Public Private Partnerships: 
Information Externality in Sequential Investments

Marco Buso

Abstract

This paper studies the benefit coming from bundling two sequential activities in a context of Public Private Partnerships (PPPs). Differently from previous literature, I introduce a source of asymmetric information in the form of an externality parameter linking the building stage with subsequent operational activity.

Within this framework, PPPs allow the government to extract private information about the sign and magnitude of the externality parameter and to minimize the informational rents needed to incentivize the builder’s effort.

Our results suggest how PPPs can become those commitment devices that force governments to define more coherent and informed plans that optimize the first period welfare, improving investment to reduce unexpected ex post costs (cost overruns).

Keywords: agency theory; information externality; sequential investment; bundling

JEL classification: D86, L33, H11, H57, C61
1 Introduction

The realization of a public infrastructure with the aim of providing services to citizens represents a long-term project characterized by several complexities that must be adequately taken into account to achieve satisfactory results. Decisions about the optimal strategy to realize these investments have increasingly come to coincide with a choice between a traditional procurement (TP) mechanism and public-private partnerships (PPPs). In the case of TP, the public institution allocates the several stages of the project to different private firms ( unbundling), but it remains the owner of the infrastructure, the only financer and it is fully accountable for poor results. In the case of PPPs, a single private consortium made up of several firms is in charge of realizing and managing the infrastructure (bundling). It can assume the role of financer and the risks are optimally shared between public and private partners.

The literature has highlighted several advantages of PPPs over TP. First, the bundling mechanism can create a stronger incentive to innovate and invest in the quality of the infrastructure during the building stage (Martimort and Pouyet, 2008; Iossa and Martimort, 2008). Second, a PPP optimally allocates risks between the two sectors. Consequentially, the private partner has a stronger incentive to apply more effort during the operational stage. Third, this option is able to attract private financers that can act as monitors reducing the information asymmetry between the government and the private agent (Iossa and Martimort 2011).

Since the 1990’s, real world PPPs have become common in most developed and developing countries, in a wide array of sectors and typologies; from standard projects where the main sources of revenues were user fees (e.g., motorways, parking facilities, public transport) to very complex projects in which the private profits came essentially from government subsidies (e.g., hospitals, schools, prisons).

Differences between theory and practice arise from the interpretation of the bundling effect. The theory emphasizes the potential benefits of PPPs in exploiting the presence of ex ante positive externalities (Iossa and Martimort, 2008, Martimort and Pouyet, 2008). The practice reveals, instead, the capacity of PPPs to optimally use a particular private expertise for providing public services; i.e., the ability to recognize innovative channels linking the different stages of the project. This paper restores a connection between the theoretical background and the practical evidence. Indeed, the analysis describes a model in which the working agents privately know how and how much the two sequential investment stages are related (differently from Martimort and Pouyet, 2008 and Iossa and Martimort, 2008). Applications of this theoretical proposition to real world cases are represented by all those contractible innovations related to the building stage of the project whose impact on the succeeding managerial activity is not ex ante perfectly recognized by the government; e.g., the development of an automatic metro, the improvement of the road’s safety net,
the realization of sustainable (green) public buildings. In such cases, public administrations know their own needs, but not what is the best way to achieve their objectives (Iossa and Russo 2008). Long-term commitment devices, such as PPPs, could grant institutions, politicians and administrative workers more useful decision-making information.

The model is developed in an asymmetric information framework and it is based on the “new economics of regulation” and contract theory approach \(^2\). The theoretical methodology assumes that the principal (government) is able to write contracts contingent on realization of contractible variables that are verifiable ex post \(^3\). The “project” consists of two sequential stages that are connected through a production externality that links the operational costs with the first phase outcomes. The sources of asymmetric information are not limited to the externality parameter (hidden information), they also include the efforts of the private agents that are not directly verifiable by the principal (hidden action). The government must choose between TP and PPPs with the aim of maximizing the net social welfare produced by the investment taking into account the shadow cost of public funds induced by the transfer of rents from the public to the private agents. The final results reveal a potential ex ante advantage to choosing PPPs that increases with the extent to which future private information is uncertain to the public buyer \(^4\). The analysis has been further generalized by considering, as a robustness check, the possibility of the government executing, under TP, simultaneous contracts with both agents at the beginning of the project. The benefit of PPPs partially holds in the new framework if the shadow cost of public funds is positive.

The study detects a general theoretical result that can be applied to different practical contexts. The paper is developed in a framework of PPPs for the realization of public infrastructures. However, the analysis may be extended to all cases where a principal has to assign connected sequential activities to more informed private agents; e.g., electricity, gas, water markets \(^5\).

2 Review of related literature

The literature on PPPs is focused on identifying how problems, like incompleteness of contracts and asymmetric information, influence the organizational management of a multi-period public investment.

The incomplete contract approach is the more developed and it is based on a setting in which a contract is not able to cope with every aspect of the economic relationship between the partners. Starting from this

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\(^2\)This strand of literature applied to the procurement context is connected with the book by Laffont and Tirole (1993).

\(^3\)Even if this hypothesis seems too strong for very complex PPPs, real world experience shows that verification of quality/efficiency may be relatively easier in some sectors (e.g., highway quality) than in others (e.g., hospitals).

\(^4\)That is higher when the variance of the private information parameter is higher.

\(^5\)These sectors are characterized by sequential phases and high levels of asymmetric information that generate large inefficiencies.
insight, the model of Hart (2003) studies the pros and cons associated with a PPP, focusing on the builder’s investment decisions in the current period regarding some public infrastructures, given his commitment to running the infrastructure in a future period. Hart concludes that the PPP is the best solution if the quality of the service can be well identified, while the quality of the building shouldn’t be explicitly stated in the contract.

Following the incomplete contract methodology, Martimort and Pouyet (2008) together with Iossa and Martimort (2008) develop a two-stage model introducing an externality parameter as a connection between the stages of the project. This parameter is known from the beginning and it is negative in cases where the first stage investment increases the second stage costs, while it is positive otherwise. Their conclusion are driven by the externality variable, inasmuch as the bundling mechanism (PPPs), which internalizes costs and benefits related to the second period activity, is socially preferable only when the externality is positive. Martimort and Pouyet (2008) expand their basic model to allow for general schemes, more complete contracts, and introducing an adverse selection issue concerning operating costs. They conclude that, with a benevolent decision-maker and a privately informed operator, the optimal organizational form is still bundling (PPP) when the externality is positive.

The potential advantages of PPPs highlighted by the literature can be partially or totally neutralized in a context of future uncertainty (exogenous shocks) or by considering agency problems within the private consortium. In the first case, the PPP option implies an excessive transfer of risks to a risk adverse consortium and a lack of flexibility induced by early commitment (Iossa and Martimort, 2008, 2011, Martimort and Straub, 2012). In the second case, imperfect bundling leads to a suboptimal privately negotiated incentive structure within the consortium and reduces the scope for welfare-improving PPPs (as compared to Traditional Procurement; Greco, 2013).

Most of the existing literature assumes that the government is able to commit to a multi-period contractual relationship. When that is not the case, the government seeks to renegotiate the contract at the second stage of the project affecting, as a consequence, the first period system of incentives (Guash et al., 2007; Engel et al., 2009; Valero, 2012). This problem creates contract distortions that directly affect investment costs and successful probabilities.

With the introduction in the U.K. of the private finance initiative (PFI) program in 1992, much emphasis has been put on the financial aspect of PPPs. The achievement of “value for money” as well as the attraction of different sources of financing became the main goals of practitioners and public institutions when starting

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6 The builder’s payment depends on the operator’s cost.

7 At the first period contractual stage, the good firm has incentive to mimic the bad one, knowing that the informed principal will be able to extract its surplus at the second period. This problem is known as the “ratchet effect”.

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PPPs. Nevertheless, the theoretical literature mainly analyzes the contractual aspects and implications of these investments, while it is not common to approach the analysis from a financial viewpoint. The main related contributions come from Engel et al. (2010, 2013) and Auriol and Picard (2013). Engel et al. (2013) state the “irrelevance result” according to which there are no public financial advantages of PPPs with respect to TP due to the participation of private financing. Indeed, PPPs cannot be justified by their freeing of public funds inasmuch as public sector current saving in the form of distortionary taxation is perfectly balanced by future losses of public revenue. Auriol and Picard compare a public regime to a build-operate-transfer (BOT) contract for the realization of investment of public and private interests. They build a model within a setting characterized by asymmetric information\(^8\) and in which there is a shadow cost of public funds \((\lambda)\). The analysis highlights a trade off between the public cost of financing a project and higher prices set by concession holders. A major role in addressing this trade-off is played by distortionary taxation. Indeed, there exists a specific threshold \(\lambda_0\) such that BOT contracts are preferred if and only if \(\lambda > \lambda_0\).

Differently from the previous literature, I develop a theoretical model with the presence of both moral hazard and adverse selection. The hidden information problem is related to the externality parameter that connects the two investment stages, while the hidden action issue concerns the non verifiable agents’ efforts.

The study reveals a potential benefit associated with PPPs, in a context of contractible outcomes and incremental innovations. Like contributions to the literature of reference, the paper permits one to better understand how the government investment decision is affected when the externality parameter introduced by Iossa and Martimort (2008) and Martimort and Pouyet (2008) becomes uncertain. Furthermore, the analysis goes farther than Engel (2013) by showing how the shadow cost of public funds comes to be relevant in driving the government choice between TP and PPPs, in a context of sequential investments of public interest with multiple sources of asymmetric information.

The paper is organized as follows: Section 3 lays out the model; Sections 4 and 5 discuss the unbundling and the bundling scenarios; Section 6 analyzes the net surplus produced by the different scenarios through a welfare analysis; Section 7 concludes.

### 3 The Model

The government aims at the realization of a public infrastructure able to provide services for its citizens. The project is made up of two stages: the construction of the public asset and the provision of services.

\(^8\)The concession holder faces a much weaker information asymmetry with their own manager compared to the government.
The realized facility generates a social surplus equal to $CS = S_0 + S * I(e_1)$. The surplus can be divided into two components: a constant term ($S_0$) that depends on the realization of the basic infrastructure and a second part that linearly depends on an incremental innovative investment $I(e_1)$ that is contractible and increasing with the builder’s effort ($I(e_1) = e_1 + \epsilon$) where $\epsilon \sim g(0, \sigma_\epsilon^2)$ and $g(\epsilon) \sim [\epsilon_l, \epsilon_u]$. The effort ($e_1$) is not verifiable by the government and it implies a non-monetary disutility for the builder equal to $\psi(e_1)$ that, by assumption, satisfies the following properties: $\psi' \geq 0$, $\psi'' \geq 0$ and $\psi(0) = 0$.

In the analysis the government is assumed to be benevolent and able to commit to a long term project. It acts as a principal and writes the contracts to maximize the social welfare function.

$$W^G = S_0 + S * I(e_1) + U - (1 + \lambda)T$$

(1) The function is the sum of social surplus ($CS = S_0 + S * I(e_1)$) and the firms’ utilities ($U$), net of the government’s expenses ($T$) weighted by the shadow cost of public funds ($\lambda$) that captures distortion imposed on taxpayers to collect the money needed for the investment.\(^9\)

The first stage of the project is entrusted to a builder whose utility is defined as follows:

$$U_1 = T_1 - \psi(e_1)$$

(2) The builder is in charge of the construction of the basic infrastructure, which entails a fixed cost that is totally reimbursed and a non-monetary disutility of effort. As compensation, he receives a transfer that increases with the level of innovation introduced in the project.

The second stage activity is assigned to an operator who receives, in return for his services, the following utility:

$$U_2 = T_2 - [O - e_2(\theta) - I(e_1)\theta] - \psi(e_2)$$

(3) The return to the operator is composed of the gross transfer from the government net of the monetary cost ($C_2(\theta) = O - e_2(\theta) - I(e_1)\theta$) and a non-monetary disutility of effort ($\psi(e_2)$). The monetary cost is verifiable and observable. It is determined by: the fixed part $O$; the cost-reducing effort $e_2$; and the impact of the first stage investment on the second phase. This last effect is driven by $\theta$, which reflects the private information of

\(^9\)The model follows the approach used in the procurement model of Laffont and Tirole (1986)
the operator. This parameter defines whether or not the builder’s investment increases (negative externality \( \theta < 0 \)) or reduces (positive externality \( \theta > 0 \)) the operational costs\(^{10}\). The agent is able to acquire this information during the building stage when the main features of the infrastructure become observable (this parameter can represent the combination of private information of the builder and the operator about the cost to manage the realized asset)\(^{11}\). The government cannot directly detect the private information, but it can observe the distribution of the variable over a range of values: \( f(\theta) \sim [\theta^l, \theta^h] \) where \( \int_\theta^{\theta^h} \theta f(\theta) d\theta = \bar{\theta} \).

For the purpose of this analysis, the possible forms that \( f(\theta) \) can take have been restricted to the class of piecewise differentiable functions that allow the use of optimal control theory. A further standard requirement regards the hazard rate, \( \frac{F(\theta)}{f(\theta)} \), that is assumed monotonic with respect to \( \theta \): \( \frac{d}{d\theta} \frac{F(\theta)}{f(\theta)} \geq 0 \)\(^{12}\). In addition to monetary expenses, the operator experiences a non-monetary cost of applying effort captured by the function \( \psi(e_2) \) that, by assumption, satisfies the same properties as the builder’s effort: \( \psi' \geq 0, \psi'' \geq 0 \) and \( \psi(0) = 0 \).

For the achievement of the project, the government can choose between two possibilities: unbundling (TP) and bundling (PPPs). In the first case the two stages are managed by different firms while, in the second case, there is a single private consortium that takes care of both stages.

4 Unbundling

Within the unbundling (TP) scenario, the government chooses to undertake the two stages of the project through different agents: the builder and the operator. These players act autonomously and the government offers two distinct contracts.

In the first stage the government wants to maximize the builder’s effort, but it must cope with a problem hidden action. Therefore, it can only offer an incentive contract based on the level of observable and verifiable outcomes, i.e., a proportional transfer \( \{T_1 = I(e_1)t_1\} \) linking the builder’s compensation with the investment output \( I(e_1) \).

In the second stage the relationship between the principal and the agent is influenced by problems of both hidden information and hidden action. The government can offer a menu of incentive-feasible contracts based on the verifiable outcome and induce truthful revelation of the firm’s cost parameter, that is a revelation

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\(^{10}\)For instance, an automatic metro may reduce the need for drivers (positive externality). Nevertheless, innovative designs or materials for the construction of sustainable public buildings can increase social surplus, but also maintenance costs (negative externality).

\(^{11}\)This parameter can capture, for example, the impact of the development of an automatic metro on operational costs. The government can forecast what will be the effect of this innovation, but only the operator is able to perfectly compare the saving of costs in the form of lower drivers’ salaries with the potential increase in expenses in the form of organizational adaptations and new professional workers’ salaries.

\(^{12}\)This assumption implies that effort decreases with agent inefficiency.
mechanism \(\{t(\theta), C(\hat{\theta})\}_{\theta \in [\theta_l, \theta_h]}\) that defines the cost that the firm has to realize and the net transfer it will receive when the cost parameter \(\hat{\theta}\) is announced\(^\text{13}\):

In this scenario, the contractual agreements are signed according to the following timeline:

![Timeline diagram](image)

The purpose of the government is to characterize the perfect Bayesian equilibria of the overall game; the solution of this problem is standard and is found using backward induction.

**Second stage of the game**

In the second stage the principal writes a contract that gives the operator incentive to apply effort in exchange for the minimum payment of money: a rent-efficiency trade off.

\[
\max_{e_2(\theta)} W^G_2 = \int_{\theta_l}^{\theta_h} \{U_2(\theta) - (1 + \lambda)[t_2(\theta) + C_2(\theta)]\}dF(\theta)
\]

s.t.

1- \[
\frac{dU_2}{d\theta} = -\psi'(e_2(\theta))
\]

2- \[
U_2(\theta^h) = 0
\]

**Proof.** See Appendix 2

The government maximizes the net social welfare related to the second period taking into account the agent’s information constraints. The contract is offered at the ex post stage, once the agent already knows his type. As a consequence, the participation constraint is binding for the most inefficient operator, who must receive at least his reservation utility (normalized to 0) to accept the contract [2]. Additionally, the government must structure the transfers to optimally induce the truthful revelation of the private agent’s information [1]\(^\text{14}\).

The problem’s solution includes the following optimal level of effort:

\(^\text{13}\)As usual, we know from the revelation principle that any regulatory mechanism is equivalent to a direct revelation mechanism that induces a truthful revelation of the firm’s cost parameter. This regulatory mechanism can then implement the optimum through a menu of linear contracts (Laffont and Tirole, 1993).

\(^\text{14}\)Otherwise, the efficient type would have incentive to mimic the inefficient agent.
$$e_2^U(\theta) = \psi^{-1}[1 - \frac{\lambda}{1 + \lambda} \frac{F(\theta)}{f(\theta)} \psi''(e_2^U(\theta))]$$  \hspace{1cm} (4)$$

The result is standard for the literature of reference. Indeed, it underscores the role played by asymmetric information that lowers the equilibrium value of effort, compared to the first best results (Appendix 1). If the monotone hazard rate property $d(F(\theta)/f(\theta))/d\theta \geq 0$ holds, the solution for effort is decreasing in $\theta$. Therefore, the distortion is lower the more efficient the firm. On the other hand, all the firms, except the least productive, receive positive utility, and the more efficient the agent, the higher his information rent.

**First stage of the game**

At the beginning of the first period the government is able to propose a contract that specifies both the parameters defining first period social welfare and the expected second stage value function ($[V_2]$ - the discounted surplus related to the managerial activity; see Appendix 2).

$$\max_{e_1} \int_0^h \{[(S_0 + I(e_1) * S) + U_1(e_1) - (1 + \lambda)T_1] + [V_2]\} g(\epsilon) d\epsilon$$

s.t.

1- \hspace{0.5cm} e_1 = \arg\max_{e_1} E[U_1] \\
2- \hspace{0.5cm} E[\epsilon_T - \psi(e_1)] \geq 0$$

**Proof.** See Appendix 2

The objective function of the government is composed of the social surplus deriving from the realization of the infrastructure, the expected second stage value function, and the builder’s payoff net of the government’s costs, weighted by the shadow cost of public funds. The random shock $\epsilon$ is realized after the conclusion of the contract. Therefore, the participation constraint is defined ex ante [2]. The incentive compatibility constraint [1] takes into account the optimal effort choice for a given contract that comes from the maximization of the agent’s ex ante utility. The problem’s solution includes the following result:

$$\psi'(e_1^U) + \lambda \frac{dT^*}{de_1} = S + (1 + \lambda)\bar{\theta}$$  \hspace{1cm} (5)$$

where $\frac{dT^*}{de_1} = \psi'(e_1^U) + \psi''(e_1^U)e_1^U$
The first order condition equalizes the expected marginal benefit (right hand side) with the marginal cost (left hand side). Increasing the level of effort creates a current benefit for the society as well as a possible future saving of operating costs when the expected externality between the two stages is positive ($\theta > 0$). On the other hand, a greater level of effort makes the operator suffer a higher non-monetary disutility, and marginally increase the transfer at the optimum to the private agent. The main parameters entering Equation 5 are the expected externality value ($\theta$) and the shadow cost of public funds ($\lambda$). $\theta$ affects the marginal benefit negatively or positively, depending on whether the investment realized during the first phase increases or decreases (in expectation) the costs needed to manage the infrastructure. $\lambda$ captures the distortion imposed on taxpayers when public money is transferred to the private builder ($\lambda \frac{dT^*}{de_1}$). Moreover, it enlarges the expected positive or negative impact of the externality parameter $((1 + \lambda)\theta)$.

### 4.1 A Robustness check: simultaneous contracts

The previous analysis describes what normally happens in real world situations in which public procurement contracts related to multi-stage projects are signed and outcomes of previous stages are already observed by both parties. As an alternative to this strategy, the government could define ex ante, i.e., before investment begins, all the contracts with multiple agents. This option is theoretically feasible, but, practically, it is not implemented. Indeed, linking an agent’s obligations to future outcomes of different contracts is normally not allowed by the legal system.

In this section, I expand the previous framework, allowing for the possibility of writing simultaneous contracts at the beginning. This is a theoretical exercise that functions as a robustness check on the preceding analysis of the unbundling structure. In this scenario, the timeline of the game is as follows:

![Timeline Diagram]

**Proof.** See Appendix 2

The results finally obtained are described by the following equations.

$$\psi'(c_1^S) + \lambda \frac{dT^*}{de_1} = S + (1 + \lambda)\theta \quad (6)$$

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15 If the expected externality is negative ($\theta < 0$), the total level of expected marginal benefit decreases.
where \( \frac{dT}{de_1} = \psi'(e_1^S) + \psi''(e_1^S)e_1^S \)

\[
e_2^S(\theta) = \psi^{-1}\left[1 - \frac{\lambda}{1 + \lambda} f(\theta)\psi''(e_2^S(\theta))\right]
\]  

(7)

Equation 6 describes the builder’s optimal effort. Compared to the sequential contracts scenario, the marginal benefit is different. Indeed, the positive or negative impact of the first stage investment to the second stage costs is driven by the real value of \( \theta \) which is actually defined in the ex ante contract between the government and the operator. Equation 7 describes the operator’s optimal effort, which does not change from the sequential contracts case\(^{16}\).

5 Bundling

Within this setting the approach and the initial assumptions are very similar compared to the unbundling scenario. There is a single private agent (consortium) that sustains a cost over the two periods dependent on the same parameters as before, and based on an ex ante information structure that does not change with the new environment. The consortium receives compensation for its activities, which is defined as the sum of the builder’s and the operator’s utilities:

\[
U_B = T_1(e_1) + T_2(\theta) - C(\theta) - \psi(e_1) - \psi(e_2(\theta))
\]  

(8)

The government can offer, in this case, a menu of incentive-feasible contracts based on verifiable outcomes that must induce truthful revelation of the operator’s cost parameter and enhance first period investment, i.e., a triplet, \( \{t_1(\hat{\theta}), t_2(\hat{\theta}), C_2(\hat{\theta})\}_{\theta \in [\theta_l, \theta_h]} \), which respects the incentive constraints and defines costs and transfers when the private parameter \( \hat{\theta} \) is announced.

In this scenario, the time-line of the game takes the following form:

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16 In this section, I implicitly assume that the government cannot commit to leaving the agent with a negative ex post utility. This hypothesis is plausible given that the operator gets the private information before the activity starts. Nevertheless, if this is not the case, the government’s optimal strategy consists of offering a fixed price contract: \( t_2(C_2) = a - C_2 \), where \( a = \int_{\theta_l}^{\theta_h} \psi(e_2(\theta)) + C_2(\theta) \, d\theta \). The operator, as residual claimant, makes the efficient decision, receives no rents in expectation and takes the risk of having a negative ex post utility. Taking the analysis in this direction does not change the core results of the analysis.
The screening strategy proposed by the government forces the consortium to truthfully reveal its private information when it becomes observable. The government maximizes the net social welfare produced by the project over the two periods taking into account the incentive-feasible constraints.

\[
\begin{align*}
\max_{e_1, e_2(\theta)} & \int_{\Theta} \left\{ [S_0 + I(e_1) \ast S] + U_B - (1 + \lambda)[I(e_1)t_1 + t_2(\theta) + C_2(\theta)] \right\} f(\theta) d\theta \right\} f(\epsilon) d\epsilon \\
\text{s.t.} & \\
1- & e_1 = \arg\max_{e_1} E[U_B] \\
2- & \frac{dE[U_B]}{d\theta} = -\psi'(e_2(\theta)) \\
3- & E[e[I(e_1)t_1 - \psi(e_1) + t_2(\theta) - \psi(e_2(\theta))] \geq 0
\end{align*}
\]

**Proof.** See Appendix 3

The government’s goals are the maximization of social surplus and the agents’ extraction of rent. On the other hand, it must take into consideration the firms’ incentives and interests, which are embodied in the three constraints. The first equation incents the agent to apply effort during the building stage [1]. The second equation represents the mechanism needed to obtain a truthful revelation of the private information parameter [2], while the third equation reflects the participation constraint, which is binding for the more costly type [3]. The maximization solution leads to the following outcomes:

\[
\psi'(e_1^B) + \lambda \frac{dT_B}{de_1} = S + (1 + \lambda)\theta
\]

where \( \frac{dT_B}{de_1} = \psi'(e_1^B) \)

\[
e_2^B(\theta) = \psi'^{-1}\left[1 - \frac{\lambda}{1 + \lambda} \frac{F(\theta)}{f(\theta)} \psi''(e_2(\theta))\right]
\]

The operator’s level of effort (Equation 9) does not change at the optimum, compared to its level under unbundling\textsuperscript{17}. Equation 10 reports the builder’s effort. Differently from the unbundling scenario, the contract with the consortium is made before the start of the investment. Therefore, thanks to the revelation

\textsuperscript{17}As in the unbundling case with simultaneous contracts, in this section, I implicitly assume that the agent cannot commit not to exit the contract when he discovers \( \theta \). This is plausible given that the operator gets the private information before the activity starts. Nevertheless, if it is not the case, the government’s optimal strategy consists of offering a second term fixed price contract: \( t_2(C_2) = a - C_2 \), where \( a = \int_\Theta \{\psi(e_2(\theta)) + C_2(\theta)\} d\theta \). In such a situation, the bundling mechanism could further increase the builder’s effort, inasmuch as the distortion induced by adverse selection will disappear. Alternatively, if the analysis is generalized to relax this assumption, the potential benefits of PPPs will increase.
mechanism, the information becomes contractible since the first stage and the optimal level of builder’s effort can be set on the basis of the real value of $\theta$ announced by the operator. A further difference with the unbundling context (both sequential and simultaneous contracts) comes from the distortionary cost that the society incurs for a marginal transfer from the government to the private agent. In the case of PPPs, the principal has the opportunity to leave the consortium with no ex ante private rent at the optimum lowering the total marginal cost (left hand side of Equation 9). This is possible because the government has the further ability, compared to TP, of optimally setting the incentive compatibility transfers, while keeping the ex ante participation constraint (over the two periods) binding.

Similarly to the previous case, the main parameters entering the outcomes equations are the shadow cost of public funds $\lambda$ and the externality parameter $\theta$. $\lambda$ increases the marginal cost of effort, and it magnifies the effect of the externality. $\theta$ captures the positive or negative impact of the first stage investment on the second period costs. In contrast to the unbundling scenario, in this case, the builder’s investment decision is directly affected by $\theta$, and not only by its average value.

6 Welfare analysis

In this section we compare the bundling and the unbundling scenarios in terms of ex ante social welfare, with the purpose of identifying the factors that drive the choices of governments facing informational settings similar to the one modeled in this paper.

The Welfare analysis is performed using the expected value of the objective function of the government over the two periods: $E_{\theta,\epsilon}[S_0 + S \cdot I(e_1) + U_1 + U_2 - (1 + \lambda)(t_1 + t_2 + C_2)]$. In this paragraph, it is assumed that $\psi(e_1) = \frac{e_1^2}{2}$ and $\psi(e_2) = \frac{e_2^2}{2}$. These functions respect the initial assumptions of the model and allow us to compute the value of the agents’ efforts at the optimum. The analysis is reported in Appendix 4. The result is summarized by the following formula, which describes the difference in value functions between the levels of social welfare produced under bundling and unbundling:

$$V_B - V_U = W_n = \{RS\} + \{IE\} = \left\{ \frac{\lambda}{2(1 + \lambda)(1 + 2\lambda)} (S + (1 + \lambda)\bar{y})^2 \right\} + \left\{ \frac{(1 + \lambda)}{2} \sigma_\theta^2 \right\}$$

where

$$\begin{align*}
RS &= \frac{\lambda}{2(1 + \lambda)(1 + 2\lambda)} (S + (1 + \lambda)\bar{y})^2 - \text{Rent Saving} \\
IE &= \frac{(1 + \lambda)}{2} \sigma_\theta^2 - \text{Information Externality}
\end{align*}$$
The result can be decomposed in two effects that are explained in detail in the following definitions:

**Rent saving effect (RS):** This effect is always positive or equal to zero. It reflects the ex ante marginal benefit to the society \((S + (1 + \lambda)\theta)\) deriving from the builder's higher investment under bundling, when \(\lambda > 0\). PPPs allow the principal to recover the first period incentive rents during the lifetime of the investment without affecting the second period incentive compatibility constraint. As a consequence, the government can optimally align the agent's incentives and maximize the total extraction of rents\(^{18}\). This effect is meaningful inasmuch as the contracts are costly for society \((\lambda \geq 0)\). Moreover, the additional transfer of risks to the private consortium does not affect the agent's utility, because of the assumption of risk neutrality. The introduction of a coefficient of risk aversion would have changed the final results.

**Information externality effect (IE):** This effect is always positive or equal to zero. PPPs commit the government to define a more informed investment plan, taking into account every short term and long term relationships between the builder's investment and the future stages of the project. Precisely, bundling the two tasks allows the government to internalize the operator’s private information in the builder's innovative investment. This effect increases with the uncertainty of the private information parameter; therefore, if the variance decreases, the private information is less valuable to the operator, and there is a lower benefit from choosing PPPs.

These effects lead to the following proposition, which summarizes the main result of the paper:

**Proposition 1:** In a context characterized by non-verifiable effort and hidden information on \(\theta\) (the externality is uncertain for the government), bundling strictly dominates unbundling when the following conditions are satisfied:

- Long term contracts based on verifiable outcomes can be signed;
- Agents are risk-neutrals;
- Either the shadow cost of public funds or the variance of \(\theta\) is different from zero.

Proposition 1 emphasizes the role of the externality parameter \(\theta\) as well as the influence of the shadow cost of public funds \(\lambda\). Figures 1 and 2 help one better understand the effects of these parameters on the government welfare function.

\(^{18}\)The rent efficiency trade off is more slack.
The graphs display the net welfare surplus of the government ($W_n = V_B - V_U$) and its components (the RS and the IE effects) with respect to $\lambda$ and $\bar{\theta}$ (the expected externality parameter).

The IE effect linearly increases with $\lambda$, while the RS dynamic is less clear. When the externality is positive or equal to 0, the RS effect initially rises with the shadow cost of public funds, but it can exhibit negative dependence for sufficiently high values of $\lambda^{19}$. On the other hand, when the externality is negative, the threshold value of $\lambda$ that determines a change in the direction of this relationship is lower$^{20}$ (Proof: See Appendix 5). The net surplus $W_n$ that is the result of the two effects is higher than 0 for every value of $\lambda$, and...

$^{19}$ When $\lambda$ is very high, the level of optimal innovative investment decreases; there is less need of incentive rents. Therefore, the potential benefit of bundling in terms of rent-saving is lower.

$^{20}$ When the externality is negative, a rise in $\lambda$ increases the negative effect of first stage investment on operational costs. This negative impact is internalized by the government in the form of lower builder’s investment. As a consequence, the incentive transfers decrease and the potential benefit of bundling in terms of rent-saving is lower.
it follows a trend that is either always increasing or initially increasing and decreasing thereafter, depending on which of the two effects dominates. Figure 2 shows the dynamics of $W_n$, IE and RS effects with respect to $\theta$. The highest gain in the form of net surplus by choosing PPPs is reached when the expected externality is positive or very negative (Proof: See Appendix 5).

When PPPs are compared to TP mechanisms, several factors must be taken into consideration, such as: the degree of completeness of the contracts; the presence of asymmetric information; the degree of agents’ risk aversion; the level of production uncertainty. This model does not aim to completely explain what drives a government’s choice between PPPs and TP, but it develops two points previously introduced by the theoretical literature on PPPs: the externality parameter (Iossa and Martimort, 2008 and Martimort and Pouyet, 2008) and the basic public finance of PPPs (Engel et al., 2013; Auriol and Picard, 2013).

Differently from the previous analyses, it has been assumed that the government does not know ex ante the real impact of the first stage investment on operational costs. This assumption is plausible in contexts where the first period investment is not standard, but its short term and long term effects are ex ante measurable. Thus, the government cannot rely on previous experience, while the agent is able, through its better market knowledge, to assess the long term implications of the investment. Within this framework, bundling dominates unbundling not only when the first stage investment decreases the operational costs (Iossa and Martimort, 2008; and Martimort and Pouyet, 2008), but also when the externality is negative. Indeed, bundling two tasks allows the government to extract through the contract, ex ante, the private information of the operator. Considering this externality, the principal has the opportunity to establish a more informed investment plan capable of either incenting the builder’s effort, when the externality is positive, or avoiding excessive operational costs and rents, when the externality is very negative. It is no longer the sign that is the main driver of the analysis, but the results are finally determined by the variance of the externality parameter and by the size of the expected externality value (either negative or positive)\textsuperscript{21}. Whether it is the sign, the size or the variance of the externality that matters for the optimal organizational choice depends on the presence of asymmetric information about $\theta$, on the level of completeness of the contract and on the degree of risk aversion of the private agents.

As a further contribution, this paper focuses on the impact of PPPs on the government budget. Engel et al. (2013) show that PPPs do not release public funds. Therefore, the distortionary cost of taxation is not a rationale for the use of PPPs. This paper goes farther in the analysis, introducing a situation of multiple asymmetric information in multi stage public investment. In such a context, PPPs commit the agent not to exit the contract, accounting for the fact that he can experience a second period incentive.

\textsuperscript{21}Fig. 2 shows how the net welfare function of the government increases when the size of the expected externality (positive or negative) is greater.
transfer of money. As a consequence, the government can optimize first stage investment, maximizing total rent extraction. Thus, the relevance of $\lambda$ is driven by the asymmetry of information, and it depends on the government’s ability to optimally enhance the builder’s investment at minimum cost. Elements that do not affect the analysis are, instead, the share of private financing and the nature of the agents’ revenues (user fees or government transfers)\footnote{These final results are in accordance with Engel et. al. (2007) and come from a different channel with respect to the one highlighted by Auriol and Picard (2013).}.

6.1 A Robustness check: simultaneous contracts

In the unbundling scenario, the government can theoretically execute the contracts with both the builder and the operator before starting the project. In such a framework, the welfare function produced under unbundling is modified. As a consequence, the difference in value functions between the two scenarios changes (Appendix 5), and this is summarized by the following formula:

$$V_B - V_U^S = \{RS\} + \frac{\lambda}{(1 + 2\lambda)} \{IE\} = \left\{ \frac{\lambda}{2(1 + \lambda)(1 + 2\lambda)} (S + (1 + \lambda)\theta)^2 \right\} + \left\{ \frac{\lambda(1 + \lambda)}{2(1 + 2\lambda)} \theta^2 \right\} \quad (12)$$

When compared to the standard case, we observe that the two effects still hold in the new environment, but the IE impact is weakened. Indeed, if simultaneous contracts can be written, the government is also able to extract the private information of the operator in the unbundling scenario ex ante. Thus, the benefits of PPPs decrease. What drives the result described by Equation 12 is the hidden action problem that complicates the building stage. In fact, under PPPs, the government can optimally make the first stage investment at minimum cost, while, under TP, it is more costly for the government to define the incentive mechanism and, hence, the impact of the externality on the optimal builder’s investment is also reduced. The main consequences of this change are described by the following proposition:

**Proposition 2:** In a context characterized by simultaneous contracts, non-verifiable effort and hidden information on $\theta$, bundling strictly dominates unbundling when the following conditions are satisfied:

- Long term contracts based on verifiable outcomes can be signed;
- Agents are risk-neutral;
- The shadow cost of public funds ($\lambda$) is strictly positive.

Proposition 2 differs from Proposition 1 essentially in the role played by the shadow cost of public funds.
Indeed, where $\lambda$ is equal to 0, the two scenarios are identical. In conclusion, when simultaneous contracts are written, the difference in value functions between bundling and unbundling is less dependent on the variance of the externality parameter ($\sigma^2_\theta$), but more related to the shadow cost of public funds ($\lambda$).

7 Conclusion

This paper discusses the choice between PPPs and TP mechanisms for the realization and management of long term projects in contexts where the future consequences of short term investment are uncertain and governments do not share all the relevant information known to private agents.

The study focuses on the role of the externality between the building and the operational stage, treating it as the main source of asymmetric information. Differently from previous results in the literature, in the current analysis, the comparison between PPPs (bundling) and TP (unbundling) does not depend only on how and how much the different activities of the project are related, but it is also related to prior government knowledge. More precisely, when the effects of first stage investment on the operational stage are very uncertain, the government is pushed toward PPPs, which permit it to extract, ex ante, the agent’s private information and to optimally control, as a consequence, the builder’s investment.

When looking at real world cases, this effect can help us to understand why cost overruns are lower when PPPs are used (EPEC 2011, EIB 2005). In infrastructure projects, estimated costs are often different from their ex post realizations. One explanation for this gap derives from the substantial uncertainty that characterizes ex ante evaluations. PPPs allow the government to collect more information at the beginning of the project. As a consequence, ex ante evaluations are more precise and large cost overruns less likely.

Another result of the analysis is that, the shadow cost of public funds ($\lambda$), is relevant to social welfare. In fact, when asymmetric information is introduced in projects with interrelated sequential stages, PPPs facilitate government’s task, allowing it to put in place an incentive mechanism that takes the two stages of the project into account at the beginning. Accordingly, incentives are more aligned, and performance rents become less costly to society.

The implication is that PPPs are particularly advantageous when long term risks associated with public investments are not easily assessed ex ante by governments, but performance can be verified ex post. Neither standardized projects nor R & D investments meet this requirement. What matters is both the ability of private agents to use their special expertise to evaluate innovative projects and the ability of governments to execute contracts on the basis of future outcomes. (All projects that involve new applications of existing innovations are suitable examples of the case of interest.) The advantage of PPPs increases when the initial
investment is particularly important, in expectation, to the second period activity, or in situations of public budget stress. In both cases, PPPs imply substantial gains in the form of fewer cost overruns and lower incentive rents.

Several extensions could enrich the current analysis.

Different degrees of contracts’ completeness could be introduced. Depending on the ability of governments to write contracts based on verifiable future outcomes, the analysis would be more or less driven by the sign of the externality (incomplete contract - Iossa and Martimort, 2008; Martimort and Pouyet, 2008) or by the variability of the externality (more complete contracts).

In this paper, I have shown how governments, using PPPs, can optimally put in place an incentive mechanism maximizes the agents’ rent extraction. As a consequence, the private consortium is left with a binding ex ante participation constraint, while its ex post utility, compared to the TP scenario, is more likely to decline. This additional uncertainty does not represent a cost in a context of risk neutrality, but if agents are risk averse the benefit of PPPs’ in terms of rent saving will be lower.

In the model it has been assumed that there is no divergence of objectives within the consortium. Therefore, perfect sharing of information and profits among consortium participants can be attained. As a further refinement, this assumption could be relaxed (Greco 2013). In such a case, PPPs would guarantee neither the complete extraction of information nor the perfect cost-minimizing alignment of incentives. The degree to which the results came to be relevant in this more general framework would depend on the information structure within the private consortium.

It is important to highlight that the preceding study is based on the assumption that the government can commit to long term contracts. If this is not the case, the principal has the incentive to extract all the second stage rent after the revelation of the operator type, upon execution of the initial contract. Anticipating this strategy, the consortium seeks to maximize its first period payoff, and a separating contract is not always implementable (ratchet effect)\(^ {23}\). What would be an interesting refinement of the preceding analysis would be an extension designed to assess the robustness of the final results to different degrees of government commitment.

Finally, the paper has been developed using the PPP as a benchmark. Nevertheless, the model detects a general theoretical result that can be applied to different frameworks. Electricity, gas and water markets are feasible contexts given the complexity of the sequential supply system and the presence of high levels of asymmetric information. Following these extensions, the analysis can provide a larger contribution in terms of real world applications.

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\(^ {23}\) This problem has already been investigated by Valéro (2012), who shows how, even under government opportunism, there is the possibility of welfare improving PPPs.
References


Appendix 1: First Best Benchmark

The government maximizes the total welfare function, there are no problems of hidden information or hidden action

\[
\max_{e_1, e_2(\theta)} \int_{-\infty}^{\infty} \{S_0 + I(e_1) * S - (1 + \lambda)[I(e_1)t_1 + t_2(\theta) + C_2(\theta)] \\
+ I(e_1)t_1 + t_2(\theta) - \psi(e_1) - \psi(e_2(\theta))\} g(\epsilon) d\epsilon
\]

The government can totally extract agents’ rents

\[
\max_{e_1, e_2(\theta)} \int_{-\infty}^{\infty} \{S_0 + I(e_1) * S - (1 + \lambda)[\psi(e_1) + \psi(e_2(\theta)) + C_2(\theta)]\} g(\epsilon) d\epsilon
\]

Optimizing w.r.t. \(e_1\) and \(e_2(\theta)\) yields respectively

\[
\psi'(e_1^{FB})(1 + \lambda) = S + (1 + \lambda)\theta
\]
\[
\psi'(e_2^{FB}(\theta)) = 1
\]

Appendix 2: Proof of the Unbundling Problem

Let us solve the problem backward

Second stage of the game

The government maximizes the second stage welfare function taking into account the operator’s incentive constraints:

\[
\max_{e_2(\theta)} \int_{\theta}^{\theta_h} \{t_2(\theta) - \psi(e_2(\theta)) - (1 + \lambda)[t_2(\theta) + O - e_1(\theta) - e_2(\theta)]\} dF(\theta)
\]

s.t.

\[
1 - \frac{dU_2}{de_2(\theta)} = -\psi'(e_2(\theta))
\]
The ex post participation constraint is binding for the least efficient agent. The incentive compatibility constraint that derives from the application of the envelope theorem allows us to compute the agent’s utility

\[ U(\theta) = \int_0^{\theta_h} \psi(e_2(\tilde{\theta})) d\tilde{\theta} + U(\theta_h) \]

Integrating by parts we can compute the expected rent granted to the operator by the principal

\[ \int_0^{\theta_h} \{U(\theta)\} f(\theta) d\theta = \int_0^{\theta_h} \{ \int_{\theta_h}^{\theta} \psi(e_2(\tilde{\theta})) d\tilde{\theta} \} f(\theta) d\theta = \int_0^{\theta_h} \{ \frac{F(\theta)}{f(\theta)} \psi'(e_2(\theta)) \} f(\theta) d\theta \]

Substituting the constraints into the government’s function, we obtain the principal’s optimization problem:

\[ max_{e_2(\theta)} W_2 = \int_0^{\theta_h} \{ -(1+\lambda)[O - e_1\theta - e_2(\theta) + \psi(e_2(\theta))] - \lambda \frac{F(\theta)}{f(\theta)} \psi'(e_2(\theta)) \} f(\theta) d\theta \]

Optimizing w.r.t. \( e_2 \) yields to the optimal level of effort, just like is reported in the text

\[ \psi'(e_2^U(\theta)) = 1 - \frac{\lambda}{1+\lambda} \frac{F(\theta)}{f(\theta)} \psi''(e_2^U(\theta)) \]

Substituting the optimal level of effort in the objective function of the government and solving the integral, we obtain the value function of the government

\[ V_2 = -O + (1+\lambda)e_1 \tilde{\theta} - \int_0^{\theta_h} \{ (1+\lambda)[-e_2^U(\theta) + \frac{(e_2^U(\theta))^2}{2}] + \frac{\lambda}{1+\lambda} \frac{F(\theta)}{f(\theta)} \psi''(e_2^U(\theta)) \} f(\theta) d\theta \]

**First stage of the game**

The government maximizes the sum of the first stage welfare function and the second stage value function taking into account the builder’s incentive constraints:
\[ \max_{e_1} \int_{t_1}^{e_1} \{S_0 + I(e_1) \ast S - (1 + \lambda)[I(e_1)T] + I(e_1)t_1 - \psi(e_1) + V_2\} g(\epsilon) d\epsilon \]

s.t.

1- \[ \frac{dE_e[U_1]}{de_1} = 0 \]

2- \[ E_e[I(e_1)t_1 - C_1 - \psi(e_1)] \geq 0 \]

From the incentive compatibility constraint we can compute the marginal transfer at the equilibrium \( t_1 = \psi'(e_1) \); from the participation constraint we obtain the ex ante expected utility: \( E_e[U_1] = e_1t_1 - \psi(e_1) \).

Substituting into the government’s function, we get the principal’s optimization problem:

\[ \max_{e_1} W_1 = e_1 + S - (1 + \lambda)[e_1 \psi'(e_1)] + e_1 \psi'(e_1) - \psi(e_1) + V_2 \]

Optimizing w.r.t. \( e_1 \) yields

\[ \psi'(e_1^U) + \lambda[\psi'(e_1^U) + \psi''(e_1^U)e_1^U] = S + (1 + \lambda)\bar{\theta} \]

where \( [\psi'(e_1^U) + \psi''(e_1^U)e_1^U] = \frac{dT^*}{de_1} \)

A Robustness check: simultaneous contracts

From a theoretical point of view, the government can offer both the operator’s and the builder’s contracts before the investment start. Within this framework, the government maximizes the total welfare function taking into account the incentive constraints:

\[ \max_{e_1, e_2(\theta)} \int_{e_1}^{e_2(\theta)} \{S_0 + I(e_1) \ast S - (1 + \lambda)[I(e_1)t_1 + t_2(\theta) + C_2(\theta)] + I(e_1)t_1 + t_2(\theta) - \psi(e_1) - \psi(e_2(\theta))\} f(\theta) d\theta \} g(\epsilon) d\epsilon \]

s.t.

1- \[ \frac{dE_e[U_1]}{de_1} = 0 \]
Substituting the constraints into the government’s objective function we obtain the principal’s maximization problem:

$$\max_{e_1, e_2(\theta)} W_S = \int_{\theta_1}^{\theta_2} \left\{ g \left( S_0 + e_1 S - (1 + \lambda)[e_1 t_1 + \psi(e_2(\theta)) + O - e_1 \theta - e_2(\theta)] + e_1 t_1 - \psi(e_1) - \lambda \left[ \frac{F(\theta)}{f(\theta)} \psi'(e_2(\theta)) \right] \right\} f(\theta)d\theta$$

Optimizing w.r.t. $e_1$ and $e_2(\theta)$ yields respectively

$$\psi'(e_S^1) + \lambda \frac{dF}{de_1} = S + (1 + \lambda) \theta$$

$$\psi'(e_S^2(\theta)) = 1 - \frac{\lambda}{1+\lambda} \left[ \frac{F(\theta)}{f(\theta)} \psi''(e_2^S(\theta)) \right]$$

**Appendix 3: Proof of the Bundling Problem**

The government maximizes the total welfare function taking into account the consortium’s incentive constraints:

$$\max_{e_1, e_2(\theta)} \int_{\epsilon_1}^{\epsilon_2} \left\{ g \left( S_0 + I(e_1) + S - (1 + \lambda)[I(e_1) t_1 + t_2(\theta) + O - I(e_1) \theta - e_2(\theta)] + I(e_1) t_1 + t_2(\theta) - \psi(e_1) - \psi(e_2(\theta)) \right\} f(\theta)d\theta \right\} g(\epsilon)d\epsilon$$

s.t.

1- \( \frac{dE[U_p]}{de_1} = 0 \)

2- \( \frac{dE[U_p]}{d\theta} = -\psi'(e_2(\theta)) \)

3- \( E[I(e_1) t_1 + t_2(\theta) - \psi(e_1) - \psi(e_2(\theta))] \geq 0 \)
From the incentive compatibility constraint related to the first phase of the project we obtain \( t_1 = \psi'(e_1) \). Substituting in the participation constraint we get the following ex ante utility: \( E[U_B] = c_1 \psi'(e_1) + t_2(\theta^h) - \psi(e_1) + \psi(e_2(\theta^h)) = 0 \). Given that the government aims at the agent’s rent extraction and considering that the consortium is not protected by a limited liability constraint, the principal can set the second period transfer equal to \( t_2(\theta^h) = \psi(e_1) + \psi(e_2(\theta^h)) - e_1 \psi'(e_1) \). As a consequence, the participation constraint is ex ante binding for the least efficient operator, while the private incentive compatibility constraints remain effective. The obtained government’s welfare function is defined as follows:

\[
\max_{e_1, e_2(\theta)} W_B \equiv \int_{\theta_l}^{\theta_h} \left\{ S_0 + e_1 S - (1 + \lambda)[\psi(e_1) + \psi(e_2(\theta)) + O - e_1 \theta - e_2(\theta)] - \lambda \frac{F'(\theta)}{f(\theta)} \psi''(e_2(\theta)) \right\} f(\theta) d\theta
\]

Optimizing w.r.t. \( e_1 \) and \( e_2(\theta) \) yields respectively

\[
\psi'(e_1^B) + \lambda \frac{dT_1^B}{de_1} = S + (1 + \lambda)\theta
\]

where \( \frac{dT_1^B}{de_1} = \psi'(e_1^B) \)

\[
\psi'(e_2^B(\theta)) = 1 - \frac{\lambda}{1 + \lambda} \frac{F'(\theta)}{f(\theta)} \psi''(e_2^B(\theta))
\]

**Appendix 4 - Welfare Analysis**

The expected function that is used to perform the comparative statics analysis is the following one:

\[
\int_{\theta_l}^{\theta_h} \left\{ S_0 + e_1 S - (1 + \lambda)[e_1 t_1 + t_2(\theta) + O - e_1 \theta - e_2(\theta)] + (e_1 t_1 - \psi(e_1)) + (t_2(\theta) - \psi(e_2(\theta))) \right\} f(\theta) d\theta
\]

Using the new effort functions the first order conditions in the bundling case become:

\[
e_1^B = \frac{S + (1 + \lambda)\theta}{1 + \lambda}
\]
\[ e^B_2(\theta) = 1 - \frac{\lambda}{1+\lambda} \frac{E(\theta)}{f(\theta)} \]

Substituting in the government objective formula we obtain the value function under bundling:

\[
V_B = \int_\theta^\infty \left\{ S_0 + Se^B_1 - (1 + \lambda)\left[ \frac{(e^B)^2}{2} + \frac{(\theta)^2}{2} \right] + O - e^B_1 \theta - e^B_2(\theta) \right\} - \lambda\frac{E(\theta)}{f(\theta)} \psi''(e_2(\theta)) \right\} f(\theta) d\theta
\]

Differences between the two scenarios come from the builder’s effort; hence the analysis is developed using only factors dependent on \( e^B_1 \)

\[
V_B = \int_\theta^\infty \left\{ S_0 + \theta S - (1 + \lambda)\left[ \frac{S^2}{2(1+\lambda)^2} + \frac{\theta^2}{2} + \frac{\theta S}{1+\lambda} - S^2 \right] \right\} f(\theta) d\theta
\]

The new efforts functions applied to the unbundling case yields respectively

\[
e^U_1 = \frac{S + (1+\lambda)\theta}{1+2\lambda}
\]
\[ e^U_2(\theta) = 1 - \frac{\lambda}{1+\lambda} \frac{E(\theta)}{f(\theta)} \]

Substituting in the government objective formula we obtain the value function under unbundling:

\[
V_U = \int_\theta^\infty \left\{ S_0 + Se^U_1 - (1 + 2\lambda) - (1 + \lambda)\left[ \frac{(e^U)^2}{2} + \frac{(\theta)^2}{2} \right] + O - e^U_1 \theta - e^U_2(\theta) \right\} - \lambda\frac{E(\theta)}{f(\theta)} \psi''(e_2(\theta)) \right\} f(\theta) d\theta
\]

Differences between the two scenarios come from the builder’s effort; hence the analysis is developed using only factors dependent on \( e^U_1 \)

\[
V_U = \int_\theta^\infty \left\{ \frac{S^2}{1+2\lambda} - S\left[ \frac{(1+\lambda)\theta S}{1+2\lambda} + \frac{(1+\lambda)\theta S}{2(1+2\lambda)} \right] - (1 + \lambda)\left[ \frac{\theta S}{1+2\lambda} - \frac{\theta S}{1+2\lambda} \right] \right\} f(\theta) d\theta
\]
\[ V_U = \int_\theta^\infty \left\{ \frac{S^2}{2(1+2\lambda)} + \frac{((1+\lambda)\theta S)}{1+2\lambda} - \frac{(1+\lambda)\theta S}{2(1+2\lambda)} \right\} f(\theta) d\theta
\]
\[ V_U = \frac{S^2}{2(1+2\lambda)} + \frac{(1+\lambda)\theta S}{1+2\lambda} + \frac{\theta^2(1+\lambda)^2}{2(1+2\lambda)} \]

The net welfare gain of governments when using PPPs is equal to:

\[ V_B - V_U = \frac{\lambda S^2}{2(1+\lambda)(1+2\lambda)} + \frac{\lambda S}{1+2\lambda} + \frac{(1+\lambda)\theta^2}{2(1+2\lambda)} + \frac{\lambda(1+\lambda)}{2(1+2\lambda)}\theta^2 \]

\[ V_B - V_U = \frac{\lambda}{2(1+\lambda)(1+2\lambda)} (S^2 + (1+\lambda)^2\theta^2 + 2(1+\lambda)\theta S) + \frac{(1+\lambda)}{2} \theta^2 \]

\[ V_B - V_U = \frac{\lambda}{2(1+\lambda)(1+2\lambda)} (S + (1+\lambda)\theta)^2 + \frac{(1+\lambda)}{2} \theta^2 \]

A Robustness check: simultaneous contracts

Differences between the two scenarios come from the builder’s effort; hence the analysis is developed using only factors dependent on \(e_1^S\)

\[ V_U^S = \int_0^{\theta} \left\{ \frac{S^2}{1+2\lambda} - \frac{S}{1+2\lambda} - \left( \frac{S^2}{2(1+2\lambda)} + \frac{(1+\lambda)\theta S}{2(1+2\lambda)} + \frac{\theta^2(1+\lambda)^2}{2(1+2\lambda)} \right) - (1+\lambda)\left[ - \frac{\theta S}{1+2\lambda} - \frac{\theta^2(1+\lambda)^2}{2(1+2\lambda)} \right] \} f(\theta) d\theta \]

\[ V_U^S = \int_0^{\theta} \left\{ \frac{S^2}{2(1+2\lambda)} + \frac{(1+\lambda)\theta S}{2(1+2\lambda)} + \frac{\theta^2(1+\lambda)^2}{2(1+2\lambda)} \} f(\theta) d\theta \]

\[ V_U^S = \frac{S^2}{2(1+2\lambda)} + \frac{(1+\lambda)\theta S}{2(1+2\lambda)} + \frac{(1+\lambda)^2}{2(1+2\lambda)} \mathbb{E}[\theta^2] \]

The net welfare gain of governments when using PPPs is equal to:

\[ V_B - V_U^S = \frac{\lambda S^2}{2(1+\lambda)(1+2\lambda)} + \frac{\lambda S}{1+2\lambda} + \frac{\lambda(1+\lambda)}{2(1+2\lambda)} \mathbb{E}[\theta^2] \]

\[ V_B - V_U^S = \frac{\lambda}{2(1+\lambda)(1+2\lambda)} (S^2 + (1+\lambda)^2\theta^2 + 2(1+\lambda)\theta S) + \frac{\lambda(1+\lambda)}{2(1+2\lambda)} \sigma^2 \theta \]

\[ V_B - V_U^S = RS + \frac{\lambda}{(1+2\lambda)} I E \]

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Appendix 5 - Comparative Statics Analysis

Comparative statics analysis of the RS effect with respect to $\lambda$

$$RS = \frac{\lambda}{2(1+\lambda)(1+2\lambda)}(S + (1 + \lambda)\bar{\theta})^2$$

$$\frac{dRS}{d\lambda} = (S + (1 + \lambda)\bar{\theta})^2 \left( \frac{2(1+\lambda)(1+2\lambda) - 2(3+4\lambda)\lambda}{4(1+\lambda)^2(1+2\lambda)^2} \right) + \frac{\lambda}{2(1+\lambda)(1+2\lambda)} 2\theta(S + (1 + \lambda)\bar{\theta})$$

$$\frac{dRS}{d\lambda} = (S + (1 + \lambda)\bar{\theta})^2 \left( \frac{2(1+3\lambda+2\lambda^2)-(6+8\lambda)\lambda}{4(1+\lambda)^2(1+2\lambda)^2} \right) + \frac{\lambda}{2(1+\lambda)(1+2\lambda)} 2\theta(S + (1 + \lambda)\bar{\theta})$$

$$\frac{dRS}{d\lambda} = (S + (1 + \lambda)\bar{\theta})^2 \left( \frac{1-2\lambda^2}{2(1+\lambda)^2(1+2\lambda)^2} \right) + \frac{\lambda}{2(1+\lambda)(1+2\lambda)} 2\theta(S + (1 + \lambda)\bar{\theta})$$

- $(S + (1 + \lambda)\bar{\theta})^2 (\frac{1-2\lambda^2}{2(1+\lambda)^2(1+2\lambda)^2}) < 0$ if $\frac{1}{\sqrt{2}} \leq \lambda \leq 1$
- $\frac{\lambda}{2(1+\lambda)(1+2\lambda)} 2\theta(S + (1 + \lambda)\bar{\theta}) < 0$ if $\bar{\theta} < 0$ (negative externality)

Comparative statics analysis of the IE effect with respect to $\bar{\theta}$

$$RS = \frac{\lambda}{2(1+\lambda)(1+2\lambda)}(S + (1 + \lambda)\bar{\theta})^2$$

$$\frac{dRS}{d\theta} = \frac{\lambda^2}{2(1+\lambda)(1+2\lambda)} 2(S + (1 + \lambda)\bar{\theta})$$

- $\frac{dRS}{d\theta} > 0$ if $S > -(1 + \lambda)\bar{\theta}$
- $\frac{dRS}{d\theta} = 0$ if $S = -(1 + \lambda)\bar{\theta}$
- $\frac{dRS}{d\theta} < 0$ if $S < -(1 + \lambda)\bar{\theta}$

Comparative statics analysis of the IE effect with respect to $\lambda$

$$IE = \frac{(1+\lambda)}{2} \sigma^2_{\theta}$$

$$\frac{dIE}{d\lambda} = \frac{\sigma^2_{\theta}}{2} \geq 0$$
Comparative statics analysis of the IE effect with respect to $\sigma_\theta^2$

\[ IE = \frac{(1+\lambda)}{2} \sigma_\theta^2 \]

\[ \frac{dIE}{d\sigma_\theta^2} = \frac{(1+\lambda)}{2} \geq 0 \]