Judicial Errors and Innovative Activity

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Abstract: We analyze the effect of errors in law enforcement on the innovative activity of firms. If successful, the innovative effort allows to take new actions that may be ex-post welfare enhancing (legal) or decreasing (illegal). Deterrence in this setting works by affecting the incentives to invest in innovation, what we call average deterrence. Type-I errors, through over-enforcement, discourage innovative effort while type-II errors (under-enforcement) spur it. The ex-ante expected welfare effect of innovations shapes the optimal policy design. Accuracy, in this setting may be undesirable, when it would influence the innovative effort in the wrong way. This result is in contrast with the traditional model, where accuracy is always welcome since it enhances marginal deterrence. When innovations are ex-ante welfare enhancing, they can be sustained by laissez-faire or, if the enforcement effort is exogenous, through better (type-I) accuracy. When instead the innovative effort is ex-ante welfare decreasing, it is discouraged through positive enforcement and (type-II) accuracy. Finally, when the enforcer can selectively reduce type-I and type-II errors, he will always concentrate accuracy on one of them only, depending on the expected impact of innovations on welfare, adopting asymmetric protocols of investigation.

Keywords: norm design, innovative activity, enforcement, Type I and Type II errors.

JEL classification: D73, K21, K42, L51.

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1 Introduction

The purpose of this paper is to study the effect of errors in law enforcement on the innovative activity of firms. Norms are often required to rule delicate issues where innovative effort is involved, as in the regulation of genetically modified organisms, or in the application of antitrust norms in high-tech industries. In these settings a widespread concern refers to the impact of the design and enforcement of norms on innovative activity. Indeed, a recurrent theme in competition policy is that antitrust should prevent abuses of dominant firms without chilling competition on the merits. The novelty of the issues brought to the attention of enforcers by innovation makes errors more likely than in standardized situations, and renders the analysis of judicial errors a major concern in law enforcement.

The traditional approach in the Law and Economics literature is not fit to address these problems. It considers private agents choosing among a set of feasible actions, some of which are socially damaging and unlawful. In this setting the feasible actions are perfectly known and implementable by the individuals, the only restraint from taking harmful acts being the expected fines associated to illegal practices. The analysis focusses on the ability of law enforcement to discourage individuals from committing the most harmful actions, which represents the very notion of marginal deterrence[1].

This setting does not correspond to the issues we want to analyze. Suppose the legislator wants to regulate the production of genetically modified seeds, or the design of new products by dominant companies. In both cases the traditional problem, where private agents choose among a set of known actions corresponds to the final stage of a process that requires first to devote resources to research in order to identify the possible innovative solutions, among which the choice will be made in the end. Two main issues characterize these new setting: firstly, private agents have a richer set of decisions, as they initially choose whether and how much to invest in innovation and then, if research has been successful, to pick out one of the innovative actions available; secondly, during the innovative process the private agents not only discover how to implement the new actions, but they also learn their social consequences and therefore whether *ex post* the new actions will be considered as lawful or unlawful according to the prescriptions of the norm. [2]

[1] See Stigler (1970), Shavell (1992), Mookherjee and Png (1994), among others.

[2] This setting has some elements in common with the so called "activity level" model: see, for instance, Shavell (1980) and (2006) and Shavell and Polinsky (2000). According to this approach, private benefits and social harms depend on two different decisions of private agents: a level of activity (how long the individual drives) and a level of precaution (the speed). This literature has mainly focussed on a comparison of different liability rules (strict vs fault-based). In our paper the role of innovative effort resembles the activity while the new actions parallel precaution. The information structure, however, is different in our setting, since innovative effort is taken before uncertainty resolves, while in the "activity level" model uncertainty plays no role.

In our analysis we associate the lawfulness of an action to its social consequences, that is whether it enhances or reduces welfare *ex post*.[3] However, agents are often unable to evaluate *ex ante* with certainty the social consequences of the innovative actions. Uncertainty may be rooted in the very nature of the firm's research activity so that some features of the innovation are unknown until discovery. For instance, in the example of the biotech firm, experiments with a new GM seed may promise higher yields but may also pose unknown risks to public health, that can be properly verified only once the research has been concluded. Alternatively, uncertainty may derive from the interaction of the innovation, whose properties may be controlled and planned with sufficient confidence by the firm, with the economic or social environment at the time the innovation is introduced. The future features of this environment may depend on the choices of many other agents and cannot be assessed *ex ante* with certainty. In our second example, a dominant software company may invest in research to tie a new software application into a new operating system for PC's: beyond the initial intent of the company, the efficiency and foreclosure effects of this new software will depend, at the time of the commercial introduction, on the supply of alternative packages and applications by the competitors, that may be only imperfectly foreseen at the time of the research investment.

In this class of situations, deterrence works through an additional channel, by affecting the initial incentives to invest in research: if private agents expect a very restrictive treatment of the results of their innovative effort, they will have lower incentives to commit resources to research. As a result, innovations will be discovered and possibly chosen with a lower probability. Deterrence in this case acts on the new actions "on average", reducing the likelihood that any of them will be taken, rather than selectively at the margin. For this reason we label this effect of law enforcement *average deterrence* to distinguish it from the traditional marginal deterrence effect. Notice that while more marginal deterrence is always welcome, and is therefore constrained only by the associated enforcement costs, average deterrence is only desirable when the innovative effort leads to new actions that *ex ante* entail an expected welfare loss.

This approach has been first proposed by Immordino, Pagano and Polo (2006) who analyze the optimal enforcement policy and the optimal flexibility of the norms, comparing benevolent and selfish (corrupt) enforcers. In the present paper we adopt the same framework and consider the case of benevolent enforcers that may commit judicial errors.

[3] Although we argue that in many instances the ultimate reason why a norm considers an action as illegal lies with its social consequences, all the arguments we develop in the paper hold true even adopting a more formalistic definition of legality, based on whatever a norm prescribes to be lawful or not. All that matters in applying our analysis is that the elements that make an action legal or unlawful according to the norm cannot be assessed with certainty at the time the investment in the innovative activity is chosen.

Judicial errors and their reduction, i.e. accuracy, are a central concern in the general literature on law enforcement. They have been analyzed by Kaplow (1994), Kaplow and Shavell (1994, 1996), Polinsky and Shavell (2000) and Png (1986) among others, focussing on the negative impact of such errors on marginal deterrence. A more specific literature on competition policy enforcement, in particular collusion and abuse of dominance, studies the cost of an inappropiate intervention by a competition authority.[4] In a model of collusion, Schinkel and Tuinstra (2006), find that the incidence of anti-competitive behavior increases in both types of enforcement errors: type II errors decrease expected fines, while type I errors encourage industries to collude when they face the risk of a false conviction. Therefore, in their framework as in the general literature on law enforcement accuracy is always desirable.

We contribute to the literature by showing that accuracy may be undesirable and that when the enforcer can selectively reduce errors, he will never choose a positive level of accuracy for both types of errors opting for an asymmetric protocol of investigation.

Following the literature, we distinguish two types of errors: the enforcer may mistakenly convict an innocent or mistakenly acquit a guilty. The first case corresponds to a type I error a case of over-enforcement or false positive, while the second entails a type II error and involves under-enforcement and false negative. Type I errors, inducing over-enforcement, reduce the profits expected from the innovative activity limiting the incentives to innovate. Conversely, type II errors, through under-enforcement, boost innovative activity and reduce average deterrence. In our setting then type I and type II errors affect deterrence in opposite ways so that, do to the ex ante uncertainty on the social desirability of innovation accuracy may be undesirable.

When we consider the investigation activity in our setting, the enforcer has two tasks: he first has to recognize properly the new actions chosen, and secondly he has to assess their lawfulness, that is their welfare consequences. Due to the novelty of the innovations, this latter task is the more compelling exercise. Examining the seeds, it is simpler to recognize a GM variety rather than assessing their effects on public health; similarly, the enforcer can easily check that a new operating system has some applications tied in, but it is much harder to see the foreclosure potential of this tying strategy. Hence, we assume that judicial errors may be committed when assessing the welfare consequences of innovative actions.

We assume that the enforcer can control different sets of instruments: the level of fines, the probability of recognizing the actions chosen (which depends on the enforcement effort) and the probability of correctly assessing the consequences of the chosen actions (which depends on the accuracy effort). In this framework we distinguish different cases, that may

[4] For a thorough assessment of the cost of erroneous antistrust interventions, or non interventions see the report prepared for the Office of Fair Trading by Lear (2006).

better fit specific situations.

Exogenous (general) versus endogenous (specialized) enforcement. If the enforcer monitors at the same time a wide and diversified range of private conducts (e.g. safety conditions of various products or industrial strategies of dominant firms), it cannot fine-tune its activity to verify the specific practice ruled by a given norm (e.g. the GM seeds or the tying policy). In this case, the probability of identifying a specific practice will depend on the resources devoted to general monitoring, and will be the same for any illegal private behavior, i.e. it will be exogenous in our policy analysis. Alternatively, the enforcer might be able to allocate resources to a specialized monitoring activity making the enforcement effort endogenous in the analysis.[5]

Common accuracy on both types of error versus specific type I and type II accuracies. The enforcer can improve accuracy by committing more resources to decrease both types of errors at the same rate or can selectively affects each of them.

To illustrate this point, consider the following example, drawn from antitrust. Suppose that the welfare effects of a given practice of a dominant firm depend on four fundamental variables: market shares, entry conditions, demand elasticity and cost efficiencies. If we choose a very low level of accuracy, we might just consider the market shares, concluding if the market share is high that the practice is harmful while infering that welfare is enhanced (or unaffected) by the practice if the firm has a small market share. If however we opt for greater accuracy, and we want to assess additional variables we might proceed in different ways.

For instance, we could consider entry conditions only if the market share is high; at this stage we might conclude that the welfare effects are positive if a large market share is combined with easy entry, or we might instead further proceed by considering demand elasticity when high market shares and hard entry have been detected. High elasticity (coupled with high market share and hard entry) then would lead us again to conclude for positive welfare effects while a low demand elasticity would move us to consider cost efficiencies. Finally, these would lead to an assessment of positive welfare effects if sufficiently high. This example shows an asymmetric protocol of investigation in which further levels of accuracy are implemented by requiring a more compelling standard of proof for negative welfare effects. In this case, therefore, the enforcer will be quite accurate in assessing that the practice is unlawful while quite rough in concluding that it is legal: consequently, type I errors are reduced while type II errors are more likely. From this example it is easy to construct a protocol of investigation that instead selectively reduces type I errors only.

[5] The traditional example of general versus specialized monitoring refers to patrolling a highway, an activity that allows to identify with the same probability any breach of the driving rules, as opposed to the use of remote speed control facilities, that instead allow to elicit cases of excessive speed only. On this issue see ...

The opposite case of common accuracy corresponds instead to the following protocol of investigation. Once the market share is assessed, in order to improve accuracy we consider entry conditions, demand elasticity and eventually cost efficiencies, drawing the conclusion on welfare once considered the realized values of all these variables. We might conclude, for instance, contrary to the outcome of the asymmetric protocol considered above, that the practice is welfare reducing even if the market shares are limited if substantial switching costs determine a very low demand elasticity, etc. In this case, adding a new piece of evidence, i.e. increasing accuracy, reduces symmetrically the probability of committing either type of error.

In both cases, more resources are needed to verify the wider set of evidence required. But the different protocols imply that investigations go more in depth symmetrically or asymmetrically. In this paper we study first the case in which the enforcer can only set a common level of accuracy for the two types of errors and then refine the analysis considering the case of different levels of type I and type II accuracies.

The main findings of our analysis can be summarized in the following way.

Consider first the case of exogenous enforcement and common accuracy. Since type I errors occur when the innovative actions are welfare-increasing (good state) while type II errors arise in the opposite case (bad state), the probability of the bad state determines which type of error is the more likely. If, for instance, the bad state is more likely, type II errors will be relatively more frequent and a greater (common) accuracy will have the net expected effect of reducing under-enforcement, the impact associated to type II errors. We show that a positive level of accuracy is optimal in two polar cases: i) when the bad state is relatively likely and the welfare loss due to the innovation may be substantial. The innovative effort is undesirable and accuracy has the prevailing effect of reducing type II errors and under-enforcement; ii) conversely, when the bad state is unlikely and the expected welfare effect of the new actions is positive. The innovative effort is desirable and accuracy has the predominant effect of reducing type I errors and over-enforcement sustaining the incentives to innovate.

Consider then the case where the enforcer can differently affect type I and type II errors. In this case when the innovation is *ex ante* welfare enhancing, the innovative activity will be sustained by spending in type I accuracy (that reduces over-enforcement) but no in type II accuracy (which maximizes under-enforcement). When the new actions are *ex ante* welfare reducing the enforcer is willing to limit the innovative effort by reducing under-enforcement and boosting over-enforcement. Hence, he will invest in type II accuracy but no in type I accuracy. Hence, the asymmetric protocols of investigations described above are optimal.

Finally, when the enforcement effort becomes endogenous we obtain one more result.

When the innovation is *ex ante* welfare enhancing it is optimal to choose not to enforce any prohibition. In other words, *laissez faire* is the less costly way to sustain the innovative effort. In the opposite case when the new actions are *ex ante* harmful, the innovative effort is discouraged by a mixture of enforcement effort and type II accuracy.

Our paper is organized as follows. Section 2 presents the model. Section 3 analyzes the case where the enforcer is not able to affect accuracy separately for type I and type II errors, and Section 4 the case when the enforcer can separately affect errors. Section 5 concludes. All proofs are in the Appendix.

2 The model

We consider a model with a profit-maximizing firm, and a benevolent enforcer that may commit mistakes. The firm can either choose one among several known and lawful actions or invest in learning to identify a new action, whose social effects are *ex ante* unknown.

The key issue that we wish to explore is: what is the optimal design of fines, enforcement and accuracy when private innovative activity is important and enforcers are subject to judgement errors?

The firm can choose the *status quo* action a_0 (planting traditional seeds, offering an untied application) with associated profits $\Pi(a_0)$ and welfare $W(a_0)$: we normalize these two measures to zero, i.e. $\Pi(a_0) = W(a_0) = 0$. Action a_0 is the most profitable among the known and legal actions that the firm is able to implement without investing in learning. It is correctly recognized by the enforcer in its own nature (a_0) and social consequences ($W(a_0)$).

Alternatively, the firm can consider a new action a (*innovation*), with associated profit $\Pi(a) = \Pi > 0$.[6] Depending on the state of nature s, the social consequences of the new action differ. With probability β, a bad state $s = b$ occurs, where the new action has a negative social externality, $W(a) = W_b(a) = \underline{W} \leqslant 0$.[7] In this case, private incentives conflict with social welfare. With probability $1-\beta$, instead, a good state $s = g$ materializes and the new action improves welfare, $W(a) = W_g(a) = \overline{W} > 0$. In this state, there is no conflict between private and social incentives, since the innovation improves both the profits

[6] In this paper we consider just one possible new action as a result of the learning effort, rather than a set of new actions. This latter case, that is analyzed in Immordino, Pagano and Polo (2006), allows to consider also the traditional issue of marginal deterrence, i.e. the choice of the firm of one among many illegal actions. Since the distinguishing feature of our approach is in the effect of deterrence on innovative activity (what we later call as average deterrence), we focus on this latter effect considering a single new action.

[7] In the comparative statics of the equilibrium we will vary $\underline{W}$ to change the social loss of the new actions.

of the firm and social welfare. Nature chooses which state of the world occurs; hence, the probability β of the bad state (social harm) is an *ex ante* measure of the likelihood of misalignment between public interest and firms' objectives. In our example, β is the prior probability that the GM seeds will be hazardous to public health, or that the new tied application, when introduced in the market, will foreclose alternative software packages.

While the firm knows from the beginning how to implement the status quo action a_0, carrying out the new action requires an investment in learning (experiment with GM seeds, create a new tied application), which accordingly will be referred to as "*innovative activity*". If the investment is successful, the firm will discover how to implement the new action a. In this case, the firm also learns the state of nature s, that is whether its innovation is socially harmful or beneficial. Proceeding with our example, the biotech company learns not only how to produce new GM seeds, but also the dangers that they pose to public health. And the software company, once the new application is created, is able to predict whether in the current market conditions it will enable to foreclose the alternative packages or not.

The amount of resources I that the firm invests in the innovative activity determines its chances of success: for simplicity, the firm's probability $p(I)$ of learning how to carry out the new action a is assumed to be linear in I, i.e. $p(I) = I$ with $I \in [0, 1]$. The cost of learning is increasing and convex in the firm's investment. For simplicity we assume $c(I) = c\frac{I^2}{2}$ with

$$c > \Pi \tag{1}$$

to ensure an internal solution.

The institutional framework in the design and enforcement of norms is as follows. The legislator writes the norm, which specifies under what circumstances the actions are legal or not. The enforcement officials commit to a certain fine and seek evidence on firms' actions and on the associated social consequences. We assume that enforcers are benevolent but may make errors.[8]

The norm identifies some circumstances that make the new action legal or unlawful. In general we can adapt this framework to a wide range of formal frameworks: for instance, the norm may state that the new action is illegal whenever it occurs together with contingencies $x_1, ..., x_n$, a case that reminds more or less articulated per se rules.[9] Alternatively, illegality

[8] Immordino, Pagano and Polo (2006) study how corrupt officials influence the design and enforcement of norms in the presence of private innovative activity. In the present setting there is no real difference between the authority and the official. Hence, we refer to them as "the enforcer".

[9] Drawing from antitrust, for instance, a very simple per se rule would consider as illegal an action as the practice of resale price maintenance when adopted by a firm with a market share larger than $x\%$. A more articulated rule would consider resale price maintenance as illegal when adopted by a dominant firm, where this latter is identified by certain thresholds in market shares (x_1), entry conditions (x_2) and demand elasticity (x_3).

may be related to the effects of the action, as required under a rule of reason approach.[10] It is important to stress that our analysis can be adapted to either of the two cases. All that matters is that, at the time of the innovative investment, the elements that the norm identifies in order to assess the legality of the new action are not known with certainty. It may be the case that the factual elements $x_1, ..., x_n$ specified in the norm, or instead the effects of the new action, are not observed ex ante. With this important caveat in mind, we consider a norm written as follows, that allows us to simplify greatly the notation in the analysis:

> The action a_0 is lawful; the (new) action a is illegal if *ex post* socially damaging, i.e. if $W(a) < 0$. The illegal action is sanctioned according to a fine f chosen in the interval $[0, F]$.

For instance, the norm prohibits to commercialize hazardous seeds or to adopt practices that foreclose the market to competitors.

In order to enforce the norm the enforcer has therefore to identify the action chosen (a_0 or a) and the social consequences of the action (0 or $W(a)$). Obtaining evidence on these elements requires to commit resources. We define respectively as enforcement and accuracy the activities devoted to obtain evidence on the action chosen and on its consequences (legality). By increasing the resources dedicated to enforcement (accuracy) the enforcer obtains with a higher probability hard evidence on the action chosen (on its consequences and legality).

Given the fine f, the expected fine depends on the probability of enforcement, i.e. on the ability of the enforcer to find hard evidence on the action chosen, and on the accuracy in assessing the social consequences of the action.

More specifically, the probability of enforcement is positively affected by the amount of resources E devoted to monitoring firms' actions: with probability $q(E)$ the enforcer obtains hard evidence that the firm took action a. For simplicity, we assume the probability $q(E)$ to be linear in E, i.e. $q(E) = E$. The cost of the enforcement effort is convex, implying decreasing returns to enforcement: $g'_E > 0$ and $g''_E > 0$ for $E \in [0, 1]$, with $g_E(0) = g'_E(0) = 0$ and $\lim_{E \to 1} g(E) = \lim_{E \to 1} g'(E) = \infty$. With probability $1 - q(E)$, instead, the enforcement effort does not produce enough evidence to prove that the firm took action a. In the benchmark model the level of enforcement effort is positive and exogenous while endogenous enforcement is considered later on.

[10] For a discussion on an effect-based interpretation of antitrust norms, see Gual et al. (2005).

Once the enforcer has successfully identified the action chosen by the firm, he still has to identify its social consequences (lawfulness). We assume that the enforcer is more accurate in assessing the effects of the status quo rather than the new action. Judicial errors occur only when assessing the effects (legality) of the new action a, while the status quo action a_0 is correctly recognized as legal. This different degree of accuracy reflects the more compelling task of assessing new rather then well known phenomena.

More precisely, the enforcer receives a signal $\sigma = \{b, g\}$ on the state of nature, i.e. on the social consequences of the new action. With probability α_I the signal is incorrect when the true state of the world is the good one: in this case the enforcer considers action a as unlawful when the good state occurs, committing a "type I error". Conversely, with probability α_{II} the signal is incorrect when the true state is the bad one, and a "type II error" occurs, i.e. the enforcer will fail to identify a as unlawful when the true state is the bad one. Hence,

$$\alpha_I = \Pr(\sigma = b \,|s = g) \quad \text{and} \quad \alpha_{II} = \Pr(\sigma = g \,|s = b)$$

We assume that the signals received are informative, i.e. $\alpha_I \leqslant \frac{1}{2}$ and $\alpha_{II} \leqslant \frac{1}{2}$.

The level of accuracy of the enforcer can be improved by committing more resources to obtain a more precise assessment of the effects. As we argued in the introduction, accuracy means reducing type I, type II or both types of errors. By adopting different protocols of investigation and standards of proof and by committing more resources, the enforcer can reduce selectively type I or type II errors or can symmetrically improve the assessment reducing both types of errors.

We assume that the cost of a given probability α_I of type I error is $g_I(\frac{1}{2} - \alpha_I)$, where $\alpha_I = \frac{1}{2}$, i.e. a completely uninformative signal, corresponds to the lowest accuracy, with $g'_I > 0$ and $g''_I > 0$ for $\alpha_I \in \left[0, \frac{1}{2}\right]$, and with $g_I(0) = g'_I(0) = 0$ and $\lim_{\alpha_I \to 0} g(.) = \lim_{\alpha_I \to 0} g'(.) = \infty$. Similarly, for the cost of decreasing type II errors we assume: $g_{II}(\frac{1}{2} - \alpha_{II})$ with $g'_{II} > 0$ and $g''_{II} > 0$ for $\alpha_{II} \in \left[0, \frac{1}{2}\right]$, with $g_{II}(0) = g'_{II}(0) = 0$ and $\lim_{\alpha_{II} \to 0} g(.) = \lim_{\alpha_{II} \to 0} g'(.) = \infty$.[11]

The timing of the model is the following. At time 0 nature chooses the state of the world $s = \{g, b\}$ which is not observed by any agent. Agents know that the probability of the bad state is $\beta > 0$. At time 1, the authority writes the norm which determines the fine f, and commits to the effort devoted to enforcement E and to accuracy α_I and α_{II}. At time 2, the firm chooses the innovative activity I and learns with probability $p(I) = I$ how to implement the new action a and its payoffs $\Pi(a)$ and $W(a)$ (state of the world), knowing the norm, the fine, the enforcement probability E and the probabilities of error α_I and α_{II}.

[11] When $\alpha_I = \alpha_{II} = \alpha$ the same assumptions apply to $g_\alpha(\frac{1}{2} - \alpha)$.

At time 3, the firm chooses an action, conditional on what it learnt in the previous stage. Finally, at time 4 the action chosen determines the private profits and the social welfare; the official collects evidence (with errors) and possibly levies fines.

Finally, we assume the following ranking among payoffs:

$$\overline{W} > \Pi > F > 0. \tag{2}$$

The first inequality implies that in the good state social gains exceed private ones, or, equivalently, that the new action in good state increases consumers' surplus as well as producer's surplus. According to the second inequality, the profits from the new action exceeds the maximum fine even when this is inflicted with certainty, implying that the firm, if the innovative effort is successful, always prefers to choose the new action (incomplete deterrence). Even in this case, however, some room for deterrence remains through the effects of the enforcement policy on the innovative activity I and on the probability to take the new action.

We analyze the firm's choices and the design of the optimal policy starting from the case of common accuracy and exogenous enforcement, moving then to the other cases.

3 Common accuracy: $\alpha_I = \alpha_{II} = \alpha$

We first consider the case where the enforcer can choose only a common level of accuracy for type I and type II errors. We solve the game backwards starting from the firm's choice of the action and of the innovative activity, moving then to the design of the optimal policy.

3.1 Firm's choices: actions and innovative activity

At stage 3, depending on whether its innovative activity was successful or not, the firm chooses an action. If the innovative activity was unsuccessful, under our assumptions the firm chooses the status quo action a_0 with associated profits $\Pi(a_0) = 0$ and welfare $W(a_0) = 0$. If instead the innovative activity was successful, the firm is able to take the new action a. If the action is not socially harmful ($s = g$) the action a is lawful. Nevertheless, with probability α_I the authority perceived state of the world is the bad one ($\sigma = b$). Then, when the firm chooses the profit maximizing action a (that gives also the maximum welfare $\overline{W}$) expected profits are equal to $\Pi - E\alpha_I f$. If, alternatively, the firm chooses the action a_0 profits are equal to 0 and there is no error in enforcement. Assumption (2) implies that $\Pi - E\alpha_I f > 0$ for any fine f, enforcement E and probability of type I error α_I. The firm will then choose the new action a.

If instead the new action is socially harmful ($s = b$), and therefore unlawful, the fine is inflicted only with probability $E(1-\alpha_{II})$ since with probability α_{II} the enforcer receives the wrong signal $\sigma = g$. In this case when the firm chooses the new action a (that gives also the minimum welfare $\underline{W}$) expected profits are equal to $\Pi - E(1-\alpha_{II}) f$. Again due to assumption (2), $\Pi - E(1-\alpha_{II}) f > 0$ and the firm will choose the unlawful action a.

At stage 2, knowing the enforcement and accuracy efforts, the firm chooses its innovative activity I so as to maximize its expected profits, given the optimal actions that it will choose at stage 3. The firm learns how to carry out the new project with probability $p(I) = I$ and its expected profits at this stage are:

$$E\Pi = I\left\{\beta\left[\Pi - E(1-\alpha_{II}) f\right] + (1-\beta)\left[\Pi - E\alpha_I f\right]\right\} - c\frac{I^2}{2}, \tag{3}$$

where the first term is the expected gain from innovative activity (net of the expected fines), positive by assumption (2) and the second term is the cost of innovative activity. We can rewrite expected profits (3) as:

$$E\Pi = I\left\{\Pi - [\beta(1-\alpha_{II}) + (1-\beta)\alpha_I]Ef\right\} - c\frac{I^2}{2}. \tag{4}$$

The optimal innovative activity $\widehat{I}$, taking into account that in the present setting $\alpha_I = \alpha_{II} = \alpha$, can be obtained from the first order condition:[12]

$$\widehat{I}(E, f, \alpha) = \frac{\Pi - [\beta(1-\alpha) + (1-\beta)\alpha]Ef}{c} \tag{5}$$

Note that $\widehat{I}(E, f, \alpha)$ is greater than zero thanks to assumption (2) and smaller than one for assumption (1). The effect of errors on the innovative activity is given by:

$$\frac{\partial \widehat{I}}{\partial \alpha} = -\frac{(1-2\beta)Ef}{c} \gtreqless 0 \Leftrightarrow \beta \gtreqless \frac{1}{2}. \tag{6}$$

This result can be explained as follows: type I errors occur in the good state and correspond to over-enforcement, lowering the expected profits; conversely, type II errors occur in the bad state and entail under-deterrence and higher expected profits. When the probability α of committing an error is the same for the two errors, type I errors are more frequent than type II errors if the good state is more likely, i.e. $\beta < \frac{1}{2}$. Over-enforcement in this case is the predominant effect, reducing the expected profits and the investment $\widehat{I}$ in innovative activity. The opposite holds true if the bad state is relatively likely, i.e. $\beta > \frac{1}{2}$. The effect of the enforcement effort E and of the fine f is always to depress the firm's innovative activity:

$$\frac{\partial \widehat{I}}{\partial E} = -\frac{[\beta(1-\alpha) + (1-\beta)\alpha]f}{c} \leqslant 0, \quad \frac{\partial \widehat{I}}{\partial f} = -\frac{[\beta(1-\alpha) + (1-\beta)\alpha]E}{c} \leqslant 0. \tag{7}$$

[12] The second order condition is satisfied as well.

We may summarize our main findings with the following Proposition.

Proposition 1: *A higher enforcement effort E, and a higher fine f always deter innovative activity. In contrast, a higher probability of judicial errors (lower accuracy) α deters innovative activity if and only if the good state is relatively more likely, i.e. iff $\beta < \frac{1}{2}$.*

We now move to the analysis of the optimal policy starting from the case of exogenous enforcement.

3.2 Enforcer's choices: exogenous enforcement E

The expected welfare, once taken into account the firm's optimal choices of actions and innovative activity, is:

$$\mathrm{E}W = \widehat{I}(E, f, \alpha)\Delta\mathrm{E}(W) - g_E(E) - g_\alpha(\frac{1}{2} - \alpha) - c\frac{\widehat{I}(E, f, \alpha)^2}{2}, \tag{8}$$

where

$$\Delta\mathrm{E}(W) \equiv [\beta\underline{W} + (1-\beta)\overline{W}]$$

is the expected welfare gain (or loss) stemming from the new action a, and the last three terms capture the public cost of enforcement and accuracy and the private costs of innovative activity. Notice that the expected welfare gains from the new action depends on the probability of the bad state β and on the social loss in the bad state $\underline{W}$: these two parameters will be key in the analysis of the equilibrium policy.

We start arguing that Becker's reasoning applies to this model. The optimal sanction, indeed, is F, the maximum feasible sanction. For any f less than F, we can raise f and reduce E (or increase α for $\beta < \frac{1}{2}$) so as to keep $[\beta(1-\alpha) + (1-\beta)\alpha]Ef$ unchanged, implementing the same level of innovative activity $\widehat{I}(E, f, \alpha)$ but reducing the enforcement cost $g_E(E)$ (and accuracy cost $g(\frac{1}{2} - \alpha)$), with a net increase in welfare.

When the enforcement effort E is exogenous, the optimal level of accuracy is given by the first-order condition:

$$\frac{\partial \mathrm{E}W}{\partial \alpha} = [\Delta\mathrm{E}(W) - c\widehat{I}]\frac{\partial \widehat{I}}{\partial \alpha} + g'_\alpha \geqslant 0. \tag{9}$$

The first term captures how accuracy affects *average deterrence* – the extent to which an increase in accuracy α affects innovative activity, reducing or increasing the probability of the new action, whether legal or not. This effect can be positive or negative, depending on whether the innovative activity has a positive or negative marginal social value $\Delta\mathrm{E}(W) - c\widehat{I}$ and depending on the effect of an increase of accuracy on innovative activity $\frac{\partial \widehat{I}}{\partial \alpha}$. The second term of condition (9) is the marginal cost of accuracy.

When the marginal social value of innovative activity, $\Delta\mathrm{E}(W) - c\widehat{I}(.)$ and the marginal impact of accuracy on innovative activity $\frac{\partial \widehat{I}}{\partial \alpha}$ are both positive or both negative, condition (9) will entail a corner solution i.e. $\alpha^* = \frac{1}{2}$. On the contrary, when $\Delta\mathrm{E}(W) - c\widehat{I}(.)$ and $\frac{\partial \widehat{I}}{\partial \alpha}$ are opposite in sign, (9) holds with equality and the optimal accuracy will be determined as an internal solution, i.e. $\alpha^* \in \left(0, \frac{1}{2}\right)$.

In the following Proposition we characterize the optimal policy for different parameters regions referring to β, the likelihood of the bad state, and $\underline{W}$, the social loss in the bad state. If we substitute the equilibrium level of innovative activity $\widehat{I}(E, F, \alpha)$ as in (5) into $\Delta\mathrm{E}(W) - c\widehat{I}(.) = 0$ and solve, we obtain:

$$\underline{W}_0(\underset{-}{E}, \underset{+/-}{\alpha}, \underset{+}{\beta}) = \frac{\Pi - (1-\beta)\overline{W} - [\beta(1-\alpha) + (1-\beta)\alpha]EF}{\beta}$$

Hence, given the policy parameters α and E and the likelihood of the bad state β, when $\underline{W} = \underline{W}_0(E, \alpha, \beta)$ the marginal social value of innovative activity is nil. In the following analysis of the optimal policy a particular role is played by

$$\underline{W}_0(E, \frac{1}{2}, \beta) = \frac{\Pi - (1-\beta)\overline{W}}{\beta} - \frac{EF}{2\beta},$$

corresponding to the value of the welfare loss in the bad state that gives a zero marginal social value of innovative activity when no accuracy is implemented. The locus $\underline{W}_0(E, \frac{1}{2}, \beta)$ is increasing and concave in β.

Then, we can state the following result.

Proposition 2: *In the space $(\underline{W}, \beta)$ the enforcer implements a positive level of accuracy, i.e. $\alpha^* \in \left(0, \frac{1}{2}\right)$, only in the following two parameter regions:*

i) When the marginal social value of the innovative activity is negative and the bad state relatively likely, i.e. for $\underline{W} < \underline{W}_0(0, \frac{1}{2}, \beta)$ and $\beta \in (\frac{1}{2}, 1)$; in this case $\alpha^* \to \frac{1}{2}$ when $\underline{W} \underset{-}{\to} \underline{W}_0(E, \frac{1}{2}\beta)$;

ii) When the marginal social value of the innovative activity is positive and the bad state relatively unlikely, i.e. for $\underline{W} > \underline{W}_0(E, \frac{1}{2}, \beta)$ and $\beta \in \left(0, \frac{1}{2}\right)$; in this case $\alpha^* \to \frac{1}{2}$ when $\underline{W} \underset{+}{\to} \underline{W}_0(E, \frac{1}{2}\beta)$.

Moreover, when $\alpha^* \in \left(0, \frac{1}{2}\right)$, $\frac{\partial \alpha^*}{\partial \underline{W}} \gtreqless 0$ and $\frac{\partial \alpha^*}{\partial E} \gtreqless 0$ if and only if $\beta \gtreqless \frac{1}{2}$.

Proof. See the Appendix. ■

Proposition 2 shows that we implement some level of accuracy, i.e. $\alpha^* \in \left(0, \frac{1}{2}\right)$ in two polar cases. When the marginal social value of innovative activity is negative and the bad state is relatively likely, as in region *i*), type II errors are more frequent than type I errors, inducing on average under-deterrence: since innovative activity is socially undesirable and errors foster innovative activity, the enforcer implements some accuracy to reduce under-deterrence and innovative activity, i.e. $\alpha^* < \frac{1}{2}$. In this case, if the new action a determines more social harm in the bad state ($\underline{W} \downarrow$) more average deterrence is obtained by increasing accuracy ($\alpha^* \downarrow$) and reducing the predominant effect of under-enforcement, i.e. $\frac{\partial \alpha^*}{\partial \underline{W}} > 0$. Conversely, if the innovative activity is ex ante desirable and the bad state is relatively unlikely, as in region *ii*), type I errors are more frequent and on average discourage the innovative activity: hence it is optimal to implement some accuracy to limit errors and sustain innovation. When in this region the social loss in the bad state is larger ($\underline{W} \downarrow$) the innovative activity becomes marginally less desirable and less accuracy is selected ($\alpha^* \uparrow$). Hence, we obtain $\frac{\partial \alpha^*}{\partial \underline{W}} < 0$.

Proposition 2 implies also that in some circumstances the optimal policy requires to choose no accuracy: this is the case when the innovative activity is desirable but accuracy would depress it, or conversely when a social loss is expected from innovation that, in turn, would be sustained by accuracy. This result is in sharp contrast with the traditional model, where accuracy is always desirable.

The comparative statics of accuracy with respect to the level of enforcement is interesting. When E is exogenous, the optimal policy allows to implement second best solutions working solely through accuracy. When the marginal social value of the innovative activity is negative and the bad state relatively likely, as in region *i*), the innovative activity is undesirable and accuracy and enforcement work in the same direction. An exogenous increase in enforcement, *ceteris paribus*, tends to reduce innovative activity. Then the enforcer can marginally save in enforcement costs by reducing accuracy, i.e. $\frac{d\alpha^*}{dE} > 0$. Conversely, in region *ii*) the innovative activity is socially desirable and an increase in E works in the wrong direction. Since accuracy in this case more often reduces type I errors, the enforcer is willing to implement more accuracy to limit over-enforcement and sustain the innovative activity. Hence, more enforcement induces more accuracy (lower α^*). In this case we have $\frac{d\alpha^*}{dE} < 0$.

We illustrate the results in Figure 1.

[Figure 1 about here]

3.3 Enforcer's choices: endogenous enforcement

We now consider the case where the enforcer can calibrate the enforcement effort to the different practices analyzed (the GM seeds or the tying policy) instead of implementing general monitoring that induces a common E on all the practices. The first order conditions to identify the optimal policy (E, α) are now

$$\frac{\partial \mathrm{E}W}{\partial \alpha} = [\Delta \mathrm{E}(W) - c\widehat{I}]\frac{\partial \widehat{I}}{\partial \alpha} + g'_{\alpha} \geqslant 0$$

and

$$\frac{\partial \mathrm{E}W}{\partial E} = [\Delta \mathrm{E}(W) - c\widehat{I}]\frac{\partial \widehat{I}}{\partial E} - g'_{E} \leqslant 0. \quad (10)$$

In both expressions, the first term captures the *average deterrence* effect of α or E on the innovative activity. This effect can be positive or negative, depending on whether the innovative activity has a positive or negative marginal social value $\Delta \mathrm{E}(\widehat{W}) - c\widehat{I}$. Notice that while $\frac{\partial \widehat{I}}{\partial E}$ is always negative, the sign of $\frac{\partial \widehat{I}}{\partial \alpha}$ depends on the likelihood of the bad state β, as widely discussed in the previous section. The second term in the expressions is the marginal cost of deterrence. In an interior solution the optimal enforcement level equates the average deterrence to its marginal cost.

Let us define the relevant locus:

$$\underline{W}_0(0, \frac{1}{2}, \beta) = \frac{\Pi - (1-\beta)\overline{W}}{\beta}$$

that is increasing and concave in β. We assume that the cost of enforcement and accuracy are sufficiently convex.[13] Then, we can state the following result.

Proposition 3: *In the space $(\underline{W}, \beta)$ we can distinguish the following regions:*

i) When the marginal social value of the innovative activity is non negative, i.e. for $\underline{W} \geqslant \underline{W}_0(0, \frac{1}{2}, \beta)$ and for any β, the optimal policy entails "laissez faire", $E^* = 0$, $\alpha^* = 1/2$;

ii) When the marginal social value of the innovative activity is negative and the bad state relatively likely, i.e. for $\underline{W} < \underline{W}_0(0, \frac{1}{2}, \beta)$ and $\beta \in (\frac{1}{2}, 1)$, the optimal policy prescribes positive enforcement and accuracy, i.e. $E^* > 0$ and $\alpha^* \in \left(0, \frac{1}{2}\right)$. Enforcement and accuracy increase as the welfare loss in the bad state increases, i.e. $\frac{dE^*}{d\underline{W}} < 0$ and $\frac{d\alpha^*}{d\underline{W}} > 0$ (when $\underline{W} \to \underline{W}_0(0, \frac{1}{2}, \beta)$ we have $E^* \to 0$ and $\alpha^* \to \frac{1}{2}$)*;*

iii) When the marginal social value of the innovative activity is negative and the bad state relatively unlikely, i.e. for $\underline{W} < \underline{W}_0(0, \frac{1}{2}, \beta)$ and $\beta \in (0, \frac{1}{2}]$ the optimal policy requires positive enforcement and no accuracy, i.e. $E^* > 0$ and $\alpha^* = 1/2$. Enforcement increases

[13] More formally, we assume that

$$g''_E g''_{\alpha} \geqslant [\Delta \mathrm{E}(W) - c\widehat{I}]^2 \left(\frac{\partial \widehat{I}}{\partial E \partial \alpha}\right)^2. \quad (11)$$

This assumption is a sufficient condition for concavity.

as the welfare loss in the bad state increases, i.e. $\frac{dE^*}{d\underline{W}} < 0$ (*when* $\underline{W} \to \underline{W}_0(0, \frac{1}{2}, \beta)$ we have $E^* \to 0$).

Proof. See the Appendix. ■

When the bad state is very unlikely and/or the social loss $\underline{W}$ limited, i.e. when the marginal social value of the innovative activity is non negative (region i), even if the norm were to define the new action a as illegal when welfare reducing, it would be optimal not to enforce such a prohibition: $E^* = 0$. The norm in this case should consider the new action a as legal ("*laissez faire*" or "*per se legality rule*"). Compared to the case of exogenous (and positive) enforcement discussed in Proposition 2, where some accuracy was implemented (region ii), with endogenous enforcement the optimal policy entails saving on enforcement and accuracy costs simply by allowing the new action a. In other words, decreasing enforcement is a better way to foster the innovative activity than it was spending on accuracy to decrease (the prevailing type I) errors.

When instead the social loss increases, the optimal enforcement E^* is positive and increasing in the social loss $\underline{W}$. In this case the main goal of the policy is to discourage the innovative activity. When the bad state is relatively unlikely (region iii) the predominant effect of errors is over-enforcement and accuracy is undesirable. Conversely, when the bad state is relatively likely (region ii) errors lead more often to under-enforcement and enforcement and accuracy both improve average deterrence. When the costs of enforcement and accuracy are sufficiently convex, as assumed, we prefer to use a mix of the two instruments rather than a single one. In this case when the social loss in the bad state becomes worse, calling for less innovative activity, both enforcement and accuracy are increased.

4 Different accuracies: $\alpha_I \neq \alpha_{II}$

We now consider the case when the enforcement policy is able to implement accuracy separately for type I and type II errors.

4.1 Firm's choices: actions and innovative activity

The firm's choice of actions and innovative activity can be borrowed from the previous case. If the innovative activity effort is not successful and the firm does not learn how to implement a, the status quo action a_0 is chosen. If however the firm learns the new action it chooses a in any state of nature. In stage 2 the firm selects the investment in innovative activity I maximizing the expected profits. The optimal innovative activity when the probability of type I and type II errors are set separately is:

$$\widehat{I}(E, f, \alpha_I, \alpha_{II}) = \frac{\Pi - [\beta\,(1-\alpha_{II}) + (1-\beta)\alpha_I]Ef}{c} \tag{12}$$

The effect of the probability of type I error α_I on innovative activity is given by:

$$\frac{\partial \widehat{I}}{\partial \alpha_I} = -\frac{(1-\beta)Ef}{c} \leqslant 0. \tag{13}$$

Since a type I error corresponds to over-enforcement, when type I errors become more likely the expected profits are reduced and the incentives to exert innovative activity fall accordingly. The effect of the probability of error α_{II} on innovative activity is given by:

$$\frac{\partial \widehat{I}}{\partial \alpha_{II}} = \frac{\beta Ef}{c} \geqslant 0. \tag{14}$$

In contrast to type I, type II errors correspond to an under-enforcement bias that favors the innovative activity. Notice that the likelihood of the bad state β affects the magnitude but not the sign of these effects as it was in the case of a common probability α of both types of error.

Finally, the enforcement effort E and the fine f also depress the firm's innovative activity as in (7):

$$\frac{\partial \widehat{I}}{\partial E} = -\frac{[\beta\,(1-\alpha_{II}) + (1-\beta)\alpha_I]f}{c} \leqslant 0, \quad \frac{\partial \widehat{I}}{\partial f} = -\frac{[\beta\,(1-\alpha_{II}) + (1-\beta)\alpha_I]E}{c} \leqslant 0.$$

We summarize our main findings with the following Proposition.

Proposition 4: *The innovative activity is deterred by a higher enforcement effort E, a higher fine f, a higher level of type II accuracy (lower α_{II}) and a lower level of type I accuracy (higher α_I).*

We now move to the analysis of the optimal policy starting from the case of exogenous enforcement.

4.2 Enforcer's choices: exogenous enforcement

In the present setting the expected welfare, once taken into account the firm's optimal choices, is:

$$\mathrm{E}W = \widehat{I}(E, f, \alpha_I, \alpha_{II})\Delta\mathrm{E}(W) - g_E(E) - g_I(\frac{1}{2} - \alpha_I) - g_{II}(\frac{1}{2} - \alpha_{II}) - c\frac{\widehat{I}(E, f, \alpha_I, \alpha_{II})^2}{2}, \tag{15}$$

where $\Delta\mathrm{E}(W) \equiv [\beta\underline{W} + (1-\beta)\overline{W}]$ is again the expected welfare change due to the new action a, while the last four terms capture the public cost of enforcement and accuracy and the private costs of the innovative activity. As in the previous model, Becker's argument on maximum fines applies and we can substitute therefore the optimal fine F.

When the enforcement effort E is exogenous, the optimal policy requires to set the level of type I and type II accuracy. The first-order conditions are:

$$\frac{\partial \mathrm{E}W}{\partial \alpha_I} = [\Delta \mathrm{E}(W) - c\widehat{I}]\frac{\partial \widehat{I}}{\partial \alpha_I} + g_I' \geqslant 0. \tag{16}$$

and

$$\frac{\partial \mathrm{E}W}{\partial \alpha_{II}} = [\Delta \mathrm{E}(W) - c\widehat{I}]\frac{\partial \widehat{I}}{\partial \alpha_{II}} + g_{II}' \geqslant 0, \tag{17}$$

The first term in both expressions refers to the average deterrence of the errors, i.e. on their marginal impact on the innovative activity. As before this effect can be positive or negative, depending on whether the innovative activity has a positive or negative marginal social value $\Delta \mathrm{E}(W) - c\widehat{I}$.

We can distinguish two cases. When $\Delta \mathrm{E}(W) - c\widehat{I}(.) > 0$ condition (16) will hold as an equality and the optimal type I accuracy α_I^* will be determined as an internal solution, i.e.$\alpha_I^* \in \left(0, \frac{1}{2}\right)$ while the optimal type II accuracy will be determined as a corner solution, i.e. $\alpha_{II}^* = \frac{1}{2}$. Consequently, the second order conditions boil down to:

$$\frac{\partial^2 \mathrm{E}W}{\partial \alpha_I^2} = -c\left(\frac{\partial \widehat{I}}{\partial \alpha_I}\right)^2 - g_I'' < 0.$$

Intuitively, when the innovative activity is *ex ante* welfare enhancing, we reduce type I errors that, through over-enforcement, would otherwise limit the innovative activity while we maximize type II errors and the associated under-enforcement effect. For instance, when the GM seeds are expected to improve welfare on average, we want to examine quite seriously possible negative arguments on public health before concluding that the new plants should be prohibited, while we (almost) take for granted their positive impact without dedicating large resources in assessing it precisely, a further example of asymmetric protocol of investigation. As a result, type I errors are reduced while type II errors are more likely, leading to more frequent approvals of the new seeds.

The case of a negative marginal social value of the innovative activity ($\Delta \mathrm{E}(W) - c\widehat{I}(.) < 0$) leads, with a parallel argument, to $\alpha_I^* = \frac{1}{2}$ and $\alpha_{II}^* \in \left(0, \frac{1}{2}\right)$, i.e. to minimize under-enforcement and maximize over-enforcement. In this case the second order conditions require

$$\frac{\partial^2 \mathrm{E}W}{\partial \alpha_{II}^2} = -c\left(\frac{\partial \widehat{I}}{\partial \alpha_{II}}\right)^2 - g_{II}'' < 0.$$

As in the previous case, we analyze the optimal policies for different values of the welfare losses in the bad state $\underline{W}$, and the likelihood of the bad state β. If we substitute the equilibrium level of the innovative activity $\widehat{I}(E, F, \alpha_I, \alpha_{II})$ as in (12) into $\Delta\mathrm{E}(W) - c\widehat{I}(.) = 0$ and solve, we obtain:

$$\underline{W}_0(\underset{-}{E}, \underset{-}{\alpha_I}, \underset{+}{\alpha_{II}}, \underset{+}{\beta}) = \frac{\Pi - (1-\beta)\overline{W} - [\beta(1-\alpha_{II}) + (1-\beta)\alpha_I]EF}{\beta}$$

Hence, given the policy $(E, \alpha_I, \alpha_{II})$, when $\underline{W} = \underline{W}_0(.)$ the marginal social value of the innovative activity is nil. In the analysis of the optimal policy a particular role is played by

$$\underline{W}_0(E, \frac{1}{2}, \frac{1}{2}, \beta) = \frac{\Pi - (1-\beta)\overline{W}}{\beta} - \frac{EF}{2\beta}, \tag{18}$$

corresponding to the value of the welfare loss in the bad state that gives a zero marginal social value of the innovative activity when no type I and type II accuracy is implemented. As before, this locus is increasing and concave in β.

Then, we can state the following result.

Proposition 5: *In the space $(\underline{W}, \beta)$ we can distinguish the following regions:*

i) When the marginal social value of the innovative activity is negative, i.e. for $\underline{W} < \underline{W}_0(E, \frac{1}{2}, \frac{1}{2}, \beta)$, the optimal policy prescribes to reduce type II errors, i.e., $\alpha_I^* = \frac{1}{2}$ and $\alpha_{II}^* \in \left(0, \frac{1}{2}\right)$. Moreover, α_{II}^* is increasing in $\underline{W}$ and converges to $\alpha_{II}^* = \frac{1}{2}$ when $\underline{W} \underset{-}{\to} \underline{W}_0(E, \frac{1}{2}, \frac{1}{2}, \beta)$.

ii) When the marginal social value of the innovative activity is zero, i.e. for $\underline{W} = \underline{W}_0(E, \frac{1}{2}, \frac{1}{2}, \beta)$, the optimal policy entails no accuracy, i.e. $\alpha_I^* = \alpha_{II}^* = \frac{1}{2}$.

iii) When the marginal social value of the innovative activity is positive, i.e. for $\underline{W} > \underline{W}_0(E, \frac{1}{2}, \frac{1}{2}, \beta)$, the optimal policy requires to reduce type I errors, i.e., $\alpha_I^* \in \left(0, \frac{1}{2}\right)$ and $\alpha_{II}^* = \frac{1}{2}$. Moreover, α_I^* is decreasing in $\underline{W}$ and converges to $\alpha_I^* = \frac{1}{2}$ when $\underline{W} \underset{+}{\to} \underline{W}_0(E, \frac{1}{2}, \frac{1}{2}, \beta)$.

Proof. See the Appendix. ■

This result derives from the different effects of type I and type II errors on the innovative activity, that is reduced by the former and enhanced by the latter. The enforcer then selectively improves accuracy in order to influence the innovative activity in the desirable direction: increasing $\widehat{I}$ when ex ante its marginal social value is positive and reducing it otherwise. We can observe that the optimal policy never implements accuracy on both types of error. Hence, whenever investigations can be focussed on assessing more carefully either the negative or the positive effects of the action, resources will be concentrated only on one side of the problem. Turning back to the antitrust example discussed in the introduction, if the innovative activity increases the expected welfare, the enforcer should adopt a protocol of investigation that selectively proceeds with further investigations as long as the interim assessment suggests a social harm, while it stops the investigation as soon as a positive welfare effect can be argued. This procedure allows to reduce type I errors while being biased towards type II errors. Conversely, when the innovative activity is *ex ante* socially harmful the enforcer should follow a protocol that investigates in depth when the interim results suggest a welfare improvement while concluding the inspection (with a negative result) if the preliminary assessment suggests a welfare loss. We argue that these asymmetric protocols of investigation often characterize the way in which antitrust authorities handle cases in practice. Our result suggests that these asymmetric protocols are indeed consistent with the optimal policy.

If we compare the optimal policy in the previous case, where a common probability α was set for both types of errors, and the present one where the enforcer is able to fine tune accuracy on each type of errors, we note that the likelihood of the bad state β now plays no specific role. With a common α the probability of the bad state β determines whether a type I or a type II error was more likely, driving the choice of (common) accuracy; when instead the enforcer can set different levels of accuracies for the two types of errors, the policy can be made implicitly contingent on the state of nature, in the sense that we can decide the optimal accuracy in the good (α_I^*) and in the bad (α_{II}^*) state, no matter how likely the states of nature (and the associated error types) are.

4.3 Enforcer's choices: endogenous enforcement

We now consider the case of endogenous enforcement effort E when accuracy is set separately for the two types of error. The first order conditions to identify the optimal policy $(E, \alpha_I, \alpha_{II})$ are now:

$$\frac{\partial \mathrm{E}W}{\partial E} = [\Delta\mathrm{E}(W) - c\widehat{I}]\frac{\partial \widehat{I}}{\partial E} - g'_E \leqslant 0. \tag{19}$$

$$\frac{\partial \mathrm{E}W}{\partial \alpha_I} = [\Delta\mathrm{E}(W) - c\widehat{I}]\frac{\partial \widehat{I}}{\partial \alpha_I} + g'_I \geqslant 0 \tag{20}$$

and

$$\frac{\partial \mathrm{E}W}{\partial \alpha_{II}} = [\Delta\mathrm{E}(W) - c\widehat{I}]\frac{\partial \widehat{I}}{\partial \alpha_{II}} + g'_{II} \geqslant 0 \tag{21}$$

The three derivatives have the same structure, adding the marginal effect of the policy variables on the innovative activity (*average deterrence*) and its marginal cost. The optimal choice of the policy variables, therefore, depends on the sign of the marginal social value of the innovative activity, $\Delta\mathrm{E}(W) - c\widehat{I}$. Let us start by identifying the relevant cases and guess the features of the optimal policy according to the value of the marginal social value of the innovative activity. As before, the different cases will correspond to regions above, on or below the locus:

$$\underline{W} = \underline{W}_0(0, \frac{1}{2}, \frac{1}{2}, \beta) = \frac{\Pi - (1-\beta)\overline{W}}{\beta}$$

When $\Delta \mathrm{E}(W) - c\widehat{I} > 0$, i.e. below $\underline{W}_0(0, \frac{1}{2}, \frac{1}{2}, \beta)$, condition (19) is negative and no enforcement effort is exerted, i.e. $E^* = 0$; then, no accuracy is implemented, that is $\alpha_I^* = \alpha_{II}^* = \frac{1}{2}$. The same policy should occur as an internal solution for $\Delta \mathrm{E}(W) - c\widehat{I} = 0$, i.e. along the locus $\underline{W}_0(0, \frac{1}{2}, \frac{1}{2}, \beta)$. When instead we are above $\underline{W}_0(0, \frac{1}{2}, \frac{1}{2}, \beta)$ and the marginal social value of the innovative activity is negative, (19) and (21) admit an internal solution while (20) implies a corner solution. Hence, the optimal policy should be $E^* > 0$, $\alpha_I^* = \frac{1}{2}$ and $\alpha_{II}^* \in \left(0, \frac{1}{2}\right)$. The following Proposition establishes the optimal policy in the different regions.[14]

Proposition 6: *In the space $(\underline{W}, \beta)$ we can distinguish the following regions:*

i) When the marginal social value of the innovative activity is non negative, i.e. $\underline{W} \geqslant \underline{W}_0(0, \frac{1}{2}, \frac{1}{2}, \beta)$, the optimal policy entails "laissez faire", i.e. $E^* = 0$, $\alpha_I^* = \alpha_{II}^* = 1/2$.

ii) When the marginal social value of the innovative activity is negative, i.e $\underline{W} < \underline{W}_0(0, \frac{1}{2}, \frac{1}{2}, \beta)$, the optimal policy prescribes a positive enforcement effort and type II accuracy: $E^* > 0$, $\alpha_I^* = 1/2$ and $\alpha_{II}^* \in \left(0, \frac{1}{2}\right)$. Both the enforcement effort and type II accuracy increase when the marginal social value of the innovative activity becomes more negative.

Proof. See the Appendix. ■

Figure 2 illustrates the results.

[Figure 2 about here]

When the innovative activity is *ex ante* welfare enhancing, the optimal policy is aimed at sustaining the innovative effort. With endogenous enforcement the enforcer does not implement any prohibition, i.e. $E^* = 0$ (Proposition 6.i), while in case of an exogenous and positive level of enforcement (Proposition 5.iii) he has to sustain the innovative activity through accuracy by reducing over-enforcement ($\alpha_I^* < 1/2$) and maximizing under-

[14] Again we assume that the cost of enforcement and accuracy are sufficiently convex:

$$g_E'' g_{II}'' \geqslant [\Delta \mathrm{E}(W) - c\widehat{I}]^2 \left(\frac{\partial \widehat{I}}{\partial E \partial \alpha_{II}} \right)^2. \tag{22}$$

enforcement ($\alpha_{II}^* = \frac{1}{2}$). This result qualitatively resembles the analogous comparison between exogenous and endogenous enforcement in the case of common accuracy (Proposition 2.ii and 3.i).

When the marginal social value of the innovative activity is negative, we use both enforcement and type II accuracy to deter the innovative activity, using these two tools as complements. Again, we find here, with a more explicit reference to type II errors, a result qualitatively similar to the case of common accuracy: indeed, the joint use of enforcement and accuracy in that setting (Proposition 3.ii) occurred when, due to the high probability of the bad state, type II errors were relatively more frequent.

The results obtained in the model with different levels of type I and type II accuracy bring to mind qualitatively those derived in the case of a common error probability: since the key objective of the enforcer in our model is sustaining or discouraging the innovative activity, accuracy is set in order to calibrate under and over–enforcement and to affect the innovative activity. When the innovative activity is socially valuable, under-enforcement is welcome while over-enforcement is detrimental. With a common probability of error α, accuracy is driven by whichever of the two effects (states of the world) is more likely: when the innovative activity is welfare decreasing, we improve accuracy (on both types of errors) since under-enforcement comes out as the predominant effect of errors. With different probabilities α_I and α_{II}, instead, we reduce accuracy on type I error and improve accuracy on type II errors in order to maximize over-enforcement and minimize under-enforcement.

5 Conclusions

In this paper we have analyzed the effect of judicial errors on the innovative activity following the approach introduced in Immordino, Pagano and Polo (2006). The traditional model of law enforcement and accuracy assumes that there is a set of privately convenient but socially damaging actions that are illegal, one of which is selected by the private agent by comparing the expected benefits and fine. Marginal deterrence, in this setting, is the key effect.

In our model the agents first have to invest resources in learning and research effort - which we call the innovative activity - and then, if successful, are able to choose a new action that, at the time of the investment, may be welfare enhancing (legal) or reducing (illegal). The enforcement and accuracy policy, determining the probability of being fined, affects the expected profits from the new action and the incentives to exert the innovative activity. The focus of the analysis is therefore shifted to the impact of enforcement on the innovative activity, what we call average deterrence, since it influences the probability of taking the new action whether legal or not. The basic instruments of the enforcer are the level of fines, the enforcement effort, which affects the probability of finding hard evidence on the actions chosen, and the accuracy effort, which reduces the probability of wrongly assessing the social consequences (legality) of the actions. We consider four different environments, corresponding to the cases of exogenous versus endogenous enforcement effort and to the cases of a common accuracy on any type of error versus different levels of type I and type II accuracy.

In this framework we analyze the impact of judicial errors and accuracy and their optimal setting. Type I errors, which imply over-enforcement, reduce the expected profits from the new actions and discourage the innovative activity, while type II errors, through under-enforcement, sustain the incentive to invest in learning. The expected welfare effect of the innovative activity drives the design of the optimal policy: when the innovative activity is *ex ante* welfare enhancing the policy should sustain the innovative activity, while it should reduce the incentives to innovate when this activity is *ex ante* socially damaging.

When innovation is socially desirable, the optimal policy prescribes not to enforce any norm (*laissez faire*) if the enforcement effort is endogenous, or to improve (type I) accuracy in order to reduce over-enforcement. Conversely, when the innovative activity is welfare-reducing it should be discouraged. We can reach this result through (type II) accuracy in order to reduce under-enforcement and by exerting more enforcement effort (as long as it is endogenous).

Our contribution to the literature is twofold. First, we show that, contrary to the usal result in L&E, accuracy may be undesirable. Second, when the enforcer can set accuracy separately for the two types or errors, he will never choose a positive level of accuracy for both types of errors. This corresponds, for instance, to sequential protocols of investigation that deepen the inspection as long as a positive (negative) interim result is obtained while stopping the analysis with a prohibition (approval) if a negative (positive) interim conclusion is reached. We argue that these asymmetric procedures are often observed, for instance in antitrust practices.

Appendix

Proof of Proposition 2. We organize the proof as follows. First, we state the conditions for the existence of the optimal policy as an internal or a corner solution. In the first case we also derive the comparative statics results with respect to the level of the social loss $\underline{W}$ and the level of the enforcement E. Then we identify in the $(\underline{W}, \beta)$ space the regions where the two types of equilibria exist, deriving the properties of the optimal policies at the boundaries of these regions.

The first order conditions for an internal maximum

$$\frac{\partial \mathrm{E}W}{\partial \alpha} = [\Delta \mathrm{E}(W) - c\widehat{I}]\frac{\partial \widehat{I}}{\partial \alpha} + g'_\alpha = 0$$

are satisfied at $\alpha^* \in \left(0, \frac{1}{2}\right)$ whenever the first term is negative, i.e. when $[\Delta \mathrm{E}(W) - c\widehat{I}]$ and $\frac{\partial \widehat{I}}{\partial \alpha}$ are opposite in sign. The second order condition for an internal maximum is satisfied given $\frac{\partial^2 \mathrm{E}W}{\partial \alpha^2} = -c\left(\frac{\partial \widehat{I}}{\partial \alpha}\right)^2 - g''_\alpha < 0$. An internal solution occurs also when $[\Delta \mathrm{E}(W) - c\widehat{I}] = 0$, implying $\alpha^* = \frac{1}{2}$ since $g'_\alpha(\frac{1}{2}) = 0$. When instead $[\Delta \mathrm{E}(W) - c\widehat{I}]$ and $\frac{\partial \widehat{I}}{\partial \alpha}$ have the same sign we have

$$\frac{\partial \mathrm{E}W}{\partial \alpha} = [\Delta \mathrm{E}(W) - c\widehat{I}]\frac{\partial \widehat{I}}{\partial \alpha} + g'_\alpha > 0$$

and a corner solution $\alpha^* = \frac{1}{2}$.

In case of an internal solution we can apply the implicit function theorem obtaining:

$$\frac{d\alpha^*}{d\underline{W}} = -\frac{\partial^2 \mathrm{E}W / \partial \alpha \partial \underline{W}}{\partial^2 \mathrm{E}W / \partial \alpha^2}.$$

Note that $sign\frac{d\alpha^*}{d\underline{W}} = sign\frac{\partial^2 EW}{\partial \alpha \partial \underline{W}} = sign\frac{\partial \widehat{I}}{\partial \alpha}$ implying that $\frac{d\alpha^*}{d\underline{W}} \gtreqless 0 \Leftrightarrow \beta \gtreqless \frac{1}{2}$. Analogously,

$$\frac{d\alpha^*}{dE} = -\frac{\partial^2 \mathrm{E}W / \partial \alpha \partial E}{\partial^2 \mathrm{E}W / \partial \alpha^2}$$

and therefore $sign\frac{d\alpha^*}{dE} = sign\frac{\partial^2 \mathrm{E}W}{\partial \alpha \partial E} = -c\frac{\partial \widehat{I}}{\partial E}\frac{\partial \widehat{I}}{\partial \alpha} = sign\frac{\partial \widehat{I}}{\partial \alpha} \gtreqless 0 \Leftrightarrow \beta \gtreqless \frac{1}{2}$.

Finally, at an internal maximum an infinitesimal change in $\underline{W}$ induces the following effect on the marginal social value of the innovative activity:

$$\frac{d\left[\Delta E(W) - c\widehat{I}(.)\right]}{d\underline{W}} = \left[\frac{\partial \mathrm{E}W}{\partial \underline{W}} - c\frac{\partial \widehat{I}}{\partial \alpha}\frac{\partial \alpha^*}{\partial \underline{W}}\right] = \beta\left[1 - \frac{c\left(\partial \widehat{I}/\partial \alpha\right)^2}{c\left(\partial \widehat{I}/\partial \alpha\right)^2 + g''_\alpha}\right] > 0, \tag{23}$$

while at a corner solution we have

$$\frac{d\left[\Delta E(W) - c\widehat{I}(.)\right]}{d\underline{W}} = \frac{\partial \mathrm{E}W}{\partial \underline{W}} = \beta > 0 \tag{24}$$

Hence, in both cases when the social loss in the bad state is reduced ($\underline{W} \uparrow$) the marginal social value of the innovative activity increases.

We are now able to characterize the optimal policy by studying the following six parameter regions:

i) $\underline{W} = \underline{W}_0(E, \frac{1}{2}, \beta)$ for any β. In this case $[\Delta \mathrm{E}(W) - c\widehat{I}] = 0$ and the internal solution implies $\alpha^* = \frac{1}{2}$. Notice that we have consistency of the optimal policy $\alpha^* = \frac{1}{2}$ and of the value of the parameter $\underline{W} = \underline{W}_0(E, \frac{1}{2}, \beta)$ that induces it.

ii) $\underline{W} > \underline{W}_0(E, \frac{1}{2}, \beta)$ for $\beta \in \left(0, \frac{1}{2}\right)$. Due to (23) we know that $\left[\Delta E(W) - c\widehat{I}(.)\right]$ becomes positive and $\partial \widehat{I}/\partial \alpha$ is negative according to (6). Then, the first order condition (9) is solved as an equality for $\alpha^* \in \left(0, \frac{1}{2}\right)$. Moreover, $\frac{d\alpha^*}{d\underline{W}} < 0$ in an internal solution. When $\underline{W} \underset{+}{\rightarrow} \underline{W}_0(E, \frac{1}{2}, \beta)$ we obtain $\alpha^* \underset{-}{\rightarrow} \frac{1}{2}$ since g'_α is smooth.

iii) $\underline{W} > \underline{W}_0(E, \frac{1}{2}, \beta)$ for $\beta \in (\frac{1}{2}, 1)$. In this region $\left[\Delta E(W) - c\widehat{I}(.)\right] > 0$ due to (24) and $\partial \widehat{I}/\partial \alpha > 0$: hence we have always a corner solution $\alpha^* = \frac{1}{2}$ in this region.

iv) $\underline{W} < \underline{W}_0(E, \frac{1}{2}, \beta)$ for $\beta \in \left(0, \frac{1}{2}\right)$. This case is specular to case *iii)* since $\left[\Delta E(W) - c\widehat{I}(.)\right] < 0$ and $\partial \widehat{I}/\partial \alpha < 0$, implying a corner solution $\alpha^* = \frac{1}{2}$.

v) $\underline{W} < \underline{W}_0(E, \frac{1}{2}, \beta)$ for $\beta \in (\frac{1}{2}, 1)$. This case is specular to case *ii)*, $\left[\Delta E(W) - c\widehat{I}(.)\right] < 0$ and $\partial \widehat{I}/\partial \alpha > 0$. Then, the first order condition (9) is solved as an equality for $\alpha^* \in \left(0, \frac{1}{2}\right)$. Moreover, $\frac{d\alpha^*}{d\underline{W}} > 0$ in an internal solution. When $\underline{W} \underset{-}{\rightarrow} \underline{W}_0(E, \frac{1}{2}, \beta)$ we obtain $\alpha^* \underset{-}{\rightarrow} \frac{1}{2}$ since g'_α is smooth.

vi) $\beta = \frac{1}{2}$ for any $\underline{W}$. When $\beta = \frac{1}{2}$ we have $\partial \widehat{I}/\partial \alpha = 0$ and the first order condition (9) is solved as an equality for $\alpha^* = \frac{1}{2}$, since $g'_\alpha(0) = 0$. Notice that in this case $\frac{d\alpha^*}{d\underline{W}} = 0$ for any $\underline{W}$. ■

Proof of Proposition 3. We follow the same steps as in the proof of Proposition 2. The first order conditions for an internal maximum

$$\frac{\partial \mathrm{E}W}{\partial \alpha} = [\Delta \mathrm{E}(W) - c\widehat{I}]\frac{\partial \widehat{I}}{\partial \alpha} + g'_\alpha = 0$$

and

$$\frac{\partial \mathrm{E}W}{\partial E} = [\Delta \mathrm{E}(W) - c\widehat{I}]\frac{\partial \widehat{I}}{\partial E} - g'_E = 0. \tag{25}$$

are satisfied at $E^* > 0$ and $\alpha^* \in \left(0, \frac{1}{2}\right)$ if $[\Delta \mathrm{E}(W) - c\widehat{I}] < 0$ and $\frac{\partial \widehat{I}}{\partial \alpha} > 0$, i.e. $\beta \in \left(\frac{1}{2}, 1\right)$.

The second order conditions for an internal maximum are satisfied given:

$$\begin{aligned}\frac{\partial^2 EW}{\partial E^2} &= -c\left(\frac{\partial \widehat{I}}{\partial E}\right)^2 - g_E'' < 0 \\ \frac{\partial^2 EW}{\partial \alpha^2} &= -c\left(\frac{\partial \widehat{I}}{\partial \alpha}\right)^2 - g_\alpha'' < 0\end{aligned}$$

and the determinant of the Hessian matrix

$$|H| = \left[c\left(\frac{\partial \widehat{I}}{\partial E}\right)^2 + g_E''\right]\left[c\left(\frac{\partial \widehat{I}}{\partial \alpha}\right)^2 + g_\alpha''\right] - \left[[\Delta \mathrm{E}(W) - c\widehat{I}]\frac{\partial^2 \widehat{I}}{\partial E \partial \alpha} - c\frac{\partial \widehat{I}}{\partial E}\frac{\partial \widehat{I}}{\partial \alpha}\right]^2 > 0$$

if we assume enough convexity as in assumption (11). Applying the implicit function theorem we get:

$$\frac{dE^*}{d\underline{W}} = \frac{\beta}{|H|}\left\{\frac{\partial \widehat{I}}{\partial E}g_\alpha'' + [\Delta \mathrm{E}(W) - c\widehat{I}]\frac{\partial^2 \widehat{I}}{\partial E \partial \alpha}\frac{\partial \widehat{I}}{\partial \alpha}\right\} < 0 \tag{26}$$

and

$$\frac{d\alpha^*}{d\underline{W}} = \frac{\beta}{|H|}\left\{\frac{\partial \widehat{I}}{\partial \alpha}g_E'' + [\Delta \mathrm{E}(W) - c\widehat{I}]\frac{\partial^2 \widehat{I}}{\partial E \partial \alpha}\frac{\partial \widehat{I}}{\partial E}\right\} > 0 \tag{27}$$

since $\frac{\partial^2 \widehat{I}}{\partial E \partial \alpha} = -\frac{(1-2\beta)f}{c} \gtreqless 0 \Leftrightarrow \beta \gtreqless \frac{1}{2}$. We can have also an internal solution $E^* = 0$ and $\alpha^* = \frac{1}{2}$ if $[\Delta \mathrm{E}(W) - c\widehat{I}] = 0$.

If $[\Delta \mathrm{E}(W) - c\widehat{I}] < 0$ and $\frac{\partial \widehat{I}}{\partial \alpha} < 0$, i.e. $\beta \in \left(0, \frac{1}{2}\right)$ we have an internal solution $E^* > 0$ and a corner solution $\alpha^* = \frac{1}{2}$. In this case

$$\frac{dE^*}{d\underline{W}} = \frac{\beta\frac{\partial \widehat{I}}{\partial E}}{c\left(\frac{\partial \widehat{I}}{\partial E}\right)^2 + g_E''} < 0 \tag{28}$$

Finally, if $[\Delta \mathrm{E}(W) - c\widehat{I}] > 0$ we have a corner solution $E^* = 0$ and $\alpha^* = \frac{1}{2}$.

When the social loss in the bad state $\underline{W}$ slightly varies the marginal social value of the innovative activity varies as well:

$$\frac{d\left[\Delta E(W) - c\widehat{I}(.)\right]}{d\underline{W}} = \frac{\partial \Delta \mathrm{E}(W)}{\partial \underline{W}} - c\left[\frac{\partial \widehat{I}}{\partial E}\frac{dE^*}{d\underline{W}} + \frac{\partial \widehat{I}}{\partial \alpha}\frac{d\alpha^*}{d\underline{W}}\right].$$

According to the different (internal or corner) solutions we have $\frac{dE^*}{d\underline{W}} \leqslant 0$ and $\frac{d\alpha^*}{d\underline{W}} \geqslant 0$. When both E^* and α^* are determined as internal solutions we have, by substituting (26) and (27) and simplifying:

$$\frac{d\left[\Delta E(W) - c\widehat{I}(.)\right]}{d\underline{W}} = \beta\left\{g_E'' g_\alpha'' - [\Delta \mathrm{E}(W) - c\widehat{I}]^2\left(\frac{\partial \widehat{I}}{\partial E \partial \alpha}\right)^2\right\} > 0$$

if we assume enough convexity as stated in. (11). When E^* is determined as an internal solution while $\alpha^* = \frac{1}{2}$ is a corner solution we obtain, by substituting (28):

$$\frac{d\left[\Delta E(\underline{W}) - c\widehat{I}(.)\right]}{d\underline{W}} = \beta \frac{g''_E}{c\left(\frac{\partial \widehat{I}}{\partial E}\right)^2 + g''_E} > 0.$$

Finally, when we have a corner solution for $E^* = 0$ and $\alpha^* = \frac{1}{2}$ we get

$$\frac{d\left[\Delta E(\underline{W}) - c\widehat{I}(.)\right]}{d\underline{W}} = \beta > 0.$$

Hence, in all cases, when the social loss in the bad state falls the expected marginal social value of the innovative activity increases taking into account the optimal adjustment of the policy parameters.

We are now able to characterize the optimal policy by studying the following five parameter regions:

i) $\underline{W} = \underline{W}_0(0, \frac{1}{2}, \beta)$ for any β. The first order conditions are solved as an equality for $E^* = 0$ and $\alpha^* = \frac{1}{2}$, since $g'_\alpha(0) = g'_E(0) = 0$ and the second order conditions hold. Hence, we have consistency of the optimal policy and of the value of the parameters $\underline{W} = \underline{W}_0(0, \frac{1}{2}, \beta)$ that induces it;

ii) $\underline{W} > \underline{W}_0(0, \frac{1}{2}, \beta)$ for any β. In this region the marginal social value of the innovative activity is positive and we have a corner solution $E^* = 0$, $\alpha^* = \frac{1}{2}$. In fact (10) is strictly negative implying $E^* = 0$. No enforcement implies $\frac{\partial \widehat{I}}{\partial \alpha} = 0$ so that (9) is solved as an equality $\alpha^* = \frac{1}{2}$;

iii) $\underline{W} < \underline{W}_0(0, \frac{1}{2}, \beta)$ for $\beta \in (0, \frac{1}{2})$. In this region the marginal social value of the innovative activity is negative and $\frac{\partial \widehat{I}}{\partial \alpha} < 0$. Looking at the first order conditions we then have an interior solution for the enforcement $E^* \in (0, 1)$ and a corner solution for the accuracy $\alpha^* = \frac{1}{2}$.

iv) $\underline{W} < \underline{W}_0(0, \frac{1}{2}, \beta)$ for $\beta = \frac{1}{2}$. This particular value of β implies $\frac{\partial \widehat{I}}{\partial \alpha} = 0$ then the first order conditions entails $E^* \in (0, 1)$ and $\alpha^* = \frac{1}{2}$;

v) $\underline{W} < \underline{W}_0(0, \frac{1}{2}, \beta)$ for $\beta \in (\frac{1}{2}, 1)$. In this region $\frac{\partial \widehat{I}}{\partial \alpha} > 0$, this together with a negative social value of the innovative activity implies two interior solutions $E^* \in (0, 1)$ and $\alpha^* \in \left(0, \frac{1}{2}\right)$. In this case $\frac{d\alpha^*}{d\underline{W}} > 0$ and $\frac{dE^*}{d\underline{W}} < 0$, that is, when the social loss become worse and worse ($\underline{W} \downarrow$) the enforcer increases E^* and accuracy ($\alpha^* \downarrow$), using the two instruments as complements. ■

Proof of Proposition 5. Since with a positive enforcement $\frac{\partial \widehat{I}}{\partial \alpha_I} < 0$ and $\frac{\partial \widehat{I}}{\partial \alpha_{II}} > 0$ the first order conditions for an internal maximum

$$\frac{\partial \mathrm{E}W}{\partial \alpha_I} = [\Delta \mathrm{E}(W) - c\widehat{I}]\frac{\partial \widehat{I}}{\partial \alpha_I} + g_I' = 0 \tag{29}$$

and

$$\frac{\partial \mathrm{E}W}{\partial \alpha_{II}} = [\Delta \mathrm{E}(W) - c\widehat{I}]\frac{\partial \widehat{I}}{\partial \alpha_{II}} + g_{II}' = 0, \tag{30}$$

are jointly satisfied iff $[\Delta \mathrm{E}(W) - c\widehat{I}] = 0$ at the optimal policy. In this case $\alpha_I^* = \alpha_{II}^* = \frac{1}{2}$ since $g_I'(0) = g_{II}'(0) = 0$. This case corresponds to $\underline{W} = \underline{W}_0(E, \frac{1}{2}, \frac{1}{2}, \beta)$ as defined in (18). The second order conditions for an internal maximum of both variables are satisfied since:

$$\frac{\partial^2 \mathrm{E}W}{\partial \alpha_I^2} = -c\left(\frac{\partial \widehat{I}}{\partial \alpha_I}\right)^2 - g_I'' < 0,$$

$$\frac{\partial^2 \mathrm{E}W}{\partial \alpha_{II}^2} = -c\left(\frac{\partial \widehat{I}}{\partial \alpha_{II}}\right)^2 - g_{II}'' < 0.$$

and the Hessian matrix determinant:

$$|H| = c\left(\frac{\partial \widehat{I}}{\partial \alpha_I}\right)^2 g_{II}'' + c\left(\frac{\partial \widehat{I}}{\partial \alpha_{II}}\right)^2 g_I'' + g_I'' g_{II}'' > 0.$$

If $[\Delta \mathrm{E}(W) - c\widehat{I}] > 0$ at the optimal policy we have an internal solution for α_I^* and a corner solution for α_{II}^*, while the opposite holds true if $[\Delta \mathrm{E}(W) - c\widehat{I}] > 0$. Turning back to the case $\underline{W} = \underline{W}_0(E, \frac{1}{2}, \frac{1}{2}, \beta)$, totally differentiating the first order conditions and solving the system of equations we obtain:

$$\begin{aligned} \frac{d\alpha_I^*}{d\underline{W}} &= \left[\beta \frac{\partial \widehat{I}}{\partial \alpha_I} g_{II}''\right] \div |H| < 0 \\ \frac{d\alpha_{II}^*}{d\underline{W}} &= \left[\beta \frac{\partial \widehat{I}}{\partial \alpha_{II}} g_I''\right] \div |H| > 0 \end{aligned}$$

Hence, if $\underline{W}$ falls slightly below $\underline{W}_0(E, \frac{1}{2}, \frac{1}{2}, \beta)$ the optimal unconstrained policy would require to increase α_I^* and reduce α_{II}^*: since at $\underline{W} = \underline{W}_0(E, \frac{1}{2}, \frac{1}{2}, \beta)$ the type I probability α_I^* is already at the upper bound $\frac{1}{2}$, it cannot be further increased, and we move to the corner solution $\alpha_I^* = \frac{1}{2}$; conversely, α_{II}^* can be reduced below $\frac{1}{2}$. This corresponds to the equilibrium we have described above when the marginal social value of the innovative activity is negative. Indeed, in the region $\underline{W} < \underline{W}_0(E, \frac{1}{2}, \frac{1}{2}, \beta)$ we have

$$\frac{d\alpha_{II}^*}{d\underline{W}} = -\frac{\partial^2 \mathrm{E}W / \partial \alpha_{II} \partial \underline{W}}{\partial^2 \mathrm{E}W / \partial \alpha_{II}^2} > 0$$

since $\frac{\partial^2 EW}{\partial \alpha_{II}^2} < 0$ for the second order conditions and $\frac{\partial^2 EW}{\partial \alpha_{II} \partial \underline{W}} = \beta \frac{\partial \widehat{I}}{\partial \alpha_{II}} > 0$. Hence, when we decrease $\underline{W}$ below $\underline{W}_0(E, \frac{1}{2}, \frac{1}{2}, \beta)$ the constrained optimum entails $\frac{d\alpha_I^*}{d\underline{W}} = 0$ (corner solution) and $\frac{d\alpha_{II}^*}{d\underline{W}} > 0$ (internal solution). In order to check the consistency of this exercise, let use consider how the marginal social value of the innovative activity varies in this region:

$$\frac{d\left[\Delta E(W) - c\widehat{I}(.)\right]}{d\underline{W}} = \left\{\frac{\partial \mathrm{E}W}{\partial \underline{W}} - c\left[\frac{\partial \widehat{I}}{\partial \alpha_I}\frac{\partial \alpha_I^*}{\partial \underline{W}} + \frac{\partial \widehat{I}}{\partial \alpha_{II}}\frac{\partial \alpha_{II}^*}{\partial \underline{W}}\right]\right\} =$$

$$\beta\left[|H| - c\left(\frac{\partial \widehat{I}}{\partial \alpha_{II}}\right)^2 g_I''\right] = \beta\left[c\left(\frac{\partial \widehat{I}}{\partial \alpha_I}\right)^2 g_{II}'' + g_I'' g_{II}''\right] > 0.$$

Hence, when $\underline{W} < \underline{W}_0(E, \frac{1}{2}, \frac{1}{2}, \beta)$ the marginal social value of the innovative activity $\Delta\mathrm{E}(W) - c\widehat{I}(.)$ becomes negative at the optimal policy $\alpha_I^* = \frac{1}{2}$ and $\alpha_{II}^* \in \left(0, \frac{1}{2}\right)$, and when $\Delta E(W) - c\widehat{I}(.) < 0$ the optimal policy, looking at the first order conditions (16) and (17), entails a corner solution for α_I^* and an internal solution for α_{II}^*. Finally, since $\underline{W} \underset{-}{\to} \underline{W}_0(E, \frac{1}{2}, \frac{1}{2}, \beta)$ implies $\Delta E(W) - c\widehat{I}(.) \underset{-}{\to} 0$ we get $\alpha_{II}^* \underset{-}{\to} \frac{1}{2}$ since g_{II}' is smooth and $g_{II}'(0) = 0$.

A parallel argument can be applied to the case of a departure of $\underline{W}$ above $\underline{W}_0(E, \frac{1}{2}, \frac{1}{2}, \beta)$ proving the results. ■

Proof of Proposition 6. When $\underline{W} = \underline{W}_0(0, \frac{1}{2}, \frac{1}{2}, \beta)$ we have $\Delta\mathrm{E}(W) - c\widehat{I} = 0$ and the three first order conditions are solved as an equality at $E^* = 0$, $\alpha_I^* = \alpha_{II}^* = \frac{1}{2}$ consistently with the definition of the threshold. The second order conditions are

$$\frac{\partial^2 EW}{\partial E^2} = -c\left(\frac{\partial \widehat{I}}{\partial E}\right)^2 - g_E'' < 0$$

$$\frac{\partial^2 EW}{\partial \alpha_I^2} = -c\left(\frac{\partial \widehat{I}}{\partial \alpha_I}\right)^2 - g_I'' < 0$$

$$\frac{\partial^2 EW}{\partial \alpha_{II}^2} = -c\left(\frac{\partial \widehat{I}}{\partial \alpha_{II}}\right)^2 - g_{II}'' < 0$$

and

$$|H_2| = g_I''\left[c\left(\frac{\partial \widehat{I}}{\partial E}\right)^2 + g_E''\right] > 0$$

$$|H_3| = -g_I'' g_{II}''\left[c\left(\frac{\partial \widehat{I}}{\partial E}\right)^2 + g_E''\right] < 0$$

Hence, we have an internal maximum at the locus $\underline{W}_0(0, \frac{1}{2}, \frac{1}{2}, \beta)$. To show that this is unique, let us totally differentiate the marginal social value of the innovative activity with respect to the social loss in the bad state $\underline{W}$:

$$\begin{aligned} d\left[\Delta E(W) - c\widehat{I}(.)\right] &= \left\{\frac{\partial \Delta \mathrm{E}(W)}{\partial \underline{W}} - c\left[\frac{\partial \widehat{I}}{\partial E}\frac{\partial E^*}{\partial \underline{W}} + \frac{\partial \widehat{I}}{\partial \alpha_I}\frac{\partial \alpha_I^*}{\partial \underline{W}} + \frac{\partial \widehat{I}}{\partial \alpha_{II}}\frac{\partial \alpha_{II}^*}{\partial \underline{W}}\right]\right\} d\underline{W} \\ &= \beta\left[1 - \frac{c\left(\frac{\partial \widehat{I}}{\partial E}\right)^2}{c\left(\frac{\partial \widehat{I}}{\partial E}\right)^2 + g_E''}\right] > 0 \end{aligned}$$

as can be easily checked by totally differentiating the first order conditions obtaining $\frac{\partial E^*}{\partial \underline{W}} < 0$ and $\frac{\partial \alpha_I^*}{\partial \underline{W}} = \frac{\partial \alpha_{II}^*}{\partial \underline{W}} = 0$. Hence, when the social loss in the bad state falls below the locus the marginal social value of the innovative activity becomes negative at the optimal policy and vice-versa.

Let us consider now the equilibria in these two regions. For $\underline{W} > \underline{W}_0(0, \frac{1}{2}, \frac{1}{2}, \beta)$ the marginal social value of the innovative activity is positive and we have a corner solution $E^* = 0$, $\alpha_I^* = \alpha_{II}^* = \frac{1}{2}$ since $\frac{\partial \widehat{I}}{\partial \alpha_I} = \frac{\partial \widehat{I}}{\partial \alpha_{II}} = 0$ when $E = 0$.

When $\Delta \mathrm{E}(W) - c\widehat{I} < 0$ we have a corner solution $\alpha_I^* = \frac{1}{2}$ while (19) and (21) admit an internal solution. Checking the second order conditions for E and α_{II} we have $\frac{\partial^2 EW}{\partial E^2} < 0$ and $\frac{\partial^2 EW}{\partial \alpha_{II}^2} < 0$. The Hessian matrix determinant is:

$$|H_2| = \left[c\left(\frac{\partial \widehat{I}}{\partial E}\right)^2 + g_E''\right]\left[c\left(\frac{\partial \widehat{I}}{\partial \alpha_{II}}\right)^2 + g_{II}''\right] - \left[[\Delta \mathrm{E}(W) - c\widehat{I}]\frac{\partial^2 \widehat{I}}{\partial E \partial \alpha_{II}} - c\frac{\partial \widehat{I}}{\partial E}\frac{\partial \widehat{I}}{\partial \alpha_{II}}\right]^2$$

that is positive, as required, in a left neighborhood of $\underline{W}_0(0, \frac{1}{2}, \frac{1}{2})$ and, for $\underline{W} < \underline{W}_0(0, \frac{1}{2}, \frac{1}{2}, \beta)$, when, g_E'' and g_{II}'' are sufficiently large. More formally, we can assume (22). Finally, the comparative statics with respect to the social loss in the bad state gives

$$\frac{dE^*}{d\underline{W}} = \beta \frac{\frac{\partial \widehat{I}}{\partial E} g_{II}'' + [\Delta \mathrm{E}(W) - c\widehat{I}]\frac{\partial^2 \widehat{I}}{\partial E \partial \alpha_{II}}\frac{\partial \widehat{I}}{\partial \alpha_{II}}}{|H_2|} < 0$$

and

$$\frac{d\alpha_{II}^*}{d\underline{W}} = \beta \frac{\frac{\partial \widehat{I}}{\partial \alpha_{II}} g_E'' + [\Delta \mathrm{E}(W) - c\widehat{I}]\frac{\partial^2 \widehat{I}}{\partial E \partial \alpha_{II}}\frac{\partial \widehat{I}}{\partial \alpha_E}}{|H_2|} > 0$$

Hence, when the social loss in the bad state gets worse ($\underline{W} \downarrow$) the enforcement effort and the type II accuracy are increased, showing a complementarity relationship. ■

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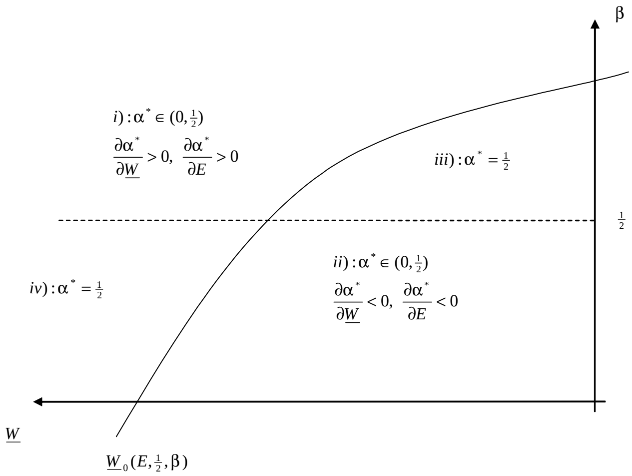

Figure 1: Optimal common accuracy – Proposition 2

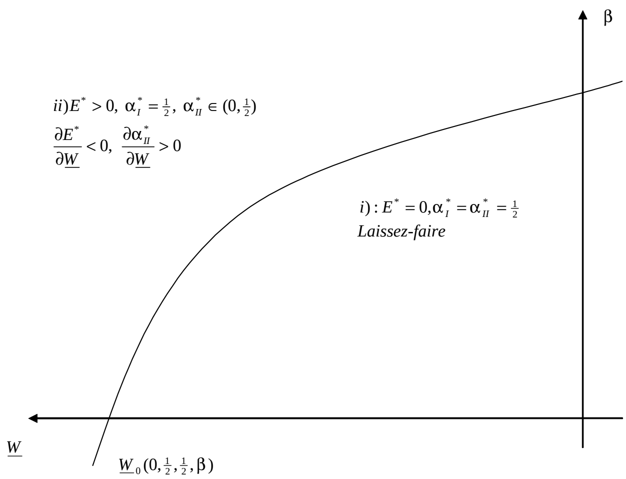

Figure 2 – Optimal enforcement and type-I and type-II accuracies – Proposition 6