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# Product Differentiation by Competing Vertical Hierarchies

Matteo Bassi
*Competition and Markets Authority and CSEF Southampton Row, London WC1B 4AD, U.K.*
*matteo.bassi@gmail.com*

Marco Pagnozzi
*Department of Economics and Statistics and CSEF Università di Napoli Federico II Via Cintia (Monte S. Angelo), 80125 Napoli, Italy*
*pagnozzi@unina.it*

Salvatore Piccolo
*Department of Economics and CSEF, Università Cattolica di Milano, Via Necchi 5, 20123 Milan, Italy*
*salvapiccolo@gmail.com*

*We revisit the choice of product differentiation in the Hotelling model, by assuming that competing firms are vertically separated, and that retailers choose products' characteristics. The "principle of differentiation" does not hold because retailers with private information about their marginal costs produce less differentiated products in order to increase their information rents. Hence, information asymmetry within vertical hierarchies may increase social welfare by inducing them to sell products that appeal to a larger number of consumers. We show that the socially optimal level of transparency between manufacturers and retailers depends on the weight assigned to consumers' surplus and trades off two effects: higher transparency reduces price distortion but induces retailers to produce excessively similar products.*

## 1. Introduction

Product positioning is one of the most important strategic choices for firms (see, e.g., Kotler and Keller, 2008), because the characteristics of the products sold in a market affect customers' willingness to pay for them. But these characteristics also affect the level of differentiation between products sold by competing firms, and hence the intensity of competition.

The economic literature has extensively studied the relationship between product differentiation, market competition, and welfare (see, e.g., Lancaster, 1990). These models, however, usually treat firms as *black boxes* and neglect the interplay between the resolution of agency conflicts within vertical hierarchies and product positioning. When suppliers deal with privately informed retailers, the rents enjoyed by the latter

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(as a price for truthful information revelation) are likely to affect product differentiation and pricing decisions, and ultimately social welfare.[1]

The objective of this paper is to analyze firms' choice of their products' characteristics, when retailers rather than manufacturers make this choice, and to investigate how the amount of retailers' private information with respect to manufacturers affects this choice. We consider a simple model with two competing vertical hierarchies, each composed of one manufacturer and one retailer (or, alternatively, by one supplier and one manufacturer), that sell substitute products. Manufacturers sell a fundamental input to retailers through contracts based on two-part tariffs,[2] and retailers choose the degree of "horizontal" differentiation of their final products.

Following the classic Hotelling (1929) model, to capture product differentiation we assume that the vertical hierarchies locate on a line; consumers are distributed along the line and pay a transportation cost to reach a retailer and purchase its product. Each point on the line may be interpreted as a possible variety of a product (e.g., a different amount of a product's characteristic). The point at which each consumer is located represents its most preferred variety whereas a hierarchy's location represents the variety it chooses to produce.[3] This model captures the idea that different consumers prefer different varieties of a product, and vertical hierarchies can choose the degree of differentiation among their products by choosing their specific characteristics.

According to the *principle of differentiation*, firms tend to differentiate their products in order to soften price competition. Although differentiating from competitors reduces competition (strategic effect), it may also reduce a firm's market share (demand effect). When the first effect prevails, firms maximize differentiation and locate as far away as possible from each other (e.g., in the Hotelling model with quadratic transportation costs firms locate at the opposite ends of the line—see D'Aspremont et al., 1979). This result is consistent with the observation that firms often search for market niches to position their products with respect to competitors' products.

Firms' incentives to differentiate their products, however, may be limited by various factors, like the elasticity of demand and its concentration on particular varieties of products (De Palma et al., 1985; Economides, 1986; Neven, 1986; Böckem, 1994; Tabuchi and Thisse, 1995; Rath and Zhao, 2001), R&D externalities between firms (Mai and Peng, 1999), uncertainty about consumers' preferences (Rhee et al., 1992) products' characteristics (Bester, 1998; Christou and Vettas, 2005) or costs (Matsushima and Matsumura, 2003), and collusion (Friedman and Thisse, 1993; Colombo, 2012).[4]

We discuss a new reason that may induce vertically separated firms to produce less differentiated products: asymmetric information between manufacturers and retailers. When retailers have private information about their marginal costs of production, they have an incentive to choose product characteristics that appeal to a larger number of consumers in order to increase sales and, hence, their information rent. Therefore, asymmetric information within firms induces retailers to choose less differentiated products (than manufacturers), even though this increases competition with rivals. And the

1. There is a large empirical literature showing that asymmetric information affects strategic decision in industries where firms are vertically separated (see, e.g., Lafontaine and Slade, 1997; Lafontaine and Shaw, 1999).

2. Two-part tariffs are commonly used in franchise contracts (see, e.g., Blair and Lewis, 1994; Gal-Or, 1991, 1999; Martimort, 1996). We also consider contracts with linear prices in an extension of our model.

3. Transportation costs can be interpreted as the loss of utility of a consumer that purchases a variety that is different from its most preferred one.

4. By contrast, Meagher and Zauner (2004) show that, with uncertainty about consumer locations, firms have a stronger incentive to differentiate their products.

availability of less differentiated products increases consumers' welfare because it reduces transportation costs.[5]

Our model can be interpreted as a simplified representation of various industries, such as traditional and business-format franchising (see, e.g., Lafontaine and Slade, 1997),[6] and assembling industries. For example, in the low-cost clothing industry, large manufacturers (like Promod, Benetton and H&M) propose to (exclusive) franchisee retailers with exclusive territories a vast catalogue of products, and retailers differentiate from competitors by choosing both their range of products and where to physically locate their shops.[7] Our results are consistent with the fact that retailers of different manufacturers tend to locate relatively close to each other and to sell relatively similar products. Similarly, in the European retail gasoline market, downstream firms (gas stations) buy the fundamental input from exclusive gas suppliers and differentiate their final product by providing loyalty programs and in-station services such as car washing, engine check-ups, etc. Other examples of industries where upstream suppliers distribute their products through exclusive retailers who choose locations include fast-food (e.g., McDonald's and Burger King), hotels (e.g., InterContinental, Wyndham, Marriott), ice creams (e.g., Langnese-Iglo and Schöller in Germany), video rentals (Mortimer, 2002), beer distribution (Asker, 2008), and cars.

In many other industries, oligopolistic suppliers sell a fundamental input to downstream firms, which then obtain different versions of a final product, that appeal to different types of consumers.[8] For example, producers of personal computers buy central processors from upstream oligopolistic manufacturers and combine them with other components (keyboard, RAM memory, hard disk, etc.), whose characteristics depend on the market segment that they aim to target (e.g., business versus leisure customers).[9] Similarly, in the Swiss watch and clock industry, producers buy fundamental inputs (like the movement mechanism) from few manufacturers, and assemble them to create differentiated watches.[10] Of course, because in these industries retailers also buy inputs from other nonexclusive suppliers, our model only highlights one channel through which asymmetric information affects product differentiation and neglects other effects that may arise in a more general environment.

Our paper contributes to the literature on the effects of vertical relationships on firms' locations.[11] In a model with complete information, Matsushima (2004) shows that retailers may not choose maximal differentiation in order to increase competition between nonexclusive manufacturers that pay transportation costs to supply retailers. (See also Kourandi and Vettas, 2010.) By contrast, we show that retailers may not choose maximal differentiation even without competition between manufacturers and

5. This can be interpreted as consumers acquiring products that are relatively more similar to their most preferred one.

6. These industries have inspired a large literature on competing vertical hierarchies (see, e.g., Bonanno and Vickers, 1988; Caillaud et al., 1995; Martimort, 1996; Gal-Or, 1999; Kuhn, 1997; among many others).

7. In our model, we assume that each manufacturer contracts with a single retailer. Although in many franchising industries upstream suppliers deal with multiple exclusive downstream outlets, they often eliminate intra-brand competition (between outlets of the same supplier) by granting exclusive territories.

8. Our results may also be applied to other vertical relations, such as procurement contracting, executive compensations, patent licensing, and credit relationships, when there are noncontractable product differentiation activities, like investments in product design, advertising campaigns, R&D, etc.

9. The industry of central processors is dominated by two firms, AMD and Intel, which jointly produced 99% of the processor sold in 2011. Usually, although not always, PC producers purchase processors from a single manufacturer.

10. See "The Swiss Watch Industry Today," available at http://www.fhs.ch/en/history.php.

11. Pepall and Norman (2001) analyze the incentives to vertically separate for firms located on the Hotelling line.

transportation costs for manufacturers, due to the presence of asymmetric information. Brekke and Straume (2004) and Liang and Mai (2006) show how, with complete information, the distribution of bargaining power between manufacturers and retailers affects the degree of product differentiation.

Building on our positive analysis, we also characterize the socially optimal level of asymmetric information between manufacturers and retailers—that is, transparency—which we model as the dispersion of retailers' private information. In practice, the level of transparency within vertical hierarchies can be affected by various regulatory policies. For example, firms' accounting standards determine the quality and quantity of information reported to the public. When a retailer is subject to stricter accounting standards, it must produce more detailed balance sheets and financial reports, which can be observed by suppliers, whereby reducing private information. More generally, Diamond and Verrecchia (1991) show that mandated disclosing rules and tight accounting standards reduce information asymmetries between market participants.

Although asymmetric information between manufacturers and retailers tends to reduce social welfare because it induces firms to choose inefficiently high prices,[12] a regulator who can control the level of asymmetric information may choose not to impose full transparency. In fact, in our model, the socially optimal level of transparency depends on the weight assigned to consumers' surplus.

Reducing transparency has two opposite effects on social welfare. On one hand, lower transparency induces manufacturers to charge inefficiently high prices, thus reducing welfare: the *price distortion effect*. On the other hand, lower transparency induces retailers to produce relatively more differentiated products. This reduces consumers' transportation costs and increases production efficiency (because more consumers buy from the most efficient firm), thus increasing welfare: the *product differentiation effect*.[13] When the weight assigned to consumers' surplus is relatively low, the product differentiation effect prevails and increasing asymmetric information increases welfare (as long as the market is fully covered). This result offers a novel contribution to the previous literature on the welfare effects of vertical contracting, that assumes an exogenous level of product differentiation (see, e.g., Gal-Or, 1999; Martimort, 1996). By contrast, when the weight assigned to consumers' surplus is relatively high, the price distortion effect prevails and increasing asymmetric information reduces welfare.

Hence, on the normative ground, our analysis offers a justification for regulatory policies that allow for lower or imperfect transparency,[14] and is consistent with the heterogeneity of accounting transparency standards across countries (Elull et al., 2011): regulatory authorities that assign different weights to consumer welfare and industry profits may impose very different transparency standards.

We also consider the choice of products' characteristics by manufacturers, rather than retailers, and show that they always maximize product differentiation, exactly as vertically integrated firms. Similarly, the principle of differentiation holds when contracts between manufacturers and retailers are based on linear prices, because in this case

12. The literature on vertical hierarchies shows how the presence of privately informed retailers exacerbates the standard double marginalization result leading to equilibrium prices that are excessively high compared to marginal costs—see Blair and Lewis (1994), Gal-Or (1991b, 1999), Kastl et al. (2011), and Martimort (1996).

13. When transparency increases, retailers choose products that are too similar from a social welfare's point of view, because their characteristics are most preferred by fewer potential consumers.

14. Of course, because we only analyze a simplified model, our analysis neglects other potentially important effects of transparency and asymmetric information.

retailers choose locations to maximize downstream profits rather than information rents. Therefore, our results hinge on retailers choosing locations and manufacturers offering nonlinear contracts.

The rest of the paper is organized as follows. Section 2 presents the model. In Section 3, we analyze the choice of the degree of product differentiation by retailers with private information. Section 4 considers the effects of transparency on social welfare. We consider various extensions in Section 5 and discuss the robustness of our results to alternative timings. Finally, Section 6 concludes. All proofs are in the Appendix.

## 2. The Model

### 2.1. Players and Environment

We consider the "linear city" model introduced by Hotelling (1929). There is a unit mass of consumers uniformly distributed with density 1 over the interval [0, 1]. There are two vertical hierarchies, each composed by one manufacturer and one exclusive retailer.[15] Each manufacturer supplies a fundamental input to his retailer, that is used to produce a homogeneous final good. For simplicity, manufacturers' cost of production is normalized to zero. Manufacturer $M_1$ and retailer $R_1$ are located at $a_1$, whereas manufacturer $M_2$ and retailer $R_2$ are located at $(1 - a_2)$. Without loss of generality, we assume that $a_1 + a_2 \leq 1$—that is, that $M_1$ and $R_1$ are always located to the left of $M_2$ and $R_2$. Hence, $a_1 = a_2 = 0$ represents maximal differentiation between products. The location of a vertical hierarchy is chosen by the retailer. Each vertical hierarchy can choose only one location—that is, it produces a single variety of the good.

Each consumer has a valuation $v$ for a single unit of the good. For simplicity, we assume that $v$ is large enough, so that each consumer always buys one unit, regardless of the price. Consumers pay a quadratic transportation cost to reach the vertical structures. Specifically, a consumer located at $x \in [0, 1]$ pays $t(x - a_1)^2$ to buy from $R_1$ and $t(1 - a_2 - x)^2$ to buy from $R_2$.

Given the retail prices of the goods produced by the two vertical hierarchies, $p_1$ and $p_2$, a consumer located at $x$ buys from $R_1$ if and only if

$$p_1 + t\,(x - a_1)^2 < p_2 + t\,(1 - a_2 - x)^2\,.$$

Therefore, in an interior solution, the demand for the good sold by $R_i$ is

$$D^i\left(p_i, p_j\right) = \frac{1 + a_i - a_j}{2} + \frac{p_j - p_i}{2t\left(1 - a_i - a_j\right)}, \quad i, j = 1, 2, \quad i \neq j.$$

Before being offered a contract from his manufacturer, $R_i$ privately observes his constant marginal cost of production $\theta_i$, which is distributed uniformly on the compact support $\Theta \equiv [\mu - \sigma, \mu + \sigma]$, so that its c.d.f. is $F(\theta_i) = \frac{\theta_i - (\mu - \sigma)}{2\sigma}$ with mean $\mu$ and variance $\frac{\sigma^2}{3} > 0$. We interpret $\sigma$ as a measure of the level of asymmetric information between manufacturers and retailers and we assume that $\sigma \leq \frac{t}{4}$. This assumption ensures that retailers' marginal costs cannot differ too much, so that in a symmetric equilibrium each

15. For example, retailers are spin-offs of manufacturers and there are (unmodeled) fixed costs of setting up retailers. Our results remain unchanged if manufacturers choose among multiple competing retailers.

retailer's demand is always positive (see the proof of Proposition 1).[16] We also assume that $\sigma < \mu$, so that marginal costs are always positive.

When $\sigma = 0$ there is full transparency and both retailers' marginal costs are equal to $\mu$. When $\sigma \neq 0$ there is asymmetric information between manufacturers and retailers. For example, this asymmetric information may arise because, when contracting with manufacturers, retailers are privately informed about the costs of other inputs such as labor, energy, and rental costs, that are not directly related to the provision of the manufacturers' essential input and are provided by other independent suppliers. Alternatively, asymmetric information may arise because retailers are privately informed about their production efficiency.

### 2.2. Contracts

Contracts between manufacturers and retailers are private—that is, they cannot be observed by competitors. We assume that manufacturers offer menus of two-part tariffs and use the *Revelation Principle* to characterize the equilibrium of the game. [17] Therefore, $M_i$ offers a contract $C_i \equiv \{w_i(m_i), T_i(m_i)\}_{m_i \in \Theta}$ to $R_i$, which is a direct revelation mechanism that specifies a (linear) wholesale price $w_i(m_i)$ and a (fixed) franchise fee $T_i(m_i)$ both contingent on $R_i$'s report $m_i$ about his cost $\theta_i$. In the Appendix we prove that, with private contracts, our model with two-part tariffs contracts is equivalent to a model in which manufacturers control retail prices (Resale Price Maintenance [RPM]) instead of using two-part tariffs—that is, $M_i$ offers a contract which specifies a retail price $p_i(\cdot)$ and a lump-sum transfers $T_i(\cdot)$ contingent on $R_i$'s report $m_i$.

### 2.3. Timing

The timing of the game is as follows:

1. Retailers simultaneously and independently choose their locations, and locations are publicly observable.
2. Retailers privately observe their costs.
3. Manufacturers offer private contracts and retailers choose whether to accept them. If $R_i$ accepts $M_i$'s offer, he reports his type to $M_i$ and pays the franchise fee.
4. Retailers simultaneously choose retail prices and the market clears.

Hence, retailers choose locations, and they do so before contracting with manufacturers (see, e.g., Matsushima, 2004). In Section 5.1, we analyze various different timings and in Section 5.2 we assume that manufacturers, rather than retailers, choose locations in period 1.

### 2.4. Equilibrium Concept

The solution concept is Perfect Bayesian Equilibrium. Because contracts are private, we have to make an assumption on retailers' beliefs about their competitors' behavior. Following most of the literature on private contracts (e.g., Caillaud et al., 1995; Martimort,

16. When this assumption is not satisfied, the most inefficient retailers may be excluded from the market in equilibrium. If a symmetric equilibrium with this feature exists, it has the same qualitative characteristics of the one with full participation in Proposition 1.
17. See Martimort (1996) for a version of the revelation principle in games with competing hierarchies.

1996), we assume that agents have *passive beliefs*—that is, that, regardless of the contract offered by his own manufacturer, a retailer always believes that the other manufacturer offers the equilibrium contract, and that each retailer expects the rival retailer to truthfully report his type to the manufacturer in a separating equilibrium. This assumption captures the idea that, because manufacturers are independent and act simultaneously, a manufacturer cannot signal to his retailer information that he does not posses about the other manufacturer's contract.[18] Finally, because players are *ex ante* symmetric in our model, we focus on symmetric equilibria.

## 3. Equilibrium Analysis

In this section, we characterize the equilibrium of our model.[19]

### 3.1. Retail and Wholesale Prices

First consider retailers' choice of retail prices. Given a wholesale price $w_i(m_i)$ and the locations $a_i$ and $(1 - a_j)$, $R_i$ chooses his price to solve

$$\max_{p_i \geq 0} D^i(p_i, p_j^e)(p_i - w_i(m_i) - \theta_i),$$

where $p_j^e$ denotes the expected equilibrium retail price of the competitor. Hence, the price that maximizes $R_i$'s expected profit is

$$p_i^*(w_i(m_i), \theta_i) = \frac{\theta_i + w_i(m_i) + p_j^e + t\left(1 - a_i - a_j\right)\left(1 + a_i - a_j\right)}{2}, \quad i, j = 1, 2. \tag{1}$$

Substituting the expected equilibrium retail prices (obtained by letting $m_i = \theta_i$ and taking expectations with respect to $\theta_i$ in the system defined by equation (1) evaluated at the equilibrium wholesale prices), yields

$$p_i^*(w_i(m_i), \theta_i) = \frac{\mu + \theta_i + w_i(m_i)}{2} + \frac{w_i^e}{6} + \frac{w_j^e}{3} + t\left(1 - a_i - a_j\right)\left(1 + \frac{a_i - a_j}{3}\right), \tag{2}$$

where $w_i^e$ is the expected equilibrium wholesale price.

Consider now manufacturers' choice of contracts, given the locations chosen by retailers. In a separating equilibrium, a manufacturer's contract must be incentive feasible—that is, it must satisfy the retailer's participation and incentive compatibility constraints.

Following a standard convention in the screening literature, $R_i$'s expected utility when his cost is $\theta_i$ and he reports $m_i$ is

$$u_i(\theta_i, m_i) = (p_i^*(w_i(m_i), \theta_i) - w_i(m_i) - \theta_i)D^i(p_i^*(w_i(m_i), \theta_i), p_j^e) - T_i(m_i). \tag{3}$$

18. See Pagnozzi and Piccolo (2012) for an analysis of the role of beliefs when contracts between manufacturers and retailers are private.

19. All details of the analysis are in the proofs of Lemma 1 and Proposition 1 in the Appendix.

For a contract to be incentive compatible, truthfully reporting $m_i = \theta_i$ must maximize $R_i$'s utility—that is, the following local first- and second-order incentive constraints must hold[20]

$$\left.\frac{\partial u_i\,(m_i,\theta_i)}{\partial m_i}\right|_{m_i=\theta_i} = 0 \quad \Leftrightarrow \quad \dot{T}_i\,(\theta_i) = -D^i(p_i^*(w_i(\theta_i),\theta_i),\, p_j^e)\dot{w}_i(\theta_i) \quad \forall\theta_i, \tag{4}$$

$$\left.\frac{\partial^2 u_i\,(m_i,\theta_i)}{\partial m_i^2}\right|_{m_i=\theta_i} \leq 0 \quad \Leftrightarrow \quad \dot{w}_i(\theta_i) \geq 0. \tag{5}$$

Moreover, letting $u_i(\theta_i) \equiv u_i(\theta_i,\theta_i)$ denote $R_i$'s utility when he reports his true cost (i.e., the information rent), the participation constraint is

$$u_i\,(\theta_i) \geq 0 \quad \forall\theta_i. \tag{6}$$

Therefore, $M_i$ solves the following maximization program:

$$\max_{w_i(\cdot),u_i(\cdot)} \int_{\theta_i} \left\{D^i(p_i^*(w_i(\theta_i),\theta_i),\, p_j^e)\left(p_i^*(w_i(\theta_i),\theta_i) - \theta_i\right) - u_i\,(\theta_i)\right\} dF\,(\theta_i),$$

subject to conditions (4), (5), and (6). Following Laffont and Martimort (2000, Ch. 3), we first ignore the constraint (5), and then check that it is actually satisfied in the equilibrium that we characterize.

Because $u_i(\theta_i)$ is decreasing and the participation constraint binds when $\theta_i = \mu + \sigma$, $R_i$'s information rent is

$$u_i\,(\theta_i) = \int_{\theta_i}^{\mu+\sigma} D^i(p_i^*(w_i(x),x),\, p_j^e)dx. \tag{7}$$

This rent is increasing in consumers' demand for the good sold by $R_i$. The reason is that a retailer with a low marginal cost obtains a higher utility by mimicking retailers with higher marginal costs when those retailers sell a higher quantity on average—that is, the information rent of a type is increasing in the quantity sold by less efficient types. Because the demand for the good sold by $R_i$ is increasing in $a_i$ (because locating closer to the center attracts more consumers), this provides an incentive for a retailer to produce a product that is more similar to his competitor's product.[21]

Using expression (7), integrating by parts, and substituting in $M_i$'s objective function yields the simplified program

$$\max_{w_i(\cdot)} \int_{\theta_i} \left\{D^i(p_i^*(w_i(\theta_i),\theta_i),\, p_j^e)\left(p_i^*(w_i(\theta_i),\theta_i) - \theta_i - \frac{F\,(\theta_i)}{f\,(\theta_i)}\right)\right\} dF\,(\theta_i). \tag{8}$$

**Lemma 1:** *Given retailers' locations $a_1$ and $a_2$, in equilibrium $M_i$ chooses the wholesale price*

$$w_i^*(\theta_i) = \theta_i - \mu + \sigma,$$

20. In the Appendix, we show that these conditions are also sufficient for global incentive compatibility—that is, $u_i(\theta_i,\theta_i) \geq u_i(m_i,\theta_i)\ \forall(m_i,\theta_i) \in \Theta^2$.

21. In other words, a retailer has an incentive to locate closer to the center and to increase demand for his product in order to reduce the manufacturer's incentive to implement price distortions designed to limit his rent.

*and $R_i$ chooses the retail price*

$$p_i^*(w_i^*(\theta_i), \theta_i) = \theta_i + \sigma + t\left(1 - a_i - a_j\right)\left(1 + \frac{a_i - a_j}{3}\right), \quad i = 1, 2. \tag{9}$$

When $\sigma = 0$, there is complete information because retailers' costs are deterministic—that is, $\theta_i = \mu$—and $w_i^*(\theta_i) = 0$. Hence, with secret contracts, manufacturers choose a wholesale price equal to their marginal cost (which is normalized to zero) in order to enhance retailers' ability to compete with rivals, and then extract the whole surplus by setting a franchisee fee equal to downstream profits. A wholesale price equal to marginal cost also allows manufacturers to avoid the double marginalization problem that arises with linear wholesale prices (see Section 5.3).

When $\sigma > 0$, manufacturers choose a wholesale price higher than their marginal cost in order to increase the retail price and hence reduce demand, thereby decreasing retailers' information rent. Specifically, with asymmetric information, the wholesale price is equal to the inverse hazard rate $\frac{F(\theta_i)}{f(\theta_i)} = \theta_i - \mu + \sigma$, which is a measure of the information rent that manufacturers provide to types that are more efficient than $\theta_i$, when they marginally reduce the wholesale price of type $\theta_i$.

The wholesale price is increasing in $\sigma$ because of the distortion effect of asymmetric information: when the support of retailers' marginal costs is larger, there are more mimicking opportunities for retailers and hence larger distortions are needed to optimally trade off efficiency and rents. Moreover, the wholesale price is also increasing in $\theta_i$, because this makes the allocation of a high-cost retailer less attractive to a low-cost retailer and reduces the latter's incentive to misreport his marginal cost. Hence, retail prices are increasing in $\sigma$ and $\theta_i$. Finally, $\frac{\partial p_i^*}{\partial a_i} < 0$ and $\frac{\partial p_i^*}{\partial a_j} < 0$: competition increases if any of the two retailers chooses a location closer to the center, and this induces each retailer to lower the retail price.

By equations (3) and (7), the equilibrium franchise fee is

$$T_i^*(\theta_i) = D^i(p_i^*(w_i^*(\theta_i), \theta_i), p_j^e)(p_i^*(w_i^*(\theta_i), \theta_i) - w_i^*(\theta_i) - \theta_i) \\ - \int_{\theta_i}^{\mu+\sigma} D^i(p_i^*(w_i(x), x), p_j^e)dx.$$

This expression is composed by two terms. The first represents $R_i$'s downstream profit, whereas the second can be interpreted as a discount based on sales that retailers obtain from manufacturers. Hence, more efficient retailers obtain more favorable contracts—that is, lower wholesale prices and fixed fees—whereas the most inefficient type obtains no rent.[22]

Notice that locations do not affect the equilibrium wholesale price, but they do affect the equilibrium franchise fee indirectly, through their effect on demand and retail prices. The reason is that, because contracts are secret, manufacturers have an incentive to undercut each other and, hence, they choose the lowest possible wholesale price, which is equal to the marginal cost of eliciting information from retailers—that is, the inverse hazard rate (as discussed earlier). By contrast, franchise fees depend on retailers' profit and, hence, on the degree of product differentiation.

22. By the Taxation Principle, the equilibrium with direct mechanisms that we have characterized is equivalent to the equilibrium of an alternative game with indirect mechanisms where manufactures offer menus of nonlinear payment schedules that only depend on the level of sales announced by retailers (see Laffont and Martimort, 2000). These payment schedules are consistent with franchise contracts based on sales.

### 3.2. Retailers' Choice of Location

Before observing his marginal cost, in order to maximize his expected information rent, $R_i$ solves

$$\max_{a_i} \int_{\theta_i} \int_{x \geq \theta_i} \left[ \int_{\theta_j} D^i(p_i^*(w_i^*(x), x), p_j^*(w_j^*(\theta_j), \theta_j)) dF(\theta_j) \right] dx dF(\theta_i).$$

Two contrasting effects determine retailers' choice of locations. On one hand, if a retailer produces a product that is more similar to the competitor's product, the competitor reacts by reducing his retail price: price competition is more intense with less differentiated products. This (standard) strategic effect tends to reduce demand and, hence, the retailer's information rent. On the other hand, however, choosing a location closer to the center allows a retailer to attract more customers, because they have to pay a lower transportation cost to acquire the retailer's product. This sales effect tends to increase information rent.[23]

The next proposition characterizes the equilibrium of the game with and without asymmetric information.

**Proposition 1:** *Suppose that firms' locations are chosen by retailers. If $\sigma = 0$, every location $a^* \in [0, \frac{1}{2}]$ is a symmetric equilibrium and retailers sell at a retail price equal to $\mu + t(1 - 2a^*)$.*

*If $\sigma > 0$, there is a unique symmetric equilibrium in which both retailers choose*

$$a^*(\sigma) = \frac{1}{2} - \frac{1}{2}\sqrt{\frac{\sigma}{t}},$$

*and sell at a retail price*

$$p^*(\theta_i) = \theta_i + \sigma + \sqrt{t\sigma}.$$

When there is no uncertainty about the retailers' costs ($\sigma = 0$), retailers obtain no information rents and are indifferent between any location. Arguably, in this case there are two natural symmetric equilibria: (i) $a^* = \frac{1}{2}$ and (ii) $a^* = 0$. Following the approach of Martimort and Stole (2009), the first equilibrium is the limit, for $\sigma \to 0$, of the unique symmetric equilibrium with asymmetric information $a^*(\sigma)$.[24] In this equilibrium, both retailers locate at the center of the interval and equally share demand (because they also have the same marginal cost).[25] In the second equilibrium, vertical hierarchies act as integrated firms and maximize total profits by locating at the opposite extremes

23. With linear demand and constant marginal costs, the trade-off between these two effects would be present even with asymmetric information about demand rather than costs. Specifically, if retailers are privately informed about demand (say, for instance, about the uncertain size of the Hotelling line) manufacturers still need to provide positive rents in order to induce truthful information revelation by retailers, and these rents are increasing in the quantity sold by each retailer, so that wholesale prices are still upward distorted. Suppose, for example, that retailers' costs are common knowledge while the length of the Hotelling line, say $\alpha$, is random and unknown to both manufacturers and retailers. It can be shown that, in this case, all our qualitative conclusions hold if retailers receive private i.i.d. signals (say $s_i$, $i = 1, 2$) about $\alpha$and the conditional distribution function $F(\alpha|s_i)$ is linear. For correlated signals or the case where retailers are informed about $\alpha$ see, for example, Gal-Or (1999) and Martimort and Piccolo (2010).

24. Martimort and Stole (2009) refine the set of equilibria of a common agency game with complete information by considering the equilibrium of a game with asymmetric information and letting asymmetric information vanish.

25. This may be considered a less plausible equilibrium in our model (than the equilibrium with maximal differentiation), because it requires that retailers choose the locations that *minimizes* manufacturers' profit (because the retail price is equal to $\mu$ when $a = \frac{1}{2}$), even if they obtain no benefit from doing so.

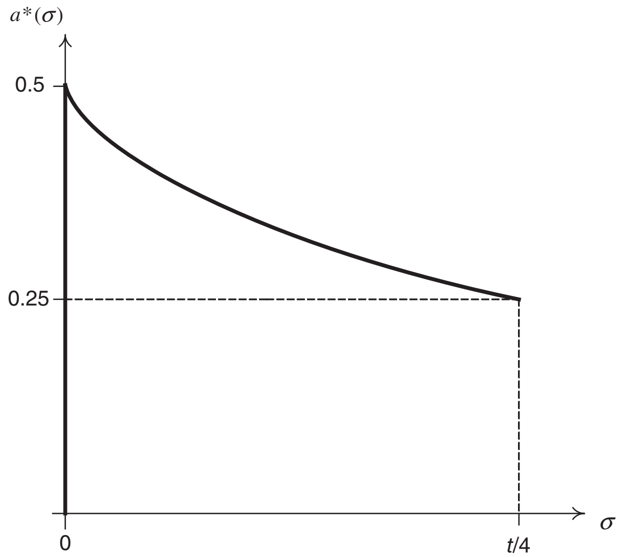


*FIGURE 1. RETAILERS' CHOICE OF LOCATION*

of the interval and charging a retail price equal to $t + \mu$ (see, e.g., Tirole, 1988).[26] By choosing maximally differentiated products, firms enjoy market power over consumers distributed around their locations—the *principle of differentiation* (D'Aspremont et al., 1979).

By contrast, with asymmetric information ($\sigma > 0$), retailers never choose maximal differentiation among their products, because of the presence of information rents. Retailers would jointly prefer to choose maximal differentiation—that is, $a_1 = a_2 = 0$—because these are the symmetric locations that maximize their total rents. However, if a retailer located approximately at 0 moves further away form the center of the interval, the other retailer has an incentive to locate closer to the center in order to increase his information rent. In other words, when products are very differentiated, the sales effect dominates. Hence, retailers face a "prisoners' dilemma" when choosing their locations. More generally, retailers' choices of locations are strategic substitutes.

Notice that $a^*(\sigma)$ is decreasing in $\sigma$. A higher $\sigma$ implies higher retail prices, because more private information creates more price distortion, so that retailers have a lower incentive to produce less differentiated products to increase sale. So retailers can produce more differentiated products, which increase rents for the strategic effect described earlier. As $\sigma \to 0, a^*(\sigma) \to 1/2$: when asymmetric information vanishes, retailers tend to eliminate product differentiation altogether, because they have the strongest incentive to increase sales in order to obtain an information rent. Figure 1 summarizes firms' choices of product differentiation as a function of $\sigma$.

Of course, a higher $t$ implies a higher $a^*(\sigma)$—that is, less product differentiation—and higher retail prices, because when the transportation cost is high, consumers perceive products as more differentiated, thus reducing price competition among firms.

26. Because this equilibrium maximizes manufacturers' profits, to implement it manufacturers may commit to pay an arbitrarily small transfer if retailers locate at the extremes of the interval.

When $\sigma > 0$, besides the symmetric equilibrium that we have analyzed, there is a continuum of other equilibria such that $a_1 + a_2 = 1 - \sqrt{\sigma/t}$. However, in *any* equilibrium the distance between the retailers' location is constant, and equal to $\sqrt{\sigma/t}$. Therefore, the symmetric equilibrium is also the one that minimizes transportation costs.

In the Appendix, we also show that both $R_i$'s and $M_i$'s expected equilibrium payoffs increase in $\sigma$. An increase in the level of asymmetric information increases retailers' rents. An increase in the level of asymmetric information, however, also increases the wholesale price and induces retailers to locate further away from each other, thus increasing the equilibrium retail price (see Proposition 1). This strategic effect always dominates so that, when $\sigma$ increases, manufacturers' profit increase because they compete less aggressively with rivals.

## 4. Transparency Policy

In order to better understand the implications of our equilibrium analysis on the effects of welfare policies that promote market transparency, in this section we characterize the degree of information asymmetry between manufacturers and retailers that maximizes (expected) welfare.

Specifically, we interpret $\sigma$ as a measure of (the inverse of) the level of transparency—that is, the amount of retailers' private information with respect to manufacturers. When $\sigma = 0$, there is full transparency because retailers have no private information; when $\sigma > 0$ retailers have private information: a higher $\sigma$ represents lower transparency because it increases the variance of retailers' private information.[27] In principle, the level of transparency can be affected by a regulator who chooses firms' accounting standards: more restrictive accounting standards for retailers allow manufacturers to obtain more precise information about their marginal costs.

Because retailers are indifferent about their locations when $\sigma = 0$, the optimal level of transparency depends on the selection of equilibrium with complete information. We consider two equilibria, which we believe are the natural ones in our environment: (i) the equilibrium pinned down by the function $a^*(\sigma)$ as $\sigma \to 0$, in which retailers locate in the middle of the interval (Proposition 2); and (ii) the equilibrium preferred by manufacturers, in which retailers locate at the extremes of the interval (Proposition 3).[28] The first is the unique equilibrium when asymmetric information never vanishes completely (which is arguably the case in the real world, for example, because enforcing full transparency is too costly).[29] The second equilibrium, where locations maximize manufacturers' profit, provides a benchmark to analyze the divergence of the manufacturer's and the social planner's objectives (notice that in a standard model these objectives coincide because both the manufacturer and the social planner would like to eliminate double marginalization). We will show that, regardless of which of these two equilibria is actually chosen, full transparency does not necessarily maximize social welfare.

27. Lower transparency may also be interpreted as a riskier technology, that generates higher uncertainty about retailers' marginal costs.

28. As we show in the proof of Proposition 1, in addition to the unique symmetric equilibrium, when $\sigma > 0$ there are multiple asymmetric equilibria. In our analysis, we only consider symmetric equilibria for two reasons. First, because retailers are *ex ante* identical, there is no reason to expect them to behave asymmetrically. Second, in any asymmetric equilibrium one retailer obtains lower profits than the other, which makes coordination between retailers harder to achieve.

29. See, for example, the empirical results of Lafontaine and Slade (1997) and Lafontaine and Shaw (1990).

### 4.1. Social Welfare

We assume that welfare is a weighted sum of consumers' surplus (i.e., the difference between $v$, consumers' transportation costs and retail prices) and firms' profits (i.e., the sum of manufacturers' profits and retailers' information rents). Given marginal costs $\theta_1$ and $\theta_2$, retail prices $p_1$ and $p_2$, and locations $a_1$ and $a_2$, the welfare function is

$$W(\cdot) \equiv \lambda \left[ v - \int_0^{D^1(p_1,p_2)} (p_1 + t\,(x - a_1)^2) dx - \int_{D^1(p_1,p_2)}^1 (p_2 + t\,(1 - a_2 - x)^2) dx \right] +$$
$$(1-\lambda)\left[D^1(p_1, p_2)(p_1 - \theta_1) + \left(1 - D^1(p_1, p_2)\right)(p_2 - \theta_2)\right]$$
$$= \lambda v - D^1(p_1, p_2)\,[(2\lambda - 1)p_1 + (1-\lambda)\theta_1] - \lambda \int_0^{D^1(p_1,p_2)} t\,(x - a_1)^2\,dx +$$
$$-\left(1 - D^1(p_1, p_2)\right)[(2\lambda - 1)\,p_2 + (1-\lambda)\theta_2] - \lambda \int_{D^1(p_1,p_2)}^1 t\,(1 - a_2 - x)^2\,dx,$$

where $\lambda \in [\frac{1}{2}, 1]$ is the weight assigned to consumers' surplus and $(1 - \lambda)$ is the weight assigned to firms' profit.[30]

When $\lambda = \frac{1}{2}$, the welfare function treats consumers and firms symmetrically and, hence, total welfare only depends on transportation and production costs (because total demand is fixed). In this case, an increase in retail prices does not affect welfare but only induces a transfer from consumers to retailers. By contrast, when consumers' surplus and firms' profits are weighted differently—that is, $\lambda \neq \frac{1}{2}$—price distortions do affect welfare. Therefore, even if demand is fixed, our welfare analysis does take into account some of the effects of the allocative distortions caused by asymmetric information. When $\lambda = 1$, the welfare function only considers consumers' surplus and, hence, is decreasing in retail prices and transportation costs. *Ceteris paribus* , a higher $\lambda$ implies a relatively higher concern for reducing retail prices.

### 4.2. Optimal Transparency

Proposition 1 implies that the level of transparency affects retailers' choice of product characteristics and prices. Hence, given the equilibrium choices of locations and retail prices, the optimal level of transparency is

$$\arg\max_{\sigma} \int_{\theta_1} \int_{\theta_2} W(\cdot)\, dF\,(\theta_1)\, dF\,(\theta_2)\,.$$

Our analysis of Section 3 suggests that firms do not choose maximally differentiated products with asymmetric information, and this may have a positive effect on welfare.

We first consider the choice of a regulator who only aims at minimizing transportation costs or production costs, and hence does not care directly about firms' prices. This allows us to decompose the effects that asymmetry between manufacturers and private information by retailers have on these two components of welfare.

30. The assumption that the regulator never assigns a higher weight to firms' profit than to consumers' surplus is consistent with the consumer protection policies adopted in various countries (see, e.g., "EU Consumer Policy Strategy 2007-2013" ).

**Lemma 2:** *Regardless of whether retailers locate at $a^*(0) = \frac{1}{2}$ or at $a^* = 0$ when $\sigma = 0$: the level of transparency that minimizes total production costs is $\sigma = \frac{t}{4}$, and it induces retailers to choose $a^{**} = \frac{1}{4}$; the level of transparency that minimizes transportation costs is $\sigma = t - \frac{t\sqrt{3}}{2}$, and it induces retailers to choose $a^{**} \in (\frac{1}{4}, \frac{1}{2})$.*

With symmetric firms and no uncertainty about marginal costs, firms minimize transportation costs by locating at $\frac{1}{4}$ and $\frac{3}{4}$—that is, by choosing $a_1 = a_2 = \frac{1}{4}$. With asymmetric firms, however, transportation costs are lower if retailers locate closer to the center, because this reduces the number of contested consumers (those located between the two firms) who pay high transportation costs to purchase from the most efficient firm. By contrast, production costs are lower if retailers locate further away from each other, because this increases the number of contested consumers who purchase from the most efficient firm. Hence, a regulator has an incentive to choose a higher level of transparency that results in less product differentiation to reduce transportation costs, but a lower level of transparency that results in more product differentiation to maximize production efficiency.

In order to analyze the optimal level of transparency, we first assume that, when $\sigma = 0$, retailers locate in the middle of the interval.

**Proposition 2:** *Suppose that retailers locate at $a^*(0) = \frac{1}{2}$ when $\sigma = 0$. There exist $\underline{\lambda}$ and $\overline{\lambda}$, with $\overline{\lambda} > \underline{\lambda} > \frac{1}{2}$, such that:*

- *If $\lambda \in [\frac{1}{2}, \underline{\lambda}]$, the optimal level of transparency is $\frac{t}{4}$, and it induces retailers to choose $a^{**} = \frac{1}{4}$.*
- *If $\lambda \in (\underline{\lambda}, \overline{\lambda})$, the optimal level of transparency is $\sigma(\lambda, t) \in (0, \frac{t}{4})$, with $\frac{\partial \sigma(\lambda,t)}{\partial \lambda} < 0$, and it induces retailers to choose $a^{**}(\sigma(\lambda, t)) \in (\frac{1}{4}, \frac{1}{2})$, with $\frac{\partial a^*(\sigma(\lambda,t))}{\partial \lambda} > 0$.*
- *If $\lambda \in [\overline{\lambda}, 1]$, full transparency maximizes social welfare and induces retailers to choose minimal differentiation.*

Reducing the dispersion of retailers' private information has two opposite effects on social welfare. On one hand, asymmetric information reduces welfare because it induces manufacturers to distort prices upward in order to minimize retailers' information rents—*a price distortion effect*. On the other hand, asymmetric information affects retailers' choice of product differentiation: minimizing asymmetric information induces retailers to locate too close to the center, compared to the locations that minimize transportation and production costs (thus producing products that appeal to fewer consumers and reducing production efficiency)—*a product differentiation effect*. The relative strength of these effects depends on the weight assigned to consumers' surplus. As shown in Figure 2, on balance the optimal level of transparency increases (i.e., the optimal level of $\sigma$ decreases) as $\lambda$ increases, and it induces retailers to locate closer to the center and produce less differentiated products.

When the weight on consumers' surplus is relatively high, welfare is maximized by minimizing $\sigma$ and reducing prices, even though this induces retailers to locate at the center of the interval and produce undifferentiated products. In this case, the price distortion effect prevails. By contrast, when the weight on consumers' surplus is relatively low, welfare is maximized by maximizing $\sigma$ and reducing the level of transparency. In this case, the product differentiation effect prevails. When the weight on consumers' surplus takes intermediate values, welfare is maximized by a strictly positive, but not maximal, degree of asymmetric information. Therefore, a regulator prefers retailers to

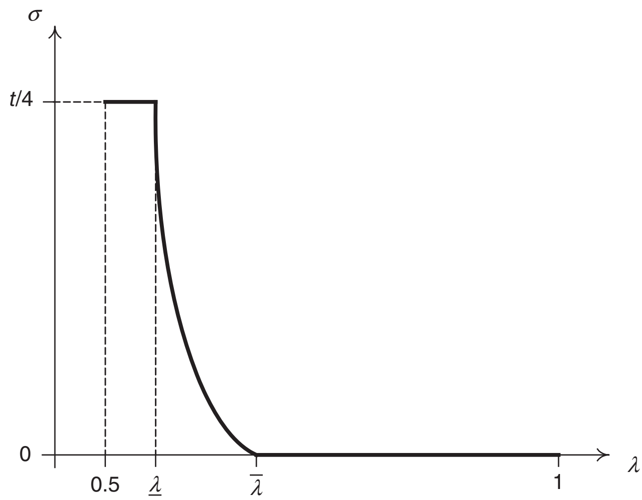


*FIGURE 2. SOCIALLY OPTIMAL TRANSPARENCY WHEN RETAILERS CHOOSE MINIMAL DIFFERENTIATION IF $\sigma = 0$*

have some private information, if the weight assigned to consumers' surplus is not too much higher than the weight assigned to firms' profit in the social welfare function.

Suppose now that, when $\sigma = 0$, retailers locate at the extremes of the interval. In this case, any level of asymmetric information (i.e., any level on transparency different from 0) generates higher social welfare than full transparency.

**Proposition 3:** *Suppose that retailers locate at $a^* = 0$ when $\sigma = 0$. For any $\lambda \in [\frac{1}{2}, 1]$, social welfare is always higher with asymmetric information (for any $\sigma > 0$) than with full transparency ($\sigma = 0$).*

When retailers choose maximal differentiation with complete information, social welfare is never maximized by completely eliminating asymmetric information between manufacturers and retailers. The reason is that, with a positive level of asymmetric information, retailers choose less differentiated products, thus intensifying price competition and increasing welfare. Therefore, in this case, even if a regulator is not able to fine tune $\sigma$ and achieve the optimal level of transparency,[31] he always prefers that retailers have some private information.

Propositions 2 and 3 imply that, regardless of whether retailers locate at $a = 0$ or $a = \frac{1}{2}$ when $\sigma = 0$, full transparency does not always maximize social welfare. Hence, our analysis offers a justification for regulatory policies that impose relatively low standards of transparency to firms' accounting reports. Moreover, it suggests that different transparency standards may reflect different weights that governments and/or regulatory authorities assign to consumer welfare and industry profits.[32] Indeed, variations in the political bias toward customer protection rights and lobbing activities by firms may considerably affect the preferred level of market transparency.

31. The analysis in the proof of Proposition 3 immediately implies that, if retailers locate at $a = 0$ when $\sigma = 0$, the optimal level of transparency is (i) $\frac{t}{4}$ if $\lambda \leq \underline{\lambda}$; (ii) $\sigma(\lambda, t) \in (0, \frac{t}{4})$ if $\lambda \in (\underline{\lambda}, \overline{\lambda})$; (iii) indeterminate if $\lambda \geq \overline{\lambda}$ (because the welfare function is strictly increasing for $\sigma \to 0$).

32. Of course, because demand is inelastic in our model, our welfare results should be interpreted cautiously.

## 5. Extensions

In this section, we extend our model in some natural directions. First, we discuss the implications of alternative timings and bargaining power on the results of our analysis. Second, we analyze the case where manufacturers, instead of retailers, choose locations. Finally, we analyze the case where manufacturers use linear wholesale prices rather than two-part tariffs. We will show that in the last two cases the principle of differentiation holds, which suggests that the result of Proposition 1 hinges on retailers choosing locations and manufacturers offering two-part tariffs.

### 5.1. Alternative Timings and Bargaining Power

If retailers choose locations after having observed their marginal costs, the equilibrium outcome depends on whether contracts are offered before or after location choices are made. If contracts are signed before retailers choose locations, and neither the wholesale price nor the fixed fee can be contingent on these choices (or, more generally, when locations are not observable by manufacturers or not verifiable), the main qualitative insight of our analysis hold because a retailer's choice of location depends on his private information. Specifically, in a symmetric equilibrium, retailers still have an incentive not to choose maximal differentiation in order to maximize their actual information rents (rather than their expected information rent as in our main model). By contrast, if contracts are offered after retailers choose locations, these choices signal retailers' private information in a separating equilibrium. This eliminates retailers' rents and leads to the same indeterminacy discussed in Proposition 1 for the case where $\sigma = 0$. Finally, if manufacturers offer contracts before retailers observe their costs (but location choices are made *ex ante*), the equilibrium outcome remains the same as in Proposition 1 if retailers are protected by limited liability—see, for example, Laffont and Martimort (2000, Ch. 3).

In our model, manufacturers have all the bargaining power when they contract with retailers. If instead, retailers have all the bargaining power and offer wholesale contracts to manufacturers, the presence of vertical hierarchies have no effect on location choices (with secret contracts). The reason is that, in this case, there is a unique equilibrium where retailers offer a wholesale price equal to the manufacturers' cost (which is normalized to zero) and a fixed fee equal to zero. Therefore, when making their location choices, retailers maximize expected profits. However, as shown by Matsushima and Matsumura (2003), with cost uncertainty retailers do not choose maximal differentiation, but for reasons that are very different from those discussed in our model.[33] Clearly, in a model where manufacturers and retailers bargain over contracts, the equilibrium locations will depend on the players' relative bargaining powers.

### 5.2. Manufacturers' Choice of Location

Suppose that, instead of being chosen by retailers, locations are chosen by manufactures in period 1. All other assumptions are as in our main model.

**Proposition 4:** *In the unique symmetric equilibrium, manufacturers locate at* 0 *and* 1*—that is, they choose maximal differentiation.*

33. See the discussion in Section 5.2.

When retailers are privately informed about their marginal costs of production, they obtain information rents that are decreasing in the level of differentiation between firms' products. And these rents reduce manufacturers' profit. Hence, in order to minimize retailers' information rents, manufacturers have an incentive to produce more differentiated products.

Matsushima and Matsumura (2003) show that, when they are uncertain about their rivals' costs, vertically integrated firms do not choose maximal differentiation, because prices are more volatile when they locate closer to each other, and this increases expected profits because the indirect profit function is convex in prices (i.e., firms are risk lovers). But when firms are vertically separated and retailers have private information, even with cost uncertainty manufacturers' incentive to reduce retailers' information rent prevails, and they choose to locate as far away from each other as possible.

Notice that the manufacturers' choice of location does not depend on the level of transparency. Therefore, when products' characteristics are chosen by manufacturers, asymmetric information between manufacturers and retailers only has a price distortion effect on welfare, and no product differentiation effect. In this case, welfare is maximized by full transparency. However, Proposition 3 implies that welfare is higher when products' characteristics are chosen by retailers with private information rather than by manufacturers, even if there is no distortion of prices above marginal costs when manufacturers choose products' characteristics.

Of course, if both manufacturers and retailers can affect the choice of location, the equilibrium will depend on their relative bargaining power, because manufacturers always prefer maximal differentiation whereas retailers do not.

### 5.3. Linear (Wholesale) Prices

In this section, we consider the impact on product positioning of contracts based on linear prices.[34] We assume that $M_i$ offers a contract $\{w_i(m_i)\}_{m_i\in\Theta}$ to $R_i$, which is a direct revelation mechanism that specifies a (linear) wholesale price $w_i(m_i)$ contingent on $R_i$'s report $m_i$ about his cost $\theta_i$.

Consider a separating equilibrium in which retailers choose the retail price $p^*(\theta_i)$. Given that $M_i$ offers to $R_i$ a menu of linear wholesale prices $w_i(m_i)$, because retailers solve the same program as with two-part tariffs, $R_i$ chooses the retail price $p_i^*(w_i(m_i), \theta_i)$ defined in equation (2).

For a contract to be incentive compatible, truthfully reporting $m_i = \theta_i$ must maximize $R_i$'s utility—that is, the following local first-order incentive constraint must hold[35]

$$-D_i(p_i^*(w_i(\theta_i), \theta_i), p_j^e)\dot{w}_i(\theta_i) = 0. \tag{10}$$

This condition has a simple interpretation. Because with linear prices $M_i$ cannot use the fixed fee to screen $R_i$'s types, an incentive compatible contract either specifies a wholesale price that equalizes (expected) demand to zero—that is, $D_i(p_i^*(w_i(\theta_i), \theta_i), p_j^e) = 0$—which however, would lead $R_i$ to always lie so as to be offered the lowest possible wholesale

34. The literature has often considered linear contracts in vertical relationships (see, e.g., Inderst and Valletti, 2009).

35. Condition (10) follows from the definition of $R_i$'s expected utility when his cost is $\theta_i$ and he reports $m_i$ in (3) and the first order condition (4) (because $T_i = 0$ with linear prices). See Martimort and Semenov (2006).

price, or it entails a pooling allocation—that is, $\dot{w}_i(\theta_i) = 0$ for any $\theta_i$—which implies that $M_i$ does not extract any information from $R_i$.[36]

Therefore, with linear wholesale prices, there are only pooling equilibria in which the manufacturer chooses a wholesale price that does not depend on the retailer's cost. Letting this unique wholesale price be $w_i$, $M_i$ solves

$$\max_{w_i} \int_{\theta_i} D_i(p_i^*(w_i, \theta_i), p_j^e) w_i dF(\theta_i),$$

where $p_j^e$ is the expected equilibrium retail price. The first-order condition equalizes the expected marginal wholesale revenue to the manufacturer's marginal cost

$$\int_{\theta_i} \left[ \frac{\partial D_i(p_i^*(w_i, \theta_i), p_j^e)}{\partial p_i} \frac{\partial p_i^*(w_i, \theta_i)}{\partial w_i} w_i + D_i(p_i^*(w_i, \theta_i), p_j^e) \right] dF(\theta_i) = 0.$$

Hence, $M_i$ sets a wholesale price equal to

$$w_i\left(a_i, a_j\right) \equiv \frac{2t}{7}\left(7 + a_i - a_j\right)\left(1 - a_i - a_j\right),$$

and $R_i$'s location problem is

$$\max_{a_i} \int_{\theta_i} D_i(p_i^*(w_i\left(a_i, a_j\right), \theta_i), p_j^e) \left(p_i^*\left(w_i\left(a_i, a_j\right), \theta_i\right) - w_i\left(a_i, a_j\right) - \theta_i\right) dF(\theta_i).$$

Notice that, in contrast to the case of two part-tariffs, with linear prices $R_i$ maximizes (expected) profits rather than rents. This is because manufacturers cannot use fixed fees to extract retailers' profit. Hence, retailers do not earn information rents but obtain a share of downstream profit (part of which is however extracted by the manufacturers via the wholesale price).

**Proposition 5:** *With linear wholesale prices, in the unique symmetric equilibrium retailers locate at $a_1 = a_2 = 0$, with and without asymmetric information.*

The intuition for the result is similar to that of Proposition 4: because with linear prices retailers do not maximize sales but (downstream) profits, they have an incentive to soften competition as much as possible by locating at the endpoints of the Hotelling line. Hence, the principle of differentiation holds when manufacturers use linear prices.

## 6. Conclusions

We have analyzed the effect of vertical contracting on product differentiation and discussed a novel reason that may induce firms to produce less differentiated products. When privately informed retailers are offered two-part tariffs and choose the degree of product differentiation with respect to competitors (through marketing or product positioning strategies), they have an incentive to choose product characteristics that appeal to a larger number of consumers, in order to increase their information rent. This also tends to increase consumers' surplus. By contrast, the *principle of differentiation* applies under linear wholesale prices or when locations are chosen by the manufacturers.

Market transparency affects social welfare. A regulatory agency that controls the degree of asymmetric information between manufacturers and retailers may prefer not to

36. As in our main model with two-part tariffs, this result also holds with uncertainty about demand instead of costs.

impose full transparency. A lower level of transparency reduces product differentiation and may increase welfare, even if it induces firms to choose inefficiently high prices. Therefore, in contrast to common wisdom, asymmetric information may be socially beneficial in our model.

Our analysis offers a justification for regulatory policies that allow lower or imperfect standards of transparency, and only impose minimum disclosure rules to firms.

## Appendix A

**Proof of Lemma 1:** Given the wholesale price $w_i(m_i)$ and locations $a_i$ and $(1-a_j)$, the first-order necessary and sufficient condition for $R_i$'s maximization program is

$$\frac{\partial D^i(p_i, p_j^e)}{\partial p_i}(p_i - w_i(m_i) - \theta_i) + D^i(p_i, p_j^e) = 0, \tag{A1}$$

where $p_j^e$ is $R_j$'s expected equilibrium retail price. Replacing the definition of $D^i(\cdot,\cdot)$ and rearranging terms yields (1)—that is,

$$p_i^*(w_i(m_i), \theta_i) = \frac{\theta_i + w_i(m_i) + p_j^e + t\left(1 - a_i - a_j\right)\left(1 + a_i - a_j\right)}{2}. \tag{A2}$$

Hence, in equilibrium when $m_i = \theta_i$, $R_i$'s expected retail price is

$$p_i^e \equiv \mathbb{E}\left[p_i^*\left(w_i^*(\theta_i), \theta_i\right)\right] = \frac{\mu + w_i^e + p_j^e + t\left(1 - a_i - a_j\right)\left(1 + a_i - a_j\right)}{2}, \quad i, j = 1, 2, \tag{A3}$$

where $w_i^*(\theta_i)$ is the equilibrium wholesale price and $w_i^e$ is the expected equilibrium wholesale price of $M_i$. Solving the system of equations defined by (A3) for $p_i^e$ and $p_j^e$ yields

$$p_i^e = \mu + \frac{2}{3}w_i^e + \frac{1}{3}w_j^e + t\left(1 - a_i - a_j\right)\left(1 + \frac{a_i - a_j}{3}\right), \qquad i, j = 1, 2.$$

Substituting this into (A2) yields (2).

Consider now $M_i$'s maximization problem

$$\max_{w_i(\cdot), T_i(\cdot)} \int_{\theta_i} \left[w_i(\theta_i) D^i(p_i^*(w_i(\theta_i), \theta_i), p_j^e) + T_i(\theta_i)\right] dF(\theta_i).$$

First, assume that $\sigma = 0$, so that there is no uncertainty about retailers' costs and $\theta_i = \mu$. In this case, $M_i$ fully extracts $R_i$'s surplus by charging the franchise fee

$$T_i^* = D^i(p_i^*(w_i(\mu), \mu), p_j^e)(p_i^*(w_i(\mu), \mu) - w_i(\mu) - \mu),$$

and $M_i$'s maximization problem becomes

$$\max_{w_i(\cdot)} D^i(p_i^*(w_i(\mu), \mu), p_j^e)(p_i^*(w_i(\mu), \mu) - \mu).$$

It is straightforward to show that the objective function is concave in $w_i$ and is maximized at $w_i^* = 0$—that is, the manufacturers' marginal cost of production that we have normalized to zero. Replacing this into (2) yields the equilibrium retail price (9) when $\theta_i = \mu$.

Next, assume that $\sigma > 0$. The contract offered to $R_i$ must satisfy the global incentive compatibility constraint (requiring that $R_i$ maximizes his information rent)

$$u_i(\theta_i) \equiv \max_{m_i \in \Theta} \left\{ (p_i^*(w_i(m_i), \theta_i) - w_i(m_i) - \theta_i) D^i(p_i^*(w_i(m_i), \theta_i), p_j^e) - T_i(m_i) \right\}. \quad \text{(A4)}$$

Using the first-order condition of the retailer's problem (A1), the first- and the second-order conditions for an incentive compatible contract are

$$\dot{w}_i(m_i) D^i(p_i^*(w_i(m_i), \theta_i), p_j^e) + \dot{T}_i(m_i)\Big|_{m_i=\theta_i} = 0, \quad \text{(A5)}$$

$$\dot{w}_i(m_i)^2 \frac{\partial D^i(p_i^*(\cdot), p_j^e)}{\partial p_i} \frac{\partial p_i^*(w_i(m_i), \theta_i)}{\partial w_i} + \ddot{w}_i(m_i) D^i(p_i^*(w_i(m_i), \theta_i), p_j^e) + \ddot{T}_i(m_i)\Bigg|_{m_i=\theta_i} \leq 0. \quad \text{(A6)}$$

Differentiating (A5) with respect to $\theta_i$,

$$\dot{w}_i(\theta_i) \left[ \frac{\partial D^i(p_i^*(\cdot), p_j^e)}{\partial p_i} \left( \frac{\partial p_i^*(w_i(\theta_i), \theta_i)}{\partial w_i} \dot{w}_i(\theta_i) + \dot{p}_i^*(w_i(\theta_i), \theta_i) \right) \right] + \ddot{w}_i(\theta_i) D^i(p_i^*(w_i(\theta_i), \theta_i), p_j^e) = -\ddot{T}_i(\theta_i),$$

and substituting in (A6) yields

$$\dot{w}_i(\theta_i) \frac{\partial D^i(p_i^*(w_i(\theta_i), \theta_i), p_j^e)}{\partial p_i} \dot{p}_i^*(w_i(\theta_i), \theta_i) \leq 0.$$

Because $\dot{p}_i^*(w_i(\theta_i), \theta_i) > 0$ by (1), this is equivalent to condition (5).

To show that $u_i(\theta_i)$ is decreasing and the participation constraint binds when $\theta_i = \mu + \sigma$ notice that, from the definition of $u_i(\theta_i)$ and the envelope theorem,

$$\dot{u}_i(\theta_i) = -D^i(p_i^*(w_i(\theta_i), \theta_i), p_j^e).$$

Therefore, $\dot{u}_i(\theta_i) < 0$. Moreover, because the most inefficient firm obtains no rent, $u_i(\mu + \sigma) = 0$.

Next, we determine the equilibrium wholesale price. The first-order necessary and sufficient condition associated to $M_i$'s relaxed maximization program (8) is

$$\frac{\partial D^i\left(p_i^*(w_i(\theta_i), \theta_i), p_j^e\right)}{\partial p_i} \frac{\partial p_i^*(w_i(\theta_i), \theta_i)}{\partial w_i} \left[ p_i^*(w_i(\theta_i), \theta_i) - \theta_i - \frac{F(\theta_i)}{f(\theta_i)} \right] + $$

$$+ D^i\left(p_i^*(w_i(\theta_i), \theta_i), p_j^e\right) \frac{\partial p_i^*(w_i(\theta_i), \theta_i)}{\partial w_i} = 0. \quad \text{(A7)}$$

Notice that (A1) implies

$$D^i\left(p_i^*(w_i(\theta_i), \theta_i), p_j^e\right) = -\frac{\partial D^i\left(p_i^*(w_i(\theta_i), \theta_i), p_j^e\right)}{\partial p_i} (p_i^*(w_i(\theta_i), \theta_i) - w_i(\theta_i) - \theta_i).$$

Replacing this expression into (A7) yields

$$\frac{\partial D^i\left(p_i^*(w_i(\theta_i), \theta_i), p_j^e\right)}{\partial p_i} \left[ w_i(\theta_i) - \frac{F(\theta_i)}{f(\theta_i)} \right] = 0,$$

which gives the equilibrium wholesale price, $w_i^*(\theta_i) = \frac{F(\theta_i)}{f(\theta_i)} = \theta_i - \mu + \sigma$. Because $\dot{w}_i^*(\theta_i) > 0$, the local second-order incentive compatibility constraint (5) is satisfied. Finally, inserting the equilibrium wholesale price into (2) yields the equilibrium retail price (9).

The global incentive constraint (A4) implies that the equilibrium contract must satisfy the following inequality:

$$u_i(\theta_i) - u_i\left(\theta_i, \theta_i'\right) \geq 0 \quad \forall(\theta_i, \theta_i') \in \Theta^2$$

$$\Leftrightarrow \quad (p_i^*(w_i^*(\theta_i), \theta_i) - w_i^*(\theta_i) - \theta_i)D^i(p_i^*(w_i^*(\theta_i), \theta_i), p_j^e) - T_T^e(\theta_i) \geq$$
$$(p_i^*(w_i^*(\theta_i'), \theta_i) - w_i^*(\theta_i') - \theta_i)D^i(p_i^*(w_i^*(\theta_i'), \theta_i), p_j^e) - T_i^*(\theta_i')$$

$$\Leftrightarrow \quad \int_{\theta_i}^{\theta_i'} \left\{\dot{w}_i^*(x)D^i(p_i^*(w^*(x), \theta_i), p_j^e) + \dot{T}_i^*(x)\right\} dx \geq 0,$$

where $T_i^*(x)$ is the equilibrium fixed fee. Substituting $\dot{T}_i^*(x) = -\dot{w}^*(x)D^i(p_i^*(w_i^*(x), x), p_j^e)$,

$$\int_{\theta_i}^{\theta_i'} \left\{\dot{w}_i^*(x)D^i(p_i^*(w^*(x), \theta_i), p_j^e) - \dot{w}_i^*(x)D^i(p_i^*(w^*(x), x), p_j^e)\right\} dx =$$
$$-\int_{\theta_i}^{\theta_i'} \left\{\dot{w}_i^*(x) \int_{\theta_i}^{x} \frac{\partial D^i(p_i^*(w^*(x), y), p_j^e)}{\partial p_i} \frac{\partial p_i^*(w^*(x), y)}{\partial y} dy\right\} dx \geq 0.$$

Suppose, without loss of generality, that $\theta_i' > \theta_i$ (so that $x > \theta_i$). Condition (5)—that is, $\dot{w}_i^*(x) > 0$—and the fact that $\frac{\partial p_i^*(.)}{\partial y} > 0$ and $\frac{\partial D^i(.)}{\partial p_i} < 0$ guarantee that the global incentive constraint holds. □

**Proof of Proposition 1:** Using the equilibrium prices in Lemma 1, $R_i$'s expected information rent is

$$\int_{\mu-\sigma}^{\mu+\sigma} \int_{\theta_i}^{\mu+\sigma} \int_{\mu-\sigma}^{\mu+\sigma} D^i(p_i^*(w^*(x), x), p_j^*(w^*(\theta_j), \theta_j))dF(\theta_j)dx dF(\theta_i)$$
$$= \frac{\sigma}{6}\left(3 + a_i - a_j - \frac{\sigma}{t\left(1 - a_i - a_j\right)}\right).$$

First, assume that $\sigma = 0$. Because $R_i$'s expected information rent is equal to zero, any location $a_i = a_j = a^*$ represents a symmetric equilibrium for retailers. Replacing into (9) yields the equilibrium retail price.

Next, assume that $\sigma > 0$. $R_i$'s optimization program is

$$\max_{a_i} \left(3 + a_i - a_j - \frac{\sigma}{t\left(1 - a_i - a_j\right)}\right),$$

yielding the first-order necessary and sufficient condition[37]

$$t\left(1 - a_i - a_j\right)^2 = \sigma.$$

37. The second-order condition is satisfied because the second-order derivative of $R_i$'s objective function is negative.

The relevant solution for $a_i$ is

$$a_i(a_j) = 1 - \sqrt{\frac{\sigma}{t}} - a_j,$$

which implies that $a_i(a_j)$ is decreasing in $a_j$—that is, location choices are strategic substitutes.

Hence, in a symmetric equilibrium, each retailer chooses $a^*(\sigma) = \frac{1}{2} - \frac{1}{2}\sqrt{\frac{\sigma}{t}}$. (Of course, any other couple of locations $a_i$ and $a_j$ such that $a_i + a_j = 1 - \sqrt{\frac{\sigma}{t}}$ is an equilibrium of our game.) Using equation (9), the equilibrium retail price is

$$\begin{aligned} p^*(\theta_i) &= \theta_i + \sigma + t\,(1 - 2a^*\,(\sigma)) \\ &= \theta_i + \sigma + \sqrt{t\sigma}. \end{aligned} \tag{A8}$$

Finally, because the equilibrium price is increasing with respect to marginal costs, retailers' (equilibrium) demand is always nonnegative regardless of realized costs—that is, $D^i(p^*(\theta_i), p^*(\theta_j)) \geq 0 \quad \forall(\theta_i, \theta_j) \in \Theta^2$—if

$$D^i(p^*(\mu + \sigma), p^*(\mu - \sigma)) = \frac{1}{2} - \sqrt{\frac{\sigma}{t}} \geq 0 \quad \Leftrightarrow \quad \frac{\sigma}{t} \leq \frac{1}{4}. \qquad \square$$

**Equilibrium Payoffs**. By the results of Proposition 1, $R_i$'s expected equilibrium rent is

$$\int_{\mu-\sigma}^{\mu+\sigma} \int_{\theta_i}^{\mu+\sigma} \left[\int_{\mu-\sigma}^{\mu+\sigma} D^i(p_i^*(x), p_j^*(\theta_j))dF(\theta_j)\right] dx dF\,(\theta_i) = \frac{1}{2}\sigma\left[1 - \frac{1}{3}\sqrt{\frac{\sigma}{t}}\right],$$

and $M_i$'s expected equilibrium profit is

$$\int_{\mu-\sigma}^{\mu+\sigma} \int_{\mu-\sigma}^{\mu+\sigma} D^i(p_i^*(\theta_i), p_j^*\left(\theta_j\right))\left(p_i^*(\theta_i) - \theta_i - u^e\,(\theta_i)\right) dF\,\left(\theta_j\right) dF\,(\theta_i) = \frac{1}{6}\left[3\sqrt{t\sigma} + \sigma\sqrt{\frac{\sigma}{t}}\right].$$

Simple computations show that both expressions are increasing in $\sigma$ in the range of parameters under consideration.

**Proof of Lemma 2:** If retailers locate at $a = 0$ when $\sigma = 0$, total production costs are $\mu$. If retailers locate at $a^*(\sigma)$ when $\sigma \geq 0$, expected total production costs are

$$\int_{\mu-\sigma}^{\mu+\sigma} \int_{\mu-\sigma}^{\mu+\sigma} \left[\left(\frac{1}{2} + \frac{\theta_2 - \theta_1}{2t\,(1 - 2a^*\,(\sigma))}\right)\theta_1 + \left(\frac{1}{2} - \frac{\theta_2 - \theta_1}{2t\,(1 - 2a^*\,(\sigma))}\right)\theta_2\right] dF\,(\theta_1)\, dF\,(\theta_2) =$$

$$= \mu - \frac{\sigma^{\frac{3}{2}}}{3\sqrt{t}}.$$

Because this function is strictly decreasing in $\sigma$, it is minimized by the lowest possible level of transparency $\sigma = \frac{t}{4}$. Then, by Proposition 1, retailers choose $a^* = \frac{1}{4}$.

If retailers locate at $a^*(\sigma)$ when $\sigma \geq 0$ , expected transportation costs are

$$\int_{\mu-\sigma}^{\mu+\sigma}\int_{\mu-\sigma}^{\mu+\sigma}\left[\int_0^{\frac{1}{2}+\frac{\theta_2-\theta_1}{2t(1-2a^*)}} t\,(x-a^*)^2\,dx+\int_{\frac{1}{2}+\frac{\theta_2-\theta_1}{2t(1-2a^*)}}^{1} t\,(1-a^*-x)^2\,dx\right]dF\,(\theta_1)\,dF\,(\theta_2)=$$

$$=\frac{\sigma}{6}\sqrt{\frac{\sigma}{t}}-\frac{\sqrt{t\sigma}}{4}+\frac{\sigma}{4}.$$

The first-order necessary and sufficient condition for minimizing this function is

$$\frac{1}{4}+\frac{1}{4}\sqrt{\frac{\sigma}{t}}-\frac{1}{8}\sqrt{\frac{t}{\sigma}}=0 \quad\Leftrightarrow\quad \sigma=t-\frac{t\sqrt{3}}{2}.$$

In this case, by Proposition 1, retailers choose $a^*=\frac{3}{4}-\frac{\sqrt{3}}{4}$.

If retailers locate at $a=0$ when $\sigma=0$, total transportation costs are

$$2\int_0^{\frac{1}{2}} tx^2dx=\frac{1}{12}t.$$

It is straightforward to show that these are higher than transportation costs when $\sigma = t-\frac{t\sqrt{3}}{2}$. □

Proof of Proposition 2: The socially optimal level of $\sigma$ maximizes

$$V(\sigma)\equiv\int_{\mu-\sigma}^{\mu+\sigma}\int_{\mu-\sigma}^{\mu+\sigma} W(\cdot)\,dF\,(\theta_1)\,dF\,(\theta_2)$$

$$=\lambda v-\int_{\mu-\sigma}^{\mu+\sigma}\int_{\mu-\sigma}^{\mu+\sigma}\left[\int_0^{\frac{1}{2}+\frac{\theta_2-\theta_1}{2t(1-2a^*)}}\Big((2\lambda-1)p^*(\theta_1)+\lambda t\,(x-a^*)^2+(1-\lambda)\theta_1\Big)\,dx+\right.$$

$$\left.-\int_{\frac{1}{2}+\frac{\theta_2-\theta_1}{2t(1-2a^*)}}^{1}\Big((2\lambda-1)p^*(\theta_2)+\lambda t\,(1-a^*-x)^2+(1-\lambda)\theta_2\Big)\,dx\right]dF\,(\theta_1)\,dF\,(\theta_2)$$

$$=\lambda\left(v-\mu-\frac{t}{12}\right)+\sigma-\sqrt{\sigma t}\left(\frac{21}{12}\lambda-1\right)+\lambda\sigma\left(\frac{1}{6}\sqrt{\frac{\sigma}{t}}-\frac{9}{4}\right),$$

for $\sigma\in[0,\frac{t}{4}]$. The first-order derivative of this objective function is

$$\frac{\partial V(\sigma)}{\partial\sigma}=\frac{1}{8}\left(8-18\lambda+(4-7\lambda)\sqrt{\frac{t}{\sigma}}+2\lambda\sqrt{\frac{\sigma}{t}}\right),$$

and it is strictly decreasing in $\lambda$.

We have the following cases:

- If $\lambda\leq\frac{16}{31}$, $\frac{\partial V(\sigma)}{\partial\sigma}>0$ for $\sigma\in[0,\frac{t}{4}]$ (because $\frac{\partial V(\sigma)}{\partial\sigma}|_{\lambda=\frac{16}{31}}>0$ for every $\sigma\in[0,\frac{t}{4}]$) and, hence, $V(\sigma)$ is maximized at $\sigma=\frac{t}{4}$.
- If $\lambda\geq\frac{4}{7}$, $\frac{\partial V(\sigma)}{\partial\sigma}<0$ for $\sigma\in[0,\frac{t}{4}]$ (because $\frac{\partial V(\sigma)}{\partial\sigma}|_{\lambda=\frac{4}{7}}<0$ for every $\sigma\in[0,\frac{t}{4}]$) and, hence, $V(\sigma)$ is maximized at $\sigma=0$.

- If $\lambda \in (\frac{16}{31}, \frac{4}{7})$, the function $V(\sigma)$ has a unique maximum $\sigma(\lambda, t)$ for $\sigma \in [0, \frac{t}{4}]$,[38] which is defined by

$$\left.\frac{\partial V(\sigma)}{\partial \sigma}\right|_{\sigma=\sigma(\lambda,t)} = 0 \quad \Leftrightarrow \quad \sigma(\lambda, t) = t\left[\frac{9}{2} - \frac{1}{2\lambda}\sqrt{16 - 80\lambda + 95\lambda^2} - \frac{2}{\lambda}\right]^2.$$

(The optimal level of transparency is continuous because $\sigma(\lambda, t)|_{\lambda=\frac{4}{7}} = 0$ and $\sigma(\lambda, t)|_{\lambda=\frac{16}{31}} = \frac{t}{4}$.)

Letting $\underline{\lambda} \equiv \frac{16}{31}$ and $\frac{4}{7} \equiv \overline{\lambda}$ yields the statement. Retailers' locations are obtained by substituting the optimal $\sigma$ in $a^*(\sigma)$. □

**Proof of Proposition 3:** Suppose that, at $\sigma = 0$, retailers' choose the locations that maximize manufacturers' profits—that is, $a^* = 0$. In this case, the retail price is $\mu + t$, and the welfare function is

$$\begin{aligned} W(\cdot)|_{\sigma=0} &= \lambda v - (2\lambda - 1)(\mu + t) - (1 - \lambda)\mu - \lambda t \int_0^{\frac{1}{2}} x^2 dx - \lambda t \int_{\frac{1}{2}}^1 (1 - x)^2 dx \\ &= \lambda(v - \mu) + t\left(1 - \frac{25}{12}\lambda\right). \end{aligned}$$

First, if $\lambda \leq \frac{16}{31}$, $V(\sigma)$ is strictly increasing in $\sigma$ (see the proof of Proposition 2). Moreover, when $\sigma \to 0$ retailers locate at $a^* \approx \frac{1}{2}$ and social welfare is

$$\lim_{\sigma \to 0} V(\sigma) = \lambda\left(v - \mu - \frac{1}{12}t\right) \geq W(\cdot)|_{\sigma=0}, \tag{A9}$$

with strict inequality for $\lambda \neq \frac{1}{2}$. Hence, $V(\sigma) > W(\cdot)|_{\sigma=0}$ for every $\sigma \in (0, \frac{t}{4}]$.

Second, for $\lambda \geq \frac{16}{31}$, $V(\sigma)$ is minimized at either $\sigma = 0$ or $\sigma = \frac{t}{4}$ (see the proof of Proposition 2).[39] The result follows from (A9) and

$$V\left(\frac{t}{4}\right) - W(\cdot)|_{\sigma=0} = \frac{t}{12}(7\lambda - 3) > 0. \qquad \square$$

**Proof of Proposition 4:** $M_i$'s optimization program is

$$\max_{a_i}\left\{\int_{\mu-\sigma}^{\mu+\sigma}\int_{\mu-\sigma}^{\mu+\sigma} D^i(p_i^*(\theta_i), p_j^*(\theta_j))(p_i^*(\theta_i) - \theta_i)\, dF(\theta_j)\, dF(\theta_i) - \int_{\mu-\sigma}^{\mu+\sigma} u_i(\theta_i)\, dF(\theta_i)\right\},$$

where by definition

$$u_i(\theta_i) = \int_{\theta_i}^{\mu+\sigma}\int_{\mu-\sigma}^{\mu+\sigma} D^i(p_i^*(x), p_j^*(\theta_j)) dF(\theta_j) dx.$$

Using equilibrium prices $p_i^*(\cdot)$ from Lemma 1 and the definition of $D^i(\cdot, \cdot)$,

$$\int_{\mu-\sigma}^{\mu+\sigma} u_i(\theta_i)\, dF(\theta_i) = \frac{\sigma\left[3 - \frac{\sigma}{t} - 2a_i - 4a_j - a_i^2 + a_j^2\right]}{6(1 - a_i - a_j)},$$

38. For $\lambda \in (\frac{16}{31}, \frac{4}{7})$, $\frac{\partial^2 V(\sigma)}{\partial \sigma^2}|_{\sigma=\sigma(\lambda,t)} < 0$. It is straightforward to show that there is no other local internal maximum and that, for $\lambda \in (\frac{16}{31}, \frac{4}{7})$, $V(\sigma(\lambda, t))$ is greater than both $V(\frac{t}{4})$ and $V(0)$.

39. Specifically, $V(.)$ is minimized at $\sigma = 0$ if $\lambda \in (\frac{16}{31}, \frac{9}{17}]$ and at $\sigma = \frac{t}{4}$ if $\lambda \in [\frac{9}{17}, \frac{4}{7})$.

and

$$\int_{\mu-\sigma}^{\mu+\sigma}\int_{\mu-\sigma}^{\mu+\sigma} D^i(p_i^*(\theta_i),\, p_j^*(\theta_j))\left(p_i^*(\theta_i)-\theta_i\right) dF(\theta_j)\, dF(\theta_i) =$$

$$= \frac{1}{18}\left(3+a_i-a_j\right)\left[3\sigma + t\left(3+a_j-a_i\right)\left(1-a_i-a_j\right)\right].$$

There exists an equilibrium with maximal differentiation because, if $M_j$ chooses $a_j = 0$, differentiating $M_i$'s objective function with respect to $a_i$ yields

$$\frac{1}{18}\left[\frac{3\sigma^2}{t(1-a_i)^2} - t\left(3a_i^2+10a_i+3\right)\right],$$

which is negative for $\sigma \le \frac{t}{4}$.

To see that there is no other symmetric equilibrium, consider the first-order necessary condition for $M_i$'s problem, evaluated at a symmetric equilibrium with $a > 0$,

$$\frac{\sigma^2 - t^2\left(1-12a^2+16a^3\right)}{t(1-2a)^2} = 0. \tag{A10}$$

For $\sigma = 0$, the left-hand-side of this condition is always negative (confirming the principle of differentiation with full transparency). Letting $\rho = \frac{\sigma}{t}$, condition (A10) is equivalent to

$$\rho^2 - 1 + 12a^2 - 16a^3 = 0. \tag{A11}$$

Notice that

$$\rho^2 - 1 + 12a^2 - 16a^3\big|_{a=0} = \rho^2 - 1 < 0,$$

and

$$\rho^2 - 1 + 12a^2 - 16a^3\big|_{a=\frac{1}{2}} = \rho^2 > 0.$$

Hence, by continuity, (A11) has at least one real root in (0, $\frac{1}{2}$). Differentiating again $M_i$'s expected utility with respect to $a_i$ and evaluating it at the interior symmetric equilibrium, the second-order derivative of $R_i$'s expected profit is

$$\frac{1}{9}\frac{3\rho^2 - 1 + 8a - 24a^2 + 32a^3 - 16a^4}{t(1-2a)^3}. \tag{A12}$$

Substituting (A11) into (A12) yields

$$\frac{4}{9}\frac{(1+a)(2-a)}{t(1-2a)} > 0.$$

Hence, every solution of the first-order condition (A14) is a minimum of the retailers' maximization program, which shows that there are no interior symmetric equilibria.

**Proof of Proposition 5:** Recall that $R_i$'s expected utility is

$$u_i\left(a_i, a_j\right) \equiv \int_{\mu-\sigma}^{\mu+\sigma} D_i(p_i^*(w_i\left(a_i,a_j\right),\theta_i),\, p_j^e)\left(p_i^*\left(w_i\left(a_i,a_j\right),\theta_i\right) - w_i\left(a_i,a_j\right) - \theta_i\right) dF(\theta_i). \tag{A13}$$

Differentiating (A13) with respect to $a_i$ and evaluating this derivative at $a_j = 0$ we have

$$\left.\frac{\partial u_i\left(a_i, a_j\right)}{\partial a_i}\right|_{a_j=0} = -\frac{1}{1176}\frac{420t^2 - 49\sigma^2 - 168t^2a_i^2 + 240t^2a_i^3 + 36t^2a_i^4 - 528t^2a_i}{t\left(1-a_i\right)^2},$$

which is strictly negative for $\frac{\sigma}{t} \leq \frac{1}{4}$ and $a_i \in [0, 1]$. Hence, with linear prices there exists a symmetric equilibrium with maximal differentiation.

Next, we show that there is no symmetric equilibrium where retailers locate in the interior of the unit line. First, it can be shown that

$$\left.\frac{\partial u_i(a_i, a_j)}{\partial a_i}\right|_{a_i=a_j=a} = 0 \quad \Leftrightarrow \quad 7\rho^2 + 192a - 48a^2 - 192a^3 - 60 = 0, \tag{A14}$$

where $\rho = \frac{\sigma}{t}$. Notice that

$$7\rho^2 + 192a - 48a^2 - 192a^3 - 60\big|_{a=0} = 7\rho^2 - 60 < 0,$$

and that

$$7\rho^2 + 192a - 48a^2 - 192a^3 - 60\big|_{a=\frac{1}{2}} = 7\rho^2 > 0.$$

Hence, by continuity, equation (A14) has at least one real solution in $(0, \frac{1}{2})$. Differentiating again $R_i$'s expected utility with respect to $a_i$ and evaluating it at the interior symmetric equilibrium, the second-order derivative of $R_i$'s expected profit is

$$\left.\frac{\partial^2 u_i\left(a_i, a_j\right)}{\partial a_i^2}\right|_{a_i=a_j=a} = \frac{49\rho^2 + 912a - 1728a^2 + 960a^3 + 192a^4 - 156}{t\left(1-2a\right)^3}. \tag{A15}$$

Substituting (A14) into (A15) yields

$$24\frac{26a + 2a^2 + 11}{(1-2a)\,t} > 0.$$

Hence, every solution of the first-order condition (A14) is a minimum of the retailers' maximization program, which shows that there are no interior symmetric equilibria.

Finally, for $a_1 = a_2 = 0$, the equilibrium wholesale price is

$$w^{**} = 2t,$$

and the equilibrium retail price is

$$p^{**}\left(\theta_i\right) = \frac{1}{2}\left(6t + \mu + \theta_i\right). \qquad \square$$

***Resale Price Maintenance (RPM)*** We show that, with secret contracts, two-part tariffs contracts are equivalent to RPM contracts in which $M_i$ offers a menu $(p_i(m_i), T_i(m_i))_{m_i\in\Theta}$ to $R_i$ where, given $R_i$'s report $m_i$ to $M_i$, $p_i(m_i)$ represents the retail price at which $R_i$ has to sell to final consumers and $T_i(m_i)$ is the franchise fee paid by $R_i$ to $M_i$.

We consider a symmetric separating equilibrium, and let $p_i^*(\theta_i)$ be the retail price chosen by $M_i$ when $R_i$'s cost is $\theta_i$, given the locations chosen by retailers. Let $R_i$'s information rent when his cost is $\theta_i$ and he reports $m_i$ be

$$u_i\,(m_i, \theta_i) \equiv (p_i(m_i) - \theta_i) D^i(p_i\,(m_i)\,, p_j^e) - T_i(m_i), \quad i = 1, 2,$$

where $p_j^e \equiv \mathbb{E}[p_j^*(\theta_j)]$. Let $u_i(\theta_i) \equiv u_i(\theta_i, \theta_i)$ denote $R_i$'s rent when he reports his true type. The contract offered to $R_i$ must satisfy the global incentive compatibility constraint

$$u_i(\theta_i) \equiv \max_{m_i \in \Theta} \left\{ (p_i(m_i) - \theta_i) D^i(p_i(m_i), p_j^e) - T_i(m_i) \right\}.$$

For a contract to be incentive compatible, truthfully reporting $m_i = \theta_i$ must maximize $R_i$'s utility—that is, the following local first- and second-order incentive constraints must hold

$$\left. \frac{\partial u_i\,(m_i, \theta_i)}{\partial m_i} \right|_{m_i = \theta_i} = 0 \quad \Leftrightarrow \quad \dot{u}_i\,(\theta_i) = -D^i(p_i\,(\theta_i)\,, p_j^e), \tag{A16}$$

and

$$\left. \frac{\partial^2 u_i\,(m_i, \theta_i)}{\partial m_i^2} \right|_{m_i = \theta_i} \leq 0 \quad \Leftrightarrow \quad -\frac{\partial D^i(p_i\,(\theta_i)\,, p_j^e)}{\partial p_i} \dot{p}_i\,(\theta_i) \geq 0 \quad \Rightarrow \quad \dot{p}_i\,(\theta_i) \geq 0, \tag{A17}$$

where we have used a standard envelope condition. Conditions (A16) and (A17), together with the participation constraint

$$u_i\,(\theta_i) \geq 0, \quad \forall \theta_i \in \Theta, \tag{A18}$$

define the set of incentive-feasible allocations for $M_i$ and $R_i$. Therefore, $M_i$ solves the following optimization program

$$\max_{\{p_i(\cdot), u_i(\cdot)\}} \int_{\mu-\sigma}^{\mu+\sigma} \left\{ D^i(p_i\,(\theta_i)\,, p_j^e)\,(p_i(\theta_i) - \theta_i) - u_i\,(\theta_i) \right\} dF\,(\theta_i)\,,$$

subject to (A16), (A17), and (A18).

To solve this program, we first ignore the constraint $\dot{p}_i(\theta_i) \geq 0$, and then check that it is actually satisfied in the equilibrium that we characterize. It follows that $u_i(\theta_i)$ is decreasing, and the participation constraint is binding when $\theta_i = \mu + \sigma$. Hence, $R_i$'s information rent is

$$u_i\,(\theta_i) = \int_{\theta_i}^{\mu+\sigma} D^i(p_i\,(x)\,, p_j^e) dx. \tag{A19}$$

Using expression (A19), integrating by parts, and substituting in $M_i$'s objective function yields the simplified program

$$\max_{p_i(\cdot)} \int_{\mu-\sigma}^{\mu+\sigma} \left\{ D^i(p_i\,(\theta_i)\,, p_j^e) \left[ p_i(\theta_i) - \theta_i - \frac{F\,(\theta_i)}{f\,(\theta_i)} \right] \right\} dF\,(\theta_i)\,.$$

Using the definition of $D^i(.,.)$, the first-order condition is

$$\frac{1 + a_i - a_j}{2} + \frac{p_j^e - 2p_i^*(\theta_i) + \theta_i + \frac{F(\theta_i)}{f(\theta_i)}}{2t\left(1 - a_i - a_j\right)} = 0$$

$$\Leftrightarrow \quad p_i^*(\theta_i) = \frac{1}{2} t \left(1 - a_i - a_j\right)\left(1 + a_i - a_j\right) + \frac{1}{2} p_j^e + \theta_i - \frac{1}{2}\,(\mu - \sigma)\,. \tag{A20}$$

Taking expectations with respect to $\theta_i$,

$$p_i^e \equiv \mathbb{E}\left[p_i^*(\theta_i)\right] = \frac{1}{2}t\left(1 - a_i - a_j\right)\left(1 + a_i - a_j\right) + \frac{1}{2}p_j^e + \frac{1}{2}(\mu + \sigma), \quad i, j = 1, 2.$$

Solving the system for $p_i^e$ and $p_j^e$ yields

$$p_i^e = \mu + \sigma + t\left(1 - a_i - a_j\right)\left(1 + \frac{a_i - a_j}{3}\right), \quad i, j = 1, 2.$$

Substituting this equation in (A20) yields exactly (9), the retail price chosen by $R_i$ with two-part tariff contracts. Therefore, our model is equivalent to a model in which a manufacturer can control retail prices.

Notice that the equilibrium retail price (9) satisfies $\dot{p}^*(\theta_i) > 0$, so that the local second-order incentive compatibility constraint holds at the equilibrium. Finally, consider the global incentive compatibility constraint. Let the equilibrium franchise fee be

$$T^*(\theta_i) = (p^*(\theta_i) - \theta_i)\int_{\mu-\sigma}^{\mu+\sigma} D^i(p^*(\theta_i), p^*(\theta_j))dF(\theta_j) - \int_{\theta_i}^{\mu+\sigma}\int_{\mu-\sigma}^{\mu+\sigma} D^i(p^*(x), p^*(\theta_j))dF(\theta_j)dx.$$

The equilibrium contract must satisfy the following inequality

$$\begin{aligned}
&u_i(\theta_i) \geq u_i(\theta_i, \theta'), \quad \forall(\theta_i, \theta') \in \Theta^2 \\
\Leftrightarrow \quad &\int_{\theta_i}^{\theta'}\left\{\dot{T}^*(x) - \frac{\partial(p^*(x) - \theta_i)\int_{\mu-\sigma}^{\mu+\sigma} D^i(p^*(x), p^*(\theta_j))dF(\theta_j)}{\partial x}\right\}dx \geq 0 \\
\Leftrightarrow \quad &\int_{\theta_i}^{\theta'}\left\{\dot{T}^*(x) - \dot{p}^*(x)\left(\int_{\mu-\sigma}^{\mu+\sigma} D^i(p^*(x), p^*(\theta_j))dF(\theta_j) - \frac{p^*(x) - \theta_i}{2t(1 - 2a^*)}\right)\right\}dx \geq 0. \quad \text{(A21)}
\end{aligned}$$

Because $\dot{p}^*(\theta_i) = 1$ and the local first-order incentive compatibility implies

$$\begin{aligned}
\dot{T}^*(\theta_i) &= -\frac{\partial\left[(p^*(\theta_i) - \theta_i)\int_{\mu-\sigma}^{\mu+\sigma} D^i(p^*(\theta_i), p^*(\theta_j))dF(\theta_j)\right]}{\partial\theta_i} \\
&= \dot{p}^*(\theta_i)\left(\int_{\mu-\sigma}^{\mu+\sigma} D^i(p^*(\theta_i), p^*(\theta_j))dF(\theta_j) - \frac{p^*(\theta_i) - \theta_i}{2t(1 - 2a^*)}\right), \quad \forall\theta_i \in \Theta,
\end{aligned}$$

the left-hand side of (A21) is

$$\int_{\theta_i}^{\theta'} \frac{x - \theta_i}{2t(1 - 2a^*)}dx. \tag{A22}$$

Suppose, without loss of generality, that $\theta' > \theta_i$. Then $x > \theta_i$ and (A22) is positive. Hence, the global incentive compatibility constraint holds at equilibrium. □

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