The Social Value of Debt in the Market for Corporate Control*

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Abstract

The free-rider problem in takeovers is in essence a (coordination) failure to "negotiate" mutually beneficial terms. Means for bidders to unilaterally seize gains could solve this issue but target shareholders prefer to limit such means. We show that takeover debt can bridge this divide in that debt constraints can serve as Pareto-improving "sharing rules." In this theory (i) leveraged buyouts are privately and socially optimal and (ii) competing bidders raise more debt, amplifying gains to target shareholders and in takeover efficiency. At its best, leveraging bids neutralizes the free-rider problem—to *mutual* benefit.

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1 Introduction

In this paper we argue that debt finance is key to unlocking the market for corporate control. Leveraging bids, according to our theory, can simultaneously increase bidder profits, effect larger value improvements in target firms, and benefit target shareholders; at its best, it fully neutralizes the free-rider problem. The underlying mechanism is that the agency problem between bidders and lenders mitigates the distributional conflict between bidders and target shareholders.

Since Berle and Means (1932) a paradigm in corporate governance is that diffuse ownership becomes separated from control and creates managerial discretion. Indeed, many models in corporate governance, notably of active blockholders, start with the premise that small shareholders' passiveness enables managerial misbehavior. Furthermore, under passive owners, other governance mechanisms, such as incentive compensation or boards of directors, can be captured by the managers they are meant to discipline.[1]

A classic counterpoint, credited to Manne (1965), is that a takeover (threat) mitigates agency problems caused by diffuse ownership. Grossman and Hart (1980) issue a fundamental critique of this claim: coordination problems that limit dispersed shareholders's role in governance also predispose them to free-riding behavior in takeovers. This frustrates takeovers unless bidders possess means to exclude target shareholders from part of the takeover surplus. Whether such exclusion mechanisms can restore takeovers as an efficient governance instrument is an overarching question in the tender offer literature, and of importance for the governance of "Berle-Means" corporations.

It is, in principle, doubtful that exclusion mechanisms restore efficiency. Because exclusion mechanisms unilaterally redistribute wealth to bidders, target shareholders are inclined to impose limits on exclusion– such as on the ability to dilute minority interests or to secretly accumulate toeholds in targets—even if doing so frus-

[1] See, e.g., Bebchuk and Fried (2004).

trates some takeovers. Further, in light of this resistance, bidders may extract gains through "hidden" means that come with deadweight losses (e.g., Burkart, Gromb, and Panunzi, 1998).

We show that the exclusion mechanism identified by Müller and Panunzi (2004), takeover leverage, can be the exception. In a setting in which firm value depends on the bidder's incentives to improve it, takeover leverage cannot only increase bidder profits but also target firm value and target shareholder wealth. We illustrate these insights first with a simple example and afterwards explain them in general terms and relative to the existing literature.

A bidder can increase the share value of a currently widely held firm from its current value of 0 to $V_l = 10$ or $V_h = 20$ by providing effort $e \in \{l, h\}$ at cost $C_l = 2$ or $C_h = 10$.[2] Denote $\Delta_V \equiv V_h - V_l = 10$ and $\Delta_C \equiv C_h - C_l = 8$. Total takeover surplus is larger under the high effort level: $S_h \equiv V_h - C_h = 10 > S_l \equiv V_l - C_l = 8$. To gain control, the bidder must buy $r \geqslant .5$ of the shares, paying rP, so her profit is $r(V_e - P) - C_e$. Because free-riding shareholders do not tender their shares unless $P \geqslant V_e$, this profit collapses to $-C_e$. Hence, no takeover occurs unless the bidder has access to an exclusion device.

Suppose the bidder can divert $B \in [2, 10]$ of the firm value as private benefits once she is in control, thus diluting the post-takeover share value to $V_e - B$. As Grossman and Hart (1980) show, her profit with dilution is $B - C_e$. So she maximizes B and minimizes C_e; in this example, $B = 10$ and $C_l = 2$. The bidder can implement this by buying $r = .5$ shares. With this equity stake, she has no incentive to incur $C_h = 10$, as $.5\Delta_V < \Delta_C$. Total surplus is then $S_l = 8$ and target shareholders get $V_l - B = 0$. The takeover is profitable but neither efficient nor beneficial to target shareholders. Indeed, the latter would limit the bidder to $B = 2$.

Alternatively, suppose the bidder can raise debt $D \in [2, 10]$ against target assets. Her profit in this setting is isomorphic to that in the setting with dilution: $D - C_e$.

[2] While we study a framework with costly effort, all our results also obtain if the moral hazard problem is modeled as inefficient diversion of private benefits.

Analogously, she would like to set $D = 10$ and $C_l = 2$. But crucially, for $D = 10$, she would never incur $C_l = 2$, as $r(10-10)-2 < 0$ for all r. Foreseeing this, lenders do not finance the bidder unless she has incentives to incur the high effort $C_h = 10$. This is feasible: for $r = 1$ and $D = 10$, $r(V_h - D) \geqslant C_h$ holds. Contrary to the setting with dilution, the maximum surplus $S_h = 10$ is achieved—and it accrues to target shareholders through $V_h - D = 10$. Moreover, the latter do not want to limit the bidder to $D = 2$. If limited to $D = 2$, she would choose $r = .5$ and $C_l = 2$, as $.5(V_l - 2) - C_l > 0 > .5(V_h - 2) - C_h$. Lenders would still be willing to finance the takeover, but target shareholders' profit would drop to $V_l - D = S_l = 8$.

As the comparison to dilution in the example highlights, the benefits of leverage originate in the agency problem between bidder and lenders. By using takeover debt, a bidder cashes out "private benefits" in exchange for pledging future target income to lenders (whose seniority dilutes post-takeover share value). At the same time, lenders only supply funds if the bidder has incentives to generate sufficient income. Hence, to raise more funds from lenders, the bidder must scale up her incentives by acquiring a larger equity stake. This coupling of debt finance to ownership concentration forces bidder profits and firm value to move in tandem, which in turn can benefit the target shareholders.

Our framework extends Grossman and Hart (1980) along two dimensions: a pre-bid stage in which bidders can raise financing for a takeover and a post-takeover stage in which incentives to improve firm value depend on ownership and capital structure. If using debt financing, a successful bidder acquires levered equity. Her post-takeover incentives increase in her equity stake and decrease in leverage, the latter insofar as it creates a debt overhang problem. The transaction that is strictly first-best for any set of parameters is for the bidder to buy all shares with no debt.

Previous work suggests that the free-rider problem pushes bidders away from the first-best financing structure. Abstracting from debt, Burkart, Gromb, and Panunzi (1998) show that bidders acquire as little equity as needed; assuming exogenous post- takeover values, Müller and Panunzi (2004) show that bidders raise as much debt as they can. In both cases, the bidders' objective is to reduce post-takeover share value. These papers predict that (i) buyouts are partial and (ii) takeover leverage is antithetical to target share value maximization.

We show that these implications are overturned when endogenous value creation and debt financing interact. In such a setting, the agency problem between lenders and bidders constrains what combinations of financing and bids are feasible: Bidders cannot minimize ownership (for given debt) nor maximize debt (for given ownership) independently. To raise more debt, a bidder must buy a larger equity stake to avoid debt overhang. In consequence, she cannot extract larger gains unless she commits to generate a higher firm value.

Although debt financing increases total firm value, how much debt a bidder issues depends on whether her additional private gains (extracted through debt) exceed her additional cost of effort (induced by the larger stake). She may not be able to extract enough to recoup the additional costs because the potential for debt overhang limits debt capacity: the bidder must leave a wedge between debt and firm value such that equity is sufficiently "in the money" and her incentives to supply effort are preserved. This wedge is larger when the bidder's stake is smaller or the optimal "in-the-money" effort is larger.

This wedge is the post-takeover share value and hence what the bidder must offer target shareholders to succeed. That is, from an ex ante perspective, the wedge needed to preserve ex post incentives is the part of the value improvement the bidder cannot extract through debt but has to "leave on the table," thus splitting the gains between her and target shareholders. Who benefits from an increase in debt financing therefore depends on its impact on the wedge. Target shareholders benefit as long as the wedge increases, while the bidder benefits as long as it increases less than total surplus. We show that these conditions can be met simultaneously, in which case takeover debt—and the constraints imposed on it by agency frictions—amount to a Pareto-improving "sharing rule." In sharp contrast to Burkart, Gromb, and Panunzi (1998) and Müller and Panunzi (2004), our theory produces full, leveraged buyouts as maximizing not only bidder profits but also post-takeover firm and share value.

Bidding competition boosts the benefits of takeover debt for target shareholders and takeover efficiency. A bidder selects the debt level that maximizes her profit. By contrast, target shareholders prefer the highest debt level and corresponding equity concentration at which the bidder just breaks even. The latter debt level leads to the highest takeover surplus conditional on lender participation, and this surplus accrues entirely to target shareholders while the bidder only recoups her costs. Competition pushes rival bidders toward this debt level, and thereby increases (i) takeover debt, (ii) bidder stakes, and (iii) post-takeover firm and share values, all while (iv) reducing bidder returns. The increase in debt amplifies the positive impact of competition on takeover efficiency, as it enables bidders to create more value while recovering costs through issuing more debt.

The contribution of our paper to the corporate governance literature is to develop a fuller picture of the role debt plays in disciplinary takeovers of Berle-Means firms, i.e., takeovers that seek to restore managerial incentives by reversing the separation of control and ownership. As said, we do so by merging the key elements of Burkart, Gromb, and Panunzi (1998) and Müller and Panunzi (2004). Most closely related to our analysis is a short extension in Section 6 of the working paper version of Müller and Panunzi (2004) (Müller and Panunzi, 2003) that is not included in the published article. The purpose of the extension is to establish that, like exogenous bankruptcy, debt overhang concerns limit use of debt financing. It stops short, however, of tracing out the full welfare implications of debt in this setting and how this sets debt apart from other exclusion mechanisms.

Exclusion mechanisms occupy a central role in the literature on hostile takeovers since Grossman and Hart (1980). Known mechanisms are dilution (Grossman and

Hart, 1980), toeholds (Shleifer and Vishny, 1986), noise trading (Kyle and Vila, 1991), and leverage (Müller and Panunzi, 2004).[3] There are few comparative studies because the various mechanisms are equivalent in standard tender offer models, and if anything, comparisons tend to emphasize how *similar* they are (Müller and Panunzi, 2004; Burkart and Lee, 2015). A takeaway of the current paper is that leverage possesses a quality that sets it apart from the other mechanisms when value creation is endogenous. We argue that this makes debt financing a uniquely powerful catalyst in the market for corporate control.

Burkart, Gromb, Mueller, and Panunzi (2014) also study takeover financing subject to the free-rider problem, but from a different angle. They examine the impact of investor protection laws on wealth-constrained bidders who *depend* on outside funds. They show that better investor protection need not promote takeovers, despite relaxing funding constraints, as long as the free-rider condition is binding. If competition pushes bids above post-takeover share values, however, better investor protection can prevent that inefficient but wealthier bidders win the contest.[4]

Last, all of our results hinge on a crucial interaction between ownership structure (à la Jensen and Meckling, 1976) and debt overhang (Myers, 1977). While standard debt overhang models focus on debt capacity D, the in-the-moneyness of equity $V - D$ usually plays no role. We show that the latter splits the gains from concentrating ownership between bidder and target shareholders. It is hence key to our insight that takeover debt can overcome free-riders' failure to "bargain" for mutual benefit.

The paper proceeds as follows. Section 2 describes the model and the equilibrium

[3] Two-tiered offers (Bebchuk, 1987) and freeze-out rules (Yarrow, 1985; Amihud et al., 2004) can also overcome the free-rider problem. They operate on a different principle: Rather than shifting rents from target shareholders to bidders directly, they eliminate the gains retaining shareholders (hope to) get relative to tendering shareholders. However, the effectiveness of these mechanisms is non-robust to the introduction of uncertainty (Müller and Panunzi, 2004; Dalkir, Dalkir, and Levit, forthcoming).

[4] Outside of the literature exploring the free-rider problem, takeover models have studied means of payment as a way to mitigate asymmetric information problems (Hansen, 1987; Fishman, 1989; Berkovitch and Narayanan, 1990; Eckbo et al., 1990) and financing as a commitment to aggressive bidding in takeover contests (Chowdhry and Nanda, 1993). Reconsidering signaling incentives in a tender offer model, Burkart and Lee (2015) show that means of payment offer no signaling potential when the free-rider condition binds.

derivation. Section 3 studies the welfare effects of takeover debt. Section 4 discusses our results in the light of existing explanations for highly leveraged tender offers, like those provided for the 1980s takeover wave. Section 5 concludes the paper.

2 Model

2.1 Assumptions

Consider a widely held firm ("target") facing a single potential acquirer ("bidder"). If the bidder gains control, she can generate a value improvement $V(e) \geqslant 0$, relative to the value under the current management, which is normalized to zero. Generating value requires unobservable effort e, which imposes a private cost $C(e)$ on the bidder. The bidder seeks control through a tender offer and must acquire at least half of the target shares to gain control. When faced with an offer, the incumbent management is assumed to be unwilling or unable to counterbid.

Each atomistic target shareholder is non-pivotal for the takeover outcome. The resulting free-riding behavior frustrates takeovers, unless the bidder has means of "excluding" target shareholders from (part of) the post-takeover value (Grossman and Hart, 1980). We focus on the exclusion mechanism identified by Müller and Panunzi (2004): an acquisition financed in part with debt backed by the target's post-takeover value. Shareholders are excluded from post-takeover value pledged to lenders because debt is senior. The bidder extracts this value when issuing the debt prior to the bid. For simplicity, we normalize pre-takeover leverage to zero.

Our model has three stages. In stage 1, the bidder issues a debt amount $D \geqslant 0$ and makes a take-it-or-leave-it tender offer to acquire target shares at a price p per share. The offer is conditional, that is, it becomes void if less than half the shares are tendered. We abstract from exclusion mechanisms other than debt, so the takeover fails unless $D > 0$.

In stage 2, target shareholders non-cooperatively decide whether to tender their shares. The shareholders are homogeneous and atomistic such that no one is pivotal. Specifically, we assume a unit mass of shares dispersed among an infinite number of shareholders whose individual holdings are equal and indivisible.[5] The shareholders' tendering strategies are functions $s : (D, p) \to \{\text{accept}, \text{reject}\}$.

In stage 3, if less than half the shares are tendered, the takeover fails. Otherwise, the bidder pays αp for the fraction α of shares acquired, obtains control, and chooses an effort level $e \geqslant 0$ to maximize her post-takeover utility $U(\alpha, D, e)$. So, her post-takeover strategy is a function $e : (\alpha, D) \to \mathbb{R}^+$. Last, the firm value and all payoffs are realized.

We assume that the value improvement function V is linear in effort, $V(e) = \theta e$, where θ is the marginal return to effort. The cost function C is twice differentiable, strictly increasing, and strictly convex, i.e., $C'(e) > 0$ and $C''(e) > 0$ for all $e \geqslant 0$. We also assume $C(0) = 0$, $\lim_{e \to 0} C'(e) = 0$, and $\lim_{e \to +\infty} C'(e) = +\infty$ to restrict attention to takeovers that (would) have strictly positive but finite value. Our focus on linear V is without loss of generality in that all results can be directly translated to concave value improvement functions.[6]

Before analyzing the model, we comment on two modeling choices. First, unlike Burkart, Gromb, and Panunzi (1998), we model bidders' post-takeover moral hazard problem as costly effort rather than costly diversion. Diversion would add a source of bidder gains without changing the main insights; indeed, takeover debt would have the benefit of reducing inefficient diversion. Second, the timing of effort is irrelevant. The analysis is virtually identical if bidders exert effort prior to (in preparation for) a bid, as long as the effort is unobservable.

[5]These assumptions are standard in tender offer models exploring the free-rider problem. When they are relaxed, Grossman and Hart (1980)'s result that target shareholders extract all the gains in security benefits becomes diluted (Holmström and Nalebuff, 1992).

[6]Suppose $V : [0, +\infty) \to \mathbb{R}$ is a twice differentiable, strictly increasing, and concave function. The game we consider is isomorphic to a game in which the bidder, instead of choosing e, chooses y where $\theta y = V(e)$. In the latter game, the bidder's post-takeover objective function is $\alpha[\theta y - D]^+ - C(V^{-1}(\theta y))$, where V^{-1} denotes the inverse function of V. Since the inverse of a strictly increasing, strictly concave function is a strictly increasing, strictly convex function, the composition $C \circ V^{-1}$ satisfies the assumptions postulated for C in our model.

2.2 Equilibrium

We derive the equilibrium by backward induction in three subsections corresponding to the stages of the game. Our focus lies on the fraction of shares tendered α and takeover debt D, which result in the post-takeover ownership and capital structure. Contrary to typical financing models, there are no wealth constraints that necessitate outside financing. Debt D and "outside equity" $1 - \alpha$ are the result of frictions that lead to a trilateral interaction between takeover financing (stage 1), tendering decisions (stage 2), and effort choice (stage 3).

Stage 3: Effort choice

Suppose the bidder has acquired a fraction $\alpha \geqslant 1/2$ of the target shares with takeover debt (that has a face value) D. She then chooses effort e to maximize the value of her equity stake in the levered firm net of private costs, $U(\alpha, D, e) \equiv \alpha[V(e) - D]^+ - C(e)$.

This objective function is not globally concave in e. Let e_D satisfy $V(e_D) = D$. For $e \in [0, e_D)$, equity is "out of the money" since $V(e) < D$, and so $U(\alpha, D, e) = -C(e)$ which is strictly decreasing in e. For $e \geqslant e_D$, $U(\alpha, D, e) = \alpha[V(e) - D] - C(e)$ since equity is "in the money." Under our assumptions about V and C, this is strictly concave and the first-order condition, $\alpha V'(e) = C'(e)$, has a unique, strictly positive solution, hereafter denoted as $e^+(\alpha)$.

Because $U(\alpha, D, e)$ is not globally concave, $e^+(\alpha)$ need not be a global optimum. Specifically, given that $\frac{\partial U}{\partial e} < 0$ for $e \in [0, e_D)$, it is possible that $U(\alpha, D, e^+(\alpha)) < 0$. If so, the bidder's optimal effort is $e = 0$. To summarize the above arguments:

Lemma 1. *The bidder's optimal effort is* $e^*(\alpha, D) = e^+(\alpha) > 0$ *if*

$$\alpha[V(e^+(\alpha)) - D] - C(e^+(\alpha)) \geqslant 0 \tag{1}$$

where $e^+(\alpha)$ *is the solution to*

$$\alpha V'(e^+(\alpha)) = C'(e^+(\alpha)) \tag{2}$$

Otherwise, she makes no effort to improve value, i.e., $e^*(\alpha, D) = 0$*.*

Lemma 1 features established wisdom. Outside debt can lead to a debt overhang that undermines shareholders' incentives to invest in value improvements (Myers, 1977). Here, this occurs when condition (1) is violated. Outside equity dilutes the incentives of "inside" shareholders to improve firm value (Jensen and Meckling, 1976). Hence, firm value increases with ownership concentration. Indeed, conditional on (1), effort $e^+(\alpha)$ and firm value $V(e^+(\alpha))$ are increasing in α (by the envelope theorem).

The novel element of Lemma 1 is that these two effects interact in condition (1). Whether a debt overhang problem emerges depends not only on the debt level D but also on the level of ownership concentration α. The intuition is simple: The bidder's incentives derive from a *levered* equity *stake* $\alpha[V(e^+(\alpha)) - D]$. While D lowers the total value of equity, α determines the bidder's share of that total value. This has the implication that a given debt level is less likely to undermine the bidder's incentives if she owns more equity. Or put differently, a firm with more concentrated ownership can sustain a higher level of (incentive-compatible) debt. This interaction between α and D—which we refer to as the *ownership-leverage link*—will be crucial.

Stage 2: Tendering decisions

As Lemma 1 indicates, the first-best structure is fully concentrated ownership and no debt, i.e., $(\alpha, D) = (1, 0)$.[7] An ideal market for corporate control would restore this structure. We discuss next how free-riding behavior by dispersed target shareholders distorts bidders' preferences regarding α and D.

[7] This is the only structure that leads to the first-best outcome for every admissible specification of V and C. For any $D > 0$, there exist admissible V and C such that (1) is violated.

Suppose target shareholders face an offer p (partially) financed by debt in the amount of D. Being non-pivotal, an individual shareholder i accepts the offer only if $p \geqslant V(e^*(\hat{\alpha}_i, D))$ where $\hat{\alpha}_i$ denotes i's belief about the bidder's post-takeover equity stake. Given tendering decisions depend on individual beliefs, no dominant-strategy equilibrium exists. In a rational expectations equilibrium, beliefs are consistent with the outcome, so shareholders tender only if

$$p \geqslant [V(e^*(\alpha, D)) - D]^+. \tag{3}$$

That is, target shareholders tender their shares only if they extract (at least) the full increase in share value that the bidder will generate. This is known as the free-rider condition.

Previous work has analyzed two special cases of (3). Müller and Panunzi (2004) study a model with exogenous post-takeover values where (3) becomes $p \geqslant [V - D]^+$. In this setting, the bidder wants to maximize D. In contrast, Burkart, Gromb, and Panunzi (1998) consider endogenous post-takeover values but abstract from debt. In this case, (3) reduces to $p \geqslant V(e^*(\alpha, 0))$, and the bidder wants to minimize α. Both cases highlight that the bidder seeks to decrease the right-hand side of (3)—i.e., the post-takeover share value—which target shareholders extract via the price. We will show that the more general case, in which D and α are jointly chosen, overturns key predictions derived in the special cases.

Before we characterize the stage-2 subgame equilibrium, note that (3) is merely a necessary condition for a successful bid; a failed bid, in which an insufficient number of shares is tendered, can always be supported as a self-fulfilling equilibrium outcome. To focus on the interesting case, we assume that shareholders always tender when the free-rider condition is weakly satisfied, thus selecting the Pareto-dominant success equilibrium whenever it exists.

Denote the post-takeover share value that the bidder creates for a given stake and debt level by $E(\alpha, D)$ and the equity stake she acquires in equilibrium by $\alpha^*(p, D)$. In the subsequent formal result, we omit describing the subgame equilibrium for bids that can be ruled out a priori: those that fail for *any* set of beliefs ($p < E(1/2, D)$ and those the bidder could undercut without affecting any other decision ($p > E(1, D)$).

Lemma 2. *Any bid* (p, D) *with* $E(1/2, D) \leqslant p \leqslant E(1, D)$ *succeeds and* $\alpha^*(p, D) = \alpha_p$ *where* α_p *satisfies* $p = E(\alpha_p, D)$.

Proof. If $p \in [E(1/2, D), E(1, D)]$, a unique $\alpha_p \in [1/2, 1]$ exists such that $E(\alpha_p, D) = p$. Each shareholder tenders for $\hat{\alpha}_i < \alpha_p$, retains her shares for $\hat{\alpha} > \alpha_p$, and is indifferent between tendering and retaining for $\hat{\alpha} = \alpha_p$. □

Target shareholders are willing to sell shares until the post-takeover share value, which increases with the bidder's stake, reaches the bid price. As in Burkart, Gromb, and Panunzi (1998), supply is hence upward-sloping: the fraction of shares tendered increases with the price. The fraction of shares acquired by the bidder equals that at which the free-rider condition (3) holds with equality.[8]

Stage 1: Bid and financing

The bidder's ex ante profit is $\alpha[V(e) - D]^+ - C(e) - \alpha p + D$. It comprises the value of the levered equity stake she expects to acquire, net of her effort costs and takeover payment, and the debt financing she receives for the bid. She maximizes this under constraints (1), (2), and (3). The lenders' participation constraint, $V(e) \geqslant D$, must also hold but is implied by (1); they would not agree to a debt amount that stymies post-takeover incentives.

For any offer (D, p), free-rider condition (3) holds with equality, endogenously, since target shareholders will tender α_p shares such that $E(\alpha_p, D) = p$ (Lemma 2). To demarcate the novel aspect of our analysis from existing results, we first

[8]While the outcome is pinned down, the equilibrium strategy profile is not necessarily unique. The outcome obtains when each shareholder uses a mixed strategy of tendering with probability α_p, but also when a mass α_p of shareholders tender with certainty while all others retain their shares.

state how the binding free-rider condition (3) and the first-order condition (2) for optimal effort affect the bidder's strategy. Incorporating these constraints in the profit function gives

$$D - C(e^+(\alpha)).$$

This replicates the known insights that debt D enables the bidder to extract private gains and that a larger equity stake α is unattractive because it induces her to incur higher effort costs, while all gains in share value accrue to target shareholders.

The above modified profit function also shows that the bidder's stage-1 problem can virtually be reduced to choosing what equity stake α to acquire and how much debt financing D to use, and thus effectively, what post-takeover ownership and capital structure to implement.[9]

The novel element is the joint restriction imposed on D and α by debt overhang constraint (1). This cannot be slack under the optimal bid. For a given α, every D that satisfies (1) induces the same effort $e^+(\alpha)$ (Lemma 1). If (1) is slack, the bidder can hence raise D infinitesimally, while keeping her effort at $e^+(\alpha)$ and lowering the bid to keep free-rider condition (3) binding. This would increase her profit, as the modified profit function illustrates. Intuitively, she benefits from exhausting her debt capacity.

Using the binding debt overhang constraint (1) to replace D in the modified profit function collapses the bidder's optimization program to

$$\max_{\alpha \in [1/2, 1]} V(e^+(\alpha)) - C(e^+(\alpha)) - \frac{C(e^+(\alpha))}{\alpha}. \tag{P}$$

In Section 3, we use this representation to study the role of debt. Before doing so, we conclude this section by establishing equilibrium existence (though not uniqueness).

[9] In fact, consistent with this formulation of the problem, the equilibrium offer can equivalently be implemented (interpreted) as (i) a cash-equity bid with $1-\alpha_p$ being the fraction of post-takeover equity offered to target shareholders as payment combined with cash; (ii) a cash bid with a fraction $1-\alpha_p$ of the equity-funded portion being funded by "outside" investors other than the bidder; or (iii) a cash bid in which the number of shares the bidder offers to acquire is restricted to $r = \alpha_p$.

Lemma 3. *If the bidder's profit under (P) is negative, she makes no bid. Otherwise, she succeeds with a bid such that (1), (2), and (3) bind and α solves (P).*

Proof. The objective function is continuous in α and its domain is compact. Hence there exists an $\alpha \in [1/2, 1]$ that solves (P). If the profit under this solution is positive, the bidder makes a successful bid. Otherwise, she abstains from a takeover. □

3 Social value of debt

This section presents our three main results. First, we consider the effect of takeover debt on total takeover surplus. Second, we analyze how debt affects the distribution of the surplus. Last, we introduce bidding competition and study its interaction with takeover debt.

3.1 Surplus-increasing debt

In our model, the social surplus created by a successful takeover is $W(\alpha) \equiv V(e^+(\alpha)) - C(e^+(\alpha))$. While this expression depends only on the bidder's post-takeover equity stake α, the latter is linked to the debt level D through the binding debt overhang constraint

$$\alpha[V(e^+(\alpha)) - D] - C(e^+(\alpha)) = 0. \tag{1*}$$

As shown in the proof of the next proposition, (1*) implicitly defines D as a strictly increasing function of $\alpha \in [1/2, 1]$ and vice versa. This has the following implication: Provided that a bid stays profitable, an increase in takeover debt D requires a larger bidder stake α as a means to increase bidder effort e and total surplus S.

This is not a prediction about empirical correlations as debt itself is endogenous, for example, to the shapes of V and C. Instead, it is a prediction about the "causal" impact of "exogenous" variation in bidders' access to debt (allowing for endogenous adjustments in the other bid parameters). The result is hence best stated as follows:

Proposition 1. *Restrictions on takeover debt are socially suboptimal.*

Proof. Section B of the Appendix. □

Proposition 1 is somewhat surprising because the primary effect of takeover debt in our tender offer framework is merely to redistribute rents from target shareholders to bidders. Indeed, in Müller and Panunzi (2004), takeover debt has a purely redistributive effect (conditional on a bid) or, in the model extension with exogenous bankruptcy costs, is even *in*efficiently high.

Endogenous value creation is the key to this difference and the reason is twofold. On the equity side, the fact that owning a larger stake improves incentives to create value is a *dis*incentive to buy shares when faced with the free-rider problem. While the bidder is more incentivized to improve share value when acquiring more shares, target shareholders extract the resulting value increase through the bid price. Absent a countervailing effect, bidders therefore prefer to reduce α so that takeovers at best partially undo the agency problem they are meant to remedy.

On the debt side, the potential overhang problem limits the amount of financing lenders are willing to supply. Because of the ownership-leverage link discussed after Lemma 1, the bidder's debt capacity increases with her stake α. She exhausts that capacity for a given α, but to obtain more debt funding, must increase α. In essence, to qualify for more debt, the bidder must commit to generate sufficient value. Buying a larger stake, which mitigates the agency problem, offers such commitment. This *indirect* incentive to raise α offsets the aforementioned disincentive to the extent that the bidder wants to raise debt.

3.2 Pareto-improving debt

Whether more debt financing benefits a bidder is not obvious. While it allows her to extract more private gains, it also forces her to buy a larger stake that induces more effort. Hence, the bidder benefits only if the gains from the increase in debt exceed the increase in effort costs. This is not guaranteed because debt overhang constraint (1*) limits how much of the surplus created by the additional effort she can extract using debt. The flipside is that target shareholders can potentially benefit from an increase in takeover debt, even though it serves as a means to dilute them.

The bidder's profit in (P) can be written as $W(\alpha) - \frac{C^{+}(\alpha)}{\alpha}$ where $C^{+}(\alpha) \equiv C(e^{+}(\alpha))$. The subtracted term is the post-takeover share value under (1*): $\frac{C^{+}(\alpha)}{\alpha} = V(e^{+}(\alpha)) - D$. As is characteristic of the free-rider problem, the bidder's profit amounts to total surplus minus post-takeover equity value, which target shareholders extract through the price. The key feature of our framework is that this equity value is the wedge between firm value and debt which the bidder must leave such that equity is sufficiently in the money to avoid a debt overhang problem.

The wedge $\frac{C^{+}(\alpha)}{\alpha}$ determines how the total surplus is split between bidder and target shareholders. How the wedge varies with α and corresponding D hence decides whether a bidder benefits from more debt, and whether that in turn harms or benefits target shareholders.

There are two countervailing effects. On one hand, if we hold effort costs (i.e., the numerator) fixed, the wedge decreases in α. Equity incentives depend on total equity value and equity concentration, which creates a form of "incentive substitutability": a controlling shareholder with a larger stake α can reduce total equity value $V(\overline{e}) - D$ more without creating a debt overhang problem. On the other hand, subject to (1*), the optimal "in-the-money" effort $e^{+}(\alpha)$ and associated cost $C^{+}(\alpha)$ increase with α. If giving the bidder a larger stake α as an incentive for her to improve firm value more, any accompanying increase in debt D must avoid discouraging the *higher* effort needed for a *greater* improvement. This effect increases the wedge.

The first effect prevails if $\frac{\partial C^{+}(\alpha)}{\alpha} \to 0$. In this case, higher D (and corresponding α) benefit bidders but harm target shareholders. The reverse occurs if $\frac{\partial C^{+}(\alpha)}{\alpha} \to \infty$; target shareholders gain so much that bidders cannot recoup the increase in cost. In this case, bidders do not seek more debt financing. Most importantly, for parameters that fall between these polar cases, bidders and target shareholders both benefit from more takeover debt. In the proof of the next result, we derive sufficient conditions to identify classes of (cost) functions that produce the third case for some range of admissible α.

Proposition 2. *Takeover leverage can be Pareto-improving.*

Proof. Section C of the Appendix. □

In our model with a linear value improvement function $V(e) = \theta e$, two commonly used families of cost functions that can lead to Pareto-improving use of takeover debt are power functions $C(e) = \frac{c}{n}e^n$ for $n > 2$ and exponential functions $C(e) = \exp(e) - c$ for $\theta > \exp(2)$. In the latter case, increases in α and corresponding D are mutually beneficial at *every* $\alpha \in [1/2, 1)$. As a result, a *full* leveraged buyout implementing the first-best value improvement *Pareto-dominates* every alternative offer. This example provides the sharpest contrast to Burkart, Gromb, and Panunzi (1998) where $\alpha > 1/2$ is suboptimal for the bidder and to Müller and Panunzi (2004) where increases in D are pure wealth transfers from target shareholders to bidders.[10]

The "commitment" and "sharing rule" effects of debt, which underlie the Pareto gains, originate in the agency problem between bidder and lenders. In other words, it is financing *constraints* that allow the bidder and target shareholders to agree on more efficient transactions at mutually beneficial terms—despite an absence of bargaining. The lack of a similar endogenous constraint is why other exclusion mechanisms such as dilution or toeholds do not produce these benefits.

The wedge $V(e^+(\alpha)) - D = \frac{\partial C^+(\alpha)}{\alpha}$ as well as its comparative statics are defined solely by debt overhang constraint (1*), independent of free-rider condition (3). Yet, in standard financing models, attention is typically confined to a firm's debt capacity D and the wedge is per se not of interest. In a takeover setting, the free-rider problem

[10] Appendix F presents these examples formally and shows that they can generate high debt-to-equity ratios as a Pareto-improving takeover feature.

endows the wedge with significance as what the bidder must "leave on the table" for target shareholders, i.e., as a "sharing rule."

Finally, the optimality of debt does not rely on the exact "curvature" of its claim. Rather, as in Müller and Panunzi (2004), its benefit derives simply from its seniority to equity, which allows bidders to employ it as an exclusion mechanism. It is for this reason important that the debt is raised at the "transaction" level, i.e., against target assets. To give a precise illustration of this requirement, when the bidder is a buyout fund, issuing debt at the fund level would not create the benefits identified in Propositions 1 and 2.[11]

3.3 Leveraging competition

When maxing out debt is not optimal for a (single) bidder, the solution to her tender offer problem (P) is a partial acquisition $\alpha^* \in [1/2, 1)$. From a social perspective, she then raises too little debt. Indeed, unlike in the model of Müller and Panunzi (2004) with exogenous value improvements, target shareholders would benefit from a higher debt level and the corresponding ownership concentration: the bidder would create more surplus but make less profit. Importantly, this difference between models with exogenous and endogenous post-takeover values also manifests itself in the effect of competition.

Consider two bidders competing in a second-price, sealed-bid auction. If bidder $i \in \{1, 2\}$ succeeds, she can generate value improvement $V_i(e_i)$ at private cost $C_i(e_i)$. Let $E_i^*(\alpha_i)$ denote the post-takeover share value that bidder i generates when she acquires α_i shares. It is hardly surprising that introducing a more efficient rival improves efficiency. Less obvious is that the presence of a weaker rival also improves efficiency, as we show below. This holds if the optimal bid in the absence of the rival leads to a partial, strictly profitable takeover, i.e., $\alpha_i^* \in [1/2, 1)$ and $W_i(\alpha_i^*) - E_i^*(\alpha_i^*) > 0$ for $i \in \{1, 2\}$.

[11]By contrast, it does not matter whether outside equity financing—as which the "outside equity" component $1 - \alpha$ can be interpreted—is raised at the level of the "transaction" or the "acquirer."

To characterize the highest price a bidder is willing to offer, consider bidder i in isolation. If bidder i increases her bid p_i, target shareholders tender more shares until the free-rider condition again holds with strict equality (Lemma 2): $p_i = E^*(\alpha_{p_i})$. At her reservation price $\overline{p}_i$, her participation constraint binds such that the entire surplus accrues to target shareholders: $W(\alpha_{\overline{p}_i}) = E^*(\alpha_{\overline{p}_i})$. Taken together, binding free-rider condition and participation constraint imply $\overline{p}_i = W(\alpha_{\overline{p}_i})$.[12] Thus, bidder i's reservation price equals the surplus she would create under the largest stake that she can acquire without making a loss.

Without loss of generality, assume that bidder 2 has the higher reservation price, i.e., $\overline{p}_2 > \overline{p}_1$. Thus, bidder 2 will win the contest. In choosing her optimal offer, she now solves the stage-1 problem with the added constraint that she must also outbid her rival:

$$p_2 \geqslant \overline{p}_1 \tag{4}$$

Considering only effective competition in which bidder 1's reservation price exceeds the price bidder 2 would offer without a rival bid, i.e., $\overline{p}_1 > E_2^*(\alpha_2^*)$, we explore how the losing bidder's reservation price affects the outcome.

Proposition 3. *Fiercer competition leads to a larger post-takeover target firm value and a higher level of takeover debt.*

Proof. Section D of the Appendix. □

To win the bidding contest, bidder 2 must at least match bidder 1's reservation price $\overline{p}_1$. The higher price induces target shareholders to tender a larger fraction of the shares. This in turn induces the bidder to generate more value, and at the same time, supports a higher debt level due to the ownership-leverage link. Non-tendering and tendering shareholders benefit equally from the competition because the higher price results in a higher post-takeover share value. By contrast, bidder 2 fares worse

[12] The above argument assumes that $\alpha_{(\overline{p}_i)} < 1$. If $\alpha_{(\overline{p}_i)} = 1$, the free-rider condition is slack but the last equality nonetheless holds, i.e., $\overline{p}_i = W(1)$.

because her share of the additional value created by competition is less than the increase in effort costs; otherwise, she would offer that price already in the absence of competition.

It is worth pointing out that in the absence of competition a bidder raises more debt to acquire more (than $1/2$ of the) target shares only if it allows her to make a larger profit. This higher debt level may or may not benefit target shareholders depending on the shape of $\frac{\partial C^{+}(\alpha)}{\alpha}$ (Proposition 2). The additional debt bidder 2 raises in response to competition reduces her profit and always benefits target shareholders.

The above explanation of Proposition 3 begs the question why bidder 2 does not generate the higher post-takeover share value $\overline{p}_1$ by acquiring fewer additional shares without increasing debt. Such a strategy implies a slack debt overhang constraint. In this case, the bidder can simultaneously increase her final equity stake and debt level in such a way that she fully extracts any additional surplus created, leaving the post-takeover share value at $\overline{p}_1$. This implies that not only the free-rider condition but also the debt overhang constraint must be binding under the optimal bid, and fiercer competition (higher $\overline{p}_1$) therefore translate into larger bidder stakes and higher debt levels.

Corollary 1. *Restrictions on takeover debt make the losing bidder less competitive.*

Proof. Section E of the Appendix. □

Recall that a bidder's reservation price equals the surplus she creates under the largest stake that she can acquire without making a loss. A binding restriction on the debt level reduces her ability to fully recoup the effort cost associated with that surplus creation. To break even, she must reduce her effort and equity stake, which implies a smaller surplus, respectively reservation price. Thus, restrictions on takeover debt reduce the price that the winning bidder has to match, that is, it weakens competition.

4 Discussion

One of the more illustrious periods in financial markets were the 1980s takeovers. Three characteristics distinguished them from previous historical M&A waves. First, many 1980s takeovers sought to divest and refocus assets, partly undoing expansions from a previous merger wave, rather than pursuing greater scale or scope. Second, in contrast to the friendly mergers of earlier waves, the 1980s saw the rise of hostile takeovers and management buyouts in which bidders seek control of widely held firms by making tender offers directly to dispersed shareholders. Third, these tender offers were highly leveraged, whereas the mergers of earlier waves were mainly equity-financed.

For the first two characteristics, a widely accepted narrative is laid out in Shleifer and Vishny (1990) and Holmstrom and Kaplan (2001): Whether induced by deregulation and technological progress or a reassessment of past expansions, there was tremendous scope for valuable corporate restructuring. At the same time, managers often opposed changes due to differences in opinion or self-interest. Regardless of the managers' motivation, many saw their resistance as manifesting agency conflicts that had grown since the 1930s as a result of the rising separation of ownership from control in public firms. Tender offers—by incumbent managers or hostile raiders—(partly) reversed this separation, thereby realigning managerial incentives (or visions for the firms) with ownership.[13] This realignment motive is why the 1980s takeovers are widely considered a watershed in the history of *corporate governance*, rather than just a wave of industrial (re)organization.[14]

As for the third characteristic, the unprecedented levels of takeover leverage, the leading explanations are tax benefits and, perhaps more importantly, what

[13]In leveraged buyouts, top executives increase their ownership significantly and post-buyout boards of directors are small and dominated by active investors with substantial stakes (Kaplan, 1989).

[14]According to Mitchell and Mulherin (1996), nearly half of all major U.S. corporations received an unsolicited "hostile" takeover bid in the 1980s. Further, many firms that restructured themselves during that period without being taken over arguably reacted to the threat of a takeover.

Jensen (1986) coined the "free cash flow problem." If managers are inclined to build or preserve "empires," debt imposes discipline by requiring them to pay out the free cash flow. However, neither of these rationales calls for "bootstrapping" target assets to raise debt (Müller and Panunzi, 2004, p. 1244):

> Jensen's free-cash-flow argument...does not require that the takeover itself is leveraged; increasing leverage shortly after the deal is closed is sufficient.[15]

Müller and Panunzi (2004) explain the leveraging of bids as an exclusion mechanism. This explanation can furthermore account for the fact that the free-rider problem à la Grossman and Hart (1980) was no prohibitive obstacle to the 1980s buyouts of widely held firms. While this complements existing explanations well, some wrinkles remain (Müller and Panunzi, 2004, p. 1220):

> [A] minimal amount of debt equal to the raider's transaction cost might be sufficient to ensure that the takeover takes place. Indeed, if debt is costly and the raider's profit is limited due to bidding competition, it is precisely this minimal amount of debt that is optimal. Hence, while our model provides a role for debt in takeovers, it cannot explain LBO-style debt levels.

This does not square well with the data. On one hand, the tender offers of the 1980s were *highly* leveraged, as are leveraged buyouts today . Even taking into account the other aforementioned benefits of leverage, it is not obvious why takeover leverage was so high, with debt-capital ratios often above 80%, exceeding those in voluntary leveraged recapitalizations during that period.

On the other hand, if takeover leverage is but an exclusion mechanism, one would think that high levels of debt transferred most of the takeover surplus to bidders. It is commonly known, however, that takeover gains accrued predominantly to target

[15] Alternatively, raising the debt at the level of the acquiring firm or fund would also suffice.

shareholders, whereas bidder returns were modest and virtually vanished in the latter half of the 1980s.

Moreover, the standard explanation for decreased bidder returns in the late 1980s is intensified competition. According to above quote from Müller and Panunzi (2004), this should have decreased takeover leverage. But leverage levels did not drop and, in fact, were less sustainable; about one-third of the buyouts after 1985 defaulted on their debt (Kaplan and Stein, 1993). Still, efficiency gains from these buyouts seem to have been positive (Andrade and Kaplan, 1998) but accrued to target shareholders.

Our theory can account for these facts because it examines the impact of takeover debt on the free-rider problem not separately from the usual incentive realignment arguments for buyouts. Once the free-rider problem and managerial incentives are studied in conjunction, *high* takeover leverage can be privately and socially optimal; higher takeover leverage can *benefit* target shareholders; and under competition, bidders raise *more* takeover debt, to their own detriment but amplifying target shareholder gains.

Our theory connects well with two other observations on takeovers and leverage. One is that debt funding lets wealth-constrained bidders, whether hostile or insiders, achieve second-best incentives by increasing their equity percentage most effectively. This argument is a variation on the agency *benefits* of debt à la Innes (1990), and predicts a positive relationship between ownership concentration and debt financing. Our model yields the same prediction through a complementary mechanism: to raise more debt, a bidder has to improve incentives by buying a larger stake to counteract agency *costs* of debt, such as debt overhang.

The other observation, recently made by Axelsson et al. (2013), is that buyout leverage is unrelated to firm characteristics that drive leverage in a cross-section of matched public firms. It is instead driven by time-series variation in economy-wide credit conditions. This points to a credit demand *distinct* to buyouts. Axelsson et al.

(2013) discuss two potential explanations: Buyout firms may time the market using (overvalued) debt to take levered bets on (undervalued) equity. Further, cheap debt may allow buyout firms to overinvest at the expense of the limited partners in their funds (Axelsson et al., 2009).[16] Our theory suggests frictions in the buyout process itself as another leverage determinant: high deal leverage can be key to overcoming the free-rider problem if realigning incentives with ownership is the buyout objective. Our theory can also match Axelsson et al. (2013)'s finding that higher deal leverage correlates with higher transaction prices and lower buyout fund returns—the latter if cheap debt invites new entrants, leading to more competition.

5 Conclusion

The fundamental governance problem is how to hold managers accountable if dispersed investors lack, individually, incentives and, collectively, coordination to monitor the firms they own. A remedy proposed by Manne (1965) and others is the market for corporate control in which rival management teams (including the incumbent one) compete for control over corporate resources through takeover bids. This competition ensures that inefficient management cannot prevail.

Many argue that this vision of disciplinary takeovers was borne out in the hostile bids and leveraged buyouts of the 1980s. Much has already been written about the various features of levered takeovers, but there is a disconnect between two strands of the literature. One focuses on agency conflicts and incentive realignment and is concerned with the source of the aggregate takeover gains (e.g., Jensen, 1986). The other strand emphasizes coordination problems among dispersed investors (Grossman and Hart, 1980) and is concerned with the division of the takeover gains. Building on Burkart, Gromb, and Panunzi (1998) and Müller and Panunzi (2004, 2003), this paper explores whether bridging this gap enhances our understanding of

[16] Based on wealth constraints rather than the free-rider problem, neither the theory in Axelsson et al. (2009) nor Innes (1990)-type arguments require targets to be "bootstrapped" for the debt.

the salient features of buyouts.

Our analysis has remarkable implications regarding the role of debt in the market for corporate control: Acquisition debt dilutes post-takeover equity and pays out the debt value to the bidder upfront in the form of financing. But to obtain (more) debt, the bidder has to buy a (larger) stake that incentivizes her to create enough value to repay the debt afterwards. Acquisition debt thus promotes ownership concentration, which the free-rider problem otherwise frustrates, and hence the incentive alignment that underlies the takeover gains. At the same time, as incentive constraints limit the bidder's borrowing capacity, part of those gains accrue to target shareholders. These virtues of acquisition debt are amplified by bidding competition.

Our theory offers a reason for *takeovers* to be *highly* levered; neither a leveraged recapitalization after a takeover nor a debt issue at the buyout fund level affords the aforementioned benefits. It also reconciles that the free-rider problem was not a prohibitive obstacle to 1980s takeovers with the empirical finding that most of the gains accrued to target shareholders—despite the leverage used to dilute them. According to our theory, not even competition curbs a bidder's use of predatory debt; on the contrary, acquisition debt may increase while bidder profit decreases. Last, since the benefit of acquisition debt originates from frictions in the takeover process and bid financing, it can explain why buyout leverage may outdo leveraged recapitalizations, even if the latter pursue the same "non-buyout" incentive (or tax) benefits of debt.

Appendix

A Auxiliary results

For reference, we state the following result from one variable calculus (e.g., Rudin (1964, p. 114)):

Lemma A.1. *Let $f : (0, +\infty) \to \mathbb{R}$ be a differentiable function such that $f'(x) > 0$ for all $x \in (0, +\infty)$. Then f is strictly increasing on $(0, +\infty)$ and has a differentiable inverse function g with*

$$g'(f(x)) = \frac{1}{f'(x)}$$

for all $x \in (0, +\infty)$. If $f : (0, +\infty) \to \mathbb{R}$ is twice differentiable and such that $f''(x) > 0$ for all $x \in (0, +\infty)$ then its inverse g is also twice differentiable and we have

$$g''(f(x)) = -\frac{f''(x)}{(f'(x))^3}$$

for all $x \in (0, +\infty)$.

We now derive two auxiliary results.

Lemma A.2. *There is a unique differentiable function $e : [1/2, 1] \to \mathbb{R}_{\geqslant 0}$ such that $rV'(e(r)) = C'(e(r))$ for all $r \in [1/2, 1]$ and such that $e'(r) > 0$ for all $r \in (1/2, 1)$. If moreover $C'''(e)$ exists for all $e > 0$, then e is twice differentiable.*

Proof. Define a function $H : (0, +\infty) \to \mathbb{R}$ by $H(e) = \frac{C'(e)}{\theta}$. Clearly

$$H'(e) = \frac{C''(e)}{\theta} > 0$$

for all $e > 0$ by our assumption that $C''(e) > 0$ for all $e \geqslant 0$. Thus H satisfies the premises of Lemma A.1, and hence there is a differentiable function G such that $G(H(e)) = e$ for all $e > 0$ and $H(G(y)) = y$ for all y in the range of H. From our assumptions $\lim_{e \to 0} C'(e) = 0$ and $\lim_{e \to +\infty} C'(e) = +\infty$ and the fact that H is continuous, it follows that $[1/2, 1]$ is a subset of the range of H, i.e., $[1/2, 1] \subseteq H((0, +\infty))$. Hence we may define $e : [1/2, 1] \to (0, +\infty)$ by $e(r) := G(r)$ for all $r \in [1/2, 1]$. Then $\frac{C'(e(r))}{\theta} = H(e(r)) = H(G(r)) = r$ for all $r \in [1/2, 1]$ and the first part of the claim follows. Let $r \in (1/2, 1)$ and $e > 0$ be such that $H(e) = r$, applying Lemma A.1 once again then yields

$$e'(r) = e'(H(e)) = \frac{1}{H'(e)} = \frac{\theta}{C''(e)} > 0.$$

Moreover if C is thrice differentiable we have that

$$e''(r) = e''(H(e)) = -\frac{H''(e)}{(H'(e))^3} = -\theta^2 \frac{C'''(e)}{[C''(e)]^3}.$$

□

Lemma A.3. *Let $f : (a, b) \to \mathbb{R}$ be a function such that $f(x) = h(x)g(x)$ for all $x \in (a, b)$, where $h(x) > 0$ and $g'(x) < 0$ for all $x \in (a, b)$. Then there is at most one $x \in (a, b)$ such that $f(x) = 0$. Moreover if a point $x \in (a, b)$ such that $f(x) = 0$ exists then $f(y) > 0$ for all $y < x$ and $f(y) < 0$ for all $y > x$.*

Proof. Consider two arbitrary distinct points $x, y \in (a, b)$ with $f(x) = f(y) = 0$. Then $h(x) > 0$ and $h(y) > 0$ implies $g(x) = g(y) = 0$. Since g is differentiable, hence also continuous on $[x, y]$, the mean value theorem (Rudin, 1964, Theorem 5.10, p. 108) gives a point z with $x < z < y$ and $g'(z) = 0$. This contradicts $g'(z) < 0$. The second part clearly holds since $g(y) > 0$ for all $y < x$ and $g(y) < 0$ for all $y > x$ and since $h(x)$ is strictly positive. □

B Proof of Proposition 1

Equation (1*) defines the equilibrium debt level $\overline{D}(\alpha) \equiv V(e^+(\alpha)) - \frac{C(e^+(\alpha))}{\alpha}$. Now,

$$\overline{D}'(\alpha) = V'(e^+(\alpha))e^{+\prime}(\alpha) + \frac{1}{r^2}C(e^+(\alpha)) - \frac{1}{r}C'(e^+(\alpha))e^{+\prime}(\alpha)$$

$$
\begin{aligned}
&= \big(V'(e^+(\alpha)) - \frac{1}{\alpha}C'(e^+(\alpha))\big)e^{+\prime}(\alpha) + \frac{1}{\alpha^2}C(e^+(\alpha)) \\
&= \frac{1}{\alpha^2}C(e^+(\alpha)) > 0.
\end{aligned}
$$

The third equality holds because $\alpha V'(e^+(\alpha)) - C'(e^+(\alpha)) = 0$ by (2). The fact that $\overline{D}(\alpha)$ is strictly increasing implies the same for its inverse function. Last, note that $W(\alpha)$ is strictly increasing in α with the first-best outcome being attained for $\alpha = 1$.

C Proof of Proposition 2

Target shareholder gains. Target shareholders benefit from higher α if

$$
\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}\alpha}\frac{C(e^+(\alpha))}{\alpha} &= \frac{C'(e^+(\alpha))e^{+\prime}(\alpha)}{\alpha} - \frac{C(e^+(\alpha))}{\alpha^2} \\
&= \frac{\theta}{\alpha}\left[\frac{C'(e^+(\alpha))}{C''(e^+(\alpha))} - \frac{C(e^+(\alpha))}{C'(e^+(\alpha))}\right] \geqslant 0.
\end{aligned}
$$

The second equality holds by Lemma A.1, whereby if $e^+(\alpha) > 0$, then $e^{+\prime}(\alpha) = \frac{\theta}{C''(e^+(r))}$. A sufficient condition for the inequality to hold (globally) is log-concavity of C, i.e., $C(e)C''(e) \leqslant [C'(e)]^2$ for all $e > 0$. Power functions satisfy this property.

Bidder gains. The bidder's profit, $\pi^B(\alpha) \equiv V(e^+(\alpha)) - \left[1 + \frac{1}{r}\right]C(e^+(\alpha))$, is strictly increasing in α if

$$
\begin{aligned}
\frac{\mathrm{d}\pi^B(\alpha)}{\mathrm{d}\alpha} &= V'(e^+(\alpha))e^{+\prime}(\alpha) + \frac{1}{r^2}C(e^+(\alpha)) - \frac{1}{r}C'(e^+(\alpha))e^{+\prime}(\alpha) - C'(e^+(\alpha))e^{+\prime}(\alpha) \\
&= \left[V'(e^+(\alpha)) - \frac{1}{\alpha}C'(e^+(\alpha))\right]e^{+\prime}(\alpha) + \frac{1}{\alpha^2}C(e^+(r)) - C'(e^+(\alpha))e^{+\prime}(\alpha) \\
&= \frac{1}{\alpha^2}C(e^+(\alpha)) - C'(e^+(\alpha))e^{+\prime}(\alpha) \\
&= \frac{1}{\alpha^2}C(e^+(\alpha)) - \frac{C'(e^+(\alpha))\theta}{C''(e^+(\alpha))} \\
&= \frac{1}{\alpha^2}C(e^+(\alpha)) - \frac{C'(e^+(\alpha))C'(e^+(\alpha))}{C''(e^+(\alpha))\alpha} \\
&= \frac{1}{\alpha}\left(\frac{C(e^+(\alpha))}{\alpha} - \frac{[C'(e^+(\alpha))]^2}{C''(e^+(\alpha))}\right) > 0
\end{aligned}
$$

The second equality is obtained by rearranging terms. The third equality holds since

$\alpha V'(e^+(\alpha)) - C'(e^+(\alpha)) = 0$ by (2). The fourth equality follows from Lemma A.2. The fifth equality holds because $\alpha\theta = C'(e^+(\alpha))$ by (2). A sufficient condition for the last inequality to be satisfied (globally) is that

$$\frac{1}{r}\left(\frac{C(e)}{r} - \frac{[C'(e)]^2}{C''(e)}\right) > \frac{1}{r}\left(C(e) - \frac{[C'(e)]^2}{C''(e)}\right) \geqslant 0$$

for all $e > 0$. The strict inequality holds for all $r < 1$. The last weak inequality holds if C is log-convex, i.e., if $C(e)C''(e) \geqslant [C'(e)]^2$ for all $e > 0$. Exponential functions satisfy this property.

Appendix F uses the two families of functions identified above as examples for which Pareto-improving use of debt occurs. It is worth emphasizing that the above sufficient conditions, which the examples satisfy, are stronger than needed for Pareto improvements to be feasible.

D Proof of Proposition 3

Let $\overline{p}_1$ denote bidder 1's maximum bid. Suppose $p_2 \geqslant \overline{p}_1$. Bidder 2's profit is then $\alpha_2 E_2(\alpha_2, D_2) - C_2(e_2) - \alpha_2 p_2 + D_2$, subject to constraints (1)-(3) as in the single-bidder case. We must now distinguish two cases.

Case 1: $p_2 \leqslant E_2(1, D_2)$. Target shareholders will tender α_{p_2} shares such that free-rider condition (3) is binding (Lemma 2). Bidder 2's profit collapses to $D_2 - C_2(e_2)$, so she raises D_2 until debt overhang constraint (1) binds. As both (1) and (3) hold with equality, the problem reduces to solving (P) subject to the additional constraint $\alpha_2 \geqslant \alpha_{\overline{p}_1}$ (analogous to $p_2 \geqslant \overline{p}_1$). We refer to this as the modified problem (P'). Let α_2^{**} denote its solution.

An increase in (the degree of competition) $\overline{p}_1$ corresponds to an increase in $\alpha_{\overline{p}_1}$. This tightening of the constraint in (P') weakly (sometimes strictly) decreases bidder 2's profit and weakly (sometimes strictly) increases α_2^{**}. Any increase in α_2^{**} in turn implies an increase in the optimal bid p_2^{**} (through the binding free-rider condition) and an increase in takeover debt D_2^{**} (through the binding debt overhang constraint).

Case 2: $p_2 > E_2(1, D_2)$. Target shareholders tender all shares. Bidder 2's profit becomes $E_2(1, D_2) - C_2(e_2) - p_2 + D_2$. This collapses to $V(e^+(1)) - C_2(e_2) - p_2$ since $E_2(1, D_2) = V(e^+(1)) - D_2$. Thus, D_2 is irrelevant, although it must exceed a lower bound set by the free-rider condition: $D_2 \geqslant \underline{D}$ where $\underline{D}$ satisfies $E_2(1, \underline{D}) = p_2 \geqslant \overline{p}_1$. If $D < \underline{D}$, case 1 applies.

The effects of an increase in $\overline{p}_1$ described in the first part of the proposition all apply in case 1. In case 2, an increase in $\overline{p}_1$ causes only a decrease in bidder profits.

E Proof of Corollary 1

We must show that a restriction on D_1 weakly and sometimes strictly decreases bidder 1's maximum bid $\overline{p}_1$. We must again distinguish two cases. First, if $\overline{\alpha}_1 < 1$, $\overline{p}_1 = W_1(\overline{\alpha}_1) = E_i^*(\overline{\alpha}_1)$. In this case, $\overline{\alpha}_1$ satisfies the binding debt overhang constraint (1*), which defines a strictly increasing function $\alpha_1(D_1)$ (see proof of Proposition 1). A restriction on D_i therefore lowers α_1 and thereby $\overline{p}_1$. Second, if $\overline{\alpha}_1 = 1$, $\overline{p}_1 = W_i(1) \geqslant E_i^*(1)$. In this case, for $\overline{p}_1 > E_i^*(1)$, restricting D_1 does not affect bidder 1's maximum bid.

F Examples

Example F.1 (Power functions.)**.** Let $V(e) \equiv \theta e$ and $C(e) \equiv \frac{c}{n} e^n$ where $\theta > 0$, $c > 0$ and $n \in \mathbb{N}$ are exogenous parameters. These functions satisfy all our assumptions. It can also be shown that they generate unique solutions to (P) (proof available upon request). So, if the bidder's profit is positive under the solution to (P), there exists a unique $\langle D, \alpha, p, e \rangle$ such that $\alpha V'(e) = C'(e)$, $p = V(e) - D$, $\alpha D = \alpha V(e) - C(e)$, and $\alpha \in [1/2, 1]$ satisfying $\alpha \in \{1/2, 1\}$ or the ex ante first-order condition for (P),

$$\frac{1}{\alpha^2} C(e^+(\alpha)) = C'(e^+(\alpha)) e^{+\prime}(\alpha). \tag{F.1}$$

The specific functional form allows us to express $\langle D, \alpha, p, e\rangle$ in closed form. The first-order condition for effort $\alpha V'(e) = C'(e)$ yields $e = \left(\frac{\alpha\theta}{c}\right)^{\frac{1}{n-1}}$. The equilibrium stake α solves (F.1). One can show that this condition holds if and only if

$$\theta e^{+\prime}(\alpha)\left(\frac{n-1}{n} - \alpha\right) = 0,$$

which in turn holds if and only if $\alpha = 0$ (since $e^{+\prime}(0) = 0$) or $\alpha = \frac{n-1}{n}$. Of these, only $\alpha = \frac{n-1}{n}$ is admissible as a solution to (P). It is straightforward to verify that

$$D = \frac{(n-1)\theta}{n}\left(\frac{(n-1)\theta}{nc}\right)^{\frac{1}{n-1}}$$

and

$$p = \frac{\theta}{n}\left(\frac{(n-1)\theta}{nc}\right)^{\frac{1}{n-1}}.$$

Furthermore, the bidder's profit under the solution to (P) is positive since

$$\begin{aligned}
D - C(e^{+}(\alpha)) &= \frac{(n-1)\theta}{n}\left(\frac{(n-1)\theta}{nc}\right)^{\frac{1}{n-1}} - \frac{(n-1)\theta}{n^2}\left(\frac{(n-1)\theta}{nc}\right)^{\frac{1}{n-1}} \\
&= \theta\left(\frac{n-1}{n}\right)^{2}\left(\frac{(n-1)\theta}{nc}\right)^{\frac{1}{n-1}} \geqslant 0.
\end{aligned}$$

To sum up, there is a unique equilibrium in which

$$\langle D, \alpha, p, e\rangle = \left\langle \frac{(n-1)\theta}{n}\left(\frac{(n-1)\theta}{nc}\right)^{\frac{1}{n-1}}, \frac{n-1}{n}, \frac{\theta}{n}\left(\frac{(n-1)\theta}{nc}\right)^{\frac{1}{n-1}}, \left(\frac{\frac{n-1}{n}\theta}{c}\right)^{\frac{1}{n-1}} \right\rangle.$$

For $n = 2$, $\alpha = 1/2$. In this case, debt financing does not induce the bidder to buy more than the minimum fraction of shares needed for control and so does not improve efficiency (conditional on a bid). However, $\alpha \in (1/2, 1)$ for all $n > 2$. In all these cases, the bidder uses takeover debt to buy more shares, which raises takeover surplus and benefits target shareholders (given the log-concavity of power functions). The debt-equity ratio is $D/p = n-1$. So, in this example, high leverage ratios can be Pareto-optimal; for instance, a debt-equity ratio of 4 is generated if $n = 5$. $\lhd$

Example F.2 (Exponential functions.)**.** Let $V(e) \equiv \theta e$ and $C(e) \equiv \exp(e)$ with $\theta > \exp(2)$. These functions satisfy all our assumptions, and can be shown to entail unique solutions to (P) (proof available upon request). If the bidder's profit is positive under (P), there is a unique $\langle D, \alpha, p, e\rangle$ such that $\alpha V'(e) = C'(e)$, $p = V(e) - D$, $\alpha D = \alpha V(e) - C(e)$, and $\alpha \in [1/2, 1]$ either satisfying the ex ante first-order condition (F.1) or $\alpha \in \{1/2, 1\}$. The post-takeover first-order condition $\alpha V'(e) = C'(e)$ yields $e^+(\alpha) = \ln(\alpha\theta)$, which is stictly positive given $\alpha\theta > \frac{\exp(2)}{2} > 1$. Substituting $e^+(\alpha)$ into the profit function of (P) yields

$$\theta \ln(\alpha\theta) - (1 + 1/\alpha)\alpha\theta.$$

Differentiating with respect to α yields $\theta(1/\alpha - 1)$, which is strictly positive for all $\alpha \in [1/2, 1)$. Thus, $\alpha = 1$ is the unique solution to (P). It is straightforward to verify that

$$D = \theta \ln(\theta) - \theta$$

and

$$p = \theta.$$

Furthermore, the bidder's profit is

$$D - C(e^+(1)) = \theta(\ln(\theta) - 2),$$

which is positive since $\theta > \exp(2)$ implies $\ln(\theta) > 2$. To summarize, there is a unique equilibrium in which

$$\langle D, \alpha, p, e\rangle = \langle \theta \ln(\theta) - \theta, 1, \theta, \ln(\theta)\rangle .$$

Being weakly log-convex, this class of functions leads to full buyouts, $\alpha = 1$. Given exponential functions are also weakly log-concave, (the accompanying) increases in takeover debt are Pareto-improving. That is, the full buyout *Pareto*-dominates every $\alpha \in [1/2, 1)$. The debt-equity ratio is $D/p = \ln(\theta) - 1$. This example too can generate high leverage ratios as Pareto-optimal; for instance, a debt-equity ratio of 4 is generated if $\theta = \exp(5)$. $\lhd$

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