Monetary policies in self-confirming equilibria with uncertain models

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Introduction

- **Uncertainty**: DM faces *multiple probabilistic models* (e.g. urns) ⇒ some acts induce lotteries with uncertain probabilities.

- Imperfect observations of state realizations given by **information feedback**, which *depends on the chosen action*.

- **Partial identification**: many state distributions consistent with empirical frequencies of observed data, given own choice.

- **Selconfirming choice**: in recurrent decision problem, the stochastic steady state choice is optimal given subjective belief consistent with empirical frequencies of observations.
Relevance for policy making

A policy maker using a selfconfirming policy has learned the distribution of consequences of the chosen policy, not the data generating process (model economy).

For example, he may believe in steady state that there is a long-run trade-off between inflation and unemployment even if no such trade off exists.
Most related literature

- **SCE (conjectural eq.) idea**: No comprehensive survey. See surveys by Battigalli et al. (1992), Fudenberg & Levine (1998 book), and lit. review in Battigalli et al. (2011).

Recurrent decision problem with feedback given by observed consequences (all sets are nice Polish spaces):

- $a \in A$, actions, policies
- $s \in S$, states
- $c \in C$, consequences
- $f : A \times S \rightarrow C$ feedback/consequence function (measurable) [more could be observable, simplification]
- $\nu : C \rightarrow \mathbb{R}$ vNM utility function (bounded, measurable)
The state of nature is determined at random according to an unknown objective probability measure $\sigma \in \Delta(S)$

- $\sigma \in \Sigma \subseteq \Delta(S)$ possible stochastic models (closed w.r.t. weak* topology)
- In (some) applications, the stochastic model $\sigma \in \Sigma$ is parametrized by parameter vector $\theta \in \Theta$

$$\theta \longmapsto \sigma_\theta$$

$$\Sigma = \{\sigma_\theta : \theta \in \Theta\}$$
Information feedback: partition of states

Each action/policy $a$ yields a partition of $S$:

- $f_a : S \rightarrow C$ section of $f$ at $a$:
  
  $$ s \mapsto f_a(a) = f(a, s) $$

- When $a$ is chosen and $c$ observed ex post, the set of observationally equivalent states is
  
  $$ f_a^{-1}(c) = \{ s' \in S : f_a(s') = c \} $$

- Ex post information partition implied by $a$:
  
  $$ \mathcal{F}_a = \{ f_a^{-1}(c) : c \in C \} \subseteq 2^S $$

  $$ s \mapsto f_a^{-1}(f_a(s)) \in \mathcal{F}_a $$
Information feedback: partition of models

- In steady state some $a$ is fixed, then – given true stochastic model $\sigma$ – DM observes long-run distribution of consequences:

\[
\hat{f}_a(\sigma) = \sigma \circ f_a^{-1} \in \Delta(C)
\]
\[
\forall E \subseteq C, \hat{f}_a(\sigma)(E) = \sigma\left(f_a^{-1}(E)\right)
\]

- If long-run distribution $\gamma \in \Delta(C)$ is observed given $a$, then the set of observationally equivalent stochastic models is

\[
\{\sigma' \in \Sigma : \hat{f}_a(\sigma) = \gamma\}
\]

- Thus, action/policy $a$, yields a partitional partial identification correspondence

\[
\Sigma_a = \{\hat{f}_a^{-1}(\gamma) : \gamma \in \Delta(C)\}
\]
\[
\sigma \mapsto \Sigma_a(\sigma) = \hat{f}_a^{-1}(\hat{f}_a(\sigma)) = \{\sigma' \in \Sigma : \hat{f}_a(\sigma') = \hat{f}_a(\sigma)\}
\]
Objective expected utility, or reward

The **reward** of action $a$ given model $\sigma$ is the objective expected utility

$$R(a, \sigma) = \mathbb{E}_\sigma[v \circ f_a] = \int_S v(f(a, s))\sigma(ds)$$

**Remark** *For each $\sigma^* \in \Sigma$, expected reward $R(a, \sigma)$ is constant on $\Sigma_a(\sigma^*)$, because all models $\sigma \in \Sigma_a(\sigma^*)$ yield the same observed distribution of consequences.*
Selconfirming equilibrium with model uncertainty

DM does not know stochastic model $\sigma$ and holds subjective belief $\mu \in \Delta(\Sigma)$. The subjective value of policy $a$ given $\mu$ is

$$V(a, \mu) = \int_{\Sigma} R(a, \sigma) \mu(d\sigma) = \int_{\Sigma} \left( \int_{\lambda} v(f(a, s)) \sigma(ds) \right) \mu(d\sigma)$$

In the long run, DM keeps his policy fixed if it is subjectively optimal and his belief is confirmed by the observed distribution of consequences:

- **Definition** $(a^*, \mu^*, \sigma^*)$ is a selfconfirming equilibrium (SCE) if
  - (subjective optimality) $a^* \in \arg \max_{a \in A} V(a, \mu^*)$
  - (confirmed belief) $\mu^* (\hat{\Sigma}_{a^*}(\sigma^*)) = 1$ [i.e., supp$\mu^* \subseteq \hat{\Sigma}_{a^*}(\sigma^*)$]
Model economy (Sargent, 2008)

- **Unemployment** ($u$) and **inflation** ($\pi$) outcomes are connected to **shocks** ($w, \varepsilon$) and the government **policy** ($a$) according to

$$u = \theta_0 + \theta_1 \pi + \theta_1 a + \theta_2 w$$  \hspace{1cm} (1)

$$\pi = a + \theta_3 \varepsilon$$  \hspace{1cm} (2)

- $\theta = (\theta_0, \theta_1 \pi, \theta_1 a, \theta_2, \theta_3) \in \mathbb{R}^5$ structural coefficients of aggregate supply equation (1)

  - $\theta_1 \pi$ and $\theta_1 a$ are slope responses of unemployment to actual and planned inflation (examples: Lucas-Sargent $\theta_1 a = -\theta_1 \pi$, Samuelson-Solow $\theta_1 a = 0$)
  - $\theta_2$ and $\theta_3$ quantify shock volatilities
  - $\theta_0$ is the rate of unemployment that would (systematically) prevail without policy interventions.
Feedback

- **States** have structural and random components:
  \[ s = (w, \varepsilon, \theta) \in W \times E \times \Theta = S \]

- **Assumption** on parameter space
  \[ \Theta = \{ \theta_0 > 0, \theta_{1\pi} < 0, \theta_{1a} \leq |\theta_{1\pi}|, \theta_2 > 0, \theta_3 > 0 \} \]

- **Consequences:** \( c = (u, \pi) \in \mathbb{R}^2 \)

- **Feedback:** from the reduced form of (1)-(2):

\[
\begin{bmatrix}
  u(a, w, \varepsilon, \theta) \\
  \pi(a, w, \varepsilon, \theta)
\end{bmatrix} = \begin{bmatrix}
  \theta_0 \\
  0
\end{bmatrix} + a \begin{bmatrix}
  \theta_{1\pi} + \theta_{1a} \\
  1
\end{bmatrix} + \begin{bmatrix}
  \theta_2 & \theta_{1\pi} \theta_3 \\
  0 & \theta_3
\end{bmatrix} \begin{bmatrix}
  w \\
  \varepsilon
\end{bmatrix}
\]
Stochastic models

- **Known** joint distribution $q$ of $(\mathbf{w}, \varepsilon)$
- $\theta$ deterministic, fixed
- true distribution on $S = \mathcal{W} \times E \times \Theta$ is $\sigma = q \times \delta_\theta$
  - where $\delta_\theta$ = Dirac supported by $\theta$
  - thus, $\sigma$ parametrized by $\theta$, hence belief $\mu \in \Delta(\Theta)$
- **Assumptions** on noise

  $$\mathbb{E}_q (\varepsilon) = \mathbb{E}_q (\mathbf{w}) = \mathbb{E}_q (\varepsilon \mathbf{w}) = 0$$

  and (normalization)

  $$\mathbb{E}_q (\varepsilon^2) = \mathbb{E}_q (\mathbf{w}^2) = 1$$
Expected utility, value

Quadratic von Neumann-Morgenstern utility function:

\[ v(u, \pi) = -u^2 - \pi^2 \]

Classic linear quadratic policy framework, hence the reward (objective expected utility) function is

\[ R(a, \theta) = v(\mathbb{E}_\theta (u_a), \mathbb{E}_\theta (\pi_a)) + \text{cost}. \]

The subjective value is

\[ V(a, \mu) = -\int_{\Theta} (\mathbb{E}_\theta^2 (u_a) + \mathbb{E}_\theta^2 (\pi_a)) \mu(d\theta) + \text{cost}. \]
Partial identification: moments

It is known that the model economy is of the form (1)-(2), and that \( q \) is the distribution of shocks \((w, \varepsilon)\). With this, the inference problem is to recover vector \( \theta \), or part of it, from the long-run distribution of \((u, \pi)\).

It is sufficient to look at first and second moments:

- \( \mathbb{E}_\theta (u_a) = \theta_0 + (\theta_{1\pi} + \theta_{1a}) a \) [recall: \((\theta_{1\pi} + \theta_{1a}) \leq 0\)]
- \( \mathbb{E}_\theta (\pi_a) = a \)
- \( \text{Var}_\theta (u_a) = \theta_{1\pi}^2 \theta_3^2 + \theta_2^2 \)
- \( \text{Var}_\theta (\pi_a) = \theta_3^2 \) [recall: \( \theta_3 \neq 0 \)]
- \( \text{Cov}_\theta (u_a, \pi_a) = \theta_{1\pi} \theta_3^2 \)
Partial identification: Phillips regression

Therefore,

- the beta coefficient of the Phillips regression of unemployment over inflation is
  \[ \theta_{1\pi} = \frac{\theta_{1\pi} \theta_3^2}{\theta_3^2} = \frac{\text{Cov}_\theta (u_a, \pi_a)}{\text{Var}_\theta (\pi_a)} \]

- the residual variance of \( u_a \) (unexplained by the regression) is
  \[ \theta_2^2 = (1 - \text{Corr}_\theta^2 (u_a, \pi_a)) \text{Var}_\theta (u_a) \]

- \( \theta_3 \) is the standard deviation of inflation

- the structural coefficients \( \theta_0 \) and \( \theta_{1a} \) remain unidentified even in the long run, but there is only one degree of freedom:

  \[ \theta_0 + \theta_{1a} a = \mathbb{E}_\theta (u_a) - \frac{\text{Cov}_\theta (u_a, \pi_a)}{\text{Var}_\theta (\pi_a)} \mathbb{E}_\theta (\pi_a) \]
Partial identification correspondence

Since the distribution of shocks $q$ is known, the partially identified set given policy $a$ and the true model $\sigma = q \times \delta\theta$ is

$$\hat{\Sigma}_a(\sigma) = \hat{\Sigma}_a(q \times \delta\theta) = \{q\} \times \hat{\Sigma}_a(\theta)$$

**Proposition 1** The partial identification correspondence is

$$\hat{\Sigma}_a(\theta) = \{\theta' \in \Theta : \theta'_0 + \theta'_1a = \hat{\alpha}(a), \theta'_1\pi = \hat{\beta}, \theta'_2 = \hat{\sigma}_{u|\pi}, \theta'_3 = \hat{\sigma}_u\}$$

where $\hat{\alpha}(a)$ and $\hat{\beta}$ are the estimated alpha and beta of the Phillips regression.
Policy-dependent partially identified set
Selfconfirming equilibrium

In steady state, given planned inflation $a$, the policy maker is only uncertain about "baseline unemployment," $\theta_0$, and the direct impact of $a$ on $u, \theta_1 a$.

In a **selfconfirming equilibrium** $(a^*, \mu^*, \theta^*)$

- $a^* \in \text{arg min}_{a \in A} \int_\Theta \left( (E_\theta[u_a])^2 + (E_\theta[\pi_a])^2 \right) \mu^*(d\theta)$
- $\text{supp} \mu^* \subseteq \{ \theta \in \Theta : \theta_0 + \theta_1 a^* = \theta_0^* + \theta_1 a^*, \theta_{1\pi} = \theta_{1\pi}^*, \theta_2 = \theta_2^*, \theta_3 = \theta_3^* \}$
Equilibrium with dogmatic belief

Simplest case: $\mu^* = \delta_{\bar{\theta}}$ for some $\bar{\theta} \in \hat{\Sigma}_a^*(\theta^*)$:

$$\min_{a \in A} \mathbb{E}_{\bar{\theta}}^2(u_a) + \mathbb{E}_{\bar{\theta}}^2(\pi_a)$$

$$\text{sub} \quad \mathbb{E}_{\bar{\theta}}(u_a) = \bar{\theta}_0 + (\bar{\theta}_{1a} + \hat{\beta}^*) \mathbb{E}_{\bar{\theta}}(\pi_a)$$

FOC yields the best reply

$$B(\bar{\theta}) \equiv -\frac{\bar{\theta}_0 \left(\hat{\beta}^* + \bar{\theta}_{1a}\right)}{1 + \left(\hat{\beta}^* + \bar{\theta}_{1a}\right)^2}$$

where $\hat{\beta}^* = \theta_{1\pi}^*$. 
Equilibrium with dogmatic belief: characterization

Hence, \((a^*, \delta_{\theta}, \theta^*)\) is a SCE with dogmatic beliefs if and only if

\[
a^* = -\frac{\theta_0^* \left( \hat{\beta}^* + \bar{\theta}_1 a \right)}{1 + \left( \hat{\beta}^* + \theta_1^* a \right) \left( \hat{\beta}^* + \bar{\theta}_1 a \right)}
\]

and

\[
\bar{\theta}_0 = \theta_0^* - \frac{\theta_0^* \left( \hat{\beta}^* + \bar{\theta}_1 a \right)}{1 + \left( \hat{\beta}^* + \theta_1^* a \right) \left( \hat{\beta}^* + \bar{\theta}_1 a \right)} \left( \theta_1^* a - \bar{\theta}_1 a \right)
\]
Dogmatic equilibrium under new-classical model
Dogmatic Keynesian belief under new-classical model
Selfconfirming monetary policy, the general case

We have a kind of SCE certainty-equivalence:

**Proposition 2** \((a^*, \mu^*, \theta^*)\) is a selfconfirming equilibrium if and only if

\[
a^* = -\frac{\theta_0^* \left( \hat{\beta}^* + \mathbb{E}_{\mu^*}(\theta_{1a}) \right)}{1 + \left( \hat{\beta}^* + \theta_{1a}^* \right) \left( \hat{\beta}^* + \mathbb{E}_{\mu^*}(\theta_{1a}) \right)}
\]

and \(\text{supp} \mu^* \subseteq \)

\[
\subseteq \left\{ \theta : \theta_0 = \theta_0^* - \frac{\theta_0^* \left( \hat{\beta}^* + \mathbb{E}_{\mu^*}(\theta_{1a}) \right)}{1 + \left( \hat{\beta}^* + \theta_{1a}^* \right) \left( \hat{\beta}^* + \mathbb{E}_{\mu^*}(\theta_{1a}) \right)} (\theta_{1a}^* - \theta_{1a}) \right\}
\]
Binary-support beliefs

Two models deemed possible: new-classical $\theta^*$, Keynesian $\theta$
Binary-support belief: rewards space

Policy $a$ yields rewards pair $(R(a, \theta^*), R(a, \theta))$
The objectively optimal policy is

\[ a^o = -\frac{\theta_0^* (\hat{\beta}^* + \theta_1^*)}{1 + (\hat{\beta}^* + \theta_1^*)^2}. \]

- \( a^* = a^o \) if and only if \( \mathbb{E}_{\mu^*}(\theta_1 a) = \theta_1^* \).
- Next we show that policy hyperactivism characterizes authorities that overestimate the estimated policy multiplier, while hypoactivism characterizes authorities that underestimate such impact.
Proposition 3 In a selfconfirming equilibrium \((a^*, \mu^*, \theta^*)\),

1. \(E_{\mu^*}(\theta_{1a}) < \theta_{1a}^*\) if and only if policy \(a^*\) is hyperactive, i.e., \(a^* > a^o > 0\);
2. \(E_{\mu^*}(\theta_{1a}) = \theta_{1a}^*\) if and only if policy \(a^*\) is objectively optimal, i.e., \(a^* = a^o\);
3. \(\theta_{1a}^* < E_{\mu^*}(\theta_{1a}) < -\hat{\beta}^*\) if and only if policy \(a^*\) is hypoactive, i.e., \(0 < a^* < a^o\);
4. \(E_{\mu^*}(\theta_{1a}) = -\hat{\beta}^*\) if and only if policy \(a^*\) is zero-target-inflation, i.e., \(a^* = 0\).
For the monetary authority both kind of deviations from objective optimality, hyperactivism and hypoactivism, cause the same welfare loss:

**Proposition 4** *In a selfconfirming equilibrium \((a^*, \mu^*, \theta^*)\), the welfare loss is*

\[
R(a^o, \theta^*) - R(a^*, \theta^*) = (1 + (\hat{\beta}^* + \theta_1^*)^2)(a^* - a^o)^2.
\]
Model uncertainty and partial identification make room for *ambiguity aversion*, which should push toward "robust" policies.

But in this linear-quadratic set up, ambiguity aversion has no effect: in SCE the policy maker behaves *as if* he were a subjective expected utility maximizer, *even if* he is ambiguity averse.

For some intuition see the last picture: even with strictly convex indifference curves in the $R_{\theta^*}(a)-R_{\theta}(a)$ rewards space (ambiguity aversion), the SCE tangency conditions (at the intersection with the PI-line) are the same.

This is an instance of a more general result about the (ir-)relevance of ambiguity aversion in SCE.
Future research: analysis of interesting (non linear-quadratic) SCE models of policy choice where ambiguity aversion can play a role.

Thank you
SCE and model uncertainty


Selfconfirming monetary policy
