Quality Competition among Platforms: a Media Market Case

Maria Rosa Battaggion,* Serena Marianna Drufuca**
* Bergamo University, CRIOS Bocconi University, **
Milan University

Abstract

The present paper provides a a vertical differentiation model in a two-sided context. In fact, we solve the model and we calculate the equilibrium in terms of advertising levels, subscription fees and qualities provision, both in duopoly - two platforms of different quality - and in monopoly case. Furthermore, we investigate how competition among platforms and entry deterrence behavior affects the equilibrium, with particular focus on quality provision.

Key words: two-sided market, media, quality
JEL codes: D42, D43, L15, L82

1 Introduction

Media markets are characterized by different forms of finance. Indeed they might collect revenues from viewers/readers as well as from advertisers. The peculiar feature of media markets is that there exists a crucial interplay between the two sides of the market, namely audience [1] and advertisers. In this respect the media market represents an idiosyncratic example of a two-sided market, (see Caillaud and Jullien (2001, 2003), Armstrong (2006), Rochet and Tirole (2006) as seminal references).

The present paper aims to analyze the role of competition in a two-sided market characterized by vertical differentiation. While most of the literature focuses on horizontal differentiation (see e.g., Ambrus and Resinger (2005), Anderson and Coate (2005), Choi (2006), Gabszewicz et al. (2004), Kind et al. (2009), Peitz and Valletti (2008)), we believe that quality is a relevant feature of the media market even though it is hard to

[1] With the term "audience" we encompasse both viewers (Tv) and readers (Newspapers).

shape. For instance in broadcasting, quality is associated with the purpose of providing not only entertainment, but also education, learning and cultural excellence, without ignoring niche interests (Collins 2007). Similarly in the press, quality as accuracy, truth, impartiality and immediacy of information, helps in forming public opinion, expressing different and minority voices and performing the watchdog role for public interest. Moreover, we believe that the quality issue deeply affects the policy debate among free-to-air televisions, pay-tvs and public broadcasters, as well as the debate about newspapers subsidization.

A second important aspect we would like to deal with is the role of competition in a two-sided market scheme. Advertising is typically considered as a nuisance for the audience or, in other words, it represents a negative externality. While the audience exerts a positive externality for the advertisers. Platforms, let say broadcasters or newspapers, compete both for audience and advertisers, in order to maximize profit. Notice that platforms compete on both sides of the market, namely they should attract consumers' demand as well as advertising spaces. Therefore, competition has a broader meaning with respect to the standard industrial organization literature and might generate different results.

As already mentioned, we provide a model of platforms competition in the framework of vertical differentiation. In a context where platforms endogenously provide the quality levels, we calculate the equilibrium values of advertising, the optimal subscription fees of the viewers/readers and the quality provision in both monopoly and duopoly cases. Then, by considering a duopoly with sequential moves, we investigate the possibility of entry by a potential competitor, not only affecting the market share, but also the quality provision of each platform. In this respect we illustrate the feasibility and the profitability of entry deterrence strategy in a two-sided market.

More specifically, in our set up, readers/viewers are single-homing, while advertisers are multi-homing, meaning that platforms have monopoly power over providing access to their single-homing customers for the multi-homing side. In this respect platforms act as "bottlenecks" between advertisers and consumers, by offering sole access to their respective set of consumers. This assumption is crucial to explain the prevailing competition on consumers' side. Furthermore, this is the driving force of the "profit neutrality" result in duopoly.[2] We also model advertisers as not strategic: their payoffs do no depend on what other advertisers do, but from an advertising benefit, related to market demand. This behavior suits the case of informative advertising.

[2]For a further discussion on the role of the single-homing or multi-homing assumption see Roger (2010).

We think that the effects of endogenous quality provision with different market structures in a two-sided framework deserve a closer attention. In fact, in this set up, we have two forces at stake. Higher quality induces consumers to pay higher subscription fees to join the platform. In turn, the platform can extracts surplus on the advertisers side and "invest" them in a reduction of subscription fees, implying that advertisers cross-subsidize single-homing consumers. Therefore, given the profit neutrality, a sort of substitution between quality and advertising comes up.

To anticipate results, we provide a full characterization of the equilibrium for what concerns advertising, subscription fees, market shares and quality both in a monopoly as well as in duopoly structure. Then, we analyze the role of competition by considering potential entrance of new competitors in the market and the associated behavior of the Incumbent platform and the potential entrants. We show that the threat of entry may induce a lower differentiation in terms of quality. Finally we provide the conditions such that an entry deterrence strategy is feasible and profitable for the Incumbent platform. On top of that, we perform a numerical simulation analysis assessing the set of parameter values such that deterrence strategy is profitable.

1.1 Related literature

Our paper belongs to the literature of two-sided markets with vertical differentiation. In this stream of the literature, Armstrong (2005) and Armstrong and Weeds (2007) provide a model with endogenous quality provision in the two-sided context of digital broadcasters. By comparing competition in two different regimes, free-to-air and pay-TV, they show that programme quality is higher in the pay-tv which is also optimal by a social point of view. More recently, Lin (2011) have extended the analysis to the direct competition among two platforms, where one operate as free-to-air broadcaster, while the second one is a pay-TV broadcaster. In this framework he shows that platforms vertically differentiate their programmes according to the degree of viewers' dislike for advertising. In the same stream, Gonzales-Mestre and Martinez-Sanchez (2013) study how public-owned platforms affect the programme quality provision, the social welfare and the optimal level of advertising. Notice that, differently from our model, all the above contributions focus on the duopoly case, neglecting the monopoly behavior.

For what concerns competition between broadcasters, see in particular Crampes et al. (2009), and Peitz and Valletti (2008). The former paper examines a free-entry model of broadcasting with exogenous programme quality, while we consider competition and entry with endoge-

nous quality provision. The second paper compares advertising intensity and content programming in a market with duopoly broadcasters choosing the degree of horizontal differentiation (i.e. platforms choose the degree of programme "diversity" in the horizontal space, rather than vertical programme quality). In this perspective, our model might be interpreted as a translation to the vertical differentiation context to the Peitz, Valletti (2008) work, with also the extension to the analysis of entry competition.

Roger (2010) and Ribeiro (2012) are also close to the present work. They both consider a two-sided structure in the media market, with vertical differentiation as described by Gabszewicz, Wauthy (2012). On one side, in a slightly different context, with respect to the present model, Roger fully characterizes a duopoly equilibrium in pure strategy (and mixed one), with respect to prices, market shares and quality. While Ribeiro shows that a negligible shock on the consumers' side can be disruptive for the market equilibrium when platforms compete on two sides.

Finally, our paper is related to an older stream of the literature on industrial organization, just about vertical differentiation. In particular we are in debt with the well known work of Shaked and Sutton (1982), which illustrates market equilibrium when firms compete in a vertical differentiated framework and they are ranked according to their quality levels. We slightly modify their conditions to explain the role of entry and competition to reflect our two-sided framework.

The paper is organized as follows. Section 2 introduces the general model, while Sections 3 and 4 respectively provide the full characterization of the equilibrium in monopoly and duopoly. Section 5 deals with competition issue. Finally, Section **??** investigates the strategy of entry deterrence. Some conclusive remarks (Section 7) close the paper.

2 Set up

2.1 Viewers

There is a continuum of individuals of mass N. They constitute the buyer side in the market. If consumers join a platform they are exposed to contents[3] and to some informative advertising about market products. They can access at most one platform (single-homing). All consumers value quality of information in the sense of vertical differentiation: the quality of platforms' content is denoted by the parameter $\theta \in \Theta = [\underline{\theta}, \overline{\theta}]$. Consumers have private valuation β for information. The individual

[3]Media contents are meant in a broader sense, including both information (or hard news) and entertaiment (or soft news).

taste for quality β is distributed uniformly on an interval $[\underline{\beta}, \overline{\beta}]$. Moreover, consumers are assumed to dislike advertising. Their utility loss is δa, where a denotes advertising level and δ the disutility parameter for being exposed to advertising.

The indirect utility of individual from joining platform i of quality θ is:

$$V - \delta a_i + \beta\theta_i - s_i$$

where θ_i denotes platform i 's quality and a_i the level of advertising. Finally, s_i stands for the subscription fee or the price to access the platform i. Each individual has a a reservation utility $u_0 = 0$.

The individual indifferent between assessing a platform i or not assessing at all is characterized by:

$$\beta_{0i} = \frac{\delta a_i - V}{\theta_i} + \frac{s_i}{\theta_i} \tag{1}$$

While the individual indifferent between two platforms is described as follows:

$$\beta_{ik} = \frac{\delta\left(a_i - a_k\right)}{\left(\theta_i - \theta_k\right)} + \frac{\left(s_i - s_k\right)}{\left(\theta_i - \theta_k\right)} \tag{2}$$

for $k \neq i$.

The value of β_{0i} and β_{ik} define NB_i, namely the viewers' demand function for platform i.

2.2 Advertisers

As standard in this class of models, we assume advertising to be informative and just consumers watching the advertisement buy the good. Advertisers sell products of quality α which are produced at constant marginal costs, set equal to zero. Product quality α is distributed on a interval $[0, \overline{\alpha}]$ according to a distribution function F. Consumers have willingness to pay α for a good of quality α. Each producer has monopoly power and can therefore extract the full surplus from consumers by selling their product at price equal to α. Advertisers are allowed to multihome and they can advertise in none, one or more platforms. Advertisers have to pay to the platform i an advertising charge r_i. Therefore, advertisers' profits on platform i are:

$$\Pi_a = N\alpha_i B_i - r_i \tag{3}$$

The advertising charge r_i is endogenously determined by each platform. Due to assumption of single homing on the buyer side, each media platform behaves as a monopoly on in carrying its audience to advertiser. Therefore, the advertising charge r_i is endogenously determined in order to leave the marginal advertiser with zero profit, $\Pi_a = N\alpha_i B_i - r_i = 0$:

$$\alpha_i = \frac{r_i}{NB_i} \tag{4}$$

Thus,the amount of advertising for each platform becomes:

$$a_i = 1 - F\left(\frac{r_i}{NB_i}\right) \tag{5}$$

2.3 Platforms

Media markets are characterized by a broad range of financing regimes, both under private or public ownership:[4] free-to-air Tv under which broadcast platform are just financed trough advertising revenues, pay-TV under which broadcast stations are financed through subscription revenues and mixed regime under which broadcast platforms are financed through both subscription fees and advertising. Therefore, we consider a very general framework where platforms are financed both by advertising as well as subscription fees.

Platforms set the advertising space, the subscription prices, which might be positive or negative (subsidies) and the qualities. We assume neither constraints on advertising space nor costs of running ads. Quality however is costly to provide. We assume that this quality cost is independent of the number of units and is fixed at K, (see e.g. Mussa and Rosen (1978) and Hung, Schmitt (1988)).[5] This assumption fits very well the structure of the ICT and media markets, where there is a prominent role of fixed costs compared to marginal ones (see e.g. Shapiro and Varian (1998), Areeda and Hovenkamp (2014)).

Hence, a media platform collects revenues from both viewers and advertisers. For any platform i the objective function takes the form:

$$\Pi_i\left(s_i, a_i, r_i, \theta_i\right) = NB_i s_i + a_i r_i - K \tag{6}$$

[4] In Italy, for instance, we have a public broadcaster financed both by subscription fees (canone RAI) as well as advertising revenues. At the same time we have both free-to-air private operators, such as Mediaset, totally financed through advertising, and private pay-TVs financed through subscription fees and adevertising revenues (e.g. Sky).

[5] This cost assumption can be justified in the theory of innovation, by the idea that a better quality level depends upon an investment in R&D

2.4 Timing

We assume a three-stage game. In the first stage, platforms choose quality levels of their contents. Then, in the second stage, subscription fees and advertising spaces are set. Finally, in the third stage viewers and advertisers simultaneously decide whether to join a platform or not. Viewers might join one platform (single-homing) while advertisers might join more than one (multihoming). The game is solved backward for a monopoly structure and a duopoly one.

3 Monopoly

3.1 Monopoly: Viewers' and Advertisers' Demands

By considering the individual indifferent between accessing the monopoly platform or not accessing at all, we obtain the demand function by viewers/readers.

From (1), by assuming $V = 0$:

$$\beta_{0M} = \frac{\delta a_M}{\theta_M} + \frac{s_M}{\theta_M} \tag{7}$$

Since individuals are uniformly distributed between $\underline{\beta}$ and $\overline{\beta}$, the demand for the monopoly platform is simply given by the fraction of population with a taste for quality greater than β_{0M} :

$$NB_M = N(\overline{\beta} - \beta_{0M}) = N\left(\frac{\overline{\beta}\theta_M - s_M - \delta a_M}{\theta_M}\right) \tag{8}$$

Notice that the demand is positive if:

$$\overline{\beta}\theta_M \geq \delta a_M + s_M \tag{9}$$

From (5), the amount of advertising for the platform becomes:

$$a_M = 1 - F\left(\frac{r_M}{NB_M}\right) \tag{10}$$

Having defined the demand function of viewers and advertisers, for given prices r_M and s_M, we solve the game backwards, from stage three. Therefore by simultaneously solving equations (8) and (10) we get:

$$r_M\left(s_M, a_M, \theta_M\right) = F^{-1}(1 - a_M)N(\frac{\overline{\beta}\theta_M - s_M - \delta a_M}{\theta_M}) \tag{11}$$

This equation describes how advertising charges react to changes in subscribers' prices, advertising and qualities.

3.2 Monopoly: Platform's Subscription Fees and Advertising Level

According to the above assumptions, platform maximizes profit subject to a positivity constraint on the advertising level:

$$\begin{cases} \max_{a_H, s_H} \Pi_M = NB_M s_M + a_M r_M - K \\ \qquad\qquad s.t. a_M \geq 0 \end{cases}$$

First order conditions with respect to advertising a_M and subscription fees s_M are respectively:

$$\frac{\partial \Pi_M}{\partial a_M} = N \frac{\partial B_M}{\partial a_M} s_M + r_M + a_M \frac{\partial r_M}{\partial a_M} \leq 0 \tag{12}$$

and

$$\frac{\partial \Pi_M}{\partial s_M} = NB_M + N \frac{\partial B_M}{\partial s_M} s_M + a_M \frac{\partial r_M}{\partial s_M} = 0 \tag{13}$$

Then, according to the literature, we define the advertising revenues per viewer as $\rho(a_i)$

$$\rho(a_i) = \frac{a_i r_i}{NB_i} = \frac{a_i F^{-1}(1-a_i) NB_i}{NB_i} = a_i F^{-1}(1-a_i) \tag{14}$$

We assume $\rho(a_i)$ to be concave in the interval $a \in [0,1]$. Given that $\rho(a_i) = 0$ for $a_i = 0$ and $a_i = 1$, the function is single-peaked.

Using the definition (14) for the monopoly platform we can rewrite optimality conditions, proving the following Proposition.

Proposition 1 *The optimal advertising level of monopoly media platform is:*

$$\rho'(a_M) = \delta \tag{15}$$

Proof. Given (14) for the monopoly platform

$$\rho(a_M) = \frac{a_M r_M}{NB_M} = \frac{a_M F^{-1}(1-a_M) NB_M}{NB_M} = a_M F^{-1}(1-a_M)$$

we have:

$$r_M = \frac{NB_M \rho(a_M)}{a_M} \tag{16}$$

Therefore optimality conditions (12) and (13) rewrite into (17) and (18):

$$N s_M \frac{\partial B_M}{\partial a_M} + r_M + a_M \left[\frac{\left(NB_M \rho(a_M) + N \frac{\partial B_M}{\partial a_M} \rho(a_M) \right) a_M - NB_M \rho(a_M)}{a_M^2} \right] \leq 0 \tag{17}$$

$$NB_M + Ns_M \frac{\partial B_M}{\partial s_H} + a_M \frac{\partial r_M}{\partial s_M} = 0 \tag{18}$$

By easy calculation, (17) and (18) become respectively:

$$\frac{\partial B_M}{\partial a_M} (s_M + \rho(a_M)) + B_M \rho(a_M) \leq 0 \tag{19}$$

$$\frac{\partial B_M}{\partial s_H} (s_M + \rho(a_M)) + B_M = 0 \tag{20}$$

Given that $\frac{\partial B_M}{\partial a_M} = -\frac{\delta}{\theta_M}$ and $\frac{\partial B_M}{\partial s_M} = -\frac{1}{\theta_M}$, we get:

$$\frac{\partial B_M}{\partial a_M} = \delta \frac{\partial B_M}{\partial s_M}$$

Therefore, plugging in (19) and (20), we get the following system:

$$\begin{cases} \delta \frac{\partial B_M}{\partial s_M} (s_M + \rho(a_M)) + B_M \rho'(a_M) \leq 0 \\ \frac{\partial B_M}{\partial s_H} (s_M + \rho(a_M)) + B_M = 0 \end{cases}$$

Finally, if $a_M > 0$ the above inequality is satisfied by equality. Therefore, given that $\rho(a_M)$ is single-peaked, a_M is uniquely determined by the following condition:

$$\rho'(a_M) = \delta$$

with $\delta < \overline{\alpha}$. Otherwise it is zero. ■

The above Proposition 1 states that for a monopoly platform the best reply is to set a fixed advertising space just depending on the disutility of the viewers, as measured by parameter δ. However, the platform does not set the maximum amount of advertising. Notice that our result is in contrast with the suggestion of Peitz and Valletti (2008), where the market is covered and the monopoly advertising space would be $\rho'(a_M) = 0$.

We can now solve for the equilibrium values, as stated in the following Proposition.

Proposition 2 *With $\rho(a_M)$ concave, we obtain the equilibrium price s_M^* and demand B_M^* as function of quality, revenues per viewer and advertising level.*

Proof. By plugging the expression for B_M in the optimality condition (20) we obtain:

$$s_M^* = \frac{\overline{\beta}\theta_M - \rho(a_M^*) - \delta a_M^*}{2} \tag{21}$$

Then,

$$B_M^* = \frac{\overline{\beta}\theta_M + \rho\left(a_M^*\right) - \delta a_M^*}{2\theta_M} \tag{22}$$

■

The above Proposition 2 shows the result of profit neutrality. Revenues from the advertising side are counterbalanced by a decrease on the subscription fees. However, just half of the revenues from advertising this is involved in this pass-through effect (see equation (21)). Moreover, given that subscription fees positively depend on quality, a sort of substitutability between advertising and quality emerges.

3.3 Monopoly: Platform's Quality

In order to solve the quality stage, we maximize monopoly profits $\Pi_M\left(s_M^*, a_M^*, r_M^*, \theta_M\right)$ with respect to quality θ_M. We obtain the following FOC, subject to $\theta_M \geq 0$:

$$\frac{\partial \Pi_M}{\partial \theta_M} = N\left(\frac{\overline{\beta}^2\theta_M^2 - \left(\rho\left(a_M^*\right) - \delta a_M^*\right)^2}{4\theta_M^2}\right) = 0 \tag{23}$$

Unfortunately, in this general framework we cannot determine analytically the equilibrium value of θ_M^*.

However, considering the special case where the p.d.f. of advertisers F is uniform on $[0,1]$, we can suggest some interesting insights. By easy calculation, in the uniform case with $\rho\left(a_M\right) = a_M\left(1 - a_M\right)$, we obtain:

$$a_M^* = \frac{1-\delta}{2} \tag{24}$$

$$s_M^* = \frac{4\overline{\beta}\theta_M - (1-\delta)(1+3\delta)}{8} \tag{25}$$

$$B_M^* = \frac{\overline{\beta}\theta_M + \left(\frac{1-\delta}{2}\right)^2}{2\theta_M} \tag{26}$$

According to the equilibrium solutions of stage 3 and stage 2, the profit function - in the uniform case - becomes:

$$\Pi_M = NB_M^*(s_M^* + \rho_M^*) - K = \frac{\left(\overline{\beta}\theta_M + \left(\frac{1-\delta}{2}\right)^2\right)^2}{4\theta_M} - K \tag{27}$$

We calculate first order condition with respect to quality:

$$\frac{\partial \Pi_M}{\partial \theta_M} = N\left(\frac{1}{64\theta_M^2}\left(4\overline{\beta}\theta_M - (1-\delta)^2\right)\left(4\overline{\beta}\theta_M + (1-\delta)^2\right)\right) \tag{28}$$

Notice that (28) defines the optimal value of θ_M as function of δ. According to the fact that profit function is convex in quality, we show the following result

Proposition 3 *In equilibrium, under the technological constraint* $\theta \in (\underline{\theta}, \bar{\theta})$ *with* $\underline{\theta} = \dfrac{(1-\delta)^2}{4\overline{\beta}}$ *, the monopoly platform chooses the maximum quality.*

Proof. By computing the second order condition, we show the convexity of profit function:

$$\frac{\partial^2 \Pi_M}{\partial \theta_M^2} = \frac{1}{32} \frac{N}{\theta_M^3} (\delta - 1)^4 \geq 0 \tag{29}$$

Given convexity, we restrict ourself on the increasing slope of the profit function (27), therefore we also restrict θ in the following technological range, $\theta \in \Theta_R$:

$$\Theta_R = (\underline{\theta}, \bar{\theta}) \text{ with } \quad \underline{\theta} = \frac{(1-\delta)^2}{4\overline{\beta}}$$

For $\theta \in \Theta_R$ profit are convex and increasing in quality. Therefore to maximize profit the monopoly platform set $\theta_M^* = \bar{\theta}$. ■

Given our result on quality, we obtain equilibrium values for subscription fees and viewers' demand:

$$s_M^* = \frac{4\overline{\beta}\bar{\theta} - (1-\delta)(1+3\delta)}{8} \tag{30}$$

$$B_M^* = \frac{\overline{\beta}\bar{\theta} + \left(\frac{1-\delta}{2}\right)^2}{2\bar{\theta}} \tag{31}$$

Equilibrium profits are:

$$\Pi_M^* = \frac{\left(\overline{\beta}\bar{\theta} + \left(\frac{1-\delta}{2}\right)^2\right)^2}{4\bar{\theta}} - K \tag{32}$$

Notice that the equilibrium values depend on $\overline{\beta}$, on the technological constraint, namely the upper bound $\bar{\theta}$, and the disutility of advertising δ.

4 Duopoly

Moving to the duopoly case, we consider two platforms, namely $i = 1, 2$. Without loss of generality we assume that $i = 1$ is the low quality platform, while $i = 2$ is the high quality one. Thus we set $i = L, H$. [6] For the remaining we maintain the same assumptions as in the general set up (Section 2).

In this framework, we consider a market structure where both firms are active (meaning that the viewers' demands for platform 1 and 2 are positive) and we look for an equilibrium in the covered market. First, we first rule out the trivial case in which the low-quality platform always faces zero demand in the price game. As standard in the vertical differentiation literature (see Tirole 1988), the individuals' heterogeneity has to be sufficiently high:

$$\overline{\beta} > 2\underline{\beta} \tag{33}$$

with $\beta \in [\underline{\beta}, \overline{\beta}]$.

Second, for the market be covered, we introduce the following condition: [7]

$$\underline{\beta}\theta_L \geq \frac{\left(\overline{\beta} - 2\underline{\beta}\right)(\theta_H - \theta_L)}{3} - \left(\rho\left(a_L^*\right) - \delta a_L^*\right) \tag{34}$$

which states that in equilibrium also the consumer with the lowest taste for quality, gets some positive utility joining the low-quality platform. If we choose Θ_R such that condition (34) holds, we obtain market coverage for every quality belonging to the technological range.[8]

Therefore, we define the demand function for the high-quality NB_H and for the low-quality NB_L, respectively:

$$\begin{aligned} NB_H &= N\left(\frac{\overline{\beta} - \beta_{LH}}{\overline{\beta} - \underline{\beta}}\right) \\ &= N\left(\frac{\overline{\beta}}{\overline{\beta} - \underline{\beta}} - \frac{\delta(a_H - a_L)}{(\theta_H - \theta_L)\left(\overline{\beta} - \underline{\beta}\right)} - \frac{s_H - s_L}{(\theta_H - \theta_L)\left(\overline{\beta} - \underline{\beta}\right)}\right) \end{aligned} \tag{35}$$

[6] We relax this ex-ante assumption when we look at the choice of quality (stage 1)

[7] This condition is obtained with equilibrium results of stage 3 and 2, as it will be clearer later on. Notice that, compared to the condition ensuring market coverage in a single side framework (see Tirole 1988 p.296) there is an additional part related to the presence of externalities.

[8] In a different paper, Battaggion and Drufuca (2014), we provide comparative statics for an appropriate set of parameter values allowing us to deal also with uncovered market.

$$NB_L = N\left(\frac{\beta_{LH}-\underline{\beta}}{\overline{\beta}-\underline{\beta}}\right)$$
$$= N\left(\frac{\delta(a_H-a_L)}{(\theta_H-\theta_L)\left(\overline{\beta}-\underline{\beta}\right)}+\frac{s_H-s_L}{(\theta_H-\theta_L)\left(\overline{\beta}-\underline{\beta}\right)}-\frac{\underline{\beta}}{\overline{\beta}-\underline{\beta}}\right) \tag{36}$$

The amount of advertising for each platform becomes:

$$a_L = 1-F\left(\frac{r_L}{NB_L}\right) \tag{37}$$

$$a_H = 1-F\left(\frac{r_H}{NB_H}\right) \tag{38}$$

Profit function (6) rewrites as follow, respectively for the high-quality platform and for the low-one:

$$\Pi_H\left(s_H,s_L,a_H,a_L,r_H,r_L,\theta_H,\theta_L\right) = NB_H s_H + a_H r_H - K \tag{39}$$

$$\Pi_L\left(s_H,s_L,a_H,a_L,r_H,r_L,\theta_H,\theta_L\right) = NB_L s_L + a_L r_L - K \tag{40}$$

Analogously to the monopoly case, we solve the game backwards. Thus we omit technical details for stage 3 and 2.

Let just point out that we obtain the same result on advertising as in the monopoly solution:

Proposition 4 *For each platform i, if the profit maximizing advertising level is positive, then it is constant and it is determined by*

$$\rho'(a_i) = \delta$$

Proof. platforms maximize profits, (40) and (39), subject to $a_i \geq 0$ with $i = H, L$. The first order conditions with respect to the advertising spaces a_i and subscription fees s_i with $i = H, L$ are:

$$Ns_i\frac{\partial B_i}{\partial a_i} + r_i + a_i\frac{\partial r_i}{\partial a_i} \leq 0 \tag{41}$$

$$NB_i + Ns_i\frac{\partial B_i}{\partial s_i} + a_i\frac{\partial r_i}{\partial s_i} = 0 \tag{42}$$

Given (14) for platform H we have, $r_H = \dfrac{NB_H\rho(a_H)}{a_H}$ and:

$$\frac{\partial r_H}{\partial s_H} = \frac{1}{a_H}N\rho(a_H)\frac{\partial B_H}{\partial s_H}$$
$$\frac{\partial r_H}{\partial a_H} = \frac{[NB_H\rho' + N\rho(a_H)\frac{\partial B_H}{\partial a_H}]a_H - NB_H\rho(a_H)}{a_H^2}$$

Therefore optimality condition (42) and (41) rewrite:

$$B_H + (s_H + \rho(a_H))\frac{\partial B_H}{\partial s_H} = 0 \tag{43}$$

$$B_H \rho'(a_H) + (\rho(a_H) + s_H)\frac{\partial B_H}{\partial a_H}) \leq 0 \tag{44}$$

Since:

$$\frac{\partial B_H}{\partial s_H} = \delta \frac{\partial B_H}{\partial s_H}$$

(44) becomes:

$$\frac{\rho'(a_H)}{\delta} B_H + (\rho(a_H) + s_H)\frac{\partial B_H}{\partial s_H}) \leq 0 \tag{45}$$

Together with (43), we obtain the following conditions:

$$\begin{cases} -B_H = (s_H + \rho(a_H))\frac{\partial B_H}{\partial s_H} \\ (\frac{\rho'(a_H)}{\delta} - 1)B_H \leq 0 \end{cases} \tag{46}$$

If $a_H > 0$ the above inequality is satisfied by equality. Therefore, given that $\rho(a_H)$ is single-peaked, a_H is uniquely determined by the following condition:

$$\rho'(a_H^*) = \delta$$

Analogously for platform L, if $a_L > 0$ we get:

$$\rho'(a_L^*) = \delta$$

■

The above Proposition 4 states that, for both platforms, a fixed advertising space is the best reply. In particular, the equilibrium level of advertising depends on the advertising disutility of the viewers, suggesting that both platform just compete on viewers. In this respect, platforms act as "bottlenecks" between advertisers and consumers, by offering sole access to their respective set of consumers.

Notice that our result replicates the outcome of Armstrong and Weeds (2007) in a context of vertical differentiation but with quadratic costs. [9] We share the same insight that what really matters for competition, in two-sided markets, is the single-homing part.

Moreover, we point out that:

[9] Peitz and Valletti (2008) also find a similar result in a context of horizontal differentiation.

Remark 5 *The strategic advertising choice is the same, regardless the market structure:*

$$\rho'(a_i^*) = \delta \quad \text{for } i = H, L, M$$

However, in the duopoly structure, the total amount of advertising doubles the monopoly level. In particular in the uniform case,

$$a_L^* + a_H^* = 1 - \delta = 2a_M^*$$

The above Remark enhances that individual platform's strategic advertising choice is neutral with respect to competitive market structure.[10]

We can now compare the subscription fees and the advertising prices of the two platforms.

Proposition 6 *Platform H set a higher subscription fee and a lower advertising price, with respect to platform L: $s_H^*(\theta_H, \theta_L) > s_L^*(\theta_H, \theta_L)$ and $r_L^*(a, \rho) > r_H^*(a, \rho)$. They also share the market in a fixed proportion: $B_H^* > B_L^*$.*

Proof. In the second stage of the game, with $\rho(a_i)$ concave, we obtain the equilibrium prices s_H^*, s_L^* and r_H^*, r_L^* as function of qualities, revenues per viewer and advertising. From condition (43) for platform H and the analogous condition for platform L, we get:

$$\begin{cases} s_H = \dfrac{s_L + \overline{\beta}(\theta_H - \theta_L) - \delta(a_H - a_L) - \rho(a_H)}{2} \\ s_L = \dfrac{s_H - \underline{\beta}(\theta_H - \theta_L) + \delta(a_H - a_L) - \rho(a_L)}{2} \end{cases} \tag{47}$$

Then, the solution of the above system becomes:

$$\begin{gathered} s_H^*(\theta_H, \theta_L, \rho(a_H), \rho(a_L)) = \\ \tfrac{2}{3}\overline{\beta}(\theta_H - \theta_L) - \tfrac{1}{3}\underline{\beta}(\theta_H - \theta_L) - \tfrac{1}{3}\delta(a_H - a_L) - \tfrac{2}{3}\rho(a_H) - \tfrac{1}{3}\rho(a_L) \end{gathered} \tag{48}$$

$$\begin{gathered} s_L^*(\theta_H, \theta_L, \rho(a_H), \rho_L) = \\ \tfrac{1}{3}\overline{\beta}(\theta_H - \theta_L) - \tfrac{2}{3}\underline{\beta}(\theta_H - \theta_L) + \tfrac{1}{3}\delta(a_H - a_L) - \tfrac{1}{3}\rho(a_H) - \tfrac{2}{3}\rho(a_L) \end{gathered} \tag{49}$$

If we plug s_H^* and s_L^* in the demand function obtained at stage three, (35) and (36), we get:

$$\begin{gathered} B_H^*(\theta_H, \theta_L, \rho(a_H), \rho(a_L)) = \\ \frac{\overline{\beta}}{\overline{\beta} - \underline{\beta}} - \frac{\frac{(\underline{\beta} + \overline{\beta})(\theta_H - \theta_L) - 2\delta(a_H - a_L) - \rho(a_H) + \rho(a_L)}{3}}{(\overline{\beta} - \underline{\beta})(\theta_H - \theta_L)^2} \end{gathered} \tag{50}$$

[10] This intuition is in line with Ambrus *et al.* (2013). They show that platform ownership does not affect advertising levels, despite non trivial strategic interactions between platforms.

$$B_L^*\left(\theta_H,\theta_L,\rho(a_H),\rho\left(a_L\right)\right)= \\ \frac{\frac{\left(\underline{\beta}+\overline{\beta}\right)(\theta_H-\theta_L)-2\delta(a_H-a_L)-\rho(a_H)+\rho(a_L)}{3}}{\left(\overline{\beta}-\underline{\beta}\right)\left(\theta_H-\theta_L\right)^2}-\frac{\underline{\beta}}{\overline{\beta}-\underline{\beta}} \tag{51}$$

Finally, considering

$$r_L\left(\left(s_H,s_L,a_H,a_L,\theta_H,\theta_L\right)\right)= \\ F^{-1}(1-a_L)NB_L \tag{52}$$

$$r_H\left(s_H,s_L,a_H,a_L,\theta_H,\theta_L\right)= \\ F^{-1}(1-a_H)NB_H \tag{53}$$

we end with:

$$r_H^*\left(\theta_H,\theta_L,\rho(a_H),\rho\left(a_L\right)\right)= \\ \frac{\rho\left(a_H\right)}{a_H}\frac{1}{N}\left(\frac{\left(\overline{\beta}-\underline{\beta}\right)(\theta_H-\theta_L)^2}{3\overline{\beta}(\theta_H-\theta_L)-\left(\underline{\beta}+\overline{\beta}\right)(\theta_H-\theta_L)+2\delta(a_H-a_L)-\rho(a_H)+\rho(a_L)}\right) \tag{54}$$

$$r_L^*\left(\theta_H,\theta_L,\rho(a_H),\rho\left(a_L\right)\right)= \\ \frac{\rho\left(a_L\right)}{a_L}\frac{1}{N}\left(\frac{3\left(\overline{\beta}-\underline{\beta}\right)(\theta_H-\theta_L)^2}{\left(\underline{\beta}+\overline{\beta}\right)(\theta_H-\theta_L)-2\delta(a_H-a_L)-\rho(a_H)+\rho(a_L)-3\underline{\beta}(\theta_H-\theta_L)}\right) \tag{55}$$

If $a_L=a_H=a^*$ then $\rho(a_H)=\rho(a_L)=\rho(a^*)$, it will be straightforward to see:

$$s_H^*\left(\theta_H,\theta_L,a^*\right)=\frac{\left(2\overline{\beta}-\underline{\beta}\right)\left(\theta_H-\theta_L\right)}{3}-\rho(a^*)> \\ \frac{\left(\overline{\beta}-2\underline{\beta}\right)\left(\theta_H-\theta_L\right)}{3}-\rho(a^*)=s_L^*\left(\theta_H,\theta_L,a^*\right)$$

and

$$r_L^*\left(a,\rho\right)=\frac{1}{N}\frac{\rho(a^*)}{a^*}\frac{3\left(\overline{\beta}-\underline{\beta}\right)\left(\theta_H-\theta_L\right)}{\overline{\beta}-2\underline{\beta}}> \\ \frac{1}{N}\frac{\rho(a^*)}{a^*}\frac{3\left(\overline{\beta}-\underline{\beta}\right)\left(\theta_H-\theta_L\right)}{2\overline{\beta}-\underline{\beta}}=r_H^*\left(a,\rho\right)$$

Finally,

$$B_H^*=\frac{2\overline{\beta}-\underline{\beta}}{3\left(\overline{\beta}-\underline{\beta}\right)}>\frac{\overline{\beta}-2\underline{\beta}}{3\left(\overline{\beta}-\underline{\beta}\right)}=B_L^*$$

■

Looking at equilibrium subscription fees and market shares, B_H^* and B_L^* , it is straightforward to see a "profit neutrality" result: advertising

does not directly affect the market shares and therefore the equilibrium profits, but it just have an impact on the subscription fees.

$$s_H^*\left(\theta_H,\theta_L,a^*,\delta\right)=\frac{\left(2\overline{\beta}-\underline{\beta}\right)\left(\theta_H-\theta_L\right)}{3}-\rho(a^*) \tag{56}$$

$$s_L^*\left(\theta_H,\theta_L,a^*,\delta\right)=\frac{\left(\overline{\beta}-2\underline{\beta}\right)\left(\theta_H-\theta_L\right)}{3}-\rho(a^*) \tag{57}$$

In particular, comparing monopoly with duopoly, it is straightforward to see how the profit neutrality result is stronger under the latter. In fact, advertising revenues are entirely devoted to reduce subscription fees, while in the monopoly case we just have half displacement (see equation (21)).

We can now solve the initial stage of the game, namely the quality choice. To anticipate results, we get that profits increase in qualities differentiation as standard in vertical differentiation models with single-side. Given our assumption on costs, platforms have the incentive to maximal differentiate.

Proposition 7 *In equilibrium the high quality platform chooses a quality level, $\theta_H^*=\bar{\theta}$ and the low quality platform chooses the minimum quality level, $\theta_L^*=\underline{\theta}$.*

Proof. Rewriting profit function for H and L respectively, (39) and (40) we have:

$$\Pi_H^*\left(\theta_H,\theta_L\right)=s_H^*NB_H^*+\rho NB_H^*-K=N\frac{\left(\underline{\beta}-2\overline{\beta}\right)^2\left(\theta_H-\theta_L\right)}{9\left(\overline{\beta}-\underline{\beta}\right)}-K \tag{58}$$

$$\Pi_L^*\left(\theta_H,\theta_L\right)=s_L^*NB_L^*+\rho NB_L^*-K=N\frac{\left(\overline{\beta}-2\underline{\beta}\right)^2\left(\theta_H-\theta_L\right)}{9\left(\overline{\beta}-\underline{\beta}\right)}-K \tag{59}$$

Calculating the FOC, under the assumption of non-negativity constraint of quality we obtain:

$$\frac{\partial\Pi_H^*}{\partial\theta_H}=\frac{\left(\underline{\beta}-2\overline{\beta}\right)^2}{9(\overline{\beta}-\underline{\beta})}N>0$$

$$\frac{\partial\Pi_L^*}{\partial\theta_L}=-\frac{\left(\overline{\beta}-2\underline{\beta}\right)^2}{9(\overline{\beta}-\underline{\beta})}N<0$$

Hence

$$\theta_H^*=\overline{\theta},\qquad\theta_L^*=\underline{\theta}$$

■

To make our results comparable with the monopoly case, we also provide equilibrium results with uniform distribution in advertising.

Lemma 8 *In the special case where the p.d.f. of advertisers F is uniform on $[0,1]$ equilibrium values are:*

$$a_L^* = a_H^* = a^* = \frac{1-\delta}{2} \tag{60}$$

$$s_H^*(\theta_H, \theta_L, \delta) = \frac{\left(2\overline{\beta} - \underline{\beta}\right)\left(\overline{\theta} - \underline{\theta}\right)}{3} - \frac{1-\delta}{2}\left(\frac{1+\delta}{2}\right) \tag{61}$$

$$s_L^*(\theta_H, \theta_L, \delta) = \frac{\left(\overline{\beta} - 2\underline{\beta}\right)\left(\overline{\theta} - \underline{\theta}\right)}{3} - \frac{1-\delta}{2}\left(\frac{1+\delta}{2}\right) \tag{62}$$

Notice that, the advertising level is decreasing in the disutility parameter δ. Instead, both subscription fees s_L^* and s_H^* are increasing in δ. This result is in line with our findings about profit neutrality: a higher δ implies lower advertising revenues to be used in the reduction of fees. As expected, profits are neutral in δ, differently from the monopoly case (see equation (32))

Finally, equilibrium market shares and qualities are not affected by the assumption on F.

5 Competition

In this Section, we take into account the effects of competition on market structure and on platforms' qualities. We have already considered both monopoly and duopoly situation and their comparison. However, the above framework does not allow to deal with the potential competition and the issue of incumbency advantage. Therefore we analyze quality differentiation in a framework of sequential entry. We slightly modify our timing by considering an Incumbent platform and an Entrant platform. We split the quality choice stage: the Incumbent platform (I) sets quality first, followed by the Entrant platform (E). Technology structure and profit function are the same, but for the entry cost F, as it is standard in this literature. In this framework we focus on the existence conditions of a duopoly equilibrium and we check robustness by looking at the entry deterrence strategy by the Incumbent.

5.1 Sequential Duopoly

As already mentioned, in order to deal with a sequential equilibrium, we slightly modify the timing of the game. Nothing change for stages 3

and 2, while we separate the quality decision of the two platforms: the Incumbent platform sets quality first, followed by the Entrant platform.

After quality-choice the two platforms set simultaneously their prices for advertising and subscription fees, r_i and s_i, as in the previous setting. Hence, the equilibrium solutions for stages 3 and 2 still hold (see Proposition 6). Recall that equilibrium profits of the high-quality platform were higher with respect to the low-quality one. Therefore the Incumbent platform will exploit its advantage, behaving as the high quality one and just living room to entry at the low quality level. Equilibrium solutions of the simultaneous framework, with $E = L$ for the Entrant and $I = H$ for the Incumbent are as follows.

Equilibrium subscription fees:

$$\begin{aligned} s_I^* &= \frac{\left(2\overline{\beta} - \underline{\beta}\right)(\theta_I - \theta_E)}{3} - \rho(a^*) \\ s_E^* &= \frac{\left(\overline{\beta} - 2\underline{\beta}\right)(\theta_I - \theta_E)}{3} - \rho(a^*) \end{aligned} \tag{63}$$

Equilibrium demands:

$$\begin{aligned} NB_I^* &= N\frac{2\overline{\beta} - \underline{\beta}}{3\left(\overline{\beta} - \underline{\beta}\right)} \\ NB_E^* &= N\frac{\overline{\beta} - 2\underline{\beta}}{3\left(\overline{\beta} - \underline{\beta}\right)} \end{aligned} \tag{64}$$

Equilibrium advertising prices:

$$\begin{aligned} r_I^* &= \frac{1}{N}\frac{\rho(a^*)}{a^*}\frac{3\left(\overline{\beta} - \underline{\beta}\right)(\theta_I - \theta_E)}{2\overline{\beta} - \underline{\beta}} \\ r_E^* &= \frac{1}{N}\frac{\rho(a^*)}{a^*}\frac{3\left(\overline{\beta} - \underline{\beta}\right)(\theta_I - \theta_E)}{\overline{\beta} - 2\underline{\beta}} \end{aligned} \tag{65}$$

Equilibrium profits

$$\begin{aligned} \Pi_I^* &= N\frac{\left(2\overline{\beta} - \underline{\beta}\right)^2}{9\left(\overline{\beta} - \underline{\beta}\right)}(\theta_I - \theta_E) - K \\ \Pi_E^* &= N\frac{\left(\overline{\beta} - 2\underline{\beta}\right)^2}{9\left(\overline{\beta} - \underline{\beta}\right)}(\theta_I - \theta_E) - K - F \end{aligned} \tag{66}$$

Indeed, the Entrant platform fixes its quality in order to maximize profits given the quality choice of the Incumbent .

$$\Pi_E^* = N\frac{\left(\overline{\beta}-2\underline{\beta}\right)^2}{9\left(\overline{\beta}-\underline{\beta}\right)}\left(\theta_I-\theta_E\right)-K-F$$

$$\frac{\partial\Pi_E^*}{\partial\theta_E}|_{\theta_I} = -N\frac{\left(\overline{\beta}-2\underline{\beta}\right)^2}{9\left(\overline{\beta}-\underline{\beta}\right)} < 0$$

Given the negative sign of the derivative, platform a E has the incentive to choose the minimum quality $\underline{\theta}$.

The final stage involves the quality choice of the Incumbent platform:

$$\begin{aligned}\Pi_I^* &= N\frac{\left(2\overline{\beta}-\underline{\beta}\right)^2}{9\left(\overline{\beta}-\underline{\beta}\right)}\left(\theta_I-\theta_E^*\right)-K\\ &= N\frac{\left(2\overline{\beta}-\underline{\beta}\right)^2}{9\left(\overline{\beta}-\underline{\beta}\right)}(\theta_I-\underline{\theta})-K\end{aligned}$$

$$\frac{\partial\Pi_I^*}{\partial\theta_I} = N\frac{\left(2\overline{\beta}-\underline{\beta}\right)^2}{9\left(\overline{\beta}-\underline{\beta}\right)} > 0$$

Given the positive sign of the derivative, platform I has the incentive to choose the maximum quality. In equilibrium, profits of the sequential duopoly are:

$$\Pi_I^* = N\frac{\left(2\overline{\beta}-\underline{\beta}\right)^2}{9\left(\overline{\beta}-\underline{\beta}\right)}\left(\overline{\theta}-\underline{\theta}\right)-K \tag{67}$$

$$\Pi_E^* = N\frac{\left(\overline{\beta}-2\underline{\beta}\right)^2}{9\left(\overline{\beta}-\underline{\beta}\right)}\left(\overline{\theta}-\underline{\theta}\right)-K-F \tag{68}$$

As in the simultaneous case, we obtain a result of maximal differentiation. Revenues are not changed for both platforms, however I has the incumbency advantage to be first on the market,behaving as the high quality platform and saving entry costs.

5.2 Threat of Entry

In this Section we analyze the effect of potential competition, by means of potential entrance of new competitors. As above mentioned, a potential entrant can choose to enter either as a high-quality platform or as a low-quality one. However, since ex-post profits of platform I are higher with respect to platform E, the Incumbent would behave as the high quality one, just living room to entry at the low quality level. Given

this framework, we point out the impact of potential competition on platforms' qualities and on the vertical differentiation.

On the one hand, notice that with fixed cost of entry a potential entrant cannot profitably leapfrog the high-quality incumbent. In fact, the quality is already at maximum, therefore the only possibility is to charge lower prices with the same quality. However the cost of entry prevent this strategy to be profitable. On the other hand, the existence of positive profit for the low quality platform make convenient for a potential entrant to get in. In this case, by setting a slightly larger quality the entrant will capture all the low-quality demand. According to Shaked and Sutton (1982) in a traditional model of vertical differentiation, there are at most two firms having positive market share and covering the entire market with different qualities, for a convenient heterogeneity of the viewers.[11] We show that this condition applies to two-sided market context too. [12]

Lemma 9 *Let $2\underline{\beta} < \overline{\beta} < 4\underline{\beta}$. Then of any n platforms offering distinct qualities, exactly two will have positive market shares on the buyers' side (audience) at equilibrium. Moreover at equilibrium the market is covered.*

Proof. We have already stated that for $2\underline{\beta} < \overline{\beta}$ low-quality platform has a positive audience, see Section 4.

For $\overline{\beta} < 4\underline{\beta}$ we follow Shaked and Sutton (1982) with appropriate transformations to fit our two-sided structure.

From Section 4, we know that in equilibrium subscription fees are

$$s_H^* = \frac{\left(2\overline{\beta} - \underline{\beta}\right)(\theta_H - \theta_L)}{3} - \rho(a^*) \tag{69}$$

$$s_L^* = \frac{\left(\overline{\beta} - 2\underline{\beta}\right)(\theta_H - \theta_L)}{3} - \rho(a^*) \tag{70}$$

Looking at equilibrium subscription fees it is straightforward to see that the "profit neutrality" result still holds. Advertising revenues per viewers $\rho(a^*)$ are entirely spent in reducing subscription fees s_i^*. Due to this neutrality result, we can apply the following transformation to equilibrium demands in order to have a single price which is always positive:

[11] That is: $2a < b < 4a$, where a and b are the lower and the upper bounds of the distribution respectively (Shaked, Sutton (1982), p.5).

[12] We focus on the buyers' side which is the crucial one. In fact, according to the assumption of multi-homing advertisers the competition on this side does not affect the equilibrium values. Furthermore the optimality condition on advertising is irrespective of the number of platforms (see Remark 5).

$$p_i = s_i + \rho(a_i)$$

In this way we are able to obtain a framework similar to the one of Shaked and Sutton (1982). We consider a situation of n platforms ordered by their quality $\theta_1 < \theta_2 < ... < \theta_n$ competing for an uniform audience (same assumptions as in previous sections) covering the entire market[13].

Given the equilibrium of stage 2 ($a_1 = a_2 = ... = a_n = a^*$), indifferent viewers are defined as follows:

$$\begin{aligned}
\beta_2 &= \frac{p_2 - p_1}{(\theta_2 - \theta_1)} \\
\beta_3 &= \frac{p_3 - p_2}{(\theta_3 - \theta_2)} \\
&\vdots \\
\beta_n &= \frac{p_n - p_{n-1}}{(\theta_n - \theta_{n-1})}
\end{aligned}$$

Demands become:

$$\begin{aligned}
NB_1 &= N\left(\frac{p_2 - p_1}{(\theta_2 - \theta_1)} - \underline{\beta}\right) \\
NB_2 &= N\left(\frac{p_3 - p_2}{(\theta_3 - \theta_2)} - \frac{p_2 - p_1}{(\theta_2 - \theta_1)}\right) \\
&\vdots \\
NB_n &= N\left(\overline{\beta} - \frac{p_n - p_{n-1}}{(\theta_n - \theta_{n-1})}\right)
\end{aligned}$$

Platforms' Revenues are:

$$\begin{aligned}
R_1 &= p_1 NB_1 = p_1 N(\beta_2 - \underline{\beta}) \\
R_2 &= p_2 NB_2 = p_2 N(\beta_3 - \beta_2) \\
&\vdots \\
R_n &= p_n NB_n = p_n N(\overline{\beta} - \beta_n)
\end{aligned}$$

[13] As in Shaked and Sutton (1982), the assumption of market coverage does not change the result of the proof. However, for the sake of simplicity, we assume it throughout the proof.

Profit maximization w.r.t. quality gives the following optimality conditions:

$$
\begin{aligned}
(\beta_2 - \underline{\beta}) + p_1(\frac{-1}{(\theta_2 - \theta_1)}) &= 0 \\
(\beta_3 - \beta_2) + p_2(-\frac{1}{(\theta_3 - \theta_2)} - \frac{1}{(\theta_2 - \theta_1)}) &= 0 \\
&\vdots \\
\left(\overline{\beta} - \beta_n\right) + p_n\left(-\frac{1}{(\theta_n - \theta_{n-1})}\right) &= 0
\end{aligned}
$$

Recall from indifference conditions:

$$\beta_{n-1} = \frac{p_{n-1} - p_{n-2}}{(\theta_{n-1} - \theta_{n-1})} = p_{n-1}\frac{1}{(\theta_{n-1} - \theta_{n-1})} - p_{n-2}\frac{1}{(\theta_{n-1} - \theta_{n-1})}$$

which can be written as:

$$p_{n-1}\frac{1}{(\theta_{n-1} - \theta_{n-1})} = \beta_{n-1} + p_{n-2}\frac{1}{(\theta_{n-1} - \theta_{n-1})}$$

Hence we re-write optimality condition for $(n-1)$th platform :

$$
\begin{aligned}
(\beta_n - \beta_{n-1}) - p_{n-1}\frac{1}{(\theta_n - \theta_{n-1})} - \beta_{n-1} - p_{n-2}\frac{1}{(\theta_{n-1} - \theta_{n-1})} &= 0 \\
\beta_n - 2\beta_{n-1} - p_{n-1}\frac{1}{(\theta_n - \theta_{n-2})} - p_1\frac{1}{(\theta_{n-1} - \theta_{n-2})} &= 0
\end{aligned}
$$

This condition implies that

$$\beta_n > 2\beta_{n-1} \tag{71}$$

We do the same for the optimality condition of nth platform , obtaining:

$$\overline{\beta} - 2\beta_n - p_{n-1}\frac{1}{(\theta_n - \theta_{n-1})} = 0$$

which implies

$$\overline{\beta} > 2\beta_n \tag{72}$$

Taking conditions (71) and (72) together we get

$$\overline{\beta} \geq 2\beta_n > 4\beta_{n-1}$$

which implies

$$\overline{\beta} > 4\beta_{n-1}$$

Having assumed $\overline{\beta} < 4\underline{\beta}$ we end up with:

$$4\beta_{n-1} < \overline{\beta} < 4\underline{\beta}$$

This inequality implies:

$$\beta_{n-1} < \underline{\beta} \tag{73}$$

This inequality (73) implies that market is completely covered by $(n-1)$th and nth platforms, namely those with higher quality. This means that all other platforms face a zero market share on viewers' side.

Notice that we can also show that in a triopoly case, given $\overline{\beta} < 4\underline{\beta}$, only the two platform with highest qualities survive and cover the market.

We consider the same framework as in the duopoly case but with three platform ranked by quality $\theta_1 < \theta_2 < \theta_3$. Under market coverage, indifferent consumers are identified by:

$$\beta_{12} = \frac{\delta\left(a_2 - a_1\right) + \left(s_2 - s_1\right)}{\theta_2 - \theta_1}$$

$$\beta_{23} = \frac{\delta\left(a_3 - a_2\right) + \left(s_3 - s_2\right)}{\theta_3 - \theta_2}$$

Demands from the consumers' side are respectively:

$$NB_1 = N\frac{1}{\overline{\beta} - \underline{\beta}}\left(\beta_{12} - \underline{\beta}\right) =$$

$$N\frac{1}{\overline{\beta} - \underline{\beta}}\left(\frac{\delta\left(a_2 - a_1\right) + \left(s_2 - s_1\right) - \underline{\beta}\left(\theta_2 - \theta_1\right)}{\theta_2 - \theta_1}\right)$$

$$NB_2 = N\frac{1}{\overline{\beta} - \underline{\beta}}\left(\beta_{23} - \beta_{12}\right) =$$

$$N\frac{1}{\overline{\beta} - \underline{\beta}}\left(\frac{\delta\left(a_3 - a_2\right) + \left(s_3 - s_2\right)}{\theta_3 - \theta_2} - \frac{\delta\left(a_2 - a_1\right) + \left(s_2 - s_1\right)}{\theta_2 - \theta_1}\right)$$

$$NB_3 = N\frac{1}{\overline{\beta} - \underline{\beta}}\left(\overline{\beta} - \beta_{23}\right) =$$

$$N\frac{1}{\overline{\beta} - \underline{\beta}}\left(\frac{\overline{\beta}\left(\theta_3 - \theta_2\right) - \delta\left(a_3 - a_2\right) - \left(s_3 - s_2\right)}{\theta_3 - \theta_2}\right)$$

Resolution for stages 3 and 2 is standard. From optimality conditions we obtain:

$$s_i = \frac{-B_i}{\partial B_i / \partial s_i} - \rho$$

where $\rho = \rho(a_i)$. Since in equilibrium $a_1 = a_2 = a_3 = a^*$ and $\rho = \rho(a^*) = \rho^*$, we get the following system:

$$\begin{aligned}
s_1 &= \frac{s_2 - \underline{\beta}\,(\theta_2 - \theta_1)}{2} - \frac{\rho^*}{2} \\
s_2 &= \frac{s_3\,(\theta_2 - \theta_1) + s_1\,(\theta_3 - \theta_2)}{2\,(\theta_3 - \theta_1)} - \frac{\rho^*}{2} \\
s_3 &= \frac{s_2 + \overline{\beta}\,(\theta_3 - \theta_2)}{2} - \frac{\rho^*}{2}
\end{aligned}$$

Equilibrium access prices are:

$$\begin{aligned}
s_1^* &= \frac{1}{6\,(\theta_3 - \theta_1)} \left(\left(\overline{\beta} - \underline{\beta}\right)(\theta_2 - \theta_1)(\theta_3 - \theta_2) - 3\underline{\beta}\,(\theta_3 - \theta_1)(\theta_2 - \theta_1) \right) - \rho^* \\
s_2^* &= \frac{1}{3\,(\theta_3 - \theta_1)} \left(\overline{\beta} - \underline{\beta}\right)(\theta_3 - \theta_2)(\theta_2 - \theta_1) - \rho^* \\
s_3^* &= \frac{1}{6\,(\theta_3 - \theta_1)} \left(\left(\overline{\beta} - \underline{\beta}\right)(\theta_3 - \theta_2)(\theta_2 - \theta_1) + 3\overline{\beta}\,(\theta_3 - \theta_2)(\theta_3 - \theta_1) \right) - \rho^*
\end{aligned}$$

Given s_1^* , s_2^* and s_3^*, we check whether or not $\beta_{12} > \underline{\beta}$ under the condition of $4\underline{\beta} > \overline{\beta}$. If this is the case, platform 1 faces zero demand and platforms 2 and 3 cover the whole consumer market, confirming the result of Shaked and Sutton (1982).

$$\begin{aligned}
\beta_{12} &= \frac{(s_2^* - s_1^*)}{\theta_2 - \theta_1} \\
&= \frac{1}{6\,(\theta_3 - \theta_1)} \left(\left(\overline{\beta} - \underline{\beta}\right)(\theta_3 - \theta_2) + 3\underline{\beta}\,(\theta_3 - \theta_1) \right)
\end{aligned}$$

$$\beta_{12} - \underline{\beta} = \frac{1}{6\,(\theta_3 - \theta_1)} \left(\left(\overline{\beta} - \underline{\beta}\right)(\theta_3 - \theta_2) - 3\underline{\beta}\,(\theta_3 - \theta_1) \right)$$

Which is negative since $\left(\overline{\beta} - \underline{\beta}\right) < 3\underline{\beta}$ and $(\theta_3 - \theta_2) < (\theta_3 - \theta_1)$.

Hence $\beta_{12} < \underline{\beta}$: the two platforms with highest qualities cover the market, leaving no room for the low-quality platform. ■

Therefore assuming $2\underline{\beta} < \overline{\beta} < 4\underline{\beta}$ we know that in equilibrium the market is covered by the two highest quality platforms. Hence, we can state that a survival strategy for the low quality platform would be to drive profit to zero. In this way no other platform has the incentive to get in. Given that, we have to check how quality levels of the Incumbent (high quality) and the Entrant (low quality) might be affected.

Proposition 10 *Under the threat of entry the equilibrium quality of the Incumbent platform θ_I^* lies in the interval* $[\max(\tilde{\theta}_I, \tilde{\tilde{\theta}}_I), \bar{\theta}]$ *while the product quality choice of firm E is such that* $\theta_E^* = \theta_I^* - (K+F)\frac{9\left(\overline{\beta}-\underline{\beta}\right)}{N\left(\overline{\beta}-2\underline{\beta}\right)^2}$.

Proof. Let start with platform E. Platform E should drive its profit to zero, in order to prevent the entrance of a new platform:

$$\Pi_E^* = N\frac{\left(\overline{\beta}-2\underline{\beta}\right)^2}{9\left(\overline{\beta}-\underline{\beta}\right)}\left(\theta_I-\theta_E\right)-K-F=0 \tag{74}$$

then

$$\theta_E^* = \theta_I - (K+F)\frac{9\left(\overline{\beta}-\underline{\beta}\right)}{N\left(\overline{\beta}-2\underline{\beta}\right)^2} \tag{75}$$

Given the choice of platform E the profit of the Incumbent becomes:

$$\Pi_E^*\left(\theta_E^*\right)=0$$
$$\Pi_I^*\left(\theta_E^*\right)=\frac{3\overline{\beta}^2-3\underline{\beta}^2}{\left(\overline{\beta}-2\underline{\beta}\right)^2}K+\frac{\left(2\overline{\beta}-\underline{\beta}\right)^2}{\left(\overline{\beta}-2\underline{\beta}\right)^2}F$$

Incumbent profits are constant (independent of quality) and positive. However, we should assess a range of quality for the platform I compatible with the duopoly equilibrium, such that a second platform can just survive as a low quality. We calculate two threshold values for the Incumbent, $\widetilde{\theta}_I$ and $\tilde{\tilde{\theta}}_I$, such that the profits of the Entrant are driven to zero if it enters with the lowest quality $\underline{\theta}$ or with the highest quality $\bar{\theta}$ respectively:

$$\Pi_E^*(\widetilde{\theta}_I, \underline{\theta})=0$$
$$\widetilde{\theta}_I = \underline{\theta} + (F+K)\frac{9\left(\overline{\beta}-\underline{\beta}\right)}{N\left(\overline{\beta}-2\underline{\beta}\right)^2} \tag{76}$$

and

$$\Pi_E^*(\tilde{\tilde{\theta}}_I, \bar{\theta})=0$$
$$\tilde{\tilde{\theta}}_I = \bar{\theta} - (K-F)\frac{9\left(\overline{\beta}-\underline{\beta}\right)}{N\left(2\overline{\beta}-\underline{\beta}\right)^2} \tag{77}$$

Indeed, if $\theta_I > \widetilde{\theta}_I$ then it is possible for platform E to enter at the low level with quality θ_E^*. If, also, $\theta_I > \tilde{\tilde{\theta}}_I$ then platform E cannot leapfrog the high quality. Hence under the threat of entry a duopoly equilibrium exists for $\theta_I^* \in [\max(\tilde{\theta}_I, \tilde{\tilde{\theta}}_I), \bar{\theta}]$ and $\theta_E^* = \theta_I^* - (K+F)\frac{9\left(\overline{\beta}-\underline{\beta}\right)}{N\left(\overline{\beta}-2\underline{\beta}\right)^2}$. ■

Remark 11 *In equilibrium, under the threat of entry the quality differentiation may decrease:* $(\theta_I^* - \theta_E^*) \leq \left(\bar{\theta} - \underline{\theta}\right)$.

This statement follows the previous Proposition 10, by noting first that the Incumbent platform does not necessary reach the maximum quality. While the Entrant platform sets a quality above the minimum unless the entry cost F and K are sufficiently high. Notice that if we assume $K = 0$ and we consider the minimum $\underline{\theta} = \frac{(1-\delta)^2}{4\overline{\beta}}$ as in the monopoly case, then if $(\theta_I^* - \theta_E^*) < \left(\bar{\theta} - \underline{\theta}\right)$ certainly holds if $\bar{\theta} > F\frac{9\left(\overline{\beta}-\underline{\beta}\right)}{N\left(\overline{\beta}-2\underline{\beta}\right)^2} + \dfrac{(1-\delta)^2}{4\overline{\beta}}$.

The threat of entry shakes the equilibrium configuration. The quality of platform I might decrease, while the quality of platform E might increase. Therefore quality differentiation may shrink. In this respect there is no evidence that increasing competition positively affect the high quality of the Incumbent. Conversely, potential competition, namely the threat of entry, can boosts the quality of the Entrant from a minimum level.[14]

6 Entry Deterrence

To check the robustness of the previous equilibria, we wonder if investment in quality might be a successful deterrence strategy. More precisely, we state under which conditions an incumbent prevents entry in the market. In this way we endogenize the monopoly structure in a two-sided framework with a quality choice. The difference in equilibrium qualities between the accommodation case (duopoly) and the deterrence case (threatened monopoly), measures the effects of the potential competition.

This analysis is performed introducing a new stage of the game, where the Entrant platform has to take the decision to enter the market or stay out, while the Incumbent platform is already in. In order to distinguish from the previous case, we slightly modify the notation such that the Incumbent is defined as Platform 1 and the Entrant as platform 2.

In this framework we check whether or not deterrence is a feasible strategy. We compute profit of platform 1 in case of deterrence. If platform 1 decides to preempt the entry of the potential entrant, it behaves as a threaten monopolist. In this case, all the assumptions of the monopoly - uniform distribution of advertisers between (0,1) and $\underline{\theta} = \dfrac{(1-\delta)^2}{4\overline{\beta}}$- hold. Defining threshold values $\widetilde{\theta}_1$ and $\widetilde{\widetilde{\theta}}_1$ (see equations

[14] Notice that our insights are in the same line of Hung and Schmidt (1988) results in a traditional one-side market.

(76) and (77)), as in Proposition 10, we prove the following statement.

Proposition 12 *Given $\widetilde{\theta}_1$ and $\widetilde{\widetilde{\theta}}_1$, if:*

- $\widetilde{\theta}_1 < \widetilde{\widetilde{\theta}}_1$ *monopoly platform cannot prevent entry for $\theta \in (\underline{\theta}, \bar{\theta})$, therefore deterrence is an unfeasible strategy (a)*
- $\widetilde{\theta}_1 > \widetilde{\widetilde{\theta}}_1$ *monopoly platform can prevent entry for $\theta_1^D = \widetilde{\theta}_1 - \varepsilon$, with ε enough close to zero, therefore deterrence is a feasible strategy (b)*

Proof. (a) According to Proposition 10, to prevent the entry of a high quality platform, the incumbent should set $\theta_1 > \widetilde{\widetilde{\theta}}_1$, while it prevents entry on low quality level if $\theta_1 < \widetilde{\theta}_1$. Therefore it is straightforward to see that if $\widetilde{\theta}_1 < \widetilde{\widetilde{\theta}}_1$ it does not exist any θ_1 such that entry is prevented at both high quality and low quality levels.

(b) According to Proposition 10, we know that for $\widetilde{\theta}_1 > \widetilde{\widetilde{\theta}}_1$ it exist a value of θ_1 such that the incumbent can prevent the entry on both high quality and low quality sides. In particular for $\forall\theta \in \left(\widetilde{\widetilde{\theta}}_1, \widetilde{\theta}_1\right)$ entry can be deterred. Recalling that for a quality $\theta \geq \underline{\theta} = \dfrac{(1-\delta)^2}{4\overline{\beta}}$ the monopoly profits are increasing in quality. Hence, the incumbent optimal deterrence strategy is to set $\theta_1^D = \widetilde{\theta}_1 - \varepsilon$ close enough to $\widetilde{\theta}_1$. ■

The above proposition states under which conditions platform1 is able to deter entry. In case (a) the only equilibrium strategy is accommodation, while in case (b), entry deterrence is feasible but it is not necessarily an equilibrium. To be an equilibrium, monopoly profit in the deterrence quality θ_1^D must be higher than duopoly's one (accommodation). Otherwise, platform 1 should accommodate even if $\widetilde{\theta}_1 > \widetilde{\widetilde{\theta}}_1$.

According to Proposition **??**, if $\widetilde{\theta}_1 > \widetilde{\widetilde{\theta}}_1$ platform 1 can prevent entry for $\theta_1^D = \widetilde{\theta}_1 - \varepsilon$, with ε enough close to zero. Now, we should check when the entry deterrence strategy is profitable with respect to the accommodation strategy. We calculate deterrence profit in θ_1^D :

$$\Pi_M(\theta_1^D) = \frac{\left(\overline{\beta}\theta_1^D + \left(\frac{1-\delta}{2}\right)^2\right)^2}{4\theta_1^D} - K \tag{78}$$

Considering $\theta_1^D = \widetilde{\theta}_1 - \varepsilon$ and taking the limit of (78), we obtain:

$$\lim_{\varepsilon\to 0}\Pi_M(\theta_1^D)=\frac{\left(\overline{\beta}\left(\frac{(1-\delta)^2}{4\overline{\beta}}+(F+K)\frac{9\left(\overline{\beta}-\underline{\beta}\right)}{N\left(\overline{\beta}-2\underline{\beta}\right)^2}\right)+\left(\frac{1-\delta}{2}\right)^2\right)^2}{4\left(\frac{(1-\delta)^2}{4\overline{\beta}}+(F+K)\frac{9\left(\overline{\beta}-\underline{\beta}\right)}{N\left(\overline{\beta}-2\underline{\beta}\right)^2}\right)}-K \tag{79}$$

We compare (79) with the duopoly profit (accommodation case) as previously calculated in Proposition 10:

$$\Pi_1=\frac{3\overline{\beta}^2-3\underline{\beta}^2}{\left(\overline{\beta}-2\underline{\beta}\right)^2}K+\frac{\left(2\overline{\beta}-\underline{\beta}\right)^2}{\left(\overline{\beta}-2\underline{\beta}\right)^2}F \tag{80}$$

Under the assumptions $K=0$, $N=1$ and $2\underline{\beta}<\overline{\beta}<4\underline{\beta}$ we compare (78) and (80). There exists a threshold value of the fixed cost of entry $\underline{F}\left(\delta,\overline{\beta},\underline{\beta}\right)$ which makes the accommodation and deterrence profits equal. According to these values, we define the condition under which deterrence is profitable with respect to accommodation. Indeed, for $F<\underline{F}\left(\delta,\overline{\beta},\underline{\beta}\right)$ accommodation profits are lower then the deterrence ones, making preemption a profitable strategy.

6.1 Entry Deterrence: a Numerical Simulation

Unfortunately we are not able to find an analytical solution for $\underline{F}\left(\delta,\overline{\beta},\underline{\beta}\right)$ and in turn the conditions such that $F<\underline{F}\left(\delta,\overline{\beta},\underline{\beta}\right)$ holds. However we can perform a numerical simulation to ascertain the existence of a set of parameter values such that entry deterrence strategy is profitable with respect to the accommodation.

Given that $\underline{F}$ depends upon $\delta,\overline{\beta},\underline{\beta}$ we start by restricting the set of values of $\overline{\beta},\underline{\beta}$ according to Lemma 9. Doing that we set the value of $\overline{\beta}$ as a function of different values of $\underline{\beta}$. Then, we compute deterrence profits, (79), and accommodation ones, (80), for every combination of $\underline{\beta}$ and $\overline{\beta}\left(\underline{\beta}\right)$.[15] This simulation has been repeated for three different values of $\delta\in[0,1)$, namely 0.01, 0.5 and 0.9. The numerical simulation does not make any remarkable difference according to the δ change, therefore we just show the results for $\delta=0.5$.

Table 1a: Monopoly ($\Pi_M(\theta_1^D)$) and Duopoly (Π_1) Profits for $\delta=0.5$

[15] We also checked the values of $\underline{\beta}$ as a function of different values of $\overline{\beta}$, (see Table 1b and 2b in Appendix 8) , without any relevant difference in the results. Thus, for the sake of exposition we focus on the $\overline{\beta}\left(\underline{\beta}\right)$ case.

$\underline{\beta}$	$\overline{\beta} = 2.00001\underline{\beta}$	$\overline{\beta} = 3\underline{\beta}$	$\overline{\beta} = 4\underline{\beta}$
0.00001	$\Pi_M(\theta_1^D) = \frac{\left(1.8\times10^{11}F+0.125\right)^2}{3.6\times10^{16}F+12500.}$ $\Pi_1 = 9.0001\times10^{10}F$	$\Pi_M(\theta_1^D) = \frac{(54.0F+0.125)^2}{7.2\times10^{6}F+8333.3}$ $\Pi_1 = 25.0F$	$\Pi_M(\theta_1^D) = \frac{(27.0F+0.1}{2.7\times10^{6}F+}$ $\Pi_1 = 12.25F$
0.5	$\Pi_M(\theta_1^D) = \frac{\left(1.8\times10^{11}F+0.125\right)^2}{7.2001\times10^{11}F+0.25000}$ $\Pi_1 = 9.0001\times10^{10}F$	$\Pi_M(\theta_1^D) = \frac{(54.0F+0.125)^2}{144.0F+0.16667}$ $\Pi_1 = 25.0F$	$\Pi_M(\theta_1^D) = \frac{(27.0F+0.12}{54.0F+0.1}$ $\Pi_1 = 12.25F$
1	$\Pi_M(\theta_1^D) = \frac{\left(1.8\times10^{11}F+0.125\right)^2}{3.6\times10^{11}F+0.12500}$ $\Pi_1 = 9.0001\times10^{10}F$	$\Pi_M(\theta_1^D) = \frac{(54F+0.125)^2}{72F+8.3333\times10^{-2}}$ $\Pi_1 = 25.0F$	$\Pi_M(\theta_1^D) = \frac{(27F+0.125}{27F+0.062}$ $\Pi_1 = 12.25F$
5	$\Pi_M(\theta_1^D) = \frac{\left(1.8\times10^{11}F+0.125\right)^2}{7.2001\times10^{10}F+2.5000\times10^{-2}}$ $\Pi_1 = 9.0001\times10^{10}F$	$\Pi_M(\theta_1^D) = \frac{(54F+0.125)^2}{\frac{72}{5}F+1.6667\times10^{-2}}$ $\Pi_1 = 25.0F$	$\Pi_M(\theta_1^D) = \frac{(27F+0.125}{\frac{27}{5}F+0.012}$ $\Pi_1 = 12.25F$
12	$\Pi_M(\theta_1^D) = \frac{\left(1.8\times10^{11}F+0.125\right)^2}{3.0\times10^{10}F+1.0417\times10^{-2}}$ $\Pi_1 = 9.0001\times10^{10}F$	$\Pi_M(\theta_1^D) = \frac{(54F+0.125)^2}{6F+6.9444\times10^{-3}}$ $\Pi_1 = 25.0F$	$\Pi_M(\theta_1^D) = \frac{(27F+0.1}{\frac{9}{4}F+5.2083}$ $\Pi_1 = 12.25F$

A closer look at the deterrence and accommodation profits (see Table 1a) helps grab more insights. The key point is that Π_1 decreases in the taste for quality $\overline{\beta}\left(\underline{\beta}\right)$, which appears to be a quite counterintuitive finding. This is the result of a pass-trough effect of a change in the profit of the entrant platform. Indeed, as already described in Proposition 10, in case of potential entry, the platform 2 has to drive profit to zero in order to not induce the entry of further competitors. Given that the entrant platform's profit would have been increasing in $\overline{\beta}$ (see equation 74), the quality of the platform 2 must to increase in order to reduce differentiation and meet again the zero-profit condition, as stated in equation (75). Hence, due to a lower differentiation, Π_1 decreases. Therefore, accommodation profits for the incumbent platform are decreasing in $\overline{\beta}$. In the case of a decrease in $\underline{\beta}\left(\overline{\beta}\right)$ analogous results hold (see Table 1b and Table 2b in Appendix 8). For what concerns the deterrence case, Table 1a shows a similar path in $\Pi_M(\theta_1^D)$. Hence, monopoly profits are decreasing in $\overline{\beta}\left(\underline{\beta}\right)$. To better figure out this result, we focus on the relationship between quality and $\overline{\beta}$ in the entry deterrence case. From Proposition (**??**) we have $\theta_1^D = \widetilde{\theta}_1 - \varepsilon$. By taking the limit of θ_1^D for ε close to zero, we get:

$$\theta_1^D = \underline{\theta} + (F+K)\frac{9\left(\overline{\beta}-\underline{\beta}\right)}{N\left(\overline{\beta}-2\underline{\beta}\right)^2} = \frac{(1-\delta)^2}{4\overline{\beta}} + (F+K)\frac{9\left(\overline{\beta}-\underline{\beta}\right)}{N\left(\overline{\beta}-2\underline{\beta}\right)^2} \tag{81}$$

We By simple calculation from equation (81) we have:

$$\frac{\partial \theta_1^D}{\partial \overline{\beta}} < 0$$

Given that monopoly profits are increasing in quality, it is straightforward to see that deterrence profits $\Pi_M(\theta_1^D)$ are decreasing in $\overline{\beta}$.The intuition is similar to the accommodation case. Given that the entrant's profits are increasing in $\overline{\beta}$, the incumbent platform need a lower quality to prevent entry. Therefore for the platform 1 the cost of deterrence is increasing in $\overline{\beta}$. The same happens in case of a decrease of $\underline{\beta}\left(\overline{\beta}\right)$ (see table 1b).

According to profit functions, we are able to calculate the value of $\underline{F}\left(\delta, \overline{\beta}, \underline{\beta}\right)$. As we said, we just report the result for $\delta = 0.5$ in Table 2a.

Table 2a: Values of $\underline{F}\ \left(\delta, \overline{\beta}(\underline{\beta})\right)$ for $\delta = 0.5$

$\underline{\beta}$	$\overline{\beta} = 2.00001\underline{\beta}$	$\overline{\beta} = 3\underline{\beta}$	$\overline{\beta} = 4\underline{\beta}$
0.00001	$\underline{F} = 1.3889 \times 10^{-17}$	$\underline{F} = 7.5 \times 10^{-8}$	$\underline{F} = 2.0408 \times 10^{-7}$
0.5	$\underline{F} = 1.1236 \times 10^{-12}$	$\underline{F} = 1.5153 \times 10^{-2}$	$\forall F > 0$
1	$\underline{F} = 9.375 \times 10^{-8}$	$\forall F > 0$	$\forall F > 0$
5	$\forall F > 0$	$\forall F > 0$	$\forall F > 0$
12	$\forall F > 0$	$\forall F > 0$	$\forall F > 0$

First, notice that for sufficiently high levels of $\overline{\beta}$ deterrence profits are always larger than the accommodation ones for every values of F. Therefore in these cases, without calculating the threshold value $\underline{F}$, we can state that deterrence strategy dominates accommodation. Second, for the remaining cases, the above table 2a shows the values of $\underline{F}$ such that deterrence and accommodation profits equal. Therefore, for $0 < F < \underline{F}$ deterrence is admissible and profitable, as shown in the graphical example below (Figure1). Otherwise accommodation strategy is preferable.

Given that deterrence $(\Pi_M(\theta_1^D))$ and accommodation (Π_1) profits move in the same direction according to changes in $\overline{\beta}$ or $\underline{\beta}$ (see Table 1a), we where expected an ambiguous effect on the threshold value $\underline{F}$. Coversely, has shown in Table 2a, it is immediate to see a clear path also for $\underline{F}$ which is increasing in $\overline{\beta}\left(\underline{\beta}\right)$. Apparently, when $\overline{\beta}$ goes up, the decrease of deterrence profits is softened by a reduction of deterrence costs trough the drop of accommodation profits. [16] This could be explained a sort of position advantage of the incumbent platform.

[16] Coversely in the case of an increase in $\underline{\beta}(\overline{\beta})$, we obserse a decrease of $\underline{F}$ due to a raise of the opportunity costs of deterrence.

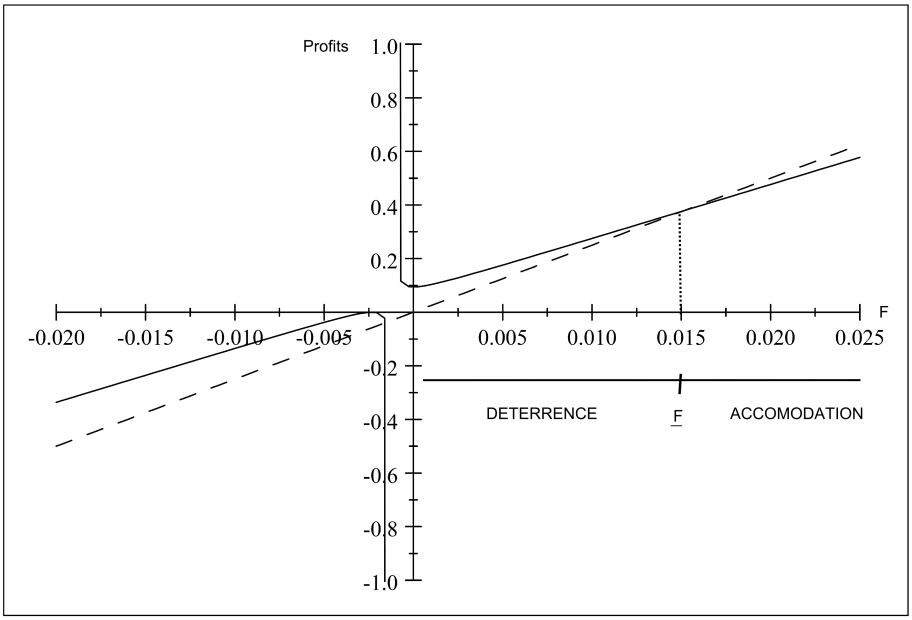

Figure 1: Figure 1: Deterrence (solid line) and accomodation (dashed line) profits.

7 Conclusion

This paper provides an analysis of vertical differentiation of two-sided platforms where competition prevails on one side of the market, namely on the consumers.

On the one hand, we illustrate a model with endogenous quality provision both in a monopoly as well as in a duopoly structure. For both market structures, we provide a full characterization of the equilibrium for what concerns advertising, subscription fees, market shares and qualities. For what concern the monopoly case, we focus on the increasing part of the profit function, by appropriate restrictions on the technological constraint of quality. This assumption is kept throughout the analysis. However, it could be worthy a further research on the decreasing part of monopoly profit. We also restrict the technological constraint in order to have market coverage at equilibrium in the duopoly case.

In the comparison between the two market structures, we point out three main results. First, for each platform, if the profit maximizing advertising level is positive, then it is constant and it is determined just by the disutility parameter δ. This means that the strategic advertising choice is the same, regardless the market structure. However, in the

duopoly structure, the total amount of advertising doubles the monopoly level. Second, in duopoly there is a full profit neutrality effect: there is a pass-through of advertising revenues into lower pay-per-view prices. This effect is reduced in the monopoly case. This result is strongly related to the issue of competitive bottleneck and prevailing competition on consumers' side. Finally, the monopoly platform chooses the maximum quality while duopoly platforms choose to maximally differentiate.

On the other hand, we have analyzed the role of competition by considering potential entry in the two-sided market and the associated behavior of an incumbent platform. We consider three different situations: a sequential duopoly, a sequential duopoly threaten by the possibility of entry by new competitors and a last case where the incumbent platform may decide to prevent the entry of a second one.

In the first case, as in the simultaneous duopoly, we obtain a result of maximal differentiation. Revenues are not changed for both platforms, however the Incumbent has the advantage to be on the market, saving entry costs. Dealing with second case, we extend the Shaked and Sutton (1982) result, we prove to a two-sided structure: under some conditions on individuals' heterogeneity, we show that of any n platforms offering distinct qualities, exactly two will have positive market shares on the buyers' side (audience) at equilibrium, covering the market. Therefore, the existence of positive profit for the low quality platform makes convenient for a potential entrant to get in. In fact, setting a slightly larger quality, any entrant will capture all the low-quality demand. Hence, a survival strategy for the low quality platform would be to drive profit to zero. According to this behavior, we show that the threat of entry shakes the equilibrium configuration of the sequential duopoly. The quality of Incumbent platform might decrease, while the quality of the entrant platform might increase. Therefore, quality differentiation may shrinks.

Finally, we provide the conditions such that an entry deterrence strategy is feasible and profitable for the Incumbent platform. We show that there exists a threshold value of the fixed cost of entry $\underline{F}(\delta, \overline{\beta}, \underline{\beta})$ which define when deterrence is profitable with respect to accommodation. Since we are not able to analytically characterized $\underline{F}(\delta, \overline{\beta}, \underline{\beta})$, we perform a numerical simulation. On the base of these results, we show that there is room for deterrence to be a profitable strategy.

References

[1] A.Ambrus, M. Reisinger (2005), "Platform competition and welfare: media markets reconsidered", mimeo.

[2] A.Ambrus, E. Calvano, M. Reisinger (2013),"Either or Both Com-

petition: A "Two-Sided" Theory of Advertising with Overlapping Viewers", mimeo

[3] S.Anderson, S.Coate, (2005), "Market Provision of Broadcasting: a Welfare Analysis", *Review of Economic Studies*, 72: 947-972.

[4] M. Armstrong, (2006), "Competition in two-sided markets", *Rand Journal of Economics*, 37, 3: 668-691.

[5] M. Armstrong, H. Weeds (2007) "Programme quality in subscription and advertising-funded television", mimeo.

[6] Areeda, Hovenkamp,(2014), "Antitrust Law: An Analysis of Antitrust Principles and their Application", Aspen Publishers, 4th ed.

[7] M.R.Battaggion, S.M.Drufuca, (2014), "Vertical Differentiation in Two-Sided Media Market: a Welfare Analysis", mimeo

[8] B.Caillaud, B.Jullien (2001), "Competing cybermediaries", *European Economic Review*, 45: 797-808

[9] B.Caillaud, B.Jullien (2003), "Chicken & Egg: Competition among Intermediation Service Providers", The *Rand Journal of Economics*, 34, 2: 309-328.

[10] J.P.Choi, (2006), "Broadcast competition and advertising with free entry: Subscription vs. free-to-air", *Information Economics and Policy*, 18, 2: 181-96.

[11] R. Collins, (2007). "The BBC and public value." *Medien und Kommunikationswissenschaft*, 65(2) pp. 164–184.

[12] C.Crampes, C.Harichtabalet, B.Jullien, (2009), "Advertising, Competition and Entry in Media Industries", *Journal of Industrial Economics*, 57, 1: 7-31.

[13] J. Gabszewicz, D. Laussel, N. Sonnac, (2004), "Programming and Advertising Competition in the Broadcasting Industry", *Journal of Economics & Management Strategy*, vol. 13(4): 657-669.

[14] H. Kind, T. Nilssen, L.Sørgard, (2009), "Business Models for Media Firms: Does Competition Matter for How They Raise Revenue", *Marketing Science*, 28, 6: 1112-1128.

[15] I. Kohlschein, (2004), Economic distortions caused by public funding of broad-casting in Europe, mimeo: 1-34.

[16] S Kremhelmer, H Zenger, (2008), "Advertising and the screening role of mass media", *Information Economics and Policy*, Vol.20, 2: 107–119.

[17] A. Mangani, (2003), "Profit and audience maximization in broadcasting markets", *Information Economics and Policy*, vol.15, 3: 305-315.

[18] M. Mussa, S. Rosen (1978), "Monopoly and product quality," *Journal of Economic Theory*, Elsevier, vol. 18(2), pages 301-317.

[19] M. Peitz, T. Valletti, (2008), "Content and advertising in the me-

dia: pay-tv versus free-to-air", *International Journal of Industrial Organization*, 4: 949-965.

[20] J.C. Rochet, J.Tirole, (2006), "Two-sided markets: a progress report", *Rand Journal of Economics*, 37, 3: 645-667.

[21] A.Shaked, J.Sutton, (1982), "Relaxing Price Competition Through Product Differentiation", *Review of Economic Studies*, XLIX: 3-13.

[22] C.Shapiro, H.Varian, (1998), *Information Rules: A strategic Guide to the Network Economy*,

[23] J.Tirole, (1988), *The Theory of Industrial Organization* , MIT Press, Cambridge (Mass.).

8 Appendix

Table 1b: Monopoly ($\Pi_M(\theta_1^D)$) and Duopoly (Π_1) Profits for $\delta = 0.5$

$\overline{\beta}$	$\underline{\beta}=\frac{1}{4}\overline{\beta}$	$\underline{\beta}=\frac{1}{3}\overline{\beta}$	$\underline{\beta}=\frac{1}{2.00001}\overline{\beta}$
0.00001	$\Pi_1 = 12.25F$ $\Pi_M(\theta_1^D) = \frac{(27.0F+0.125)^2}{1.08\times10^7F+25000.0}$	$\Pi_1 = 25.0F$ $\Pi_M(\theta_1^D) = \frac{(54.0F+0.125)^2}{2.16\times10^7F+25000.0}$	$\Pi_1 = 9.0001\times10^{10}F$ $\Pi_M(\theta_1^D) = \frac{(1.8\times10^{11}F+0.125}{7.2001\times10^{16}F+2500}$
0.5	$\Pi_1 = 12.25F$ $\Pi_M(\theta_1^D) = \frac{(27.0F+0.125)^2}{216.0F+0.5}$	$\Pi_1 = 25.0F$ $\Pi_M(\theta_1^D) = \frac{(54.0F+0.125)^2}{432.0F+0.5}$	$\Pi_1 = 9.0001\times10^{10}F$ $\Pi_M(\theta_1^D) = \frac{(1.8\times10^{11}F+0.125)}{1.44\times10^{12}F+0.5}$
1	$\Pi_1 = 12.25F$ $\Pi_M(\theta_1^D) = \frac{(27F+0.125)^2}{108F+0.25}$	$\Pi_1 = 25F$ $\Pi_M(\theta_1^D) = \frac{(54F+0.125)^2}{216F+0.25}$	$\Pi_1 = 9.0001\times10^{10}F$ $\Pi_M(\theta_1^D) = \frac{(1.8\times10^{11}F+0.125)}{7.2001\times10^{11}F+0.2}$
5	$\Pi_1 = 12.25F$ $\Pi_M(\theta_1^D) = \frac{(27.0F+0.125)^2}{21.6F+0.05}$	$\Pi_1 = 25F$ $\Pi_M(\theta_1^D) = \frac{(54.0F+0.125)^2}{43.2F+0.05}$	$\Pi_1 = 9.0001\times10^{10}F$ $\Pi_M(\theta_1^D) = \frac{(1.8\times10^{11}F+0.125)}{1.44\times10^{11}F+0.05}$
12	$\Pi_1 = 12.25F$ $\Pi_M(\theta_1^D) = \frac{(27.0F+0.125)^2}{9.0F+2.0833\times10^{-2}}$	$\Pi_1 = 25F$ $\Pi_M(\theta_1^D) = \frac{(54.0F+0.125)^2}{18.0F+2.0833\times10^{-2}}$	$\Pi_1 = 9.0001\times10^{10}$ $\Pi_M(\theta_1^D) = \frac{(1.8\times10^{11}F+0.}{6.0001\times10^{10}F+2.08}$

Table 2b:Values of $\underline{F}\left(\delta, \underline{\beta}(\overline{\beta})\right)$ for $\delta = 0.5$

$\overline{\beta}$	$\underline{\beta} = \frac{1}{4}\overline{\beta}$	$\underline{\beta} = \frac{1}{3}\overline{\beta}$	$\underline{\beta} = \frac{1}{2.00001}\overline{\beta}$
0.00001	$\underline{F} = 5.102\times10^{-8}$	$\underline{F} = 2.5\times10^{-8}$	$\underline{F} = 6.9444\times10^{-18}$
0.5	$\underline{F} = 3.0226\times10^{-3}$	$\underline{F} = 1.4726\times10^{-3}$	$\underline{F} = 4.0093\times10^{-13}$
1	$\underline{F} = 9.0989\times10^{-3}$	$\underline{F} = 4.3611\times10^{-3}$	$\underline{F} = 1.1236\times10^{-12}$
5	$\forall F > 0$	$\forall F > 0$	$\forall F > 0$
12	$\forall F > 0$	$\forall F > 0$	$\forall F > 0$