Relational Procurement*

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Abstract

Procurement is often repeated in time, and gains from trade on non-contractible dimensions can be realized through reputation/relational contracting: the buyer can penalize or exclude in future procurements sellers that performed poorly in the past. In such a dynamic framework, keeping optimal number of eligible sellers and contract duration endogenous, we find that: (i) when non-contractible quality has moderate relative importance for the buyer, a general trade-off between reputation for quality and collusion emerges, as shorter contract duration (more frequent re-auctioning) and restricted participation (smaller number of eligible suppliers) facilitate non-contractible quality provision, but also supplier collusion; (ii) when quality is very important, optimal procurement requires selecting only one supplier and sticking with him ('efficiency price' contract); (iii) when auctions are compulsory (as for many public administrations) and non-contractible quality is important, collusion between few eligible sellers may be desirable for the buyer and welfare maximizing.

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1 Introduction

Non-contractible dimensions are present in different measure in every economic exchange.[1] For example in procurement of complex IT services it is often impossible to fully specify all the requirements that are of value for a buyer. R&D activities are often at the frontier of technological knowledge, effort towards improvements is difficult to measure and it is often impossible, to check whether the contractor has behaved as promised. Similarly, in health procurement it is often the case that the more critical is quality of procured goods and services, the more difficult is to correctly specify required properties of services to be procured and whether procurement comply with them.

It is well known that when these non-contractible dimensions are important in terms of gains from trade, letting suppliers compete on price - say, in an auction - may lead to a very inefficient outcome, for a buyer and in general.[2] Exchanges, however, are often regularly repeated, particularly in procurement where buyers need to be served over time. Reputational forces may then help governing transactions on non-contractible dimensions. An opportunistic supplier that overstates the non-contractible quality of an experience good or that purposely reduces non-verifiable but ex-post observable qualitative aspects to cut costs and bust profits can be punished by its buyer(s). Clearly, this cannot take place under any infringement of contractual terms, but the buyer can exercise some of his discretion to deliberately hinder an "unfaithful" contractor. With this respect, the most effective and, probably, the most natural punishment consists in excluding the supplier from (some) future trade(s). This form of punishment is certainly available in private contracting where the buyer his generally free to exclude any buyer from its selecting process. In the case of public procurement this decision is still viable to some extent, but it may be partially limited by ruling laws which often restrict civil servants' discretion so as to avoid corruption. However, some national legislations do provide room for discretion in exclusion of dubious providers and leave some accountability in the hands of public buyers.[3]

This paper analyzes repeated procurement processes (recurrent auctions or other forms of search) where non-contractible dimensions are an important source of gains from trade. In particular, quality is non-contractible in our analysis and will be interpreted in broad sense capturing all value-enhancing decisions that a supplier is free to take during the contract execution and which the buyer observes but cannot directly enforce because they are not verifiable. Among the most interesting instances of this type of decisions we list many type of investments for innovation, R&d activities and, in general, effort. Certainly, there are normally other dimensions in the relationships

[1] Reasons why some dimensions of exchanges are not explicitly contractible include complexity and prohibitive legal cost of verification; see Hart (1995) for an in depth discussion and Tirole (1999) for an evaluation of the debate on contracts incompleteness.

[2] See Manelli and Vincent (1995), among others.

[3] This is the case in some countries such as for example the US, UK and Hong Kong. See for example the 2004 US Public Procurement Guidelines. Similarly, the Hong Kong Government Works Branch Technical Circular, 12/90, "Prequalification of tenderers for public works contracts" for unusual scope or complexity of work. Recently, a new two-stages procedure for EU procurement has been introduced. It contemplates a pre-qualification stage where the public buyer has some discretion to exclude suppliers, followed by an auction. See EC, Directive 2004/18/EC, "On the coordination of procedures for the award of public works contracts, public supply contracts and public service contracts". For a broad discussion on procurement strategies and national legislations see Albano et al. (2006).

that are under the control of the supplier but that are contractible and can then be governed with a proper design of the contract by the buyer and with an adequate price paid for procurement. In addition, the buyer can also condition the procurement environment. In particular, she can decide to vary the amount of search done before proposing a contract, namely it has the possibility to fix the number of suppliers in the pool admitted to participate at the recurrent auctions. Furthermore, she can also decide the length of the procurement contract and consequently also the frequency of the selection procedure. As previously discussed, we will also allow the buyer to punish with exclusion from future procurement contests firms that have decided to offer low levels of quality along the contractual relationship.

We show that, in a dynamic procurement process the buyer may want to restrict the number of potential trading partners at the cost of reduced screening and more expensive procurement. This is done to boost non-contractible quality. Indeed, by restricting competition, the buyer leaves firms sufficient future rents so that they can find profitable to build reputational commitments for future interactions and prefer to provide acceptable levels of non-contractible quality in the current relationship (and refrain from moral hazard). Duration of supply contracts is also a crucial aspect in dynamic procurement. Abstracting from (important) technological aspects such as the rate of obsolescence, a shorter duration of supply contracts implies more frequent re-selection or search. We then show that with higher frequency of interaction it is easier for a buyer to obtain high non-contractible quality levels from sellers by threatening exclusion from future trade. Indeed, with more frequent contracting the threat of exclusion is closer in time and gains from "cheating" are smaller so that larger implementable quality can be expected. Furthermore, shortening contract length also reduces the buyer's risk to be locked-in with undesirably low quality but at the same time these desirable effects for the buyer have to be contrasted with the buyer's costs of running and organizing more frequent auctions and with firms' inability to spread possible fixed (and relation-specific) costs of procurement over a longer contract.

Considering pros and cons we then show that in repeated procurement a buyer may profit from suppliers' stake on future profits and induce larger non-contractible quality by restricting the pool of firms admitted to compete for the procurement contract and reducing the contract length or, which is equivalent, increasing the frequency of recurrent auctions. With respect, we also show that these two important dimensions of procurement which would be normally independent instruments for the buyer in case of fully contractible quality, turn out to be closely related (either in terms of complementarity or substitutability) when quality is non contractible.

Non-contractible dimensions thus induce the buyer to run frequent auctions within a limited number of potential suppliers. However, it is also well known that this environment that the buyer designs to (at least partially) govern non-contractible quality is also the most favorable environment for inducing and sustaining collusive behavior between the selected suppliers. Hence, we illustrate a rather general and possibly disappointing trade-off between reputation for non-contractible quality and collusion in dynamic procurement. Longer duration of supply contracts - less frequent auctions - together with larger pool of competing suppliers both deter collusion among eligible suppliers but also reduce non-contractible quality levels obtainable from them. Symmetrically, shorter contracts - more frequent auctions - and a smaller pool of suppliers both facilitate suppliers' collusion but also the enforcement of non-contractible quality standards.[4]

This trade-off may show up being rather disappointing because it seems to mark limits on the remedies that can be put at work for non-contractible quality provision. However, our analysis clarifies that collusion itself can directly interact with firms' incentives to provide non-contractible quality. In fact, by increasing the selling price, collusion clearly increases the expected gains from participating in procurement auctions which, as usual, can also be seen as the cost of being excluded when the buyer reacts to low levels of non-contractible quality. Hence, we show that the seeming trade-off between non-contractible quality and collusion, that naturally seems to emerge in repeated procurement, may in fact reveal to be only apparent because larger future rents associated with collusion make firms ready to offer larger non-contractible quality. This somehow provocative result suggests that the buyer may not be necessarily concerned by the spontaneous formation of cartels and may even foster establishment of consortia among firms that may alternatively compete. Indeed, reinterpreting our result on collusion and non-contractible quality, cooperation among firms in the form of consortia or joint-ventures is expected to provide larger levels of quality. Hence, lessening competition by reducing the pool of potential suppliers and shortening contract duration that both induce a collusive or legal agreement between eligible suppliers may leave the buyer better off. For example, this suggests that in the procurement of innovation cooperation among firms and in the limit also collusion, do not necessarily hinder innovation when R&D activity is not fully-contractible.

Further exploring this point, we also note that it is often the case in repeated procurement that participating firms have better knowledge on their relative abilities and efficiency than the buyer. Hence, in addition to the previous effects of agreements among firms, consortia and collusion may also have a desirable sorting effect whereby more efficient firms may be selected for supplying. This may not necessarily translate into lower prices for the buyer, and certainly does not in the case of collusion. Rather, larger efficiency in production in these cases tend to increase firms rent and then further boosts implementable non-contractible quality.

We then conclude our analysis discussing how a buyer concerned by non-contractible quality, cost of procurement and, possibly, also directly by efficiency in supplying should design the procurement environment (i.e. the number of competing firms and the exclusion rule for low quality provision) together with the contractible dimensions of the relationship (i.e. the contract duration and auction frequency) to effectively compose its objectives.

1.1 Related literature

Manelli and Vincent (1995) analyze the adverse effects of competition in procurement with non-contractible quality. They cast their model in terms of adverse selection (sellers are of different quality which is reflected in the produced good), focus on single transaction, and show that when

[4]Interestingly some recent studies do confirm this intuition. For example, studying a data set for train operating companies in UK Affuso and Newbery (2002) show that (discretionary) investment is stimulated by shorter rather than longer contracts. Notwithstanding a standard hold-up problem associated with contract renewal that should point in opposite direction, the authors suggest that frequent re-procurement with short contracts disciplines suppliers who care for future re-award of the franchise.

gains from trade are concentrated on non-contractible dimensions, auctions deliver the worst possible outcome, as they select firms producing the good at the lowest cost but with the lowest level of non-contractible quality. Sequential take-it-or-leave-it offers to randomly selected sellers is then a better mechanism. We clearly differentiate from this paper by considering an environment where quality is an effort decision of procuring firms and not an innate characteristic of suppliers and also considering a dynamic context with repeated procurement. Reaching similar conclusions on the desirability of limited competition but for a different reason than the "bad selection" effect in Manelli and Vincent (1995), our paper complements their analysis.[5] In a different context where unexpected contingencies may make renegotiation necessary, Bajari and Tadelis (2001) also show that with relevant non-contractible dimensions auctioning with fixed-price contracts may be dominated by bilateral negotiation over cost-plus contracts. The literature on optimal procurement of innovative goods and services, where pre-auction non-contractible R&D investments are crucial, including for example Taylor (1995), Fullerton and McAfee (1999), and Che and Gale (2003), is also related to our work. In these analyses even though procurement is not repeated, limiting participation is optimal, as in our model, but for a different reason than reputational effects. Reducing the number of participant increases each participant's probability of winning the award, and thereby encourages pre-auction non-contractible R&D effort-investment.

Our environment on repeated purchase with non-contractible quality relates our analysis to the industrial organization literature on reputation and competition which studies whether firms' reputational commitments to high non-contractible quality can be compatible with a competitive environment and that has been initiated by the seminal works on experience good markets of Macaulay (1963), Klein and Leffler (1981), Shapiro (1983), and Allen (1984). These early analyses were concerned with the compatibility of "quality-assuring" reputational equilibria - requiring rents that make the effort of maintaining reputation worthwhile also with free entry in the market - but did not analyze in detail firms' competitive interaction (firms' incentives to steal business from each other).[6] Stiglitz (1989) also raised the question how could reputation be compatible with perfect competition that should eliminate any future supracompetitive gains. More recently, Kranton (2003) offers a model that captures this dilemma. In presence of moral hazard on quality and competition, high quality equilibria are unfeasible and he suggests restricting competition in industries where non-contractible quality is important. In a different model Bar-Isaac (2005) confirms Kranton and Stiglitz's view at the limit, but shows that at intermediate levels of competition a further increase in competition (number of firms and substitutability) may well increase equilibrium product quality. Hoerner (2002) offers the first elegant answer to Stiglitz's question: in his model with heterogeneous consumers, adverse selection and moral hazard, high prices signal high quality and make competition compatible with (in fact necessary for) reputation to work.[7]

[5] On the desirability of auction when all relevant dimension of the trade relationship are contractible see Bulow and Klemperer (1996).

[6] In Klein and Leffler and Shapiro firms face a perfectly elastic demand at the quality assuring price; in Allen consumers are randomly allocated among the firms charging the lowest price weakly above the "quality-assuring" one.

[7] In our model, and with auctions in general, (price) signalling is impossible because the lower price is chosen by the mechanism, and the trade off quality-competition reappears.

In the context of procurement Laffont and Tirole (1993, chapter 4) explain in a two-stage model that when demand is fixed or inelastic, as in the case of many procurement situation in which the buyer buys a fixed amount, one way to stimulate quality provision is linking future business to current quality. Reputation can be exploited to threaten suppliers, especially when the buyer can potentially deal with many suppliers. Kim (1998) and Doni (2004) also study a repeated procurement auctions with moral hazard on non-contractible quality, and show that it may be good to restrict participation and threaten exclusion if the level of non-contractible quality is too low. Even if Shapiro (1983) noted that the frequency of interaction may facilitate the operation of reputational mechanisms with experience goods, the effects of contract duration and auction frequency have been largely neglected in the procurement literature. Contrary to these papers, we consider the possibility to adjust the frequency of procurement, its interaction with the decision to restrict the pool of potential suppliers and also the potential of cartels and consortia in sorting the several trade-offs between quality, procurement costs and efficiency.[8] Ellman (2006) analyses a contract length trade-off associated to the presence of investment which becomes specific to the contractual relationship but does not look at the role of threats in case of repeated purchase.[9]

Our paper also contributes to the literature on incomplete explicit contracts (the "Hart and Moore paradigm"), and in particular on how dynamic interaction allows to complement these by implicit/relational contracts, from the early contributions of Bull (1987) and McLeod and Malcomson (1989, 1998), to the recent ones like Levin (2003) and Fuchs (2005). As in our paper, the focus of this literature is how relational contracting allows parties to enforce and govern agreements on observable but not verifiable dimensions (effort, investment, quality). Within this rich and growing literature, our work is closest in spirit to the contributions more directly focussing on the interaction between explicit and implicit contracts, i.e. on how explicit contracts should be structured or modified to optimize the joint outcome of explicitly contracted dimensions and implicit effects, like Baker, Gibbons and Murphy (1994, 2002), Pearce and Stacchetti (1998), Halonen (2002), Che and Yoo (2001), Blonski and Spagnolo (2003), and Rayo (2004), among others. From a technical point of view our paper contributes this literature by studying relational contracting with multiple competing agents, adverse selection between the agents (here the firms) and the principal (the buyer) and considering the possibility that the agents collude or are induced to cooperate by the principal. In this context we also add as important variables that the principal can control the length of contractual interaction with the selected agent and the level of competition among agents.[10]

We conclude this literature review with noticing that there are at least two approaches to reputation in markets, as convincingly emphasized by Bar-Isaac (2003). A first approach views "reputation as beliefs" where uninformed players infer intrinsic qualities of contracting parties by their behavior so that a firm's reputation consists in buyer's beliefs about its quality-type. See for example Kreps and Wilson (1982), Milgrom and Roberts (1982). A second non-exclusive view,

[8] Some authors have explicitly dealt with collusion in repeated auctions among asymmetrically informed firms but uniquely considering full contractibility. See for example Aoyagi (2003) and Blume and Heidhues (2004).

[9] The importance of contracting timing has been recently emphasized by Guriev (2005).

[10] Fehr et al. (2004) show experimentally how in a dynamic environment, when non contractible aspects become important, agents do not search for the best offer each period but rather stick to the same partner with whom they try to cooperate.

which is the one employed in our paper and is related to the previously discussed literature on markets with experience goods and implicit contracting, considers reputation as a self-sustaining commitment to provide desirable but non-contractible quality by a credible threat on the side of the other partner in case reputational commitment is violated.

The rest of the paper is organized as follows. Section 2 presents the model setup. Section **??** analyzes procurement and quality with competing firms. Section **??** discusses the effect of collusion on implementable quality. Section 5 illustrates optimal procurement. Section 6 extends the base model and discusses its main assumptions. Finally, Section 7 concludes.

2 Model Setup

A buyer needs to procure a unit of a good (or a service) at any period and her per-period valuation of the good $V(q)$ is increasing in the procured quality $q \geq 0$. Amongst the N potential suppliers, any firm i can procure a unit of the good with quality q by incurring in a per-period cost $\theta_i + \psi(q)$ where $\theta_i \in \Theta \equiv [\underline{\theta}, \overline{\theta}]$ is a firm-specific (in-)efficiency parameter and $\psi(\cdot)$ is a positive real valued function increasing in q, with $\psi(0) = 0$. The per-period (social) value of quality $s(q) \equiv V(q) - \psi(q)$ is concave in q with a unique maximum at q^* (subscripts will denote derivatives). Supplied quality is observable for contracting parties but is not verifiable for third parties and then it is not-contractible.

The buyer is not fully informed on each firm's cost θ_i so that she may run an auction or any awarding process to selects a supplier. An auction awards a procurement contract to a bidder i that offers the lowest acceptable bid b_i for the contract (we will indicate that firm as the contractor) and maps the vector of all bids $\mathbf{b}$ into the payment b_w for contract execution. Since quality is not enforceable by a court, a contract specifies the number of procurement periods $x \geq 1$ and the payment b_w that the supplier receives for all the x periods. The buyer may also set a reservation price $r \geq 0$ so that acceptable bids must satisfy $r \geq b_i$ (if $r < b_i$ for any i, the buyer does not award the contract). She may also decide to limit the number of bidders that are eligible and admitted to the auction process. We denote with $n \leq N$ the number of firms admitted in this pool so that any of the participating firms knows that the (maximal) number of competitors at the auction stage is $n-1$.[11]

The supplier that is awarded the contract (i.e. the contractor) sets the level of quality it will provide once and for all the duration x of the contract.[12] Although quality is not contractible, the buyer may react to low procured quality with decisions that affect future contractual relationships with the current supplier. In particular, if the quality provided by the contractor does not satisfy the buyer, in a sense that we clarify next, then the latter can discretionally penalize the contractor in future auctions. We model this possibility with an exclusion rule that contemplates a *minimum quality requirement* $\underline{q}$ so that if the firm procures a quality $q < \underline{q}$, the buyer can discretionally decide to exclude this seller for (at least) the next $T \geq 0$ auctions. In addition, we will consider

[11] In Section 5 we will consider the possibility that the buyer asks a participation fee to suppliers admitted to the pool of participating n firms.

[12] Our result would be unaffected if firms could be free to choose a different quality level at *any* period.

the possibility that the buyer promises to deliver a discretionary bonus B and retains any "bond" P posted in her hands by the contractor in case the latter turns out to be cheating (i.e. providing quality $q < \underline{q}$).

The relationship between the buyer and any potential contractor is thus characterized by some verifiable (and court-enforceable) ingredients: the terms of the contract itself (i.e. duration x and price b_w) and the rules governing the auction (i.e. reservation price r and number n of eligible suppliers). We will refer to (n, r, b_w, x) as the *contractible terms.* Furthermore, the procurement relationship is characterized by terms that are *non-contractible* and discretionary: the procured quality q, exclusion rule $\sigma \equiv (\underline{q}, T)$ and the buyer's decision concerning the bonus and the bonds.

Time horizon is infinite and all the players have a constant and common discount factor equal to $\delta \leq 1$. Finally, running any procurement contract implies a contract-specific investment with a total (for the buyer and the supplier) costs $K \geq 0$. For example, K may be a fixed set-up cost that each contractor incurs to procure the good of whatever quality and that has to be paid anew in any contractual relationship. Alternatively, it may also be composed by the buyer's cost for organizing an auction or interacting with a new supplier.

We will consider stationary strategies so that a strategy for the buyer consists in setting the contractible elements (x, n, r, b_w) of the procurement relationship and publicly announcing the exclusion rule σ, the discretionary bonus B and the required bond P. A strategy for any firm i is composed by a participation decision, a bid at any awarding process and a decision on procured quality in any awarded contract.[13] Hence, we can describe the **timing of the game** as follows.

$\mathbf{t = -2}$: *The buyer sets the contractible terms and publicly announces the non-contractibles.*
$\mathbf{t = -1}$: *She randomly selects the n eligible suppliers amongst the N potential sellers.*
$\mathbf{t = 0}$: *An infinite repetition of the following* ***stage game*** *(or auction game) takes place:*

– *At time $t_1 \geq 0$, an auction publicly awards the procurement contract to a contractor who obtains the payment b_w and sets procured quality q;*

– *At any period $t \in \{t_1, t_1 + x - 1\}$ the winner procures the good;*

– *At time $t_1 + x$,* if *$q < \underline{q}$ the buyer retains any bonus or bonds, the contractor is excluded for at least T next auctions and replaced by a new firm,* otherwise *the buyer pays B, returns P and the pool of eligible suppliers remains unchanged with a new stage game starting.*

Although relatively simple, this dynamic game may turn to be intractable. We thus introduce some simplifying assumptions. Then, having emphasized the main ideas and results in this clear-cut environment, in Section 6 we will discuss these assumptions, alternative model specifications and their role on our results.

Assumption 1 *The cost parameter θ_i of any firm i is drawn anew at any period from a time-invariant and independent distribution with density $f(\theta_i)(> 0$ for any $\theta_i \in \Theta)$ which is common knowledge.*

[13] As shown in MacLeod and Malcomson (1989) and Levin (2003), restricting to stationary contracts where buyer's and firms' behavior does not change along the equilibrium path is without loss. For simplicity, in the exposition we will treat n and x as continuous variables.

Assumption 2 *Firms are fully informed.*

Assumption 3 *The buyer needs to be served at any period, otherwise she obtains a very negative payoff $-k$ with $k >> 0$.*

Assumption 1 could be substituted by considering a θ_i which is drawn anew at any auction stage but remains the same for the x periods of the contract, without altering our qualitative results. On the contrary, a model where each firm is characterized by a persistent level of efficiency would be much less tractable because the buyer would then learn firms' efficiency and firms would anticipate that their actions signal information. Also Assumption 2 is certainly not without loss of generality. It significantly simplifies our analysis (particularly when dealing with colluding firms) and can be justified on the basis that procuring firms frequently interact and then tend to know each other well. As we will discuss in Section 6, the underlying trade-offs we discuss in this simple model persist also with persistent efficiency and asymmetrically informed firms. Assumption 3 simplifies the analysis of the reservation price and will be also discussed in Section 6.

Also simplifying the exposition, we have not explicitly modelled the many other elements of a typical procurement relationship that are contractible (such as certain elements of quality). However, a more general interpretation of our model is that the buyer evaluates all these contractible elements with a scoring rule that is here simply represented by firms' bids (which are then scores for all contractible elements). In Section 6 we will explore the possibility for the buyer to use scoring rules also on non-contractible quality (irrelevant in the current model setup).

3 Implementable quality

We first briefly illustrate the benchmark of contractible quality that will be useful in the sequel. If the buyer can directly control and contract procured quality she directly sets the desired quality. For an awarding price b_w the profit of the winning firm i is $b_w - \theta_i - C(q)$ where $C(q)$ is the cost for procuring quality q in any of the x contractual periods plus the quality-independent (expected) costs $E[\theta]$ for the remaining $x-1$ periods of supply after the first one (recall that at the bidding stage each firm i knows only the current realization of θ_i).

By standard arguments on price competition, at any auction the contract is awarded to the most efficient firm who wins by offering and being paid a price b_w equal to the minimum between the reservation price r and the second most efficient firm's (total) cost.[14] When the buyer admits n firms at the auction stage, for any vector of firms' cost-efficiency $\boldsymbol{\theta} \equiv (\theta_1, ..., \theta_n)$, we will indicate with $\theta'(n) \equiv \min \boldsymbol{\theta}$ the cost parameter of the most efficient firm in the pool of n eligible suppliers (i.e. the first order statistics of $\boldsymbol{\theta}$). Furthermore, the risk of being left with no procurement forces the buyer to set a reservation price high enough to guarantee participation for any realization of firms costs, i.e. $r = \overline{\theta} + C(q)$ when required quality is q. Hence, the awarded price b_w can be written

[14] Interpreting K as a fixed set up cost for procurement, the winning bid would be augmented by K so that the buyer's surplus and the winning firm's profits would be as in the text.

as $b_w = \theta''(n) + C(q)$ where $\theta''(n)$ is the cost of the second most efficient firm.[15] Anticipating this winning bid b_w, the buyer sets n, x and q to maximize her objective function[16],

$$S(q, x, n) \equiv \sum_{t=0}^{\infty} \delta^{tx} E\left[\frac{1-\delta^x}{1-\delta} V(q) - b_w - K\right].$$

Lemma 1 (Contractible quality) *If quality is contractible the buyer asks quality the optimal quality q^*, she admits all potential suppliers at the auction stages, i.e. $n^* = N$ and she sets the minimal contract length i.e. $x^* = 1$ if $E[\theta] - E[\theta''(n)] \geq K$ and the maximal contract length i.e. $x^* = \infty$ otherwise.*

The price paid by the buyer and asked by the most efficient firm b_w is decreasing in n since $\theta''(n)$ is a also (weakly) decreasing in n. Hence, as expected, the buyer will foster maximal competition. Concerning contract duration, on the one hand a longer contract determines smaller costs for organizing the auctions which become less frequent (or a better management of scale economies). On the other hand, a larger x also implies that for a longer period (i.e. along the current contract) the buyer will be stuck with a firm that may no longer be the most efficient one in the pool of n eligible suppliers. Hence, the buyer compares the cost K of organizing a new auction (or a new contract) with the price reduction that can be obtained after the first period of procurement by auctioning off a new contract, i.e. $E[\theta] - E[\theta''(n)]$ (≥ 0). Note that this price reduction is increasing in n so that stronger competition induces (weakly) shorter contracts, thus showing substitutability between n and x. Finally, optimal contractible quality is efficient and independent of other contractual terms.

From now on we will abandon the assumption of contractible quality. Consider first a single period interaction. In this case the buyer would only be able to implement nil quality since none of the firms is ready to supply costly but not-contractible quality. However, procurement over an infinite horizon allows to rely on reputation forces so that the buyer can implement larger quality.[17]

For any exclusion rule σ, the contractor may choose to satisfy the quality requirement $\underline{q}$ or not (i.e. to provide $q \geq \underline{q}$ or $q < \underline{q}$). Clearly, none of the firms has incentive to provide a quality larger than $\underline{q}$ because it will not be credited for extra quality and in equilibrium procured quality will be either $q = \underline{q}$ or $q = 0$. If the contractor decides to comply with the buyer's quality requirement, it obtains an over-all expected profit equal to

$$b_w - \theta - C(q) + B + \pi(n)\frac{\delta^x}{1-\delta^x},$$

[15] Precisely, $\theta''(n)$ is the minimum between the θ of the second most efficient firm and $\overline{\theta}$ (as prescribed by the reservation price). If the buyer contracts with a single firm (who then asks the reservation price), then $\theta''(n) = \overline{\theta}$.

[16] Since the framework is stationary, the buyer's optimal strategy is time invariant and she sets the terms of the procurement process once and for all the stages of the game.

[17] As usual, multiple equilibria exists in a repeated game. We are interested in existence of equilibria in which procured quality is $q > 0$.

where $\pi(n)$ is the expected profits from any future auction for a firm that in the current contract complies with the quality requirement.[18] As we will show below, this profit depends on the number n of competing firms. Alternatively, the winning firm may decide to cheat, thus saving quality costs, but it then loses some of future profits as prescribed by the exclusion rule σ and also the benefit B and the posted bonds P. In this case the profits are

$$b_w - \theta - C(q) + \psi\left(q\right)\frac{1-\delta^x}{1-\delta} - P + \pi_0,$$

where π_0 is the expected *stream* of future profits determined by the exclusion rule σ.

The contractor will be ready to satisfy the quality requirement if the expected gain of future profits by providing the required quality is larger than the immediate cost saving the firm obtains in the current contract by cheating on quality, i.e.

$$\pi(n)\frac{\delta^x}{1-\delta^x} - \pi_0 + (B+P) \geq \psi(\underline{q})\frac{1-\delta^x}{1-\delta}. \tag{1}$$

In the following we will state that a minimum quality requirement $\underline{q}$ is implementable, if it satisfies condition (1).

Proposition 1 (Non contractible quality) *An optimal exclusion rule σ so that $\overline{q}\left(x,n\right) > 0$ implies $\pi_0 = 0$ and the buyer cannot increase $\overline{q}\left(x,n\right)$ by promising bonuses and accepting bonds.*

The maximal quality $\overline{q}\left(x,n\right)$ the buyer can implement is decreasing in n and x with $\overline{q}\left(x,n\right) = 0$ if either $n = N$ or $x = \infty$.

The maximal implementable quality $\overline{q}\left(x,n\right)$ is derived equating the two sides of contractor's incentive constraint (1) so that the buyer can implement a minimum quality requirement if it is not too large, i.e. if $\underline{q} \leq \overline{q}\left(x,n\right)$.

The possibility to obtain a strictly positive $\overline{q}\left(x,n\right)$ relies on the contractor's cost of exclusion from future relationships with the buyer. With this respect whenever the number of firms admitted in the pool of n is strictly smaller than the number of all potential suppliers, then a rule σ that excludes a cheating firm for a single period effectively excludes it forever so that $\pi_0 = 0$. Indeed, when all other firms are ready to supply the required quality $\underline{q}$ a firm that cheats will never be able to enter again the pool of n firms since no firm will be excluded in the future and the pool of competing firms will remain unchanged. Clearly, for this mechanism to be effective it must be that at least one firm is excluded from the pool and that the contract does not last for ever.

Furthermore, since the expected profit from future competitions $\pi(n)$ is decreasing in n it is clear that a larger pool of competitors makes the deviation to a low quality provided more interesting. Similarly it happens when the duration of the contract is larger since the contract saves a larger quality cost by cheating and also because future profits are more "distant" in the future. In both cases the left hand side of (1) is smaller and the right hand side larger, implying that the maximal implementable quality that does not induce a contractor to cheat on quality is smaller when n and x are large.

[18] Clearly, if a firm prefers to comply with quality, it will do so forever.

Finally, bonus and bonds cannot help the buyer governing quality due to her own incentive constraint. Although bonus and bonds increase the stake for the contractor to provide the required quality, they also increase "by the same" amount the temptation by the buyer to cheat on the implicit agreement retaining bonds and not providing the bonus even if the firm procures the required quality. Indeed, the buyer has an incentive to always discard the current contractor, retain the bonus and the posted bonds and substitute it with any one of the $N-n$ other firms.[19]

Proposition 1 shows that if the buyer wants to increase procured quality she must rely on two instruments: the contract length x and the dimension of the pool of competing firms. In particular, if the desired quality is $q' > \overline{q}(x,n)$, the buyer must necessarily reduce x and / or n at levels x' and n' so that the maximal implementable quality increases and becomes $\overline{q}(x',n') \geq q'$.

4 Reputation and collusion: a trade-off

Let us assume first that firms compete so that the most efficient firm wins with a bid $b_w \equiv \theta''(n) + C(q)$ where q is the quality other competing suppliers would be ready to offer in case they were asked to procure. In this case, the expected profit for any future auction is $\pi(n) = \pi''(n)$ where

$$\pi''(n) \equiv \{E[\Delta\theta(n)] - [\psi(\underline{q}) - \psi(q)]\frac{1-\delta^x}{1-\delta}\}\frac{1}{n}, \tag{2}$$

and $E[\Delta\theta(n)] = E\left[\theta''(n) - \theta'(n)\right]$ is the informational rent that firms can expect when winning any of the future auctions, the second term in the curly bracket is the extra cost the contractor incurs in procuring required quality $\underline{q}$ (as compared with other potential suppliers's quality q) and $1/n$ is the probability of being the most efficient firm out of the n eligible firms. (Clearly, if potential suppliers plan to respect the minimum quality requirement so that $q = \underline{q}$ then $\pi''(n) = E[\Delta\theta(n)]/n$.) As shown in the Appendix, this profit satisfies the properties discussed in Section 3 and it is also interesting to notice that for low heterogeneity of firm's cost (so that $E[\Delta\theta(n)]$ is small, independently of n) implementable quality is necessarily small.

In light of the discussion of Proposition 1, the buyer then faces a trade-off when setting the required non-contractible quality $\underline{q}$: it must reduce x and n so as to increase $\overline{q}(x,n)$ but by so doing it increases the cost of establishing the contractual relationships more frequently (i.e. the cost K accrues more often) and it renounces to lower prices since the winning price b_w is larger due to softer competition (i.e. $\theta''(n)$ is a decreasing function of n).

Corollary 1 (Quality and competing firms) *When quality is not contractible and firms compete, the buyer optimally restricts the pool of eligible firms $n < n^* = N$ and the contract duration $x \leq x^*$ as compared with the case of contractible quality.*

The optimal x is $x^* = 1$ if $E[\theta] - E\left[\theta''(n)\right] \geq K$ in which case n and x are substitutes, otherwise $x \in (1, x^*)$ and n and x are complements.

[19] This parallels the well known results in MacLeod and Malcomson (1989) and Levin (2003). Formally, the buyer's incentive constraint requires $-B \geq P$ so that it must be $B = P = 0$.

This Corollary shows that since quality matters for the buyer and it is not contractible, indeed she prefers excluding some firms thus reducing competition and also possibly limiting the length of the procurement contract, as compared with what she would prefer were quality contractible or were a nil quality preferable.[20] In fact, it is only by restricting n and x that she can obtain a strictly positive maximal implementable quality $\overline{q}(x,n)$ and then also set a strictly positive minimum quality requirement $\underline{q}$. Interestingly, the Corollary 1 also shows that, contrary to the contractible quality case, the optimal n and x can be complement in the sense that any event which causes an optimal reduction of firms admitted at any auction also fosters a reduction of the contract length. The result in the previous Corollary thus sheds some light on a relationship which is often neglected in the literature, namely that between competition and contract length in providing rents when the buyer needs to induce the suppliers to provide non-contractible quality.[21]

In light of the results in Corollary 1 it is now important to realize that in a context of repeated interactions, reducing the number of competing firms and increasing the frequency of interactions are (among) the most effective conditions that foster and strengthen collusion between potential suppliers. Since collusion in procurement is far from being a simple theoretical curiosity and, on the contrary, it is a pervasive phenomenon, the results in Corollary 1 point out that, controlling for procured quality (reducing n and x), the buyer may risk to induce suppliers in the pool of n firms to collude. We now explicitly address this possibility and illustrate what may happen to the procurement process determined above (with associated maximum implementable quality $\overline{q}(x,n)$ and exclusion rule σ) if competition cannot be simply assumed.

To account for this possibility, we indicate with $\hat{\pi}(n)$ the expected profit that a contractor can obtain if a cartel is in place. In this case, the most efficient firm is awarded the contract and all the other potential suppliers in n abstain from bidding or submit losing bids, so that collusion takes place with bid rotation. For the cartel to be sustainable at any auction stage, the second most efficient firm (i.e. the one with the highest incentive to deviate) should not prefer to undercut the currently most efficient one. If it does not deviate from the agreement, it can expect the collusive payoff. Otherwise, by deviating it obtains an immediate rent $D(n) \equiv \overline{\theta} - \theta''(n)$ but the cartel breaks down reverting to competition. Hence, a cartel is viable and more likely to form if the following cartel incentive compatibility constraint is verified

$$\hat{\pi}(n)\frac{\delta^x}{1-\delta^x} \geq D(n) + \tilde{\pi}''(n)\frac{\delta^x}{1-\delta^x}, \tag{3}$$

[20] Although Manelli and Vincent (1995) consider a screening and static contexts, they show that the buyer prefers an auction (i.e. a $n = N$) if the procured good is a standardized one and bilateral bargaining (i.e. a sequence of contracting with $n = 1 < N$) when the value of non contractible quality is large.

[21] An interesting implication of these results refers to the process of international liberalization which often obliges local governments to allow foreign firms to public tendering (e.g. in the EU in case of large projects). Foreign participation is generally seen as competition and welfare enhancing effect of non discriminating and open markets. However, our analysis points outs that with non-contractibility, this may not be the case. On the contrary, the additional competitive pressure of foreign firms corresponds to an uncontrolled (by the buyer) expansion of firms n with consequent reduction of implementable quality.

where $\tilde{\pi}''(n)$ is the competitive profit firms can expect after the cartel breaks down.[22]

Introducing the possibility that suppliers collude leads to the following trade-off for the buyer.

Proposition 2 (A trade-off: reputation vs. collusion) *Reducing the contract length x and /or the number of potential suppliers n allows the buyer to increase implementable non-contractible quality, but at the same time it facilitates collusion among suppliers.*

First note that a necessary condition for constraint (3) is $\hat{\pi}(n) \geq \pi''(n)$. Hence, it follows that reducing x the left hand side in (3) increases more than the right hand side. Consider now the effect of reducing n. Since $D(n) \equiv \overline{\theta} - \theta''(n)$ is decreasing in n, one simple needs to show that $\hat{\pi}(n) - \pi''(n)$ is also decreasing in n, so that reducing n has the same qualitative effect of a reduction of x. From (2) we know that $\pi''(n)$ is decreasing in n since a larger n determines a lower probability of being the most efficient firm in the future *and* also since it determines a lower rent in case the firm wins ($\Delta\theta\,(n)$ is decreasing in n). The first effect is also clearly present in $\hat{\pi}(n)$ for any collusive agreement. However, the second effect is smaller with collusion. Indeed, the most efficient firm that wins with a collusive agreement asks a price (weakly) higher than that it would ask with competition which is $b_w = \theta''\,(n) + C(q)$ and the collusive price is also less sensitive to n than the competitive price.

Corollary 1 illustrates that prominence of quality for the buyer may induce her to both restrict the pool n of bidders at any auction and reduce the length of contracts x, thus having more frequent auctions. However, Proposition 2 shows that the smaller are n and x, the larger is the scope for and stability of collusion. Hence, the buyer ends up with a trade-off: trying to implement larger implementable quality with reputational forces, the buyer increases the risk of inducing suppliers to collude.

How the buyer addresses this trade-off between reputation forces for non contractible quality and collusion? What is then optimal procurement in light of this trade-off?

5 Procuring non contractible quality

Proceeding into the analysis of optimal procurement it is first important to discuss what are the incentive for firms in a cartel to abide the quality requirement of the buyer. The (colluding) contractor will respect any quality requirement $\underline{q}$ if the expected rent provided by the cartel $\hat{\pi}(n)$ satisfies constraint (1) on incentive compatibility for quality. Comparison of the rents with and without collusion (respectively $\hat{\pi}(n)$ and $\pi''(n)$) leads to the following.

Proposition 3 (Implementable Quality with Collusion) *Given contract length x and n potential suppliers, the maximal implementable quality when firms collude is larger than with competing firms.*

[22] The deviating firm also decides if cheating on quality or not so that $\pi''(n) \leq \tilde{\pi}''(n) \leq (\hat{\pi}(n))$ when the deviating firm cheats on quality.

Firms collude since they can expect a larger profit as compared with competition so that they are more reluctant to give up those larger (future) profits for an immediate but once and for all gain by shirking on quality. Hence, if the buyer sets n and x so low that firms are induced to collude, the maximal implementable quality $\overline{q}$ (discretely) jumps at a higher level. Recognizing this, the buyer may actually benefit out of collusion. Indeed, if quality is sufficiently important in the sense that $s(q)$ is steeply increasing, then the buyer may want to further restrict n and/or x, exactly because this induces collusion, larger rents and then increases implementable quality. It is also worth noticing that inducing collusion is here (formally) equivalent to negotiating with an internally stable consortium composed with n firms. In this interpretation constraint (3) guarantees internal incentive compatibility of the consortium which is then independent of any legal obligation among the partners in the agreement.

Hence, the result in Proposition 3 shows that if non-contractible quality is important for the buyer, then an efficient antitrust authority that systematically detects cartels may be undesirable for the buyer, as well as rules that forbid consortia of suppliers. On the contrary, an effective competition-"watchdog" or such prohibition on consortia increase the buyer' surplus when she does not aspire to obtain large implementable quality.

In light of results in Corollary 1 and Proposition 3 one could ask how the buyer should organize procurement (setting $\underline{q}$, x and n) to maximize her expected surplus $S(\underline{q}, x, n)$.

Proposition 4 (Optimal procurement) *The buyer either prefers running competitive auctions with many competing firms (as in Corollary 1) or, if "quality is important", she sets* $n = 1$ *and negotiates with a single firm.*

As expected, when the buyer does not care too much for non-contractible quality, many competing firms and auctions organized as described in Corollary 1 are optimal. When instead the buyer cares for non contractible quality (in the sense that $s(q)$ is steeply increasing in q), then letting the contractor earn a larger rent than with competing firms allows to obtain a larger q. However, contrary to what expected from Proposition 3 and although collusion lets the buyer implement a larger quality than competition, Proposition 4 shows that procurement from a cartel is never the best procurement mode.[23] To understand why this is the case consider the possibility that the buyer restricts the pool of potential suppliers n to a single firm (so that it bilaterally negotiates with that firm) and compare the rent with a single supplier with that of colluding firms, respectively $\pi(1)$ and $\hat{\pi}(n)$. When a firm becomes the contractor out of a collusive ring or as the single admitted firm (i.e. with $n = 1$) it can ask a very high price (possibly also the reservation price r). However, on one hand the colluding contractor is more efficient than the single supplier (efficiency is respectively measured by $E[\theta'(n)] \leq E[\theta]$) and then it expects a larger rent. On the other hand, it is clear that the probability to be the contractor when several (although colluding) firms are admitted at the auction is smaller than in the case the firm is the only one contracting with the buyer (becoming the contractor is a certain event in this case). In the proof we show that this latter effect dominates so that $\pi(1) \geq \hat{\pi}(n)$ for any n.

[23]In the next Proposition ?? we will instead show that collusion may be optimal when the buyer also cares for procurement efficiency.

Hence, whenever the buyer cares for quality, any expected surplus S she can procure with a cartel, can be replicated negotiating with a single firm. By so doing she can also optimally adjust the contract length x with no need to assure cartel stability (i.e. that n and x are such that (3) is verified). Furthermore, since by $\pi(1) \geq \hat{\pi}(n)$ certain levels of high quality can be sustained with a single firm but not with a cartel, so that when quality "matters a lot" negotiating with a single firm is preferable a fortiori.

We now complete the picture of optimal procurement with non-contractible quality by considering the possibility that the buyer also cares for efficiency of the procurement process. There are several (non-exclusive) reasons that may induce a buyer to be concerned also with efficiency. For example, a public buyer may be given the mandate to maximize social welfare and not simply the surplus S as in the previous analysis (or a weighted average of S and contractors' profit π). Alternatively, maximizing the difference between the value of procurement and its price (as in S), the buyer may increase her surplus by asking a participation fee to the firms that are admitted in the pool of n potential suppliers, as in the case of "selective tendering". In this way she internalizes the contractors' expected rent π which, as discussed above, depends on the efficiency of the procuring firm.[24]

Keeping the analysis consistent with the model setup in Section 2, here we follow this latter interpretation. To this end, let τ be the fee the buyer asks for firms' participation in the pool of n potential suppliers and assume firms have a zero outside option if they are not allowed to participate. We can then define the set of rational prices for participation with $\tau = \gamma\pi$ where the parameter $\gamma \in [0, 1]$ captures the fraction of firms' surplus that can be appropriated by the buyer.

When the buyer cares for non-contractible quality but at the same time also for efficiency (according to the parameter γ), procuring from a small pool of potential suppliers n implies a large cost in terms of *inefficient production* since, as discussed above, expected efficiency of the winning firm reduces when restricting the number of suppliers. With this respect, the advantage of a cartel or of a consortium of suppliers consists in the possibility of implementing a large quality associated with a (moderately) large number of potential suppliers n which are on average more efficient than a single firm (i.e. $E[\theta] \geq E[\theta'(n)]$). In general, for any $n > 1$ a cartel is ready to provide larger quality than with n competing firms and, at the same time, it produces more efficiently than a single firm would do. Clearly, this latter effect is most powerful the more effective the cartel is in sorting out the most efficient firm (in our setup this is always the case since firms are fully informed). However, it is clear that the potential efficiency improvements in production with $n > 1$ firms (colluding or not) are large as compared with a single firm procurement.

The next proposition illustrates optimal procurement when the buyer cares for efficiency measured by the parameter γ.

Proposition 5 (Optimal procurement with efficiency concerns) *(i) If the buyer's concern for efficiency is limited (i.e. $\gamma \leq \underline{\gamma}$), optimal procurement is as in Proposition 4. (ii) If efficiency is*

[24] Similarly, relaxing Assumption 3 the buyer is no more constrained to set the reservation price r at the highest value that guarantees certain procurement. In this case, setting a smaller reservation price may as well serve the buyer to appropriate part of the contractor's rent.

very important (i.e. $\gamma \geq \overline{\gamma}(< 1)$), she negotiates with a single firm when "quality is important" and runs collusion-inducing auctions otherwise. (iii) For other values of efficiency (i.e. $\underline{\gamma} < \gamma < \overline{\gamma}$), she negotiates with a single firm when "quality is important", she runs collusion-inducing auctions for intermediate value of quality and runs competitive auctions otherwise.

Collusion leaves large rents to the firms and rents are necessary to induce them to provide high level of quality. However, if the buyer does not care for efficiency in production, then all what she can obtain in terms of quality with collusion can be replicated by admitting a single firm to the auction. This guarantees the maximal rent and then the highest implementable quality. However, a single firm comes at the cost of inefficient production because the expected cost of the single supplier is higher than the expected cost of the most efficient firm when n firms are admitted at the auction stage. This is not an issue when the buyer doesn't care (too much) for efficiency since it is exactly for this reason that she can procure higher quality, as shown in Proposition 4. On the contrary, if the buyer's concern for efficiency are significant, then a trade off arise.

Restricting n and x so that collusion emerges may become optimal because it allows for the best balance balance between higher implementable quality, as compared with competition, and higher efficiency in production, as compared with the single buyer. In fact, comparing optimal procurement inducing collusion with optimal procurement and competing firms, these two schemes are equivalent with respect to efficiency in production which, in both cases, is allocated to the most efficient firm. Hence, comparison between these two alternatives ultimately rests on their properties in terms of implementable quality (higher with collusion) and price for procurement (smaller with competition). When the buyer significantly cares for efficiency, the price paid is a transfer to the contractor that has small impact on her payoff (cancels out if $\gamma = 1$) so that procurement from competing firms tends to be dominated, unless quality is of limited value.[25]

It is also interesting to notice that if there is little heterogeneity in firms' costs (i.e. $E\left[\theta''(n)\right]$ and $E\left[\theta'(n)\right]$ are close independently of n), then implementable quality with competition is also small since because informational rents $\pi''(n)$ are small. If this is the case, then auctioning with competing firms is dominated either by bilateral negotiation or collusion-inducing procurement even if the buyer's concern for efficiency is very low. On the contrary, if firms are very heterogeneous, then the buyer can implement a high quality also with many competing firms. Hence, collusion inducing procurement shows its maximal strength for intermediate values of heterogeneity in costs, exactly because it mediates and provides the right balance between quality with efficiency.

The optimality of collusion-inducing procurement that is "guessed" in Proposition 3 and effectively shown in Proposition 5 is somehow provocative and unexpected. However, as we previously discussed, it should be noticed that collusion can be seen as a metaphor for a consortium among the n suppliers that requires internal stability (independently of any legal obligation among the partners in the agreement). With this respect the practitioners' literature on procurement often

[25] Optimal procurement remains a second-best in the current model due to firms' private information (and non contractible-quality). When $n = 1$ the buyer may design an optimal screening contract. Hence, the surplus discussed in the text may be a lower bound for the case $n = 1$ but an optimal contract may negatively impair quality since it reduces the firm's rent. For relational contracts with asymmetric information and a single agent see Levin (2003).

stresses the benefits of consortia and joint bidding, but considers these agreements almost systematically as black boxes. Our analysis, on the contrary, allows to illustrate both the benefits but also the possible limitations of firms' agreements in terms of a stability requirement according to which a consortium may deliver benefits to the buyer only if any of its members finds profitable to abide the agreement. Furthermore, in some cases consortia are ruled out by law, so that the optimal solution for the buyer can only be implemented with collusion.[26] In other cases, consortia are admissible and do not even need to be self enforcing since they can relay on complete contracting among partners (even if quality is non verifiable to thrid-parties and only the firms know their respective efficiency). In these cases when the cartel is optimal for procurement, a consortium is even better since it guarantees larger rents and the same level of efficiency.[27]

6 Discussion and Extensions

The analysis in the previous Section contributes to the literature on the mode of transaction for procurement. For example, Manelli and Vincent (1995) have analyzed sequential bargaining with take-it-or-leave-it offers designed by the buyer when non contractible quality is sufficiently important. It is worth noticing that the few papers that have dealt with the choice of the mode of transaction have limited the analysis mainly to a framework with no repetition. As we have previously emphasized, a main ingredient of procurement is the need to repeat the procuring process over time and, with non-contractible quality, the level of competition (i.e. the number n of firms admitted at the auction) is only one relevant dimension in the procurement process. Indeed, the duration of the relationship is important. A long term relationship creates an implicit incentive so that procuring firms have incentives to establish reputation and the buyers may prefer long lasting contracts when quality is not contractible. Hence, by combining these two elements in the choice of a trading procedure (the degree of competition and the length of the awarded contract) we create a bridge with the important strand of literature dealing with trust and reputation formation in long-term relationships (Fehr, Brown and Falk 2004).

An interesting policy implication of our results refers to the process of international liberalization which often obliges local governments to allow the participation of foreign firms to national public tendering (for example in the EU in case of large public projects). In addition to abiding no discrimination agreements, foreign firm participation is generally seen as competition and welfare enhancing effect of open markets. However, our analysis points outs that when some important ingredients are not fully contractible, this may not be the case. The additional competitive pressure of foreign firms, here corresponds to an uncontrollable (by the buyer) expansion of firms that have

[26] For example, in the case of public procurement, the Italian antitrust authority recently ruled out consortia, unless firms in the consortium are able to prove that they could not technically supply the buyer if they were obliged to operate independently.

[27] If the buyer does not care for efficiency and quality is important, then negotiating with a single firm is still optimal since, for the same reason illustrated with respect to the cartel, it guarantees larger rents and quality than any consortium.

to be admitted to the pool of eligible contractors, with a consequent reduction of implementable quality.

Cartels, consortia and sub-contracting. Cartels or consortia may act even more efficiently than previously discussed. Since after the first period the winning firm may no more be the most efficient among the colluding firms, the cartel can further boost efficiency delegating (or subcontracting) production to the firm that is period-by-period the most efficient one. In this case, clearly, the optimality of procuring with a cartel or a consortium increases with respect to both competition and single supplier. On the other hand, for simplicity we have considered only consortia or cartels that comprise all the n firms admitted at the bidding stage. Clearly, this is not the unique possibility, in that the buyer may induce only partial cartellization, for example allowing consortia of maximum two firms. If this is the case, the gains of consortia and cartels in terms of quality (with respect to competing firms) and efficiency (with respect to single firm) are lowered but they could be "fine tuned" to the buyer's interest.

Sub-contracting often takes place in procurement and if it allows to increase the firms' expected rent (see before), then also implementable quality may increase. However, the effects of sub-contracting is more intricate than that because responsibility for quality provision may remain in the hands of the main contractor so that incentives for quality may well result diluted. We plan to investigate this interesting relationship between sub-contracting and non-contractible quality in procurement in a future work.

In alternative to collusion with bid rotation, firms may be able to use undetectable side transfers thus sharing the collusive surplus at any auction. In case the winning firm shirks on quality it will be excluded but it can still be compensated by the future (colluding) contractors. The number of firms that belongs to the collusive ring increases with time but collusion with quality shirking can be still an equilibrium if firms use transfers that also decline with time. Although this is a theoretical possibility, it seems to be less of practical relevance than bid rotation discussed in the previous pages (especially in the case of consortia).

Finally, a cartel or a consortium may also help firms monitor buyer's behavior. In particular the common organization may allow all insider firms (not only those procuring) to verify quality so that the cartel or consortium may itself punish a buyer that claims law quality contrary to reality. This possibility is known in the literature as multilateral relational contracting (see Levin, 2002), to be contrasted with the case of bilateral relational contracting analyzed in our paper. Since with multilateral relations the cartel or consortium can pool the reaction of many firms when the buyer deviates, this may discipline the latter and also let implement larger non-contractible quality.

Private information and renegotiation. The literature on repeated games with asymmetric information (see for example Compte, 1998 and Kandori and Matsushima, 1998) and that on collusion in repeated auctions (see Aoyagi, 2003 and Blume and Heidhues, 2004) have highlighted (among other results) the complexities that could arise in our model by introducing asymmetric information among bidders. Notwithstanding the intricacies of collusion in repeated auctions with asymmetrically informed bidders we expect that our simple trade-offs still hold in such a sophisticated environment. Consider for example the properties of production with collusion or in a consortium. It is clear that being the members privately informed, the efficiency properties of the cartel would be weakened. On the other hand, some information sharing among members can be expected (centralized, for example, as in many paper dealing with collusion in auctions). Hence, one can still expect higher prices in collusion than with competing firms and at the same time larger efficiency within the cartel than with single firm procurement, thus preserving the main drivers of our results.

Relatedly, in our framework, the buyer does not learn from auctions and contracting because, in order to highlight the drivers of our results, we deliberately assumed that firms' efficiency is reshuffled anew at any period (or equivalently at any auction). For sure the absence of any persistence in efficiency for firms is not very realistic and eliminates a cost in dismissing cheating firms that could be important for the supplier. In particular, when excluding a firm that has decided not to abide the minimum quality requirement, the buyer may realize that although unfaithful, that firm may be very efficient and thus ready to procure at a significantly lower price than other firms in the market. In this case exclusion could be less of a scarecrow for efficient firms thus pointing to an intrinsic trade-off between efficiency and quality. On one hand, less efficient firm would be conscious that they can be easily discarded and substituted and this provide the right incentives for quality provision. On the other hand, more efficient firms know that the buyer would be reluctant to discard them and are then less disciplined to provide high non contractible quality.

Finally, although in our model changing efficiency during the contract execution seems to point to the possibility of beneficial renegotiation, it is worth emphasizing that the trade-off between efficiency and non-contractible quality is already accounted for by the buyer (also) with the choice of contract duration. Hence, a fortiori with any specific cost for renegotiation this would not take place in the current setup.[28]

Subjective quality evaluation. Performance measures are often subjective. For example, firms could chose and observe their investment in quality, though the realized quality observed by the buyer may be subject to noise, so that both parties have private information on what they observe. The theory of relational contracts with subjective performance measures has been developed recently by Levin (2003), McLeod (2003) and Fuchs (2006). A common theme in these theories is that to induce the principal to report truthfully the perceived quality and act consequently according to the prescriptions of the relational contract, the optimal contract must make the principal indifferent between reporting different performance levels of the agent. In bilateral relationships (i.e. a principal with a single agent) this tends to induce inefficiencies (sometimes obtained with "money burning"): when the agent's performance is poor and the contract prescribes a punishment for the agent, the principal cannot gain from that punishment, otherwise it would be induced to report bad performance of the agent more often.

The exclusion strategy of the buyer we analyzed is such that the buyer does not gain from

[28] For renegotiation to occur we would need unless unexpected shocks that are not modeled in the current framework. Irrelevance of renegotiation is clearly also true if efficiency is persistent within the contract execution. See later on this.

punishing a firm that did not perform, as all firms are identical and replacing one with another brings nothing to him.[29] This means that if instead of the observable quality we would have assumed a subjective quality assessment from the buyer as described above, we would have found analogous equilibria and related results. All our results can be replicated qualitatively unchanged under the assumption of subjective evaluation by the buyer. The reason why no additional inefficient "money burning" is required –already pointed out in Fuchs (2006)– is that, differently than in bilateral relationships, the presence of competing agents allows the principal to punish the incumbent for its poor performance without gaining anything from punishing, rather, having a competing agent to profit from it. This maintains incentives for truthful reporting by the buyer.

Discontinuing procurement and scoring rules. Assumption 3 requires that the buyer procures the good at any point in time. This clearly puts her in a weak position with colluding firms or when contracting with a single firm. In fact, to guarantee procurement, she has to set a reservation price r which is high enough to reimburse the cost of the least efficient firm. Hence, colluding firms can extract (almost) all the surplus by asking a very high price and similarly the single firm can ask r. Abandoning this assumption, r can be set at a lower value even if this may discontinue procurement for certain realization of costs. However, consider a (small) reduction of r from its value in the previous analysis $r < \overline{\theta} + C(q)$. It is clear that the reduction of r has a direct effect in limiting colluding firms' rents so that the buyer pays less for procurement but implementable quality also reduces. Hence, the buyer may actually avoid reducing r. Furthermore, as long as the price paid with collusion is larger than with competing firms (a necessary condition for collusion itself), our qualitative results on optimal procurement still hold.

In principle, firms' offers could be formed by a price bid and other properties of the supply such as contractible but also non-contractible quality. Hence, in addition to what employed in the previous sections where the procurement price implicitly accounted for contractible elements in the supply relationship, the buyer could instead rank the offers according to a scoring rule which is a function of the price and contractible together with non-contractible quality dimensions. By so doing the buyer may then exclude the winning firm in case the latter does not provide the promised score.[30] However, this form of competition with bid-quality offers and scoring rule is irrelevant as long as there is no cost heterogeneity among firms for quality, as in our model.

Minimum price. The buyer may decide also to use a minimum price L in addition to the reservation price r so that admissible bids are $b_i \in [L, r]$. It is interesting to notice that minimum prices are used in procurement when the buyer cannot restrict and pre-select the potential suppliers, particularly in public procurement and, on the contrary, are rarely employed in private procurement when the buyers can limit the number of eligible suppliers. As we now discuss these observations

[29] Bonds and bonuses would not changed the set of implementable outcomes also with subjective evaluation, as shown by Levin (2003).

[30] The possibility to use these scoring rules may be limited by the fact that the assignment of the contract (i.e. a contractible dimension) turns out to be determined also by non-contractible dimensions. In case of public procurement, for example, this may not be viable.

are consistent with our previous analysis.

Clearly, when firms collude the presence of a minimum price is irrelevant as well as when a single supplier is admitted. When instead firms compete and the buyer does not care for efficiency, a larger minimum price is substitute to a reduction in n. To see this let us simplify notation indicating the minimum price with $L = l + C(\underline{q})$ so that for any level of required quality $\underline{q}$ firms obtain a reimbursement of costs $C(\underline{q})$ plus a (minimum) additional price $l \geq 0$.[31] Firms' expected rents are now indicated with $\pi''(n, l)$ which is increasing in l and also (weakly) larger than the rent with competition and no minimum price $\pi''(n)$ as defined in (2).[32] Hence, if the buyer wants to procure a large implementable quality, she can increase l (and/)or reduce n obtaining a similar effect in on the quality incentive constraint (1). With this respect, the unique effect of a minimum price is to allow implement an even larger quality than in the case of a single buyer. To see this consider the extreme case in which the buyer sets $l \geq \overline{\theta}$ (although this is not necessarily optimal). The buyer can then control firm's rent entirely with l and since she pays a price that does not depend on contractor's efficiency, she optimally restrict $n = 1$. Hence, a larger implementable quality can be obtained granting a larger rent to the single firm with larger l.

Things are different when the buyer also cares for efficiency as analyzed in the previous section. In fact, with a minimum price (and competing firms) the current contractor is one of the (possibly many) firms characterized by a cost parameter $\theta \leq l$ since all these firms will bid the same price L and will then have the same probability to win the contract. Furthermore, the larger is l, the larger is the set of these firms and the smaller the expected efficiency of the actual contractor. Hence, it follows that the same trade-offs illustrated for the proof of Proposition 5 also apply when the buyer can use a minimum price and the result remain qualitatively unaffected. It is also worth noticing that in this case when procuring with a cartel or a consortium is desirable, then the buyer cannot use a large l together with many firms in the pool (i.e. large n) since both these decisions make the cartel less stable. The buyer may then be obliged to reduce n and limit (or avoid the use of) the minimum price.

7 Concluding Remarks

@TO BE REWRITTEN

In this paper we have analyzed the relationships between reputation, non-contractible quality and collusion in a repeated procurement context. Repetition in the procurement relationship allow the emergence of reputation as an incentive device inducing firms to supply acceptable levels of quality. Restricting participation and contractual length, the buyer increases firms' incentives to provide quality and hence, maximal implementable quality. On the other hand, running more

[31] We will not consider here endogenous minimum prices that depend on firms' bids such it is often the case with average bid methods used in procurement to avoid "wild-bids" and "gambling for resurrection".

[32] With probability $1/n$ any firm is the most efficient one earning a rent $\theta'(n)$ with the minimum price and $\theta''(n) - \theta'(n) (\leq l - \theta'(n))$ without it. With complementary probability $1 - 1/n$ the firm is not the most efficient one and earns nothing without the minimum price. On the contrary, with a minimum price if the firm's type is $\tilde{\theta} \leq l$, it can still earn a profit $l - \tilde{\theta}$ since all firms with $\theta \leq l$ bid the same (minimum) price.

frequent auctions among few bidders facilitates collusive agreements among suppliers. We have analyzed this trade-off showing that when non-contractible quality and variability in suppliers' efficiency are both important, short contract duration and a collusive agreement between a few eligible sellers may maximize welfare and leave the buyer better off. We find this result interesting also because it shows that in the procurement of innovation cooperation among firms and in the limit also collusion, do not necessarily clash with innovation when R&D activity is not fully-contractible.

Finally, we also show that if quality is a major concern, the buyer can do even better by negotiating with a single firm, even if this may clash with efficiency in production. Hence, we show that the optimal procurement strategy involves a subtle balance between firms' rents, incentives for quality, collusion and efficiency.

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Appendix A

Proof of Lemma 1. Let $\theta''(n) \equiv \min\left\{\boldsymbol{\theta}/\theta'(n) \cup \overline{\theta}\right\}$ and

$$C(q) \equiv \psi(q)\frac{1-\delta^x}{1-\delta} + \delta\frac{1-\delta^{x-1}}{1-\delta}E[\theta]$$

so that the winning bid of the most efficient firm is $b_w = \theta''(n) + C(q)$. The buyer's objective function can then be written

$$\begin{aligned} S(q,x,n) &\equiv \sum_{t=0}^{\infty}\delta^{tx}\left[\tfrac{1-\delta^x}{1-\delta}V(q) - b_w - K\right] \\ &= \tfrac{1}{1-\delta}s(q) - \tfrac{1}{1-\delta^x}\left[E\left[\theta''(n)\right] + \tfrac{\delta(1-\delta^{x-1})}{1-\delta}E[\theta] + K\right]. \end{aligned}$$

Since $E\left[\theta''(n)\right]$ is decreasing in n then $S(q,x,n)$ is decreasing in n. The sign of the derivative $\partial S(q,x,n)/\partial x$ is equal to the sign of $K + E\left[\theta''(n)\right] - E[\theta]$. Finally, the optimal q maximizes per-period net value of quality $s(q)$. ■

Proof of Proposition 1. Assume first there are no discretionary bonus and bonds, i.e. $B = P = 0$. We will then show that this is without loss of generality.

If $n = N$ then for any σ, the maximal implementable quality is $\overline{q} = 0$. In fact, if the buyer decides to always keep all firms at the competitive stage, then there is no punishment for cheating firms, i.e. $\pi(n)\frac{\delta^x}{1-\delta^x} = \pi_0$ and from (1) $\underline{q}$ is implementable if and only if $\underline{q} = 0$. Since we are interested in analyzing the possibility the buyer can implement strictly positive quality, from now on we will consider $n < N$. A cheating contractor is replaced with anyone of the $N - n$ firms initially excluded from participation and the pool of n competing firms remains unchanged as long contractors satisfy the quality requirement. Hence, also the set of $n - N$ excluded firms remains unchanged: no firm can enter the pool of n competing firm unless one of these is excluded for cheating. This is clearly with no cost for the buyer since firms are identical from the buyer's view point, so that for any σ with $T \geq 1$ we have $\pi_0 = 0$.[33]

Now, let $\bar{q}(x, n)$ be implicitly defined with

$$\pi(n)\frac{\delta^x}{1-\delta^x} - \pi_0 + (B + P) = \psi\left(\underline{q}\right)\frac{1-\delta^x}{1-\delta} \tag{4}$$

so that for any quality requirement $\underline{q} > \bar{q}(x, n)$, any contractor would prefer to shirk on quality so that $\underline{q}$ is implementable only if $\underline{q} \leq \bar{q}(x, n)$.

Consider now the properties of $\bar{q}(x, n)$. That $\bar{q}(x, n) = 0$ for $x = \infty$ is immediate from (4) and from implicit differentiation we obtain $\frac{\partial \bar{q}}{\partial x} \leq 0$. As it will be shown in the proofs of the next results (see that of Corollary 1) and Proposition 4) the expected profit $\pi(n)$ is a decreasing function of n so that $\frac{\partial \bar{q}(x,n)}{\partial n} \leq 0$.

Finally, we consider the possibility that the buyer contemplates a discretionary bonus $B \geq 0$ that she is free to pay to the contractor and that the latter posts in the hands of the buyer some "bonds" $P \geq 0$ which buyer can discretionary decide to retain. We next show that $\underline{q}$ is implementable if and only if $\underline{q} \leq \bar{q}(x, n)$ also in this case (the approach is similar to the analysis in MacLeod and Malcomson, 1989 and Levin, 2003 on bonuses and bonds).

We have to control for the buyer's incentives not to deviate by retaining P and not paying B even if the contractor has provided the required quality. Since procured quality is not observable outside the contractual relationship, none of the potential suppliers observes q. Furthermore, we know that the buyer can always discard the current contractor and substitute it with any one of the $N - n$ other firms. It then follows that the buyer's incentive compatibility constraint is $-B \geq P$ which immediately shows that the unique admissible values of B and P are $B = P = 0$. ■

Proof of Corollary 1. Let the probability that a firm i with cost θ_i is the most efficient firm be denoted as $\Pr(\theta_i \leq \theta_j, \forall j \neq i | \theta_i)$ and, from independence, we have $\Pr(\theta_i \leq \theta_j, \forall j \neq i | \theta_i) = [1 - F(\theta_i)]^{n-1}$, so that the ex-ante probability of being the lowest cost firm at any stage game is $\Pr(\theta_i \leq \theta_j, \forall j \neq i) = 1/n$.

The most efficient firm wins with a price b_w and, if it decides to comply with the quality requirement, it obtains a rent

$$\theta''(n) + C(q) - \theta'(n) - C(\underline{q}) = \Delta\theta(n) - [\psi(\underline{q}) - \psi(q)]\frac{1-\delta^x}{1-\delta}$$

so that the expected rent $\pi(n)$ from future auctions is as in (2). In alternative, it may decide to shirk on quality thus saving the cost $\psi(\underline{q})\frac{1-\delta^x}{1-\delta}$. Hence, it prefers not to shirk on quality if $\underline{q} \leq \bar{q}(x, n)$

[33] We will not consider coalitional deviations on quality. However, note that also with a coalition of all n firms in the pool, exclusion is forever and $\pi_0 = 0$, as long as $n \leq N - n$. If $n \geq N/2$, after T periods cheating contractors re-enter the pool. In this case we may have $0 < \pi_0 \leq \pi(n)\frac{\delta^x}{1-\delta^x}$.

where $\bar{q}\,(x,n)$ is implicitly defined with

$$\Delta\theta\,(n)\,\frac{\delta^{x}}{n(1-\delta^{x})}=\psi(\bar{q})\frac{1-\delta^{x}}{1-\delta}+[\psi(\bar{q})-\psi(q)]\frac{\delta^{x}}{n(1-\delta)}. \tag{5}$$

We now need to show that with b_w and procuring $q=\underline{q}$ the most efficient firm indeed wins the auction. For this it suffices to consider the incentives of the second most efficient firm. Clearly, if it decides to procure quality $\underline{q}$, it cannot win the auction since its offer would be no less than b_w. However, it could lower its price by deciding to offer $q<\underline{q}$. Since it would then be excluded, we will consider the most profitable deviation, i.e. $q=0$. This deviation is not profitable as long as

$$\theta''\,(n)+C(\underline{q})-\theta''\,(n)-C\,(0)\leq\pi(n)\frac{\delta^{x}}{1-\delta^{x}}$$

which shows that the second most efficient firm cannot win by undercutting and cheating on quality whenever $\underline{q}\leq\bar{q}\,(x,n)$.

Consider now the properties of $\bar{q}\,(x,n)$. Note that $F_{\theta'(n)}(\theta)=1-(1-F(\theta))^{n}$ with $\frac{\partial F_{\theta'(n)}(\theta)}{\partial n}\geq 0$ and $F_{\theta''(n)}(\theta)=1-(1-F(\theta))^{n-1}(1+F(\theta)(n-1))$ with $\frac{\partial F_{\theta''(n)}(\theta)}{\partial n}\geq 0$, thus both for $\theta''\,(n)$ and $\theta'\,(n)$, increasing n amounts to a first order stochastic dominance effect. However, since $F_{\theta''(n)}(\theta)=nF_{\theta'(n-1)}(\theta)-(n-1)F_{\theta'(n)}(\theta)$ so that $E\,[\Delta\theta\,(n)]=nE\left[\theta'\,(n-1)-\theta'\,(n)\right]$, we have $\frac{\partial E[\theta''(n)-\theta'(n)]}{\partial n}\leq 0$. Hence, $\partial\bar{q}/\partial n\leq 0$ and $\lim_{n\to\infty}E[\theta''\,(n)-\theta'\,(n)]=0$. Differentiating $\frac{\partial\bar{q}}{\partial x}$ from the proof of Proposition 1 with respect to n, we obtain that if $\psi_{qq}\leq 0$ then $\frac{\partial^{2}\bar{q}(x,n)}{\partial x\partial n}\geq 0$.

We now turn to the buyer's optimization program $(\mathcal{P}_c)$ which consists in maximizing the (expected) surplus $S\left(\underline{q},x,n\right)$ with respect to n, x and $\underline{q}$ subject to the implementability constraint $\overline{q}\,(x,n)\geq\underline{q}$.

First, for any $\overline{q}\,(x,n)$ the buyer always prefers to have the quality-implementability constraint binding, i.e. $\overline{q}\,(x,n)=\underline{q}$. Indeed, suppose to the contrary that she set $\underline{q}$ so that $\overline{q}\,(x,n)>\underline{q}$. She could increase n (and possibly also x) thus paying a lower price b_w still obtaining the same level of procured quality $\underline{q}$.[34] Hence, $(\mathcal{P}_c)$ becomes

$$\max_{\{x,n\}}\quad S\left(\overline{q}\,(x,n)\,,x,n\right).$$

Differentiating we obtain

$$\begin{array}{l}\frac{\partial S(\overline{q}(x,n),x,n)}{\partial n}=\frac{1}{1-\delta}s_q(\overline{q}\,(x,n))\frac{\partial\overline{q}}{\partial n}-\frac{1}{1-\delta^{x}}\frac{\partial E[\theta''(n)]}{\partial n}\leq 0\\ \frac{\partial S(\overline{q}(x,n),x,n)}{\partial x}=\frac{1}{1-\delta}s_q(\overline{q}\,(x,n))\frac{\partial\overline{q}}{\partial x}+\log(\delta)\,\frac{\delta^{x}}{(1-\delta^{x})^{2}}\left\{E[\theta]-E\left[\theta''\,(n)\right]-K\right\}\leq 0\end{array} \tag{6}$$

where $\frac{\partial\overline{q}}{\partial n}\leq 0$, $\frac{\partial\overline{q}}{\partial x}\leq 0$ and $\frac{\partial E[\theta''(n)]}{\partial n}\leq 0$.

From the f.o.c. on n, comparing with Lemma 1 and considering the results in Proposition 1, the buyer optimally sets $n<n^{*}=N$.

[34] Relatedly, in the equilibrium in which any contractor would offer $q=0$ it must be $\underline{q}=0$ since otherwise one can increase n and x thus reducing the price for procurement with no effect on quality.

From the f.o.c. on x it follows that if $E[\theta] - E\left[\theta''(n)\right] \geq K$, then $\frac{\partial S}{\partial x} \leq 0$ and the optimal contract length is $x = 1$ (as with contractible quality). When instead $E[\theta] - E\left[\theta''(n)\right] < K$, the optimal x is interior and from Lemma 1 and Proposition 1 it follows that the buyer optimally sets $x < x^* = \infty$.

When the optimal solutions for x and n are interior, combining the two f.o.c.s we obtain that the sign of $\frac{\partial n}{\partial x}$ is equal to the sign of

$$\{E[\theta] - E\left[\theta''(n)\right] - K\}/(\frac{\partial E\left[\theta''(n)\right]}{\partial n})$$

so that if the optimal contract length is $x^* = 1$ then $\frac{\partial n}{\partial x} \leq 0$ (the two instruments n and x are substitutes as with contractible quality) and, if instead the optimal contract length is larger than 1, then $\frac{\partial n}{\partial x} \geq 0$ (n and x are complements). ■

Proof of Proposition 3.

Assume n, x and $\underline{q}$ are such that (i) the cartel is stable, i.e.

$$\hat{\pi}(n)\frac{\delta^x}{1-\delta^x} \geq D(n) + \pi''(n)\frac{\delta^x}{1-\delta^x}$$

and (ii) competing firms are induced to provide the required quality, i.e.

$$\pi''(n)\frac{\delta^x}{1-\delta^x} \geq \psi(\underline{q})\frac{1-\delta^x}{1-\delta}.$$

Since collusion implies a larger rent than competition $\hat{\pi}(n) \geq \pi''(n)$, from the two previous inequality it follows that

$$\hat{\pi}(n)\frac{\delta^x}{1-\delta^x} \geq \psi(\underline{q})\frac{1-\delta^x}{1-\delta}$$

and the colluding contractor will certainly provide quality.

Consider now condition (i) and assume the colluding contract prefers to provide quality (i.e. the last inequality is verified). Then, it may be the case that $\underline{q}$ is implementable with colluding but not with competing firms.[35] ■

Proof of Proposition 4.

Following the logic used for Proposition 3, the buyer can implement the largest quality setting $n = 1$. In fact, consider n, x and $\underline{q}$ so that (3) is satisfied and $\hat{\pi}(n)\frac{\delta^x}{1-\delta^x} \geq \psi(\underline{q})\frac{1-\delta^x}{1-\delta}$.[36] Then $\underline{q}$ is implementable also when the buyer contracts with a single firm. To show this one must prove that $\pi(1) \geq \hat{\pi}(n)$ for any n. Indeed, since the single firm will ask a price r, the price paid by the buyer with collusion cannot be higher.[37] Assume the cartel is able to ask for the maximal price r so that

[35] Also notice that since procured quality is not observable outside the contractual relationship bonds and bonuses are ineffective also with colluding firms, as illustrated in the proof of Proposition 1.

[36] See the discussion at the end of this proof for firms' out-of-equilibrium profit when the cartel breaks down.

[37] It may be in fact smaller if the collusive ring prescribes temporary revision to smaller price so as to make the cartel stable, as we will discuss below.

$\hat{\pi}(n) = \frac{1}{n}\left[\overline{\theta} - E[\theta'(n)]\right]$ and note that the expected profit of the single firm is $\pi(1) = \overline{\theta} - E[\theta]$. Then, although $\overline{\theta} - E[\theta'(n)] \geq \overline{\theta} - E[\theta]$, with the cartel the firm earns a rent with probability $1/n$ whilst the single firm always earns a rent, even if it is not the largest one. In particular, with probability $1/n$ the single firm will have the same efficiency as the most efficient firm in the cartel (i.e. $E[\theta'(n)]$), thus earning the same rent, and with complementary probability it will not be as efficient but still earning a rent. Hence, we obtain $\pi(1) \geq \hat{\pi}(n)$ so that it is $\pi(1)\frac{\delta^x}{1-\delta^x} \geq \psi(\underline{q})\frac{1-\delta^x}{1-\delta}$.

Hence, the buyer needs to compare three possible modes of procurement contemplating: competition, collusion (both cases with at least two suppliers) or a single supplier. The winning price $b_w = \theta_w + C(q)$ has a component $C(q)$ common to the three modes but it differs in the way first period efficiency translates into informational rents since with competition $\theta_w = \theta''$, with collusion $\theta_w = \hat{\theta} \geq \theta''$ and $\theta_w = \overline{\theta} \geq \hat{\theta}$ with a single firm. For each procurement mode the buyer has then to determine the optimal n, x and $\underline{q}$ to maximize

$$S(\underline{q}, x, n) = \frac{1}{1-\delta}s(\underline{q}) - \frac{1}{1-\delta^x}\left[E[\theta_w] + \frac{\delta(1-\delta^{x-1})}{1-\delta}E[\theta] + K\right]$$

where $\underline{q} \leq \overline{q}(n,x)$ is the constraint on implementable quality as in (1). We will indicate the set of n and x that induce competition (i.e. satisfying (3) with reverse inequality) with P'' and that of n and x inducing collusion with $\hat{P}$ (i.e. satisfying (3)). For the case of single supplier $n = 1$ we will indicate with x_1 the optimal contract length.

From what stated above, we have that

$$\overline{q}(1,x) > \overline{q}(n,x) \text{ for } (n,x) \in \hat{P},$$
$$\overline{q}(n,x) > \overline{q}(\tilde{n},\tilde{x}) \text{ for } (n,x) \in \hat{P} \text{ and } (\tilde{n},\tilde{x}) \in P''.$$

To see why inequalities are strict, consider first the comparison between collusion and competition and recall that when n and x are low enough that (3) is satisfied, collusion takes place. For a given x, let $\hat{n}_0$ the value such that (3) is satisfied as an equality. At $\hat{n}_0$ contractor's expected profits jumps from $\pi''(\hat{n}_0)$ to $\hat{\pi}(\hat{n}_0)$ with $\hat{\pi}(\hat{n}_0) - \pi''(\hat{n}_0) = D(\hat{n}_0)\frac{1-\delta^x}{\delta^x} > 0$ and then $\overline{q}(\hat{n}_0, x) > \overline{q}(\hat{n}_0 + 1, x)$ for any x. Similarly, for any $(2,x) \in \hat{P}$ we have $\overline{q}(2,x) < \overline{q}(1,x)$.

For the sequel of the proof notice that $S(\underline{q}, x, n)$ is weakly increasing in n and if, $\theta_w = \overline{\theta}$, also in x since in this case

$$\frac{\partial S\left(\underline{q}, x, n\right)}{\partial x} = \log\left(\delta\right)\frac{\delta^x}{(1-\delta^x)^2}\left\{E[\theta] - \overline{\theta} - K\right\} \geq 0.$$

Furthermore, independently of the procurement mode, constraint $\underline{q} \leq \overline{q}(n,x)$ is always binding at the optimum since, if it is not, the buyer can increase n (or x with a single supplier), thus increasing $S\left(\underline{q}, x, n\right)$ and reducing $\overline{q}(n,x)$.

Let first assume that $q^* \leq \max_{(n,x)\in P''}\overline{q}(n,x)$, so that optimal quality is implementable in all procurement modes even if quality is not contractible. It is immediate that inducing competition is the optimal procurement mode since it allows to increase n thus reducing the second term in $S\left(\overline{q}\left(x,n\right), x, n\right)$, and possibly also x (see Corollary 1 for optimal procurement with competing firms). Further increase of q with collusion or a single contractor is not required since S is decreasing in q for $q > q^*$.

Let now $\max_{(n,x)\in P''} \overline{q}(n,x) < q^* \leq \max_{(n,x)\in \hat{P}} \overline{q}(n,x)$. We prove that, although inducing collusion allows to implement a quality which is larger and closer to q^* than with competing firms, collusion is dominated by the single supplier. With collusion, optimization of

$$S(\overline{q}(n,x),x,n) = \frac{1}{1-\delta} s(\overline{q}(n,x)) - \frac{1}{1-\delta^x}\left[E[\hat{\theta}_w] + \frac{\delta(1-\delta^{x-1})}{1-\delta}E[\theta] + K\right]$$

w.r.t. $(n,x) \in \hat{P}$ requires,[38]

$$\begin{aligned} \frac{\partial S(\overline{q}(x,n),x,n)}{\partial n} &= \frac{1}{1-\delta} s_q(\overline{q}\,(x,n))\frac{\partial \overline{q}}{\partial n} = 0, \\ \frac{\partial S(\overline{q}(x,n),x,n)}{\partial x} &= \frac{1}{1-\delta} s_q(\overline{q}\,(x,n))\frac{\partial \overline{q}}{\partial x} + \log(\delta)\frac{\delta^x}{(1-\delta^x)^2}\left\{E[\theta] - \overline{\theta} - K\right\} = 0. \end{aligned}$$

Since $s_q(q^*) = 0$ and q^* is implementable with colluding firms, then n is reduced to the point in which $\overline{q}\,(x,n) = q^*$ and this determines the optimal n as a function of x, i.e. $\hat{n}(x)$. The derivative on x then becomes

$$\frac{\partial S\,(q^*,x,n)}{\partial x} = \log(\delta)\frac{\delta^x}{(1-\delta^x)^2}\left\{E[\theta] - \overline{\theta} - K\right\} \geq 0$$

(recall the curly bracket is negative) so that the optimal $\hat{x}$ is the largest x that verifies (3). Notice that for a given x, since $n = 1 < \hat{n}(x)$, then if q^* is implementable with colluding firms it is also implementable with a single supplier. The buyer will then obtain quality $\overline{q}\,(x,1) = q^*$, so that $\frac{\partial S(q^*,x,n)}{\partial x}$ is as in the previous expression. Hence, since now the buyer does not need to take care of constraint (3) (i.e. that $(n,x) \in \hat{P}$), it follows that $x_1 \geq \hat{x}$ and then $S\,(q^*,x_1,1) \geq S\,(q^*,\hat{x},\hat{n})$. In the current case, then optimal procurement requires competing firms if q^* is not too large and a single supplier otherwise.

Finally, if $\max_{(n,x)\in \hat{P}} \overline{q}(n,x) < q^*$ the implementable quality is closer to the optimal q^* with the single supplier. Hence, with arguments similar to the previous case, collusion is here dominated a fortiori. ■

The buyer may realize if a cartel breaks down and revert to optimal procurement with competition so that (1) is satisfied and firms do not shirk on quality. Alternatively, when collusion breaks down, if the buyer sticks to her collusion-inducing strategies, firms will be induced to shirk on quality so that they will be excluded from future auctions. This, in turn, implies that firms' payoff following a deviation are very low and the stability of collusion is strengthened. In other words, when collusion is desirable, it is in the buyer's interest not to revert to competition in future auctions if a deviation from collusion occurs.

MacLeod and Malcomson (1989) and Levin (2003) have shown that stationary relational contracts are without loss in terms of implementable outcomes and this explains our restriction to

[38] We consider here the highest collusive price so that $E[\hat{\theta}] = \overline{\theta}$. In our stochastic-cost environment colluding firms may be unable to systematically coordinate on the highest price (as in Rotemberg and Saloner, 1986) so that $E[\hat{\theta}] \leq \overline{\theta}$. However, the argument in the proof clearly follows also considering $E[\theta''] \leq E[\hat{\theta}] < \overline{\theta}$.

stationary strategies for the firms and the buyer. It is however worth noticing that in the case of procurement inducing collusion, we assumed the buyer does not change her strategy when she realizes that collusion has broken down due to a cartel deviation (see the right hand side of equation (3)). This is not the unique behavior one could envisage for the buyer on this occurrence.

Proof of Proposition 5. Let the buyer' surplus be $S^{\gamma}(\underline{q}, x, n) \equiv S(\underline{q}, x, n) + \tau$ which substituting becomes

$$S^{\gamma}(\underline{q}, x, n) = \frac{1}{1-\delta}s(\underline{q}) - \frac{1}{1-\delta^{x}}\left[(1-\gamma)\,E[\theta_w] + \gamma E[\theta^*] + \frac{\delta(1-\delta^{x-1})}{1-\delta}E[\theta] + K\right]$$

where $E[\theta^*]$ is the expected efficiency of the selected contractor. If $\gamma = 0$ we are back to the case discussed in Proposition 4. If instead $\gamma = 1$ the price b_w paid by the buyer is simply a transfer that cancels out from her surplus. Hence, the difference of S^{γ} with $n = 1$ and collusion is, for the same x,

$$\Delta_1 \equiv \frac{1}{1-\delta}[s(\overline{q}(1, x)) - s(\overline{q}(n, x))] - \frac{\gamma}{1-\delta^{x}}\left[E[\theta] - E[\theta'(n)]\right]$$

Similarly, the difference between S^{γ} with collusion and competition is

$$\Delta_2 \equiv \frac{1}{1-\delta}[s(\overline{q}(\hat{n}, x)) - s(\overline{q}(n'', x))] - \frac{1-\gamma}{1-\delta^{x}}\left[E[\hat{\theta}(n)] - E[\theta''(n)]\right]$$

for any $(\hat{n}, x) \in \hat{P}$ and $(n'', x) \in P''$.

Consider now result (i). For continuity of $S^{\gamma}(\underline{q}, x, n)$ on γ it is immediate that for γ close enough to zero, i.e. $\gamma \leq \underline{\gamma}$ for some constant $\underline{\gamma} \in [0, 1)$, the arguments used for the proof of Proposition 4 apply, hence the result.

As for result (ii), consider the extreme case with $\gamma = 1$. For any n, x and $\underline{q}$, the buyer obtains the same payoff S^{γ} with both competition and collusion. Since we know that $\overline{q}(n, x) > \overline{q}(\tilde{n}, \tilde{x})$ for $(n, x) \in \hat{P}$ and $(\tilde{n}, \tilde{x}) \in P''$ where $\hat{P}$ and P'' are respectively the set of (n, x) that induce collusion and competition, it follows that $\Delta_2 > 0$: auctions with competition is dominated by auctions with a cartel. We now need to compare the latter mode of procurement with bilateral contracting with a single firm. For this note that $E[\theta^*] = E[\theta]$ when $n = 1$, $E[\theta^*] = E[\theta'(n)]$ with collusion and $E[\theta'(n)] < E[\theta]$ for any $n \geq 2$. On the other hand we know $\overline{q}(1, x) > \overline{q}(n, x)$ for $(n, x) \in \hat{P}$. Hence, following the same arguments in the proof of Proposition 4 we obtain that $\Delta_1 < 0$ if $s_q(q)$ is not too large for $q \leq q^*$ and $\Delta_1 > 0$ otherwise. By continuity, result (ii) follows for any $\gamma \geq \overline{\gamma}$ with $\overline{\gamma} < 1$.

Finally let us consider the intermediate case $\underline{\gamma} < \gamma < \overline{\gamma}$ (result (iii)). Since γ is not too small, collusion-inducing auctions are not systematically dominated by the single supplier and since γ is not too high competition is not systematically dominated by collusion-inducing auctions. Hence, if $s_q(q)$ is intermediate for $q \leq q^*$, then $\Delta_1 < 0$ and $\Delta_2 > 0$ and with collusion the buyer obtains the overall optimum; if $s_q(q)$ is small $\Delta_1 < 0$ and $\Delta_2 < 0$ and competition is best; finally if $s_q(q)$ is large $\Delta_1 > 0$ and $\Delta_2 < 0$ and $n = 1$ is optimal. ■