta \left\{ p_i(\theta, e_i(\theta), e_{-i}(\theta), q_i(\theta), q_{-i}(\theta))q_i(\theta) - \psi(e_i(\theta)) - h(\theta)(1 + \sigma \dot{e}_{-i}(\theta) + \rho \dot{q}_{-i}(\theta))\psi'(e_i(\theta)) \right\} f(\theta) \, d\theta.
\]

Pointwise optimization with respect to \( q_i(\cdot) \) and \( e_i(\cdot) \) respectively lead to the following first-order conditions:

\[
q_i(\theta) = p_i(\theta) = \theta + e_i(\theta) + \sigma e_{-i}(\theta) - q_i(\theta) + \rho q_{-i}(\theta),
\]

\[
q_i(\theta) = \psi'(e_i(\theta)) + h(\theta)(1 + \sigma \dot{e}_{-i}(\theta) + \rho \dot{q}_{-i}(\theta))\psi''(e_i(\theta)) = 0.
\]

Let \( \{q^P(\theta), e^P(\theta)\}_{\theta \in \Theta} \) be the symmetric allocation obtained when both suppliers use complete contracts. Next Proposition characterizes some useful features of this allocation.

**Proposition 8** Assume \( \Delta \theta \) is small enough, \( \sigma + \rho < 1 \) and \( \psi''(e) > \max \left\{ \frac{1}{2}, \frac{1+\sigma}{2-\rho}, \frac{1+\sigma}{2(1-\sigma-\rho)} \right\} \) for all \( e \in \mathbb{R}_+ \). Then the following properties hold:

- \( e^P(\theta) \leq e^*(\theta) \) and \( q^P(\theta) \leq q^*(\theta) \) for all \( \theta \) (with equality only at \( \bar{\theta} \));
- Information rents are increasing in \( \theta \) and the local-second-order condition (17) holds.

**Incomplete Contracting:** As before, under this contractual regime the upstream supplier gives up the retail price as a screening instrument and types’ revelation must be elicited only through sales. Proceeding as before, the retailer \( R_i \)'s information rent under a QF regime can be rewritten as:

\[ U_i(\theta) = -t_i(\theta) + \max_{\theta_i \in \mathbb{R}_+} \left\{ (\theta + e_i + \sigma e_{-i}(\theta) - q_i(\theta) + \rho q_{-i}(\theta))q_i(\theta) - \psi(e_i) \right\}. \]

By incentive compatibility, we immediately obtain the following local first- and second-order conditions:

\[
\dot{U_i}(\theta) = (1 + \sigma \dot{e}_{-i}(\theta) + \rho \dot{q}_{-i}(\theta))q_i(\theta),
\]

\[
(1 + \sigma \dot{e}_{-i}(\theta) + \rho \dot{q}_{-i}(\theta))q_i(\theta) \geq 0.
\]
Incentive feasible allocations must also satisfy the usual participation constraint:

\[ U_i(\theta) \geq 0, \quad \forall \theta \in \Theta. \]  

(23)

Taking into account that \( R_i \)'s effort satisfies \( e_i(\theta) = \phi(q_i(\theta)) \), \( S_i \)'s problem \( (P_i^Q) \) can be rewritten as:

\[
(P_i^Q) : \max_{\{U_i(\cdot), q_i(\cdot)\}} \int_{\Theta} \left\{ (\theta + \phi(q_i(\theta))) + \sigma e_{-i}(\theta) - q_i(\theta) + \rho q_{-i}(\theta)q_i(\theta) - \psi(\phi(q_i(\theta))) - U_i(\theta) \right\} f(\theta) \, d\theta,
\]

subject to (21), (22) and (23).

Assuming that \( (1 + \sigma \hat{e}_{-i}(\theta) + \rho \hat{q}_{-i}(\theta)) \geq 0 \) (a condition to be also checked ex post), \( U_i(\theta) \) is increasing and thus (23) binds at \( \theta \) only. Hence we get:

\[
U_i(\theta) = \int_0^\theta (1 + \sigma \hat{e}_{-i}(x) + \rho \hat{q}_{-i}(x))q_i(x) \, dx.
\]

Inserting \( U_i(\theta) \) into the maximand of \( (P_i^Q) \) and integrating by parts yields the new expression of the relaxed program \( (P_i^{Q_0}) \):

\[
(P_i^{Q_0}) : \max_{\{q_i(\cdot)\}} \int_{\Theta} \left\{ p_i(\theta, e_i(\theta), e_{-i}(\theta), q_i(\theta), q_{-i}(\theta))q_i(\theta) - \psi(\phi(q_i(\theta))) - h(\theta)(1 + \sigma \hat{e}_{-i}(\theta) + \rho \hat{q}_{-i}(\theta))q_i(\theta) \right\} f(\theta) \, d\theta.
\]

Optimizing pointwise yields the following first-order condition:

\[ \theta + \phi(q_i(\theta)) - 2q_i(\theta) + \sigma e_j(\theta) + \rho q_j(\theta) - h(\theta)(1 + \sigma \hat{e}_j(\theta) + \rho \hat{q}_j(\theta)) = 0, \]

(24)

together with the effort optimality condition:

\[ e_i(\theta) = \phi(q_i(\theta)). \]

(25)

Let \( \{q^Q(\theta), e^Q(\theta)\}_{\theta \in \Theta} \) denote the output-effort pair when both suppliers offer an incomplete contract. Next Proposition characterizes some useful features of this allocation.

**Proposition 9** Assume \( \Delta \theta \) is small enough and \( \psi''(e) > \max \left\{ \frac{1}{2}, \frac{1+\sigma}{2-\rho}, \frac{1+2\sigma}{2(1-\rho)} \right\} \) for all \( e \in \mathbb{R}_+ \). Then the following properties hold:

- \( e^Q(\theta) \leq e^*(\theta) \) and \( q^Q(\theta) \leq q^*(\theta) \) for all \( \theta \) (with equality only at \( \theta \));

- Information rents are increasing and (22) holds.
6.1 Equilibrium Characterization

We now turn to characterize the pure strategy equilibrium choice of contracting mode. As we show, besides the extra agency costs associated to an incomplete contract which are kept small in the limit of small uncertainty, two key effects are at play once one moves from a complete to an incomplete contract in this symmetric hierarchies framework. As we have argued above, an incomplete contract makes a retailer willing to exert more effort relative to a complete mechanism since the agent is residual claimant of the full impact of his non-market activities (effort) on demand enhancing when retail price is not monitored. This in turn might affect: (i) the effort-output choice of the competing retailer via the “demand” driven horizontal externality: a direct strategic effect; and (ii) it effects own reaction function and in turn the output-effort pair of the competing hierarchy: an indirect strategic effect. Depending upon the nature of downstream externalities, i.e., cooperative versus selfish effort and complements versus substitutes goods, those effects might well go in opposite directions.

The next result extends Proposition 7 to a symmetric hierarchies environment and summarizes our results.

**Proposition 10** Assume $\Delta \theta$ is small enough, $-1 < \sigma + \rho < 1$ and that $\psi''$ is large enough. Then the following properties hold:

- If $\rho \sigma > 0$ the game $G$ displays multiple (symmetric) equilibria in the contractual choice.
- If $\rho \sigma \leq 0$ there are multiple equilibria when $|\rho| \leq \min \left\{ 1, \frac{\sigma}{\psi''} \right\}$. When $\min \left\{ 1, \frac{\sigma}{\psi''} \right\} = \frac{\sigma}{\psi''}$ the unique equilibrium entails complete contracts if $\frac{\sigma}{\psi''} \leq |\rho| \leq \min \left\{ 1, \frac{\sigma(2\psi''+1)}{2\psi''} \right\}$. No equilibria in pure strategy exist for $|\rho| > \frac{\sigma(2\psi''+1)}{2\psi''}$ whenever $\min \left\{ 1, \frac{\sigma(2\psi''+1)}{2\psi''} \right\} = \frac{\sigma(2\psi''+1)}{2\psi''}$.

The intuition underlying this result is again driven by the different impact that the two kinds of mechanisms have on retailers’ effort choices.

Let us begin by explaining why incomplete contracts can be part of a PBE of $G$. Assume then, without loss of generality, that an incomplete contract rules the hierarchy $S_{-i}R_{-i}$. We must consider three cases.

- If efforts have a cooperative value ($\sigma > 0$) and goods are complements ($\rho > 0$), then when $S_i$ moves from a complete to an incomplete contract $R_i$’s effort increases, this has a direct positive effect on $R_{-i}$’s reaction function which shifts upward, thus making $S_i$ better off as goods are complements. Moreover, a larger $e_i(\theta)$ has also a positive effect on $R_i$’s demand and so it increases $q_{-i}(\theta)$ via a higher $q_i(\theta)$. Of course, both effects are beneficial to $S_i$ who prefers an incomplete contract to a complete one in the limit of small uncertainty.

- Assume now that efforts have a selfish nature ($\sigma < 0$) and goods are substitutes ($\rho < 0$). In this case increasing $R_i$’s effort through the choice of an incomplete contract lowers $R_{-i}$’s reaction function and,
as a consequence, it relax retail competition. Moreover, it also increases \( q_i(\theta) \) so to further reduce \( q_{-i}(\theta) \). Both effects again go in the same direction of increasing \( S_i \)'s profits, and incomplete contracts remain an equilibrium of \( G \).

- Finally, in the free-riding case, i.e., \( \sigma > 0 \) and \( \rho \leq 0 \), when \( S_i \) moves from a complete to an incomplete contract the increased effort by \( R_i \) has a direct positive effect on \( R_{-i} \)'s reaction function which shifts upward thus making \( R_{-i} \) more aggressive at the market stage. However, a larger \( q_{-i}(\theta) \) calls for a higher \( e_{-i}(\theta) \), which has a positive effect on \( R_i \)'s demand. Of course, the latter effect dominates the former one when competition is not very intense, so that incomplete contract remain an equilibrium of \( G \). The opposite is true when \( |\rho| \) is large enough.

Let us now explain why complete contracts are also an equilibrium of \( G \). Assume then that a complete contract rules the hierarchy \( S_{-i} \cdot R_{-i} \). We must look at \( S_i \)'s incentive to play a complete contract (relative to an incomplete one). Again, three cases must be discussed.

- When \( \sigma > 0 \) and \( \rho > 0 \), moving from an incomplete to a complete contract reduces \( R_i \)'s incentive to exert effort due to the dichotomy result, this has a direct positive effect on \( R_{-i} \)'s effort through the RPM condition \( e_{-i}(\theta) = 2q_{-i}(\theta) - \rho q_i(\theta) - \sigma e_i(\theta) - \theta \), everything else being kept constant. Hence \( R_i \)'s reaction function shifts upward. Moreover, as \( e_{-i}(\theta) \) increases \( q_{-i}(\theta) \) increases too, thus making \( S_i \) better off since goods are complements. Both effects go in the direction of increasing \( S_i \)'s profits, as a consequence, complete contracts are part of an equilibrium for small demand uncertainty.

- When \( \sigma < 0 \) and \( \rho < 0 \), the same kind of argument drives the result. The only difference being that the effort reduction driven by the choice of a complete contract now pushes \( e_{-i}(\theta) \) down.

- Finally, in the free-riding case the lower incentive to exert effort for \( R_i \) under a complete contract has a direct positive effect on \( R_{-i} \)'s effort through the RPM condition \( e_{-i}(\theta) = 2q_{-i}(\theta) - \rho q_i(\theta) - \sigma e_i(\theta) - \theta \). This shifts upward \( R_i \)'s reaction function: a positive direct effect. However, since \( e_{-i}(\theta) \) increases, then \( q_{-i}(\theta) \) increases too, thus making \( R_{-i} \) more aggressive at the market stage and, in turn, \( S_i \) worse off. When retail competition is soft enough the former effect dominates the latter one, hence complete contracts remain part of an equilibrium of \( G \) in the limit of small uncertainty.

Since the game \( G \) admits multiple equilibria, in the next Proposition we order these equilibria according to a Pareto criterion from the suppliers’ viewpoint. As we will see the key element will be the nature of effort externalities.

Let \( \Pi^{\omega} \) denote the suppliers’ profit in a symmetric choice of contractual modes where in both hierarchies the mechanism \( \omega \in \{QF, RPM\} \) is chosen.
Proposition 11 Assume $\Delta \theta$ is small enough, $-1 < \sigma + \rho < 1$ and that $\psi''$ is large enough. When the game $G$ displays multiple equilibria in the contracting choice, then $\Pi^{Q,Q} > \Pi^{P,P}$ (resp. $\leq$) if $\sigma > 0$ (resp. $\leq$).

Incomplete contracts are thus mutually beneficial to suppliers (relative to complete ones) only when effort spillovers are positive or effort has a cooperative value. Indeed, in the limit of small uncertainty, as retailers are residual claimants for the full impact of their effort choices under incomplete contracts, consumers’ willingness to pay increases under incomplete contracting relative to complete contracts, thus improving upon productive efficiency.

6.2 Welfare Analysis

This section investigates the welfare effects of contracting incompleteness on consumers’ surplus. As we have considered a simple manufacturer-retailer economy, the analysis developed in previous sections can be used to assess the impact of retail price restrictions on third parties such as consumers.\textsuperscript{20} The question of whether this type of contracts are welfare detrimental has indeed been at the heart of a controversial debate in the antitrust and regulation literature for a long time.

Next Proposition extends the welfare results provided in Martimort and Piccolo (2006) within a isolated manufacturer-retailer economy to a competitive environment.

Proposition 12 Assume $\Delta \theta$ is small enough, $-1 < \sigma + \rho < 1$ and that $\psi''$ is large enough. Then incomplete contracts are detrimental (resp. beneficial) to consumers if $\sigma \geq 0$ (resp. $\leq$).

Interestingly, the result relates the welfare desirability of vertical contracting based on retail price restrictions to the nature of non-market activities performed by retailers, i.e., cooperative versus selfish effort. Since the implementation of complete contracts forces firms to be more aggressive at the market stage when effort has a cooperative value, contracts based only upon sales must be beneficial to consumers relative to more complex arrangements aimed at controlling retail price, as the former ones stimulate productive efficiency. The converse is obviously true when effort has a selfish nature. An interesting corollary of this result is that suppliers’ and consumers’ preferences are aligned with respect to the equilibrium contractual mode.

7 Concluding Remarks

The paper has provided a simple rationale for incomplete contracting. By taking a traditional agency perspective we have showed that, once one moves from the isolated principal-agent set-up to games where vertical organizations impose externalities on each other, incomplete contracting may emerge in equilibrium whenever agency costs are sufficiently small and some aspects of the agents’ performance are unverifiable.

\textsuperscript{20}Much scholars have indeed advocated that the sole role of Antitrust policies should be to promote consumers’ surplus. See Bork (1978, Chapter 2, pp. 51) for instance.
Strikingly, in a set-up where transaction or retailing costs are endogenously created by asymmetric information, our predictions are at the odds with the common wisdom postulating a negative relationship between transaction costs and degree of contracting incompleteness.

From a normative perspective, we provide conditions under which the welfare loss that double mark-ups inflict on consumers is minimized either by vertical arrangements imposing retail price restrictions or those controlling only sales depending on whether non-market activities performed by retailers have a cooperative or selfish value. Our results show that RPM enhances (resp. reduces) consumers’ surplus relative to QF whenever these kind of activities (effort) produce negative (resp. positive) spillovers on downstream demands. In light of these results, the analysis reveals that the concerns about social desirability of RPM, typically addressed by antitrust authorities, are justified only under certain conditions related to the underlying economic environment.

References


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8 Appendix

8.1 Proof of Proposition 1
It is obvious and thus omitted. ■

8.2 Proof of Lemma 2
To prove the claim it suffices to verify that vertical price control allows $S_1$ to learn (ex post) the realized demand state. So, suppose that $R_1$ lies at the revelation stage and announces a message $\hat{\theta} \neq \theta$, $S_1$ then supplies $q_1(\hat{\theta})$ units of inputs at the fixed fee $t(\hat{\theta})$ and expects to observe (ex post) a retail price level
\[ p_1(\hat{\theta}) = (2 + \rho)(\hat{\theta} + \rho q_1(\hat{\theta})). \] However, since \( S_2 - R_2 \) produces at \( q_2(\theta) = (\theta + \rho q_1(\hat{\theta}))/2 \), ex post \( S_1 \) will observe \( p_1(\hat{\theta}, \theta) = (2 + \rho)(\theta + \rho q_1(\hat{\theta})) \neq p_1(\hat{\theta}) \) for each \( \hat{\theta} \neq \theta \). \( R_1 \)'s deviation will then be immediately detected. \( \blacksquare \)

### 8.3 Proof of Proposition 3

As a preliminary step we show that in a properly defined neighborhood of \( \bar{\theta} \), throughout denoted \( B(\bar{\theta}) \), it must be \( q_1^Q(\theta) \leq q_1^P(\theta) \) with equality holding only at \( \bar{\theta} \). That \( q_1^Q(\bar{\theta}) = q_1^P(\bar{\theta}) = q^*(\bar{\theta}) \) is obvious. Notice then, that for all \( \theta \in B(\bar{\theta}) \), a first-order Taylor expansion of \( q_1^Q(\theta) \) around \( \bar{\theta} \) yields:

\[ q_1^Q(\theta) \approx q_1^P(\bar{\theta}) - q_1^Q(\bar{\theta})(\theta - \bar{\theta}), \]

where \( q_1^Q(\bar{\theta}) \) can be obtained by a simple application of l'Hopital's rule to the differential equation (4):

\[ \lim_{\theta \to \bar{\theta}} q_1^Q(\theta) = \frac{2 + \rho}{\rho^2} \lim_{\theta \to \bar{\theta}} \frac{1 - (2 - \rho)q_1^Q(\theta) - \dot{h}(\theta)}{h(\theta)}, \]

Using the fact that \( \dot{h}(\bar{\theta}) = -1 \) and rearranging terms in the above equation we have \( q_1^Q(\bar{\theta}) = (2 + \rho)(2 - \rho^2)^{-1} \geq q_1^P(\bar{\theta}) = (2 - \rho)^{-1} \). Therefore, for all \( \theta \in B(\bar{\theta}) \), it must be:

\[ q_1^Q(\theta) = (2 + \rho)(2 - \rho^2)^{-1} \geq (2 - \rho)^{-1} = q_1^P(\theta), \]

which in turn implies \( q_1^Q(\theta) \leq q_1^P(\theta) \) for all \( \theta \in B(\bar{\theta}) \) since \( q_1^Q(\bar{\theta}) = q_1^P(\bar{\theta}) \). Now, let \( B^c(\bar{\theta}) = \{ \theta \in \Theta | \theta \notin B(\bar{\theta}) \} \), we show that the result also holds globally for all \( \theta \in B^c(\bar{\theta}) \). Suppose, indeed, that there exists a \( \theta^* \in B^c(\bar{\theta}) \) such that \( q_1^Q(\theta^*) = q_1^P(\theta^*) \), and without loss of generality, consider that \( \theta^* \) is the lowest of such values. By using equation (4) one can easily verify that \( q_1^Q(\theta^*) = -(2 + \rho)\rho^{-2} < 0 < q_1^P(\theta^*) = (2 - \rho)^{-1} \). Hence, in a properly defined neighborhood of \( \theta^* \), say \( B(\theta^*) \), one must have \( q_1^Q(\theta) \geq q_1^P(\theta) \) (resp. <) if and only if \( \theta \leq \theta^* \) (resp. >). This yields a contradiction with the definition of \( \theta^* \).

We shall now prove the following results: Information rents are increasing in \( \theta \), the local second-order condition \( (IC_2) \) holds, program \( (P') \) has a unique solution and global incentive compatibility holds. As a preliminary step we prove that \( q_1^Q(\theta) \leq q_1^m(\theta) \) for all \( \theta \), where \( q_1^m(\theta) \) satisfies:

\[ \theta - (2 - \rho)q_1^m(\theta) - h(\theta) = 0. \] (26)

To begin with, we show that the result holds in a properly defined neighborhood of \( \bar{\theta} \), say \( B^m(\bar{\theta}) \). Differentiating equation (26) with respect to \( \theta \) we have \( \dot{q}_1^m(\bar{\theta}) = 2(2 - \rho)^{-1} \leq (2 - \rho)(2 - \rho^2)^{-1} = q_1^Q(\bar{\theta}) \). Therefore, since \( q_1^m(\bar{\theta}) = q_1^Q(\bar{\theta}) = q_1^P(\bar{\theta}) \), it must be \( q_1^m(\theta) \geq q_1^Q(\theta) \) for all \( \theta \in B^m(\bar{\theta}) \). Moreover, one can also show that the result remains true for all \( \theta \in \{ \theta \in \Theta | \theta \notin B^m(\bar{\theta}) \} \). Indeed, suppose that there exists a \( \theta^{**} \in \Theta \) such that \( q_1^m(\theta^{**}) = q_1^Q(\theta^{**}) \), and without loss of generality assume that \( \theta^{**} \) is the lowest of such values. In this case, from equation (4) one immediately gets \( q_1^Q(\theta^{**}) = 0 \). Since \( \dot{q}_1^m(\theta) = (2 - \rho)^{-1}(1 - \dot{h}(\theta)) > 0 \), we
have \( q''_{\theta^*} > q''_1(\theta^*) \) and \( q''_{\theta}(\theta) < q''_1(\theta) \) for \( \theta \leq \theta^* \), a contradiction.

By using the same logic one can immediately show that \((IC_2)\) holds too. Also, uniqueness follows directly by linearity of \( p_i(.) \) and strict concavity of \((P')\).

For global incentive compatibility constraints, let \( U_1^Q(\theta, \hat{\theta}) \) define \( R_1 \)'s profits evaluated at the allocation \( \{q_1^1(\hat{\theta}), t_1^Q(\hat{\theta})\} \), that is when the true demand state is \( \theta \) and the message sent to \( S_1 \) is \( \hat{\theta} \neq \theta \). To show that global incentive compatibility constraints hold, we must have \( \Gamma(\theta, \hat{\theta}) \equiv U_1^Q(\theta, \theta) - U_1^Q(\theta, \hat{\theta}) > 0 \) for each pair \((\theta, \hat{\theta}) \in \Theta^2\). Assume then \( \theta > \hat{\theta} \) without loss of generality, simple algebraic manipulations allow to rewrite \( \Gamma(.) \) as:

\[
\Gamma(\theta, \hat{\theta}) = \int_0^\theta \left\{ -q_1^1(s) + (\theta - q_1^Q(s) + \rho q_2^Q(\theta))q_1^Q(s) - q_1^Q(s)q_1^Q(s) \right\} ds.
\]

By using \((IC_1)\) and substituting for \( t_1^Q(s) = (s - q_1^Q(s) + \rho q_2^Q(s))q_1^Q(s) - q_1^Q(s)q_1^Q(s) \) into the above equation one obtains:

\[
\Gamma(\theta, \hat{\theta}) = \int_0^\theta 2q_1^Q(s) \left\{ \int_s^\theta (1 + \rho q_2^Q(x))dx \right\} ds.
\]

Since we have already proved that \( q_1^Q(\theta) > 0 \) and \((1 + \rho q_2^Q(\theta)) > 0 \) for all \( \theta \), it follows that \( \Gamma(\theta, \hat{\theta}) > 0 \), which concludes the proof.

Finally, from the first-order conditions \((2)\) and \((3)\) one has \( \Pi_1^P = E_\theta[q_1^P(\theta)]^2 \) and \( \Pi_1^Q = E_\theta[q_1^Q(\theta)]^2 \). Then \( \Pi_1^P > \Pi_1^Q \) follows immediately from \( q_1^Q(\theta) \leq q_1^P(\theta) \) for all \( \theta \in \Theta \).

### 8.4 Proof of Proposition 4

Observe that as we are considering \( \Delta \theta \) small, we can use the following first-order Taylor expansions in describing output and effort: \( q_1^P(\theta) \approx q_1^P(\bar{\theta}) - q_1^P(\bar{\theta})(\theta - \bar{\theta}) \) and \( q_1^Q(\theta) \approx q_1^Q(\bar{\theta}) - \hat{\theta}(\theta - \bar{\theta}) \) for all \( \theta \).

To show that \( q_1^P(\theta) \geq 0 \), simple application of l'Hopital’s rule yields:

\[
\lim_{\theta \to \bar{\theta}} q_1^P(\theta) = \lim_{\theta \to \bar{\theta}} \frac{2(q_1^P(\bar{\theta}) - \psi''(e_1^P(\bar{\theta})e_1^P(\theta)) - (2 + \rho(1 + \sigma \hat{\theta}^P(\bar{\theta})))\hat{h}(\bar{\theta})\psi''(e_1^P(\theta)) - \hat{\theta}(\bar{\theta})\psi'''(e_1^P(\theta))e_1^P(\theta))}{\rho^2 \left( \hat{h}(\bar{\theta})\psi''(e_1^P(\theta)) + \hat{\theta}(\bar{\theta})\psi'''(e_1^P(\theta))e_1^P(\theta) \right)},
\]

which, from \((10)\) together with \( \hat{h}(\bar{\theta}) = -1 \) and \( \hat{\theta}(\bar{\theta}) = 0 \), yields:

\[
(27) \quad q_1^P(\bar{\theta}) = \frac{2\psi''(2 + \rho)}{2\psi''(2 - \rho(\sigma + \rho)) - \sigma \rho - 2}.
\]

Therefore, monotonicity follows if the following restriction holds:

\[
\psi''(e) > \frac{2 + \sigma \rho}{2(2 - \rho(\sigma + \rho))} \quad \forall \ e \in \mathbb{R}_+,
\]
where \((\sigma, \rho) \in \{(\sigma, \rho) \in [-1, 1]^2 : 2 - \rho (\sigma + \rho) \neq 0\}\).

By using the definition of \(q_1^P(\theta)\) we get:

\[
\hat{q}_1^*(\overline{\theta}) = \frac{\psi''(2 + \rho)}{\psi''(4 - \rho^2) - \sigma \rho - 2}.
\]

Since \(q_1^P(\theta) - q_1^*(\theta) \approx (\hat{q}_1^*(\overline{\theta}) - \hat{q}_1^P(\overline{\theta}))(\overline{\theta} - \theta)\) for all \(\theta\), we get:

\[
q_1^P(\overline{\theta}) - \hat{q}_1^*(\overline{\theta}) = \frac{(2 + \sigma \rho)\left(2\psi'' - 1\right)(2 + \rho)\psi''}{(2\psi''(2 - \rho(\sigma + \rho)) - \sigma \rho - 2)(\psi''(4 - \rho^2) - \sigma \rho - 2)}
\]

which immediately implies \(\hat{q}_1^*(\overline{\theta}) < q_1^P(\overline{\theta})\) since \(\psi''(e) > \max\left\{\frac{1}{2}, \frac{2 + \sigma \rho}{2(2 - \rho(\sigma + \rho))}\right\}\) for all \(e \in \mathbb{R}_+\), hence \(q_1^P(\theta) \leq q_1^*(\theta)\) for all \(\theta\) with equality only at \(\overline{\theta}\).

By using the same kind of argument, we have:

\[
\hat{e}_1^P(\overline{\theta}) = \frac{(2\psi'' + 1)(2 + \rho)}{2\psi''(2 - \rho(\sigma + \rho)) - \sigma \rho - 2}.
\]

Then, since \(\psi''q_1^*(\overline{\theta}) = \hat{e}_1^*(\overline{\theta})\), one gets:

\[
\hat{e}_1^P(\overline{\theta}) - \hat{e}_1^*(\overline{\theta}) = \frac{2(2 + \rho)\psi''}{(2\psi''(2 - \rho(\sigma + \rho)) - \sigma \rho - 2)(\psi''(4 - \rho^2) - \sigma \rho - 2)}
\]

implying \(\hat{e}_1^*(\overline{\theta}) < \hat{e}_1^P(\overline{\theta})\) since \(\psi''(e) > \max\left\{\frac{1}{2}, \frac{2 + \sigma \rho}{2(2 - \rho(\sigma + \rho))}\right\}\) for all \(e \in \mathbb{R}_+\) and, as a consequence, \(e_1^P(\theta) \leq e_1^*(\theta)\) for all \(\theta\).

Notice also that program \((\mathcal{P}^P)\) displays interior solutions whenever \(\Delta \theta\) is sufficiently small. Moreover, from the above results one can show that information rents are increasing in \(\theta\). In fact, simple algebra yields:

\[
0 < \hat{U}_1^P(\theta) = \frac{(2 + \rho)(2\psi'' - 1)\psi'(e_1^P(\theta))}{2\psi''(2 - \rho(\sigma + \rho)) - \sigma \rho - 2}.
\]

The same kind of argument allows to show that the monotonicity condition holds, i.e., \(2\hat{q}_1^P(\theta)(1 + \rho q_2^P(\theta)) > 0\) for all \(\theta\). Moreover, uniqueness simply follows from linearity of \(p_i(\theta, e_1, q_i, q_j)\) for \(i = 1, 2\) and strictly concavity of \((\mathcal{P}^P)\).

Finally, let \(U_1^P(\theta, \hat{\theta})\) be \(R_1\)'s profits at \(\{p_1^P(\theta, \hat{\theta}), q_1^P(\theta, \hat{\theta}), t_1^P(\theta, \hat{\theta})\}\), namely when the true retailer’s type is \(\theta\) and the message sent to the manufacturer is \(\hat{\theta} \neq \theta\). To show that global incentive compatibility constraints hold we must have \(\Gamma_1^P(\theta, \hat{\theta}) \equiv U_1^P(\theta, \theta) - U_1^P(\theta, \hat{\theta}) > 0\) for each pair \((\theta, \hat{\theta}) \in \Theta^2\). Assume then \(\theta > \hat{\theta}\) without loss of generality, simple algebraic manipulations allow to rewrite \(\Gamma_1^P(\cdot, \cdot)\) as \(\Gamma_1^P(\theta, \hat{\theta}) = \int_{\hat{\theta}}^\theta \{\hat{e}_1^P(s) - 2\psi'(e_1^P(s, \theta))\hat{q}_1^P(s)\} ds\), where \(e_1^P(s, \theta) \equiv 2q_1^P(s) - \rho q_2^P(\theta) - \theta\). By using \((IC_1)\) and
substituting for $\hat{t}_1^P(s) \equiv 2\psi'(e_1^P(s))q_1^P(s)$ into $\Gamma^P(.)$, one obtains:

$$\Gamma^P(\theta, \hat{\theta}) = 2 \int_0^\theta \hat{q}_1^P(s) \left\{ \int_s^\theta \psi''(e(s,x))(1 + \rho \hat{q}_2^P(x))dx \right\} ds.$$  

Then, since $\hat{q}_1^P(\bar{\theta}) > 0$ whenever $\psi''(e) > \max \left\{ \frac{1}{2}, \frac{2+\sigma}{2(2-\rho(\sigma+\rho))} \right\}$ for all $e \in \mathbb{R}_+$, it follows that $\Gamma^P(\theta, \hat{\theta}) > 0$ for all $\theta$ and $\hat{\theta}$, which concludes the proof.

8.5 Proof of Lemma 5

Again, for $\Delta \theta$ small enough we consider $q_1^Q(\theta) \approx q_1^s(\bar{\theta}) - \hat{q}_1^Q(\bar{\theta})(\bar{\theta} - \theta)$ for all $\theta$. From (15), a straightforward application of l’Hopital’s rule yields:

$$\lim_{\theta \to \bar{\theta}} q_1^Q(\theta) = \lim_{\theta \to \bar{\theta}} \frac{(2 + \rho)(1 - \hat{h}(\theta)) + (2 + \rho \sigma)\phi'(q_1^Q(\theta))q_1^Q(\theta) - \hat{q}_1^Q(\theta)(4 - \rho^2)}{\rho \left( \hat{h}(\theta)(\sigma \phi'(q_1^Q(\theta)) + \rho) + \sigma h(\theta)\phi''(q_1^Q(\theta))q_1^Q(\theta) \right)} ,$$

hence, using $\hat{h}(\bar{\theta}) = -1$ and $h(\bar{\theta}) = 0$, we get:

$$\hat{q}_1^Q(\bar{\theta}) = \frac{(2 + \rho)\psi''}{\psi''(2 - \rho^2) - \sigma \rho - 1},$$

where monotonicity follows, i.e., $\hat{q}_1^Q(\bar{\theta}) \geq 0$, if the following restriction holds:

$$\psi''(e) > \frac{1 + \sigma \rho}{2 - \rho^2} \forall \ e \in \mathbb{R}_+.$$

In the limit of small uncertainty, i.e., $\Delta \theta$ small, it also follows that $q_1^Q(\theta) - q_1^s(\theta) \approx (\hat{q}_1^s(\bar{\theta}) - \hat{q}_1^Q(\bar{\theta}))(\bar{\theta} - \theta)$ for all $\theta$, and as a consequence:

$$\hat{q}_1^Q(\bar{\theta}) - \hat{q}_1^s(\bar{\theta}) = \frac{(2 + \rho)(2\psi'' - 1)\psi''}{(\psi''(2 - \rho^2) - \sigma \rho - 1)(\psi''(4 - \rho^2) - \sigma \rho - 2)},$$

which immediately implies $\hat{q}_1^s(\bar{\theta}) < \hat{q}_1^Q(\bar{\theta})$ for all $\psi''(e) > \max \left\{ \frac{1}{2}, \frac{1+\sigma \rho}{2-\rho^2} \right\}$ for all $e \in \mathbb{R}_+$. Hence $q_1^Q(\theta) \leq q_1^s(\theta)$ for all $\theta$ with equality only at $\bar{\theta}$. By using the same kind of argument one also has $e_1^Q(\theta) \approx e_1^s(\bar{\theta}) - e_1^Q(\bar{\theta})(\bar{\theta} - \theta)$. Hence:

$$e_1^Q(\bar{\theta}) = \phi'(q_1^s(\bar{\theta}))q_1^Q(\bar{\theta}) > \phi'(q_1^s(\bar{\theta}))\hat{q}_1^s(\bar{\theta}) = e_1^s(\bar{\theta}),$$

which directly implies $e_1^Q(\theta) \leq e_1^s(\theta)$ for all $\theta$.

That program $(P^{Q})$ displays interior solutions whenever $\Delta \theta$ is small enough is obvious. Hence, for $q_1^Q(\theta)$
being interior, information rents are increasing in $\theta$ since $\psi''(e) > \max\left\{ \frac{1}{2}, \frac{1+\sigma \rho}{2-\rho^2} \right\}$ for all $e \in \mathbb{R}_+$ implies:

$$0 < \dot{U}_1^Q(\theta) = \frac{q_1^Q(\theta) (2\psi'' - 1) (2 + \rho)}{2(\psi'' (2 - \rho^2) - \sigma \rho - 1)}.$$

Since $q_1^Q(\overline{\theta}) > 0$, one can check that the monotonicity condition also holds, i.e., $(1 + \rho q_2^Q(\theta))q_1^Q(\theta) > 0$. Moreover, uniqueness simply follows from linearity of $p_1(\theta, e_1, q_1, q_2)$ and strictly concavity of $(P^Q)$.

Finally, let $U_1^Q(\theta, \hat{\theta})$ define the retailer’s profits evaluated at $\{q_1^Q(\hat{\theta}), t_1^Q(\hat{\theta})\}$, when the true demand realization is $\theta$ and the message sent to $S_1$ is $\hat{\theta} \neq \theta$. To show that global incentive compatibility constraints hold we must have $\Gamma^Q(\theta, \hat{\theta}) \equiv U_1^Q(\theta, \hat{\theta}) - U_1^Q(\theta, \hat{\theta}) > 0$ for each pair $(\theta, \hat{\theta}) \in \Theta^2$. Assume then $\theta > \hat{\theta}$ without loss of generality, simple algebraic manipulations allow to rewrite $\Gamma^Q(.)$ as:

$$\Gamma^Q(\theta, \hat{\theta}) = \int_0^\theta \left\{ -\dot{q}_1^Q(s) + \dot{q}_1^Q(s)p_1(\theta, e_1^Q(s), q_1^Q(s), q_2^Q(s)) - q_1^Q(s)q_1^Q(s) \right\} ds.$$

By using $IC_1$ and substituting for $\dot{q}_1^Q(s) \equiv \dot{q}_1^Q(s)p_1(s, e_1^Q(s), q_1^Q(s), q_2^Q(s)) - q_1^Q(s)\dot{q}_1^Q(s)$ into the above equation, we get:

$$\Gamma^Q(\theta, \hat{\theta}) = \int_0^\theta \dot{q}_1^Q(s) \left\{ \int_0^\theta (1 + \rho q_2^Q(x)) dx \right\} ds.$$

Then, since we have proved above that $(1 + \rho q_2^Q(\theta)) > 0$ when $\psi''(e) > \max\left\{ \frac{1}{2}, \frac{1+\sigma \rho}{2-\rho^2} \right\}$ for all $e \in \mathbb{R}_+$, one has $\Gamma^Q(\theta, \hat{\theta}) > 0$, which concludes the proof. ■

### 8.6 Proof of Proposition 6

First, observe that $e_1^P(\theta) - e_1^Q(\theta) \approx (\dot{e}_1^P(\overline{\theta}) - \dot{e}_1^Q(\overline{\theta}))(\overline{\theta} - \theta)$ for all $\theta$. Using the definition of $\dot{e}_1^P(\overline{\theta})$ and $\dot{e}_1^Q(\overline{\theta})$ we get:

$$\dot{e}_1^P(\overline{\theta}) - \dot{e}_1^Q(\overline{\theta}) = \frac{(2\psi'' - 1)(2 + \rho)(\psi''(2 - \rho^2) - 1)}{(2\psi''(2 - \rho(\sigma + \rho)) - \sigma - 2)(\psi''(2 - \rho^2) - \sigma - 1)},$$

which immediately implies $\dot{e}_1^P(\overline{\theta}) > \dot{e}_1^Q(\overline{\theta})$ as we have assumed $\psi''(e) > \max\left\{ \frac{1}{2}, \frac{2+\sigma \rho}{2(2-\rho(\sigma+\rho))}, \frac{1+\sigma \rho}{2-\rho^2} \right\}$ for all $e \in \mathbb{R}_+$. Hence, $\dot{e}^Q(\theta) > e^P(\theta)$ for all $\theta$ with equality only at $\overline{\theta}$.

By using the same kind of argument we have $q_2^Q(\theta) - q_2^P(\theta) \approx (\dot{q}_2^P(\overline{\theta}) - \dot{q}_2^Q(\overline{\theta}))(\overline{\theta} - \theta)$ for all $\theta$. Hence:

$$\dot{q}_2^P(\overline{\theta}) - \dot{q}_2^Q(\overline{\theta}) = \frac{\sigma (2 + \rho)(2\psi'' - 1)^2}{2(2\psi''(2 - \rho(\sigma + \rho)) - \sigma - 2)(\psi''(2 - \rho^2) - \sigma - 1)},$$

which immediately yields the result since $\psi''(e) > \max\left\{ \frac{1}{2}, \frac{2+\sigma \rho}{2(2-\rho(\sigma+\rho))}, \frac{1+\sigma \rho}{2-\rho^2} \right\}$ for all $e \in \mathbb{R}_+$. 

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Similarly, we have:

\[ q^P_1(\bar{\theta}) - q^Q_1(\bar{\theta}) = \frac{(\sigma \rho)(2\psi'' - 1)(2 + \rho)\psi''}{(2\psi'' (2 - \rho(\sigma + \rho)) - \sigma \rho - 2)(\psi'' (2 - \rho^2) - \sigma \rho - 1)}, \]

yielding \( \text{sign}(q^Q_1(\theta) - q^P_1(\theta)) = \text{sign}(\sigma \rho) \) since \( \psi''(e) > \max \left\{ \frac{1}{2}, \frac{2 + \sigma \rho}{2(2 - \rho(\sigma + \rho))}, \frac{1 + \sigma \rho}{2 - \rho^2} \right\} \) for all \( e \in \mathbb{R}_+ \).

### 8.7 Proof of Proposition 7

First, observe that in the limit of small uncertainty the type-contingent profit of the upstream supplier under both contractual regimes can be obtained by using a second-order Taylor expansion of (virtual) profits around \( \bar{\theta} \):

\[ \Pi^\omega_1(\theta) \approx \Pi^\omega_1(\bar{\theta}) - \hat{\Pi}^\omega_1(\bar{\theta})(\bar{\theta} - \theta) + \frac{1}{2} \hat{\Pi}^\omega_1(\bar{\theta})(\bar{\theta} - \theta)^2 \quad \text{for each } \omega \in \{QF, RPM\}. \tag{29} \]

Observe that when \( \Delta \theta \) is sufficiently small one can also approximate the distribution of \( \bar{\theta} \) with a uniform, i.e., \( \bar{\theta} \sim U[\theta] \), we then have \( E_{\bar{\theta}}[\bar{\theta} - \theta] \approx \Delta \theta/2 \) and \( E_{\bar{\theta}}[\bar{\theta} - \theta]^2 \approx \Delta \theta^2/3 \). Taking expectations on both sides of (29) we get:

\[ \Pi^Q_1 - \Pi^P_1 \approx \left( \hat{\Pi}^P_1(\theta) - \hat{\Pi}^Q_1(\theta) \right) \frac{\Delta \theta}{2} + \left( \hat{\Pi}^Q_1(\theta) - \hat{\Pi}^P_1(\theta) \right) \frac{\Delta \theta^2}{6}. \tag{30} \]

We need then to compute each term in (30). First, consider program \((P^P)\), differentiating with respect to \( \theta \) and using the Envelope Theorem we have:

\[ \hat{\Pi}^P_1(\theta) = \psi'(e^P_1(\theta))(1 + \rho q^P_1(\theta))(1 - \hat{h}(\theta)) + h(\theta)\psi''(e^P_1(\theta))(1 + \rho q^P_1(\theta))^2. \]

From \( h(\bar{\theta}) = 0 \) and \( \hat{h}(\bar{\theta}) = -1 \), taking limits for \( \theta \to \bar{\theta} \) on both sides it follows \( \hat{\Pi}^P_1(\bar{\theta}) = 2q^1(\bar{\theta})(1 + \rho q^2_1(\bar{\theta})) \).

Using the first-order condition (8) in \( \hat{\Pi}^P_1(\theta) \), differentiating again with respect to \( \theta \), we have \( \hat{\Pi}^P_1(\bar{\theta}) = (q^P_1(\bar{\theta}) + \psi'' e^P_1(\bar{\theta}))(1 + \rho q^2_1(\bar{\theta})) \).

Consider now program \((P^Q)\), again differentiating with respect to \( \theta \) and using again the Envelope Theorem we have:

\[ \hat{\Pi}^Q_1(\theta) = (1 + \rho q^Q_2(\theta))q^Q_1(\theta) - \hat{h}(\theta)(1 + \rho q^Q_2(\theta))q^Q_1(\theta), \]

taking limits for \( \theta \to \bar{\theta} \) on both sides and using \( h(\bar{\theta}) = -1 \) we have \( \hat{\Pi}^Q_1(\bar{\theta}) = 2q^1(\bar{\theta})(1 + \rho q^2_1(\bar{\theta})) \). By using the same kind of argument, we get \( \hat{\Pi}^Q_1(\bar{\theta}) = (q^Q_1(\bar{\theta}) + \psi'' e^Q_1(\bar{\theta}))(1 + \rho q^2_1(\bar{\theta})) \).

Substituting \( \hat{\Pi}^P_1(\bar{\theta}) \) and \( \hat{\Pi}^Q_1(\bar{\theta}) \), for \( \omega \in \{QF, RPM\} \), into \( \Pi^Q_1 - \Pi^P_1 \), it follows:

\[ \Pi^Q_1 - \Pi^P_1 \approx \rho q^*(\bar{\theta}) \left( q^P_2(\bar{\theta}) - q^Q_2(\bar{\theta}) \right) \Delta \theta - (\hat{\Pi}^P_1(\bar{\theta}) - \hat{\Pi}^Q_1(\bar{\theta})) \frac{\Delta \theta^2}{6}. \]
Taking $\Delta \theta$ small enough and substituting (28) into the above equation we have:

$$
\Pi_1^Q - \Pi_1^P \approx \frac{(\sigma \rho) \Delta \theta q^*(\bar{\theta}) (2\psi'' - 1) (2 + \rho) \psi''}{(2\psi'' (2 - \rho (\sigma + \rho)) - \sigma \rho - 2)(\psi'' (2 - \rho^2) - \sigma \rho - 1)},
$$

which immediately yields the result since $\Pi_1^Q - \Pi_1^P > 0$ (resp. $< 0$) whenever $\rho \sigma > 0$ (resp. $< 0$).

To conclude the proof we must consider the case $\sigma \rho = 0$. The sign of $\Pi_1^Q - \Pi_1^P$ is now obtained by looking at the second order term of the Taylor approximation (30):

$$
\Pi_1^Q - \Pi_1^P \approx ((q_1^Q(\bar{\theta}) + \psi''e_1^Q(\bar{\theta}))(1 + \rho q_2^Q(\bar{\theta})) - (q_1^P(\bar{\theta}) + \psi''e_1^P(\bar{\theta}))(1 + \rho q_2^P(\bar{\theta}))) \frac{\Delta \theta^2}{6}.
$$

First assume $\sigma = 0$, in this case $q_1^P(\bar{\theta}) = q_1^Q(\bar{\theta})$ and $q_2^P(\bar{\theta}) = q_2^Q(\bar{\theta})$, then we have:

$$
\Pi_1^Q - \Pi_1^P \approx - (1 + \rho q_2^Q(\bar{\theta}))(\dot{e}_1^P(\bar{\theta}) - \dot{e}_1^Q(\bar{\theta})) \frac{\Delta \theta^2}{6},
$$

which yields the result since $\dot{e}_1^P(\bar{\theta}) > \dot{e}_1^Q(\bar{\theta})$. The same kind of argument allows to show that $\Pi_1^Q - \Pi_1^P \approx -\psi''(\dot{e}_1^P(\bar{\theta}) - \dot{e}_1^Q(\bar{\theta})) \Delta \theta^2 / 6 < 0$ when $\rho = 0$, and thus concludes the proof.

### 8.8 Proof of Proposition 8

We follow the same technique used in the proof of Propositions 4 and 5. First, notice that the allocation $\{q^P(\theta), e^P(\theta)\}_{\theta \in \Theta}$ solves the following system of differential equations:

$$
\begin{align*}
q^P(\theta)(2 - \rho) - 1 - e^P(\theta)(1 + \sigma) &= 0, \\
q^P(\theta) - \psi'(e^P(\theta)) - h(\theta)(1 + \sigma e^P(\theta) + \rho q^P(\theta))\psi''(e^P(\theta)) &= 0,
\end{align*}
$$

with boundary conditions $q^P(\bar{\theta}) = q^*(\bar{\theta})$ and $e^P(\theta) = e^*(\bar{\theta})$. As we have assumed $\Delta \theta$ small enough, from a first-order Taylor expansion of $q^P(.)$ around $\bar{\theta}$ we have $q^P(\theta) \approx q^*(\bar{\theta}) - q^P(\bar{\theta})(\bar{\theta} - \theta)$. From a simple application of l’Hopital’s rule we have:

$$
q^P(\bar{\theta}) = \frac{2\psi''}{2\psi'' (1 - \rho - \sigma) - (1 + \sigma)} \quad \text{and} \quad \dot{e}^P(\theta) = \frac{2\psi'' - 1}{2\psi'' (1 - \rho - \sigma) - (1 + \sigma)}.
$$

Having assumed $\sigma + \rho < 1$, monotonicity is insured if the following restriction holds:

$$
\psi''(e) > \frac{1 + \sigma}{2(1 - \rho - \sigma)} \quad \forall \ e \in \mathbb{R}_+,
$$
Now, it is easy to show that in the complete information benchmark we have:

\[ q^*(\theta) = \frac{\psi''}{\psi''(2 - \rho) - (1 + \sigma)}(1 + \sigma), \]

with \( q^*(\theta) > 0 \) since \( \psi''(e) > (1 + \sigma)/(2 - \rho) \) for all \( e \in \mathbb{R}_+ \).

Hence, as \( q^P(\theta) = q^*(\theta) \), it follows that \( q^P(\theta) > q^*(\theta) \) must imply \( q^P(\theta) \leq q^*(\theta) \) for all \( \theta < \bar{\theta} \). Simple algebra then yields:

\[ \dot{q}^P(\theta) - \dot{q}^*(\theta) = \frac{(2\psi'' - 1)(1 + \sigma)\psi''}{(\psi''(2 - \rho) - (1 + \sigma))(2\psi''(1 - \rho - \sigma) - (1 + \sigma))}, \]

which implies the result since \( \psi''(e) > 1/2 \) for all \( e \in \mathbb{R}_+ \). By using the same kind of argument one shows that \( e^P(\theta) \leq e^*(\theta) \) for all \( \theta \leq \bar{\theta} \). Finally, notice that for \( \Delta \theta \) small enough program efforts and outputs will be positive. The rest of the proof is omitted as it follows Martimort (1996).

8.9 Proof of Lemma 9

By definition the allocation \( \{q^Q(\theta), e^Q(\theta)\}_{\theta \in \Theta} \) solves the following system of differential equations:

\[ \theta + (1 + \sigma)e^Q(\theta) - q^Q(\theta)(2 + \rho) - h(\theta)(1 + \sigma)e^Q(\theta) + \rho q^Q(\theta) = 0, \]

\[ e^Q(\theta) = \phi'(q^Q(\theta))\dot{q}^Q(\theta), \]

with boundary conditions \( q^R(\theta) = q^*(\theta) \) and \( e^R(\theta) = e^*(\theta) \). Again, as we have assumed \( \Delta \theta \) small, l’Hopital’s rule yields:

\[ \dot{q}^Q(\theta) = \frac{2\psi''}{2\psi''(1 - \rho) - (1 + 2\sigma)} \quad \text{and} \quad \dot{e}^Q(\theta) = \frac{2}{2\psi''(1 - \rho) - (1 + 2\sigma)}. \]

which directly implies monotonicity if:

\[ \psi''(e) > \frac{1 + 2\sigma}{2(1 - \rho)} \quad \forall \ e \in \mathbb{R}_+. \]

Simple algebra then yields:

\[ \dot{q}^Q(\theta) - \dot{q}^*(\theta) = \frac{(2\psi'' - 1)\psi''}{(2\psi''(1 - \rho) - (1 + 2\sigma))(\psi''(2 - \rho) - (1 + \sigma))}, \]

that immediately implies \( q^Q(\theta) \leq q^*(\theta) \) for all \( \theta \) with equality holding only at \( \bar{\theta} \). The same kind of argument allows to show that \( e^Q(\theta) \leq e^*(\theta) \) for all \( \theta \) with equality holding only at \( \bar{\theta} \). Finally, \( (P^Q)' \) has interior solutions whenever \( \Delta \theta \) is small enough. The rest of the proof is omitted as it follows Martimort (1996).
8.10 Proof of Proposition 10

To begin with, we need to characterize allocations in an asymmetric play of $G$. So, suppose that $S_i$ chooses a complete contract while $S_{-i}$ plays an incomplete one. By definition the allocations $\{q_{i}^{P,Q}(\theta), e_{i}^{P,Q}(\theta)\}_{\theta \in \Theta}$ and $\{q_{-i}^{P,Q}(\theta), e_{-i}^{Q,P}(\theta)\}_{\theta \in \Theta}$ solve the following system of differential equations:

\[ q_{i}^{P,Q}(\theta) - \psi'(e_{i}^{P,Q}(\theta)) - h(\theta)(1 + \sigma e_{-i}^{Q,P}(\theta) + \rho q_{-i}^{Q,P}(\theta))\psi''(e_{i}^{P,Q}(\theta)) = 0, \]

\[ e_{i}^{P,Q}(\theta) = 2q_{i}^{P,Q}(\theta) - \sigma e_{-i}^{Q,P}(\theta) - \rho q_{-i}^{Q,P}(\theta) - \theta, \]

\[ \theta + e_{-i}^{Q,P}(\theta) - 2q_{-i}^{Q,P}(\theta) + \sigma e_{i}^{P,Q}(\theta) + \rho q_{i}^{P,Q}(\theta) - h(\theta)(1 + \sigma e_{i}^{P,Q}(\theta) + \rho q_{i}^{P,Q}(\theta)) = 0, \]

\[ e_{-i}^{Q,P}(\theta) = \phi(q_{-i}^{Q,P}(\theta)). \]

with boundary conditions $q_{-i}^{Q,P}(\overline{\theta}) = q_{i}^{P,Q}(\overline{\theta}) = q^*(\overline{\theta})$ and $e_{i}^{P,Q}(\overline{\theta}) = e_{-i}^{Q,P}(\overline{\theta}) = e^*(\overline{\theta})$.

Differentiating (36) it follows $e_{-i}^{Q,P}(\theta) = \phi'(q_{-i}^{Q,P}(\theta))q_{-i}^{Q,P}(\theta)$, then plugging $e_{-i}^{Q,P}(\theta)$ into (33)-(35) and linearizing the reduced system around the point $(\overline{\theta}, q^*(\overline{\theta}), e^*(\overline{\theta}))$ we have:

\[ 2\dot{q}_{i}^{P,Q}(\overline{\theta}) - 1 - \dot{e}_{i}^{P,Q}(\overline{\theta}) - \sigma \frac{\dot{q}_{-i}^{Q,P}(\overline{\theta})}{\psi''} - \rho q_{-i}^{Q,P}(\overline{\theta}) = 0, \]

\[ \dot{q}_{i}^{P,Q}(\overline{\theta}) - \psi''\dot{e}_{i}^{P,Q}(\overline{\theta}) + (1 + \frac{\sigma}{\psi'q_{-i}^{Q,P}(\overline{\theta}) + \rho q_{-i}^{Q,P}(\overline{\theta})})\psi'' = 0, \]

\[ 1 + \frac{1}{\psi''q_{-i}^{Q,P}(\overline{\theta}) - 2q_{-i}^{Q,P}(\overline{\theta}) + \sigma e_{i}^{P,Q}(\overline{\theta}) + \rho q_{i}^{P,Q}(\overline{\theta}) + (1 + \sigma e_{i}^{P,Q}(\overline{\theta}) + \rho q_{i}^{P,Q}(\overline{\theta})) = 0. \]

One can check that the solution of the linearized system (37)-(39) yields:

\[ e_{i}^{P,Q}(\overline{\theta}) = \frac{-(1 - 2\sigma) + 2\psi''(2\sigma + \rho) + 4(\psi'')^2(1 + \rho)}{a + b\psi'' + c(\psi'')^2}, \]

\[ q_{i}^{P,Q}(\overline{\theta}) = \frac{-2\psi''(1 - 2\sigma) + 4(\psi'')^2(1 + \rho)}{a + b\psi'' + c(\psi'')^2}, \]

and

\[ q_{-i}^{Q,P}(\overline{\theta}) = \frac{-2\psi''(1 - \sigma) + (\psi'')^2(1 + \sigma + \rho)}{a + b\psi'' + c(\psi'')^2}. \]
Moreover, \( e_{i}^{Q,P}(\bar{\theta}) = \phi'(q^*(\bar{\theta}))q^{Q,P}(\bar{\theta}) \) implies:

\[
\varepsilon_{i}^{Q,P}(\bar{\theta}) = \frac{1}{\psi''(e)} \left( \frac{-2\psi''(1-\sigma) + (\psi''(e))^2(1+\sigma + \rho)}{a + b\psi'' + c(\psi''(e))^2} \right),
\]

where we have defined \( a = (1-2\sigma) \), \( b = -2(3\sigma\rho + 2\sigma^2 + 2) \) and \( c = 4(1-\rho^2 - \sigma\rho) \). With \( c > 0 \) since we have assumed \( 1 > \sigma + \rho \). Notice when \( \psi'' \) is sufficiently large and \(-1 < \sigma + \rho < 1\), monotonicity follows under all possible contractual regimes. That is, \( q^{i,\omega'}(\theta) \geq 0 \) for all pairs \((\omega, \omega') \in \{QF, RP,M\}^2 \) and \( \theta \in \Theta \) if:

\[
\psi''(e) > \max \left\{ \frac{1 - \sigma}{1 + \sigma + \rho}, \frac{1 + 2\sigma}{2(1-\rho)}, \frac{1 + \sigma}{2(1 - \rho - \sigma)} \right\}, \quad \forall e \in \mathbb{R}_+
\]

where,

\[
\psi''_1 = \max \left\{ \psi'' \in \mathbb{R}_+ | -2(2\sigma + \rho) + 4(\psi''(1 + \rho)) = 0 \right\},
\]

and

\[
\psi''_2 = \max \left\{ \psi'' \in \mathbb{R}_+ | a + b\psi'' + c(\psi''(e))^2 = 0 \right\}.
\]

Equipped with this characterization we can conclude the proof. Again, since \( \Delta \theta \) is small enough, from a second-order Taylor approximation around \( \bar{\theta} \) the \( S_i \)'s type-contingent profit when he chooses a mechanism \( \omega \) while \( S_i \) chooses \( \omega' \) can be written as:

\[
\Pi_i^{\omega,\omega'}(\theta) \approx \Pi_i^{\omega,\omega'}(\bar{\theta}) - \Pi_i^{\omega,\omega'}(\bar{\theta})(\bar{\theta} - \theta) + \frac{1}{2}\Pi_i^{\omega,\omega'}(\bar{\theta})(\bar{\theta} - \theta)^2.
\]

Then, using \( E_{\bar{\theta}}[\Pi_i^{\omega,\omega'}(\theta)] = \Pi_i^{\omega,\omega'}(\theta) \leq E_{\bar{\theta}}[\Pi_i^{\omega,\omega'}(\theta)] = \Pi_i^{\omega,\omega'}(\theta) \leq \Pi_i^{\omega,\omega'}(\theta) \leq \Pi_i^{\omega,\omega'}(\theta) \) (resp. <) if:

\[
-(\Pi_i^{\omega,\omega'}(\bar{\theta}) - \Pi_i^{\omega,\omega'}(\bar{\theta}))(\frac{\Delta \theta}{2} + (\Pi_i^{\omega,\omega'}(\bar{\theta}) - \Pi_i^{\omega,\omega'}(\bar{\theta})) \frac{\Delta \theta^2}{6} \geq 0 \quad \text{(resp. <)}.
\]

We first show that incomplete contracts are part of a PBE of \( \mathcal{G} \) if one of the following conditions holds: (i) \( \rho \sigma > 0 \), or (ii) \( |\rho| < \sigma/\psi'' \) when \( \sigma > 0 \) and \( \rho \leq 0 \). Since players are symmetric, we only need to show that \( \Pi_i^{Q,Q} \geq \Pi_i^{P,Q} \). Consider then program \( \mathcal{P}_i^{Q_i} \), differentiating with respect to \( \theta \) and using the Envelope Theorem one can easily show that \( \Pi_i^{Q,Q}(\bar{\theta}) = q^*(\bar{\theta})(1 + \sigma e^Q(\bar{\theta}) + \rho q^Q(\bar{\theta})) \) and \( \Pi_i^{Q,Q}(\bar{\theta}) = (q^Q(\bar{\theta}) + \sigma e^Q(\bar{\theta}) + \rho q^Q(\bar{\theta})) \). By using the same kind of argument, differentiating program \( \mathcal{P}_i^{P_i} \) with respect to \( \theta \) and taking limits for \( \theta \to \bar{\theta} \) one also gets \( \Pi_i^{P,Q}(\bar{\theta}) = q^*(\bar{\theta})(1 + \sigma e^{P,Q}(\bar{\theta}) + \rho q^{P,Q}(\bar{\theta})) \) and \( \Pi_i^{P,Q}(\bar{\theta}) = (q^P(\bar{\theta}) + \sigma e^{P,Q}(\bar{\theta}) + \rho q^{P,Q}(\bar{\theta})) \). Substituting in (40) and taking \( \Delta \theta \) small we have:

\[
\Pi_i^{Q,Q} - \Pi_i^{P,Q} \approx q^*(\bar{\theta}) \left( \sigma \hat{e}_i^{P,Q}(\bar{\theta}) - e^Q(\bar{\theta}) \right) \Delta \theta,
\]
using the solutions of the linearized system of equations (37)-(39) together with $q^Q(\bar{\theta})$ and $\bar{e}^Q(\bar{\theta})$, we have:

$$\Pi_i^{Q,Q} - \Pi_i^{P,Q} \approx \frac{2\sigma \left( \sigma + \rho \psi'' \right) \Delta \theta q^*(\bar{\theta}) \left( 2\psi'' - 1 \right)^2}{(a + b\psi'' + c(\psi'')^2) \left( 2\psi''(1 - \rho - \sigma) - (1 + \sigma) \right)},$$

which yields immediately the result when $\sigma + \rho \psi'' \neq 0$.

Assume now $\sigma + \rho \psi'' = 0$, for $\Delta \theta$ small the sign of $\Pi_i^{Q,Q} - \Pi_i^{P,Q}$ is given by the second-order terms of equation (40):

$$\lim_{\rho \to -\sigma/\psi''} \left( \Pi_i^{Q,Q} - \Pi_i^{P,Q} \right) \approx \frac{\Delta \theta^2}{6} \lim_{\rho \to -\sigma/\psi''} \left( \Pi_i^{Q,Q}(\bar{\theta}) - \Pi_i^{P,Q}(\bar{\theta}) \right) = -\psi'' \frac{\Delta \theta^2}{6} < 0,$$

the same result holds when $\sigma = 0$.

We now show that complete contracts are part of a PBE of $G$ if one of the following conditions holds: (i) $\rho \sigma > 0$, or (ii) $|\rho| \leq \sigma(2\psi'' + 1)/2\psi''$ when $\sigma \geq 0$ and $\rho \leq 0$. Consider now the following difference $\Pi_i^{P,P} - \Pi_i^{Q,P}$, for $\Delta \theta$ small the same argument used above allows to obtain:

$$\Pi_i^{P,P} - \Pi_i^{Q,P} \approx q^*(\bar{\theta}) \left( \sigma(e_i^{P,Q}(\bar{\theta}) - \bar{e}^P(\bar{\theta})) + \rho(\bar{q}_i^{P,Q}(\bar{\theta}) - \bar{q}^P(\bar{\theta})) \right) \Delta \theta.$$

After simple algebra we have:

$$\Pi_i^{P,P} - \Pi_i^{Q,P} \approx \frac{\sigma \left( \sigma + 2\psi''(\sigma + \rho) \right) \Delta \theta \psi^*(\bar{\theta}) \left( 2\psi'' - 1 \right)^2}{(a + b\psi'' + c(\psi'')^2) \left( 2\psi''(1 - \rho - \sigma) - (1 + \sigma) \right)},$$

yielding the result for $\sigma + 2\psi''(\sigma + \rho) \neq 0$.

Consider now the case $\sigma + 2\psi''(\sigma + \rho) = 0$, for $\Delta \theta$ small the sign of $\Pi_i^{P,P} - \Pi_i^{Q,P}$ is given by the second-order terms of equation (40):

$$\lim_{\rho \to -\sigma/2\psi''} \left( \Pi_i^{P,P} - \Pi_i^{Q,P} \right) \approx \frac{\Delta \theta^2}{6} \lim_{\rho \to -\sigma/2\psi''} \left( \Pi_i^{P,P}(\bar{\theta}) - \Pi_i^{Q,P}(\bar{\theta}) \right) = \psi'' \frac{\Delta \theta^2}{6} > 0,$$

it is easy to check that the same result holds when $\sigma = 0$ and $\sigma + 2\psi''(\sigma + \rho) \neq 0$. This concludes the proof.

\section*{8.11 Proof of Proposition 11}

Again, as we have assumed $\Delta \theta$ small:

$$\Pi^{Q,Q} - \Pi^{P,P} \approx q^*(\bar{\theta}) \left( \sigma(e^P(\bar{\theta}) - \bar{e}^Q(\bar{\theta})) + \rho(\bar{q}^P(\bar{\theta}) - \bar{q}^Q(\bar{\theta})) \right) \Delta \theta.$$
From equations (31) and (32) we have:

$$\Pi^Q - \Pi^P \approx \frac{\sigma \Delta \theta q^*(\bar{e}) (2\psi'' - 1)^2}{(2\psi''(1 - \rho - \sigma) - (1 + \sigma))(2\psi''(1 - \rho) - (1 + 2\sigma))},$$

which directly proves that $\Pi^Q > \Pi^P$ (resp. $<\Pi^P$) if $\sigma > 0$ (resp. $<0$).

When $\sigma = 0$, the sign of $\Pi^Q - \Pi^P$ is given by the second-order terms of (40):

$$\lim_{\sigma \to 0} \left( \Pi^Q - \Pi^P \right) \approx \frac{\Delta \theta^2}{6} \lim_{\sigma \to 0} \left( \tilde{H}^Q(\bar{e}) - \tilde{H}^P(\bar{e}) \right) = -\frac{\Delta \theta^2}{6} \frac{(2\psi'' - 1)^2 \psi''}{(1 - 2\psi''(1 - \rho))^2} < 0,$$

which concludes the proof.

### 8.12 Proof of Proposition 12

Since we have considered a simple representative consumer economy where, for any given wealth level $w$, demands are derived as solution of the program,

$$\max_{(q_1, q_2, I)} \left\{ V(q_1, q_2, I, \theta) : \sum_{i=1,2} p_i q_i + I \leq w \right\},$$

with:

$$V(q_1, q_2, I, \theta) = \sum_{i,j=1,2; i \neq j} e_i (q_i + \sigma q_j) + \theta \sum_{i=1,2} q_i - \frac{1}{2} \left( \sum_{i=1,2} q_i^2 - 2\rho q_1 q_2 \right) + I.$$

Hence, as we focus only on what happens to consumers in the symmetric equilibria of contracting choices we have:

$$V(q^Q(\theta), q^Q(\theta), I, \theta) - V(q^P(\theta), q^P(\theta), I, \theta) = \frac{1}{2} \sum_{i=1,2} (q^Q(\theta) - q^P(\theta))(q^Q(\theta) + q^P(\theta)).$$

As we have assumed $\Delta \theta$ small enough, the result follows from:

$$\hat{q}^Q(\bar{e}) - \hat{q}^P(\theta) = -\frac{2\sigma \psi''(2\psi'' - 1)}{(2\psi''(1 - \rho) - (1 + \sigma))(2\psi''(1 - \rho - \sigma) - (1 + 2\sigma))},$$

which directly yields $\text{sign} (q^Q(\theta) - q^P(\theta)) = \text{sign}(\sigma)$. ■