Monetary policies in self-confirming equilibria with uncertain models

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Introduction

- **Uncertainty**: DM faces *multiple probabilistic models* (e.g. urns) $\Rightarrow$ some acts induce lotteries with uncertain probabilities.
- Imperfect observations of state realizations given by **information feedback**, which *depends on the chosen action*.
- **Partial identification**: many state distributions consistent with empirical frequencies of observed data, given own choice.
- **Selconfirming choice:** in recurrent decision problem, the stochastic steady state choice is optimal given subjective belief consistent with empirical frequencies of observations.

Relevance for policy making

A policy maker using a selfconfirming policy has *learned the distribution of consequences of the chosen policy, not the data generating process* (model economy).

For example, he may believe in steady state that there is a long-run trade-off between inflation and unemployment even if no such trade off exists.

Most related literature

- *SCE (conjectural eq.) idea:* No comprehensive survey. See surveys by Battigalli *et al.* (1992), Fudenberg & Levine (1998 book), and lit. review in Battigalli *et al.* (2011).
- *Application to monetary policy:* Sargent (1999 book) Sargent (2008).

Framework

Recurrent decision problem with feedback given by observed consequences (all sets are nice Polish spaces):

- $a \in A$, **actions**, policies
- $s \in S$, **states**
- $c \in C$, **consequences**
- $f : A \times S \to C$ **feedback**/consequence function (measurable) [more could be observable, simplification]
- $v : C \to \mathbb{R}$ **vNM utility** function (bounded, measurable)

The *state* of nature is determined at *random* according to an *unknown* objective probability measure $\sigma \in \Delta(S)$

- $\sigma \in \Sigma \subseteq \Delta(S)$ possible **stochastic models** (closed w.r.t. weak* topology)
- In (some) applications, the stochastic model $\sigma \in \Sigma$ is parametrized by parameter vector $\theta \in \Theta$

$$\theta \longmapsto \sigma_\theta$$

$$\Sigma = \{\sigma_\theta : \theta \in \Theta\}$$

Information feedback: partition of states

Each action/policy a yields a partition of S:

- $f_a : S \to C$ section of f at a:

$$s \longmapsto f_a(a) = f(a, s)$$

- When a is chosen and c observed ex post, the set of **observationally equivalent states** is

$$f_a^{-1}(c) = \{s' \in S : f_a(s') = c\}$$

- Ex post information partition implied by a:

$$\mathcal{F}_a = \{f_a^{-1}(c) :_{c \in C}\} \subseteq 2^S$$

$$s \longmapsto f_a^{-1}(f_a(s)) \in \mathcal{F}_a$$

Information feedback: partition of models

- In steady state some a is fixed, then – given true stochastic model σ – DM observes **long-run distribution of consequences**:

$$\hat{f}_a(\sigma) = \sigma \circ f_a^{-1} \in \Delta(C)$$

$$\forall E \subseteq C,\ \hat{f}_a(\sigma)(E) = \sigma\left(f_a^{-1}(E)\right)$$

- If long-run distribution $\gamma \in \Delta(C)$ is observed given a, then the set of **observationally equivalent** stochastic models is

$$\{\sigma' \in \Sigma : \hat{f}_a(\sigma) = \gamma\}$$

- Thus, action/policy a, yields a partitional **partial identification** correspondence

$$\Sigma_a = \{\hat{f}_a^{-1}(\gamma) : \gamma \in \Delta(C)\}$$

$$\sigma \longmapsto \Sigma_a(\sigma) = \hat{f}_a^{-1}(\hat{f}_a(\sigma)) = \{\sigma' \in \Sigma : \hat{f}_a(\sigma') = \hat{f}_a(\sigma)\}$$

Objective expected utility, or reward

The **reward** of action a given model σ is the objective expected utility

$$R(a, \sigma) = \mathbb{E}_\sigma[v \circ f_a] = \int_S v(f(a, s))\sigma(\mathrm{d}s)$$

Remark *For each* $\sigma^* \in \Sigma$*, expected reward* $R(a, \sigma)$ *is constant on* $\Sigma_a(\sigma^*)$*, because all models* $\sigma \in \Sigma_a(\sigma^*)$ *yield the same observed distribution of consequences.*

Selconfirming equilibrium with model uncertainty

DM *does not know* stochastic model σ and holds subjective **belief** $\mu \in \Delta(\Sigma)$. The **subjective value** of policy a given μ is

$$V(a,\mu) = \int_{\Sigma} R(a,\sigma)\mu(\mathrm{d}\sigma) = \int_{\Sigma} \left(\int_{S} v\left(f(a,s)\right)\sigma(\mathrm{d}s) \right) \mu(\mathrm{d}\sigma)$$

In the long run, DM keeps his policy fixed if it is subjectively optimal and his belief is confirmed by the observed distribution of consequences:

- **Definition** (a^*, μ^*, σ^*) *is a* **selfconfirming equilibrium** *(SCE) if*
  - *(subjective optimality)* $a^* \in \arg\max_{a \in A} V(a, \mu^*)$
  - *(confirmed belief)* $\mu^*\left(\hat{\Sigma}_{a^*}(\sigma^*)\right) = 1$ [*i.e.*, $\mathrm{supp}\mu^* \subseteq \hat{\Sigma}_{a^*}(\sigma^*)$]

Model economy (Sargent, 2008)

- **Unemployment** (u) and **inflation** (π) outcomes are connected to **shocks** (w, ε) and the government **policy** (a) according to

$$u = \theta_0 + \theta_{1\pi}\pi + \theta_{1a}a + \theta_2 w \qquad (1)$$

$$\pi = a + \theta_3 \varepsilon \qquad (2)$$

- $\theta = (\theta_0, \theta_{1\pi}, \theta_{1a}, \theta_2, \theta_3) \in \mathbb{R}^5$ structural coefficients of aggregate supply equation (1)
  - $\theta_{1\pi}$ and θ_{1a} are slope responses of unemployment to actual and planned inflation (examples: Lucas-Sargent $\theta_{1a} = -\theta_{1\pi}$, Samuelson-Solow $\theta_{1a} = 0$)
  - θ_2 and θ_3 quantify shock volatilities
  - θ_0 is the rate of unemployment that would (systematically) prevail without policy interventions.

Feedback

- **States** have structural and random components: $s = (w, \varepsilon, \theta) \in W \times E \times \Theta = S$
- **Assumption** on parameter space

$$\Theta = \{\theta_0 > 0, \theta_{1\pi} < 0, \theta_{1a} \leq |\theta_{1\pi}|, \theta_2 > 0, \theta_3 > 0\}$$

- **Consequences:** $c = (u, \pi) \in \mathbb{R}^2$
- **Feedback:** from the reduced form of (1)-(2):

$$\begin{bmatrix} \mathbf{u}(a, w, \varepsilon, \theta) \\ \boldsymbol{\pi}(a, w, \varepsilon, \theta) \end{bmatrix} = \begin{bmatrix} \theta_0 \\ 0 \end{bmatrix} + a \begin{bmatrix} \theta_{1\pi} + \theta_{1a} \\ 1 \end{bmatrix} + \begin{bmatrix} \theta_2 & \theta_{1\pi}\theta_3 \\ 0 & \theta_3 \end{bmatrix} \begin{bmatrix} w \\ \varepsilon \end{bmatrix}$$

Stochastic models

- *Known* joint distribution q of $(\mathbf{w}, \varepsilon)$
- θ deterministic, fixed
- true distribution on $S = W \times E \times \Theta$ is $\sigma = q \times \delta_\theta$
    - where δ_θ=Dirac supported by θ
    - thus, σ parametrized by θ, hence belief $\mu \in \Delta(\Theta)$
- **Assumptions** on noise

$$\mathbb{E}_q(\varepsilon) = \mathbb{E}_q(\mathbf{w}) = \mathbb{E}_q(\varepsilon\mathbf{w}) = 0$$

and (normalization)

$$\mathbb{E}_q(\varepsilon^2) = \mathbb{E}_q(\mathbf{w}^2) = 1$$

Expected utility, value

Quadratic von Neumann-Morgenstern utility function:

$$v(u, \pi) = -u^2 - \pi^2$$

Classic linear quadratic policy framework, hence the reward (objective expected utility) function is

$$R(a, \theta) = v\left(\mathbb{E}_\theta(\mathbf{u}_a), \mathbb{E}_\theta(\boldsymbol{\pi}_a)\right) + cost.$$

The subjective value is

$$V(a, \mu) = -\int_\Theta \left(\mathbb{E}_\theta^2(\mathbf{u}_a) + \mathbb{E}_\theta^2(\boldsymbol{\pi}_a)\right) \mu(\mathrm{d}\theta) + cost.$$

Partial identification: moments

It is known that the model economy is of the form (1)-(2), and that q is the distribution of shocks $(\mathbf{w}, \varepsilon)$. With this, the inference problem is to recover vector θ, or part of it, from the long-run distribution of $(\mathbf{u}, \boldsymbol{\pi})$.

It is *sufficient to look at first and second moments:*

- $\mathbb{E}_\theta(\mathbf{u}_a) = \theta_0 + (\theta_{1\pi} + \theta_{1a})\, a$ [recall: $(\theta_{1\pi} + \theta_{1a}) \leq 0$]
- $\mathbb{E}_\theta(\boldsymbol{\pi}_a) = a$
- $\mathrm{Var}_\theta(\mathbf{u}_a) = \theta_{1\pi}^2 \theta_3^2 + \theta_2^2$
- $\mathrm{Var}_\theta(\boldsymbol{\pi}_a) = \theta_3^2$ [recall: $\theta_3 \neq 0$]
- $\mathrm{Cov}_\theta(\mathbf{u}_a, \boldsymbol{\pi}_a) = \theta_{1\pi} \theta_3^2$

Partial identification: Phillips regression

Therefore,

- the beta coefficient of the *Phillips regression* of unemployment over inflation is

$$\theta_{1\pi} = \frac{\theta_{1\pi}\theta_3^2}{\theta_3^2} = \frac{\mathrm{Cov}_\theta\left(\mathbf{u}_a, \boldsymbol{\pi}_a\right)}{\mathrm{Var}_\theta\left(\boldsymbol{\pi}_a\right)}$$

- the residual variance of $\mathbf{u}_a$ (unexplained by the regression) is

$$\theta_2^2 = \left(1 - \mathrm{Corr}_\theta^2\left(\mathbf{u}_a, \boldsymbol{\pi}_a\right)\right)\mathrm{Var}_\theta\left(\mathbf{u}_a\right)$$

- θ_3 is the standard deviation of inflation
- the structural coefficients θ_0 and θ_{1a} remain *unidentified even in the long run*, but there is only one degree of freedom:

$$\theta_0 + \theta_{1a}a = \mathbb{E}_\theta\left(\mathbf{u}_a\right) - \frac{\mathrm{Cov}_\theta\left(\mathbf{u}_a, \boldsymbol{\pi}_a\right)}{\mathrm{Var}_\theta\left(\boldsymbol{\pi}_a\right)}\mathbb{E}_\theta\left(\boldsymbol{\pi}_a\right)$$

Partial identification correspondence

Since the distribution of shocks q is known, the partially identified set given policy a and the true model $\sigma = q \times \delta_\theta$ is

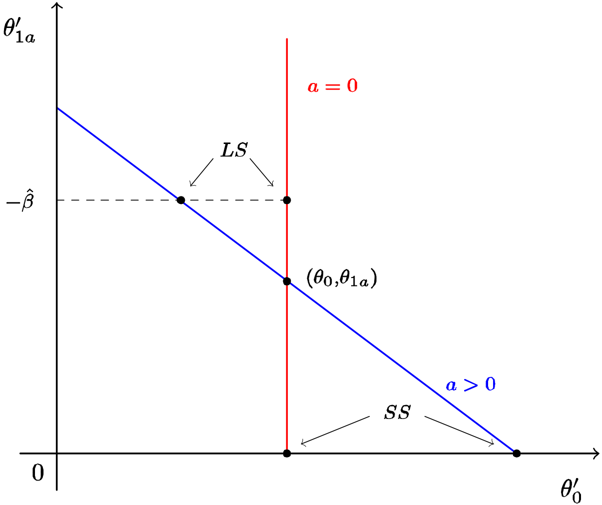
$$\hat{\Sigma}_a(\sigma) = \hat{\Sigma}_a(q \times \delta_\theta) = \{q\} \times \hat{\Sigma}_a(\theta)$$

Proposition 1 *The partial identification correspondence is*

$$\hat{\Sigma}_a(\theta) = \left\{\theta' \in \Theta : \theta'_0 + \theta'_{1a} a = \hat{\alpha}(a),\ \theta'_{1\pi} = \hat{\beta},\ \theta'_2 = \hat{\sigma}_{\mathbf{u}|\boldsymbol{\pi}},\ \theta'_3 = \hat{\sigma}_{\mathbf{u}}\right\}$$

where $\hat{\alpha}(a)$ and $\hat{\beta}$ are the estimated alpha and beta of the Phillips regression.

Policy-dependent partially identified set



Selfconfirming equilibrium

In steady state, given planned inflation a, the policy maker is only uncertain about "baseline unemployment," θ_0, and the direct impact of a on u, θ_{1a}.

In a **selfconfirming equilibrium** (a^*, μ^*, θ^*)

- $a^* \in \arg\min_{a \in A} \int_\Theta \left((\mathbb{E}_\theta[\mathbf{u}_a])^2 + (\mathbb{E}_\theta[\boldsymbol{\pi}_a])^2 \right) \mu^*(\mathrm{d}\theta)$
- $\mathrm{supp}\mu^* \subseteq \{\theta \in \Theta : \theta_0 + \theta_{1a} a^* = \theta_0^* + \theta_{1a}^* a^*,\ \theta_{1\pi} = \theta_{1\pi}^*,\ \theta_2 = \theta_2^*,\ \theta_3 = \theta_3^*\}$

Equilibrium with dogmatic belief

Simplest case: $\mu^* = \delta_{\bar{\theta}}$ for some $\bar{\theta} \in \hat{\Sigma}_{a^*}(\theta^*)$:

$$\min_{a \in A} \mathbb{E}^2_{\bar{\theta}} (\mathbf{u}_a) + \mathbb{E}^2_{\bar{\theta}} (\boldsymbol{\pi}_a)$$

$$\text{sub } \mathbb{E}_{\bar{\theta}} (\mathbf{u}_a) = \bar{\theta}_0 + \left(\bar{\theta}_{1a} + \hat{\beta}^*\right) \mathbb{E}_{\bar{\theta}} (\boldsymbol{\pi}_a)$$

FOC yields the best reply

$$B\left(\bar{\theta}\right) \equiv -\frac{\bar{\theta}_0 \left(\hat{\beta}^* + \bar{\theta}_{1a}\right)}{1 + \left(\hat{\beta}^* + \bar{\theta}_{1a}\right)^2}$$

where $\hat{\beta}^* = \theta^*_{1\pi}$.

Equilibrium with dogmatic belief: characterization

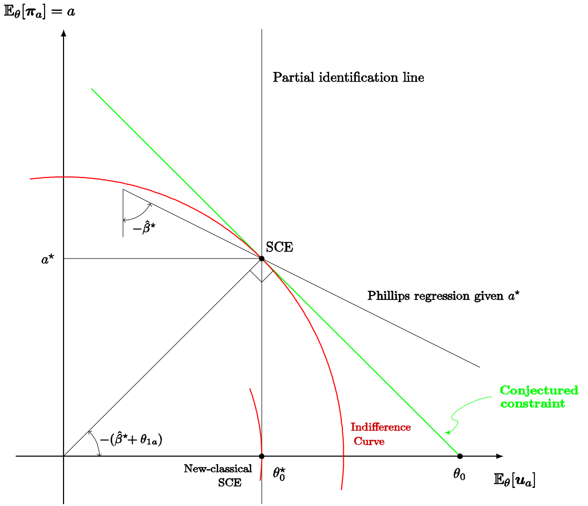
Hence, $(a^*, \delta_{\bar{\theta}}, \theta^*)$ *is a SCE with dogmatic beliefs if and only if*

$$a^* = -\frac{\theta_0^* \left(\hat{\beta}^* + \bar{\theta}_{1a}\right)}{1 + \left(\hat{\beta}^* + \theta_{1a}^*\right)\left(\hat{\beta}^* + \bar{\theta}_{1a}\right)}$$

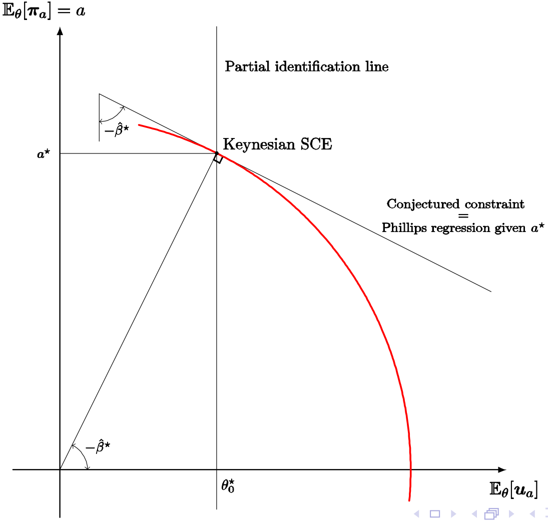
and

$$\bar{\theta}_0 = \theta_0^* - \frac{\theta_0^* \left(\hat{\beta}^* + \bar{\theta}_{1a}\right)}{1 + \left(\hat{\beta}^* + \theta_{1a}^*\right)\left(\hat{\beta}^* + \bar{\theta}_{1a}\right)} \left(\theta_{1a}^* - \bar{\theta}_{1a}\right)$$

Dogmatic equilibrium under new-classical model



Dogmatic Keynesian belief under new-classical model



Selfconfirming monetary policy, the general case

We have a kind of SCE certainty-equivalence:

Proposition 2 (a^*, μ^*, θ^*) *is a selfconfirming equilibrium if and only if*

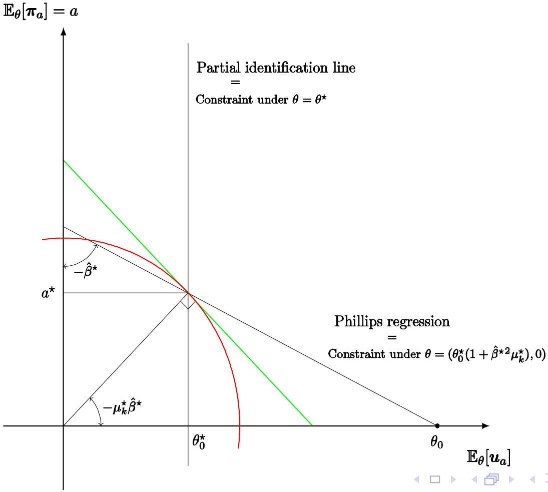
$$a^* = -\frac{\theta_0^* \left(\hat{\beta}^* + \mathbb{E}_{\mu^*}(\boldsymbol{\theta}_{1a})\right)}{1 + \left(\hat{\beta}^* + \theta_{1a}^*\right)\left(\hat{\beta}^* + \mathbb{E}_{\mu^*}(\theta_{1a})\right)}$$

and $\mathrm{supp}\mu^* \subseteq$

$$\subseteq \left\{\theta : \theta_0 = \theta_0^* - \frac{\theta_0^* \left(\hat{\beta}^* + \mathbb{E}_{\mu^*}(\boldsymbol{\theta}_{1a})\right)}{1 + \left(\hat{\beta}^* + \theta_{1a}^*\right)\left(\hat{\beta}^* + \mathbb{E}_{\mu^*}(\boldsymbol{\theta}_{1a})\right)}\left(\theta_{1a}^* - \theta_{1a}\right)\right\}$$

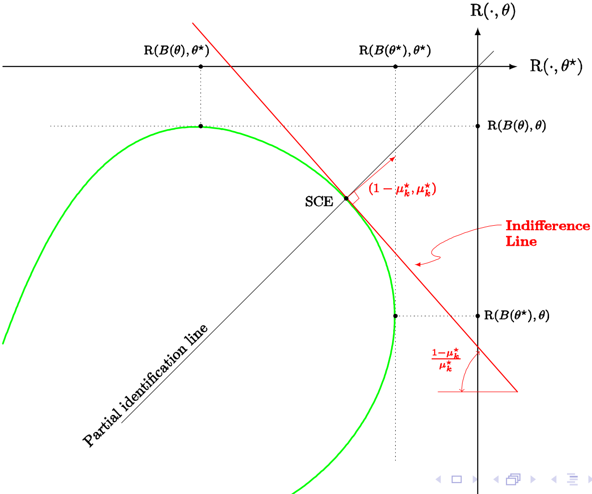
Binary-support beliefs

Two models deemed possible: new-classical θ^*, Keynesian θ



Binary-support belief: rewards space

Policy a yields rewards pair $(R(a,\theta^*), R(a,\theta))$



Objectively optimal policy

- The objectively optimal policy is

$$a^o = -\frac{\theta_0^* \left(\hat{\beta}^* + \theta_{1a}^*\right)}{1 + \left(\hat{\beta}^* + \theta_{1a}^*\right)^2}.$$

- $a^* = a^o$ if and only if $\mathbb{E}_{\mu^*}(\theta_{1a}) = \theta_{1a}^*$.
- Next we show that policy hyperactivism characterizes authorities that overestimate the estimated policy multiplier, while hypoactivism characterizes authorities that underestimate such impact.

Activism and welfare loss

Proposition 3 *In a selfconfirming equilibrium* (a^*, μ^*, θ^*),

1. $\mathbb{E}_{\mu^*}(\theta_{1a}) < \theta_{1a}^*$ *if and only if policy* a^* *is hyperactive, i.e.,* $a^* > a^o > 0$;
2. $\mathbb{E}_{\mu^*}(\theta_{1a}) = \theta_{1a}^*$ *if and only if policy* a^* *is objectively optimal, i.e.,* $a^* = a^o$;
3. $\theta_{1a}^* < \mathbb{E}_{\mu^*}(\theta_{1a}) < -\hat{\beta}^*$ *if and only if policy* a^* *is hypoactive, i.e.,* $0 < a^* < a^o$;
4. $\mathbb{E}_{\mu^*}(\theta_{1a}) = -\hat{\beta}^*$ *if and only if policy* a^* *is zero-target-inflation, i.e.,* $a^* = 0$.

For the monetary authority both kind of deviations from objective optimality, hyperactivism and hypoactivism, cause the same welfare loss:

Proposition 4 *In a selfconfirming equilibrium* (a^*, μ^*, θ^*), *the welfare loss is*

$$R(a^o, \theta^*) - R(a^*, \theta^*) = \left(1 + (\hat{\beta}^* + \theta^*_{1a})^2\right)(a^* - a^o)^2 .$$

Discussion

Model uncertainty and partial identification make room for *ambiguity aversion*, which should push toward "robust" policies.

But in this linear-quadratic set up, ambiguity aversion has no effect: in SCE the policy maker behaves *as if* he were a subjective expected utility maximizer, *even if* he is ambiguity averse.

For some intuition see the last picture: even with strictly convex indifference curves in the $R_{\theta^*}(a)$-$R_{\theta}(a)$ rewards space (ambiguity aversion), the SCE tangency conditions (at the intersection with the PI-line) are the same.

This is an instance of a more general result about the (ir-)relevance of ambiguity aversion in SCE.

Future research: analysis of interesting (non linear-quadratic) SCE models of policy choice where ambiguity aversion can play a role.

Thank you

SCE and model uncertainty

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