8 Multi-contract tendering procedures and package bidding in procurement

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8.1. Introduction

In practical procurement, the most common way to purchase multiple supply contracts – of different types, say, or for different geographical areas – is probably the simultaneous sealed-bid competitive tendering of several distinct contracts. In this competitive bidding procedure, when tenders are only economic, suppliers make a separate bid for each contract and each contract is awarded to the supplier who makes the lowest bid, at a price equal to his bid. Typically, the chance that a supplier is awarded a particular contract is independent of the bids he submits on any other contract.

This simple way of awarding supply contracts may be appropriate when the cost of supplying each contract is independent of which other contracts a supplier is serving. But, in reality, a supplier's cost of serving a contract often depends on how many, and which, other contracts he also supplies. When this is the case, the procurer should allow suppliers to submit offers that can take such relations into account. For example, when the average cost of serving two adjacent regions is substantially lower than that of serving just one of the two areas – say because part of the fixed investment required can be used for both areas – bidders should be allowed to tender offers whose conditions are valid only if they are awarded the service contracts for both adjacent regions.

In this chapter, we discuss methods for procuring multiple contracts when there are 'complementarities' among them – that is when a supplier's cost of serving each contract depends on which other contracts the same supplier is also serving. We mainly focus on sealed bidding and on situations where the private-cost component of serving contracts prevails (see Chapter 6). Multi-contract dynamic auctions are discussed in Chapter 9.

We argue that the nature and magnitude of complementarities should drive the choice of the most appropriate form of competitive tendering for the procurer.

Positive complementarities, the most common and interesting in procurement, arise when, for some potential suppliers, the total cost of serving a set of contracts is lower than the sum of the costs of serving each single contract in the set alone. An example of products with strong positive complementarities is ICT devices and software, where large suppliers can offer very low prices thanks to significant economies of scale.

Negative complementarities, more rare in procurement, arise in the opposite situation, when for a supplier the cost of serving a group of contracts is higher than the sum of the costs of the single contracts. This is normally the case when a bidder's capacity constraints are relatively tight so that, as the number of contracts supplied increases towards this capacity, total cost increases steeply. An example of negative complementarities is energy provision: since energy cannot be stored and power plants have limited capacity, production constraints represent rather rigid bounds for a producer, who has to drastically increase production costs in order to provide a quantity beyond such limit.

When two contracts generate strong positive complementarities for a supplier, he may be willing to lower his offer for one of the contracts only if he is sure he will also be awarded the other contract. Therefore, the procurer should adopt a tendering procedure in which bidders are also allowed to bid for a group, or a 'package', of contracts, as well as for single contracts, in order to increase his saving.

We first discuss the simultaneous ascending auction (adapted to procurement), which has been successfully used around the world, for example, to allocate radio spectrum for mobile-phone licences and pollution licences. Although this tendering format does not allow package bidding, it does allow bidders to place their offers while observing their competitors' bids and, hence, helps them in selecting which contracts to bid on in the presence of mild complementarities. However, if complementarities are sufficiently important, suppliers may still be unwilling to bid aggressively in a simultaneous ascending auction and, therefore, other formats that allow package bidding should be considered.

We briefly discuss the procurement version of the Vickrey auction, which has been widely analysed theoretically and which enjoys remarkable efficiency properties. However, because of the practical problems related to its implementation (due, e.g., to its complicated pricing rule), the Vickrey auction is not in general advisable for procurement. We then consider the procurement version of the sealed-bid 'menu auction', a sealed-bid tendering procedure in which bidders are allowed to bid on packages of contracts, and winning bidders receive the price they bid for the contract(s) they are awarded. This competitive tendering procedure is becoming increasingly popular for procurement, because it is relatively easy to implement and its rules are readily understood by bidders.

Package auctions can become computationally complex because a large number of possible packages arises from the combination of even a small number of initial contracts. We suggest some practical solutions to mitigate such complexity. Package bidding may also bring in other inefficiencies, because it may induce suppliers only interested in single contracts to bid less aggressively.

When negative complementarities among contracts are likely to prevail, in a standard sealed-bid tendering that allows offers only on individual contracts, a supplier may be exposed to the risk of winning more contracts than he desires at a given price because, even if he is interested in winning just one of several similar contracts, he may still want to bid on more contracts in order to increase his chance of winning. As with positive complementarities, the procurer should favour tendering procedures where bidders can explicitly submit offers conditional on being awarded a certain set of contracts, so that the price a winning supplier is paid can depend on which contracts he is awarded. This encourages bidders to submit competitive offers for single contracts, without facing the risk of being assigned too many contracts at a low price.

When contracts are homogeneous 'shares' of a whole supply and complementarities are negative, as for example in electricity supply, the procurer may choose simpler versions of sealed-bid tendering procedures with package bidding, where suppliers are asked to bid 'supply schedules' – that is, combinations of quantities (number of contracts) and prices at which they are willing to supply those quantities. These 'competitive tendering of shares' are simple to implement and can award contracts using different rules to determine prices. We discuss which pricing rules are appropriate in different situations, taking into account their potential costs and benefits for attracting participants and inducing strategic/collusive bidding ('demand reduction').

Chapters 7 and 11 argue that, when there are capacity constraints, the higher the number of contracts procured and the smaller their size, the greater is the number of small suppliers that can participate in the tendering. But although many small contracts can increase competition, in the absence of package bidding they also reduce the ability of larger supplier to exploit economies of scale, by exposing them to the risk of only winning few very small contracts. There is then a trade-off, in choosing the number and size of contracts to tender, between encouraging participation of small suppliers and allowing large suppliers to exploit economies of scale. Allowing for package bidding helps resolving this trade-off, because even if only very small contracts are offered, large firms can still exploit economies of scale by bidding for large packages of contracts. So package bidding with many small contracts may be the ideal solution to foster participation of many heterogeneous suppliers, allowing each of them to express their competitive strength by offering on their own ideal combination of contracts, improving the outcome for the procurer.

The rest of the chapter is structured as follows. Section 8.2 discusses auctions and sealed-bid tendering procedures which are appropriate when contracts display positive complementarities. The case of negative complementarities is discussed in section 8.3. Section 8.4 analyses the effects of the tendering rules on bidders' participation. A few examples conclude the chapter.

8.2. Contracts with strong positive complementarities

Positive complementarities arise when two or more contracts are worth more together than separately or when, equivalently, due for example to economies of scale, a bidder's total cost of supplying a group of contracts is lower than the sum of the costs of supplying each of the contracts in the group separately. This, in principle, can allow bidders to supply contracts at relatively low prices, and the procurer to obtain substantial savings.

In the sealed-bid tendering most commonly used to award heterogeneous contracts in procurement, suppliers are not allowed to offer on groups of contracts (or, in general, to place a bid on a contract conditional on also winning another contract).

But with strong positive complementarities, a bidder may be willing to lower his price for a particular contract only if he is sure to be also awarded another contract (or even more than one), because the cost of a single contract when both of them are supplied is lower than when only one of the contracts is supplied. In this case, in a competitive tendering where suppliers are not allowed to bid for a specific *group* of contracts, the bidder may be unwilling to offer aggressively on a single contract because he is unsure whether he will also be able to win the other contract. This is often called *exposure* problem, because the bidder is exposed to the risk of winning only some components of a desired package for which the price offered was calculated. Should such risk be perceived as too high, suppliers may be discouraged from bidding aggressively. In this situation, allowing package bidding could lead to much higher savings for a procurement agency.

To evaluate the potential extreme consequences of the exposure problem, consider the following example with two bidders, 1 and 2, and two different lots A and B. Figures in the cells represent the lowest prices that bidders' are

	Lot A	Lot B	Package (A,B)
Bidder 1	300	300	300
Bidder 2	100	100	500

willing to accept for supplying the single lots or the two lots together.
As the figures suggest, the two lots are positive complements for bidder 1: due, for example, to substantial economies of scale, bidder 1 can provide the two lots at the same cost at which he is able to provide a single lot. This is the case if, for instance, bidder 1 has a fixed cost of production equal to 300 and marginal cost equal to 0. Bidder 2 can provide each one of the lots at price 100, but because of a binding capacity constraint, his cost for providing both lots is higher than the sum of the costs of producing each single lot. So bidder 2 has decreasing returns to scale in production. Assume that bidders know each other's production costs.

Suppose that, as it is standard in procurement practice, the procurer is buying the two lots simultaneously, asking bidders to place sealed-bids for the lots. Suppose also that bidders are only allowed to bid for single lots, and not for the package of two lots.

Bidder 1, who is the most efficient bidder (for the two lots), could provide the highest savings for the procurer. So the efficient (i.e., cost-minimizing) allocation consists in awarding both lots to bidder 1. However, bidder 2 is willing to bid a price as low as 100 for at least one of the lots. Therefore, in order to be sure to win both lots beating bidder 2, bidder 1 has to offer a price no higher than 100 for each lot. But this is not profitable for him (because the total price he would receive would be lower than his cost of supplying the two lots). So the allocations of the contracts cannot be efficient (even if there is no incomplete information on the costs). As a consequence, bidder 1 may prefer not to participate in the competition at all, which would drastically increase the price paid by the procurer.

This extreme conclusion depends on bidder 1 facing a bidder with sharply decreasing returns to scale. But the exposure problem also arises in much more general contexts. Suppose, for example, that bidder 1 participates in the competitive tendering without any information about bidder 2's costs. Then bidder 1 may be unwilling to offer a price lower than 300 (i.e., his cost of production for one lot) on any single lot, for fear of ending up winning that lot only (and hence having to supply it at a price lower than his cost), in case bidder 2 places a lower winning bid only on the other lot. (Notice that this argument does not depend on the actual costs of bidder 2 and, in particular, on whether bidder 2 has increasing, decreasing or constant returns to scale).

As a consequence, even if bidder 1 is able to supply both lots at a cost of 300, the sum of his bids on each lot may be much higher than that, and possibly not lower than 600. This, again, could increase the price paid by the procurer and generate an inefficient allocation.

8.2.1. Simultaneous ascending auction (SAA)

A simultaneous ascending auction (SAA) is a dynamic auction format that can also be used to partially address the exposure problem (besides the problems addressed in Chapter 6). This format allows bidders to place their offers while observing their competitors' bids and, hence, helps them in selecting which contracts to bid on in the presence of complementarities.

The SAA, developed by Milgrom, Wilson and McAfee, is the format successfully adopted by the American Federal Communications Commission in a number of auctions that, starting from 1994, have been used to sell radio spectrum for mobile-phone licences. The format was also used by various European governments to sell 3G mobile-phone licences in 2000/2001. This auction is very similar to a standard ascending auction (used, e.g., to sell paintings by Sotheby's and Christies), except that several items are auctioned at the same time and bidders can choose which object(s) to bid on. In procurement auctions, the price decreases on each contract independently, but none of the contracts is awarded until no one is willing to bid again on any of the contracts. So the auction only stops when no further offer is submitted on any contract, and each contract is assigned to the highest proposed price discount.

This auction is organized in rounds, with the procurer communicating, at the end of each round and for each contract, the lowest price received and the highest acceptable offer for the next round. Typically, though not necessarily, the highest acceptable offer is fixed by decreasing of a certain percentage the lowest price submitted in the last round. To avoid bidders remaining idle during the auction (because, for example, a bidder may be reluctant to place offers until he first observes his rivals' bids), an activity rule can be introduced. This rule requires a bidder in each round to hold the lowest bid on a certain number of contracts, or else make a new lower bid. This induces participants to bid from the beginning of the auction and reduces the auction length.[1] The activity rule may vary during the auction. For example, to facilitate price discovery when bidders are uncertain about the contracts' values, it may be less strict in the early stages of the auction and tighter in later rounds.

The SAA has many advantages. First, it is a simple and transparent procedure which encourages price discovery. As the auction progresses, bidders can observe the price offered by their opponents' and, hence, they can condition subsequent bids on this new information. And if the contracts to be procured have common and uncertain cost components, a dynamic mechanism such as the SAA better reveals a supplier's private information on this component to his opponents and, therefore, reduces bidders' information rents. This induces more aggressive bidding and allows the procurer to pay a lower price.

The second main advantage of the SAA is that bidders can choose the most desirable subsets of contracts on which to bid, given their opponents' offer. And since bidders have the flexibility to shift their bids across groups of contract when relative price levels change, a supplier can stop offering on a group of complement any contracts he initially intended to obtain if he realizes, as the auction progresses, that he will not be able to win one of the contracts in that group. This mitigates the exposure problem and helps bidders assemble the most desirable group of contracts they can obtain. For example, suppose a bidder considers two contracts to be positive complements (and, hence, is willing to receive a lower price if he is awarded both contracts). The bidder may confidently start bidding on both contracts because he knows that, if the price of one of the contracts becomes too low, he may stop bidding on the other contract too and/or switch to other

[1] A further way to keep under control the pace and length of the auction is to fix the daily number of rounds.

contracts that are more attractive at the new prices.[2] Moreover, since during the auction participants can shift their bids across substitute contracts, the final prices of contracts with similar characteristics should be similar.

But although the SAA limits the exposure problem, it does not completely solve it. Consider again the example discussed in the previous paragraph. Assume the procurer is running a SAA auction and the price has reached 300 on each of the lots. At this point bidder 1 may prefer not to bid any further and drop out of the auction because, although he is willing to provide both lots at a price lower than the current total price of 600, he may be afraid of bidding less than 300 on a single lot, in case he fails winning the other lot too. In general, even in a SAA, bidder 1 may fail to bid aggressively for fear of being unable to win both lots and having to take one lot only. The reason is precisely that the SAA does not allow suppliers to place bids on groups of contracts. As discussed above, this may discourage aggressive offers and induce an inefficient allocation.

Two other problems which emerged in running a SAA are *demand reduction* and *collusion*. Demand reduction, which will be discussed in more detail in section 8.3.1, arises when a bidder prefers to bid on fewer contracts than he actually desires in order to reduce competition and maintain high prices on the contracts he is actually bidding on. Collusion is typically easy in the SAA just because of its transparency. Participants may use their bids during the auction to coordinate on a collusive outcome by, for example, signalling their willingness to concentrate on a certain subset of contracts. And since a bidder can observe his opponents' bids, he can detect and punish during the auction another bidder who tries to deviate from a collusive agreement. This reduces the competitiveness of the auction and generates higher prices for the procurer. This theme is the subject of Chapters 14 and 15 and therefore will not be discussed any further here.

8.2.2. Sealed-bid tendering with package bidding

When there are strong positive complementarities between contracts, because of the exposure problem suppliers may be unwilling to bid aggressively, even in a SAA. In this case, the procurer should consider

[2] Moreover, to further mitigate the exposure problem, a supplier is sometimes allowed to withdraw a bid, paying a fee equal to the difference between the final winning bid and his withdrawn bid, if this difference is positive.

allowing package bidding. In a package (or combinatorial) tendering procedure, a bidder can make offers conditional on being awarded a specific group of contracts, called a package, as well as for a single contract.

Package bidding has often been used in practice. For example, it has been employed in the assignment of airport slots, in truckload transportation, bus routes and procurement. In the United States, package bidding was also proposed to allocate radio spectrum for mobile-phone licences.

Clearly, combinatorial offers allow bidders to better express their preferences and eliminate the exposure problem because, by only bidding for a package of contracts that he considers complements, a supplier can be sure that he will be awarded a contract if and only if he is also awarded the other contracts in the package. This induces suppliers to bid more aggressively and can lead to lower prices paid by the procurer.

Practical conclusion 1

Package bidding should be introduced if it appears likely that, for a significant number of bidders, there are substantial positive complementarities among some of the contracts/lots.

We are now going to analyse the specific rules that can be adopted in a combinatorial tendering procedure. In this chapter we will concentrate on sealed bidding. Dynamic package auctions will be analysed in Chapter 9.

8.2.2.1. Vickrey auction in procurement

In a procurement version of the Vickrey auction, the sealed-bid tendering procedure should be such that the buyer asks each bidder to report his production cost, for each contract to be procured and for each possible group of contracts. Then the buyer awards the contracts in order to minimize the total price (i.e., the sum of the winning bids). A winning bidder receives for each contract or group of contracts that he wins a price equal to the cost he declared, less the total (called social) cost for all contracts, plus the total cost for all contracts that would have been paid if that bidder had not been present.[3]

It is the multi-contract Vickrey auction (rather than the uniform-price auction discussed later in this chapter) that incorporates the main strategic

[3] Equivalently, the price received by a winning bidder is the difference between (i) the sum of the bids that would win if that bidder does not participate in the auction and (ii) the sum of the other bidders' actual winning bids.

feature of the second-price single-contract tendering competition; namely that, when bidders have private valuations, it is the dominant strategy for them to submit an offer equal to their true cost for each of the contracts, and each group of contracts to be procured (see Chapter 1). So bidders' strategies are very simple and the tender always efficient, as it awards the contracts to the bidders who value them the most. And, since suppliers are allowed to bid on packages of contracts, the Vickrey auction solves the exposure problem arising when contracts exhibit positive complementarities.

It's worth showing the auction pricing rule considering the following example.

	Lot A	Lot B	Package (A,B)
Bidder 1	300	300	350
Bidder 2	250	250	400

If both bidders behave according to their dominant strategy and report their true costs, the procurer awards both lots to bidder 1 (which is the efficient allocation). The total cost of the contracts is 350; while, if bidder 1 had not been present, the total cost of the contracts would have been 400 (bidder 2's cost for the two contracts). Therefore, bidder 1 is paid by the procurer a price equal to $350 - 350 + 400 = 400$.

Given the above considerations, it could be tempting to believe that the Vickrey auction is the perfect solution to purchase multiple contracts because of its remarkable theoretical properties. However, due to its complexity the design is not used in procurement activity. Moreover, like in the single-contract case, bidders may be unwilling to report their true cost to the procurer, because they may fear he will exploit this information in future negotiations. Furthermore, the Vickrey auction may also result in more efficient bidders with a low production cost receiving a lower price for a contract than bidders with a high production cost (which often appears unfair). Finally, it may result in the procurer paying a high price. The Vickrey auction also has the undesirable property that increasing the number of bidders may actually *increase* the price paid by the procurer.[4] For all these reasons, it is probably better for a procurer to consider other

[4] E.g., Milgrom (2004).

types of combinatorial tendering procedures, when he has to purchase multiple contracts in the presence of positive complementarities.

8.2.2.2. Sealed-bid tendering with menu (pay-as-bid) package bidding

This is a sealed-bid tendering in which each bidder submits a separate price for each contract or package of contracts he may want to supply. The word menu derives from the fact that packages on which suppliers can submit offers are defined by the procurer who, in so doing, may introduce some constraints on possible bids, a point which is discussed in the following paragraph. The buyer selects the feasible combination of offers that maximises his savings (i.e., minimises the total price tendered). Each winning supplier receives the price he bids for the contract or package of contracts he is awarded. This procurement mechanism has been used, for example, in the IBM–Mars auction and in the London bus routes auction.

To understand how this competitive tendering works, consider again our previous example. In a menu package bidding tendering, if bidder 1 knows the production costs of bidder 2, he can place a bid slightly lower than 400 for the package of two contracts, so that it will never be profitable for bidder 2 to underbid him. Then the contracts are efficiently allocated to bidder 1, who has an incentive to participate in the auction. Even if, more realistically, bidder 1 does not know the production cost of his opponent, he is still willing to bid aggressively even in the presence of positive complementarities, because he knows that, by placing a bid for the package of two contracts which is not lower than 350 (his total production cost), he will never be awarded the two contracts at a price which is unprofitable for him, regardless of his and his opponent's bids for the individual contracts.

From a theoretical point of view, this form of competitive tendering has been analysed under the restrictive hypothesis that each bidder knows his opponents' production costs. Its properties under the more realistic assumption that bidders do not know their opponents' costs have not been fully analysed yet. However, when compared to the Vickrey auction, the pricing rule of the menu package bidding tendering is much more intuitive and easier to understand for bidders. Therefore, from a practical point of view, such tendering procedure should be preferred to the Vickrey auction to procure multiple contracts in the presence of complementarities and small, or absent, common and uncertain cost components.

We recapitulate the main considerations made in the following practical conclusion.

Practical conclusion 2

When positive complementarities among contracts are likely to prevail, and common and uncertain cost components are small or absent, favour sealed-bid tendering with menu package bidding.

Example 8.1. Procurement of fresh fruit and vegetables in Italy.

In 2005, Consip, the Italian Procurement Agency for Public Administrations, designed a competitive tendering with package bidding to procure fresh fruit and vegetables for the Public Administration. The end-users of this purchasing service were wide-ranging, as they were from different PA sectors and differed in size and location. Thus, logistics was a major concern for an effective supply execution, which induced Consip to look at the wholesale market as the relevant one when designing the competitive tendering. The Italian wholesale market was characterized by a geographically homogeneous firm distribution, large fragmentation both at the regional and the provincial level, and strong competition. Given the underlying competitiveness of the market, in order to achieve high savings, Consip needed to allow larger players to exploit geographical synergies within the macro-area level. However, the strength of geographical synergies was not clear, and with weak synergies a combination of small local suppliers might have been more efficient. For this reason supply was divided into 24 geographical lots, grouped in 6 macro-areas. Package bids were allowed for these macro-areas, to let large suppliers exploit synergies within such areas and compete against local suppliers bidding on single contracts. Bidding competition would have then established and contracts efficiently allocated to large suppliers, if positive synergies were strong, to small local suppliers otherwise.

8.2.2.3. Issues with package bidding

Constraints on bids

A distinguishing feature of competitions with package bidding is their potential computational complexity, since even in very simple procurement designs the number of packages to be considered may be extremely high, and the number of bids that suppliers report to the procurer very large. More specifically, with combinatorial bidding the number of offers is an exponential function of the number of contracts being procured: if n is the number of contracts, then $2^n - 1$ is the total number of possible bids that a participant can submit. So if N is the number of bidders, the number of offers that can be received by the procurer is $N(2^n - 1)$. For example, if $N = 3$ and $n = 4$, the buyer can receive $15 \times 3 = 45$ bids. And when the number of contracts increases from $n-1$ to n, the number of possible bids increases by $N2^{n-1}$, namely linearly in N but much faster, exponentially, in n.

It should then be clear that managing a combinatorial tendering procedure can in principle be highly demanding for the procurer, because of the cost of processing a high number of offers and determining the winners. For this reason, the computational tractability of the design could be, in principle, a concern when allowing combinational bids.

These considerations may justify the introduction of an upper bound to the total number of bids that a participant can place. Constraints on bids to reduce the complexity problem can take different forms; in what follows we exemplify few possible ones.

Is the change OK?

1. If $(2^n - 1)$ is the maximum possible number of offers, a generic upper bound, say $n^* < (2^n - 1)$, can be imposed independently of where bids are made. The bound can further specify whether offers can all be combinational or not.
2. A limit on bids for packages can be introduced. For instance, if $n = 5$, bidders could be allowed to submit at most one offer for each package with 5, 4, 3, 2 and 1 items.
3. A structure of offers, such as the so called 'Sunflower', can be introduced. In this case, bidders may be asked to submit as many offers as they want, but with a unique non-empty common intersection. The idea is simple: with regard, for example, to bus routes, bidders would be asked to identify their most important routes (the non-empty intersection) so that all (possibly package) offers that they make will have to include those routes.

Free-rider problem

With package bidding, suppliers seeking only a single contract may *free ride* – that is, they may prefer to submit high prices, relying upon other participants bidding aggressively. Free riding may allow a package bidder to win the competition even when it would be more efficient to allocate the contracts separately and, hence, it may result in low savings for the procurer. To illustrate the issue, also known as the *threshold problem*, consider the following example with three bidders and two lots. (As usual, numbers represent bidders' production costs.)

	Lot A	Lot B	Package (A,B)
Bidder 1	90	90	300
Bidder 2	90	90	300
Bidder 3	0	0	200

In this example, total savings for the procurer would be maximized by bidder 1 winning lot A (or B) at price 90 and bidder 2 winning lot B (or A) at price 90.

Consider a dynamic reversed auction with package bidding, a generalization of the SAA, and suppose that the following table summarizes the bidding situation at round *t*, where a 0 stands for no offer having been submitted.

Bids at round *t*

	Lot A	Lot B	Package (A,B)
Bidder 1	110	0	0
Bidder 2	0	110	0
Bidder 3	0	0	210

Given the current bids, both contracts would be allocated to bidder 3. This allocation, however, would be inefficient and economically unattractive for the procurer.

Bidders 1 and 2 could win the auction, beating bidder 3, by reducing their offers so that the total price for the two lots is lower than 200; but each bidder would prefer the other to bear the cost of doing so. As a consequence, bidder 1 may be unwilling to lower his own offer, if he expects bidder 2 to lower his offer down to 90. In general, to win one of the lots, each bidder may rely upon the other supplier offering a low price, to induce bidder 3 to drop out of the competition. But if none of them lowers his offer, the two lots may end up being assigned to bidder 3.[5]

When the possibility of free riding is a major concern, and if an open auction is not too costly to implement, the procurer should favour SAA with no package bidding.

[5] When the lots to be procured are identical, even if bidders 1 and 2 do not want to *free ride*, they may still fail to coordinate their bids on different lots and, hence, they may induce an inefficient allocation.

8.3. Contracts with strong negative complementarities

As we observed above, supply contracts that exhibit strong negative complementarities are somewhat less common in procurement. They typically arise when suppliers have limited and rigid production capacities, so that their costs are steeply increasing in the quantity supplied, even at relatively low production levels. An example is given by procurement for electricity, where firms have a rigid production capacity that makes it extremely expensive or even impossible for suppliers to extend electricity provision beyond a certain level.

With negative complementarities suppliers prefer to be awarded few contracts. But since bidders are now exposed to the risk of winning more contracts than they wish at a given price, in multi-contract lowest price sealed-bid competitions, where offers made on separate lots are independent, they might bid very cautiously (or not participate at all). Therefore, to encourage entry and competition the buyer should choose tendering processes where suppliers are not exposed to the risk of winning different number of contracts at the same per-contract price. Again, both dynamic auctions and package bidding should reduce bidders' exposure to such risk.

As suggested in section 8.2.1 (for positive complementarities) one possibility would be to choose a simultaneous ascending auction. This would allow observing which lots each supplier is likely to win and take this into account when formulating new bids. Such auction would partially protect suppliers from the risk of being awarded the 'wrong' set of contracts relative to their price bid. However, the implementation of dynamic auctions, such as a SAA, can be too expensive and complex for some procurements, and may facilitate collusive behaviour (see Chapter 14.). If the SAA is too costly, collusion is a problem, and dynamic auctions are not needed for the reasons discussed in Chapter 6, then sealed-bid tendering with package bidding should be considered.

8.3.1. Sealed-bid conditional tendering

When lots are different, a sealed-bid tendering competition similar to the *menu package tendering* procedure described in section 8.2.2.2 is probably the easiest way to reduce bidders' exposure to the risk of winning too many contracts at the wrong price. As discussed above, in a lowest price *menu tendering* competition a bid on a package prevails on the sum of single bids made on the lots underlying the same package bid if the bidder is awarded all such lots in the package. However, while with *positive complementarities* a package bid prevails because of a buyer's savings optimisation, with *negative complementarities* such priority should be made an absolute rule, namely should apply independently of price considerations, in order to effectively 'protect' bidders against the risk of winning too many contracts at the price offered to serve one of them.

The allocation procedure then minimizes the buyer's expenditure taking into account such a constraint, that is, the package bid (absolute) priority.

In this 'constrained' *menu (pay-as-bid) package tendering* bids on single contracts will of course be more aggressive than those on packages of several contracts, which will have the 'insurance' function mentioned above, and the efficient outcome for the procurer will be likely to involve many suppliers simultaneously serving one or very few of the contracts/lots each.[6]

To understand the working of this tendering format, consider the following example with two bidders, 1 and 2, and two different lots A and B. Figures in the cells represent the lowest prices that bidders are willing to accept for supplying the single lots or the two lots together.

	Only A	Only B	(A,B)
Bidder 1	290	320	1,000
Bidder 2	330	370	1,300

In this example supplying both lots for bidder 1 and 2 implies much higher production costs than the simple sum of supplying lots singularly. Then the 'constrained' *menu (pay-as-bid)* package tendering allows suppliers to bid aggressively on each of the lots without running the risk of being awarded both lots at the prices bid for the single lots: by submitting a package bid on lot A and B, bidders can 'protect' themselves against such a risk. Bidder 1, who can offer the lowest price on each of the two single lots, knows that he can bid competitively both on lot A and lot B, whatever his knowledge about bidder 2's costs, since he is aware that, by placing a package bid on the two lots no lower than €1,000,000 (his total production cost), he will never be awarded the two contracts at a price

[6] However, this form of competitive tendering suffers for the same problem of the standard *menu (Pay-as-Bid) package tendering* procedure described in section 8.2.2.2, i.e., it has not been analysed from a theoretical point of view yet.

which is unprofitable for him, regardless of his and his opponent's bids for the individual contracts.

As the example showed, also in the context of *negative complementarities* the presence of package bidding (but in this case strengthened with an absolute priority rule) reduces the exposure of suppliers to the risk of winning too many contracts. Once eliminated the risk of winning too many contracts, bidders would be willing to compete aggressively on both lots in order to increase the likelihood of winning a contract.

Practical conclusion 3

With negative complementarities, if contracts/lots are not homogeneous, collusion appears unlikely (see Chapters 7 and 14), and the common-cost component is important and uncertain (see Chapter 6), favour SAA. Otherwise, use sealed-bid tendering with menu package bidding.

8.3.2. Homogeneous supply contracts and 'tendering on shares'

When contracts are perfect substitutes for suppliers the negative complementarities are simply 'negative returns to scale', that is, per-unit production cost that are increasing in the supplied quantity. Then only the number of awarded lots matters (not exactly which ones) and a tendering competition with package bidding becomes simpler since it can be implemented by letting each bidder tender a 'supply function' – that is a schedule of prices that depends only on the number of contracts/lots awarded. To see how such a supply function implicitly expresses package bidding, consider how it can be expressed using the same format employed for the example above (where lots were different). Consider the following supply function submitted by bidder 1:

$B(1) = (200, 500).$

The supply function is increasing and indicates that bidder 1 is willing to serve the first contract at €200,000 and the second one at €500,000; or, in other words, one contract at €200,000 and two contracts at €700,000. Then, this supply function can be expressed by the table format used above in the following way (remember that lots A and A' are now identical):

	Only A	Only A'	Package (A,A')
Bidder 1	200	200	700

This is precisely what happens in most electricity supply tendering processes (as well as in Treasury auctions for selling government bonds). When lots are homogeneous, there are negative complementarities (decreasing return to scale) and suppliers can submit a whole supply function, the procurer can select the parts of the supply functions of the lowest cost suppliers until total supply equals total demand. When the auctioneer is selling a good, these competitive tendering mechanisms are called 'auctions of shares', hence in our procurement applications we can name them 'tendering on shares'.

The price at which contracts are awarded in tendering on shares may differ. Specifically, if each winning supplier is paid a price equal to his bid for the contract he is awarded, the tendering is called 'discriminatory'; if each winning supplier is paid the same price, the tendering is called 'uniform'.

8.3.2.1. Discriminatory tendering on shares

To illustrate how a discriminatory tendering on shares works, suppose a buyer wants to procure in a single competition five identical contracts/shares/lots for the provision of electricity. There are three potential suppliers and each of them is required to submit his, sealed bid, supply schedule specifying the price at which he is willing to provide each of the lots.

For example, assume that supplier 1 bids the following supply price schedule:

$$B(1) = (150;\ 200;\ 400;\ 700;\ 1{,}200).$$

This means that supplier 1 is willing to provide a first lot at the price €150,000, a second additional lot at €200,000, and so on up to €1,200,000 for the fifth lot. Similarly, assume that suppliers 2 and 3 bid the following schedules:

$$B(2) = (100;\ 200;\ 400;\ 700;\ 1{,}300).$$

$$B(3) = (150;\ 300;\ 500;\ 500;\ 1{,}100).$$

The procurer then ranks all the bids for the single lots from lowest to highest, and awards the five lots to the suppliers who made the five lowest bids. Therefore, in our example suppliers 1 and 2 obtain two lots each and supplier 3 obtains one lot. Each winning supplier receives a price equal to his bid. Hence, supplier 1 receives €150,000 for providing the first lot and a price of €200,000 for the second lot; supplier 2 receives a price of €100,000 for the first lot and a price of €200,000 for the second lot and supplier 3 receives a price of €150,000 for the only lot he wins.

A problem with a discriminatory-price tendering is that suppliers may be paid different prices for lots that are identical. First, this may be considered unfair by bidders and may be a potential source of legal problems, particularly in public procurement. Second, when significant and uncertain common-cost components are present, and bidders are highly heterogeneous in terms of available information, less informed bidders may pay much higher prices than better informed ones, and this risk (of a winners' curse) may induce them not to participate or to bid extremely cautiously.

8.3.2.2. Uniform price tendering on shares

The term 'uniform' refers to the fact that with this mechanism the procurer buys all contracts at the same price, which is determined by equating demand and supply (the price is then equal to the lowest losing bid, or the highest winning bid).

Consider again the previous example with the same set of supply functions tendered (note though that in general the same set of suppliers will make different bids under different tendering rules). After having ranked from lowest to highest all the bids for the single lots, the procurer awards the five lots to the suppliers who made the lowest bids, but now all suppliers are paid for each of the awarded contracts the same price, equal to the sixth lowest bid. Therefore, suppliers 1 and 2 obtain two lots each and supplier 3 obtains one lot, and the 'uniform price' is given by €300,000.

More in general, if the procurer is interested in buying K contracts, the K lowest bids will win and, for each lot, the winners will all pay a price equal to the $(K+1)$st lowest offer.

An important characteristic of uniform-price tendering is that, when the contracts have relevant and uncertain common-cost components, it encourages participation by small and less-informed suppliers. Since the final price is the same for all the assigned lots, and depends on the bids and the information of all winning suppliers, including better-informed ones, less-informed suppliers (like new entrants or smaller firms) are less exposed to the risk of a winner's curse – which is particularly intense when firms are heterogeneous and some of them are better informed – and, hence, are willing to bid more aggressively. For these reasons, less-informed suppliers are also more likely to participate in a uniform-price tendering.[7]

[7] Though rather obvious, it is worth remarking that one should not deduce from the example we discussed that the discriminatory-price tendering is always preferable to the uniform-price tendering simply because in the former each winning bidder is paid a price equal to his bid, which is lower than the marginal losing bid (paid by all winners in a uniform-price tendering). This is because different

The distinguishing feature of uniform-price tendering, that all lots are assigned at the same price, is also usually perceived as fair, since all awarded contracts are perfect substitutes both for the buyer and the suppliers.

8.3.2.3. Unilateral and coordinated demand reduction

With both the pricing rules described, suppliers' exposure problem is greatly reduced, because they can specify the minimum price at which they are willing to supply a given group of contracts. However, a well-known bidding phenomenon that might occur in uniform-price tendering is the so called 'demand reduction', which in case of procurement becomes 'supply reduction'. This arises when participants shift their supply schedule upwards and therefore bid higher prices.[8]

Consider, for example, a buyer offering two identical contracts for electricity provision, and two bidders: bidder 1 is an efficient firm who can supply one lot for €300,000 and a second lot for €300,000 and bidder 2 is a less efficient firm who is capable of supplying one lot for €500,000 and a second lot for €1,000,000. Moreover, suppose each bidder knows his competitor's costs. Bidders' lowest acceptable prices for supplying the single lots or the two lots together are then the following (remember that lots A and A' are still identical):

	Only A	Only A'	(A, A')
Bidder 1	300	300	600
Bidder 2	500	1,500	1,500

Suppose supplier 2 bids competitively and, hence, offers to supply the first lot at price €500,000 and the second lot at price €1,000,000. Then if bidder 1 too bids competitively (i.e., he offers to supply the first lot at price €300,000 and the second lot at price €300,000), he is awarded both lots and receives the tendering price of €500.000 for each of them, making a total profit of €400,000. However, bidder 1 could do better by manipulating his bid and offering, for example, to supply the first contract at price €300,000 and the second at price above €1,000,000. Because of the uniform price rule, in this case bidder 1 is awarded one lot only, but receives a price of €1,000,000, making a higher total profit of €700,000. Clearly, the strategic behaviour of

designs will induce different bidding behaviour and, in particular, suppliers would be willing to offer lower prices in a uniform-price tendering, which may well yield higher savings for the procurer.

[8] Wilson (1979) and Ausubel and Cramton (1998).

bidder 1 greatly increases the price paid by the procurer, and also reduces efficiency as the second lot is then served by a supplier with higher costs.

In procurement, demand reduction is particularly likely when there is a large supplier among bidders that, by manipulating its supply, can produce a substantial effect on the final price. In particular, a large supplier realizes that bidding aggressively (i.e., close to her production cost for each contract) lowers the price of all contracts that she wins. This induces him to bid less aggressively, which in the case of procurement consists in offering to supply lower quantities at higher prices. This strategic behaviour – that leads to fewer contracts won at higher prices and higher profits – is akin to that of a monopolist, who prefers to sell a lower quantity charging a higher price.

In the example described above, one of the bidders has an incentive to unilaterally manipulate his bid. However, bidders may also find it attractive to coordinate their strategies. In a uniform-price tendering there can also be outcomes that appear collusive, because they induce a price that is much higher than if contracts were sold as an indivisible package. This happens because bidders can implicitly (or explicitly) agree to determine a very high price for the procured contracts, by each submitting very steep supply functions – that is, by both bidding very high prices for a small number of contracts and very low prices for a large number of contracts. This makes it unprofitable for other bidders to try to obtain a number of contracts higher than their collusive share, by deviating from the collusive agreement.

Consider the following example, in which bidders are now symmetric. Figures in the cells represent the lowest prices that bidders are willing to accept for serving the single lots or the two lots together (with lots A and A' still identical):

	Only A	Only A'	(A, A')
Bidder 1	300	300	600
Bidder 2	300	300	600

If both bidders behave competitively (offering for each lot the minimum price they are willing to obtain) they are awarded one lot each at price €300,000. Therefore, they both make no profits. But bidders can do much better by coordinating to manipulate their bids. If, for example, they offer to supply one contract at price €300,000 and a second contract at price €1,000,000, they are still awarded one lot each, but they are paid €1,000,000 each (which is the marginal losing bid), making a higher profit of €700,000. Notice that no bidder has an incentive to deviate from this behaviour and try to win two lots, because in doing so it would reduce the price to €300,000 and, hence, obtain no profit. Once again, this strategic behaviour greatly increases the price paid by the procurer. As a consequence, in a uniform-price tendering suppliers' bids can be much higher than their costs and, from the point of view of a procurer, expected savings can be particularly low.

There is evidence of demand reduction in electricity markets, spectrum auctions, and in experiments.[9] Typically, the presence of a large supplier bidding against smaller firms suggests a higher risk of unilateral demand reduction. Coordinated demand reduction is more likely when suppliers are able to implicitly or explicitly collude.

In discriminatory auctions demand reduction is less of a problem. For example, assume, as in our previous example, two bidders are trying to sustain a 'collusive' division of the lots being procured by bidding a low price for their share of the lots, and much higher prices for the other lots. (This makes it unprofitable for a bidder to try to obtain more lots than his 'collusive' share). In a discriminatory auction, this strategy is much less profitable than in a uniform-price auction, because each supplier is simply paid the price she offers on the lots she wins.

We can now suggest the following practical conclusion, concerning the two formats discussed above.

Practical conclusion 4

With strong negative complementarities and homogeneous supply contracts, if the common-cost component is relevant and uncertain, potential suppliers are heterogeneous in terms of available information and/or participation is a major concern, then favour uniform-price tendering. Otherwise, favour discriminatory tendering.

Both the uniform-price and discriminatory tendering procedures can be easily implemented when the number of identical contracts is very large and their size very small. In electricity procurement (as well as in sales auctions of Treasury bonds) the size of a single contract is typically very small, and this is why the procedures are denominated tendering/auctions of shares.[10]

[9] Kagel and Levin (2001), List and Lucking-Reiley (2000), Wolfram (1998), and Wolak (2003)
[10] Wilson (1979)

In this kind of tendering bidders have the opportunity to submit their supply schedule with a very high degree of accuracy, which attracts potential suppliers and helps the procurer in selecting the best providers from a large set of possibly very heterogeneous market suppliers.

8.4. Number of lots, package bidding and participation

When designing a procurement, the buyer must decide the number and size of lots/contracts in which the supply is split (see Chapter 7). The procurer is usually facing many heterogeneous potential suppliers, typically very many small ones, several medium-large sized ones, and a few very large ones. Also, the procurer typically does not know exactly which type of potential supplier is more efficient for a specific procurement, nor the strength of economies or diseconomies of scale and of other possible complementarities among potential lots/contracts.

When positive complementarities are expected to be relevant for larger suppliers, in the absence of package bidding there is a natural tension between lots aggregation, which allows larger suppliers to fully exploit economies of scale, and lots fragmentation, which favours entry by many smaller firms. Note that small firms are often more flexible and innovative, and so may sometimes be more cost effective than large ones, even if they cannot exploit economies of scale.

The flexibility in lots aggregation allowed by package bidding lets the market endogenously choose the optimal aggregation of contracts and scale of supply, at the same time encouraging participation of small potential suppliers and allowing the exploitation of economies of scale. The buyer can greatly reduce the minimum size of contracts/lots, and thereby maximize the number of smaller suppliers otherwise excluded, without hindering the ability of larger suppliers to bid on large sets of contracts in case they are characterized by positive complementarities. This allows all types of firms to express their different competitive advantages and the market to effectively decide who should be awarded the contracts.

Practical conclusion 5

When package bidding is allowed, reducing the size of the contracts to be procured encourages the participation of small firms without preventing large firms from exploiting economies of scale.

Example 8.2. The procurement of road paintings in Sweden.

The Swedish National Road Administration (SNRA) usually awards 50–60 contracts per year for the updating of road markings on national roads and each contract is valid for one year. In 2001, the SNRA implemented a field test applying a combinatorial tendering procedure. There were 8 potential bidders in the market. Two firms were relatively large and operated on a national basis. The other firms were more or less local, operating in adjacent counties only. The main aim behind the test was to make it easier for firms – both SMEs and large firms – to express in their bids the true production costs for various packages of contracts. This would in turn have the potential of lowering the SNRA's costs and increase economic efficiency. The SNRA set bidding rules which made the combinatorial bidding possible, allowing firms to submit offers on individual contracts and on any arbitrary number of contracts bundled at the bidder's discretion. In addition, the SNRA gave individual firms the option to put an upper bound on the maximum number of lots a firm could take on in case it won 'too many contracts'. On average 4.7 bids were submitted on each contract. The SNRA's cost was reduced and the number of firms that won contracts increased. In sum, the result indicates that combinatorial bidding increases competition because, compared to more conventional mechanisms, it allows SMEs to enter the auction lowering the procurer's cost.

Example 8.3. The procurement of telecommunication services in Italy.

In 2002, Consip implemented a combinatorial tendering procedure to procure telecommunication services. Two different lots were purchased: lot A was for fixed telecommunication services and lot B was for mobile telecommunication services. The market was characterized by two incumbents, Telecom Italia and Wind, which were the current providers respectively for fixed and mobile telecommunication services, and some potential entrants, among which the larger ones were Albacom and Vodafone. Furthermore, the two incumbents were active both in the fixed and mobile telecommunication service markets, whereas all the potential entrants were active either in the fixed or in the mobile telecommunication services market. With such bidders, the main goal was to design a tendering mechanism to encourage both participation of potential entrants and the emergence of synergies, if any, between fixed and mobile telecommunication services. Therefore, Consip decided to keep mobile and fixed telecom services as distinct lots, so that Albacom and Vodaphone could participate, and implement a combinatorial tendering design that allowed Telecom and Wind to express possible cost synergies. (For legal reasons the best non-combinatorial offer for the single lot had the preference. Hence, a combinatorial bid could win only if it was the best both in lot A and B.)

The design was successful: all four potential participants placed a bid and the threat of a competitive offer by potential entrants pushed the incumbents to offer aggressively. Both Telecom and Wind submitted package bids which were slightly lower than their offers on single lots and which lost to a combination of bids on single lots (because of the rule described above). This revealed that only small synergies existed between the two services.

Bibliographical notes

The theory and practice of multi-object competive bidding has long been focused on how to sell perfectly divisible assets such as state bonds (Wilson, 1979; Back and Zender, 1993). For an exposition to the main themes, such as the exposure and the threshold problems, and results on multi-object competitive bidding see the excellent books by Krishna (2002), Klemperer (2004), Janseen (2004) and Milgrom (2004). The work by Rassenti, Smith and Bulfin (1982) has been pioneer on combinatorial bidding while Rothkopf, Pekec and Harstad (1998) later analysed the issue of computionally complexity. Menu-Auctions, a sealed-bid specification of package bidding, were first introduced by Bernheim and Whinston (1986). For a more general, and recent, discussion on package bidding see Pekec and Rothkopf (2003) and Milgrom (2004). The volume by Cramton, Shoham and Steinberg (2006), is the most exhaustive effort on combinatorial tendering procedures, putting together practical as well as theoretical contributions by economists, computer scientists and operations reaserch experts. Applications of package bidding to procurement are more recent; an interesting and successuful example is illustrated in the paper by Epstein et al. (2004).

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