

# Dealer Attention, Liquidity Spillovers, and Endogenous Market Segmentation \*

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## Abstract

We consider a model in which two securities are traded by two different pools of dealers. As the payoffs of these securities are correlated, dealers in one security can learn information by monitoring the price of the other security. But monitoring requires attention and attention is costly. The number of dealers in one security who monitors the price of the other security sets the level of attention to this security. We show that variations in the level of attention to a security is a source of positive covariation in liquidity across securities. We also endogenize the levels of attention by analyzing dealers' decision in each security to monitor the price of the other security. We find that monitoring decisions by dealers operating in different securities are complements. For this reason, for identical parameters, two possible levels of attention can emerge in equilibrium: no attention at all or maximal attention. Thus, for a fixed value of the correlation between the fundamentals of the two securities, the markets for both securities can be strongly segmented or strongly integrated in equilibrium. Moreover, a decline in the cost of obtaining price information does not necessarily result in a greater level of attention in equilibrium.

*Keywords:* Commonality in liquidity, Market segmentation, Cross-Price pressure effects, Transparency, Value of market data.

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# 1 Introduction

As asset payoffs are correlated, market-makers have an incentive to price a security by using prices of other, related, securities. For instance, market-makers in a stock can learn information from prices of other stocks in the same industry. Similarly, domestic market-makers in a cross-listed stock have an incentive to watch its price in the foreign country or market-makers in CDSs can obtain information from the stock price of the firm on which the CDS is written.

However, cross asset price monitoring is costly for several reasons. Sometimes, as in OTC markets, price information is simply not readily available in real time or available only to a few privileged market-makers.<sup>1</sup> Second, even when price information is disseminated in real time, this information is not free as it is sold by data vendors (Reuters, Bloomberg etc...) and market participants often complain about the size of the fees charged for this information.<sup>2</sup> Last, in fast markets, prices are changing quickly and tracking price changes in all securities that could potentially be informative requires attention, which is costly. Thus, in choosing which price information to follow or to buy, market-makers face a trade-off between the value of monitoring a specific security and the cost of doing so. In this paper, we show that this trade-off has important implications. For instance, it can explain comovements in liquidity between securities or generate segmentation between securities with highly correlated payoffs, even when the cost of attention is small.

To this end, we consider a model in which two securities with correlated payoffs are traded by two different pools of risk averse market-makers. Each market-maker is active in one security and is perfectly informed on the price of this security. He may also follow the price of the other security, in which case we say that the market-maker is a “pricewatcher.” The fraction of pricewatchers in a given security sets *the level of attention* by market-makers in this security to the other security. We obtain four main findings.

First, pricewatchers are a source of liquidity spillovers across securities. To understand this point, consider a market-maker in security  $X$  who monitors price movements in security  $Y$ . These movements can be due to uninformative trades or news about fundamentals known only to market-makers in security  $Y$ . When the market for security  $Y$  is more liquid, trades contribute less to price volatility relative to news about fundamentals. Thus, market-makers in security  $X$  obtain more reliable information from the price of security  $Y$  when the liquidity of this security improves. This improvement spills over to the liquidity of security  $X$  because its market-makers price security  $X$  more aggressively when they face less uncertainty on its payoff. As this spillover mechanism is bi-directional, it is even stronger when market-makers

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<sup>1</sup>For instance, a market-maker in a stock may be an employee of a trading firm also active in the CDS market. In this way, the market-maker may be privy of information on trades in the CDS market not available to his competitors.

<sup>2</sup>For instance, the fee charged by Nasdaq for the dissemination of corporate bond prices has been very controversial. For accounts of these debates, see, for instance, “Latest Market Data Dispute Over NYSE’s Plan to Charge for Depth-of-Book Data Pits NSX Against Other U.S. Exchanges,” Wall Street Technology, May 21, 2007; the letter to the SEC of the Securities Industry and Financial Markets Association (SIFMA) available at [http://www.sifma.org/regulatory/comment\\_letters/41907041.pdf](http://www.sifma.org/regulatory/comment_letters/41907041.pdf), and “TRACE Market Data Fees go to SEC,” Securities Industry News, 6/3/2002.

in security  $Y$  also monitor the price of security  $X$ .

Second, variations in attention by market-makers active in one security are a source of covariation in liquidity across securities. Indeed, under some parameter restrictions, an increase in the fraction of pricewatchers in one security improves its liquidity. In this case, the spillover mechanism we just described implies that this improvement in liquidity raises the liquidity of the other security as well.<sup>3</sup> This effect sparks a chain reaction that amplifies the initial effect of the change in the fraction of pricewatchers on the liquidity of both securities. We show through numerical simulations that this amplification effect and therefore comovements in illiquidity are strong when the fraction of pricewatchers in both markets is high.

Third, in the presence of pricewatchers in both markets, the liquidity of one security depends on the belief about this liquidity by pricewatchers active in the *other* security. As there is no fundamental anchor for this belief, multiple equilibria with differing liquidity levels can emerge. For instance, suppose that market-makers in security  $X$  expect the market for security  $Y$  to be highly liquid. Then, the market for security  $X$  is more liquid as market-makers feel that they can rely on security  $Y$  as an extra source of information. Thus, the price of security  $X$  is more informative, which works to make security  $Y$  liquid and confirms the belief of market-makers in security  $X$  about the liquidity of security  $Y$ . But, by the same token, the belief that security  $Y$  is illiquid would be self-fulfilling.

Last, we endogenize the number of pricewatchers in each security and show that, *for identical parameter values*, the markets for the two securities can be either strongly integrated (as all market-makers are price watchers) or strongly segmented (as no market-maker is a price watcher). The reason is that the value of being a pricewatcher operating in security  $X$  increases in the fraction of pricewatchers in security  $Y$  and vice versa. Indeed, when the fraction of pricewatchers in security  $Y$  increases, the liquidity of this security improves and therefore its price conveys more precise information. Hence, pricewatchers in market  $X$  are willing to pay more to observe this price. For this reason, the value of price monitoring is higher when all market-makers watch prices than when no market-maker watches prices. This happens despite the fact that the value of being a pricewatcher declines in the number of pricewatchers operating in the same security, as usual in models of information acquisition (e.g., Admati and Pfleiderer (1986)).<sup>4</sup>

Hence, there is a range of costs for price information such that two levels of attention are sustainable in equilibrium. In the first equilibrium, attention is high: all market-makers are pricewatchers because the value of monitoring the other security is high relative to the cost of attention. In the second equilibrium, attention is low: no market-maker acquires information on the price of the other security because the value of monitoring the other security appears

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<sup>3</sup>The positive covariation in liquidity also obtains if an increase in the fraction of pricewatchers in one security reduces the liquidity of this security. This situation can occur when the fraction of pricewatchers is small because pricewatchers have a larger informational advantage when they are less numerous.

<sup>4</sup>Usually, in models of information acquisition, the value of information declines when more investors acquire information (see Admati and Pfleiderer (1986)). This is also the case here: when more market-makers in market  $X$  monitor market  $Y$ , the value of monitoring this market declines. But the value of monitoring market  $X$  increases for market-makers in  $Y$ .

low relative to the cost of attention. This situation is self-sustaining in, say, security  $X$  since the value of price information on  $Y$  is low because there are no pricewatchers in security  $Y$  and vice versa. Thus, a decrease in the cost of attention is not sufficient to stir market-makers away from this equilibrium, unless this decrease is large.

This result has interesting implications. First, it implies that fads, traditions, or other coordination devices may determine the degree of integration between two securities, independently of the correlation in the payoffs of these securities. Second, a decrease in the cost of attention (due for instance to better information linkages between markets) does not per se entail greater market integration, unless the cost is very low. Third, market-makers operating in related but opaque segments may undervalue the benefit of greater market integration. Indeed, in the low attention equilibrium, the value of getting price information is low. Thus, data vendors will perceive a weak demand and will therefore lack incentives to collect and disseminate price information. In this case, regulatory intervention is needed. A case in point is the U.S. corporate bond market where real time dissemination of bond prices took off only under regulatory pressure (see Bessembinder et al. (2006)).

Our paper is related to several strands of research. First, it contributes to the literature on comovements in liquidity across assets. A series of papers have shown that liquidity is positively correlated across stocks (e.g., Chordia et al. (2000), Hasbrouck and Seppi (2001), Huberman and Halka (2001)), and between stocks and bonds (Chordia et al. (2005)). This phenomenon has important implications for asset pricing (see Amihud et al. (2005) for a survey) but its causes are not well understood. Brunnemeier and Pedersen (2007) show how funding constraints for liquidity providers can generate comovements in liquidity. Our model describes another mechanism, based on limited attention.

Second our paper relates to recent empirical papers on the impact of limited attention on liquidity (Corwin and Coughenour (2008) and Boulatov et al.(2009)). On the NYSE, each stock is assigned to a market-maker (a “specialist”). Typically specialists are assigned several stocks. Corwin and Coughenour (2008) argues that time and processing constraints limit a specialist’s ability to monitor multiple stocks simultaneously. Thus, specialists face an attention allocation problem. Corwin and Coughenour (2008) shows empirically that the liquidity of a stock declines when its specialist’s attention to other stocks increases. Their findings suggest that modeling dealer attention is important to understand liquidity.<sup>5</sup> Our paper is an effort in this direction.

Next, our paper relates to the literature on multimarket trading (e.g., Chowdry and Nanda (1991)) and cross asset price pressures (e.g., King and Wadhvani (1990), Pasquariello (2007), Bernhardt and Taub (2008), Pasquariello and Vega (2009)). Chowdry and Nanda (1991) focuses on the case in which the same security trades in two different markets and market-makers in one market cannot condition their price on the price in the other market. Pasquariello (2007), Bernhardt and Taub (2008) and Pasquariello and Vega (2009) consider models of trading with multiple assets and assume that market-makers in one security can condition their price on

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<sup>5</sup>Peng and Xiong (2006) develop a model of trading in which investors’ limited attention is a source of return comovements.

the prices of all other securities.<sup>6</sup> Our results are not directly comparable to these models because the assumptions of our model are different in many ways. For instance, we consider risk averse market-makers while these models consider risk neutral market-makers. But the most important difference is that we consider the effect of varying the number of market-makers in a given market who monitor the price in another market and we endogenize the decision of market-makers to monitor this price. These features raise issues that do not arise in other models. For instance we can relate the level of comovements in illiquidity to the fraction of pricewatchers and show that segmentation can arise even if the cost of observing prices is small.

Our set-up is closer to King and Wadhvani (1990) who study the transmission of information across markets. They analyze how news in one market are transmitted to another market and they explicitly assume that prices are the conduit through which information is transmitted. However, they do not link the informativeness of the price in one market to the liquidity of this market, as we do. We show that this linkage leads to multiple equilibria with different levels of price informativeness and liquidity, a possibility which does not arise in King and Wadhvani (1990). It is well-known that participation externalities result in multiple equilibria with differing levels of liquidity (see Admati and Pfleiderer (1988), Pagano (1989), and Dow (2004) for example). However, to the best of our knowledge, the coordination problem that arises in our model, which involves market-makers operating in different securities, has not been analyzed before in the literature.

The rest of the paper is organized as follows. Section 2 describes the model. In Section 3, we consider the case in which the fraction of pricewatchers is fixed. We show how liquidity spillovers and multiple equilibria arise in this set-up. Then, in Section 4.1, we endogenize the fraction of pricewatchers and show how a low attention or a high attention equilibrium can emerge for the same parameter values. Proofs are collected in the Appendix.

## 2 The model

We consider two securities,  $D$  and  $F$ , traded by two distinct pools of traders. As in King and Wadhvani (1990), we assume that the payoffs of these securities,  $v_D$  and  $v_F$ , are linear functions of two “fundamentals,” namely:

$$v_D = \delta_D + d \times \delta_F + \eta \tag{1}$$

$$v_F = f \times \delta_D + \delta_F. \tag{2}$$

The random variables  $\delta_D$ ,  $\delta_F$  and  $\eta$  are independent and have a normal distribution, with mean zero. The variance of  $\delta_j$  is 1 and the variance of  $\eta$  is  $\sigma_\eta^2$ . Thus, securities  $F$  and  $D$  are two securities that load, with possibly different weights, on the same risk factors (see below for more specific interpretations). To fix things, we assume that  $f = 1$  and  $d \in [0, 1]$ , so that the payoffs of the two securities are positively correlated.

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<sup>6</sup>Bernhardt and Taub (2008) compare equilibrium outcomes when informed investors can condition their market orders in one security on the prices of all other securities and when they cannot.

In each market, there are two types of traders: (i) a continuum of risk-averse speculators and (ii) liquidity traders. The aggregate demand of liquidity traders in market  $j$  is  $u_j \sim N(0, \sigma_u^2)$ . Liquidity demands in both markets are independent. The net order imbalance from liquidity traders is absorbed by speculators, who are therefore best viewed as market-makers. Hence, in the rest of the paper, we refer to speculators as being market-makers and to  $u_j$  as being the size of the *liquidity shock* in market  $j$ .<sup>7</sup>

Market-makers are specialized: they only trade in one security. Market-makers specialized in security  $j$  have perfect information on factor  $\delta_j$  but no information on factor  $\delta_{-j}$ . Yet, a fraction  $\mu_j$  of market-makers in each market have real time information on prices in the other market. We refer to these market-makers as being “insiders” or *pricewatchers*. Other market-makers are called outsiders. We index decisions of insiders by  $I$  and decisions of outsiders by  $O$ .

Each market-maker has a CARA utility function with risk tolerance  $\gamma$ . Thus, if market-maker  $i$  in market  $j$  holds  $x_{ij}$  shares of the risky security, her expected utility is

$$E[U(\pi_{ij})|\delta_j, \Omega_j^k] = E[-\exp\{-\gamma^{-1}\pi_{ij}\}|\delta_j, \Omega_j^k], \quad (3)$$

where  $\pi_{ij} = (v - p_j)x_{ij}$  and  $\Omega_j^k$  is the *price information* available at date  $j$  to a market-maker with type  $k \in \{I, O\}$ .

As usual in rational expectations models, the clearing price in each market reflects both traders’ private and public information. In particular, the price in market  $j$  will reflect insiders’ information about the price in market  $-j$ . For this reason the price in market  $j$  is an additional source of information for outsiders in this market. As market-makers submit price contingent demand functions, they all act as if they were observing the clearing price in their market. Thus, we have  $\Omega_j^I = \{p_j, p_{-j}\}$  and  $\Omega_j^O = \{p_j\}$ . We denote the demand function of an insider by  $x_j^I(\delta_j, p_j, p_{-j})$  and that of an outsider by  $x_j^O(\delta_j, p_j)$ . In each period, the clearing price in security  $j$ ,  $p_j$ , is such that the demand for this security is equal to its supply, i.e.,

$$\int_0^{\mu_j} x_j^I(\delta_j, p_j, p_{-j}) di + \int_{\mu_j}^1 x_j^O(\delta_j, p_j) di + u_j = 0, \quad \text{for } j \in \{D, F\}. \quad (4)$$

**Interpretation.** There are several ways to interpret the two securities in our model. For instance, securities  $D$  and  $F$  could be two stock market indexes of two different countries as in King and Wadhvani (1990). In this case, assuming  $\sigma_\eta^2 > 0$  and  $d \in [0, 1]$  captures the existence of asymmetries in the dependence between related markets. Another possibility is that these securities represent the stock of a cross-listed firm and its ADR in the foreign market,<sup>8</sup> or the same security traded by different pools of market-makers with distinct information and different

<sup>7</sup>As will become clear from the discussion that follows, the adopted information structure implies that market makers’ aggressiveness in market  $j$  positively depends on the informativeness of the equilibrium price in market  $-j$ . To obtain this result the factor structure for payoffs appears crucial. Indeed, adopting an information structure in which asset payoffs are *not* the sum of (potentially) common factors, prevents insiders’ aggressiveness in market  $j$  from depending on the illiquidity of market  $-j$ . Thus, if we modeled the information structure à la Admati (1985), the results of our model no longer hold.

<sup>8</sup>In the cross-listing case,  $\delta_j$  can be viewed as the cash-flow generated in country  $j$  by the cross-listed firm.

clientele bases.<sup>9</sup> In this latter case, as these securities have the same payoff, it is natural to assume  $d = f = 1$  and  $\sigma_\eta^2 = 0$ .<sup>10</sup>

In each of these cases, market-makers will have specialized information. For instance, market-makers in country  $j$  will be well informed on local fundamental news but not on foreign fundamental news as in King and Wadhvani (1990).<sup>11</sup> As a result, the dependence between the payoffs traded in the two markets will vary depending on the value of  $d$  and  $\sigma_\eta^2$ . In particular, denoting by  $\rho_{v_D, v_F; \delta_D}^2$  and  $\rho_{v_D, v_F; \delta_F}^2$ , respectively the correlation coefficient between  $v_D$  and  $v_F$  conditional on the information held by market makers in market  $D$  and  $F$  we have

$$\rho_{v_D, v_F; \delta_D}^2 = \frac{d^2}{\sigma_\eta^2 + d^2}, \quad \rho_{v_D, v_F; \delta_F}^2 = \frac{1}{\sigma_\eta^2 + 1}. \quad (5)$$

A lower (higher) value of  $d$  ( $\sigma_\eta$ ) reduces the dependence between market  $D$  and market  $F$ . Indeed, as  $d$  approaches zero,  $v_D$  tends increasingly to load on  $\delta_D$  only. As  $\sigma_\eta$  increases, residual uncertainty has a larger impact on the payoff of security  $D$ , and the latter is increasingly less dependent on fundamental information. As a consequence, the two markets become less dependent.

### 3 Information-based liquidity spillovers

We start by characterizing the equilibrium in the two markets in the general case where  $d > 0$  and  $\sigma_\eta^2 > 0$ . Under these assumptions both assets payoffs load on common factors and it is therefore natural to assume that a positive fraction of market-makers in each market monitors the price in the other market ( $\mu_F > 0$  and  $\mu_D > 0$ ). This case is complex to analyze. Indeed, pricewatchers are better informed than outsiders. Thus, there is asymmetric information among market-makers in each market and outsiders can extract information from the price in their market. As usual, we will focus on the linear rational expectations equilibria of the model. This enables us to obtain the following result:

**Proposition 1.** *Suppose that  $\mu_F > 0$  and  $\mu_D > 0$ . If there exists a linear rational expectations equilibrium then the prices in market  $D$  and  $F$  are given by:*

$$p_j = \omega_j + A_j \omega_{-j}, \text{ for } j \in \{F, D\}, \quad (6)$$

where  $\omega_j = \delta_j + B_j u_j$ ,  $A_j = (B_{-j}^2 \sigma_u^2)^{-1} \gamma \mu_j B_j > 0$ , and

$$B_D = \frac{\text{Var}[v_D | \delta_D, \omega_F]}{\gamma} \times \frac{\gamma^2 \mu_D \rho_{v_D, \omega_F}^2 + \sigma_u^2 \text{Var}[v_D | \delta_D, \omega_F]}{\gamma^2 \mu_D^2 \rho_{v_D, \omega_F}^2 + \sigma_u^2 \text{Var}[v_D | \delta_D, \omega_F] (1 - \rho_{v_D, \omega_F}^2 (1 - \mu_D))} \quad (7)$$

$$B_F = \frac{\text{Var}[v_F | \delta_F, \omega_D]}{\gamma} \times \frac{\gamma^2 \mu_F \rho_{v_F, \omega_D}^2 + \sigma_u^2 \text{Var}[v_F | \delta_F, \omega_D]}{\gamma^2 \mu_F^2 \rho_{v_F, \omega_D}^2 + \sigma_u^2 \text{Var}[v_F | \delta_F, \omega_D] (1 - \rho_{v_F, \omega_D}^2 (1 - \mu_F))}, \quad (8)$$

<sup>9</sup>For instance, in bond markets, some market-makers may cater mainly to institutions whereas others market-makers cater mainly to retail investors.

<sup>10</sup>Yet another possibility, it that security  $F$  is a credit default swap (CDS) and security  $D$  the stock issued by the firm on which the CDS is written.

<sup>11</sup>Equivalently, in the case of the CDS market, market-makers in CDS are often affiliated with lenders and therefore better informed on the likelihood of defaults (and size of associated losses) than market-makers in the stock market (see Acharya and Johnson (2007))

$$\rho_{v_D, \omega_F}^2 = \frac{d^2}{(\sigma_\eta^2 + d^2)(1 + B_F^2 \sigma_u^2)}, \quad \rho_{v_F, \omega_D}^2 = \frac{1}{1 + B_D^2 \sigma_u^2}. \quad (9)$$

Moreover, insiders' and outsiders' demands in each market are:

$$\begin{aligned} x_j^I(p_j, \omega_{-j}) &= a_j^I (E[v_j | \delta_j, p_{-j}] - p_j) = a_j^I (\delta_j - p_j) + b_j^I \omega_{-j} \\ x_j^O(p_j, \hat{\omega}_j) &= a_j^O (E[v_j | \delta_j, p_j] - p_j) = a_j^O (\delta_j - p_j) + b_j^O \hat{\omega}_j, \end{aligned}$$

with  $a_j^I = \gamma \text{Var}[v_j | \delta_j, \omega_{-j}]^{-1}$ ,  $a_j^O = \gamma \text{Var}[v_j | \delta_j, \hat{\omega}_j]^{-1}$ ,  $\hat{\omega}_j = A_j \omega_{-j} + B_j u_j$ , and expressions for  $b_j^I$ ,  $b_j^O$  are provided in the appendix.

To interpret this result, observe that, in equilibrium, the price in market  $j$  can be written as:

$$p_j = \omega_j + A_j(p_{-j} - A_{-j}\omega_j) = (1 - A_j A_{-j})\omega_j + A_j p_{-j}. \quad (10)$$

Thus, the price of security  $j$  is positively related to the price of the other security as  $A_j > 0$ . Now, consider market-makers in market  $D$ . Insiders in this market observe the prices of securities  $D$  and  $F$ . Equation (10) shows that, from this observation, insiders in market  $D$  infer a signal  $\omega_F$  about  $\delta_F$ . Thus, their demand depends both on their specialized information,  $\delta_D$  and  $\omega_F$ , i.e., the additional information that insiders in security  $D$  glean from monitoring the price of security  $F$  in real time. For this reason, the clearing price in market  $D$  is informative about  $\delta_F$ . It is easily seen, using equation (6), that the signal provided by the price of security  $D$  about  $\delta_F$  to outsiders is  $\hat{\omega}_D = A_D \omega_F + B_D u_D$ . Thus, their demand depends on  $\delta_D$  and  $\hat{\omega}_D$ , i.e., the additional information that outsiders in security  $D$  obtain from the clearing price in market  $D$ . The same reasoning explains why insiders' and outsiders' demands in market  $F$  depend on  $\omega_D$  and  $\hat{\omega}_F$ , respectively.

In Proposition 1, we have expressed insiders and outsiders' demands in each market as functions of their specialized information and the signals that they extract from the prices that they observe. Of course, it is also possible to directly write insiders' and outsiders' demands in terms of prices and private signals. Using investors' demand functions in Proposition 1 and equation (10), we obtain:

$$\begin{aligned} x_j^I(\delta_j, p_j, p_{-j}) &= a_j^I \delta_j - \beta_{j2}^I p_j + \beta_{j3}^I p_{-j}, \\ x_j^O(\delta_j, p_j) &= \lambda_j^O (\delta_j - p_j), \end{aligned}$$

with  $\beta_{j2}^I = a_j^I + A_{-j} \beta_{j3}^I$ ,  $\beta_{j3}^I = (1 - A_j A_{-j})^{-1} b_j^I$ , and  $\lambda_j^O = a_j^O - b_j^O > 0$ . Thus, insiders' demand functions are negatively related to the price in the market in which they specialize but positively related to the price of the other security (as both securities have positively correlated payoffs).

**Market illiquidity.** Using equation (6), we deduce that the price impact of a liquidity shock in market  $j$  on the price of this market is given by:

$$\frac{\partial p_j}{\partial u_j} = B_j. \quad (11)$$

Thus, we refer to  $B_j$  as the “illiquidity of market  $j$ .”<sup>12</sup> According to (7) and (8) the observability of  $\omega_{-j}$  has two effects on market  $j$ ’s illiquidity: a direct effect due to *learning* from cross-order flow observability (which leads to payoff uncertainty reduction), which is captured by the first term of the expressions; an indirect effect due to the increase in *adverse selection*, which is captured by the second term. To see this, consider for instance (7). The first term of the expression is given by

$$\frac{\text{Var}[v_D|\delta_D, \omega_F]}{\gamma}.$$

As  $\omega_F$  potentially contains information about  $v_D$ , its observability contributes to reduce the conditional variance of market  $D$ ’s payoff, lowering the illiquidity of that market. Using standard formulae for the correlation coefficient, the second term in the expression can be rewritten as follows:

$$\frac{\gamma^2 \mu_D \rho_{v_D, \omega_F}^2 + \sigma_u^2 \text{Var}[v_D|\delta_D, \omega_F]}{\gamma^2 \mu_D^2 \rho_{v_D, \omega_F}^2 + \sigma_u^2 \text{Var}[v_D|\delta_D, \omega_F](1 - \rho_{v_D, \omega_F}^2(1 - \mu_D))} = \frac{\gamma^2 \mu_D (\text{Var}[v_D|\delta_D] - \text{Var}[v_D|\delta_D, \omega_F]) + \sigma_u^2 \text{Var}[v_D|\delta_D, \omega_F] \text{Var}[v_D|\delta_D]}{\gamma^2 \mu_D^2 (\text{Var}[v_D|\delta_D] - \text{Var}[v_D|\delta_D, \omega_F]) + \sigma_u^2 \text{Var}[v_D|\delta_D, \omega_F] (\text{Var}[v_D|\delta_D] - \text{Var}[v_D|\delta_D, \omega_F](1 - \mu_D))}. \quad (12)$$

Manipulating (12):

$$\begin{aligned} \gamma^2 \mu_D \rho_{v_D, \omega_F}^2 + \sigma_u^2 \text{Var}[v_D|\delta_D, \omega_F] &> \\ \gamma^2 \mu_D^2 \rho_{v_D, \omega_F}^2 + \sigma_u^2 \text{Var}[v_D|\delta_D, \omega_F](1 - \rho_{v_D, \omega_F}^2(1 - \mu_D)) &\text{ if and only if } \mu_D < 1. \end{aligned}$$

The above inequality implies that as a two-tiered market generates adverse selection, the negative effect due to cross order flow observability on illiquidity is lessened by the positive effect due to the risk-premium outsiders demand in their transactions to be compensated for the losses they expect to make vis-à-vis insiders. Thus, adverse selection acts as a multiplier of illiquidity. If  $\mu_D = 1$ , all market makers in  $D$  are insiders and this multiplier disappears.

**Market illiquidity and cross-security price informativeness.** Insiders in market  $j$  watch price movements in market  $-j$  as prices in this market constitute a source of information on factor  $\delta_{-j}$ . For insiders, observation of the price in market  $-j$  provides a signal  $\omega_{-j}$  on factor  $\delta_{-j}$ . Thus, from the point of view of insiders, the informativeness of the price in market  $-j$  can be measured by the precision of their forecast about factor  $\delta_{-j}$  after observing  $\omega_{-j}$ . Setting  $j = D$ , and using the definition of  $\omega_F$ , we obtain:

$$\begin{aligned} \text{Var}[v_D|\delta_D, \omega_F] &= \text{Var}[v_D|\delta_D] (1 - \rho_{v_D, \omega_F}^2) \\ &= (\sigma_\eta^2 + d^2) \left( 1 - \frac{d^2}{(\sigma_\eta^2 + d^2)(1 + B_F^2 \sigma_u^2)} \right). \end{aligned} \quad (13)$$

Thus, an increase in the illiquidity of market  $F$  reduces the informativeness of the price in this market for insiders in security  $D$ . Intuitively, as security  $F$  becomes more illiquid, it is more difficult for pricewatchers in security  $D$  to tell whether price movements in security  $F$  are due to liquidity shocks or the arrival of fundamental news. This lowers the conditional

<sup>12</sup>Thus,  $B_j$  is conceptually similar to Kyle’s lambda.

correlation between  $\omega_F$  and  $v_D$ , increasing the dispersion of insiders' forecast in market  $D$ . A similar reasoning implies that other things equal, the informativeness of the signal insiders in market  $F$  gather from market  $D$  decreases in  $B_D$ .

**Equilibrium existence.** In the appendix we show that all the equilibrium parameters that characterize prices and traders' strategies in each market, depend on the values of the illiquidities  $B_D$  and  $B_F$ . Hence, equilibrium existence revolves around the solution of the system of equations (7) and (8) which can be expressed as

$$B_D = f(B_F, \mu_D), \quad B_F = g(B_D, \mu_F). \quad (14)$$

Denoting by  $B_j^*(\mu_j, \mu_{-j})$  the equilibrium level of illiquidity in market  $j$  for given fractions of insiders in both markets, an equilibrium will thus be a couple of illiquidities  $B_D^*(\mu_D, \mu_F)$  and  $B_F^*(\mu_D, \mu_F)$  such that

$$B_D^* = f(B_F^*(\mu_D, \mu_F), \mu_D), \quad B_F^* = g(B_D^*(\mu_D, \mu_F), \mu_F).$$

The system (14) together with the above discussion suggest that when the illiquidity of market  $-j$  increases, watching the price of security  $-j$  is less informative for insiders in market  $j$  and, as a consequence, the illiquidity of security  $j$  is affected. Hence, the illiquidities of markets  $D$  and  $F$  are interdependent. To analyze the effect of this interdependence for the equilibrium of the two markets, we proceed in two steps: first we look at the case in which illiquidity spillovers are one-directional. In this case, the analysis is greatly simplified and the equilibrium can be computed in closed form. We then turn our attention to the case in which spillovers are bi-directional.

### 3.1 One-way spillovers

As argued in Section 2 the magnitude of  $d$  parametrizes the dependence between the two payoffs  $v_D$  and  $v_F$ . According to (5), for  $d = 0$ ,  $v_D$  only loads on  $\delta_D$  while  $v_F$  depends on both factors. As a consequence, pricewatchers in  $D$  learn nothing from  $\omega_F$  and the observation of  $\omega_F$  is useless. On the contrary, insiders in market  $F$  benefit from the observation of  $\omega_D$  (while outsiders suffer from the presence of adverse selection). As a result, the analysis of the model is greatly simplified, allowing us to obtain the following result:

**Corollary 1.** *If  $d = 0$ , there exists a unique equilibrium in linear strategies where*

$$B_D^* = \frac{\sigma_\eta^2}{\gamma} \quad (15)$$

$$B_F^* = \frac{\sigma_\eta^4 \sigma_u^2}{\gamma(\gamma^2 + \sigma_\eta^4 \sigma_u^2)} \times \frac{(\gamma^2 + \sigma_\eta^4 \sigma_u^2)(\gamma^4 \mu_F + \sigma_\eta^4 \sigma_u^4)}{\gamma^4 \mu_F (\gamma^2 + \sigma_\eta^4 \sigma_u^2) + \sigma_\eta^4 \sigma_u^4 (\gamma^2 \mu_F + \sigma_\eta^4 \sigma_u^2)}. \quad (16)$$

**Proof.** The result follows immediately by substituting  $d = 0$  into (7) and rearranging (8).  $\square$

If  $v_D$  does not depend on  $\delta_F$  the only source of uncertainty that remains on  $D$ 's market makers comes from the presence of the residual uncertainty term  $\eta$ . Thus, the larger is  $\sigma_\eta^2$ , the

higher is the unpredictability of  $v_D$  and the more illiquid market  $D$  becomes. This, in turn, has two contrasting effects on  $B_F$  as a poorer market  $D$  signal worsens  $F$ 's insiders' forecast but also ameliorates the adverse selection problem that outsiders in market  $F$  face.

As  $d$  increases away from zero,  $v_D$  depends increasingly more on  $\delta_F$  and observing  $\omega_F$  is more rewarding. Note, however, that according to (7):

$$B_D|_{d=0} \equiv \frac{\sigma_\eta^2}{\gamma} < B_D|_{d>0, \mu_D=1} \equiv \frac{\sigma_\eta^2}{\gamma} + \frac{d^2}{\gamma} \frac{B_F^2 \sigma_u^2}{1 + B_F^2 \sigma_u^2},$$

so that when all market makers in  $D$  are pricewatchers, an increase in dependence across payoffs *reduces* liquidity, the more, the stronger is the dependence between payoffs (i.e., the higher is  $d$ ). The intuition is immediate: as  $d$  increases away from zero, a new information shock about which investors can only receive an imperfect signal affects  $v_D$ . This increases the uncertainty about the final liquidation value, making the market more illiquid.

Inspection of (16) shows that an increase in the fraction of insiders in market  $F$  has a *negative* effect on the illiquidity of that market:

$$\frac{\partial B_F^*}{\partial \mu_F} = -\frac{\sigma_\eta^4 \sigma_u^2}{\gamma(\gamma^2 + \sigma_\eta^4 \sigma_u^2)} \times \frac{\gamma^2 \sigma_\eta^4 \sigma_u^4 (\gamma^2 + \sigma_\eta^4 \sigma_u^2) (\gamma^4 + \sigma_\eta^4 \sigma_u^4)}{(\gamma^4 \mu_F (\gamma^2 + \sigma_\eta^4 \sigma_u^2) + \sigma_\eta^4 \sigma_u^4 (\gamma^2 \mu_F + \sigma_\eta^4 \sigma_u^2))^2} < 0.$$

The above effect illustrates a well-known property of financial markets in which the observability of a given price which is informative about an asset cannot be restricted. As a wider fraction of market participants becomes informed, the price reflects with an increasing precision the information held by insiders. As a consequence, traders' information becomes more homogeneous and the adverse selection problem faced by outsiders is less severe. As is well-known (see, e.g., Admati and Pfleiderer (1986)), this implies that the value of observing  $p_D$  *decreases* in  $\mu_F$ . We can easily verify that this result also applies to the present context. To this end we define the *maximum* fee  $\phi_F(\mu_F, B_D)$  that a market-maker in market  $F$  would be willing to pay to monitor prices in market  $D$  (for given  $\mu_F$ ) as follows:

$$E[U((v_F - p_F) x_F^I - \phi_F)] = E[U((v_F - p_F) x_F^O)].$$

Using the specification of market-makers' utility functions and the fact that all variables have a normal distribution, we obtain that:<sup>13</sup>

$$\phi_F(\mu_F, B_D) = \frac{\gamma}{2} \ln \frac{\text{Var}[v_F | \delta_F, \hat{\omega}_F]}{\text{Var}[v_F | \delta_F, \omega_D]} > 0. \quad (17)$$

The value of being informed on prices in market  $D$  for a market-maker in market  $F$  is strictly positive because it enables an insider in market  $F$  to better forecast the payoff of asset  $F$  compared to outsiders. As a consequence, insiders can take better positions than outsiders (e.g., conditional on asset  $F$  payoff being high, they are more likely to have a large long position on average). The value of price information is increasing in the ratio of the precision of an insider's

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<sup>13</sup>Our expression for the value of information is standard in models of information acquisition with normally distributed variables and CARA utility functions (see for instance Admati and Pfleiderer (1986)). Thus, for brevity we omit the derivation of this result, which can be obtained upon request.

forecast to the precision of an outsider's forecast (i.e.,  $\text{Var}[v_F|\delta_F, \omega_D]^{-1}/\text{Var}[v_F|\delta_F, \hat{\omega}_F]^{-1}$ ) because an insider's informational advantage increases in this ratio. Using Corollary 1, we obtain the following lemma:

**Lemma 1.** *If  $d = 0$  the value of monitoring  $p_D$  for market makers in security  $F$  is given by:*

$$\phi_F(\mu_F, B_D) = \frac{\gamma}{2} \ln \left( 1 + \frac{\gamma^2 \sigma_\eta^4 \sigma_u^4}{\sigma_\eta^4 \sigma_u^2 (\gamma^4 \mu_F^2 + \sigma_\eta^4 \sigma_u^4) + \gamma^6 \mu_F^2} \right), \quad (18)$$

and is a decreasing function of  $\mu_F$ .

**Proof.** Substituting the expressions for  $B_D$  and  $B_F$  obtained in Corollary 1 into the expressions for  $\text{Var}[v_F|\delta_F, \omega_D]$  and  $\text{Var}[v_F|\delta_F, \hat{\omega}_D]$  provided in the appendix, and rearranging yields (18).  $\square$

Thus, if informational spillovers are unidirectional our model delivers the standard result that the value of information is *decreasing* in the fraction of traders monitoring  $p_D$ . However, as shown above, assuming  $d = 0$  implies that the equilibrium illiquidity in market  $D$  is independent from the one of market  $F$ . Therefore, an increase in  $\mu_F$  affects  $\phi_F$  only through the impact that it has on the severity of the adverse selection problem faced by outsiders. If, on the other hand,  $d > 0$ , informational spillovers are bidirectional and insiders in market  $D$  also find it useful to observe  $\omega_F$ . This, in turn, is likely to have a positive effect on the informativeness of  $p_D$ , thereby positively affecting the value of information in market  $F$ .

### 3.2 Two-way spillovers

Suppose that  $d > 0$ , and that  $\mu_j = 1$ , so that all market makers in each market  $j$  observe the price in market  $-j$  and no adverse selection affects the two markets. In this situation it is easy to see that (7) and (8) respectively reduce to

$$B_D = f(B_F, 1) = \frac{\sigma_\eta^2(1 + B_F^2 \sigma_u^2) + d^2 B_F^2 \sigma_u^2}{\gamma(1 + B_F^2 \sigma_u^2)}$$

$$B_F = g(B_D, 1) = \frac{B_D^2 \sigma_u^2}{\gamma(1 + B_D^2 \sigma_u^2)}.$$

Inspection of the above equations shows that the illiquidity of market  $j$  is a continuous, strictly increasing function of  $B_{-j}$  and that  $\sigma_\eta^2 \leq f(B_F, 1) \leq (1 + \sigma_\eta^2)/\gamma$ , while  $0 \leq g(B_D, 1) \leq 1/\gamma$  which in turn implies that:

**Corollary 1.** *If  $\mu_j = 1$  for all  $j$ : (i) there are positive illiquidity spillovers across markets and (ii) there always exists an equilibrium in linear strategies where  $B_F^* \in (0, 1/\gamma)$  and  $B_D^* \in (\sigma_\eta^2/\gamma, (1 + \sigma_\eta^2)/\gamma)$ .*

As argued in Section 3, when  $d > 0$  and market makers in both markets observe price information, the illiquidities of markets  $D$  and  $F$  are interdependent: an increase in  $B_F$  makes it more difficult for pricewatchers in  $D$  to tell apart noise from information in  $\omega_F$ , lowering

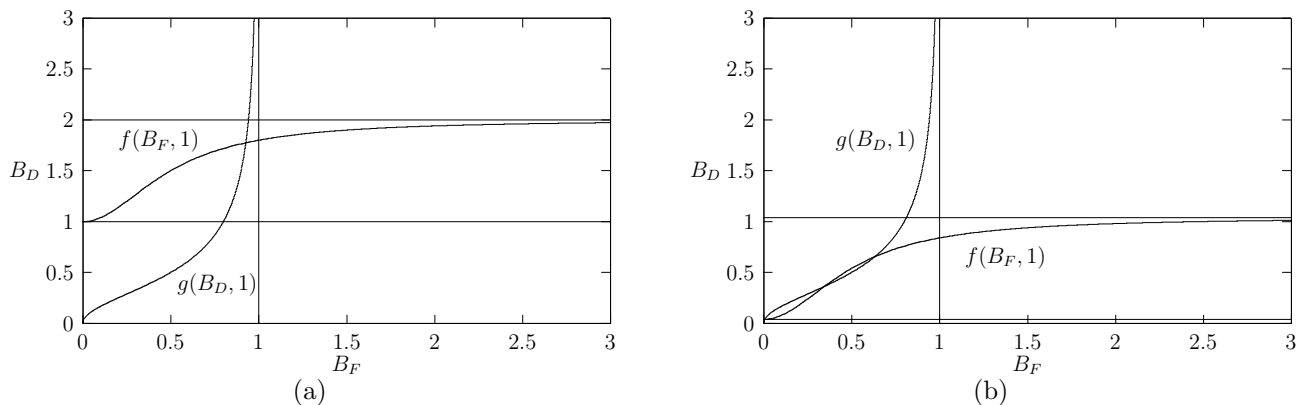


Figure 1: Equilibrium determination with no adverse selection: uniqueness (panel (a)) and multiplicity (panel (b)).

their aggressiveness when pricing  $v_D$ . Absent adverse selection effects (as  $\mu_j = 1$ ), this impairs market  $D$ 's depth. As  $B_F$  depends on  $B_D$  this feeds back into market  $F$ 's illiquidity, further reducing the latter market's depth. As a result, multiple equilibria with different levels of market depth can arise. In figure 1 we illustrate the determination of the equilibrium in two cases. In panel (a) we set  $\gamma = \sigma_\eta = d = 1$  and  $\sigma_u = 2$  and obtain a unique equilibrium. In panel (b) we set  $\sigma_\eta = 0.2$ , while keeping the other parameters' values at their previous levels. In this case we obtain three equilibria. Our numerical simulations show that the pattern illustrated in figure 1 is general: for  $\sigma_\eta$  "low," we obtain three equilibria with different levels of illiquidity. For  $\sigma_\eta$  "high," only the high illiquidity equilibrium survives. The intuition is straightforward: a high level of residual uncertainty in market  $D$  makes  $v_D$  less dependent on fundamental information; as a result observing  $\omega_F$  is less important to price  $v_D$  and the two markets are less dependent. Conversely, a lower level of  $\sigma_\eta$  increases the dependence between the two markets, potentially leading to multiple equilibria.

When  $\mu_j < 1$ , the analysis is more complicated. Indeed, in this case an increase in  $B_F$  has two countervailing effects: on the one hand, it impairs learning from  $\omega_F$ , thus positively affecting  $B_D$ ; on the other hand, it renders less severe the adverse selection problem faced by outsiders in market  $D$ , thus negatively affecting  $B_D$ . A simple calculation shows that both effects can be captured by analyzing the impact that a change in the conditional correlation coefficient between  $v_D$  and  $\omega_F$  has on  $B_D$  (symmetrically, looking at the impact that a change in the conditional correlation coefficient between  $v_F$  and  $\omega_D$  has on  $B_F$ ). Indeed, according to (9), an increase in  $B_F$  unambiguously lowers the correlation coefficient between  $v_D$  and  $\omega_F$ ; this, in turn, increases  $\text{Var}[v_D|\delta_D, \omega_F]/\gamma$ :

$$\frac{\text{Var}[v_D|\delta_D, \omega_F]}{\gamma} = \frac{(\sigma_\eta^2 + d^2)(1 - \rho_{v_D, \omega_F}^2)}{\gamma},$$

while lowering the second component of  $B_D$ :

$$\frac{\partial}{\partial \rho_{v_D, \omega_F}^2} \left( \frac{\gamma^2 \mu_D \rho_{v_D, \omega_F}^2 + \sigma_u^2 \text{Var}[v_D | \delta_D, \omega_F]}{\gamma^2 \mu_D^2 \rho_{v_D, \omega_F}^2 + \sigma_u^2 \text{Var}[v_D | \delta_D, \omega_F] (1 - \rho_{v_D, \omega_F}^2 (1 - \mu_D))} \right) = \frac{(\sigma_\eta^2 + d^2)(1 - \mu_D)(1 - \rho_{v_D, \omega_F}^2) \sigma_u^2 (\gamma^2 \mu_D (1 + \rho_{v_D, \omega_F}^2) + (\sigma_\eta^2 + d^2)(1 - \rho_{v_D, \omega_F}^2) \sigma_u^2)}{(\gamma^2 \mu_D^2 \rho_{v_D, \omega_F}^2 + \sigma_u^2 \text{Var}[v_D | \delta_D, \omega_F] (1 - \rho_{v_D, \omega_F}^2 (1 - \mu_D)))^2} > 0.$$

As a result, the overall impact of bilateral spillover effects is in general ambiguous and we obtain the following result

**Corollary 2.** *If  $\mu_j < 1$ , a sufficient condition for the existence of positive illiquidity spillovers across markets is that  $\sigma_u^2 > \gamma^2$  and  $\text{Var}[v_D | \delta_D] \geq 1$ .*

The set of conditions of the above Corollary (a high dispersion in noise traders' demand together with a high coefficient of risk aversion and a high uncertainty on the asset payoff in market  $D$ ) make sure that the adverse selection effect is weak compared to the learning effect. As a result, an increase (decrease) in market  $j$ 's illiquidity yields a corresponding increase (decrease) in the illiquidity of market  $-j$ .

Hence, our model can generate positive co-movements in liquidity across securities. These comovements are usually explained by a tightening of financial constraints for liquidity providers acting simultaneously in several markets (see for instance Brunnermeier and Pedersen (2009)). The reason for our finding is different since, in our framework, market-makers acting in each security are distinct and face no financial constraints. Instead, when a market becomes illiquid, its price as a source of information is less reliable. Thus, uncertainty for market-makers in related securities increases and their liquidity drops as well.

With this in mind in the next section we simplify our analysis, concentrating on the case  $\sigma_\eta = 0$ . This enables us to provide a more accurate analytical characterization of the equilibrium in the two markets.

## 4 Symmetric markets

In this section we analyze the case of maximum interdependence between the two payoffs, setting  $\sigma_\eta = 0$  and  $d = 1$ , so that informational spillovers exist across *both* markets, and allowing  $0 \leq \mu_j \leq 1$ . In this case, the two assets are identical but traded in different markets, as in Chowdry and Nanda (1991), and we obtain the following lemma:<sup>14</sup>

**Lemma 2.** *If  $\sigma_u^2 > 4\gamma^2$  then there exists at least one linear rational expectations equilibrium (i.e., the system of equations (14) admits at least one solution) with  $B_j > 0$ . Moreover, in every equilibrium  $B_j^* \in ((\gamma^2 \mu_j + \sigma_u^2)/\gamma, 1/\gamma)$ .*

<sup>14</sup>If investors could submit limit orders in both assets simultaneously, arbitrage would preclude any deviations in prices between the two securities. Grossman (1990), Section 4 explains why this is difficult in reality. In any case, this problem does not arise if  $f \neq d$  and/or  $\sigma_\eta^2 > 0$ .

Thus, in the presence of adverse selection, the set of parameter values for which a linear rational expectations equilibrium exists is non empty. A second question relates to the cardinality of the equilibrium set. In Section 3.2 we argued that a decrease in  $\sigma_\eta$  makes the two markets more interdependent. This, in turn, may yield multiple linear rational expectations equilibria. In the symmetric market analyzed in this section,  $\sigma_\eta = 0$ , and it is possible to show that existence of an equilibrium also implies multiplicity. To illustrate this, we focus on the special case in which  $\mu_D = \mu_F = \mu$ . In this case, the two securities are perfectly identical in all respects and it is therefore natural to focus on symmetric equilibria in which  $B_F^*(\mu, \mu) = B_D^*(\mu, \mu) = B^*(\mu)$ .

**Corollary 3.** *If  $0 < \mu < 1$  and  $\sigma_u^2 > 4\gamma^2$ , there are two symmetric linear rational expectations equilibria. In these equilibria, the level of illiquidity in each market is either low  $B^*(\mu) = B^{*L}(\mu)$  or high  $B^*(\mu) = B^{*H}(\mu)$  ( $B^{*L}(\mu) \leq B^{*H}(\mu)$ ). Moreover,*

$$\frac{\partial B^{*H}(\mu)}{\partial \mu} < 0, \text{ whereas, } \frac{\partial B^{*L}(\mu)}{\partial \mu} > 0. \quad (19)$$

*In either case market illiquidity is less than when  $\mu_F = \mu_D = 0$  ( $B^{*H}(\mu) < 1/\gamma$ ). When  $\mu = 1$ , we have*

$$B^{*H}(1) = \frac{\sigma_u^2 + \sigma_u(\sigma_u^2 - 4\gamma^2)^{1/2}}{2\gamma\sigma_u^2}, \quad B^{*L}(1) = \frac{\sigma_u^2 - \sigma_u(\sigma_u^2 - 4\gamma^2)^{1/2}}{2\gamma\sigma_u^2}. \quad (20)$$

Thus, when  $\mu_D = \mu_F = \mu > 0$  and  $\sigma_u^2 > 4\gamma^2$ , the model has two equilibria differing in the levels of illiquidity of securities  $D$  and  $F$ . We refer to the equilibrium in which the level of illiquidity is the largest as being the high illiquidity equilibrium. The reason for this result is that the liquidity of security  $j$  depends in part on the belief about this liquidity by pricewatchers active in security  $-j$  and this belief can be self-fulfilling. Indeed, suppose that market-makers in security  $-j$  expect the market for security  $j$  to be illiquid. Then, they expect the price of this security not to be very informative (see equation (13)). As a consequence, they behave as if they had little additional information, which makes the market for security  $-j$  less liquid. But, as a result, the price of this security is not very informative, which works to make security  $j$  illiquid and confirms the belief of market-makers in security  $-j$  about the liquidity of security  $j$ .

To illustrate Corollary 3, consider the following numerical example where we set  $\sigma_u \in \{1, 2\}$  and  $\gamma = 0.49$ .<sup>15</sup> Table 1 below provides equilibrium values for some variables of interest for different values of  $\mu_D$  and  $\mu_F$ .

The numerical example shows that the two equilibria have very different properties. Summarizing:

1. *Impact of an increase in  $\sigma_u^2$  on  $B_j$ .* Other things equal, an increase in the variance of liquidity shocks makes the price of one security less informative for market-makers in the

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<sup>15</sup>We have checked through simulations that the conclusions of this example are not specific to the case in which  $\mu_D = \mu_F = \mu$ . In fact, even when  $\mu_D \neq \mu_F$ , there are typically one high illiquidity equilibrium and one low illiquidity equilibrium displaying qualitative properties similar to those discussed in relation to the above example.

	$\sigma_u^2 = 1$						$\sigma_u^2 = 4$											
	$\mu_D = \mu_F = 0$			$\mu_D = \mu_F = 1/2$			$\mu_D = \mu_F = 1$			$\mu_D = \mu_F = 0$			$\mu_D = \mu_F = 1/2$			$\mu_D = \mu_F = 1$		
	<i>H</i>	<i>L</i>	<i>L</i>	<i>H</i>	<i>L</i>	<i>L</i>	<i>H</i>	<i>L</i>	<i>H</i>	<i>L</i>	<i>H</i>	<i>L</i>	<i>H</i>	<i>L</i>	<i>H</i>	<i>L</i>		
$B_j^*$	2.04, 2.04	1.8, 1.8	0.27, 0.27	1.22, 1.22	0.82, 0.82	2.04, 2.04	1.98, 1.98	0.06, 0.06	1.9, 1.9	0.13, 0.13								
$\text{Var}[\delta_{-j} \omega_{-j}]^{-1}$	1.24, 1.24	1.31, 1.31	14.77, 14.77	1.66, 1.66	2.49, 2.49	1.06, 1.06	1.06, 1.06	64.92, 64.92	1.07, 1.07	15.59, 15.59								
$A_j^*$	0, 0	0.13, 0.13	0.9, 0.9	0.4, 0.4	0.6, 0.6	0, 0	0.03, 0.03	0.97, 0.97	0.06, 0.06	0.94, 0.94								

Table 1: Illiquidity, insiders' precision and cross-security price pressure in the symmetric model for  $j \in \{D, F\}$ , when  $\sigma_u \in \{1, 2\}$ , and  $\gamma = 0.49$ . For  $\mu_j > 0$  we display the equilibrium parameter values for the high and low illiquidity equilibrium (respectively denoted by  $H$  and  $L$ ).

other security (see equation (13)). Thus, one would expect securities to become more illiquid in equilibrium. This is indeed the case in the high illiquidity equilibrium. For instance, when  $\mu_D = \mu_F = 1/2$ ,  $B^{*H}(0.5) = 1.8$  when  $\sigma_u^2 = 1$  and  $B^{*H}(0.5) = 1.98$  when  $\sigma_u^2 = 4$ . However, this intuition does not apply in the low illiquidity equilibrium. In this case, counter-intuitively, illiquidity decreases with  $\sigma_u^2$ . For instance, when  $\mu_D = \mu_F = 1/2$ ,  $B^{*L}(0.5) = 0.27$  when  $\sigma_u^2 = 1$  and  $B^{*L}(0.5) = 0.06$  when  $\sigma_u^2 = 4$ . The fact that the liquidity of one security depends in part on the beliefs of pricewatchers active in the other market is key here. Indeed suppose that market-makers believe that illiquidity decreases after an increase in  $\sigma_u^2$ . If they expect the reduction in illiquidity to be large enough so that  $B^{*L}\sigma_u^2$  decreases then price informativeness increases (see equation (13)) and market-makers' beliefs about the evolution of liquidity is self-fulfilling.

2. *Impact of an increase in  $\mu_j$  on  $B_j$ .* The spillover mechanism described in Corollary 2 suggests that illiquidity in both markets should be reduced when  $\mu_D$  and  $\mu_F$  increase. Table 1 and the second part of Corollary 3 show that this intuition is correct in the high illiquidity equilibrium. For instance, when  $\sigma_u^2 = 1$ ,  $B^{*H}(0.5) = 1.8$  and  $B^{*H}(1) = 1.22$ . However, the intuition does not apply in the low illiquidity equilibrium where the illiquidity increases in the fraction of pricewatchers. For instance, when  $\sigma_u^2 = 1$ ,  $B^{*L}(0.5) = 0.27 < B^{*L}(1) = 0.82$ . Again, the reason for this difference is that liquidity is in part driven by market-makers' beliefs.

The above example suggests that an increase in the fraction of insiders in one market negatively affects the equilibrium level of illiquidity in that market along the high illiquidity equilibrium. The following result formally proves this fact. Let

$$\kappa = \left( 1 - \left( \frac{\partial f(B_{-j}^*, \mu_j)}{\partial B_{-j}} \right) \left( \frac{\partial f(B_j^*, \mu_{-j})}{\partial B_j} \right) \right)^{-1}.$$

Observe that either  $\kappa \geq 1$  or  $\kappa < 0$  since

$$\left( \frac{\partial f(B_{-j}^*, \mu_j)}{\partial B_{-j}} \right) \left( \frac{\partial f(B_j^*, \mu_{-j})}{\partial B_j} \right) > 0.$$

Moreover,  $\kappa = 1$  if  $\mu_{-j} = 0$  because in this case  $(\partial f(B_j^*, \mu_{-j})/\partial B_j) = 0$ . If  $\mu_{-j} > 0$ , then  $\kappa \neq 1$ , and we obtain the following result.<sup>16</sup>

**Corollary 4.** *If  $\kappa \geq 1$ , the effect of a change in the fraction of insiders on the equilibrium level of illiquidity in market  $j$  is given by*

$$\frac{dB_j^*}{d\mu_j} = \kappa \frac{\partial f(B_{-j}^*, \mu_j)}{\partial \mu_j}. \quad (21)$$

*If  $\sigma_u^2 \geq 4\gamma^2$  and  $\kappa \geq 1$ , an increase in the fraction of insiders in market  $j$  reduces the illiquidity of this market ( $(dB_j^*/d\mu_j) < 0$ ) and the illiquidity of market  $-j$  ( $(dB_{-j}^*/d\mu_j) < 0$ ).*

<sup>16</sup>Given the symmetry of the current setup we use  $f(\cdot)$  to denote both the function that determines  $B_j$  as a function of  $B_{-j}$  and  $\mu_j$  and the one that determines  $B_{-j}$  as a function of  $B_j$  and  $\mu_{-j}$ .

To understand this finding, consider first the effect of a small increase in the fraction of insiders in market  $j$ , for a *fixed level of the illiquidity in the other market*. This effect is measured by  $(\partial f(B_{-j}^*, \mu_j) / \partial \mu_j)$ . We call this effect the *direct* effect of a change in the fraction of insiders in market  $j$  since it ignores the fact that a change in this fraction also affects the illiquidity of the other market. The sign of this direct effect is determined by two opposite forces. On the one hand, an increase in the fraction of insiders in market  $j$  raises the level of asymmetric information in this market since insiders are better informed than outsiders. On the other hand, it lowers the level of uncertainty on future payoffs. The first effect raises illiquidity while the second effect decreases illiquidity. The second effect always prevails when  $\sigma_u^2 \geq 4\gamma^2$ .<sup>17</sup> In this case, an increase in attention to security  $-j$  by market-makers in security  $j$  translates first in a higher liquidity for security  $j$ .

Now, if  $\mu_j > 0$  and  $\sigma_u^2 \geq 4\gamma^2$ , the higher liquidity in market  $j$  leads to an increase in the liquidity of market  $-j$  as implied by Corollary 2. This indirect effect of the change in the fraction of insiders in market  $j$  triggers a chain reaction since the higher liquidity in market  $F$  raises the liquidity of market  $j$  which in turn further raises the liquidity of market  $-j$  etc. . . (Corollary 2). This chain reaction amplifies the direct effect of an increase in the fraction of insiders in market  $j$ . If  $\kappa \geq 1$ , the cumulative effect of this chain reaction remains bounded and is equal to  $\kappa$ , which can be seen as the multiplier effect.

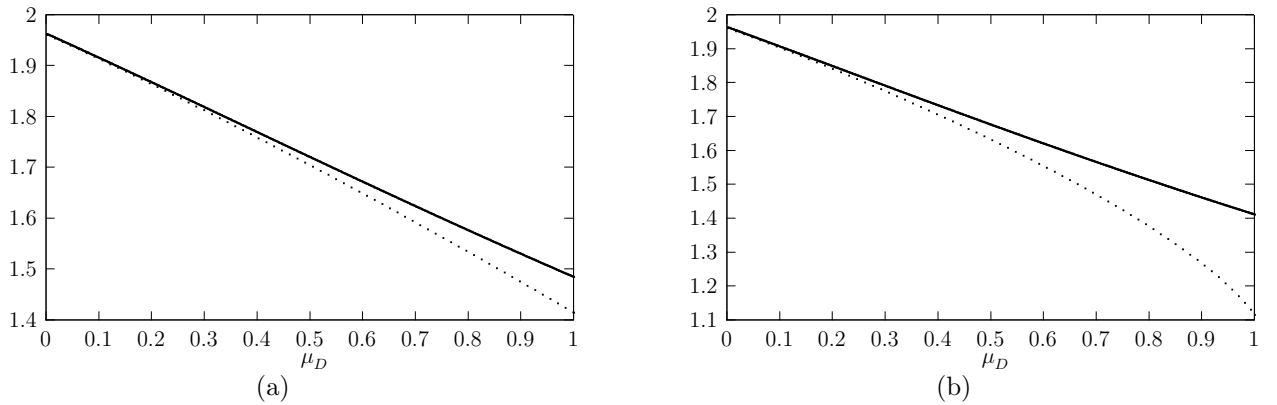


Figure 2: The figure displays the illiquidity of security  $D$  as a function of  $\mu_D$  when  $\mu_F = 0.5$  (in panel (a)) and when  $\mu_F = 0.9$  (panel (b)) when  $B_F$  is fixed at its equilibrium value for  $\mu_D = 0.001$  (bold curve) and when instead it adjusts to its equilibrium value for each value of  $\mu_D$  (dotted curve). Parameters' values are as follows:  $\sigma_u = 1$  and  $\gamma = 0.49$ .

Numerical simulations reveal that the size of the multiplier increases in both  $\mu_F$  and  $\mu_D$  (when  $\mu_F > 0$  and  $\mu_D > 0$ ). As an illustration, consider Figure 2. In this figure we plot the illiquidity of security  $D$  as a function of  $\mu_D \in [0.001, 1]$  when  $\mu_F = 0.5$  (panel (a)) and  $\mu_F = 0.9$  (panel (b)) when  $B_F$  is fixed at its equilibrium value for  $\mu_D = 0.001$  (bold curve) and when  $B_F$  adjusts to its equilibrium value for each value of  $\mu_D$  (dotted curve). Equivalently, the

<sup>17</sup>Again the condition  $\sigma_u^2 > \gamma^2$  is sufficient but not necessary. The same findings can be obtained for  $\mu$  high enough.

dotted (bold) curve depicts the equilibrium level of  $B_D$  as  $\mu_D$  varies taking (without taking) into account the multiplier effect. Thus, the difference between the two curves quantifies the size of the multiplier. Clearly this difference is higher when  $\mu_D$  increases and  $\mu_F$  is higher (compare panels (a) and (b)).

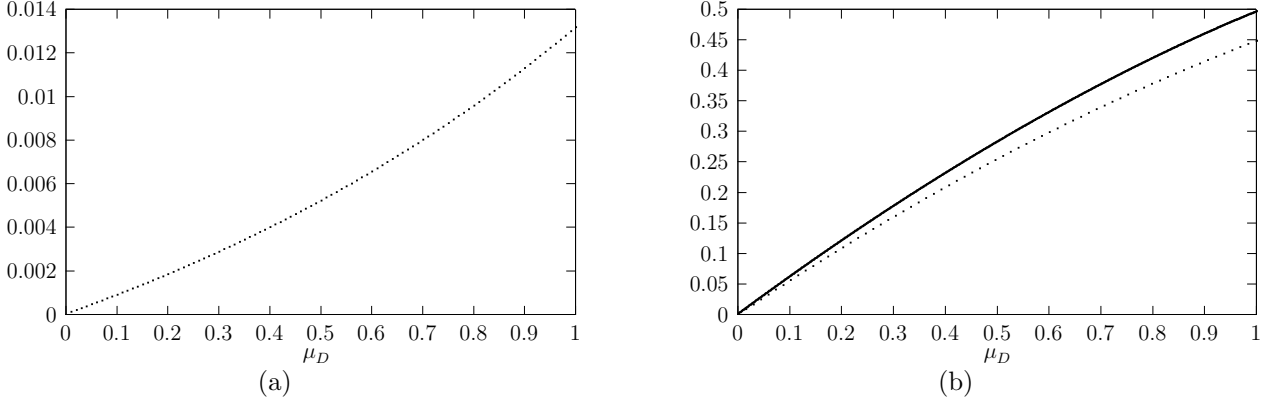


Figure 3: Panel (a) in the above figure displays the covariance between the illiquidity in the  $F$  and  $D$  markets as a function of  $\mu_D$  in the high illiquidity equilibrium. Parameters' values are as follows:  $\sigma_u = 1$  and  $\gamma = 0.7$ . Panel (b) displays the evolution of cross-security price pressure from market  $F$  to market  $D$  as a function of  $\mu_D$  when  $\mu_F = 0.5$  (dotted curve) and when  $\mu_F = 0.9$  (bold curve). Parameters' values are as follows:  $\sigma_u = 1$  and  $\gamma = 0.49$ .

Corollary 4 implies that variations in the fraction of pricewatchers in one market will generate positive covariations in the illiquidity of both markets. To show this point, we ran the following numerical simulation when  $\sigma_u = 1$  and  $\gamma = 0.7$ . In this case, two equilibria are obtained and we focus on the high illiquidity equilibrium (for which  $\kappa \geq 1$ ). For a given value of  $\mu_D$ , we compute the illiquidity of securities  $F$  and  $D$  for  $\mu_F$  varying in  $[0, 1]$ . Then, assuming that  $\mu_F$  is uniformly distributed over  $[0, 1]$ , we compute the covariance between  $B_D^*(\mu_D, \mu_F)$  and  $B_F^*(\mu_F, \mu_D)$ . Figure 3 (panel (a)) reports the value of this covariance when  $\mu_D$  increases from 0 to 1 when  $\sigma_u = 1$  and  $\gamma = 0.7$ . Clearly the comovement between the illiquidity of securities  $F$  and  $D$  is positive and increasing in the fraction of pricewatchers in market  $D$ .

**Cross-security price pressures.** Equation (6) implies that a liquidity shock ( $u_{-j}$ ) in one market will move the price in the other market. We refer to this effect as a cross-security price pressure effect. Formally, this cross-security price pressure can be measured by  $(\partial p_j / \partial u_{-j})$ . Using equation (6), we obtain

$$\frac{\partial p_j}{\partial u_{-j}} = A_j B_{-j} = \frac{\gamma \mu_j B_j^*(\mu_j, \mu_{-j})}{(B_{-j}^*(\mu_{-j}, \mu_j)) \sigma_u^2} > 0, \quad (22)$$

where the second equality follows from the expression for  $A_j$  in Proposition 1. Thus, a liquidity shock in market  $-j$  will affect prices in market  $j$ , even though market-makers operating in each market are distinct and liquidity shocks are uncorrelated across assets. Again, the reason is that market-makers in market  $j$  infer information from the price in market  $-j$  and they cannot

tell whether the price change is due to a liquidity shock (which is uninformative) or a change in the information of market-makers in market  $-j$ .

The model generates two predictions regarding the size of the cross price pressure effect in market  $j$ . First, it should decrease in the illiquidity of market  $-j$  other things equal as shown by equation (22). Second, it should increase both in the fraction of insiders in market  $j$  and the fraction of insiders in market  $-j$  as shown in Figure 3 (panel (b)).<sup>18</sup>

As explained in the introduction, cross-price pressure effects arise in other models of cross-market trading, in particular King and Wadhvani (1990), Bernhardt and Taub (2009) and Pasquariello and Vega (2009). But these predictions are specific to our model.

**Effects of transparency.** The model can also be used to shed light on the effects of increased market transparency. To this end, consider the case in which initially the market for securities  $F$  and  $D$  is very opaque in the sense that no market-makers for a given security can observe the price for the security (i.e.,  $\mu_j = 0$ ). An increase in transparency *for both securities* means that it becomes easier for market-makers in one security to observe prices in the other security. Thus, it corresponds to an increase in  $\mu_j$ . Corollary 3 implies that this increase should result in a lower illiquidity of both markets as information flows between the securities reduce market-makers' uncertainty on securities' values. Moreover, it should raise the tendency for the liquidity of securities  $F$  and  $D$  to comove, as explained previously.<sup>19</sup>

The first prediction is consistent with empirical findings regarding the effects of greater transparency in the U.S. corporate bond markets. In 2002, the Nasdaq began to publicly report transactions for a subset of all corporate bonds listed in the U.S. Several studies (namely, Bessembinder et al. (2006), Edward et al. (2005), Goldstein et al. (2007)) show that this increase in transparency was followed by a significant decline in trading costs for the bonds affected by the program relative to other comparable bonds. These empirical findings are usually interpreted in light of bargaining models. Transparency helps investors to be better informed on current market conditions and thereby to obtain better prices from market-makers. Bessembinder et al. (2006) offers a different explanation. They argue that the decline in trading costs following the increased transparency in the U.S. bond market is due to an improvement in the precision with which bonds' values are estimated, as we do here. However, they do not relate the informational content of bond prices to their illiquidity, which is key for our second prediction: after the dissemination of bond prices in US markets, co-movements in liquidity should be greater because an increase in liquidity for one bond makes its price more informative to price other bonds and therefore increases the liquidity of other bonds.

## 4.1 Endogenous segmentation

In our model, the markets for securities  $F$  and  $D$  will appear to be fully segmented if the fraction of pricewatchers in both markets is zero. In contrast, they will be strongly inter-related (e.g., co-movement in illiquidities and cross-price pressures will be high) if all market-makers

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<sup>18</sup>This is clear when  $\mu_F = \mu_D = \mu$  since in this case  $B_j^*(\mu_j, \mu_{-j}) = B_{-j}^*(\mu_{-j}, \mu_j)$ . Thus  $\partial p_j / \partial u_{-j} = (\gamma \mu_j / \sigma_u^2)$ .

<sup>19</sup>These two implications do not depend on whether we focus on the high or the low illiquidity equilibria.

are pricewatchers. Thus, the degree of market segmentation is determined by the fraction of pricewatchers and therefore it depends on market-makers' monitoring decision.

Intuitively, one would expect the fraction of pricewatchers to increase and securities to be less segmented as the cost of monitoring prices declines. Our main finding in this section is that this intuition does not necessarily apply. Indeed, as the cost of monitoring declines, there is a range of cost for which there are two equilibria: an equilibrium in which the markets for securities  $F$  and  $D$  are fully segmented and an equilibrium in which the markets for securities  $F$  and  $D$  are fully integrated. Thus, the level of segmentation between the two markets can remain high even if the cost of monitoring prices declines and appears relatively low.

The reason for this finding is as follows. Intuitively, the value of monitoring the price of security  $-j$  increases in the informativeness of this price. Now, as explained previously, this informativeness is higher when the illiquidity of security  $-j$  is lower. Thus, an increase in the fraction of pricewatchers in security  $-j$  makes the value of monitoring the price of this security higher, since it is associated with an increase in its liquidity (Corollary 4). For this reason, an increase in the fraction of pricewatchers in one market creates a positive externality for pricewatchers in the other market. As pricewatchers do not internalize this cross-market externality in making their monitoring decisions, market-makers can be trapped in a low attention equilibrium, even if the cost of monitoring becomes small. We now formalize this intuition.

Extending the definition given in Section 3.1, the value of information in market  $j$  is given by

$$\phi_j(\mu_j, B_{-j}) = \frac{\gamma}{2} \ln \frac{\text{Var}[v_j | \delta_j, \hat{\omega}_j]}{\text{Var}[v_j | \delta_j, \omega_{-j}]} > 0. \quad (23)$$

Using Proposition 1, we obtain the following corollary.

**Corollary 5.** *The value of monitoring the price of security  $-j$  for market-makers in security  $j$  is:*

$$\phi_j(\mu_j, B_{-j}) = \frac{\gamma}{2} \ln \left( 1 + \frac{B_{-j}^2 \sigma_u^4}{\gamma^2 \mu_j^2 (1 + B_{-j}^2 \sigma_u^2) + B_{-j}^4 \sigma_u^6} \right). \quad (24)$$

*This value decreases in the fraction of pricewatchers in market  $j$  and decreases in the illiquidity of market  $-j$  when  $\sigma_u > \mu_j(\gamma^2 \mu_j / \sigma_u + 1)$ .*

According to (24), *other things equal*, an increase in the fraction of pricewatchers in market  $j$  reduces the value of being a pricewatcher in this market. This is the same effect which we discussed in Section 3.1. We refer to this effect as the “information leakage effect.” A second determinant of the value of monitoring the price of market  $-j$  in real time is the illiquidity of market  $-j$ . As  $B_{-j}$  increases, the price in market  $-j$  is less informative (see equation (13)). As a consequence, pricewatchers in market  $j$  form a less precise forecast of factor  $\delta_{-j}$ . This effect tends to decrease the value of being a pricewatcher. However, as pricewatchers have a less precise information, they trade less aggressively on this information and therefore the price of security  $j$  becomes less informative about factor  $\delta_{-j}$ . This effect tends to increase the value of monitoring security  $-j$ . The first effect dominates when  $\sigma_u > \mu_j(\gamma^2 \mu_j / \sigma_u + 1)$  and therefore

in this case  $\partial\phi_j(\mu_j, B_{-j})/\partial B_{-j} < 0$ .<sup>20</sup> We refer to this effect as the “illiquidity effect.”

We now compare the value of price information in two polar cases: (a)  $\mu_D = \mu_F = 1$  and (b)  $\mu_D = \mu_F = 0$ . We refer to the first case as the high attention regime and the second case as the low attention regime as  $\mu_j$  can be seen as the level of attention by market-makers in market  $j$  to prices in market  $-j$ . When  $\mu_D = \mu_F = 1$ , we focus on the high illiquidity equilibrium.

Recall that, in equilibrium, market liquidity is higher in the high attention regime than in the no attention regime since  $B^{*H}(1) < B^*(0)$  (see Corollary 3). Thus, both the illiquidity effect and the leakage effect are maximal in the high attention regime. As they have opposite impacts on the value of monitoring prices, this value can be higher or smaller in the high attention regime. However, the next lemma shows that its is always higher in the high attention regime.

**Lemma 3.** *The value of monitoring prices in market  $-j$  for market-makers in market  $j$  is strictly higher in the high attention regime than in the low attention regime:*

$$\phi_j(1, B^{*H}(1)) > \phi_j(0, B^*(0)).$$

Let  $C$  be the cost for market-makers in one security to monitor the price in the other security. In centralized markets with real time dissemination of price information, this cost includes the fee charged by data vendors and the time cost of monitoring this information. In OTC markets,  $C$  represents the cost of searching for price information as it is not readily available in real time. Now suppose that the fractions of investors in markets  $D$  and  $F$  are respectively  $\mu_D$  and  $\mu_F$  and consider whether a market-maker in market  $j$  should become an insider. Her decision rule is simple. If  $\phi_j(\mu_j, B_{-j}^*(\mu_D, \mu_F)) > C$ , it is optimal for the market-maker to be an insider since the value of monitoring the price in the other market is higher than the cost. If  $\phi_j(\mu_j, B_{-j}^*(\mu_D, \mu_F)) < C$ , it is optimal for the market-maker not to monitor the price in the other market and finally for  $\phi_j(\mu_j, B_{-j}^*(\mu_D, \mu_F)) = C$ , the market-maker is just indifferent. Given these observations, the equilibrium fractions of insiders in each market,  $(\mu_D^*, \mu_F^*)$ , will be:

$$\begin{aligned} \mu_j^* &= 1 \text{ if } \phi_j(1, B_{-j}^*(1, 1)) > C \text{ for } j \in \{D, F\} \\ \mu_j^* &= 1 \text{ and } 0 < \mu_{-j}^* < 1 \text{ if } \phi_j(1, B_{-j}^*(1, \mu_{-j}^*)) > C \text{ and } \phi_{-j}(\mu_{-j}^*, B_{-j}^*(1, \mu_{-j}^*)) = C \\ 0 &< \mu_j^* < 1 \text{ and } 0 < \mu_{-j}^* < 1 \text{ if } \phi_j(\mu_j^*, B_{-j}^*(\mu_j^*, \mu_{-j}^*)) = C \text{ for } j \in \{D, F\} \\ \mu_j^* &= 0 \text{ and } 0 < \mu_{-j}^* < 1 \text{ if } \phi_j(0, B_{-j}^*(0, \mu_{-j}^*)) < C \text{ and } \phi_{-j}(\mu_{-j}^*, B_{-j}^*(1, \mu_{-j}^*)) = C \\ \mu_j^* &= 0 \text{ and } \mu_{-j}^* = 0 \text{ if } \phi_j(0, B_{-j}^*(0, 0)) = C \end{aligned}$$

Observe that  $\phi_j(1, B^{*H}(1))$  and  $\phi_j(0, B^*(0))$  are independent of  $j$  because when  $\mu_D = \mu_F$ , the two market are symmetric. Hence, let  $\phi_1 \stackrel{def}{=} \phi_j(1, B^{*H}(1))$  and  $\phi_0 \stackrel{def}{=} \phi_j(0, B^*(0))$ . We obtain the following result.

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<sup>20</sup>To establish this condition, we consider variations in  $B_{-j}$  only in the interval  $\left[\frac{\gamma\sigma_\delta^2}{\gamma^2\mu_j + \sigma_\delta^2\sigma_u^2}, \frac{\sigma_\delta^2}{\gamma}\right]$  since equilibrium values of  $B_{-j}^*$  are in this interval.

**Proposition 2.** *If  $\phi_0 < C < \phi_1$ ,  $\mu_D^* = \mu_F^* = 1$  and  $\mu_D^* = \mu_F^* = 0$  are two possible equilibrium levels of attention when market-makers' monitoring decisions are endogenous.*

**Proof.** Suppose that  $\mu_D^* = \mu_F^* = 1$ . Then in this case, the value of monitoring market  $j$  for a market-maker in market  $-j$ , given the actions of other market-makers, is  $\phi_j(1, B^{H^*}(1)) = \phi_1$ . As this value is higher than  $C$ , monitoring is optimal. Hence  $\mu_D^* = \mu_F^* = 1$  is an equilibrium. Now suppose that  $\mu_D^* = \mu_F^* = 0$ . Then in this case, the value of monitoring market  $j$  for a market-maker in market  $-j$ , given the actions of other market-makers, is  $\phi_j(0, B^*(0)) = \phi_0$ . As this value is lower than  $C$ , not monitoring is optimal. Hence  $\mu_D^* = \mu_F^* = 1$  is an equilibrium.  $\square$

Thus, for the *same* parameters value, the high and no attention regimes can emerge in equilibrium. In other words, the markets for the two securities can appear strongly inter-related or fully segmented. As an illustration, suppose that  $\sigma_\delta = \sigma_u = 1$ ,  $\gamma = 1/4$  as in Table 1. In this case, we get:  $\phi_1 = 0.007$  and  $\phi_0 = 0.34$ . Thus, for any value of  $C \in [0.007, 0.34]$ , the two markets can look either fully segmented or fully integrated, depending on whether market-makers in both markets coordinate on the high or the low attention equilibria. The liquidity of both markets and the informativeness of prices are higher if market-makers coordinate on the high attention equilibrium ( $B^{H^*}(1) = 3.73$  and  $B^*(0) = 4$  – see Table 1). Interestingly this coordination problem can arise even if the cost of monitoring prices is relatively low.

## 5 The general case

In this section we investigate, using numerical simulations, the generality of the results obtained in the symmetric case.

**Positive illiquidity spillovers.** Figure 4 displays the illiquidity of security  $D$  as a function of  $\mu_D$  when  $\mu_F = 0.5$  (in panel (a)) and when  $\mu_F = 0.9$  (panel (b)) when  $B_F$  is fixed at its equilibrium value for  $\mu_D = 0.001$  (bold curve) and when instead it adjusts to its equilibrium value for each value of  $\mu_D$  (dotted curve). Parameters' values are as follows:  $\sigma_u = 1$ ,  $\sigma_\eta = 0.77$  and  $\gamma = 1$ .

**Strength of the positive feedback effect.** From our discussion in Section 4.1 we know that  $\phi_j(\mu_j, B_{-j})$  can be increasing in  $\mu_j$ . However, as the model considered so far concentrates on the symmetric solution, we have obtained this result only for  $\mu_D = \mu_F = \mu$ . In this section we disentangle the feedback effects by assuming  $\mu_F$  fixed and looking at the impact that an increase in  $\mu_D$ , through the effect it has on  $B_F$ , eventually has on  $\phi_D(\mu_D, B_F)$ . If the leakage effect is to be tamed by the positive externality coming from more aggressive pricing in market  $F$ , then we should expect that  $\phi_D(\mu_D, B_F)$  is increasing in  $\mu_D$ . In Figure 5 we plot the value of information in the two markets for a given level of attention in market  $F$  (i.e.,  $\mu_F \in \{.1, .9\}$ ). According to panel (a) and panel (b) as the level of attention increases in market  $D$ , the value of information increases in *both* markets, confirming that the positive feedback effect can make

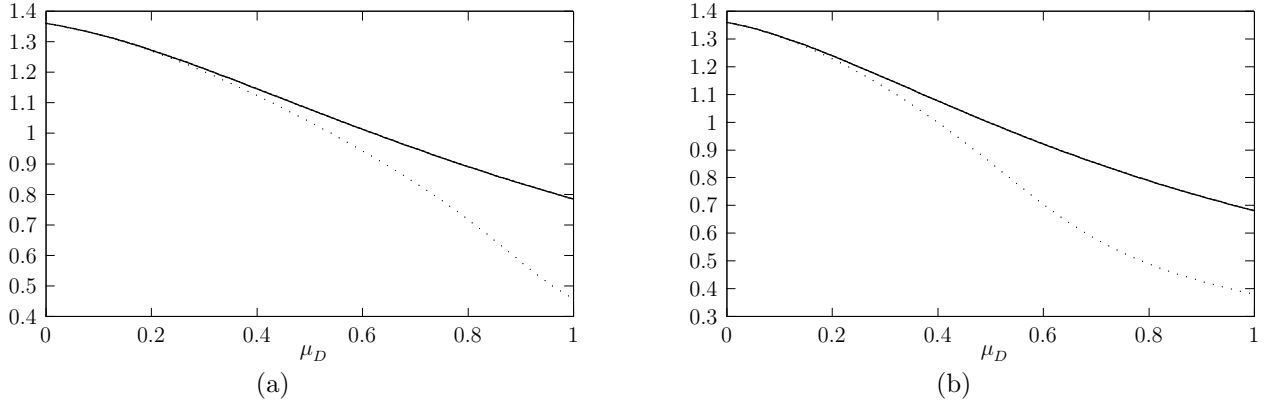


Figure 4: The figure displays the illiquidity of security  $D$  as a function of  $\mu_D$  when  $\mu_F = 0.5$  (in panel (a)) and when  $\mu_F = 0.9$  (panel (b)) when  $B_F$  is fixed at its equilibrium value for  $\mu_D = 0.001$  (bold curve) and when instead it adjusts to its equilibrium value for each value of  $\mu_D$  (dotted curve). Parameters' values are as follows:  $\sigma_u = 1$ ,  $\sigma_\eta = 0.77$  and  $\gamma = 1$ .

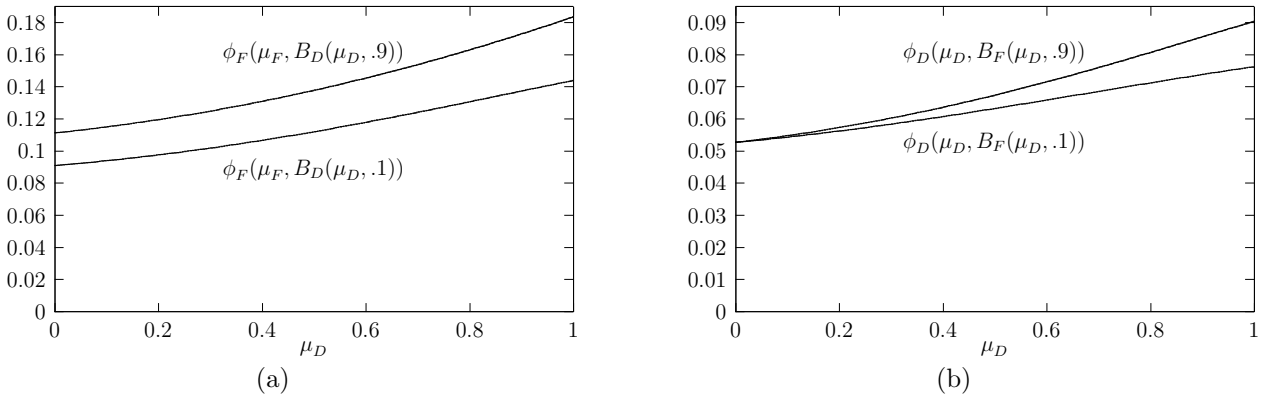


Figure 5: The figure illustrates the relevance of the positive feedback effect on the value of information in the two markets. In panel (a) we plot  $\phi_F$  as a function of  $\mu_D$ , for  $\mu_F \in \{0.1, 0.9\}$ . In panel (b) we plot  $\phi_D$  as a function of  $\mu_D$ , for  $\mu_F \in \{0.1, 0.9\}$ . Other parameter values are as follows:  $\sigma_\eta = 1$ ,  $\sigma_u = 1$ ,  $\gamma = 1$ , and  $d = 1$ .

the value of price information higher in the high attention regime compared to the low attention regime.

## 6 Conclusions

*To be written*

# Appendix

## Proof of Proposition 1

Let us assume that at any linear equilibrium, the price in market  $j$  has the form  $p_j = \delta_j + B_j u_j + A_j \omega_{-j}$ , where  $\omega_{-j} = \delta_{-j} + B_{-j} u_{-j}$ , and  $B_j, A_j$  are parameters to be determined. Note that this implies that  $\{\delta_j, \omega_{-j}\}$  is a sufficient statistic for (observationally equivalent to)  $\{\delta_j, p_{-j}, p_j\}$  and, defining  $\hat{\omega}_j = B_j u_j + A_j \omega_{-j}$ ,  $\{\delta_j, \hat{\omega}_j\}$  is a sufficient statistic for  $\{\delta_j, p_j\}$ .

### Step 1. Equilibrium in market $j$ .

**Insiders.** An insider's demand function in market  $j$ ,  $x_j^I(\delta_j, p_j, p_{-j})$ , maximizes

$$E \left[ -\exp \left\{ - \left( (v_j - p_j) x_j^I \right) / \gamma \right\} \mid \delta_j, p_j, p_{-j} \right].$$

We deduce that

$$\begin{aligned} x_j^I(\delta_j, p_j, p_{-j}) &= \gamma \frac{E[v_j \mid \delta_j, p_{-j}, p_j] - p_j}{\text{Var}[v_j \mid \delta_j, p_{-j}]} \\ &= a_j^I (E[v_j \mid \delta_j, p_{-j}, p_j] - p_j), \end{aligned}$$

with  $a_j^I = \text{Var}[v_j \mid \delta_j, p_{-j}]^{-1}$ . To fix ideas, suppose  $j = D$ . As  $\{\delta_D, \omega_F\}$  is a sufficient statistic for  $\{\delta_D, p_F, p_D\}$ , we deduce (using well-known properties of normal random variables) that

$$\begin{aligned} E[v_D \mid \delta_D, p_F, p_D] &= E[v_D \mid \delta_D, \omega_F] \\ &= \delta_D + d \frac{1}{(1 + B_{-j}^2 \sigma_u^2)} \omega_F, \end{aligned} \quad (25)$$

and

$$\begin{aligned} a_D^I &= \gamma \text{Var}[v_D \mid \delta_D, \omega_F]^{-1} \\ &= \gamma \left( \frac{1 + B_F^2 \sigma_u^2}{d^2 B_F^2 \sigma_u^2 + \sigma_\eta^2 (1 + B_F^2 \sigma_u^2)} \right). \end{aligned} \quad (26)$$

Thus,

$$x_D^I(\delta_D, \omega_F) = a_D^I (\delta_D - p_D) + b_D^I \omega_F, \quad (27)$$

where

$$b_D^I = d a_D^I \frac{\sigma_\delta^2}{\sigma_\delta^2 + B_F^2 \sigma_u^2}. \quad (28)$$

Similarly, for market  $F$  we obtain

$$x_F^I(\delta_F, \omega_D) = a_F^I (\delta_F - p_F) + b_F^I \omega_D, \quad (29)$$

where  $\omega_D = \delta_D + B_D u_D$ , and

$$a_F^I = \gamma \left( \frac{1 + B_D^2 \sigma_u^2}{B_D^2 \sigma_u^2} \right), \quad b_F^I = a_F^I \frac{1}{1 + B_D^2 \sigma_u^2}, \quad (30)$$

implying that  $B_D > 0$  for an equilibrium to be well defined.

**Outsiders.** An outsider's demand function in market  $j$ ,  $x_j^O(\delta_j, p_j)$ , maximizes:

$$E \left[ -\exp \left\{ - \left( (v_j - p_j) x_j^O \right) / \gamma \right\} \mid \delta_j, p_j \right].$$

We deduce that

$$\begin{aligned} x_j^O(\delta_j, p_j) &= \gamma \frac{E[v_j \mid \delta_j, p_j] - p_j}{\text{Var}[v \mid \delta_j, p_j]} \\ &= a_j^O (E[v_j \mid \delta_j, p_j] - p_j), \end{aligned}$$

with  $a_j^O = \text{Var}[v_j \mid \delta_j, p_j]^{-1}$ . Define  $\hat{\omega}_j = A_j \omega_{-j} + B_j u_j$ . As  $\{\delta_j, \hat{\omega}_j\}$  is a sufficient statistic for  $\{\delta_j, p_j\}$ , fixing  $j = D$ , we deduce (using well-known properties of normal random variables) that

$$\begin{aligned} E[v_D \mid \delta_D, p_D] &= E[v_D \mid \delta_D, \hat{\omega}_D] \\ &= \delta_D + d \frac{A_D \sigma_\delta^2}{A_D^2 (\sigma_\delta^2 + B_D^2 \sigma_u^2) + B_D \sigma_u^2} \hat{\omega}_D, \end{aligned} \quad (31)$$

and

$$\begin{aligned} a_D^O &= \gamma \text{Var}[v_D \mid \delta_D, \hat{\omega}_D]^{-1} \\ &= \gamma \frac{A_D^2 (\sigma_\delta^2 + B_D^2 \sigma_u^2) + B_D^2 \sigma_u^2}{d^2 \sigma_u^2 \sigma_\delta^2 (A_D^2 B_D^2 + B_D^2) + \sigma_\eta^2 (A_D^2 (\sigma_\delta^2 + B_D^2 \sigma_u^2) + B_D^2 \sigma_u^2)}. \end{aligned} \quad (32)$$

Thus,

$$x_D^O(\delta_D, \hat{\omega}_D) = a_D^O (\delta_D - p_D) + b_D^O \hat{\omega}_D, \quad (33)$$

where

$$b_D^O = d a_D^O \frac{A_D \sigma_\delta^2}{A_D^2 (\sigma_\delta^2 + B_D^2 \sigma_u^2) + B_D^2 \sigma_u^2}. \quad (34)$$

Similarly, for market  $F$  we obtain:

$$x_F^O(\delta_F, \hat{\omega}_F) = a_F^O (\delta_F - p_F) + b_F^O \hat{\omega}_F, \quad (35)$$

where

$$a_F^O = \gamma \frac{A_F^2 (\sigma_\delta^2 + B_D^2 \sigma_u^2) + B_F^2 \sigma_u^2}{f^2 \sigma_u^2 \sigma_\delta^2 (A_F^2 B_D^2 + B_F^2)}, \quad b_F^O = f a_F^O \frac{A_F \sigma_\delta^2}{A_F^2 (\sigma_\delta^2 + B_D^2 \sigma_u^2) + B_F^2 \sigma_u^2}. \quad (36)$$

**Clearing price in market  $j$ .** The clearing condition in market  $j$  imposes

$$\int_0^{\mu_j} x_j^I(\delta_j, p_j, p_{-j}) di + \int_{\mu_j}^1 x_j^O(\delta_j, p_j) di + u_j = 0$$

Let  $a_j = \mu_j a_j^I + (1 - \mu_j) a_j^O$ . Using equations (27) and (33), we solve for the equilibrium price and we obtain

$$p_j = \delta_j + \left( \frac{\mu b_j^I + (1 - \mu_j) b_j^O A_j}{a_j} \right) \omega_j + \left( \frac{(1 - \mu_j) b_j^O B_j + 1}{a_j} \right) u_j, \quad (37)$$

Identifying the parameters in the price, in equilibrium, we must have:

$$B_j = \frac{1}{a_j - (1 - \mu_j)b_j^O} \quad (38)$$

$$A_j = \mu_j B_j b_j^I. \quad (39)$$

Hence, at a linear rational expectations equilibrium equations (26), (28), (30), (32), (34), (36), together with (38) and (39) must be satisfied with  $B_j > 0$  (as otherwise the demand function of insiders in market  $F$  is not well defined). As these equations must hold for both markets, we have a system of 12 equations with 12 unknowns. To solve for the equilibrium we use recursive substitution and after standard algebra obtain the expressions (7) and (8) for  $B_D$  and  $B_F$  in the proposition.  $\square$

### Proof of Corollary 1

As argued in the text, both  $B_D$  and  $B_F$  are continuous and strictly increasing in  $B_F$  and  $B_D$  in the intervals  $[0, 1/\gamma]$  and  $[\sigma_\eta^2/\gamma, (1 + \sigma_\eta^2)/\gamma]$ . Hence, they must cross at least once in the region

$$\Theta \equiv \left[0, \frac{1}{\gamma}\right] \cap \left[\frac{\sigma_\eta^2}{\gamma}, \frac{1 + \sigma_\eta^2}{\gamma}\right] \subset \mathbb{R}_+ \times \mathbb{R}_+,$$

which in turn implies (1) the existence of at least one equilibrium with finite depth in  $\Theta$ ; (2) the impossibility of equilibrium existence in any other region  $\mathbb{R}_+ \times \mathbb{R}_+ - \Theta$ .  $\square$

### Proof of Corollary 2

Differentiating (7) with respect to  $\rho_{v_D, \omega_F}^2$  yields

$$-\frac{\text{Var}[v_D|\delta_D]\mu_D}{\gamma(\gamma^2\mu_D^2\rho_{v_D, \omega_F}^2 + \sigma_u^2\text{Var}[v_D|\delta_D, \omega_F](1 - \rho_{v_D, \omega_F}^2(1 - \mu_D)))^2} \times \\ (\gamma^4\mu_D^2\rho_{v_D, \omega_F}^4 + \text{Var}[v_D|\delta_D, \omega_F]\sigma_u^2(\text{Var}[v_D|\delta_D](1 - \rho_{v_D, \omega_F}^2)\sigma_u^2 - \gamma^2(1 - \mu_D - \rho_{v_D, \omega_F}^2(1 + \mu_D)))).$$

An increase in  $B_F$  lowers  $\rho_{v_D, \omega_F}^2$ . Hence, for this to yield an increase in  $B_D$  we need the sign of the above derivative to be negative. Keeping in mind that  $\text{Var}[v_D|\delta_D] = \sigma_\eta^2 + d^2$ , the sufficient condition proposed in the corollary ensures exactly this. Note also that the derivative of  $B_F$  with respect to  $\rho_{v_F, \omega_D}^2$  has a similar expression. In that case, however, owing to our assumptions  $\text{Var}[v_F|\delta_F] = 1$ . Hence, the only requirement for  $B_F$  to be increasing in  $B_D$  is that  $\sigma_u^2 > \gamma^2$ .  $\square$

### Proof of Lemma 2

For fixed values of  $\mu_D$  and  $\mu_F$ , consider the mapping  $\mathbf{F}$  from  $\mathbb{R}^2$  into  $\mathbb{R}^2$  which transforms  $(B_D, B_F)$  into  $\{f(B_D, \mu_D), f(B_F, \mu_F)\}$  and let

$$\Theta = \left[\frac{\gamma\sigma_\delta^2}{\gamma^2\mu_j + \sigma_\delta^2\sigma_u^2}, \frac{\sigma_\delta^2}{\gamma}\right].$$

Note that  $\lim_{B_{-j} \rightarrow \infty} f(B_{-j}, \mu_{-j}) = \sigma_\delta^2 / \gamma$ . Hence, given that for  $\sigma_\delta^2 \sigma_u^2 > 4\gamma^2$ ,  $B_j$  is increasing in  $B_{-j}$ , this implies that  $B_j < \sigma_\delta^2 / \gamma$ , for all  $B_{-j}$ . Furthermore, given that for  $\sigma_\delta^2 \sigma_u^2 > 4\gamma^2$   $B_j$  is decreasing in  $\mu_j$ , it must be that

$$\gamma (B_j \gamma \mu_j - \sigma_\delta^2) + B_j \sigma_\delta^2 \sigma_u^2 > 0 \Leftrightarrow B_j > \frac{\gamma \sigma_\delta^2}{\gamma^2 \mu_j + \sigma_\delta^2 \sigma_u^2}, \quad (40)$$

which yields a lower bound for  $B_j$ . To prove (40) we proceed by contradiction and assume

$$\gamma (B_j \gamma \mu_j - \sigma_\delta^2) + B_j \sigma_\delta^2 \sigma_u^2 < 0 \Leftrightarrow B_j < \frac{\gamma \sigma_\delta^2}{\gamma^2 \mu_j + \sigma_\delta^2 \sigma_u^2}.$$

Given that for  $\sigma_\delta^2 \sigma_u^2 > 4\gamma^2$  we know that  $B_j$  is decreasing in  $\mu_j$ , then it must be that

$$\frac{\gamma \sigma_\delta^2}{\gamma^2 \mu_j + \sigma_\delta^2 \sigma_u^2} < B_j|_{\mu_j=1} \equiv \frac{\sigma_\delta \sigma_u (\sigma_\delta \sigma_u - (\sigma_\delta^2 \sigma_u^2 - 4\gamma^2)^{1/2})}{2\gamma \sigma_u^2}.$$

Rearranging the above expression we obtain

$$2\gamma^2 ((\gamma^2 \mu_j + \sigma_\delta^2 \sigma_u^2) (2\gamma^2 \mu_j + \sigma_\delta^2 \sigma_u^2) + 2\gamma^2 \sigma_\delta^2 \sigma_u^2) < 0,$$

which is clearly impossible. Thus, we know that

$$\mathbf{F} : \Theta \times \Theta \rightarrow \Theta \times \Theta.$$

As  $\Theta$  is compact and  $\mathbf{F}$  is continuous, we deduce from Brower's fixed point theorem that  $\mathbf{F}$  has at least one fixed point. Existence of at least one rational expectations equilibrium follows.  $\square$

## Proof of Corollary 4

**Step 1.** First, consider the effect a small increase in the fraction of insiders in market  $j$ . The direct effect of this small increase on the illiquidity of this market is given by  $(\partial f(B_{-j}^*, \mu_j) / \partial \mu_j)$ . But, in turn, this change in the illiquidity of market  $j$  triggers a change in the illiquidity of market  $-j$  through the spillover mechanism that we just described. The size of this change is

$$\left( \frac{\partial f(B_j^*, \mu_{-j})}{\partial B_j} \right) \left( \frac{\partial f(B_{-j}^*, \mu_j)}{\partial \mu_j} \right).$$

But then the change in the illiquidity of market  $-j$  affects the illiquidity of market  $j$  by an amount

$$\left( \frac{\partial f(B_{-j}^*, \mu_j)}{\partial B_{-j}} \right) \left( \frac{\partial B_{-j}}{\partial B_j} \right) \left( \frac{\partial B_j}{\partial \mu_j} \right),$$

which again affects the illiquidity of market etc... Thus, if

$$\left( \frac{\partial f(B_{-j}^*, \mu_j)}{\partial B_{-j}} \right) \left( \frac{\partial f(B_j^*, \mu_{-j})}{\partial B_j} \right) < 1,$$

the total effect of a small increase in the fraction of insiders in market  $j$  on the illiquidity of this market is given by:

$$\frac{dB_j^*}{d\mu_j} = \kappa \frac{\partial f(B_{-j}^*, \mu_j)}{\partial \mu_j}$$

with

$$\kappa = \left( 1 - \left( \frac{\partial f(B_{-j}^*, \mu_j)}{\partial B_{-j}} \right) \left( \frac{\partial f(B_j^*, \mu_{-j})}{\partial B_j} \right) \right)^{-1}.$$

**Step 2.** Recall that  $B_j = f(B_{-j}, \mu_j)$  for  $j \in \{D, F\}$ . Differentiation  $f(B_{-j}, \mu_j)$  with respect to  $\mu_j$  shows that the sign of  $\frac{\partial f}{\partial \mu_j}$  is identical to the sign of

$$z(\mu_j) \stackrel{def}{=} -\gamma^4(\sigma_\delta^2 + B_{-j}^2\sigma_u^2)\mu_j^2 + B_{-j}^4\sigma_u^6(\gamma^2 - \sigma_u^2\sigma_\delta^2) \\ - \mu_j B_{-j}^2\sigma_u^2(\gamma^2 B_{-j}^2\sigma_u^2(\sigma_\delta^2 + B_{-j}^2\sigma_u^2) + B_{-j}^2\sigma_u^4 + \gamma^2\sigma_\delta^2 B_{-j}^2\sigma_u^4),$$

which is negative if  $\gamma^2 \leq \sigma_u^2\sigma_\delta^2$ . Thus,  $(\partial f(B_{-j}^*, \mu_j)/\partial \mu_j) > 0$  if  $\gamma^2 \leq \sigma_u^2\sigma_\delta^2$ . and therefore  $(dB_j^*/d\mu_j) > 0$  if  $\kappa < 1$ .

**Note:** The function  $z(\cdot)$  decreases in  $\mu_j$  and is strictly negative for  $\mu_j = 1$  and strictly positive for  $\mu_j = 0$  when  $\gamma^2 > \sigma_u^2\sigma_\delta^2$ . Thus, when  $\gamma^2 > \sigma_u^2\sigma_\delta^2$ , there exists  $\bar{\mu}_j$  such that:  $z(\bar{\mu}_j) = 0$ . For  $\mu_j > \bar{\mu}_j$ , we have  $(\partial f/\partial \mu_j) < 0$  and for  $\mu_j < \bar{\mu}_j$ , we have  $(\partial f/\partial \mu_j) > 0$ .  $\square$

### Proof of Corollary 3

In a symmetric equilibrium we have  $B_D^*(\mu) = B_F^*(\mu) = B^*(\mu)$  where, using equations (7) and (8),  $B^*(\mu)$  solves:

$$\varphi(B^*) \equiv (B^{*2}\sigma_u^4 + \gamma^2\mu^2)(\sigma_\delta^2(\gamma\mu - B^*\sigma_u^2) + \gamma\sigma_u^2 B^{*2}) + \gamma^2\sigma_\delta^2\mu(1 - \mu)(\gamma\mu - B^*\sigma_u^2) = 0. \quad (41)$$

**Step 1.** Consider first the case  $\mu = 1$ . In this case, using equation (41),  $B^*$  solves:

$$\sigma_\delta^2(\gamma\mu - B^*\sigma_u^2) + \gamma\sigma_u^2 B^{*2} = 0$$

Thus, there are two possible equilibrium values for  $B^*$ :

$$B^{*H}(1) = \frac{\sigma_\delta^2\sigma_u^2 + \sigma_\delta\sigma_u(\sigma_\delta^2\sigma_u^2 - 4\gamma^2)^{1/2}}{2\gamma\sigma_u^2}, B^{*L}(1) = \frac{\sigma_\delta^2\sigma_u^2 - \sigma_\delta\sigma_u(\sigma_\delta^2\sigma_u^2 - 4\gamma^2)^{1/2}}{2\gamma\sigma_u^2}.$$

Moreover,  $B^{*L}(1) < B^{*H}(1) < B^*(0)$ .

**Step 2.** Now consider the case  $\mu < 1$  and let:

$$B_1 = \frac{\sigma_\delta^2\sigma_u^2 + \sigma_\delta\sigma_u(\sigma_\delta^2\sigma_u^2 - 4\gamma^2\mu)^{1/2}}{2\gamma\sigma_u^2} \\ B_0 = \frac{\sigma_\delta^2\sigma_u^2 - \sigma_\delta\sigma_u(\sigma_\delta^2\sigma_u^2 - 4\gamma^2\mu)^{1/2}}{2\gamma\sigma_u^2},$$

which are well defined since  $\sigma_\delta^2\sigma_u^2 > 4\gamma^2\mu$ . It is immediate that:

$$\varphi(0) = \gamma^3\mu^2\sigma_\delta^2 > 0 \quad (42)$$

$$\varphi(B_1) < 0 \quad (43)$$

$$\varphi(B_0) < 0 \quad (44)$$

As  $B_0 < B_1$  and since  $\varphi(\cdot)$  is a quartic in  $B$ , there exist at least two solutions  $B^{*L}$  and  $B^{*H}$  to  $\varphi(B) = 0$  with:

$$B^{*L} \in (0, B_0) \text{ and } B^{*H} \in (B_1, \infty).$$

We can in fact get tighter intervals for  $B^{*L}$  and  $B^{*H}$ . To see this, observe that

$$\sigma_\delta^2 (\gamma\mu - B^{*L}\sigma_u^2) + \gamma\sigma_u^2(B^{*L})^2 > 0.$$

Thus, at equilibrium it must be that the second component of (41) is negative or that  $B^{*L} > \gamma\mu/\sigma_u^2$ . Given that, as one can verify

$$\frac{\gamma\mu}{\sigma_u^2} < \frac{\sigma_\delta^2\sigma_u^2 - \sigma_\delta\sigma_u(\sigma_\delta^2\sigma_u^2 - 4\gamma^2\mu)^{1/2}}{2\gamma\sigma_u^2},$$

we can conclude that

$$B^* \in \left( \frac{\gamma\mu}{\sigma_u^2}, B_0 \right). \quad (45)$$

Let's turn the attention to the second equilibrium and define:

$$B_2 = \frac{\sigma_\delta^2\sigma_u^2 + \sigma_\delta\sigma_u(\sigma_\delta^2\sigma_u^2 - \gamma^2\mu)^{1/2}}{2\gamma\sigma_u^2}.$$

It is immediate that  $B_2 > B_1$ . Thus, with the additional restriction  $\sigma_\delta^2\sigma_u^2 > 4\gamma^2\mu$ , calculations show that  $\varphi(B_2) > 0$ . Thus, Hence, we can conclude that

$$B^{*H} \in (B_1, B_2). \quad (46)$$

**Step 3.** To prove that  $B^{*H}$  and  $B^{*L}$  are the unique real solutions to  $\varphi(B) = 0$ , we need to show that the function  $\varphi(\cdot)$  changes slope only once in the positive ortant  $\mathbb{R}_{++}$ . This is equivalent to show that the cubic  $\varphi'(B)$  has a unique real root. We thus start by differentiating  $\varphi(B)$ :

$$\varphi'(B) = \sigma_u^2 (2\gamma\mu B(\sigma_u^2\sigma_\delta^2 + \gamma^2\mu) - 3B^2\sigma_u^4\sigma_\delta^2 + 4\gamma B^3\sigma_u^4 - \gamma^2\sigma_\delta^2\mu). \quad (47)$$

The cubic equation  $\varphi'(B) = 0$  possesses a unique real root if and only if its discriminant  $\nabla$  is negative. Setting  $\kappa \equiv \sigma_\delta\sigma_u$  and computing yields:

$$\begin{aligned} \nabla = 4\gamma^2\mu\sigma_u^4 & \left( -32\gamma^8\mu^5 - 96\gamma^6\mu^4\kappa^2 - 3\gamma^4\mu\kappa^4 (36(1-\mu) + 29\mu^2) \right. \\ & \left. - 9\kappa^6 \left( 3(\kappa^2 - 4\gamma^2\mu) - \mu \left( \kappa^2 - \frac{14}{9}\gamma^2\mu \right) \right) \right). \end{aligned}$$

As one can verify, the latter component of the discriminant is negative if and only if

$$\kappa^2 > 6\gamma^2\mu > \frac{2\gamma^2\mu(54-7\mu)}{9(3-\mu)},$$

and the result follows.

**Step 4.** Finally we show that  $(\partial B^{*H}(\mu)/\partial\mu) < 0$  whereas  $(\partial B^{*L}(\mu)/\partial\mu) > 0$ . By implicit differentiation of (41) we have

$$\frac{\partial B}{\partial\mu} = -\frac{\partial\varphi(B)/\partial\mu}{\partial\varphi(B)/\partial B}. \quad (48)$$

Note that according to Step 2, the denominator in the above expression is negative (positive) in the high (low) depth equilibrium. Hence, the sign of  $\partial B/\partial \mu$  depends on the sign of  $\partial \varphi(B)/\partial \mu$ . We obtain

$$\frac{\partial \varphi(B)}{\partial \mu} = \gamma (2\gamma^2 \mu (\sigma_\delta^2 + B^2 \sigma_u^2) + B \sigma_u^2 \sigma_\delta^2 (B \sigma_u^2 - \gamma)). \quad (49)$$

Again, according to Step 2, we know that any symmetric equilibrium of the market has  $B > \gamma \mu / \sigma_u^2$ , for all  $\mu \leq 1$  (when  $\mu = 1$ , direct inspection of the solutions confirms that  $B > \gamma / \sigma_u^2$ ). This implies that the sign of (49) is positive. As a consequence, in the high (low) depth equilibrium the sign of (48) is positive (negative), confirming our claim.  $\square$

**Proof of Corollary 5** *To be written.*

**Proof of Lemma 3**

We have

$$\phi_j(1, B^{*H}(1)) = \frac{\gamma}{2} \ln \left( 1 + \frac{B^{*H}(1)^2 \sigma_u^4 \sigma_\delta^2}{\gamma^2 (\sigma_\delta^2 + B^{*H}(1)^2 \sigma_u^2) + B^{*H}(1)^4 \sigma_u^6} \right), \quad (50)$$

and

$$\phi_j(0, B^*(0)) = \frac{\gamma}{2} \ln \left( 1 + \frac{\sigma_\delta^2}{B^*(0)^2 \sigma_u^2} \right) = \frac{\gamma}{2} \ln \left( 1 + \frac{\gamma^2}{\sigma_\delta^2 \sigma_u^2} \right)$$

Thus,

$$\phi_j(1, B^{*H}(1)) > \phi_j(0, B^*(0)) \Leftrightarrow \frac{B^{*H}(1)^2 \sigma_u^4 \sigma_\delta^2}{\gamma^2 (\sigma_\delta^2 + B^{*H}(1)^2 \sigma_u^2) + B^{*H}(1)^4 \sigma_u^6} > \frac{\gamma^2}{\sigma_\delta^2 \sigma_u^2}. \quad (51)$$

We deduce that  $\phi_j(1, B^{*H}(1)) > \phi_j(0, B^*(0))$  if and only if

$$-\gamma^2 \sigma_u^6 B^{*H}(1)^4 + (\sigma_u^4 \sigma_\delta^4 - \gamma^4) \sigma_u^2 B^{*H}(1)^2 - \gamma^4 \sigma_\delta^2 > 0. \quad (52)$$

Now observe that

$$B^{*H}(1)^2 = \frac{(B^{*H}(1) \sigma_u^2 - \gamma) \sigma_\delta^2}{\gamma \sigma_u^2}. \quad (53)$$

Thus, we can rewrite condition (52) as

$$-\gamma \sigma_u^2 \sigma_\delta^2 (B^{*H}(1) \sigma_u^2 - \gamma)^2 + (\sigma_u^4 \sigma_\delta^4 - \gamma^4) (B^{*H}(1) \sigma_u^2 - \gamma) - \gamma^5 > 0.$$

It can be checked that this inequality holds true if  $B^{*H}(1)$  belongs to

$$\left( \frac{\gamma}{\sigma_u^2} + \frac{\sigma_u^4 \sigma_\delta^4 - \gamma^4 - ((\sigma_u^4 \sigma_\delta^4 - \gamma^4)^2 - 4\gamma^6 \sigma_u^2 \sigma_\delta^2)^{1/2}}{2\gamma \sigma_u^4 \sigma_\delta^2}, \frac{\gamma}{\sigma_u^2} + \frac{\sigma_u^4 \sigma_\delta^4 - \gamma^4 + ((\sigma_u^4 \sigma_\delta^4 - \gamma^4)^2 - 4\gamma^6 \sigma_u^2 \sigma_\delta^2)^{1/2}}{2\gamma \sigma_u^4 \sigma_\delta^2} \right).$$

We now verify that this is the case. First, we check that  $B^{*H}(1)$  is always larger than the lower bound, that is:

$$\frac{\sigma_\delta^2 \sigma_u^2 + \sigma_\delta \sigma_u (\sigma_\delta^2 \sigma_u^2 - 4\gamma^2)^{1/2}}{2\gamma \sigma_u^2} > \frac{\gamma}{\sigma_u^2} + \frac{\sigma_u^4 \sigma_\delta^4 - \gamma^4 - ((\sigma_u^4 \sigma_\delta^4 - \gamma^4)^2 - 4\gamma^6 \sigma_u^2 \sigma_\delta^2)^{1/2}}{2\gamma \sigma_u^4 \sigma_\delta^2}.$$

Setting  $\sigma_\delta \sigma_u = \kappa$ , rearranging and simplifying we obtain that the above inequality is satisfied if and only if

$$\kappa^3 (\kappa^2 - 4\gamma^2)^{1/2} > \gamma^2 (2\kappa - \gamma^2) - \left( (\kappa - \gamma^4)^2 - 4\gamma^6 \kappa \right)^{1/2}. \quad (54)$$

However, under the parameter restriction imposed in Corollary 3 the l.h.s. of (54) is positive, while the r.h.s. is negative, and the result follows.<sup>21</sup> Next, we check that  $B^{*H}(1)$  is always smaller than the upper bound, that is:

$$\frac{\sigma_\delta^2 \sigma_u^2 + \sigma_\delta \sigma_u (\sigma_\delta^2 \sigma_u^2 - 4\gamma^2)^{1/2}}{2\gamma \sigma_u^2} < \frac{\gamma}{\sigma_u^2} + \frac{\sigma_u^4 \sigma_\delta^4 - \gamma^4 + ((\sigma_u^4 \sigma_\delta^4 - \gamma^4)^2 - 4\gamma^6 \sigma_u^2 \sigma_\delta^2)^{1/2}}{2\gamma \sigma_u^4 \sigma_\delta^2}.$$

Rearranging the above inequality we have

$$\sigma_u^3 \sigma_\delta^3 (\sigma_u^2 \sigma_\delta^2 - 4\gamma^2)^{1/2} < \gamma^2 (2\sigma_u^2 \sigma_\delta^2 - \gamma^2) + ((\sigma_u^4 \sigma_\delta^4 - \gamma^4)^2 - 4\gamma^6 \sigma_u^2 \sigma_\delta^2)^{1/2}. \quad (55)$$

And setting  $\sigma_\delta \sigma_u = \kappa$  we rewrite (55) as follows:

$$\kappa^3 (\kappa^2 - 4\gamma^2)^{1/2} < \gamma^2 (2\kappa^2 - \gamma^2) + \left( (\kappa^4 - \gamma^4)^2 - 4\gamma^6 \kappa^2 \right)^{1/2}.$$

Squaring both sides in the above inequality and rearranging yields

$$-4\gamma^2 \kappa^6 < 2\gamma^4 (\kappa^2 (\kappa^2 - 4\gamma^2) + \gamma^8) + 2\gamma^2 (2\kappa^2 - \gamma^2) \left( (\kappa^4 - \gamma^4)^2 - 4\gamma^6 \kappa^2 \right)^{1/2}. \quad (56)$$

While the l.h.s. of (56) is negative, under the parameter restriction of Corollary 3 the r.h.s. is positive (since  $\kappa^2 > 4\gamma^2$ ), and the result follows.  $\square$

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<sup>21</sup>In particular, the r.h.s. of (54) is negative for all  $\sigma_u^2 \sigma_\delta^2 > \gamma^2 \sqrt{6}$ , a condition that is satisfied by the parameter restriction imposed in Corollary 3.

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