Learning whether other Traders are Informed*

Snehal Banerjee
Northwestern University
Kellogg School of Management
snehal-banerjee@kellogg.northwestern.edu

Brett Green
UC Berkeley
Haas School of Business
bgreen@haas.berkeley.edu

Current draft: June 2013
First draft: August 2012

Abstract

We develop a dynamic model in which some investors are uncertain about whether others are informed and gradually learn about them by observing prices and dividends. The model gives rise to a rich set of implications for return dynamics. For instance, expected returns and return volatility are both stochastic and persistent, even though fundamentals and signals are i.i.d. The price reaction to information about dividends is asymmetric: the price reacts more strongly to bad news than it does to good news. The model also generates volatility clustering in which large return realizations, which are associated with dividend surprises, are followed by higher future volatility and higher expected returns. Finally, the relation between information quality and returns varies endogenously over time and depends on the degree of disagreement across investors.

JEL Classification: G12, G14
Keywords: Asset Prices, Trade, Learning, Asymmetric Information, Rational Expectations

*We thank Bradyn Breon-Drish, Mike Fishman, Ron Kaniel, Paul Pfleiderer, Costis Skiadas, Viktor Todorov, Johan Walden and participants at the Miami Behavioral Finance Conference (2012), the Stanford/Berkeley Joint Seminar, the Utah Winter Finance Conference (2013), and the University of Illinois at Urbana Champaign for their comments and suggestions.
1 Introduction

As early as Keynes (1936), it has been recognized that investors face uncertainty not only about fundamentals, but also about the underlying characteristics and trading motives of other market participants. Asset pricing models have focused primarily on the former, taking the latter as common knowledge. For instance, in Grossman and Stiglitz (1980), uninformed investors know the number of informed investors in the market and the precision of their signals. Similarly, each agent in Hellwig (1980) is certain about both the number other agents and the distribution of their signals.\footnote{Likewise, in Kyle (1985), the uninformed market maker not only knows about the existence of an informed strategic investor, but also the precision of that trader’s private signal (though not the signal realization).} Arguably, this requires an unrealistic degree of sophistication on the part of market participants — it seems unlikely that investors who are uncertain about fundamentals, know, with certainty, whether other investors are privately informed.

In this paper, we develop a framework in which investors are uncertain about, and gradually learn, whether others are traders informed. We show that these features give rise to a rich set of implications for return dynamics that are consistent with empirical evidence, but are inherently absent in standard rational expectations models. First, we show that expected returns and return volatility are both stochastic and persistent despite i.i.d. dividends and signals. Second, prices react more strongly to negative news than to positive news. In fact, the price may even decrease following positive signals about dividends. Third, the model can generate volatility clustering in which large (positive or negative), unexpected return realizations in the current period are followed by higher return volatility and higher expected returns in the next period. Finally, the relation between information quality and return moments varies endogenously over time and depends on the degree of disagreement across investors.

In order to explain the mechanism underlying these predictions, a brief overview of the model is useful. There are two groups of investors: the uninformed ($U$) and the potentially informed ($\theta$). Both groups of investors have mean-variance preferences and trade competitively in a centralized market by submitting limit orders. There is a single a risky asset in fixed supply that pays i.i.d. dividends. Each period, $\theta$ investors observe a signal prior to submitting their order. The $\theta$ investors can be one of two types: either informed ($\theta = I$) or not informed ($\theta = NI$). If $\theta$ investors are informed, the signal is informative about next period’s dividend. If $\theta$ investors are not informed, the signal is spurious — however, $NI$ investors incorrectly believe their signal is informative. $U$ investors are uncertain about $\theta$ and, hence, whether the price is informative. By observing prices and subsequent dividends,
uninformed investors update their beliefs about \( \theta \) investors over time. These beliefs, in turn, affect their willingness to participate in the market and hence the market clearing price.

In our analysis, we first consider a static version of the model in which there is uncertainty about whether other traders are informed, but there is no learning along this dimension. In equilibrium, the price and residual demand reveals the realization of the signal to the uninformed traders, but they are uncertain about whether or not it is informative. Because of this uncertainty, a surprise in the signal (in either direction) increases the uninformed investors’ posterior variance about fundamentals. As a result, the equilibrium price is (i) non-linear in the signal, and (ii) depends on the probability that uninformed investors assign to the presence of informed traders.

The key additional feature of the dynamic setting is that, over time, uninformed investors update their beliefs about whether others are informed using realized prices and dividends. When a dividend realization is in line with the signal revealed through the price, uninformed investors increase the likelihood that others are informed. The endogenous evolution of their beliefs (combined with (i) and (ii) above) generates our first main result: return moments that are both stochastic and persistent even though fundamentals and signals are i.i.d. As mentioned earlier, this uncertainty and learning generates a number of additional implications, which we discuss in more detail and connect to the empirical literature below.

**Asymmetric Price Reaction to News.** Prices react asymmetrically to news due to the nature of the uninformed investors filtering problem. When there is a negative surprise, an uninformed investor’s conditional expectation is lower and her conditional variance is higher, both of which lead to a decrease in the price. However, when there is a positive surprise, the conditional expectation is higher but so is the conditional variance, and these have off-setting effects on the price. As a result, prices are more sensitive to bad news, or negative surprises, than to good news. When the magnitude of a positive surprise in the signal is small relative to the overall risk concerns, the effect on the conditional variance dominates and the price decreases with additional good news.

Asymmetric price reactions have been well documented in the empirical literature. Campbell and Hentschel (1992) document asymmetric price reaction to dividend shocks at the aggregate stock market level through a volatility feedback channel. At the firm level, using a sample of voluntary disclosures, Skinner (1994) documents that the price reaction to bad news is, on average, twice as large as that for good news. Skinner and Sloan (2002) document that the price response to negative earnings surprises is larger, especially for growth stocks. Our model’s predictions are consistent with these results, especially if investors in growth, or glamor, stocks face more uncertainty about whether other investors in these stocks are
informed.

**Volatility Clustering.** Since Mandelbrot (1963), a large number of papers have documented the phenomenon of volatility clustering for various asset classes, and at different frequencies (see Bollerslev, Chou, and Kroner, 1992 for an early survey). In our model, volatility clustering is a result of how an uninformed investor updates her beliefs in response to realized dividends. Since an uninformed investor forms her conditional expectation of next period’s dividends based on the signal of the potentially informed investors, a dividend realization that is far from her conditional expectation (i.e., a large dividend surprise) leads her to revise her belief about the informativeness of the signal downwards. In other words, large surprises in dividend realizations, which are accompanied by large absolute return realizations, reduces the likelihood that the potentially informed investors are informed. In turn, this increases the uninformed investor’s uncertainty about fundamentals and, therefore, leads to higher volatility and higher expected returns in future periods.

**Information Quality and Return Moments.** The empirical evidence documenting the firm-level relation between information quality and expected returns has been mixed. While some papers document a negative relation between information quality and expected returns (e.g., Easley and O’Hara, 2005; Francis, Nanda, and Olsson, 2008), others find either limited or no evidence of a relation (e.g., Core, Guay, and Verdi, 2008; Duarte and Young, 2009).

In our model, the relation between the information quality of the signal and return moments depends on whether investors agree on the interpretation of the signal. For instance, if investors agree on the informativeness of the signal (i.e., if the uninformed investors put a high probability on the other traders being informed), then higher information quality reduces uncertainty about fundamentals and, intuitively, leads to lower expected returns and return volatility. However, if the uninformed investors believe that other investors are not likely to be informed, the opposite relation obtains. A more informative signal for the potentially informed investors induces them to trade more aggressively. From an uninformed investor’s perspective, this introduces more noise to current and future prices leading to higher expected returns and volatility. Since the uninformed investors’ beliefs about whether the others are informed evolves over time, the relation between information quality and expected returns (and volatility) varies endogenously in our model. As such, our model may help reconcile the apparently conflicting empirical evidence documented about this relation in the cross-section of firms.

An additional feature of the model is that the presence of noisy supply shocks (or noise
traders) is unnecessary to generate trade and prevent a fully revealing equilibrium.\(^2\) Although the signal of the potentially informed is perfectly revealed in equilibrium, uninformed investors are unsure whether the signal conveys payoff relevant information and trade occurs due to the lack of a common prior (i.e., the potential existence of traders who incorrectly perceive their information). We view this as an appealing feature of the model, both for its tractability and its empirical relevance.\(^3\) Furthermore, our model of potentially informed investors is arguably closer to Black (1986)’s notion of noise traders than aggregate supply shocks: “Noise trading is trading on noise as if it were information. People who trade on noise are willing to trade even though from an objective point of view they would be better off not trading. Perhaps they think the noise they are trading on is information.” Nevertheless, our results do not rely on this particular specification. In Section 6, we show how a model with a common prior and noisy aggregate supply generates qualitatively similar results. This suggests the mechanism driving our results is the uncertainty, and subsequent learning, about the characteristics of other traders in the market.

The rest of the paper is organized as follows. We discuss the related literature in the next section. Section 3 presents the setup of the general model. In Section 4, we solve the static version of the model, which allows us to highlight the intuition for many of our results transparently. Section 5 analyzes the dynamic model. In Section 6, we consider two alternative specifications of the model. Section 7 concludes. Unless otherwise specified, all proofs are in Appendix A.

2 Related Literature

While the majority of the RE literature has focused on linear-normal equilibria, a number of papers, including most recently Breon-Drish (2012) and Albagli, Hellwig, and Tsyvinski (2011), have explored the effects of relaxing the assumption that fundamental shocks and signals are normally distributed.\(^4\) Our paper contributes to this literature by developing

\(^2\) Odean (1998) shows a similar result in a model where the presence of overconfident traders is common knowledge. More generally, investors in our model may “agree to disagree,” or exhibit differences of opinion (see Morris (1995) for a discussion of this feature).

\(^3\) A number of laboratory studies have demonstrated the tendency for agents to overestimate the precision of their knowledge. See Odean (1998) for a discussion of this literature. For more recent empirical evidence of overconfidence in financial markets, see, for example, Odean (1999), Barber and Odean (2000), Grinblatt and Keloharju (2000). Biais, Hilton, Mazurier, and Pouget (2005) document that investors exhibit a similar type of over-confidence (referred to as miscalibration) in a simulated market setting, and that this behavior leads to trading losses.

a model in which the non-linearity arises endogenously. Specifically, even though shocks to fundamentals and signals are normally distributed in our model, since the uninformed investor is uncertain about whether other investors are informed, her beliefs about the price signal are given by a mixture of normals distribution.\footnote{In a series of papers, Easley, O’Hara and co-authors analyze the probability of informed trading (PIN) in a sequential trade model similar to Glosten and Milgrom (1985) (e.g., Easley, Kiefer, and O’Hara, 1997a; Easley, Kiefer, and O’Hara, 1997b; Easley, Hvidkjaer, and O’Hara, 2002). In these papers, the risk-neutral market maker updates her valuation of the asset based on whether a specific trade is informed or not, but does not face uncertainty about the presence of informed traders in the market. In contrast, the uninformed investor in our model must update their beliefs, not only about the value of the asset, but also about the probability of other investors being informed, which leads to non-linearity in prices.}

A related non-linearity arises in the incomplete information, regime switching models of David (1997), Veronesi (1999), David and Veronesi (2008, 2009), and others, in which a representative investor updates her beliefs about which macroeconomic regime she is currently in using signals about fundamental shocks (e.g., dividends). In these models, the non-linearity in representative investor’s filtering problem leads to time-variation in uncertainty and, consequently, variation in expected returns and volatility. Stochastic volatility also arises in noisy rational expectations models, like Fos and Collin-Dufresne (2012), in which noise trader volatility is stochastic and persistent. In contrast, these features arise \textit{endogenously} in our model even though shocks to both fundamentals and news are i.i.d., and are driven by how uninformed investors learn to use the price to update their beliefs about fundamentals.

Cao, Coval, and Hirshleifer (2002) show how limited or costly participation by investors can also generate stochastic volatility, as well as large price movements in response to little, or no, apparent information.\footnote{Other papers that study the informational effects of limited participation include Romer (1993), Lee (1998), Hong and Stein (2003), and Alti, Kaniel, and Yoeli (2012).} Because of participation costs, sidelined investors update the interpretation of their private signals based on what they learn from prices, and only enter the market once they are sufficiently confident. In our model, the friction is purely informational — uninformed investors trade less aggressively because they are uncertain about the trading motives of other investors, and consequently, the informativeness of the price.

Our model is related to a number of papers in microstructure literature, in which investors face multiple dimensions of uncertainty. Gervais (1997) considers a static Glosten and Milgrom (1985) model in which the market maker is uncertain about the precision of informed trader’s signal. Romer (1993), Avery and Zemsky (1998) and Gao, Song, and Wang (2012) consider models in which the proportion of informed traders is uncertain (but is not learned over time). Li (2011) considers a generalization of the continuous-time, Kyle-model of Back (1992) that allows for uncertainty about whether the strategic trader is informed or
not. In contrast to these papers, which focus on the market microstructure implications of multidimensional uncertainty (e.g., market depth, insider’s profit), we focus primarily on the asset pricing implications. Importantly, since our model considers risk-averse investors, we are able to analyze the effect of the uninformed investor’s non-linear learning problem on risk-premia and expected returns.

Finally, our model contributes to the differences of opinion (DO) literature, which has been important in generating empirically observed features of price and volume dynamics (e.g., Harrison and Kreps, 1978; Harris and Raviv, 1993; Kandel and Pearson, 1995; Scheinkman and Xiong, 2003; Banerjee and Kremer, 2010). The DO models in the literature have largely ignored the role of learning from prices, since investors agree to disagree about fundamentals, and therefore find the information in the price irrelevant.7 In our model, investors may exhibit differences of opinion (since all potentially informed investors believe their signals are payoff relevant), but uninformed investors still condition on prices to update their beliefs about fundamentals. In this sense, our model bridges the gap between the RE and DO approaches.

3 Model Setup

In this section, we present the setup of the model and discuss some of our assumptions.

Payoffs. There are two assets: a risk-free asset and a risky asset. The gross risk-free rate is normalized to \( R ≡ 1 + r > 1 \). The risky asset pays a stream of dividends \( d_t \sim N(\mu, \sigma^2) \), which are i.i.d., and investors observe the realization of \( d_t \) at date \( t \). For notational convenience, we normalize \( \mu \) to zero.8 The aggregate supply of the risky asset is constant and equal to \( Z \). Denote the price of the risky asset at date \( t \) by \( P_t \), and denote the dollar return on the risky asset by \( \frac{Q_{t+1} - RP_t}{P_t} \).

Preferences. Investor \( i \) has mean-variance preferences over next period’s wealth, and trades competitively i.e., is a price taker. In particular, she submits a limit order, \( x_{i,t} \), such that

\[
x_{i,t} = \arg \max_x E_{i,t}[W_{i,t}R + xQ_{t+1}] - \frac{\alpha}{2} \text{var}_{i,t}[W_{i,t}R + xQ_{t+1}],
\]

and where \( E_{i,t}[\cdot] \) and \( \text{var}_{i,t}[\cdot] \) denote her conditional expectation and variance, respectively, given date her \( t \) information set, \( W_{i,t} \) denotes her wealth, \( \alpha \) denotes her risk-aversion, and \( x_{i,t} \) denotes her trading position in the risky asset at date \( t \). Given these preferences, investor

7One exception is Banerjee, Kaniel, and Kremer (2009), in which investors agree to disagree but use the price to update their beliefs about higher order expectations, which is useful for them to speculate against each other.

8In the numerical example of Section 5.4, we set \( \mu \) to a non-zero value to ensure that, except for the most extreme negative shocks prices are non-negative.
i’s optimal demand for the risky asset is given by

\[ x_{i,t} = \frac{E_{i,t}(Q_{t+1})}{\alpha \text{var}_{i,t}(Q_{t+1})} = \frac{E_{i,t}(P_{t+1} + d_{t+1}) - RP_t}{\text{var}_{i,t}(P_{t+1} + d_{t+1})}. \]  

(2)

The optimal demand is analogous to an investor’s demand in an overlapping generations (OLG) model (e.g., Spiegel, 1998; Biais, Bossaerts, and Spatt, 2010; Banerjee, 2011). This parsimonious structure retains the key feature of a dynamic framework — namely, an investor’s optimal portfolio depends not only on her beliefs about the fundamental dividend, but also on her beliefs about future prices. Finally, the market clearing condition in the risky asset is given by

\[ \sum_i x_{i,t} = Z. \]  

(3)

An equilibrium consists of a price process, \( P_t \), and investor demands \( x_{i,t} \), such that for all \( i, t \), (i) investor demands are optimal given their beliefs and information described below and (ii) markets clear.

**Information and Beliefs.** There are two groups of investors: the uninformed (denoted by \( U \)) and the potentially informed (denoted by \( \theta \)). Investors within each group are identical and behave competitively, so for ease of exposition, we will often refer to the representative investor for each group (i.e., investor \( U \) is the representative investor for all uninformed investors, and investor \( \theta \) represents all potentially informed investors). The \( \theta \) investors are either informed (i.e., \( \theta = I \)) or not informed (i.e., \( \theta = NI \)), where the prior probability of being informed is \( \pi_0 \equiv \text{Pr}(\theta = I) \). Specifically, at date \( t \), investor \( \theta \) receives a signal \( S_{\theta,t} \) of the form:

\[ S_{\theta,t} = \begin{cases} 
  d_{t+1} + \varepsilon_t & \text{if } \theta = I \\
  u_{t+1} + \varepsilon_t & \text{if } \theta = NI,
\end{cases} \]

where \( \varepsilon_t \sim \mathcal{N}(0, \sigma^2_\varepsilon) \) and \( u_{t+1} \) is distributed identically to \( d_{t+1} \) and where \( (\varepsilon_t, u_{t+1}, d_{t+1}) \) are mutually independent for all \( t \). It is convenient to parametrize the information quality of the informed investors’ signal (i.e., \( S_{I,t} \)) by the Kalman gain, \( \lambda \), where

\[ \lambda \equiv \frac{\text{cov}[S_{I,t}, d_{t+1}]}{\text{var}[S_{I,t}]} = \frac{\sigma^2}{\sigma^2 + \sigma^2_\varepsilon}. \]

Note that \( \lambda \) is decreasing in the noise of the signal (i.e., \( \sigma^2_\varepsilon \)) and takes values between zero and one. When \( \lambda = 0 \), \( S_{I,t} \) is completely uninformative; investors learn nothing about future
dividends by observing it. Conversely, when \( \lambda = 1 \), \( S_{I,t} \) perfectly reveals the realization of next period’s dividend. Unless otherwise noted, we assume \( \lambda > 0 \).

Investor \( U \) does not know \( \theta \) and so is unsure whether the other investors have payoff relevant information about the asset. However, she has rational expectations about the joint distribution of signals and fundamentals. Since there are no additional sources of noise, one expects that in equilibrium, \( U \) will be able to infer \( S_{\theta,t} \) from the price (i.e., \( P_t \)) and the aggregate residual supply (i.e., \( Z - x_{\theta,t} \)) and use this to update her beliefs about fundamentals.\(^9\) We say that an equilibrium is a *signal-revealing*, if the equilibrium price and allocations reveal \( S_{\theta,t} \) (but not \( \theta \)) to the \( U \) investor.\(^10\) In Section 4, we show that the unique equilibrium in the static model is signal-revealing. In Section 5, we characterize properties of signal-revealing equilibria in the general (dynamic) model and show that when the \( U \) investor faces no uncertainty about \( \theta \) (i.e., \( \pi_0 = 0 \) or \( \pi_0 = 1 \)), the unique stationary equilibrium of the dynamic model is again, signal-revealing.

Let \( \pi_t \) denote the probability that investor \( U \) attributes to investor \( \theta \) being informed at date \( t \), i.e., \( \pi_t \equiv \Pr_{U,t}(\theta = I) \). Then, investor \( U \)’s conditional beliefs about the value \( d_{t+1} \) next period are given by

\[
\begin{align*}
\mathbb{E}_{U,t}[d_{t+1}] &= \pi_t \lambda S_{\theta,t}, \quad \text{and} \\
\text{var}_{U,t}[d_{t+1}] &= \frac{\pi_t \sigma^2 (1 - \lambda) + (1 - \pi_t) \sigma^2}{\pi_t (1 - \pi_t) (\lambda S_{\theta,t})^2}.
\end{align*}
\]

Equation (5) highlights the first key feature of the model: investor \( U \)’s conditional variance depends on the realization of the signal \( S_{\theta,t} \). Note that if \( U \) were certain that \( \theta \) was informed (i.e., \( \pi_t = 1 \)), then her conditional expectation of \( d_{t+1} \) should depend on \( S_{\theta,t} \). On the other hand, if \( U \) were certain that \( \theta \) was not informed (i.e., \( \pi_t = 0 \)), then her conditional expectation should be unaffected by \( S_{\theta,t} \). In either of these cases, since \( U \) is certain about \( \theta \), her conditional variance is constant. However, when \( U \) is uncertain about \( \theta \), she is uncertain about how to update her conditional expectation given \( S_{\theta,t} \), and this leads to additional uncertainty about dividends. Moreover, since larger realizations of signals (i.e., larger \( |S_{\theta,t}| \)) lead to bigger revisions in beliefs if \( \theta \) is informed, but no revisions otherwise, the additional uncertainty increases with the magnitude of the signal (i.e., increases with \( S^2_{\theta,t} \)). As discussed in Section 4, the dependence of the \( U \) investor’s conditional variance on the realization of the signal plays an important role in our model.

Investor \( U \) will also update her beliefs (i.e., learn) about whether \( \theta \) is informed based

\(^9\)The equilibrium price alone does not necessarily reveal the signal \( S_{\theta,t} \), since \( P_t \) is non-monotonic in \( S_{\theta,t} \). We follow Kreps (1977) and allow the \( U \) investor to condition her order on both price and quantity.

\(^{10}\)Formally, that \( S_{\theta,t} \) is measurable with respect to \( U \)’s information set at date \( t \).
on the realization of \( d_{t+1} \) using Bayes rule. The posterior belief of \( \theta = I \), conditional on a realization of \( d_{t+1} \), is given by

\[
\pi_{t+1} = \frac{\pi_t \Pr(S_{\theta,t} | \theta = I, d_{t+1})}{\pi_t \Pr(S_{\theta,t} | \theta = I, d_{t+1}) + (1 - \pi_t) \Pr(S_{\theta,t} | \theta = NI, d_{t+1})} = \frac{\pi_t \phi \left( \frac{S_{\theta,t} - d_{t+1}}{\sigma_c} \right)}{\pi_t \phi \left( \frac{S_{\theta,t} - d_{t+1}}{\sigma_c} \right) + \frac{1 - \pi_t}{\sqrt{\sigma^2 + \sigma_c^2}} \phi \left( \frac{S_{\theta,t} - 0}{\sqrt{\sigma^2 + \sigma^2}} \right)},
\]

where \( \phi(\cdot) \) is the probability distribution function for a standard normal random variable. Equation (6) highlights the second key feature of the model. U’s beliefs about \( \theta \) in period \( t \) influence her beliefs in period \( t + 1 \), which creates a link across periods and generates persistence in an otherwise i.i.d. model.

Regardless of type, investor \( \theta \) believes that her signal is informative with probability one. This implies that the conditional beliefs of the potentially informed investor are symmetric across types. For \( \theta \in \{I, NI\} \), investor \( \theta \)’s conditional beliefs about the value \( d_{t+1} \) next period are given by

\[
\mathbb{E}_{\theta,t}[d_{t+1}] = \lambda S_{\theta,t}, \quad \text{and} \quad \text{var}_{\theta,t}[d_{t+1}] = \sigma^2(1 - \lambda).
\]

**Discussion of Assumptions.** In our model, investors exhibit differences of opinion since \( \theta = NI \) investors believe their signals are informative even though they are not. This lack of a common prior generates trade (e.g., Milgrom and Stokey, 1982) and prevents prices from being fully informative (despite being signal revealing). Given the empirical evidence on the trading behavior of retail investors (see footnote 3), that some investors believe they are informed, even if they are not, seems quite plausible. The setup also facilitates a clean decomposition between the two key features of the model: **uncertainty about** whether other traders are informed, and **learning** about them over time. In the static version of the model, only the first channel is present, whereas the dynamic model incorporates both features.\(^{11}\) It is important to note that the lack of a common prior is not crucial for our results. One alternative specification is to impose a common prior across investors and incorporate aggregate shocks to supply (i.e., noise traders) as is standard in noisy rational expectations models. In Section 6.1, we analyze this alternative and show how it leads to qualitatively similar results.

As is common in the literature on asymmetric information in financial markets, we con-

\(^{11}\)In particular, the symmetry in the equilibrium trades of the I and NI investors in our specification implies that by just observing \( x_{\theta,t} \), the U investor cannot update \( \pi \) — instead, she must observe \( d_{t+1} \) in order to update her beliefs about \( \pi \). If the two types of \( \theta \) traders had equilibrium trades from different distributions, as in the noisy supply model of Appendix B, this would lead the U investor to update \( \pi \) twice — once upon observing \( x_{\theta,t} \), and then again, after observing \( d_{t+1} \).
sider a model with a single risky asset (e.g., Grossman and Stiglitz, 1980; Kyle, 1985; Wang, 1993). This assumption is made primarily for tractability. Given that there is empirical evidence for many of our predictions for both portfolio and individual asset returns (e.g., stochastic volatility, time-varying expected returns, volatility clustering), the mechanism that we highlight may be applicable at both the aggregate-level and the firm-level (due to limits to arbitrage or other frictions). One could interpret the risky asset in the model as an industry-level portfolio, which bears aggregate risk, and about which investors may have asymmetric information. While a general equilibrium model with multiple firms or industries is beyond the scope of this paper, we expect the qualitative implications of the mechanism we highlight to survive in such a setting.

4 The Static Model

In this section, we present a two date version of the model. At \( t = 0 \), \( \theta \) investors observe signals, both traders submit their orders and the market clears. At date \( t = 1 \), the dividend is realized and all consumption takes place. This simple setting will allow us to isolate the effects of uncertainty about \( \theta \) (i.e., \( \pi_0 \in (0, 1) \)) from the effects of learning about \( \theta \) (i.e., updating \( \pi_0 \)), which obtain in the dynamic setting of Section 5. The static version of the model also allows us to solve for equilibrium prices in closed form and develop the underlying intuition for the model more transparently.

4.1 Equilibrium Prices

We begin with an explicit characterization of the equilibrium price. Note that since there is only one trading period, \( U \) investors do not trade subsequent to updating their beliefs about whether others are informed. For notational convenience, we drop the time subscripts on the signal and prices (realized at \( t = 0 \)) as well as the dividend (realized at \( t = 1 \)). The uninformed investors prior, \( \pi_0 \), will be a key parameter of interest; comparative statics with respect to \( \pi_0 \) will help develop the intuition for the dynamic model, in which beliefs evolve over time.

Proposition 1. In the static model, there exists a unique equilibrium. This equilibrium is signal-revealing and the price is given by

\[
P = \frac{1}{R} \left((\kappa + (1 - \kappa)\pi_0) \lambda S_\theta - \kappa \alpha \sigma^2 (1 - \lambda)Z\right),
\]

(8)
where the weight \( k_t \) is given by
\[
\kappa = \frac{\sigma^2(1 - \pi_0 \lambda) + \pi_0(1 - \pi_0)(\lambda S_\theta)^2}{\sigma^2(1 - \lambda) + \sigma^2(1 - \pi_0 \lambda) + \pi_0(1 - \pi_0)(\lambda S_\theta)^2} \in [0, 1].
\]

The equilibrium price can be decomposed into a market expectations component and a risk-premium component, since
\[
P = \frac{1}{R} \left( \kappa \mathbb{E}_\theta [d] + (1 - \kappa) \mathbb{E}_U [d] - \kappa \alpha \sigma^2 (1 - \lambda) Z \right).
\]

The risk-aversion coefficient, \( \alpha \), and the aggregate supply of the asset, \( Z \), scale the risk-premium component, but not the expectations component. Thus, the product, \( \alpha Z \), determines the relative role of each component in the price. When risk aversion is low or the aggregate supply of the asset is small, the price is primarily driven by the expectations component. On the other hand, when risk aversion is high, or the aggregate supply of the asset is large, the risk-premium component drives the price. As such, it will be useful to characterize separately how each component of the price depends on the underlying parameters. The following corollary presents these results.

**Corollary 1.** In the static model:

(i) The expectations component of the price is increasing in \( S_\theta \), increasing in both \( \lambda \) and \( \pi_0 \) for \( S_\theta > 0 \), and decreasing both in \( \lambda \) and \( \pi_0 \) for \( S_\theta < 0 \).

(ii) The risk-premium component of the price is hump-shaped in \( S_\theta \) around zero, \( U \)-shaped in \( \pi_0 \) around \( \frac{1}{2} \left( 1 - \frac{\sigma^2}{\lambda S_\theta} \right) \), increasing in \( \lambda \) for small \( |S_\theta| \), but decreasing in \( \lambda \) for large \( |S_\theta| \).

The comparative statics on the expectations component follow from observing that the average conditional expectation is linear in \( S_\theta \) with a coefficient that is increasing in \( \pi_0 \) and \( \lambda \). The risk-premium component of prices depends on the uncertainty that investors face. In particular, note that the risk-premium component can be rewritten as
\[
-\kappa \alpha \sigma^2 (1 - \lambda) Z = -\alpha \left( \frac{1}{\text{var}_d [d]} + \frac{1}{\text{var}_U [d]} \right)^{-1} Z,
\]
which is increasing in the conditional variance of both \( U \) and \( \theta \) investors. Unlike standard RE models with linear equilibria, because the conditional variance of the uninformed investors depends on the signal realization, so too does the risk-premium component. Recall that the
conditional variance \( \text{var}_{U,t}[d] \) given in expression (5) is given by

\[
\text{var}_{U,t}[d] = \pi_0 \sigma^2 (1 - \lambda) + (1 - \pi_0) \sigma^2 + \pi_0 (1 - \pi_0) (\lambda S_\theta)^2.
\]  

(11)

As discussed in Section 3, \( \text{var}_{U,t}[d] \) increases in \( |S_\theta| \); larger realizations of \( |S_\theta| \) increase the uninformed investors’ uncertainty about fundamentals, since they are unsure about whether the signal is informative. For small realizations of \( |S_\theta| \), a more informative signal (i.e., high \( \lambda \)) reduces the posterior variance (for \( \pi_0 > 0 \)), but when \( |S_\theta| \) is large enough, a more informative signal can increase the uninformed investor’s uncertainty.

The overall effect of \( S_\theta \) on the price in our model distinguishes it from both linear rational expectations and difference of opinions models. While the expectations component of price is monotonic in \( S_\theta \), the risk-premium component is hump-shaped in \( S_\theta \) around zero. This implies that the two components reinforce each other when \( S_\theta < 0 \), but offset each other when \( S_\theta > 0 \). In other words, the market reacts asymmetrically to news about fundamentals: the price responds more strongly to bad news (i.e., \( S_\theta < 0 \)) than to good news (i.e., \( S_\theta > 0 \)).

Since the risk-premium component is bounded, the expectations component dominates when \( |S_\theta| \) is large enough. However, for \( S_\theta \) small enough, the risk-premium component dominates. This means that for small, positive news (i.e., small \( S_\theta > 0 \)), if the overall risk concerns in the market are large enough (i.e., \( \alpha Z \) is large relative to \( S_\theta \)), the price actually decreases with \( S_\theta \). Intuitively, a small, positive surprise about fundamentals can have a bigger impact on prices through the uncertainty it generates for uninformed investors than through its effect on the market’s expectations about future dividends.

The mechanism through which the asymmetry in prices arises in our model differs from those in the regime-switching models of Veronesi (1999) and others. Specifically, in Veronesi (1999), the asymmetry in price reaction is driven by uncertainty about whether the underlying state of the economy is good or bad. The investor “over-reacts” to bad news only if he believes (with sufficiently high probability) that the current state is good, and “under-reacts” to good news only if he believes that the current state is bad, because these are the instances in which the news increases uncertainty about the underlying state. In our model, the asymmetry is not state-dependent: the price is more sensitive to bad news for any \( \pi_t \in (0, 1) \) (not just when \( \pi_t \) is close to 1) and even when \( \pi_t \) is fixed. This is because the asymmetry is driven by uncertainty about the informativeness of the price signal, not the underlying fundamentals.\(^\text{12}\)

\(^{12}\)Note that in the dynamic version of our model, the uninformed investor updates \( \pi_t \) based on realizations of fundamentals (i.e., \( d_{t+1} \)), but this is not what drives the asymmetric reaction of \( P_t \) to \( S_{\theta,t} \).
4.2 Expected returns and volatility

Given the results from Proposition 1, we now turn to investigating the moments of returns. The decomposition in (9) implies that dollar returns can be expressed as

\[ Q = d - (\kappa \mathbb{E}_\theta[d] + (1 - \kappa)\mathbb{E}_U[d]) + \kappa \alpha \sigma^2(1 - \lambda)Z. \]  

(12)

Return moments are computed based on the information set of the \( U \) investor.\(^\text{13}\) We refer to *conditional* expected returns as the expected returns conditional on all information up to and including the current period (i.e., the price). *Unconditional* returns are computed based on all information prior to the current period.

**Proposition 2.** In the static model, the conditional expected return and volatility are given by

\[ \mathbb{E}[Q|P, x_\theta] = -(1 - \pi_0)\lambda \kappa S_\theta + \kappa \alpha \sigma^2(1 - \lambda)Z, \quad \text{and} \]

\[ \text{var}[Q|P, x_\theta] = \sigma^2(1 - \pi_0\lambda) + \pi_0(1 - \pi_0)(\lambda S_\theta)^2. \]  

(13)

(14)

The unconditional expected return and volatility are given by

\[ \mathbb{E}[Q] = \mathbb{E}[\kappa]\alpha \sigma^2(1 - \lambda)Z, \quad \text{and} \]

\[ \text{var}[Q] = \sigma^2(1 - \pi_0^2\lambda) + (1 - \pi_0)^2\lambda^2\text{var}[\kappa S_\theta] + (\sigma^2(1 - \lambda)\alpha Z)^2\text{var}[\kappa] \]  

(15)

(16)

To gain some intuition for the expressions in Proposition 2, we note that the expectation of (12) with respect to an arbitrary information set \( \mathcal{I} \) can be decomposed into the following two components:

\[ \mathbb{E}[Q|\mathcal{I}] = \underbrace{\mathbb{E}[\kappa(\mathbb{E}_U[d] - \mathbb{E}_\theta[d])|\mathcal{I}]}_{\text{expectations}} + \underbrace{\mathbb{E}[\kappa \alpha \sigma^2(1 - \lambda)Z|\mathcal{I}]}_{\text{risk premium}}. \]

(17)

Equation (17) highlights two key predictions of the model. First, the risk premium component of conditional expected returns is state dependent (i.e., depends on \((\pi_0, S_\theta)\)). Since the \( U \) investor is uncertain about the interpretation of \( S_\theta \), her conditional variance about \( d \) depends on both \( \pi_0 \) and \( S_\theta \), and as a result, so does \( \kappa \). The state dependence of the risk premium component of expected returns is what gives rise to expected returns and return volatility that are both stochastic and persistent in a dynamic setting (see Section 5).

\(^\text{13}\)This corresponds to the information set of an econometrician who observes the price and quantity of executed trades as well as dividends.
Second, unlike linear rational expectations models, the expectations component of expected returns need not always be zero in our model. In rational expectations models, since every investor’s beliefs satisfies the Law of Iterated Expectations, and investors share a common prior, investors cannot disagree conditional on the same information set, and so the expectations component is zero. In our model, since investors exhibit differences of opinion, the expectations component of expected returns need not be zero. The uninformed $\theta$ investors (who believe they are informed) incorrectly condition on their signal, and this introduces predictability in conditional expected returns through the $S_\theta$ term in equation (13). However, since the $S_\theta$ signals are mean-zero and i.i.d., the expectations component of unconditional expected returns is zero.

The expression for the unconditional volatility of returns given in equation (16) can be decomposed into three terms, each of which captures a different source of risk,

$$\text{var}[Q] = \sigma^2 (1 - \pi_0^2 \lambda) + (1 - \pi_0)^2 \lambda^2 \text{var}[\kappa S_\theta] + (\sigma^2 (1 - \lambda) \alpha Z)^2 \text{var}[\kappa].$$

The first term is the expectation of the conditional variance in returns and so captures the volatility in returns due to uncertainty about next period’s fundamental dividend shock $d$. The second term in (18) reflects the volatility in returns due to variation in the expectations component of conditional expected returns. Finally, the third term is volatility due to variation in the risk-premium component of conditional expected returns. Much like prices, each of these components behaves differently with changes in $\pi_0$ and other key parameters of interest, to which we now turn our focus.

4.2.1 Comparative statics on return moments

To investigate comparative statics, we start by presenting the following result.

**Proposition 3.** In the static model,

(i) The unconditional expected return is homogeneous of degree 1 (HD1) in $\sigma^2$ and $\alpha Z$.

(ii) The unconditional volatility component due to fundamental shocks is HD1 in $\sigma^2$ and HD0 in $\alpha Z$.

---

14In noisy rational expectations models, such as the one discussed in Section 6.2, predictability in returns is generated by aggregate supply shocks or noise traders. See Banerjee et al. (2009) for a more general discussion of this result and of the role of differences of opinion and noise traders in generating predictability in returns.

15Looking ahead, this will not be the case in the dynamic model, since the price will depend not only on expectations of future dividends, but also on expectations of future prices.
The unconditional volatility component due to the expectations component of returns is $HD_1 \text{ in } \sigma^2$ and $HD_0 \text{ in } \alpha Z$.

The unconditional volatility component due to the risk premium component of returns is $HD_2 \text{ in } \sigma^2$ and $\alpha Z$.

As expected, (i) implies that unconditional expected returns are increasing in the fundamental volatility and the overall risk concerns in the market (as captured by $\alpha Z$). Results (ii) through (iv) are also fairly intuitive, but they have important implications for which component drives overall volatility. In particular, when overall concerns about risk in the market are relatively high, the risk premium component of expression (18) is the key driver of overall return volatility. When $\alpha Z$ and $\sigma^2$ are relatively small, the first and second components of expression (18) drive overall volatility.

Proposition 3 is also useful for exploring comparative static results with respect to $\lambda$ and $\pi_0$. For example, (i) implies that when exploring how expected returns change with $\lambda$ and $\pi_0$, it is without loss to normalize $\sigma^2$ and $\alpha Z$. By doing so, we are left with a two-dimensional parameter space (i.e., $(\pi_0, \lambda) \in [0, 1]^2$), over which the expected return can be plotted to obtain comparative-static results that obtain for any parameter specification of the model. Figure 1(a) illustrates the result; both higher quality information (as measured by $\lambda$) and greater likelihood of an informed trader (as measured by $\pi_0$) decrease the expected return. This is because both higher quality information and a higher likelihood of an informed trader imply that the price is more informative about the fundamentals in expectation, and the uncertainty faced by the uninformed investor is lower.
Using (ii) through (iv), we can conduct a similar exercise to characterize the comparative static effects of each of the individual components of volatility. Figure 2(a) shows the volatility in returns due to fundamental dividend shocks is decreasing in \( \pi_0 \) and \( \lambda \), since an increase in either parameter reduces the uncertainty that investors face about next period’s dividend. Figure 2(b) shows that the variance in the expectations component of conditional expected returns is decreasing in \( \pi_0 \) but increasing in \( \lambda \). Recall that the expectations component of the conditional expected returns is non-zero because investors exhibit differences of opinion, and in particular, because uninformed \( \theta \) investors believe they are informed. This effect is larger when \( \pi_0 \) is smaller (since \( \theta \) investors are less likely to actually be informed) and when \( \lambda \) is larger (since uninformed \( \theta \) investors put more weight on their signals), which leads to the effect on volatility. Figure 2(c) shows the risk-premium component of volatility is non-monotonic in both \( \pi_0 \) and \( \lambda \). This is because the risk-premium component of returns is stochastic only when both \( \lambda \) and \( \pi_0 \) are strictly between zero and one.\(^{16}\)

Of course, comparative statics on the total return volatility depend on the relative magnitudes of \( \sigma^2 \) and \( \alpha Z \), which determine the relative weight on each component. For instance, Figure 1(b) presents the effect of \( \pi_0 \) and \( \lambda \) on overall volatility for a given set of parameters, for which the fundamental and expectations components dominate the risk-premium component.

## 5 The Dynamic Model

In this section, we incorporate learning by extending our analysis to a dynamic setting. This will illustrate which features of the static model are robust and highlight new insights delivered from learning dynamics, which allows us to generate testable predictions for return dynamics that we discuss in Sections 5.2 and 5.3.

There are two key differences between the static and dynamic settings. First, the price is affected not only by investors’ beliefs about fundamentals and other traders, but also their beliefs about future prices. Second, uninformed investors’ beliefs about other traders evolve stochastically over time as prices and dividends are realized. This creates an additional source of risk, which feeds back into prices. These two effects reinforce some of the results from the static setting, but overturn others.

Naturally, the degree to which these dynamic considerations affect current prices depends on the extent to which investor discount future payoffs. In particular, as one would expect,\(^{16}\)

---

\(^{16}\)If \( \pi_0 \in \{0, 1\} \), the conditional variance of \( U \) investors does not depend on \( S_\theta \). Consequently, \( \kappa \) and the risk-premium component of expected returns are constant. Similarly, when \( \lambda = 1 \), the risk-premium is zero, while when \( \lambda = 0 \), all \( \theta \) investors are (effectively) uninformed, and so the risk-premium is, again, constant.
Figure 2: The three components of volatility as they depend on the quality of information \( \lambda \) and the probability of a \( \theta \) being informed, i.e., \( \pi_\theta \). Panel (a) plots the fundamental component of volatility (i.e., \( \sigma^2(1 - \pi_\theta^2\lambda) \)), panel (b) plots the expectations component (i.e., \( (1 - \pi_\theta)^2\lambda^2 \text{var}[\kappa S_\theta] \)), and panel (c) plots the risk-premium component (i.e., \( (\sigma^2(1 - \lambda)\alpha Z)^2 \text{var}[\kappa] \)). The other parameters are set as in Figure 1.

return characteristics (i.e., expected returns, volatility) tend toward those in the static model as \( R \) increases. For our discussion and analysis, we assume a small enough value of \( R \) that allows us to highlight the differences in the implications of the dynamic and static models. We first provide a characterization of signal revealing equilibria in the dynamic setting.

**Proposition 4.** In any signal-revealing equilibrium, investor \( i \)'s optimal demand is given by expression (2), investor beliefs are given by

\[
\begin{align*}
    \mathbb{E}_{U,t}[d_{t+1}] &= \pi_t \lambda S_{\theta,t}, \\
    \mathbb{E}_{\theta,t}[d_{t+1}] &= \lambda S_{\theta,t}, \\
    \text{var}_{U,t}[d_{t+1}] &= \sigma^2(1 - \pi_t \lambda) + \pi_t(1 - \pi_t)(\lambda S_{\theta,t})^2, \text{ and } \text{var}_{\theta,t}[d_{t+1}] = \sigma^2(1 - \lambda)
\end{align*}
\]

and the price of the risky asset is given by

\[
P_t = \frac{1}{R} \left( \mathbb{E}_t[P_{t+1} + d_{t+1}] - \alpha \kappa_t \text{var}_{\theta,t}[P_{t+1} + d_{t+1}] Z \right),
\] (19)
where $\mathbb{E}_t[\cdot] \equiv \kappa_t \mathbb{E}_{\theta,t}[\cdot] + (1 - \kappa_t) \mathbb{E}_{U,t}[\cdot]$, and $\kappa_t$ and $\lambda$ are given by

$$
\kappa_t = \frac{\text{var}_{U,t}[P_{t+1} + d_{t+1}]}{\text{var}_{U,t}[P_{t+1} + d_{t+1}] + \text{var}_{\theta,t}[P_{t+1} + d_{t+1}]} \quad \text{and} \quad \lambda = \frac{\sigma^2}{\sigma^2 + \sigma_\epsilon^2}.
$$

(20)

The equilibrium price has an intuitive form; it is given by a weighted average of investors’ conditional expectations about future payoffs, adjusted for a risk-premium.\textsuperscript{17} The weight of each investor’s expectation in $\mathbb{E}_t[\cdot]$ depends on the conditional variance of her beliefs relative to those of the others.\textsuperscript{18} The price reflects only the market expectation (i.e., $P_t = \frac{1}{\mathbb{E}_t}[P_{t+1} + d_{t+1}])$ if either investors are risk-neutral (i.e., $\alpha \to 0$), or, if the aggregate supply of the risky asset is zero (i.e., $Z = 0$).

As in the static case, the characterization of the price in Proposition 4 implies that (dollar) returns can be expressed as

$$
Q_{t+1} = P_{t+1} + d_{t+1} - \mathbb{E}_t[P_{t+1} + d_{t+1}] + \alpha \kappa_t \text{var}_{\theta,t}[P_{t+1} + d_{t+1}]Z,
$$

and therefore conditional on an arbitrary information set $\mathcal{I}$, expected returns can be expressed as:

$$
\mathbb{E}[Q_{t+1}|\mathcal{I}] = \mathbb{E}[\kappa_t(\mathbb{E}_{U,t}[P_{t+1} + d_{t+1}] - \mathbb{E}_{\theta,t}[P_{t+1} + d_{t+1}])|\mathcal{I}] + \mathbb{E}[\alpha \kappa_t \text{var}_{\theta,t}[P_{t+1} + d_{t+1}]Z|\mathcal{I}].
$$

(21)

As in the static version, the conditional risk-premium is stochastic and the expectations component of conditional returns is non-zero when $\pi_t < 1$. In contrast to the static version, however, the expectations component of the unconditional expected return need not be zero in the dynamic model. As we discuss in Section 5.2, this is because the price is a non-linear function of $S_{\theta,t}$, and so while disagreement about next period’s dividend may be unconditionally zero, the disagreement about next period’s price is not.

### 5.1 Benchmark Cases

In this subsection, we consider two natural benchmarks which arise as special cases of the dynamic model. In both benchmarks, $U$ investors are not uncertain about whether $\theta$ is...\textsuperscript{17}

\textsuperscript{17}To this point, we do not have a proof of existence for the general case as doing so requires solving a non-standard fixed point problem and then verifying that the fixed point retains certain properties. However, we prove both existence and uniqueness in the static model as well as the two benchmark cases. We have also verified existence numerically for a wide range of parameters in the general case.

\textsuperscript{18}Specifically, the weight on investor $i$ is given by the precision of her conditional beliefs divided by the sum of the precisions of all investors, i.e., $\kappa_{i,t} = \frac{1}{\text{var}_{i,t}[P_{t+1} + d_{t+1}]} \sum_i \frac{1}{\text{var}_{i,t}[P_{t+1} + d_{t+1}]}$. 

18
informed. This exercise helps us highlight the key forces generating our main results. First, we characterize the equilibrium for the case in which \( \pi_t = 0 \). In this case, \( U \) investors do not condition on the price when updating their beliefs about the fundamental value of the asset and thus it is analogous to a Walrasian setting (or a standard difference of opinions setting).

Next, we consider the other extreme, when \( \pi_t = 1 \). This setting is analogous to a standard rational expectations environment.

Without uncertainty about other traders, the model’s predictions are more standard — the equilibrium price is linear and return moments are i.i.d. In the long-run steady state of the dynamic model, uncertainty about other traders is fully revealed: \( \pi_t \) converges almost surely to either 0 (if \( \theta = NI \)) or 1 (if \( \theta = I \)). This may cast doubt on whether our predictions should survive in the long run. In Section 6.1, we consider an extension of the model in which uncertainty about other traders persists even when the true underlying state is fully revealed and show that many of the interesting results survive.

**Proposition 5.** If \( \pi_0 = 0 \) and \( \theta = NI \), there exists a unique stationary equilibrium, which is signal-revealing, and the price of the risky asset is given by \( P_t = A_0 S_{\theta,t} + B_0 \), where \( A_0 \) is the unique real root to the following cubic equation:

\[
RA_0 \left( 2A_0^3 + (2 - \lambda) \lambda \right) - \lambda \left( A_0^2 + \lambda \right) = 0,
\]

where

\[
B_0 = -\frac{1}{r} \kappa_t \left( A_0^2 (\sigma^2 + \sigma_{\varepsilon}^2) + \sigma^2 (1 - \lambda) \right) \alpha Z, \quad \text{and} \quad \lambda \text{ and } \kappa_t \text{ are given by equation (20).}
\]

Since \( \mathbb{E}_t [d_{t+1} | S_{\theta,t}] = 0 \), conditional expected returns and variance in returns are respectively given by

\[
\mathbb{E}_t [Q_{t+1} | S_{\theta,t}] = \kappa_t \alpha \text{var}_t [Q_{t+1} | S_{\theta,t}] Z - RA_0 S_{\theta,t}, \quad \text{and}
\]

\[
\text{var}_t [Q_{t+1} | S_{\theta,t}] = A_0^2 \left( \sigma^2 + \sigma_{\varepsilon}^2 \right) + \sigma^2.
\]

Though \( \theta \) investors are not informed, they believe they have payoff relevant information. As a result, the price responds to realizations of \( S_{\theta,t} \). However, since the signals are spurious, prices are expected to mean-revert in the next period, and this induces mean reversion in expected returns through the \(-RA_0 S_{\theta,t}\) term in expression (22). Finally, since the equilibrium price is linear in \( S_{\theta,t} \), the risk-premium component of expected returns and the conditional volatility of returns are constant.

**Proposition 6.** If \( \pi_0 = 1 \) and \( \theta = I \), there exists a unique stationary equilibrium, which is signal-revealing, and where investor \( i \)'s optimal demand is given by expression (2), the price of the risky asset is given by \( P_t = A_1 S_{\theta,t} + B_1 \), where \( A_1 = \frac{\lambda}{R} \), \( B_1 = -\frac{1}{2r} (A_1^2 (\sigma^2 + \sigma_{\varepsilon}^2) + \sigma^2 (1 - \lambda)) \alpha Z \), and \( \lambda \) and \( \kappa_t \) are given by equation (20). Since \( \mathbb{E}_t [d_{t+1} | S_{\theta,t}] = \lambda S_{\theta,t} \),
conditional expected returns and variance in returns are respectively given by

\[ \mathbb{E}_t[Q_{t+1}|S_{\theta,t}] = \frac{1}{2} \alpha \text{var}_t[Q_{t+1}|S_{\theta,t}]Z, \quad \text{and} \quad \text{var}_t[Q_{t+1}|S_{\theta,t}] = A_1^2(\sigma^2 + \sigma_e^2) + \sigma^2(1 - \lambda). \]

When all \( \theta \) investors are informed and \( U \) investors are certain about this (i.e., \( \pi_t = 1 \)), prices are informationally efficient with respect to the signal \( S_{\theta,t} \). As a result, expected returns are constant, and reflect only the risk-premium that investors require for holding the risky asset. The conditional volatility of returns is also constant, since the equilibrium price is linear in \( S_{\theta,t} \).

5.2 Prices, Returns and Volatility

In the general case (i.e., \( \pi_t \in (0,1) \)), the price is a non-linear function of \( S_{\theta,t} \) and \( \pi_t \). As a result, the equilibrium price, which depends on conditional expectation and variance of next period’s price for each investor, cannot be characterized in closed form. Instead, we solve the general dynamic model numerically, by using an iterative procedure to compute the equilibrium price function (i.e., the fixed-point of (19)). Figure 3 illustrates the two components of the price. As in the static setting, the price \( P_t \) — the sum of the two components — is more sensitive to bad news (i.e., negative \( S_{\theta,t} \)) than it is to good news (i.e., positive \( S_{\theta,t} \)). All else equal, investors’ expectation of dividends next period, and hence the expectations component of prices as in Figure 3(a), increases in \( S_{\theta,t} \). However, a surprise in \( S_{\theta,t} \) in either direction also leads to an increase in uncertainty for the \( U \) investor, and hence the risk-premium component is hump-shaped in \( S_{\theta,t} \) as in Figure 3(b). For negative \( S_{\theta,t} \) these two effects reinforce each other, while for positive \( S_{\theta,t} \), the effects offset each other, and this leads to the asymmetric reaction of prices to \( S_{\theta,t} \). The comparative statics with respect to \( \pi_t \) are familiar from the static case — the sensitivity of the expectations component to \( S_{\theta,t} \) increases in \( \pi_t \) and the risk-premium component is \( U \) shaped in \( \pi_t \) for any \( S_{\theta,t} \).

Equation (21) highlights how expected returns can be decomposed into an expectations component and a risk-premium component. And as in the static case, the relative impact of the expectations component and the risk-premium component depend on fundamental volatility and overall risk concerns (i.e., \( \alpha Z \)). Unlike the static case, however, the expectations component of unconditional expected returns is not zero in the dynamic model. As Figure 4(a) suggests, this is because it depends not only on the difference in investors’ unconditional expectations of \( d_{t+1} \) (which is zero, as in the static setting), but also the difference in their beliefs about \( P_{t+1} \), which is not zero. This can be interpreted as a measure of disagreement between \( U \) and \( \theta \) investors about future prices, it increases in \( \lambda \) and decreases in \( \pi_t \).
Intuitively, investors disagree more when $\lambda$ is larger since each puts more weight on their own interpretation of $S_{\theta,t}$, but less when $\pi_t$ is larger. The risk-premium component of expected returns, shown in Figure 4(b), also behaves differently in the dynamic model. Recall that in the static setup, the risk-premium component decreases in $\lambda$ and $\pi_t$ since better information (i.e., higher $\lambda$) and a higher likelihood of $\theta$ being informed both lead to less uncertainty about next period’s dividend. However, in the dynamic model, the risk-premium component is non-monotonic in both $\pi_t$ and $\lambda$.

The difference in these patterns is a consequence of uncertainty about future prices, which in turn affects the risk-premium. In particular, while an increase in $\lambda$ decreases investor
uncertainty about next period’s dividends, it also makes prices more sensitive to \( S_{\theta,t} \) thereby increasing future volatility. Similarly, for a fixed \( \lambda \), the risk-premium component of the price is most sensitive to \( S_{\theta,t} \) for intermediate values of \( \pi_t \) (see Figure 3(b)), and so uncertainty about future prices is higher for intermediate values of \( \pi_t \). The interaction between dividend uncertainty and price uncertainty leads to the patterns in the risk-premium component. For high and low values of \( \pi_t \), the effect of \( \lambda \) on dividend uncertainty dominates, and so the risk-premium decreases in \( \lambda \). For intermediate values of \( \pi_t \), the effect of \( \lambda \) on dividend uncertainty dominates for low \( \lambda \) but the effect on price uncertainty dominates for high \( \lambda \), and hence the risk-premium exhibits a U-shape in \( \lambda \). Finally, all else equal, uncertainty about next period’s dividend (for the U investor) and about the risk-premium component of next period’s price are highest for intermediate values of \( \pi_t \), and hence the risk-premium exhibits a hump-shape in \( \pi_t \).

We can decompose the total volatility (i.e., unconditional variance) of returns into the expectation of the conditional variance (plotted in Figure 5(a)), and the variance of the conditional expectation (plotted in Figure 5(b)). Much like the analysis of expected returns, beliefs about future prices play an important role. First, note that the conditional variance in returns now depends not only on uncertainty about next period’s dividend (as in the static setup) but also on uncertainty about next period’s price. While uncertainty about next period’s dividend is decreasing in both \( \pi_t \) and \( \lambda \), uncertainty about next period’s price can increase in \( \lambda \) (as \( P_{t+1} \) becomes more sensitive to \( S_{\theta,t+1} \)) and is hump-shaped in \( \pi_t \), as discussed above. As a result, the expected conditional variance component of the volatility is hump-shaped in \( \pi_t \) and may increase or decrease in \( \lambda \).

\[ \text{Figure 5: The two components of unconditional return volatility as they depend on the underlying state variable and the quality of the information.} \]
As in the static case, the variance of the conditional expectation of returns is increasing in $\lambda$ — all else equal, a larger $\lambda$ increases the sensitivity of investors’ expectations to $S_{\theta,t}$ and, therefore, makes them more volatile. The effect of $\pi_t$ on the variance of the conditional expectation of returns depends on which component dominates. In particular, as equation (21) suggests, the expectations component of conditional expected returns is driven by the disagreement between $U$ and $\theta$ investors about expected future dividends and prices, and so the variance of this component decreases in $\pi_t$, as disagreement between investors falls.\footnote{The disagreement between the $U$ and $\theta$ investors is given by $|E_{U,t}[d_{t+1}] - E_{\theta,t}[d_{t+1}]| = (1 - \pi_t)\lambda|S_{\theta,t}|$.}

However, the variance of the risk-premium component of expected returns is hump-shaped in $\pi_t$ since the conditional risk-premium is most sensitive to $S_{\theta,t}$ when $\pi_t$ is near $\frac{1}{2}$. The overall effect of $\pi_t$ and $\lambda$ on the variance of returns depend on the interaction of these components, which in turn, depend on the relative impact of the expectations and risk premium components of returns.

Note that the above analysis implies that the relation between return characteristics (i.e., expected returns and volatility) and information quality (i.e., $\lambda$) depends on the state $\pi_t$, and, therefore, can vary over time. The following proposition shows the relation has the opposite sign at the two boundary values of $\pi_t$.

**Proposition 7.** When $\pi_t = 1$, expected returns and volatility are decreasing in $\lambda$. When $\pi_t = 0$, expected returns and volatility are increasing in $\lambda$.

As standard intuition suggests, when investors agree on the informativeness of $S_{\theta,t}$ (i.e., $\pi_t$ is close to one), higher information quality (higher $\lambda$) leads to lower uncertainty and therefore lower expected returns. However, if investors disagree on the interpretation of the signal (i.e., $\pi_t$ is close to zero), a signal with a higher $\lambda$ generates more uncertainty for the $U$ investor since it makes the $\theta$ investor trade more aggressively on his information. All else equal, this leads to higher volatility of current and future prices (due to higher sensitivity to $S_{\theta,t}$ shocks), which leads to higher expected returns.

Proposition 7 predicts that the relation between information quality and return moments depends on the likelihood that investors are informed (i.e., $\pi_t$), and as a result, can vary over time. This may be useful in reconciling the mixed empirical evidence on the relation between information quality and returns discussed in Section 1. The model also suggests that conditioning on an empirical proxy of $\pi_t$ (e.g., institutional ownership) may be useful in uncovering the underlying relation. Finally, since the disagreement about dividend forecasts in our model decreases in $\pi_t$, our model suggests that the relation between proxies of disagreement (e.g., analyst forecast dispersion) and expected returns may also be time-varying.
5.3 Return Dynamics and Volatility Clustering

The way in which $\lambda$ and $\pi_t$ affect the distribution of prices and returns allow us to generate predictions of the model that distinguish it from standard (linear) rational expectations models. The probability $\pi_t$ that $U$ investors attribute to $\theta$ investors being informed, is a stochastic (endogenous) state variable of the model and is persistent over time.\textsuperscript{20} As a result, in addition to generating \textit{stochastic} expected returns and volatility, the model predicts that these conditional moments are \textit{persistent}, despite the fact that shocks to fundamentals and signals are i.i.d.

The dynamic model generates predictability in future expected returns and volatility. In particular, for $\pi_t$ close to one, the model predicts \textit{volatility clustering} — return surprises in either direction are followed by an increase in both volatility and expected returns. The intuition for these results follows from the discussion in the earlier section. An unanticipated realization of $d_{t+1}$ leads the $U$ investor to revise her beliefs about $\theta$ being informed downwards (i.e., $\pi_{t+1} < \pi_t$).\textsuperscript{21} This revision in beliefs generates additional uncertainty for $U$ investors, and as a result, leads to higher future volatility and higher expected returns going forward (see Figures 4 and 5). For $\pi_t$ close to zero, the opposite result can obtain; returns in line with expectations cause the $U$ investor to revise her belief upwards, which again increases the uncertainty about other traders and hence volatility and expected returns. In this sense, no \textit{news} (i.e., little to no surprise in returns) can either be good news (when $\pi_t$ is close to one) or bad news (when $\pi_t$ is close to zero).

Figure 6 illustrates this clustering effect. Specifically, the figure plots expected returns and volatility in period $t + 1$ as a function of the current realization of $d_{t+1}$ (scaled by its standard error). Note that most realizations of $|d_{t+1} - S_{\theta,t}|$ lead to an increase in both volatility and expected returns. However, sufficiently large realizations of $|d_{t+1} - S_{\theta,t}|$ decreases expected returns as the posterior $\pi_{t+1}$ tends to zero.

5.4 Parameterized Example

In this section, we parametrize the model to gage the magnitude and economic significance of our predictions. While the analysis in the earlier sections has focused on characterizing properties of dollar returns per share (i.e., $Q_{t+1}$), in this section, we characterize properties of the rate of return (i.e., $r_{t+1} = Q_{t+1}/P_t$) in order to highlight the robustness of the results and to facilitate comparisons to the broader literature.\textsuperscript{22} The risk-free rate is set to 5%, the

\textsuperscript{20}From the perspective of the $U$ investor it is a martingale; $E_{U,t}[\pi_{t+1}] = \pi_t$.
\textsuperscript{21}This follows from the evolution of $\pi_t$ in (6) and because $\pi_t$ is initially large.
\textsuperscript{22}Because we are using normally distributed random variables, the population moments of $r_{t+1}$ are not well defined due to prices arbitrarily close to zero (see Campbell, Grossman, and Wang (1993) and Llorente,
mean of $d_{t+1}$ is set to 3.5% and the supply of the asset is normalized to $Z = 1$.

The remaining two parameters are chosen so that, in the benchmark case where it is commonly known that $I$ is informed (i.e., $\pi_t = 1$), the excess expected rate of return is 3% and the volatility in returns is 23%. In particular, we set $\alpha = 0.66$ and $\sigma = 1.18$. It is important to keep in mind that while dividends are extremely persistent in the data, they are assumed to be i.i.d. in our model. Hence, in order to match reasonable levels of expected returns and volatility, we must set the variance of dividends to be significantly higher than what is usually observed in the data.

As Figure 7 shows, for this parametrization, both the expectations component and the risk premium component play a role; excess returns are first increasing in $\pi_t$ (as in Figure 4(b)) but decreasing for larger $\pi_t$ (as in Figure 4(a)). Similarly, expected returns and volatility are decreasing in $\lambda$ for high $\pi_t$; better quality information reduces uncertainty when traders are sufficiently confident of its source. However, returns and volatility are increasing in $\lambda$ for low $\pi_t$; higher quality information leads to more volatile asset prices when traders are skeptical of the information source.

Figure 7 also gives a sense of the magnitude of the comparative statics results. For instance, an increase in $\lambda$ from 0.3 to 0.7 implies an increase in expected returns from 4.6% to 6.9% and an increase in volatility from 26% to 31% percent (for $\pi_t = 0.5$); an increase in $\pi_t$ from 0.3 to 0.7 implies a decrease in expected returns from 5.9% to 4.5% percent and a decrease in volatility from 29% to 26% percent (for $\lambda = 0.5$).

*Michaely, Saar, and Wang, 2002* for a discussion). We adopt the conventional approach and choose $\mu$ large
Figure 7: Illustrates the economic significance of the learning, after combining both the expectations component and the risk premium component, on the total of returns and volatility.

Figure 8: Illustrates the economic significance of the clustering effect. In both figures $\pi_t = 0.9$. 

(a) Expected Excess Rate of Return

(b) Volatility of Rate of Return
For the above parameters, Figure 8 provides magnitudes for the volatility clustering effect described in Section 5.3. For \( \lambda = 0.7 \), a one-standard deviation surprise in dividend realizations predicts an increase in future expected excess returns of roughly 1.5\% (from 3.5\% to 5\%) and an increase in future volatility of 3.5\% (from 23.3\% to 26.8\%). It is important to note that in the benchmark cases where \( \pi_t \in \{0, 1\} \), these plots are perfectly flat. Thus, even for small deviations from the benchmark model \( (\pi_t = 0.9) \), the clustering effect can be quite economically significant.

6 Extensions

In this section, we present two alternative specifications of the model. In the first, we generalize the setup from Section 3 (hereafter, referred to as the base model) by allowing the type of \( \theta \) investors (I or NI) to change stochastically from period to period. In addition to providing a generalization of the base model, this setup ensures that even after observing an arbitrarily long history of signals and dividends \( (t \to \infty) \), the uninformed investor still faces uncertainty about whether the other traders are informed, and therefore the effects of this uncertainty persist in the limit.

In the second alternative specification, investors share a common prior (i.e., NI investors are fully rational) but the aggregate supply of the risky asset is stochastic. This specification is useful in highlighting the mechanism in a more familiar setting — the model reduces to the noisy rational expectations model of Grossman and Stiglitz (1980) when \( \pi_0 = 1 \). This specification also helps to illustrate the robustness of our qualitative implications though it is less analytically tractable than the base model.

6.1 Uncertainty and Learning about the Likelihood of Informed Traders

The setup is as in Section 3, except that we allow \( \theta \) to change over time; each period, the probability that the \( \theta \) investor is informed is \( p \in \{p_L, p_H\} \), where \( p_L \leq p_H \).\(^{23}\) The uninformed investors have a prior \( \pi_0 = \Pr(p = p_H) \). Two immediate observations will be useful. First, the base model is a special case in which \( p_L = 0 \) and \( p_H = 1 \). Second, \( U \) investors care whether \( p = p_H \) only to the extent that it influences the probability that the \( \theta \) investor is informed in the current and future periods, which we denote by \( \hat{\pi}_t = p_H \pi_t + p_L (1 - \pi_t) \). In particular, conditional dividend moments can be computed as in (4)-(5) by replacing \( \pi_t \) with enough such that the numerical estimation of these moments is well behaved.

\(^{23}\)We thank Paul Pfleiderer for suggesting this extension of the model.
That is,
\[
\mathbb{E}_{U,t}[d_{t+1}] = \hat{\pi}_t \lambda S_{\theta,t}, \quad \text{and}
\]
\[
\text{var}_{U,t}[d_{t+1}] = \hat{\pi}_t \sigma^2 (1 - \lambda) + (1 - \hat{\pi}_t)\sigma^2 + \hat{\pi}_t (1 - \hat{\pi}_t) \lambda S_{\theta,t}^2.
\]

Given these observations, we can show there is a mapping from the two-date version of this extension to the two-date version of the base model.

**Proposition 8.** *In the two-date version of this extension, the results from Section 4 continue to hold by replacing \( \pi_0 \) with \( \hat{\pi}_0 \equiv \pi_0 p_H + (1 - \pi_0) p_L \).*

In the dynamic setting, the analysis can be generalized with similar modifications. Instead of (6), the \( U \) investor will update her beliefs about \( p \) according to
\[
\pi_{t+1} = \frac{\pi_t \left( p_H \phi_1 + (1 - p_H) \phi_2 \right)}{\pi_t \left( p_H \phi_1 + (1 - p_H) \phi_2 \right) + (1 - \pi_t) \left( p_L \phi_1 + (1 - p_L) \phi_2 \right)},
\]
where
\[
\phi_1 = \frac{1}{\sigma_c} \phi \left( \frac{S_{\theta,t+1} - d_{t+1}}{\sigma_c} \right), \quad \phi_2 = \frac{1}{\sqrt{\sigma^2 + \sigma_c^2}} \phi \left( \frac{S_{\theta,t} - 0}{\sqrt{\sigma^2 + \sigma_c^2}} \right).
\]
She will then assign probability \( \hat{\pi}_{t+1} = \pi_{t+1} p_H + (1 - \pi_{t+1}) p_L \) to the event that the \( \theta \) investors is informed in period \( t + 1 \), which will be used to compute dividend moments in \( t + 1 \).

As in the base model, in any signal revealing equilibrium, investor \( U \)’s uncertainty about the aggregate underlying state variable is resolved eventually (i.e., \( \lim_{t \to \infty} \pi_t \equiv 1_{\{p = p_H\}} \)). However, unlike the benchmark cases in Section 5.1, in the current setup \( U \) investors face uncertainty about whether other traders are informed even as \( t \to \infty \). As a result, the limiting equilibrium price function looks remarkably similar to the equilibrium price in the static version of both models.

**Proposition 9.** *Suppose that \( \pi_0 \in \{0, 1\} \) so that \( \hat{\pi}_t = p \in \{p_L, p_H\} \) is fixed for all \( t \). Then, there exists a stationary equilibrium in which the price is characterized by the solution to:

\[
P^* \equiv \frac{1}{R} \left( m + \left( p + \kappa^* (1 - p) \right) \lambda S_{\theta} - \kappa^* \left( v^* + \sigma^2 (1 - \lambda) \right) \alpha Z \right), \quad \text{where}
\]
\[
\kappa^* \equiv \frac{v^* + \sigma^2 (1 - p \lambda) + p (1 - p) \lambda S_{\theta}^2}{2v^* + \sigma^2 (1 - \lambda) + \sigma^2 (1 - p \lambda) + p (1 - p) \lambda S_{\theta}^2},
\]
\[
m^* \equiv \frac{1}{R - 1} \left( v^* + \sigma^2 (1 - \lambda) \right) \mathbb{E} [\kappa^*] \alpha Z,
\]
and \( v^* \) is characterized implicitly by
\[
v^* = \frac{1}{R^2} \left( (1 - p)^2 \lambda^2 \text{var} (\kappa^* S_{\theta}) + (v^* + \sigma^2 (1 - \lambda))^2 \alpha^2 Z^2 \text{var} (\kappa^*) \right).
\]
The above result shows that under this specification, even the limiting price is a non-linear function of the signal. As such, properties from the base model (e.g., stochastic conditional expected returns and volatility, asymmetric reaction to news) survive in the limit. This is in contrast to limiting behavior of the base model, as illustrated by the analysis of the benchmark models in Section 5.1, where the price is linear in the signal.

6.2 Common Priors and Noisy Aggregate Supply

We focus on the two-date version (normalize \( R = 1 \)) using the same setup as in Section 4, with the following exceptions: there is a noisy supply of the asset and the investors share a common prior about the signal. The latter implies that, conditional on \( \theta = NI \), investors agree that the signal is uninformative. Or, equivalently, \( NI \) investors do not observe a signal prior to submitting orders. As a result, the optimal demand for a \( \theta \) investor is given by:

\[
x_\theta = \begin{cases} 
  \frac{\lambda S_\theta - P}{\alpha \sigma^2 (1 - \lambda)} & \text{if } \theta = I \\
  \frac{\theta - P}{\alpha \sigma^2} & \text{if } \theta = NI,
\end{cases}
\]

Note that without an additional source of noise, observing price and quantities perfectly reveals both \( S_\theta \) and \( \theta \). Thus, we follow the noisy rational expectations literature and introduce aggregate supply shocks. In particular, the aggregate supply of the risky asset is \( Z + z_t \), where \( z_t \sim N(0, \sigma^2_z) \).\(^{24}\) The market clearing condition is given by:

\[
x_{\theta,t} + x_{U,t} = Z + z_t.
\]

Finally, as in the base model, we assume \( U \) investors can condition on the equilibrium price and residual supply when determining their optimal demand. In this setup, we show that there exists a rational expectations equilibrium which is characterized by the following proposition.

**Proposition 10.** There exists a rational expectations equilibrium in which the price is characterized by the solution to:

\[
P_t = (\kappa_t + (1 - \kappa_t) \pi_t \lambda_y) y_t - \kappa_t \alpha \sigma^2 (1 - \lambda) Z
\]

\(^{24}\) Alternatively, one could assume that in addition to being potentially informed, \( \theta \) investors anticipate an endowment to their wealth of \( z_t d_{t+1} \) in the next period, where \( z_t \) is known to \( \theta \) investors but not to \( U \) investors, and \( z_t \sim N(0, \sigma^2_z) \).
where \(y_t = \alpha \sigma^2 (1 - \lambda)(x_{\theta,t} - z_t) + P_t\), \(\pi_t = \Pr(\theta = I|y_t, P_t)\), and

\[\kappa_t = \frac{\sigma^2 (1 - \pi_t \lambda_y) + \pi_t (1 - \pi_t) (\lambda_y y_t)^2}{\sigma^2 (1 - \lambda) + \sigma^2 (1 - \pi_t \lambda_y) + \pi_t (1 - \pi_t) (\lambda_y y_t)^2}.\]

Analogous to the decomposition in equation (9), the price can be decomposed as:

\[P_t = (\kappa_t + (1 - \kappa_t) \pi_t \lambda_y) y_t - \kappa_t \alpha \sigma^2 (1 - \lambda) Z.\]

Figure 9 illustrates these two components. The plots suggest that uncertainty about whether others are informed has qualitatively similar implications in this setup. As in the static model of Section 4, the expectations component is monotonic in the price signal. Moreover, \(U\) investors are unsure about the informativeness of \(y_t\), which implies that their posterior variance, and therefore, the risk-premium component of price depends on the realization of \(y_t\). Thus, as in the base model, the price reacts asymmetrically to good news vs. bad news.

In contrast to the base model, \(U\) investors learn directly about \(\theta\) from the signal \(y_t\); large realizations of \(y_t\) lead to large updates in \(\pi_t\) (either towards zero or one). That is, both uncertainty and learning are present in the two date version of this specification. As a result, the risk-premium component is dampened for large realizations of \(y_t\), since for these realizations \(\pi_t\) is closer to zero or one. Though we do not numerically solve the dynamic version of this specification, clearly both uncertainty and learning will play a role in such a setting. Based on the analysis here, we expect qualitatively similar results to those in Section 5 to obtain.
7 Final Remarks

We consider a framework in which investors are uncertain about whether others are informed and gradually learn about them by observing prices and dividends. We show that these additional channels of uncertainty and learning can have important implications for return dynamics. Specifically, the model generates non-linear prices, which are more sensitive to bad news than good news; persistent, stochastic expected returns and volatility, even though shocks to fundamentals and signals are i.i.d.; and volatility clustering (i.e., big return realizations of either sign are followed by higher volatility and higher expected returns). We connect these predictions to the empirical literature and discuss several new testable implications.

We have kept the model as parsimonious as possible to highlight the key forces behind our results and to maintain tractability. For instance, specifying i.i.d. dividends facilitates the analysis using only a single state variable ($\pi_t$). This clarifies the economic intuition (all persistence in the model derives from $U$ investors’ uncertainty about $\theta$) and simplifies the numerical analysis. However, the lack of persistence in dividends limits the model’s ability to empirically match moments of the data. Developing an appropriately enriched version of the model that is suitable for calibration is left for future work. Extending this framework to study the role of information acquisition, and thereby endogenizing the information structure, also seems to be a promising direction for future work.
References


Keynes, J. (1936): The general theory of employment, interest and money, MacMillan.


Appendix - Proofs

Proof of Proposition 1. If the equilibrium did not reveal $S_{\theta,t}$, then there would exist $S_{\theta,t}^1 > S_{\theta,t}^2$ for which $P_t$ would be the same. But in this case, $x_{\theta,t}(S_{\theta,t}^1) > x_{\theta,t}(S_{\theta,t}^2)$, which implies that $S_{\theta,t}$ would be revealed in equilibrium. Moreover, since $\theta = I$ and $\theta = NI$ have symmetric optimal strategies, the equilibrium cannot reveal $\theta$. Existence and uniqueness follow from characterizing the optimal demand for $U$ and $\theta$ investors at date $t$ in terms of their beliefs about next period’s dividend $d_{t+1}$, and imposing the market clearing condition.  

Proof of Corollary 1. Taking derivatives of $\kappa_t$ we get

$$\frac{\partial}{\partial \lambda} \kappa_t = \frac{\sigma^2(1-\pi_t)(\pi_t S_{\theta,t}^t(2-\lambda))}{(\sigma^2(1-\lambda)+\sigma^2(1-\pi_t))}\frac{(2-\lambda)+\sigma^2}{\sigma^2(1-\lambda)}(1-\kappa_t)^2 \geq 0 \quad (31)$$

$$\frac{\partial}{\partial \pi_t} \kappa_t = -\frac{(\sigma^2(1-2\pi_t)\sigma^2 S_{\theta,t}^t\sigma^2\kappa_t)}{(\sigma^2(1)+\sigma^2(1-\pi_t))}\frac{(2-\lambda)+\sigma^2}{\sigma^2(1-\lambda)}(1-\kappa_t)^2 \quad (32)$$

$$\frac{\partial}{\partial S_{\theta,t}} \kappa_t = -\frac{2\pi_t(1-\pi_t)(1-\lambda)^2}{(\lambda^2+\sigma^2(1-\pi_t))\lambda S_{\theta,t}^2}\frac{(2-\lambda)+\sigma^2}{\kappa_t(1-\lambda)}(1-\kappa_t)^2 \quad (33)$$

which implies $\kappa_t$ is $U$-shaped around $S_{\theta,t} = 0$, and is hump shaped in $\pi_t$ around $\frac{1}{2} \left(1 - \frac{\sigma^2}{\lambda S_{\theta,t}^2}\right)$.

- Effect of $\lambda$: The derivative of the expectations component of $P_t$ with respect to $\lambda$ is given by

$$\frac{\partial}{\partial \lambda} ((\kappa_t + (1-\kappa_t)\pi_t))\lambda S_{\theta,t}) = (\pi_t + (1-\pi_t))\left((\kappa_t + \lambda \frac{\partial}{\partial \lambda} \kappa_t)\right) S_{\theta,t}$$

and so this component increases with $\lambda$ for $S_{\theta,t} > 0$ and decreases in $\lambda$ otherwise. The derivative of risk-premium component is given by

$$\frac{\partial}{\partial \lambda}(-\alpha \sigma^2(1-\lambda)\kappa_t Z) = -\alpha \sigma^2 Z ((1-\lambda)\frac{\partial}{\partial \lambda} \kappa_t - \kappa_t)$$

The derivative can be positive or negative depending on $|S_{\theta,t}|$. For $|S_{\theta,t}|$ small enough, the derivative is strictly positive, and so the risk-premium component of price increases in $\lambda$. But for $|S_{\theta,t}|$ large enough the derivative can be negative for intermediate $\lambda$, and so the risk-premium component of price is $U$-shaped in $\lambda$. Therefore, the risk premium is not monotone in $\lambda$. However, the risk-premium component is always lower for $\lambda = 0$ than it is for $\lambda = 1$.

- Effect of $\pi_t$: The derivative of the expectations component of $P_t$ with respect to $\pi_t$ is given by

$$\frac{\partial}{\partial \pi_t} ((\kappa_t + (1-\kappa_t)\pi_t))\lambda S_{\theta,t}) = \left((1-\kappa_t) + (1-\pi_t)\frac{\partial}{\partial \pi_t} \kappa_t\right) \lambda S_{\theta,t}$$

Inserting the expression from (32) for $\frac{\partial}{\partial \pi_t} \kappa_t$ gives:

$$\left((1-\kappa_t) + (1-\pi_t)\frac{\partial}{\partial \pi_t} \kappa_t\right) = \frac{\sigma^2(1-\lambda)((1-\pi_t)S_{\theta,t})^2 + 2(1-\lambda)\sigma^2}{(1-\lambda)\sigma^2 + (1-\pi_t)\sigma^2 + (\lambda S_{\theta,t})^2(1-\pi_t)} > 0$$
Therefore, the derivative of the expectations component of prices with respect to $\pi_t$ has the same sign as $S_{\theta,t}$.

The risk premium component of the price is $U$-shaped in $\pi_t$ around $\frac{1}{2} \left( 1 - \frac{\sigma^2}{\lambda S_{\theta,t}} \right)$, since

$$\frac{\partial}{\partial \pi_t}(-\alpha\sigma^2(1-\lambda)\kappa_t Z) = -\alpha\sigma^2(1-\lambda)Z \frac{\partial}{\partial \pi_t} \kappa_t$$

- Effect of $S_{\theta,t}$: The expectations component of $P_t$ is always increasing in $S_{\theta,t}$, since

$$\frac{\partial}{\partial S_{\theta,t}} \left((\kappa_t + (1 - \kappa_t)\pi_t)\lambda S_{\theta,t}\right) = (\kappa_t + (1 - \kappa_t)\pi_t)\lambda + (1 - \pi_t)\lambda S_{\theta,t} \frac{\partial}{\partial S_{\theta,t}} \kappa_t > 0$$

The risk-premium component of the price is hump-shaped in $S_{\theta,t}$ around zero, since

$$\frac{\partial}{\partial S_{\theta,t}}(-\alpha\sigma^2(1-\lambda)\kappa_t Z) = -\alpha\sigma^2(1-\lambda)Z \frac{\partial}{\partial S_{\theta,t}} \kappa_t$$

This establishes the comparative static results. \hfill \square

**Proof of Proposition 2.** Taking the expectation of the right-hand side (RHS) of (12) and using that $\mathbb{E}_t[d_{t+1}] = \pi_0 \lambda S_{\theta,t}$, we have

$$\mathbb{E}[Q_{t+1}] = \mathbb{E}[\mathbb{E}_t[Q_{t+1}]] = \alpha\sigma^2(1-\lambda)Z\mathbb{E}[\kappa_t] - (1 - \pi_0)\lambda \mathbb{E}[\kappa_t S_{\theta,t}]$$

Thus, it suffices to show that $\mathbb{E}[\kappa_t S_{\theta,t}] = 0$. For this, note that $\kappa_t \cdot S_{\theta,t}$ is an odd-function (of $S_{\theta,t}$) and the distribution of $S_{\theta,t}$ is symmetric around zero. Thus, $\mathbb{E}[\kappa_t S_{\theta,t}|S_{\theta,t} > 0] = -\mathbb{E}[\kappa_t S_{\theta,t}|S_{\theta,t} < 0]$, which implies $\mathbb{E}[\kappa_t S_{\theta,t}] = 0$.

Volatility of returns is given by

$$\text{var}[Q_{t+1}] = \mathbb{E}[	ext{var}_t[Q_{t+1}]] + \text{var}[\mathbb{E}_t[Q_{t+1}]]$$

$$= \mathbb{E}[\sigma^2(1 - \pi_t \lambda) + \pi_t(1 - \pi_t)(\lambda S_{\theta,t})^2] + \text{var} \left[ \alpha\sigma^2(1-\lambda)\kappa_t Z - (1 - \pi_0)\kappa_t \lambda S_{\theta,t} \right]$$

$$= \sigma^2(1 - \pi_t^2 \lambda) + (\alpha\sigma^2(1-\lambda)Z)^2 \text{var}[\kappa_t] + (1 - \pi_0)^2 \lambda^2 \text{var}[\kappa_t S_{\theta,t}]$$

$$- 2\alpha\sigma^2(1-\lambda)\lambda Z(1 - \pi_0)\text{cov}(\kappa_t, \kappa_t S_{\theta,t})$$

Recall that Stein’s Lemma implies that for $Y \sim \mathcal{N}(0, \sigma_Y^2)$, and $g(Y)$ such that $\mathbb{E}[g(Y)] < \infty$ and $\sigma_Y^2 \mathbb{E}[g'(Y)] < \infty$, we have $\text{cov}(g(Y), X) = \mathbb{E}[g(Y)]\text{cov}(Y, X)$. This implies

$$\text{cov}(\kappa_t, S_{\theta,t}) = \mathbb{E} \left[ \frac{\partial}{\partial S_{\theta,t}} \kappa_t \right] \text{var}(S_{\theta,t})$$

$$\text{var}[\kappa_t S_{\theta,t}] = \mathbb{E}[\kappa_t^2 S_{\theta,t}^2] - (\mathbb{E}[\kappa_t S_{\theta,t}])^2$$

$$= \text{cov}(\kappa_t^2 S_{\theta,t}, S_{\theta,t}) - \text{cov}(\kappa_t, S_{\theta,t})$$

$$= \left( \mathbb{E} \left[ \kappa_t^2 + 2\kappa_t S_{\theta,t} \frac{\partial}{\partial S_{\theta,t}} \kappa_t \right] - \mathbb{E} \left[ \frac{\partial}{\partial S_{\theta,t}} \kappa_t \right] \right) \text{var}(S_{\theta,t})$$

$$\text{cov}(\kappa_t, \kappa_t S_{\theta,t}) = \mathbb{E}[\kappa_t^2 S_{\theta,t}] - \mathbb{E}[\kappa_t] \mathbb{E}[\kappa_t S_{\theta,t}]$$

$$= \text{cov}(\kappa_t^2, S_{\theta,t}) - \mathbb{E}[\kappa_t] \text{cov}(\kappa_t, S_{\theta,t})$$

$$= \left( \mathbb{E} \left[ 2\kappa_t \frac{\partial}{\partial S_{\theta,t}} \kappa_t \right] - \mathbb{E}[\kappa_t] \mathbb{E} \left[ \frac{\partial}{\partial S_{\theta,t}} \kappa_t \right] \right) \text{var}(S_{\theta,t})$$

37
Since \( \frac{\partial}{\partial S_{\theta,t}} \kappa_t(S_{\theta,t}) = -\frac{\partial}{\partial S_{\theta,t}} \kappa_t(-S_{\theta,t}) \), we have that \( \mathbb{E} \left[ \frac{\partial}{\partial S_{\theta,t}} \kappa_t \right] = 0 \), and \( \mathbb{E} \left[ \kappa_t \frac{\partial}{\partial S_{\theta,t}} \kappa_t \right] = 0 \). This implies that volatility can be expressed as:

\[
\text{var}[Q_{t+1}] = \sigma^2(1 - \pi_t^2)\lambda + (\alpha\sigma^2(1 - \lambda)Z)^2\text{var}[\kappa_t] + (1 - \pi_0)^2\lambda^2\text{var}[\kappa_t S_{\theta,t}]
\]

\[
= \sigma^2(1 - \pi_t^2)\lambda + (\alpha\sigma^2(1 - \lambda)Z)^2\text{var}[\kappa_t] + \sigma^2(1 - \pi_0)^2\lambda \left( \mathbb{E} \left[ \kappa_t^2 + 2\kappa_t S_{\theta,t} \frac{\partial}{\partial S_{\theta,t}} \kappa_t \right] \right)
\]

since \( \lambda = \sigma^2/\text{var}(S_{\theta,t}) \).

**Proof of Proposition 3.** It suffices to show that \( \mathbb{E}[\kappa] \) and \( \text{var}[\kappa] \) are HD0 in \( \sigma^2 \), while \( \text{var}[\kappa S_{\theta,t}] \) is HD1 in \( \sigma^2 \). Recall that

\[
\kappa = \frac{\sigma^2(1 - \pi_0^2) + \pi_0(1 - \pi_0)\lambda^2 S_{\theta,t}^2}{\sigma^2(1 - \lambda) + \sigma^2(1 - \pi_0) + \pi_0(1 - \pi_0)\lambda^2 S_{\theta,t}^2}
\]

and by definition, \( \lambda = \frac{\sigma^2}{\sigma^2 + \sigma_t^2} \) and \( S_{\theta,t} \sim \mathcal{N}(0, \sigma^2 + \sigma_t^2) = \mathcal{N}(0, \sigma^2/\lambda) \), we have

\[
\mathbb{E}[\kappa_t] = \frac{1}{\sqrt{2\pi\sigma^2/\lambda}} \int_{-\infty}^{\infty} \frac{\sigma^2(1 - \pi_0^2) + \pi_0(1 - \pi_0)\lambda^2 s^2}{\sigma^2(1 - \lambda) + \sigma^2(1 - \pi_0) + \pi_0(1 - \pi_0)\lambda^2 s^2} \exp \left( \frac{-s^2}{2\sigma^2/\lambda} \right) ds
\]

Using a change of variables, by letting \( x = \sqrt{\frac{\lambda}{\sigma}} s \), we get that

\[
E[\kappa_t] = \frac{1}{\sqrt{2\pi\sigma^2/\lambda}} \frac{\sigma}{\sqrt{\lambda}} \int_{-\infty}^{\infty} \frac{\sigma^2(1 - \pi_0^2) + \pi_0(1 - \pi_0)\lambda^2 \frac{s^2}{\lambda} x}{\sigma^2(1 - \lambda) + \sigma^2(1 - \pi_0) + \pi_0(1 - \pi_0)\lambda^2 \frac{s^2}{\lambda} x} \exp \left( \frac{-x^2}{2} \right) dx
\]

\[
= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{(1 - \pi_0^2) + \pi_0(1 - \pi_0)\lambda x}{(1 - \lambda) + (1 - \pi_0^2) + \pi_0(1 - \pi_0)\lambda x} \exp \left( \frac{-x^2}{2} \right) dx \tag{34}
\]

And clearly (34) is independent of \( \sigma \). To see that \( \text{var}[\kappa_t] \) is also independent of \( \sigma \), note that same proof as above applies to \( E[\kappa_t^2] \).

For \( \text{var}[\kappa S_{\theta,t}] \), again using the same change of variables, we have that

\[
\mathbb{E}[\kappa_t S_{\theta,t}] = \frac{1}{\sqrt{2\pi\sigma^2/\lambda}} \int_{-\infty}^{\infty} \frac{\sigma^2(1 - \pi_0^2) + \pi_0(1 - \pi_0)\lambda^2 s^2}{\sigma^2(1 - \lambda) + \sigma^2(1 - \pi_0) + \pi_0(1 - \pi_0)\lambda^2 s^2} \exp \left( \frac{-s^2}{2\sigma^2/\lambda} \right) ds
\]

\[
= \frac{\sigma}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{(1 - \pi_0^2) + \pi_0(1 - \pi_0)\lambda x}{(1 - \lambda) + (1 - \pi_0^2) + \pi_0(1 - \pi_0)\lambda x} \exp \left( \frac{-x^2}{2} \right) dx
\]

which clearly scales with \( \sigma \) and hence \( (E[\kappa S_{\theta,t}])^2 \) scales with \( \sigma^2 \). The same change of variables can be used to show that the same is true of \( E[(\kappa S_{\theta,t})^2] \), which completes the proof.

**Proof of Proposition 4.** Optimality of \( x_{i,t} \) follows from (2), the expressions for beliefs are given by (4)–(7), and the expression for the price follows from the market clearing condition, i.e., (3).

**Proof of Proposition 5.** Given that dividends are i.i.d., and \( S_{\theta,t} \) is uncorrelated with \( d_t \), the price in any stationary equilibrium must be of the form \( P_t = P(S_{\theta,t}) \). This implies that
\( \mathbb{E}_{t}[P_{t+1}] \) and \( \text{var}_{t}[P_{t+1}] \) are constant, and \( \text{cov}[P_{t+1}, d_{t+1}] = 0 \), which in turn implies that the optimal demand for each type of investor is linear in \( S_{\theta,t} \) and \( P_t \). Market clearing then implies that \( P_t \) is linear in \( S_{\theta,t} \).

Given a conjecture for the price of the form \( P_t = A_0 S_{\theta,t} + B_0 \), the expressions for beliefs are immediate and they imply that the optimal demand for the \( U \) investor is given by

\[
x_{U,t} = \frac{B_0 - RP_t}{\alpha (A_0^2 \sigma^2 + \sigma^2)}.
\]

and for the \( \theta \) investor is given by

\[
x_{\theta,t} = \frac{B_0 + \lambda S_{\theta,t} - RP_t}{\alpha (A_0^2 \sigma^2 + \sigma^2(1 - \lambda))}.
\]

The market clearing condition implies that

\[
RP_t = B_0 + \kappa_t \lambda S_{\theta,t} - \alpha \text{var}_{\theta,t}[P_{t+1} + d_{t+1}] \kappa_t Z
\]

and matching terms with the conjectured price function, we get that \( A_0 \) solves

\[
RA_0 = \frac{\lambda(A_0^2 + \lambda)}{2A_0^2 + (2 - \lambda)\lambda}
\]

and \( B_0 = -\frac{1}{\alpha} \lambda \text{var}_{\theta,t}[P_{t+1} + d_{t+1}] Z \). This implies \( B \) solves the following cubic equation:

\[
RA_0(2A_0^2 + (2 - \lambda)\lambda) - \lambda(A_0^2 + \lambda) = 0
\]

Moreover, since the discriminant of the above equation is less than zero, there is one real root of \( A_0 \). The above also implies that \( 0 \leq A_0 \leq \frac{\lambda}{R} \). The expressions for expected returns and volatility in returns follow from plugging in the expression for price and computing the moments.

**Proof of Proposition 6.** As in the proof of Proposition 5, the price in any stationary equilibrium must be a linear function of \( S_{\theta,t} \). Given a conjecture for the price of the form \( P_t = A_1 S_{\theta,t} + B_1 \), the expressions for beliefs are immediate and they imply that the optimal demand for both types of investors is given by

\[
x_{U,t} = x_{\theta,t} = \frac{B_1 + \lambda S_{\theta,t} - RP_t}{\alpha(\sigma^2(1 - \lambda) + A_1^2 (\sigma^2 + \sigma^2))}.
\]

Since beliefs are identical, \( \kappa_t = \frac{1}{2} \), and so the market clearing condition implies that

\[
RP_t = B_1 + \lambda S_{\theta,t} - \frac{1}{2} \alpha \text{var}_{\theta,t}[P_{t+1} + d_{t+1}] Z,
\]

and matching terms with the conjectured price function, we get that \( A_1 = \frac{\lambda}{R} \), \( B_1 = -\frac{1}{\alpha} \lambda \text{var}_{\theta,t}[P_{t+1} + d_{t+1}] Z \). The expressions for expected returns and volatility in returns follow from plugging in the expression for price and computing the moments. \( \Box \)
Proof of Proposition 7. When $\pi_t = 1$, Proposition 6 implies that conditional expected return and variance in returns are constant, and so equal to the unconditional moments. Moreover,

$$
\mathbb{E}[Q_{t+1}] = \frac{1}{2}\text{var}[Q_{t+1}] = \alpha Z
$$

$$
\text{var}[Q_{t+1}] = A_1^2 (\sigma^2 + \sigma^2_\varepsilon) + \sigma^2 (1 - \lambda) = \sigma^2 (1 - \lambda (1 - \frac{1}{R^2})) .
$$

Differentiating w.r.t. $\lambda$, we get

$$
\frac{\partial}{\partial \lambda}\text{var}[Q_{t+1}] = -\sigma^2 (1 - \frac{1}{R^2})
$$

which implies that expected return and variance are decreasing in information quality.

When $\pi_t = 0$, Proposition 5 implies that

$$
\kappa = \frac{A_0^2 (\sigma^2 + \sigma^2_\varepsilon) + \sigma^2}{A_0^2 (\sigma^2 + \sigma^2_\varepsilon) + \sigma^2 + A_0^2 (\sigma^2 + \sigma^2_\varepsilon) + \sigma^2 (1 - \lambda)}
$$

$$
= \frac{A_0^2 \sigma^2 / \lambda + \sigma^2}{A_0^2 \sigma^2 / \lambda + \sigma^2 + A_0^2 \sigma^2 / \lambda + \sigma^2 (1 - \lambda)}
$$

$$
= \frac{A_0^2 + \lambda}{2A_0^2 + 2\lambda - \lambda^2} = RA_0 / \lambda,
$$

the unconditional expected return is

$$
\mathbb{E}[Q_{t+1}] = \mathbb{E}[\kappa \alpha [A_0^2 (\sigma^2 + \sigma^2_\varepsilon) + \sigma^2] Z - RA_0 S_{\theta,t}] = \sigma^2 RA_0 / (A_0^2 / \lambda + 1) \alpha Z
$$

and the unconditional variance in returns is

$$
\text{var}[Q_{t+1}] = \mathbb{E}[\text{var}_t [Q_{t+1} | S_{\theta,t}]] + \mathbb{E}_t [\text{var}_{t+1} [Q_{t+1} | S_{\theta,t}]] = \sigma^2 \left( (1 + R^2) A_0^2 / \lambda + 1 \right).
$$

Since the unique solution to $A_0$ is given by

$$
A_0 = \frac{\lambda}{6R} - \frac{12R^2 \lambda - \lambda^2 - 6R^2 \lambda^2}{3 \times 2^{2/3} R \left(72R^2 \lambda^2 + 2\lambda^3 + 18R^2 \lambda^3 + \sqrt{4(12R^2 \lambda - \lambda^2 - 6R^2 \lambda^2) + (72R^2 \lambda^2 + 2\lambda^3 + 18R^2 \lambda^3)^2} \right)^{1/3}}
$$

$$
+ \frac{\left(72R^2 \lambda^2 + 2\lambda^3 + 18R^2 \lambda^3 + \sqrt{4(12R^2 \lambda - \lambda^2 - 6R^2 \lambda^2) + (72R^2 \lambda^2 + 2\lambda^3 + 18R^2 \lambda^3)^2} \right)^{1/3}}{6 \times 2^{2/3} R},
$$

it can be shown that both the unconditional expected return and the unconditional variance in returns are increasing in $\lambda$.

Proof of Proposition 8. For any prior $p_0$, the probability that $U$ investors assign to the $\theta = I$ at $t = 0$ is $\hat{p}_0$. From here, the analysis for computing the expectation and variance of dividends from $U$’s perspective (i.e., (4)-(5)) carry forward by replacing $p_0$ with $\hat{p}_0$ and therefore $U$’s optimal demand is isomorphic. Since $\theta$’s demand does not depend on $\pi_t$ in either version of the model, the mapping for the price function then obtains from the market
Proof of Proposition 9. In the steady state, \( \hat{\pi} \in \{p_L, p_H\} \), and so the equilibrium is i.i.d. over time. Since the distribution of \( S_{\theta,t+1} \), conditional on date \( t \) information is \( S_{\theta,t+1} \sim N(0, \sigma^2 + \sigma_z^2) \) for \( i \in \{\theta, U\} \), we have that

\[
\mathbb{E}_{i,t}[P_{t+1}] = m, \quad \text{var}_{i,t}[P_{t+1}] = v, \quad \text{and} \quad \text{cov}_{i,t}[P_{t+1}, d_{t+1}] = 0, \tag{46}
\]

for constants \( m \) and \( v \). The expression for the price follows from applying the market clearing condition to the demand from \( \theta \) and \( U \) investors, and the characterization of \( m \) and \( v \) following from imposing the steady state conditions i.e.,

\[
m \equiv \mathbb{E}_t[P_t] = \frac{1}{R} \mathbb{E}_t \left[ m + (\hat{\pi} + \kappa_t (1 - \hat{\pi})) \lambda S_{\theta,t} - \kappa_t (v + \sigma^2 (1 - \lambda)) \alpha Z \right] \tag{47}
\]

\[
v \equiv \text{var}_t[P_t] = \frac{1}{R^2} \left[ (1 - \hat{\pi})^2 \lambda^2 \text{var}(\kappa_t S_{\theta,t}) + (v + \sigma^2 (1 - \lambda))^2 \alpha^2 Z^2 \text{var}(\kappa_t) + (1 - \hat{\pi}) \lambda (v + \sigma^2 (1 - \lambda)) \alpha Z \text{cov}(\kappa_t S_{\theta,t}, \kappa_t) \right] \tag{48}
\]

\[
= \frac{1}{R^2} \left( (1 - \hat{\pi})^2 \lambda^2 \text{var}(\kappa_t S_{\theta,t}) + (v + \sigma^2 (1 - \lambda))^2 \alpha^2 Z^2 \text{var}(\kappa_t) \right) \equiv F(v) \tag{49}
\]

and recognizing that \( \mathbb{E}[\kappa_t S_{\theta,t}] = 0 \) and \( \text{cov}[\kappa_t S_{\theta,t}, \kappa_t] = 0 \) since \( \kappa_t S_{\theta,t} \) is even in \( S_{\theta,t} \). Since \( \kappa_t \in [0, 1] \), we have that \( \text{var}[\kappa_t S_{\theta,t}] \leq \sigma^2 + \sigma_z^2 \) and \( \text{var}[\kappa_t] \leq 1 \). This implies that

\[
F(v) \leq \frac{1}{R^2} \left( (1 - \hat{\pi})^2 \lambda^2 (\sigma^2 + \sigma_z^2) + (v + \sigma^2 (1 - \lambda))^2 \alpha^2 Z^2 \right) \equiv A + B(v + C)^2 \tag{50}
\]

where \( A = (1 - \hat{\pi})^2 \lambda^2 (\sigma^2 + \sigma_z^2) \), \( B = \alpha^2 Z^2 \), \( C = \sigma^2 (1 - \lambda) \). Then, note that \( F(0) > 0 \) and \( F(v^*) \leq v^* \) for

\[
v^* = \frac{1 - 2BC \pm \sqrt{1 - 4AB - 4BC}}{2B}. \tag{51}
\]

As long as \( 1 - 4AB - 4BC \geq 0 \), or equivalently,

\[
4\alpha^2 Z^2 \left( (1 - \hat{\pi})^2 \lambda^2 (\sigma^2 + \sigma_z^2) + \sigma^2 (1 - \lambda) \right) = 4\alpha^2 Z^2 \sigma^2 (1 - \lambda(1 - (1 - \hat{\pi})^2)) \leq 1, \tag{52}
\]

there exists a solution \( v \) to (49), and consequently, a stationary equilibrium. □

Proof of Proposition 10. Since \( U \) investors can observe the equilibrium price and the residual supply, they can construct a signal

\[
y_t \equiv \alpha \sigma^2 (1 - \lambda)(x_{\theta,t} - z_t) + P_t = \begin{cases} 
\lambda S_{\theta,t} - \alpha \sigma^2 (1 - \lambda) z_t & \text{if } \theta = I \\
\lambda P_t - \alpha \sigma^2 (1 - \lambda) z_t & \text{if } \theta = NI,
\end{cases} \tag{53}
\]

Note that this implies that, unlike the static version of the base model, the \( U \) investor can
use $y_t$ to update her beliefs about the likelihood of $\theta$ being informed, as follows:

$$
\pi_t (y_t, P_t) = \frac{\pi_0}{\sqrt{\sigma^2_{Y,I}}} \phi \left( \frac{y_t}{\sqrt{\sigma^2_{Y,I}}} \right) + \frac{1-\pi_0}{\sqrt{\sigma^2_{Y,NI}}} \phi \left( \frac{y_t-\lambda P_t}{\sqrt{\sigma^2_{Y,NI}}} \right)
$$

(54)

where

$$
\sigma^2_{Y,I} = \lambda^2 \left( \sigma^2 + \sigma^2_z \right) + \alpha^2 \sigma^4 \left(1 - \lambda \right)^2 \sigma^2_z,
$$

and

$$
\sigma^2_{Y,NI} = \alpha^2 \sigma^4 \left(1 - \lambda \right)^2 \sigma^2_z.
$$

(55)

Moreover, conditional on $\theta = I$, $U$’s belief about $d_{t+1}$ is given by

$$
E_{U,t} [d_{t+1} | y_t, \theta = I] = \lambda_y y_t, \quad \text{var}_{U,t} [d_{t+1} | y_t, \theta = I] = \sigma^2 (1 - \lambda_y)
$$

(56)

where $\lambda_y = \frac{\text{cov}(y_t, d_{t+1})}{\text{var}(y_t)}$.

(57)

Optimal demand for the $U$ investor is then given by

$$
x_U(\theta_t - z_t, P) = Z - (x_{\theta,t} - z_t)
$$

(59)

Note that since

$$
\frac{\partial \pi_t}{\partial P_t} = \frac{\pi_t (1-\pi_t)}{\sigma^2_{Y,I} \sigma^2_{Y,NI}} \left( \alpha \sigma^2 (1 - \lambda) (x_{\theta,t} - z_t) \left( \sigma^2_{Y,I} (1 - \lambda) - \sigma^2_{Y,NI} \right) + P \left( (\sigma^2_{Y,I} (1 - \lambda)^2 - \sigma^2_{Y,NI}) \right) \right),
$$

for any realization of $x_{\theta,t} - z_t$, we have:

- If $\sigma^2_{Y,I} (1 - \lambda)^2 - \sigma^2_{Y,NI} > 0$, the derivative is increasing in $P_t$ and, for large enough values of $P_t$, it is positive, which implies $\lim_{P_t \to \infty} \pi_t = 1$, and consequently

$$
\lim_{|P_t| \to \infty} x_U = \frac{\lambda_y y_t - P_t}{\alpha \sigma^2 (1 - \lambda_y)}.
$$

- If $\sigma^2_{Y,I} (1 - \lambda)^2 - \sigma^2_{Y,NI} < 0$, the derivative is decreasing in $P_t$ and, for large enough $P_t$, it is negative. This implies that $\lim_{|P_t| \to \infty} \pi_t = 0$, and consequently

$$
\lim_{|P_t| \to \infty} x_U = \frac{0 - P_t}{\alpha \sigma^2}
$$

Since $x_U$ is continuous in $P_t$ and $\pi_t$, this implies that in either case, there exists a $P_t$ that satisfies equation (59). Rearranging equation (59) gives the expression for the price in the proposition.