Modeling Observed Inflation Expectations

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Abstract

We ask the following questions: (1) Can a standard Christiano et al. (2005)/ Smets and Wouters (2003) -type model, which can supposedly explain macro variables reasonably well, also describe the evolution of inflation expectations, as measured by forecasts from the Survey of Professional Forecasters? (2) If we augment the model with a plausible feature, namely imperfect information about the inflation target on the part of the agents as in Erceg and Levin (2003), do we achieve a better fit on both counts – macro variables and expectations? We find that the answer to both questions is negative: Standard medium scale DSGE models have difficulties explaining the evolution of inflation expectations, and that the fit worsens when we introduce imperfect information.

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1 Introduction

This paper uses inflation expectations as an observable in the estimation of a DSGE model, along with a standard set of macro variables. We ask the following questions:

1. Can a model with several nominal and real rigidities along the lines of Christiano et al. (2005), Smets and Wouters (2003), and Smets and Wouters (2007), which can supposedly explain macro variables reasonably well (see Smets and Wouters (2007)), also describe the evolution of inflation expectations?

2. If we augment the model with a plausible feature, namely imperfect information about the inflation target on the part of the agents as in (Erceg and Levin (2003)), do we achieve a better fit on both counts – macro variables and expectations?

   If we augment the model with a plausible feature, namely imperfect info about the inflation target on the part of the agents, do we achieve a better fit on both counts – macro variables and expectations?

We find that the answer to both questions is negative: Standard medium scale DSGE models have difficulties explaining the evolution of inflation expectations, and that the fit substantially worsens when we introduce imperfect information.

What is the purpose of using inflation expectations as observables in DSGE model?

I) Model comparison: Inflation expectations help discriminate across models.

- A stated rationale for imperfect information/learning models is that they offer a more plausible mechanism for expectation formation – one where the agents are not omniscient about all aspects of the economy but need to learn about some of its features. Given these premises, it seems reasonable to expect that these models should be better at describing the evolution of expectation – hence expectations are important discriminating evidence.

- Yet observed expectations are rarely formally used in previous literature (e.g., Milani (2007)).

- In particular, this paper is motivated by the literature on the time-variation in the policy makers' inflation targets. There are models with perfect info (Smets and Wouters (2003)), and others with imperfect info (Erceg and Levin (2003)). Finding out which of the two better describes observed inflation expectations seems a good way to discriminate between the two.

II) Term structure modeling: Inflation expectations are allegedly important in determining the term structure of interest rates.

- This paper makes no attempt to explain the term structure directly since we use a linear model.[1]

- But the model comparison exercise done here can be helpful indirectly in investigating how one should formulate expectation formation: Models that have a hard time generating observed inflation expecations may not be too helpful in understanding the term structure of interest rates.

III) Signal extraction: The econometrician has a relatively poor signal about the state of the economy. Agents have a richer information set. Measuring expectations is a way to exploit such information set.[2]

- This information can be exploited for forecasting . . .

[1]There are attempts to use DSGE models to explain the terms structure, e.g. Swanson and Rudebusch '08.

[2]Following the FAVAR methodology (Bernanke and Boivin '03, Bernanke, Boivin, and Eliasz '05) there are some attempts to combine factor and DSGE models with the goal of incorporating as much of the available data as possible (Giannone, Monti and Reichlin '08, Boivin and Giannoni '08). We take a different route and model expectations explicitly.

- ... and shocks identification: Using expectations as observables can lead to different 'histories' for what has happened: What may appear to be a shock from the perspective of the econometrician may not be at all an innovation from the perspective of the agents.

Note: Unlike Erceg and Levin (2003), this paper is *not* about the Great Deflation. We are interesting in assessing which model best describes the evolution of inflation expectation in the post deflation period (that is, post 1984) – a period where allegedly the policy regime has not changed. Investigating the Great Deflation, while very interesting, involves issues of changes in policy regimes that we do not address at this stage.

2 Model

The economy is described by a medium-scale New Keynesian model with price and wage rigidities, capital accumulation, investment adjustment costs, variable capital utilization, and habit formation. The model is based on work of Smets and Wouters (2003), Smets and Wouters (2007), and Christiano et al. (2005). The specific version is taken from DelNegro et al. (2007). For brevity we only present the log-linearized equilibrium conditions and refer the reader to the above referenced papers for the derivation of these conditions from assumptions on preferences and technologies.

Aggregate Supply: Firms. The economy is populated by a continuum of firms that combine capital and labor to produce differentiated intermediate goods. These firms have access to the same Cobb-Douglas production function with capital elasticity α and total factor productivity Z_t. Total factor productivity is assumed to be non-stationary, and its growth rate $z_t = \ln(Z_t/Z_{t-1})$ follows the autoregressive process:

$$z_t = (1 - \rho_z)\gamma + \rho_z z_{t-1} + \sigma_z \epsilon_{z,t}. \quad (1)$$

Output, consumption, investment, capital, and the real wage can be detrended by Z_t. In terms of the detrended variables the model has a well-defined steady state.

All variables that appear subsequently are expressed as log-deviations from this steady state.

The intermediate goods producers hire labor and rent capital in competitive markets and face identical real wages, w_t, and rental rates for capital, r_t^k. Cost minimization implies that all firms produce with the same capital-labor ratio

$$k_t - L_t = w_t - r_t^k \tag{2}$$

and have marginal costs

$$mc_t = (1-\alpha)w_t + \alpha r_t^k. \tag{3}$$

The intermediate goods producers sell their output to perfectly competitive final good producers, which aggregate the inputs according to a CES function. Profit maximization of the final good producers implies the following demand curve

$$\widehat{y}_t(j) - \widehat{y}_t = -\left(1 + \frac{1}{\lambda_f e^{\widetilde{\lambda}_{f,t}}}\right)(p_t(j) - p_t). \tag{4}$$

Here $\widehat{y}_t(j) - \widehat{y}_t$ and $p_t(j) - p_t$ are quantity and price for good j relative to quantity and price of the final good. The price p_t of the final good is determined from a zero-profit condition for the final good producers. We assume that the price elasticity of the intermediate goods is time-varying. Since this price elasticity affects the mark-up that intermediate goods producers can charge over marginal costs, we refer to $\widetilde{\lambda}_{f,t}$ as mark-up shock. Following Calvo (1983), we assume that in every period a fraction of the intermediate goods producers ζ_p is unable to re-optimize their prices. A fraction ι_p of these firms adjust their prices mechanically according to lagged inflation, while the remaining fraction $1-\iota_p$ adjusts to steady state inflation π^*. All other firms choose prices to maximize the expected discounted sum of future profits, which leads to the Phillips curve:

$$\pi_t = \frac{\beta}{1+\iota_p\beta}I\!E_t[\pi_{t+1}] + \frac{\iota_p}{1+\iota_p\beta}\pi_{t-1} + \frac{(1-\zeta_p\beta)(1-\zeta_p)}{\zeta_p(1+\iota_p\beta)}mc_t + \frac{1}{\zeta_p}\lambda_{f,t}, \tag{5}$$

where π_t is inflation and β is the discount rate.[3] Our assumption on the behavior of firms that are unable to re-optimize their prices implies the absence of price

[3] We used the following re-parameterization: $\lambda_{f,t} = [(1-\zeta_p\beta)(1-\zeta_p)\lambda_f/(1+\lambda_f)(1+\iota_p\beta)]\widetilde{\lambda}_{f,t}$.

dispersion in the steady state. As a consequence, we obtain a log-linearized aggregate production function of the form

$$\widehat{y}_t = (1-\alpha)L_t + \alpha k_t. \tag{6}$$

Equations (3), (2), and (6) imply that the labor share lsh_t equals marginal costs in terms of log-deviations: $lsh_t = mc_t$.

Aggregate Demand: Households. There is a continuum of households with identical preferences, which are separable in consumption, leisure, and real money balances. Households' preferences display (internal) habit formation in consumption, that is, period t utility is a function of $\ln(C_t - hC_{t-1})$. Households supply monopolistically differentiated labor services. These services are aggregated according to a CES function that leads to a demand elasticity $1 + 1/\lambda_w$ (see Equation (4)). The composite labor services are then supplied to the intermediate goods producers at real wage w_t. To introduce nominal wage rigidity, we assume that in each period a fraction ζ_w of households is unable to re-optimize their wages. A fraction ι_w of these households adjust their $t-1$ nominal wage by $\pi_{t-1}e^{\gamma}$, where γ represents the average growth rate of the economy, while the remaining fraction $1-\iota_p$ adjusts to steady state wage growth $\pi^* e^{\gamma}$. All other households re-optimize their wages. First-order conditions imply that

$$\begin{aligned} \tilde{w}_t &= \zeta_w \beta \mathbb{E}_t \Big[\tilde{w}_{t+1} + \Delta w_{t+1} + \pi_{t+1} + z_{t+1} - \iota_w \pi_{t-1} \Big] \\ &\quad + \frac{1-\zeta_w \beta}{1+\nu_l(1+\lambda_w)/\lambda_w} \left(\nu_l L_t - w_t - \xi_t + \frac{1}{1-\zeta_w \beta} \phi_t \right), \end{aligned} \tag{7}$$

where $\tilde{w}_t$ is the optimal real wage relative to the real wage for aggregate labor services, w_t, and ν_l would be the inverse Frisch labor supply elasticity in a model without wage rigidity ($\zeta_w = 0$) and differentiated labor. Moreover, ξ_t denotes the marginal marginal utility of consumption defined below and ϕ_t is a preference shock that affects the intratemporal substitution between consumption and leisure. The real wage paid by intermediate goods producers evolves according to

$$w_t = w_{t-1} - \pi_t - z_t + \iota_w \pi_{t-1} + \frac{1-\zeta_w}{\zeta_w} \tilde{w}_t. \tag{8}$$

Households are able to insure the idiosyncratic wage adjustment shocks with state contingent claims. As a consequence they all share the same marginal utility of consumption ξ_t, which is given by the expression:

$$(e^{\gamma} - h\beta)(e^{\gamma} - h)\xi_t = -(e^{2\gamma} + \beta h^2)c_t + \beta h e^{\gamma} I\!\!E_t[c_{t+1} + z_{t+1}] + he^{\gamma}(c_{t-1} - z_t), \quad (9)$$

where c_t is consumption. In addition to state-contingent claims households accumulate three types of assets: one-period nominal bonds that yield the return R_t, capital $\bar{k}_t$, and real money balances.[4]

The first order condition with respect to bond holdings delivers the standard Euler equation:

$$\xi_t = I\!\!E_t[\xi_{t+1}] + R_t - I\!\!E_t[\pi_{t+1}] - I\!\!E_t[z_{t+1}]. \quad (10)$$

Capital accumulates according to the following law of motion:

$$\bar{k}_t = (2 - e^{\gamma} - \delta)\left[\bar{k}_{t-1} - z_t\right] + (e^{\gamma} + \delta - 1)i_t, \quad (11)$$

where i_t is investment, δ is the depreciation rate of capital. Investment in our model is subject to adjustment costs, and S'' denotes the second derivative of the investment adjustment cost function at steady state. Optimal investment satisfies the following first-order condition:

$$i_t = \frac{1}{1+\beta}\left[i_{t-1} - z_t\right] + \frac{\beta}{1+\beta} I\!\!E_t[i_{t+1} + z_{t+1}] + \frac{1}{(1+\beta)S''e^{2\gamma}}(\xi^k_t - \xi_t), \quad (12)$$

where ξ^k_t is the value of installed capital and evolves according to:

$$\xi^k_t - \xi_t = \beta e^{-\gamma}(1-\delta) I\!\!E_t\left[\xi^k_{t+1} - \xi_{t+1}\right] + I\!\!E_t\left[(1-(1-\delta)\beta e^{-\gamma})r^k_{t+1} - (R_t - \pi_{t+1})\right]. \quad (13)$$

Capital utilization u_t in our model is variable and r^k_t in the previous equation represents the rental rate of effective capital $k_t = u_t + \bar{k}_{t-1}$. The optimal degree of utilization is determined by

$$u_t = \frac{r^k_*}{a''} r^k_t. \quad (14)$$

[4]Since preferences for real money balances are assumed to be additively separable and monetary policy is conducted through a nominal interest rate feedback rule, money is block exogenous and we will not use the households' money demand equation in our empirical analysis.

Here a'' is the derivative of the per-unit-of-capital cost function $a(u_t)$ evaluated at the steady state utilization rate. The aggregate resource constraint is given by:

$$\widehat{y}_t = (1 + g_*)\left[\frac{c_*}{y_*}c_t + \frac{i_*}{y_*}\left(i_t + \frac{r_*^k}{e^\gamma - 1 + \delta}u_t\right)\right] + g_t. \tag{15}$$

Here c_*/y_* and i_*/y_* are the steady state consumption-output and investment-output ratios, respectively, and $g_*/(1 + g_*)$ corresponds to the government share of aggregate output. The process g_t can be interpreted as exogenous government spending shock. It is assumed that fiscal policy is passive in the sense that the government uses lump-sum taxes to satisfy its period budget constraint. Finally, all stochastic processes described above are assumed to be AR(1) processes with normally distributed errors.

Monetary Policy. The central bank follows a standard feedback rule:

$$R_t = \rho_R R_{t-1} + (1 - \rho_R)\left(\psi_1 \pi_t - \psi_1 \pi_t^* + \psi_2 \hat{y}_t\right) + \sigma_r \epsilon_{R,t}, \tag{16}$$

where $\epsilon_{R,t}$ is i.i.d. and the inflation target π_t^*, defined in log-deviations from its nonstochastic steady state π^*, evolves according to

$$\pi_t^* = \rho_{\pi^*}\pi_{t-1}^* + \sigma_P \epsilon_{P,t}, \tag{17}$$

where $0 < \rho_* < 1$ and $\epsilon_{P,t}$ is an i.i.d. shock. Under perfect information, agents are able to observe π_t^* while under imperfect information it has to be inferred from the observed interest rate behavior. Agents are assumed to know the policy rule (16). They solve a signal extraction problem from the observed Taylor rule residual

$$\tilde{\pi}_t = \pi_t^* + \sigma_T \epsilon_{R,t}$$

where

$$\sigma_T = \frac{\sigma_r}{(1 - \rho_R)\,\psi_1}.$$

Private agents' forecast of the inflation target is

$$\pi_{t+s|t}^* = \rho_*^s \pi_{t|t}^*$$

It is obtained using the steady state Kalman filter

$$\pi^*_{t+1|t} = \rho_{\pi^*}\pi^*_{t|t-1} + \bar{K}\left(\tilde{\pi}_t - \pi^*_{t|t-1}\right),$$

where

$$\bar{K} = \rho_{\pi^*}\frac{V(\frac{\sigma_P}{\sigma_T}, \rho_{\pi^*})}{1 + V(\frac{\sigma_P}{\sigma_T}, \rho_{\pi^*})}$$

is the steady state Kalman gain coefficient and $\sigma_T^2 V(\frac{\sigma_P}{\sigma_T}, \rho_{\pi^*}) = E_t\left(\pi^*_t - \pi^*_{t|t-1}\right)^2$ defines the uncertainty regarding the inflation target.[5]

We also consider the alternative law of motion for inflation target π^*_t proposed in Gurkaynak et al. (2005):

$$\pi^*_t = \rho_* \pi^*_{t-1} + \chi\pi_{t-1} + \sigma_P \epsilon_{P,t}. \tag{18}$$

As above agents know the policy rule and the evolution of the unobserved inflation target. The forecast of the unobserved inflation target $\pi^*_{t+1|t}$ is obtained by the steady state Kalman filter

$$\pi^*_{t+1|t} = \rho_*\pi^*_{t|t-1} + K\left(\tilde{\pi}_t - \pi^*_{t|t-1}\right) + \chi\pi_t$$

where

$$K = \rho_* P\left(P + \sigma_R^2\right)^{-1}$$

is the kalman gain and P solves

$$P = \rho_*^2\left[P - P\left(P + \sigma_R^2\right)^{-1} P\right] + \sigma_P^2.$$

State-Space Representation of the DSGE Model. We use the method in Sims (2002) to solve the log-linear approximation of the DSGE model. We collect

[5] In detail, $\sigma_T^2 V(\frac{\sigma_P}{\sigma_T}, \rho_{\pi^*}) = P = E_t\left(\pi^*_t - \pi^*_{t|t-1}\right)^2$ solves

$$P = \rho_*^2\left[P - P\left(P + \sigma_T^2\right)^{-1} P\right] + \sigma_P^2.$$

The solution yields

$$V(\frac{\sigma_P}{\sigma_T}, \rho_{\pi^*}) = \frac{-\left[1 - \rho_*^2 - (\sigma_P/\sigma_T)^2\right] + \sqrt{\left[1 - \rho_{\pi^*}^2 - (\sigma_P/\sigma_T)^2\right]^2 + 4\left(\sigma_P/\sigma_T\right)^2}}{2}.$$

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all the DSGE model parameters in the vector θ, stack the structural shocks in the vector ϵ_t, and derive a state-space representation for our vector of observables y_t, which is composed of the transition equation:

$$s_t = \mathcal{T}(\theta)s_{t-1} + \mathcal{R}(\theta)\epsilon_t, \tag{19}$$

which summarizes the evolution of the states s_t, and of the measurement equations:

Real output growth (%, annualized)	$400(\ln Y_t - \ln Y_{t-1})$	$=$	$400(\widehat{y}_t - \widehat{y}_{t-1} + z_t)$
Hours (%)	$100 \ln L_t$	$=$	$100(L_t + \ln L^{adj})$
Labor Share (%)	$100 \ln lsh_t$	$=$	$100(L_t + w_t - \widehat{y}_t + \ln lsh_*)$
Inflation (%,annualized)	$400(\ln P_t - \ln P_{t-1})$	$=$	$400(\pi_t + \ln \pi_*)$
Interest Rates (%,annualized)	$400 \ln R_t$	$=$	$400(R_t + \ln R_*)$,
Inflation Expectations (%,annualized)	$\pi_t^{O,t+k}$	$=$	$400(\mathbb{E}_t^{dsge}[\pi_{t+k}] + \ln \pi_*)$

where LS_*, π_*, and R_* are the steady states of the labor share, the inflation rate, and the nominal interest rate, respectively, and where in the last equation $\pi_t^{O,t+k}$ represents the observed k periods ahead inflation expectations and $\mathbb{E}_t^{dsge}[.]$ are the expectations obtained from the DSGE model. The parameter L^{adj} captures the units of measured hours. It can be viewed as a re-parameterization of the steady state associated with the time-varying preference parameter ϕ_t that appears in the households' utility function.

3 Issues with Modeling Inflation Expectations

Several issues arise in modeling inflation expectation. I) Do SPF forecasts really capture agents' expectations? Why not using other measures of expectations?

- We use SPF forecasts as a measure of inflation expectations 4-quarters ahead – this is the measure most commonly used in the literature. We also check robustness to Blue Chip forecasts as well.

II) Data revisions.

- We try to address the issue of data revisions by showing the robustness of the results when we CPI as a measure of inflation (as opposed to the more commonly used GDP deflator): CPI non-seasonally adjusted is never revised; CPI seasonally adjusted has revisions, but these are fairly small compared to those for the GDP deflator. See Figure 1.

III) Information synchronization.

- SPF forecasts are generated in the middle of the quarter. SPF forecasters therefore have partial information about the state of the economy in the current quarter. We deal with this issue by checking the robustness of the results to different assumptions regarding the timing of the agents' information set. The benchmark results are obtained assuming that observed expectations are formed using current quarter information. The alternative assumption, which we call "Lagged Information" specification, is that the forecasters are only endowed with information up to the previous quarter.

IV) Heterogeneous expectations.

- ...

The other observables are: Output growth (log differences, quarter-to-quarter, in %); hours worked (log, in %); labor share (log, in %); inflation (annualized, in %, we use either GDP deflator and CPI, depending on the the corresponding inflation expectation measure); nominal interest rate (annualized, in %). See Appendix A for details. We use 97 quarters of data spanning the Volcker-Greenspan period: 1984Q2 to 2008Q2.

4 Comparing Perfect and Imperfect Information Models of Time-Varying Inflation Target

4.1 Prior Choice and Prior Predictive Checks

We first discuss the priors for the parameters of the policy rule (16) and the associated law of motion for the inflation target π_t^* (17) – see Table 1. These are the key parameters in terms of this application:

- Priors for the responses to inflation (ψ_1) and the output gap (ψ_2) in the policy rule, persistence (ρ_r), and steady state inflation target (π^*) are as in DelNegro and Schorfheide (2008).

- Prior on variance of i.i.d. policy shocks σ_r is a bit lower than usual because we have additional source of policy shocks, π_t^*. (Prior standard deviations are chosen so that overall variance of endogenous variables is roughly close to that observed in the presample 1959Q3-1984Q1)

- Key priors are those on persistence and standard deviation of the innovation to π_t^* process (they determine the Kalman gain in the II model). We follow Erceg and Levin (2003) and make the process followed by π_t^* very persistent (ρ_{π^*} prior centered at .95 with small standard deviation). In the Bechmark prior the prior on σ_{π^*} is independent from all other parameters, and is centered at .05, and is fairly loose.

- An alternative prior ("Signal-to-Noise Ratio Prior") places a prior directly on the Signal-to-Noise ratio (and hence induces dependence between σ_{π^*}, σ_r, ψ_1 and ρ_r) and is centered at the value that delivers a Kalman gain of approximately .13, the value calibrated by Erceg and Levin.

Priors on nominal rigidities parameters (top panel of Table 2): To check robustness to the degree of nominal rigidities in the economy we consider two priors (as in DelNegro and Schorfheide (2008)): "Low Rigidities" (loosely calibrated at Bils and Klenow values of average duration less than 2 quarters), and "High Rigidities" (duration about 4 quarters)

Priors on remaining parameters (bottom panel of Table 2): Priors on "Endogenous Propagation and Steady State" chosen as in DelNegro and Schorfheide (2008). Specifically, The prior for the habit persistence parameter h is centered at 0.7, which is the value used by Boldrin et al. (2001). These authors find that $h = 0.7$ enhances the ability of a standard DSGE model to account for key asset market statistics. The prior for a'' implies that in response to a 1% increase in the return to capital, utilization rates rise by 0.1 to 0.3%. These numbers are considerably smaller than the one used by Christiano et al. (2005). The 90% interval for the prior distribution on ν_l implies that the Frisch labor supply elasticity lies between 0.3 and 1.3, reflecting the micro-level estimates at the lower end, and the estimates of Kimball and Shapiro (2003) and Chang and Kim (2006) at the upper end. We use a pre-sample of observations from 1959Q3-1984Q1 to choose the prior means for the parameters that determine steady states.

The priors on standard deviations and autocorrelations are chosen so that overall variance and autocorrelations of endogenous variables is roughly close to that observed in the presample 1959Q3-1984Q1 (see Table 3). Table 3 also shows that although we use the same prior for both the models under consideration – the perfect (PI) and imperfect information (II) model – the prior predictive statistics are fairly similar across models. If anything, the prior chosen slightly disfavors the perfect information model, for which the interest rate displays too much volatility and not enough persistence relative to the data.

4.2 Model Comparison Results

Table 4 shows the log marginal likelihood for three models: Imperfect Information, Perfect Information, and the model with constant inflation target ("Fixed π^*"). For all models we use the Benchmark prior. The Dataset with Expectations uses the SPF 4-quarters ahead median forecast for the GDP deflator. For these results we assume that the expectations are generated using current quarter information.

Table 4 shows that for the dataset without expectations the II model sizably outperforms the other two (PI and "Fixed π^*"): The difference in log marginal likelihood is larger than 10. We argue that this finding is consistent with Erceg and Levin (2003), who claim that the model with imprefect information has more realistic properties in terms of the dynamics of inflation and output than the perfect information model. When SPF inflation expectations are included among the observables however, the Perfect Information model with time-varying π^* performs significantly better than both the "Fixed π^*" and, most importantly, the II model. In the remainder of the section we will provide an intuition for this result, and in the following section we will assess its robustness.

The marginal likelihood is the likelihood of observing the data under model $\mathcal{M}_i$, and is computed as the integral of the likelihood with respect to the prior:

$$p(Y|\mathcal{M}_i) \; = \; \int \underbrace{p(Y| \; \theta, \mathcal{M}_i)}_{\text{Likelihood}} \; \underbrace{p(\theta, \mathcal{M}_i)}_{\text{Prior}} \, d\theta$$

This suggests that a look at in-sample forecast errors (computed using the prior distribution for the parameters) may provide some insights for the model comparison results. Table 5 shows the median in-sample forecast errors obtained from the Kalman filter. All models (II: imperfect information, and PI: perfect information) use the same prior (*Benchmark* prior) for the DSGE parameters. The difference between the *w/o Exp* (without Inflation Expectations) and *w Exp* (with Inflation Expectations) columns is that for the latter inflation expectations are used as an additional observable in the measurement equation. For the dataset without expectations the ranking of the forecast errors between the II and PI models is not all too clear: the II model does better in terms of interest rates forecasts, but worse for output growth. The other RMSEs are very close to each other. When inflation expectations are included, we observe that: 1) for both models the RMSEs increase, with the exceptions of that for inflation. The deterioration of in-sample fit is much larger for the II model, however; 2) the PI model now performs better than the II model in all dimensions, except for interest rates where the RMSEs are basically the same.

Including inflation expectations among the observables worsens the fit of the model for all other variables (except inflation), and the deterioration is quite dramatic for the imperfect information model. In order to understand this result we ask what kind of inflation expectations the two models generate whenever actual inflation expectations are not among the observables. Figure 3 plots the projections for the 4-quarter ahead inflation forecasts generated by the II model (black solid) and the PI model (gray solid). Specifically, the time t inflation expectations projections are generated as follows: i) We provide the econometrician with time $t-1$ information on all observables (except inflation expectations), which she uses to obtain the time $t-1$ filtered states in the DSGE model's law of motion (19); ii) The law of motion (19) is also used to generate forecasts of the time t states and hence, via the measurement equation (20), of time t 4-quarter ahead inflation expectations. This exercise is performed using the prior distribution for the parameters (the bands are fairly small and for expositional simplicity we focus on the posterior mean). We use the prior distribution both to make the results comparable with those in Table 5, where we compute the a-prior forecast errors, and because we want to use the same parameter draws for both models.

To the extent that these projections of inflation expectations are roughly in line with the observed data, including measured expectations among the observables is unlikely to change the estimates of the states, and hence the forecasts for the other variables. However, if there is a large discrepancy between a model's forecasts of inflation expectations and what we observe in the data, we expect both the estimates of the states and the forecasts of the other variables to change substantially following the addition of measured expectations to the list of observables. Figure 3 also plots the actual inflation expectation data – namely, the SPF 4-quarters ahead median forecast for the GDP deflator (red dashed-and-dotted) – along with the projections. It is clear that the inflation forecasts generated by the II model are very much at odds with the data, especially in the early and late part of the sample. Those generated from the PI model are far from being fully consistent with the SPF forecasts, but are at least in the ballpark.

The above results provide some explanation for the findings in Table 5, and ultimately for the model comparison results in Table 4. But why are forecasts generated by the II model so grossly inconsistent with the SPF forecasts? Figure 4 plots the interest rate projections for the 4-quarter ahead inflation forecasts generated by the II model (black solid) and the PI model (gray solid).[6] The interest rates projections from the II and PI models are not very different from each other. Both models produce forecast errors, and these appear to be persistent, especially in the early and late part of the sample. If anything, the forecast errors from the PI model appear to be larger, consistently with the RMSEs results. However, the effect of *persistent* errors on the inflation forecasts are quite different for the two models: for the II model, a persistent positive (negative) error in predicting the interest rate is an indication that the Central Bank has lowered (raised) its target, and has a dramatic impact on expectations. But in reality expectations do not change that much – hence we have a sort of "curse of anchored expectations" for the II model.

Figure 5 plots the projections for the 4-quarter ahead inflation forecasts generated by the II model (black solid) and the PI model (gray solid) when inflation expectations are *included* among the observables. In forming the projections we still use the prior distribution for the DSGE model parameters. Figure 5 shows that when SPF expectations are included among the observables, the II model's one-period ahead projection are roughly in line with those from the data, even using the prior. However, in trying to correctly predict SPF forecasts the II model generates worse forecasts in all other dimensions (except inflation), as we have seen from Table 5.

[6] As before, the projections are computed using time $t-1$ information and are generated using the prior distribution for the parameters, and without including inflation expectations among the observables.

4.3 Robustness to the Choice of Priors, Datasets, Timing Conventions, and Policy Rules

This section investigates the robustness of the model comparison results to the choice of priors, datasets, timing conventions, and policy rules. Line (1) of Table 6 reproduces the marginal likelihood results for the II and PI models from Table 4 for the sake of comparison. Lines (2) and (3) report the model comparison results under the "'High Nominal Rigidities" prior and "Signal-to-Noise Ratio" prior described in section 4.1, respectively. We find that the "High Nominal Rigidities" prior favors the PI model and penalizes the II model. Using the "Signal-to-Noise Ratio" prior makes little difference.

Lines (4), (5), and (6) show the log marginal likelihood for the two models under different timing assumptions ("Lagged Information" specification), inflation measure ("SPF CPI "), and sample ("1980Q1 Sample"). The "Benchmark" results are obtained assuming that SPF expectations are formed using current quarter information. Under the "Lagged Information" specification the forecasters in the SPF Survey are only endowed with information up to the previous quarter. Results are robust to timing assumptions and choice of the inflation measure. In fact, the gap between the marginal likelihoods for the PI and II models widens substantially whenever we use CPI (which is less subject to revisions) as opposed to the GDP Deflator, or the alternative timing assumption. For the sample starting in 1980Q1 the two models are comparable: Of course one issue with these latter results is that we are assuming that policy has not changed since 1980Q1, which is a questionable assumption. The issue of which model bests describes the disinflation period, which is the one investigated in Erceg and Levin (2003), is very interesting and worth investigating more in future drafts of the paper.

Lines (7), (8), and (9) report the model comparison results under different specifications of the policy rule, where the policy makers target output growth as opposed to the output gap ("Output Growth"), a four-quarter moving average of inflation as opposed to current inflation ("4Q Inflation"), or where the the law of motion for the inflation target follows the rule suggested by Gurkaynak et al. (2005) ("GSS"). Under this rule the marginal likelihood gap between the II and PI models stays roughly constant or increases. Under the rule proposed by Gurkaynak et al. (2005) the gap narrows, but this is mainly because under this rule the fit of the PI model worsens somewhat.

4.4 Posterior Estimates and Variance Decomposition

Table 7 shows the posterior mean, the 5th and 95th percentiles of the posterior distribution of selected parameters for the II and the PI models for the dataset with expectations. The posterior estimates in the II model indicate that the best fitting parameter configurations are those where the "learning" component is virtually shut down by making the σ_r (standard deviation of temporary shocks) very large, therefore making the signal to noise ratio small. The persistence of the policy rule is also small in the model. This is needed to generate large forecast errors, consistently with the large values for σ_r. Finally, note that the model does not like nominal rigidities, consistently with the results in Table 6.

Table 8 shows the (unconditional) variance decomposition computed using the posterior distribution for the II and PI models obtained using the dataset that includes observed inflation expectations. The time -varying inflation target π_t^* is the main driver of inflation expectations in the PI model, while it explains very little in the II model, consistent with the posterior estimates.

5 Introducing Measurement Error

Table 9 shows the log marginal likelihood for the II and PI models for the Benchmark specification, which has no measurement error, and for specifications where the measurement error is i.i.d. ("i.i.d. Meas. Error") or follows and AR(1) process ("AR(1) Meas. Error"). The PI model is still superior to the II model when the measurement error is i.i.d. – although the two models are fairly close. The II model fits better than the PI model under AR(1) measurement error: we conjecture that the measurement error largely "takes care" of inflation expectations for the PI model.

Table 10 shows the variance decomposition for observed inflation expectations – both unconditional and 10 quarters ahead – computed using the posterior distribution for the II and PI models with both i.i.d. and AR(1) measurement error. Not surprisingly, we find that the measurement error is the most important source of variation for Expectations in the II model. For the PI model the contribution of measurement error to the variance of expectations is negligible; changes in π_t^* are the most important source of variations. Note that the unconditional variance for the II model with i.i.d. measurement error explained by the measurement error is farily small. However, that is just a consequence of the i.i.d. assumption: When computing the unconditional variance decomposition persistent shocks (like the μ_t shocks) trump the i.i.d. shocks. However, the conclusion that measurement error explains the vast majority of fluctuations in observed inflation expectations still holds true for the II model with i.i.d. measurement error, as indictaed by the fact that the latter explains 64% of the variance of inflation expectations as much as the 10 quarters ahead.

6 Conclusions

To be written.

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A Data

The data set is obtained from Haver Analytics (Haver mnemonics are in italics). We compile observations for the variables that appear in the measurement equation (20). Real output is obtained by dividing the nominal series (*GDP*) by population 16 years and older (*LN16N*), and deflating using the chained-price GDP deflator (*JGDP*). We compute quarter-to-quarter output growth as log difference of real GDP per capita and multiply the growth rates by 100 to convert them into percentages. Our measure of hours worked is computed by taking total hours worked reported in the National Income and Product Accounts (NIPA), which is at annual frequency, and interpolating it using growth rates computed from hours of all persons in the non-farm business sector (*LXNFH*). We divide hours worked by *LN16N* to convert them into per capita terms. We then take the log of the series multiplied by 100 so that all figures can be interpreted as percentage changes in hours worked. The labor share is computed by dividing total compensation of employees (*YCOMP*) obtained from the NIPA by nominal GDP. We then take the log of the labor share multiplied by 100. Inflation rates are defined as log differences of the GDP deflator and converted into annualized percentages. The nominal rate corresponds to the effective Federal Funds Rate (*FFED*), also in percent.

We use Survey of Professional Forecasters (SPF) quarterly measures of expected inflation. We consider both expectations for GDP deflator[7] and for CPI inflation. In particular, we use the median four -quarters-ahead forecast of inflation in annualized terms. Concerning the information available to the forecasters, the survey is sent out at the end of the first month of each quarter and responses deadlines occur in the middle month of each quarter. Therefore, respondents have knowledge about the BEA advance report of the National Income and Product Accounts. We also compute the revisions in GDP deflator and CPI occurred since 1982 using the real time dataset available from the Federal Reserve Bank of Philadelhia. (http://www.philadelphiafed.org/research-and-data/real-time-center/real-time-data/)

[7] In more detail, the forecast are for the GDP price index, seasonally adjusted (base year varies). Prior to 1996, the forecast variable was the GDP implicit deflator. Prior to 1992, the GNP deflator.

We also use Bluechip monthly forecasts of CPI inflation. We choose forecast horizons of 3 and 4 quarters ahead. In order to compare Bluechip and SPF quarterly forecast of CPI inflation, we use the Bluechip forecasts available in the middle month of each quarter. This roughly corresponds to the time period when SPF participants provide their forecasts.

Table 1: Priors on Policy Parameters

Parameter	Domain	Density	Para (1)	Para (2)	5%	95%
ψ_1	$\mathbb{R}^+$	Gamma	2.00	0.25	1.592	2.409
ψ_2	$\mathbb{R}^+$	Gamma	0.20	0.10	0.047	0.347
ρ_r	[0,1)	Beta	0.50	0.200	0.170	0.825
π^*	$\mathbb{R}$	Normal	4.30	2.50	0.165	8.398
σ_r	$\mathbb{R}^+$	InvGamma	0.150	4.00	0.080	0.298
ρ_{π^*}	[0,1)	Beta	0.950	0.025	0.913	0.989
Benchmark Prior						
σ_{π^*}	$\mathbb{R}^+$	InvGamma	0.050	8.000	0.032	0.078
Signal-to-Noise Ratio Prior						
$\sigma_{NR} = \frac{\sigma_P}{\sigma_T}$	$\mathbb{R}^+$	Gamma	0.180	0.150	0.001	0.380

Notes: Para (1) and Para (2) correspond to means and standard deviations for the Beta, Gamma, and Normal distributions and to s and ν for the Inverse Gamma distribution, where $p_{\mathcal{IG}}(\sigma|\nu, s) \propto \sigma^{-\nu-1}e^{-\nu s^2/2\sigma^2}$. The last two columns report the 5^{th} and 95^{th} quintile of the prior distribution.

Table 2: Priors on Non-Policy Parameters

Parameter	Domain	Density	Para (1)	Para (2)	5%	95%
Priors on Nominal Rigidities Parameters						
Low Rigidities (Benchmark)						
ζ_p	[0,1)	Beta	0.450	0.100	0.285	0.614
ζ_w	[0,1)	Beta	0.450	0.100	0.285	0.614
High Rigidities						
ζ_p	[0,1)	Beta	0.750	0.100	0.590	0.913
ζ_w	[0,1)	Beta	0.750	0.100	0.590	0.913
Priors on "Endogenous Propagation and Steady State" Parameters						
α	[0,1)	Beta	0.330	0.020	0.297	0.362
s'	$\mathbb{R}^+$	Gamma	4	1.500	1.558	6.250
h	[0,1)	Beta	0.700	0.050	0.619	0.782
a''	$\mathbb{R}^+$	Gamma	0.200	0.100	0.049	0.349
ν_l	$\mathbb{R}^+$	Gamma	2	0.75	0.779	3.121
r^*	$\mathbb{R}^+$	Gamma	1.5	1	0.106	2.883
γ	$\mathbb{R}^+$	Gamma	1.650	1	0.204	3.073
g^*	$\mathbb{R}^+$	Gamma	0.300	0.100	0.143	0.459
ι_p	[0,1)	Beta	0.5	0.280	0.089	0.969
ι_w	[0,1)	Beta	0.5	0.280	0.051	0.932
Priors on ρs and σs						
ρ_z	[0,1)	Beta	0.400	0.250	0.000	0.764
ρ_ϕ	[0,1)	Beta	0.750	0.150	0.530	0.982
ρ_{λ_f}	[0,1)	Beta	0.750	0.150	0.530	0.982
ρ_μ	[0,1)	Beta	0.750	0.150	0.530	0.982
ρ_g	[0,1)	Beta	0.750	0.150	0.530	0.982
σ_z	$\mathbb{R}^+$	InvGamma	0.200	4.000	0.160	0.596
σ_ϕ	$\mathbb{R}^+$	InvGamma	2.500	4.000	1.616	5.941
σ_{λ_f}	$\mathbb{R}^+$	InvGamma	0.300	4.000	0.105	0.395
σ_μ	$\mathbb{R}^+$	InvGamma	0.500	4.000	0.397	1.490
σ_g	$\mathbb{R}^+$	InvGamma	0.300	4.000	0.261	0.984

Notes: Para (1) and Para (2) correspond to means and standard deviations for the Beta, Gamma, and Normal distributions and to s and ν for the Inverse Gamma distribution, where $p_{\mathcal{IG}}(\sigma|\nu, s) \propto \sigma^{-\nu-1} e^{-\nu s^2/2\sigma^2}$. The last two columns report the 5^{th} and 95^{th} quintile of the prior distribution.

Table 3: Prior Implications for Moments of the Endogenous Variables

Variables	St. Dev.			Autocorr.		
	II	PI	*Data*	II	PI	*Data*
OutputGrowth	3.33	3.46	*4.33*	0.39	0.36	*0.28*
LaborSupply	2.84	2.86	*3.20*	0.93	0.92	*0.96*
LaborShare	1.37	1.38	*2.24*	0.86	0.86	*0.95*
Inflation	3.42	3.68	*2.77*	0.73	0.75	*0.88*
InterestRate	4.55	5.97	*4.30*	0.86	0.66	*0.87*
Exp. Inflation	1.49	1.69		0.90	0.90	

Notes: II: imperfect information; PI: perfect information. The pre-sample statistics (column *Data*) are in italics. These statistics are computed over the sample 1959Q3-1984Q1. Inflation expectations are not available during most of the pre-sample. The in-sample mean, standard deviation, and first-order autocorrelation of inflation expectations are 2.85, 1.21, and 0.86, respectively.

Table 4: Model Comparison

	Imperfect Information	**Perfect Information**	**Fixed π^***
Dataset w/o Expectations	-715.465	-728.498	-725.465
Dataset w Expectations	-816.232	-801.934	-816.914

Notes: The Table shows the log marginal likelihood for three models: Imperfect Information, Perfect Information, and the model with constant inflation target ("Fixed π^*"). For all models we use the Benchmark prior. The Dataset with Expectations uses the SPF 4-quarters ahead median forecast for the GDP deflator. We assume that the expectations are generated using current quarter information.

Table 5: In-sample *a-priori* RMSEs

Variable	II Model w/o Exp	II Model w Exp	PI Model w/o Exp	PI Model w Exp
Output Growth	3.67	7.70	3.39	3.74
Labor Supply	1.06	2.22	0.99	1.09
Labor Share	0.82	0.89	0.79	0.80
Inflation	1.83	1.51	1.82	1.33
Interest Rate	2.21	2.70	2.57	2.74
Exp. Inflation		0.78		0.66

Notes: The table shows the median in-sample forecast errors obtained from the Kalman filter. All models (II: imperfect information, and PI: perfect information) use the same prior (*Benchmark* prior) for the DSGE parameters. The difference between the w/o Exp (without Inflation Expectations) and w Exp (with Inflation Expectations) columns is that for the latter inflation expectations are used as an additional observable in the measurment equation.

Table 6: Robustness of Model Comparison Results

		Imperfect Information	**Perfect Information**
(1)	Benchmark	-816.232	-801.934
Robustness to Priors			
(2)	High Nominal Rigidities Prior	-826.696	-797.390
(3)	Signal-to-Noise Ratio Prior	-816.890	-802.362
Robustness to Data Sets and Timing Assumptions			
(4)	Lagged Information	-808.287	-794.616
(5)	CPI	-863.304	-780.086
(6)	1980Q1 Sample	-1053.270	-1052.465
Robustness to Policy Rule Specifications			
(7)	Output Growth	-812.930	-800.280
(8)	4Q Inflation	-820.123	-804.586
(9)	GSS	-811.528	-805.923

Notes: The Table shows the log marginal likelihood for the Imperfect Information, and Perfect Information models under different choices of priors, datasets, timing conventions, and policy rules. Line (1) reproduces the marginal likelihood results for the II and PI models from Table 4. Lines (2) and (3) reports the results under the "'High Nominal Rigidities" prior and "Signal-to-Noise Ratio" prior, respectively. Lines (4), (5), and (6) show the log marginal likelihood for the two models under different timing assumptions ("Lagged Information" specification), inflation measure ("SPF CPI "), and sample ("1980Q1 Sample"). Lines (7), (8), and (9) report the results under different specifications of the policy rule, where the policy makers target output growth as opposed to the output gap ("Output Growth"), a four-quarter moving average of inflation as opposed to current inflation ("4Q Inflation"), or where the the law of motion for the inflation target follows the rule suggested by Gurkaynak et al. (2005) ("GSS").

Table 7: Posterior Estimates for Selected Parameters

Parameter	Prior Mean	Prior Stdd	Post Mean	5% Quintile	95% Quintile
Imperfect Information					
ζ_p	0.450	0.100	0.504	0.416	0.583
ι_p	0.500	0.280	0.570	0.267	0.942
ζ_w	0.450	0.100	0.160	0.118	0.202
ι_w	0.500	0.280	0.519	0.136	0.973
ρ_r	0.500	0.200	0.380	0.259	0.507
$\rho_{\pi^\star}$	0.950	0.025	0.910	0.879	0.943
σ_r	0.150	4.000	0.687	0.599	0.778
σ_{π^*}	0.050	8.000	0.067	0.038	0.094
Perfect Information					
ζ_p	0.450	0.100	0.682	0.602	0.765
ι_p	0.500	0.280	0.268	0.037	0.469
ζ_w	0.450	0.100	0.553	0.430	0.674
ι_w	0.500	0.280	0.299	0.001	0.585
ρ_r	0.500	0.200	0.727	0.680	0.774
$\rho_{\pi^\star}$	0.950	0.025	0.966	0.942	0.990
σ_r	0.150	4.000	0.189	0.085	0.295
σ_{π^*}	0.050	8.000	0.070	0.046	0.091

Notes: The Table shows the posterior mean, the 5th and 95th percentiles of the posterior distribution of selected parameters for the II and the PI models for the dataset that includes observed inflation expectations.

Table 8: Variance Decomposition

Variables	Tech	ϕ	μ	g	λ_f	π^*	Money
Imperfect Information							
Output Growth	0.27	0.28	0.14	0.23	0.05	0.00	0.01
Labor Supply	0.01	0.86	0.10	0.01	0.01	0.00	0.00
Labor Share	0.04	0.03	0.00	0.02	0.89	0.00	0.01
Inflation	0.10	0.25	0.39	0.06	0.08	0.02	0.07
Interest Rate	0.09	0.16	0.54	0.06	0.06	0.00	0.08
Exp. Inflation	0.04	0.18	0.70	0.00	0.00	0.04	0.01
Perfect Information							
Output Growth	0.29	0.06	0.24	0.17	0.06	0.01	0.14
Labor Supply	0.04	0.07	0.73	0.03	0.03	0.01	0.05
Labor Share	0.12	0.20	0.01	0.01	0.52	0.01	0.10
Inflation	0.03	0.07	0.22	0.00	0.09	0.42	0.09
Interest Rate	0.03	0.05	0.35	0.00	0.05	0.19	0.28
Exp. Inflation	0.03	0.00	0.34	0.00	0.00	0.59	0.01

Notes: The Table shows the (unconditional) variance decomposition computed using the posterior distribution for the II and PI models obtained using the dataset that includes observed inflation expectations.

Table 9: Model Comparison Results with Measurement Error

	Imperfect Information	**Perfect Information**
Benchmark	-816.232	-801.934
i.i.d. Meas. Error	-798.115	-796.441
AR(1) Meas. Error	-779.813	-797.69

Notes: The Table shows the log marginal likelihood for the II and PI models for the Benchmark specification, which has no measurement error, and for specifications where the measurement error is i.i.d. ("i.i.d. Meas. Error") or follows and AR(1) process ("AR(1) Meas. Error ").

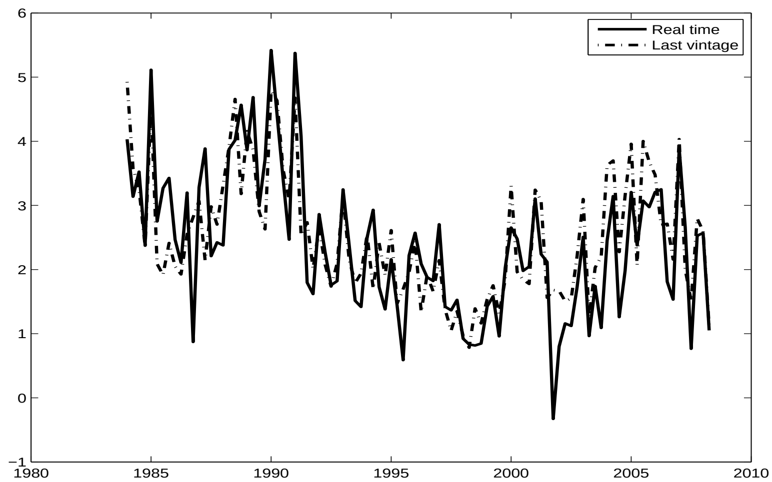
Table 10: Variance Decomposition for Observed Inflation Expectations: Models with Measurement Errors

Variables	Tech	ϕ	μ	g	λ_f	π^*	meas.	Money
Unconditional								
Imperfect Information								
i.i.d. Meas. Error	0.03	0.00	0.76	0.00	0.00	0.11	0.07	0.01
AR(1) Meas. Error	0.04	0.02	0.24	0.00	0.00	0.11	0.52	0.01
Perfect Information								
i.i.d. Meas. Error	0.06	0.00	0.29	0.00	0.01	0.53	0.04	0.02
AR(1) Meas. Error	0.07	0.00	0.33	0.00	0.01	0.44	0.06	0.02
10 Quarters Ahead								
Imperfect Information								
i.i.d. Meas. Error	0.03	0.00	0.04	0.00	0.00	0.22	0.64	0.04
AR(1) Meas. Error	0.03	0.02	0.23	0.00	0.00	0.06	0.63	0.01
Perfect Information								
i.i.d. Meas. Error	0.06	0.01	0.08	0.00	0.02	0.49	0.22	0.08
AR(1) Meas. Error	0.06	0.01	0.08	0.00	0.02	0.44	0.28	0.08

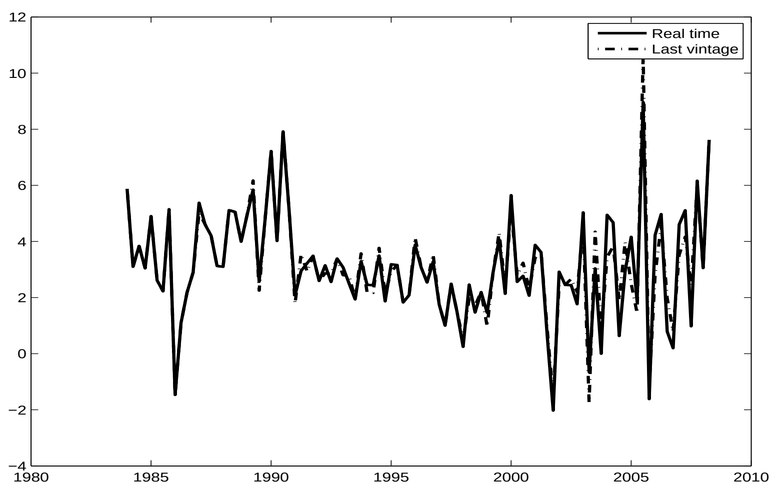
Notes: The Table shows the variance decomposition for observed inflation expectations – both unconditional and 10 quarters ahead – computed using the posterior distribution for the II and PI models with both i.i.d. and AR(1) measurement error. The posteriors are obtained using the dataset that includes observed inflation expectations.

Figure 1: Revisions in Inflation Data: Real Time vs Last Vintage

GDP Deflator



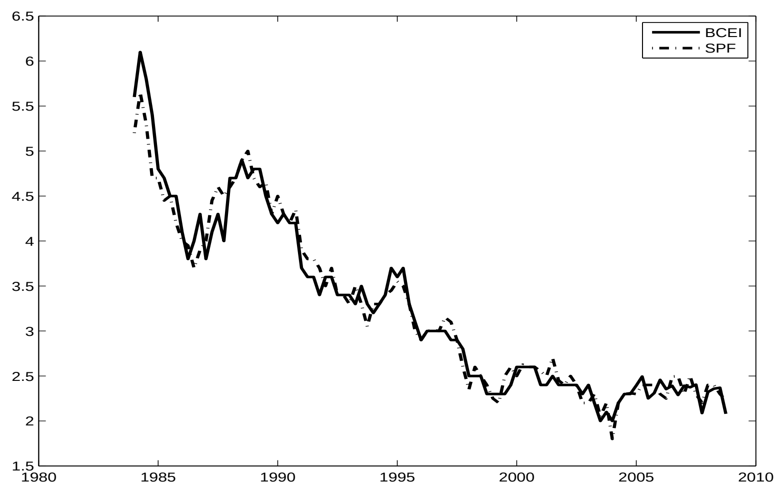
CPI



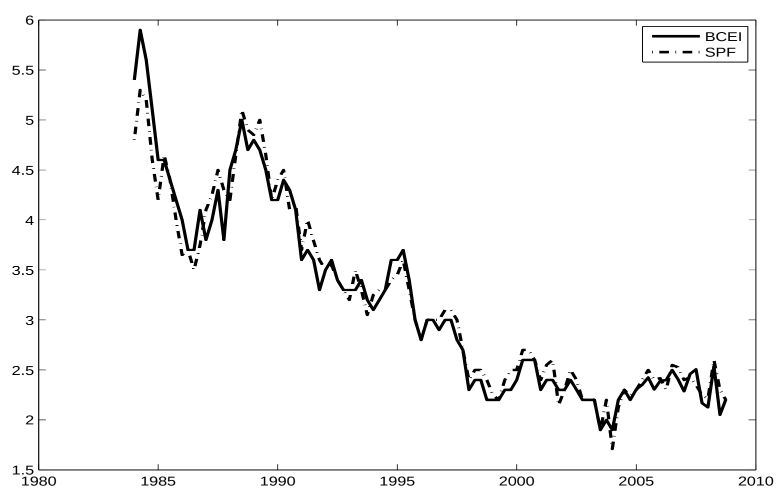
Notes:

Figure 2: Inflation Expectations: SPF vs Blue Chip

Four quarters ahead

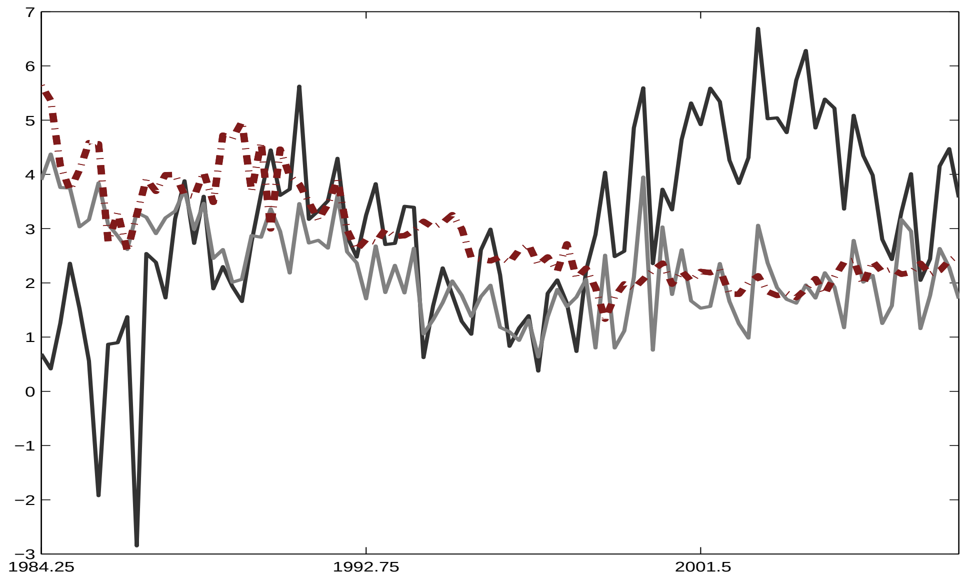


Three quarters ahead



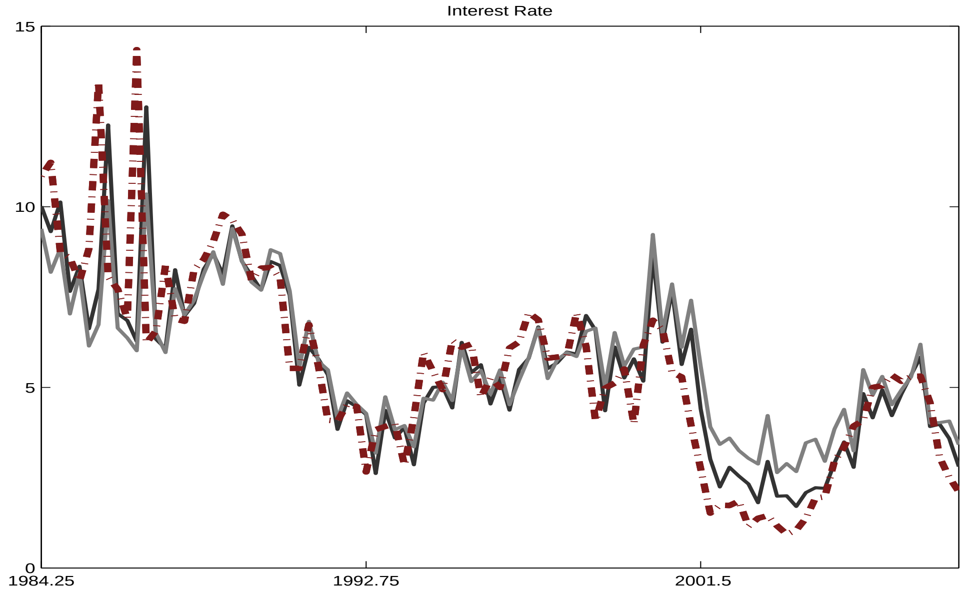
Notes:

Figure 3: Inflation Expectations: Data vs Model Prediction



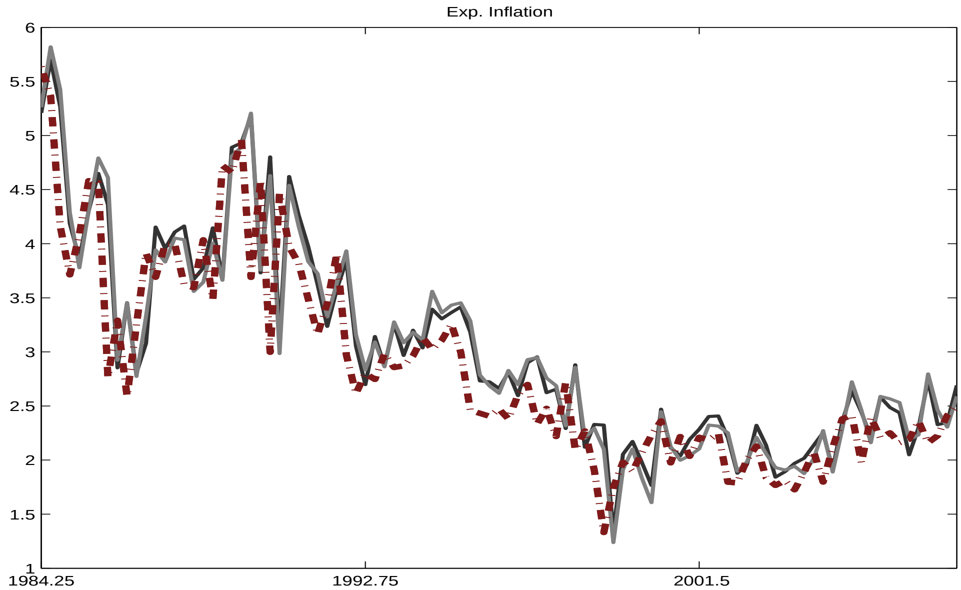
Notes: The figure plots SPF 4-quarters ahead median forecast for the GDP deflator (red dashed-and-dotted), together with the projections for the 4-quarter ahead inflation forecasts generated by the II model (black solid) and the PI model (gray solid). The projections are computed using time $t-1$ information and are generated i) using the prior distribution for the parameters, and ii) without including inflation expectations among the observables.

Figure 4: Interest Rates: Data vs Model Prediction



Notes: The figure plots the interest rate (red dashed-and-dotted), together with the interest rate projections generated by the II model (black solid) and the PI model (gray solid). The projections are computed using time $t-1$ information and are generated i) using the prior distribution for the parameters, and ii) without including inflation expectations among the observables.

Figure 5: Inflation Expectations: Data vs Model Prediction – Including Inflation Expectations In the Observables



Notes: The figure plots SPF 4-quarters ahead median forecast for the GDP deflator (red dashed-and-dotted), together with the projections for the 4-quarter ahead inflation forecasts generated by the II model (black solid) and the PI model (gray solid). The projections are computed using time $t-1$ information and are generated i) using the prior distribution for the parameters, and ii) including inflation expectations among the observables.