Insider Trading Rights: Short-termism and Trading Constraints

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Abstract

I investigate the role of trading restrictions for price efficiency in a model with short-term shareholders. I show that whenever shareholders impose trading constraints on the insider, prices will maintain a minimum level. More importantly, as trade is the mechanism that transfers information into price, the pricing function used by the market maker has non-constant sensitivity to order flows, with negative orders impacting the price less than positive orders. These two effects lower price volatility. Prices will be more efficient in good states of the world, but lose their informativeness in bad states of the world. Trading constraints will emerge in equilibrium if shareholders have preferences for short-term market prices. Moreover, in this case, an increase in cash flow volatility can potentially reduce the amount of granted trading rights, while the impact of noise variance depends on the preferences of the firm.

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1. Introduction

According to the strong form of efficient market hypothesis, stock prices should always reflect all public and private information available. The holders of this information, however, are generally insiders of companies, such as directors, managers and employees, whose fiduciary duty is to ensure that there is no harm to the company. Therefore, it is the company's best interest to push insiders to reveal positive information as quickly as possible, while companies will incentivize them to not disclose negative news. Consequently, we should expect stock prices to be less informative about future cashflows, above all in bad states of the world.

There are many reasons why firms care about their stock performance in the secondary market. If a company is expected to increase equity in the short term, it must show a healthy share price, or if a temporary positive NPV investment opportunity arises, the company must convince the financiers that the project is a sound investment before it expires. Short-term shareholders may decide to liquidate their holdings, and they will wish to do that at a higher price. All these considerations implicitly impact the cost of capital for the firm. At the same time, firms' insiders get rewarded with company shares to align their incentives to the objective of the firm. However, while the insider will buy (if he can) even more shares upon positive news, boosting share price up, if he sells them to avoid losses coming from negative news, he is in breach of that fiduciary duty with the company, having adversely impacted its price. The company can avoid this behavior by means of serious consequences for the insider, which will result in trading constraints.

Starting from these considerations, in this paper I enrich a traditional market microstructure model with trading constraints and link these trading constraints to the preferences of companies for price distributions, considering a short-term firm (shareholder), whereas past studies put more attention on a value-oriented shareholder. I demonstrate that the pricing function is characterized by different sensitivities to order flows in the presence of constraints. The restrictions prevent the price from falling below a certain level and being less volatile. Moreover, market efficiency is state-specific, with more informative prices in good states of the world relative to bad states of the world. The value of the constraints rises with the concerns of the firms about the unfavorable realizations of stock prices. In addition, cashflow volatility affects the severity of the constraints in the case of a risk averse company, whereas noise variance has an ambiguous effect on the optimal constraint.

In my model, a firm needs a manager to be hired to perform a task generating cashflows independent of manager's ability. Once he is hired, the manager learns about these cashflows and he is endowed with trading rights. These rights can either be specified by an explicit contract between the manager and the company (such as short-selling restrictions or vesting periods), or be the result of a tacit agreement among shareholders and executives (such as firing and a loss in reputation if taking unwanted actions). The manager will then maximize his trading profit in the limit the contract specifies.

The market maker for the company shares is a zero-profit agent, thus the manager can potentially trade on his competitive information advantage and extract rent from liquidity (non-strategic) traders in the market, in the same fashion of Kyle (1985). However, the market maker anticipates the constraints imposed on manager's trading strategy and will utilize a pricing function that has a non-constant sensitivity to order flows. As a result, prices stabilize above a certain value, capturing the inability of some insiders to freely dispose of their shares. Thus, prices are allowed to move in a smaller range of value, and the different sensitivity to the order flows makes prices less volatile compared to the unconstrained equilibrium.

This effect immediately translates into a differential market efficiency of prices. In fact, while in good states of the world, the insider can trade and use his order flow to transmit some information to the market, the constraints limit his ability to trade in bad state of the world. As a consequence, market prices will convey less information about company fundamentals in bad states of the world. The main result that the mechanism in the paper highlights is thus in opposition to the strong form of market efficiency, both in the statement of Fama (1970) , as well as of Jensen (1978). Nevertheless, following the classical trade-off in market microstructure, the reduced price efficiency is associated with an increase in liquidity.

Differently from past literature, which focuses more on her incentives to maximize firm value, the shareholder (the firm) in my model is price-oriented rather than value-oriented. Short-termism is fundamental for the imposition of constraints on insiders' trading ability. A risk neutral shareholder has no value for trading constraints, as she can diversify away any specific risk arising in the company. In opposition, a risk averse shareholder will benefit from shutting down all trading. My approach gives a mid-way solution among these two extreme scenarios and encompasses them. A short-term shareholder cares has preferences with different weights for positive or negative realizations of the price. In that case, there exists a region for which limiting trading is beneficial. This region can further be impacted by the variance of cash flow. In fact, if the shareholder is both risk averse and downside risk averse, an increase in the cash flow variance calls for more stringent constraints. Instead, the impact of noise trader variance has an ambiguous impact. As an increase in noise variance will be associated to more volatile order flows and more volatile prices, the firm will prefer to lift (increase) the trading constraint if upside (downside) realizations weight more than downside (upside) realizations.

The environment the actors in my model operate is characterized by ex-post disclosure of order flows from the insider to the shareholder, in line with rules of the Security and Exchange Commission (SEC), [1] according to which any manager or director of a company must disclose his or her trades within two days of a transaction. While the SEC agrees that the insider's report is helpful to investors in recognizing the long-term value of the company,[2] firms generally require executives, directors and employees to comply with insider trading policies that outline actions, transactions and penalties for trading their firm's share. Companies do not restrict themselves to the sole penalties imposed by SEC, but introduce "possible company-imposed" disciplinary actions, such as termination of employment or ineligibility of equity compensation programs.[3] Across companies, most of the policies set limits on short-sales or impossibility to hedge positions derived from incentives plans. Moreover, since December 2018, SEC requires companies to disclose any restrictions on the limit to purchase instrument or hedge or offset any decrease in the market value of equity securities granted as compensation.[4] The

[1] Section 16(a) and 23(a) of the Security Exchange Act of 1934. More information on https://www.sec.gov/about/forms/form4data.pdf

[2] https://www.sec.gov/files/forms-3-4-5.pdf

[3] Example of company-imposed actions can be found in the Insider Trading Policies of The Hershey Company (https://www.thehersheycompany.com/content/dam/corporate-us/documents/investors/insider-trading-policy.pdf) and Citi Trend, INC (https://ir.cititrends.com/static-files/ f9c51467-3c29-4044-8872-b16a9fcb75c8)

[4] https://www.sec.gov/corpfin/disclosure-hedging-employees-officers-and-directors

model presented in this paper offers insight on why companies would impose such prohibitions.

The main framework of this model is the market microstructure model of Kyle (1985), which was primarily used for its simplicity and predictability. With its trading, the insider improves price efficiency by reducing the ex-post volatility of cash flows. In fact, the higher the cash flow volatility, the lower the demand or flow value, and hence the higher the market liquidity. The presence of linear equilibrium in the model has also been shown for other distributional assumptions that satisfy given properties (Bagnoli et al. (2001)) and Huddart et al. (2001) show that the insider would use a mixed strategy and give up some profit if they have to disclose the transaction.

Although in other market-making models such as Glosten and Milgrom (1985), researchers may characterize the equilibrium in the presence of short-selling constraints (Diamond and Verrecchia (1987)), this is not the case with models à la Kyle, with the exception of Carre et al. (2019). In their model, the insider is subjected to regulatory penalties independently of the directions of the order flows. Instead, in the paper, I present a new methodology to address the imposition of directional constraints in models à la Kyle (1985). In fact, I characterize the asymptotic behavior of the price function that best-responds to the strategy of the insider when facing trading constraints. Moreover, I show that this best response can be approximated by a quadratic function within two standard deviations around the mean of the order flows. This approximation allows me to deal with the highly non-linear model and to shed light on the behavior of the pricing function. Moreover, an iteration of the maximization algorithm suggests that the optimal function is not qualitatively different from the equilibrium resulting from the second order approximation.

The point of view in this paper is the one of shareholders of the company (or the Board of Directors), or the firm itself with the aim of maintaining market prices at a sustainable level, and thus rewarding those actions that will not depress prices. Evidences toward these views are presented in Graham et al. (2005), where the authors find that managers prefer to take actions that would adversely affect long-term value to smooth earnings, and that managers would prefer to reveal information to reduce risk and raise stock prices. Moreover, limits to trading is associated with short-term price performances. Vesting equity correlates with managerial decisions to improve short-term price and maximize equity selling proceeds (Edmans et al. (2018)). Bolton et al. (2006) show that an optimal contract may foster market prices, but reduce the long-term value of the company. In my paper, the manager is focused on short-termism for two reasons: on the one hand, the manager wants to increase his trading profit; on the other hand, the shareholders want to guarantee a price that is high enough. As a result, management compensation relies on the price that can be obtained, in comparison to models where the price impact of contracts is exogenous (Bebchuk and Stole (1993); Edmans (2009)). A theoretical analysis of the relationship between market prices and short-termism is present in Piccolo (2019), but, differently from my model, his manager and speculator are two separate entities.

In the feedback effects literature[5], two papers may be considered similar and complementary to mine. Bebchuk and Fershtman (1994) consider an investment decision problem, and granting trading realign shareholders and manager risk taking. Their model is complementary to mine, as it analyzes a moral hazard problem, while I focus more on the asymmetric information. Nevertheless, they do not characterize unilateral constraints, while only focusing either on full trading or no trading rights.

The inability to trade on negative news in my model has some similarity with the model proposed by Edmans et al. (2015). In their model, the speculator will refrain to sell, as his decision will communicate to the managers information about future states of the world, thus advising them to disinvest and avoid losses. This result in bad news being incorporated slowly into prices. In my model, I find a similar patterns for news incorporation, except now it is a company best interest to delay bad news, and cashflows are exogenous to managerial decisions.

The paper also address a result in the insider trading literature about predictability of insiders order flow on the stock return. In fact, with the first paper written on it in 1968 (Lorie and Niederhoffer (1968)), there is consistent evidence of predictability of insider purchase against insider sales. Practitioners agrees that usually sales are associated with liquidity trading motives, thus their predictive power is different from purchase.[6] My paper offers another, and to my knowledge unique, and alternative explanation that is not at odds with common beliefs. Insider could sell for liquidity related reasons

[5]For a review of the literature in feedback effects, see Bond et al. (2012)

[6]Peter Lynch, former manager of Magellan Fund at Fidelity, was noted as saying that "insiders might sell their shares for any number of reasons, but they buy them for only one: they think the price will rise."

when this decision has nothing to do with company cashflows (i.e. after a sale, share prices will go up), but as long as their sale will be interpret as carrying negative news about company's fundamental, the company will prevent it.

The paper proceeds as follows. Section 2 describes the model, characterizes the financial market equilibrium, and discusses the assumptions. In Section 3, I analyze the trading game and introduce some techniques to shed lights on the pricing function. In Section 4, I characterize the contracting stage and study different preferences specifications for the shareholder, and show that the results are qualitative unchanged. Then I discuss the minimization problem associated with the cost of imposing trading restrictions. In Section 5, I summarize the main results of the paper and offer some empirical prediction.

2. The model

A shareholder (she) needs a manager (he) to operate a firm with only one project. The risk-neutral manager (also the CEO or the Insider), is compensated with wage and trading rights on his company's shares, all specified in a contract $w(p,x,v)$. If allowed, the manager can submit orders x for the company share on the market anonymously. Without loss of generality, his reservation utility is set to zero, and the market for managerial skills is perfectly competitive. The market features a noise trader who submits order flow $n \sim N(0, \sigma_n^2)$, and perfectly competitive market maker observes total order flows $q = x + n$. Although managers can submit their trades anonymously, the regulator requires the ex-post disclosures of the orders to the shareholders (in compliance with SEC Rule 10b).

The shareholder has some preference for the short-term price of the company, net of any cost to compensate the manager. Her preferences with respect to the distribution of the share price are:

$$U^{\mathscr{W}}(p) = E_w[h(p)] \tag{1}$$

$h(\cdot)$ is a function that focuses on prices resulting by imposing a contract $\mathscr{W}$. While shareholders' preferences consider only the price of the stock, I can extend the model to include both short-term and long-term shareholders. A rational reason for the short-term emphasis is the need to raise money for the company in the immediate future, so that the price needs to be high or not volatile in order for funding providers to finance the business, or any other cost of capital related issues.

2.1. Information and Trading

The cashflow of the company per share v follows a distribution $N(v_0, \sigma_v^2)$. Upon signing the contract $w(p,x,v)$, the manager knows value v with certainty, but the shareholders do not. Then he submits order $x : v \to \mathbb{R}$ with the final goal of optimizing his trading profit and compensation, and ex-post discloses the trade to the shareholder. As the shareholder is unaware of the company's value, she will refrain from trading their shares, and suffer losses due to asymmetric information. Independently of the company's cash flows, noise trader submits order flow n due to exogenous shock in his/her income.

The market maker observes total order flows $q = x + n$ and set a price p according to Bayes' rule in order to break even in expectation.

2.2. Contracts

The shareholder proposes an ("implicit or explicit") contract to the manager in the following way. The contract $\mathscr{W}$ will specify w, an ex-ante or ex-post "payment or punishment" plan, that can potentially be written upon the share price p after the round of trading, the order flow of the manager x and the value of the company v once this is known. Notice that the contract could specify a payment at the beginning of the game, a payment later in the game, trading rights and termination (in this case one can consider termination as negative amount w). The shareholder will have to commit to the contract, and cannot renegotiate on it. I will focus on particular type of contracts that are characterized by $\bar{x}$, which is the amount of shares the insider could trade. I am using contract in the broad term of the word. We could think as an explicit as well as an implicit agreement between shareholders and managers to not take actions that will negatively impact the price of the company.

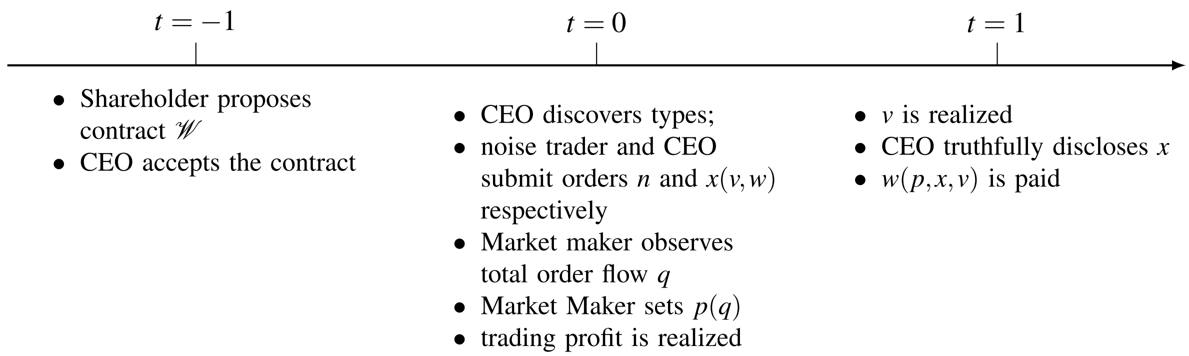

Figure 1. Timeline of the model

2.3. Model timeline, benchmark scenario and equilibrium definition

As shown in Figure 1, the game unfolds as follow:

1. At t=-1, the shareholder proposes a contract to the manager to perform the single project of the firm
2. At t=0, upon signing of the contract, manager discovers the cashflows v and submits order flow x, if the contract allows. In the meanwhile, noise traders submit order flow n. Market maker observes q, set a price p, shares are exchanged and trading profits are realized
3. At t=1, the value of the firm is realized and manager truthfully discloses x to shareholder, the payoff of the contract are realized

When a contract is implementable, it will specify some payoff w to be paid at the end of the game. In this environment, we suppose the shareholder can commit to a contract whose payoff will at the end depend on the realized value of p,x and v. The shareholder's problem is to

$$\max_{\mathscr{W}} \quad U^{\mathscr{W}}(p) \qquad (objective) \tag{2}$$

$$s.t. \quad E(w|x)+xE[v-p|x] \geq 0 \qquad PC \tag{3}$$

$$x \quad \in arg\max_{x} \quad E(w|x)+xE[v-p|x] \qquad IC \tag{4}$$

$$p = E[v|q] \qquad MM \tag{5}$$

Notice that we impose that the shareholder can commit to some payoff structure: once trading occurs, price is set and v becomes common knowledge, and the manager gets (pays) w. Notice that x depends on w and w may depend on both x and p. The shareholders could write a contract that specifies what happens if p is realized and manager trades x, and the optimal trading will so depend on it. The participation constraint of the insider has one source of uncertainty, which is the realization of noise trader order flow. This impact the price and as a consequence the "wage" of the CEO. One of the nice interpretation to the above problem is that the payoff specified by the contract has not to be limited to the value of the company. In fact, we could interpret an extremely negative w to be the loss in reputation for the CEO following a price plunge, or any loss of future compensation deriving from the company firing the manager for bad stock market performances. If the cost to implement the contract is independent from the level of w, then the shareholder can impose maximum punishment and the contract incentive compatible and the punishment is never experienced in equilibrium. [7]

Given this interpretation, it is incentive compatible to set the actual payoff from the contract to be $w = -\infty$ if $x < \bar{x}$, 0 otherwise, where $\bar{x}$ is a level of transactions that the insider is allowed to trade. Moreover, as x will be a sufficient statistics to understand if the manager adhered to the contract, , I define the following equilibrium

Definition 1. Equilibrium Definition

In this model, an equilibrium is a triple $x(v), p(q)$ and $\mathscr{W}$ such that:

1. $x(v)$ maximizes CEO payoff given pricing rule p and contract w
2. $\mathscr{W}$ maximizes shareholders' payoff given the induced trading and pricing rule
3. $p(q)$ is such that the Market maker breaks even, that is $p(q) = E[v|q]$

In a baseline model with no contracting I obtain the same solution as in Kyle (1985).

Proposition 2 (The no contract environment). *In an environment when no contract is enforceable,*

[7] I could impose a further Limited Liability constraint on the payoff structure of the contract such that

$$v \geq w \qquad LL$$

i.e. $\mathscr{W} = \{\emptyset\}$*, the strategy of the insider and the market maker are:*

$$x(v) = \frac{\sigma_v}{\sigma_n}(v - v_0)$$
$$p(q) = E(v|q) = v_0 + \frac{\sigma_n}{2\sigma_v}q$$

the insider profit from trading and the variance of the price distribution are respectively:

$$xE(v - p|x) = \frac{\sigma_v}{\sigma_n}\frac{(v - v_0)^2}{2}$$
$$var(v|p) = \frac{1}{2}\sigma_v^2$$

If the shareholder has no ex-ante information on v, the ex-post price given noise traders' order flow n will be :

$$\hat{p} = v_0 + \frac{(v - v_0)}{2} + \frac{\sigma_v}{2\sigma_n}n$$

Proof. As in Kyle (1985), with $\Sigma = \sigma_v^2$. □

The second term in the last expression captures the information that is revealed given the order flow of an insider following the optimal strategy. It is easy to see that this term is positive if $v > v_0$. However, in expectation (i.e. from the perspective of the ex-ante shareholder), this term is equal to zero. This may suggest that the shareholder would be willing to write a contract to avoid the revelation of information in the case of a negative order flow, thus imposing selling constraints to the strategy of the insider. The no-contract case serve as benchmark for the full model. Given risk-neutrality and reservation level set up to zero, this can be implemented as it is associated with a slack participation constraint for the insider, except if the cash flows of the company are equal to v_0.

3. The trading game

I now solve for the equilibrium strategies by backward induction. I analyze the problem in a situation where the shareholder can perfectly enforce a punishment in the following way: after the shareholder discloses his order flow, the punishment is set to minus infinity for all orders below $\bar{x}$, and zero otherwise. With a cut-off $\bar{x} = 0$ the shareholder is implementing selling constraints that could potentially emerge due to rights' vesting periods or for the signal that could be usually associated with negative order flow.

Under this contract, in equilibrium, manager of low quality firms will be pulled together. This contract is incentive compatible, and the shareholder will never punish the manager on the equilibrium path. I will first show the more general unfolding to the problem, but given the intractable form of the solution, I will show numerically how a linear strategy in price will be dominated by a quadratic one, and show how the quadratic strategy could approximate the Bayes' rule strategy. Imposing a cut-off level of zero is made only for ease of exposure. It may be that in equilibrium, this is either negative or positive. A negative cut off $\bar{x}$ can be considered as a initial grant of shares, followed by a short-selling constraints. A positive $\bar{x}$ should instead considered as a call option of shares the insider has to exercise.

3.1. Conditional Expectation and Bayes' rule

3.1.1. The Market Maker Problem

Suppose that the Market Maker believes that the insider is using a strategy $x(v) : v \to \mathbb{R}$ with $x'(\cdot) \geq 0$ (a monotone, non-decreasing strategy), the joint distribution of q and v after observing order flow q is given by:

$$f(q,v) = \phi(\frac{(v - v_0)}{\sigma_v})\phi(\frac{q - x(v)}{\sigma_n}) = \frac{e^{\left(-\frac{(q-x(v))^2}{2\sigma_n^2} - \frac{(v-v_0)^2}{2\sigma_v^2}\right)}}{2\pi\sigma_n\sigma_v} \tag{6}$$

as $n = q - x$ and independent from v. By applying Bayes' rule:

$$f(v|q) = \frac{1}{F_q(q)} e^{(-\frac{(q-x(v))^2}{2\sigma_n^2} - \frac{(v-v_0)^2}{2\sigma_v^2})} \tag{7}$$

Where

$$F_{(q)}(q) = \int_{-\infty}^{+\infty} e^{-\frac{(q-x(v'))^2}{2\sigma_n^2} - \frac{(v'-v_0)^2}{2\sigma_v^2}} dv' \tag{8}$$

Since market maker is competitive, he sets the price $p = E[v|q]$ and so:

$$p(q) = \frac{g_q(q)}{F_{(q)}(q)} \tag{9}$$

Where

$$g_q(q) = \int_{-\infty}^{+\infty} v' e^{-\frac{(q-x(v'))^2}{2\sigma_n^2} - \frac{(v'-v_0)^2}{2\sigma_v^2}} dv' \tag{10}$$

Equation (9) pins down $p(q)$ given $x(v)$. If the market maker (rightfully) conjectures that for some cutoff ν the strategy of the insider will be

$$x(v) = \begin{cases} f(v) & v \geq \nu \\ \bar{x} & v < \nu \end{cases}$$

(9) becomes

$$p(q) = \frac{\phi\left(\frac{q-\bar{x}}{\sigma_n}\right) \int_{-\infty}^{\nu} v\phi\left(\frac{v-v_0}{\sigma_v}\right) \mathrm{d}v + \int_{\nu}^{+\infty} v\phi\left(\frac{v-v_0}{\sigma_v}\right) \phi\left(\frac{q-f(v)}{\sigma_n}\right) \mathrm{d}v}{\phi\left(\frac{q-\bar{x}}{\sigma_n}\right) \Phi\left(\frac{\nu-v_0}{\sigma_v}\right) + \int_{\nu}^{+\infty} \phi\left(\frac{v'-v_0}{\sigma_v}\right) \phi\left(\frac{q-f(v')}{\sigma_n}\right) \mathrm{d}v'} \tag{11}$$

Lemma 3. *The pricing function used by the insider in the constrained equilibrium is characterized by an horizontal asymptote at* $E[v|v < \nu]$, *that is*

$$\lim_{q \to -\infty} p(q) = E[v|v < \nu] \tag{12}$$

Proof. See appendix A.1. □

3.1.2. Insider's problem

Given the induced constraint from the contract, the insider will only submit constraints orders, that is $x(v) \geq \bar{x}$. If insider believes that the market maker will set price $p(q)$, then the insider will solve the following problem[8]:

$$x(v) := \arg\max_{x \geq \bar{x}} x(v - E[p(x+n)|x]) \tag{13}$$

$$s.t. p(x+n) = E[v|q] \tag{14}$$

When the constraint on quantity is not binding, $x(v)$ is such that it solves

$$(v - \bar{p}) - \bar{p}'x = 0 \tag{15}$$

where

$$\begin{aligned}
\bar{p} =& E[p(x+n)|x] \\
\bar{p'} =& \frac{\partial}{\partial x} E[p(x+n)|x]
\end{aligned}$$

Moreover, the Second order condition is satisfied whenever:

$$-x\bar{p}'' - 2\bar{p}' < 0 \tag{16}$$

where

$$\bar{p''} = \frac{\partial^2}{\partial x^2} E[p(x+n)|x]$$

[8]After signing the contract the insider knows the value v of the company. His profit from trading is

$$\begin{aligned}
E[x(v-p(q))|x] =& xE[v - p(x+n)|x] \\
=& x(v - E[p(x+n)|x])
\end{aligned}$$

3.1.3. Fixed point analysis

Proposition 4. *If the market maker is using a non-decreasing pricing function and the insider is expecting a positive impact of his order flow on the price, the strategy of the insider is characterized by a cut-off* $\mathbf{v} = f^{-1}(\bar{x})$

$$x(v) = \begin{cases} f(v) & v \geq \mathbf{v} \\ \bar{x} & v < \mathbf{v} \end{cases}$$

with f(x) to be non-decreasing and continuous function.

A price function exists and it is right-continuos over $\mathbb{R}$. *Moreover, if the insider is using an non-decreasing strategy, the market maker will use an increasing function of q.*

Proof. Given equations (11), (15) and (16), an equilibrium in the trading game consists of two strategies $p(q)$ and $x(v)$ such that (11) and (15) are satisfied for all v, and (16) is verified. It is easy to see that whenever x is 0 (the insider is not trading), equation (16) is satisfied if and only if the filtering of $p(q)$ with respect to n is strictly increasing with respect to x. Notice that the all problem is a system of integro-differential equations.

I first start by analyzing the property of the optimal $x(v)$ whenever the short-selling constraint is not binding. Assuming p is non-decreasing in the order flow, then if (15) binds

$$x = \frac{v - \bar{p}}{\bar{p}'} \tag{17}$$

I will now use Implicit function theorem and Chain Rule to study the characteristics of $x(v)$ whenever the selling constraint is not binding. By deriving (15) with respect to v I have that

$$\begin{aligned} \frac{\partial}{\partial v}[(\text{v-}\bar{p})\text{-}\bar{p}\text{'x}] &= 0 \\ 1 - \bar{p}'\frac{\partial x}{\partial v} - \frac{\partial x}{v}(\bar{p}''x + \bar{p}') &= 0 \\ \frac{\partial x}{\partial v} = \frac{1}{\bar{p}''x + 2\bar{p}'} &> 0 \end{aligned} \tag{18}$$

Where the last inequality holds if and only if (16) holds. Unsurprisingly, (18) is telling us that $x(v)$ is an increasing function of v, confirming that, the better the future prospects of the firm are, the more the manager is willing to buy. I define with $\nu := sup\{v|x(v) = \bar{x}\}$. That is ν is the cutoff level on v, such that the insider is indifferent between selling $\bar{x}$ or a slightly higher amount. This also imply that the $x(v)$ is a continuous and non-decreasing function in v.

The second statement in the Proposition is trivial given Bayes' rule and continuity of the order flow space, given the support of the noise and cash-flow distribution. □

The major challenge comes into evaluating the second derivative, In fact:

$$\frac{\partial^2 x}{\partial v^2} = -\frac{x\bar{p}^{(3)} + 3\bar{p}''}{(x\bar{p}'' + 2\bar{p}')^3} \tag{19}$$

Now, since the denominator of (19) is always positive for (16), I only need to care about numerator. When p is linear (i.e. the market maker uses OLS to set prices), one can easily see that the numerator is zero and so also the optimal strategy for the insider is linear. Thus the conjecture of the insider about the strategy of the market maker feeds into the convexity of the function following the sign and magnitude of the second and third derivatives of p.

3.2. Linear Strategy for the market maker

Previous literature has focused on linear strategy as equilibrium for the unconstrained problem. In a constrained problem with $\bar{x} = 0$, I analyze a linear strategy for the Market Maker and Insider, and I argue this strategy is dominated by a more flexible strategy. This serves as an useful exercise to shed some lights to the behavior of the insider and of the market maker in the context where trading can only be positive. The market maker set price

$$p(q) = \mu + \lambda q \tag{20}$$

Under zero-profit conditions this will imply that the market maker uses a linear projection as best predictor of price given the observed quantity. When both v and q follow a Normal distribution, the linear projection coincides with the conditional expectation of v given q. Instead, when the two variables are not Gaussian, the linear projection of v onto q is the minimum variance linear predictor of v given q.

3.2.1. The insider problem

The discussion follows Kyle (1985) but one has to consider an additional constraint of the order space to be non-negative, $\bar{x} = 0$.

If the insider knows that the market maker can only set price p(q)=$\mu + \lambda q$, he solves the following problem:

$$\max_{x \geq 0} \quad x(v - \mu - \lambda x) \tag{21}$$

Which leads to the following solution:

$$x(v) = \begin{cases} \frac{v-\mu}{2\lambda} & v \geq \mu \\ 0 & v < \mu \end{cases} \tag{22}$$

In a standard fashion, for $v \geq \mu$ I have that

$$\begin{cases} \alpha = & -\mu\beta \\ \beta = & \frac{1}{2\lambda} \end{cases} \tag{23}$$

Moreover

$$v = \mu \tag{24}$$

3.2.2. *The market maker strategy when $\bar{x} = 0$*

Suppose that the market maker hypothesizes that the manager will submit orders according to the following strategy

$$x(v) = \begin{cases} \alpha + \beta v & v \geq \nu \\ 0 & v < \nu \end{cases} \tag{25}$$

Using the estimation procedure I have that

$$\begin{aligned} p =& E[v|q] \\ p =& E[v] + \frac{cov(v,q)}{var(q)}(q - E[q]) \\ p =& \left(v_0 - \frac{cov(v,q)}{var(q)} \left(\frac{\beta \sigma_v e^{-\frac{(\nu - v_0)^2}{2\sigma_v^2}}}{\sqrt{2\pi}} + (\alpha + \beta v_0)(1 - \Phi\left(\frac{\nu - v_0}{\sigma_v}\right)) \right) \right) + \frac{cov(v,q)}{var(q)} q \\ p =& \mu + \lambda q \end{aligned} \tag{26}$$

3.2.3. *Equilibrium Analysis*

To find the equilibrium coefficients $\nu, \alpha, \beta, \mu, \lambda$, one needs to solve the following system of equations:

$$\nu = \mu \tag{27}$$

$$\alpha = -\mu\beta \tag{28}$$

$$\beta = \frac{1}{2\lambda} \tag{29}$$

$$\mu = v_0 - \lambda \left(\frac{\beta \sigma_v e^{-\frac{(\nu - v_0)^2}{2\sigma_v^2}}}{\sqrt{2\pi}} + (\alpha + \beta v_0)(1 - \Phi\left(\frac{\nu - v_0}{\sigma_v}\right)) \right) \tag{30}$$

$$\lambda = \frac{cov(v,q)}{var(q)} \tag{31}$$

By (27), (28), and (29) , one can use (30) to show

$$(\nu - vo) = -\frac{\sigma_v e^{-\frac{(v-v_0)^2}{2\sigma_v}}}{\sqrt{2\pi}\left(1+\Phi\left(\frac{v-v_0}{\sigma_v}\right)\right)} \tag{32}$$

$$\frac{\partial \nu}{\partial v_0} = 1 \tag{33}$$

$$\frac{\partial \nu}{\partial \sigma_v} = -\frac{e^{-\frac{(v-v_0)^2}{2\sigma_v}}}{\sqrt{2\pi}\left(1+\Phi\left(\frac{v-v_0}{\sigma_v}\right)\right)} \tag{34}$$

From (32), ν is smaller than v_0. Moreover, while the cut-off increases with the prior of the distribution, this also decreases with the standard deviation. It is worth noticing that now the insider will like to buy in a situation for which he was willing to sell without the short-selling constraints. For the indifferent type ν, this will imply that the expected price is lower all else equal. [9]

In this scenario the insider will trade more aggressively (higher beta) to compensate for the profit he would have to give up if the signal is negative. The market maker will react by lowering λ, as now more order flows are independent from v, making the market for the share more liquid, which will in turn feed back in the insider strategy. In fact, with both v and n distributed as a standard Normal distribution, $\nu \approx -0.27$ and $\lambda \approx 0.44$ and $\beta = 1/2\lambda$. However now the mean for q is approximately is 0.55 β. The price is still unbiased in expectation. Order flows have a lower impact on price compared to the standard case, fostering liquidity, and so reducing the speed of adjustment of price to information, which is in line with Diamond and Verrecchia (1987).[10] Compared to the situation in Proposition (2), the insider is not capable anymore to perfectly trade by mimicking the distribution of noise. The semi-separation induced by the constraint, in fact, does not allow him to mimics the variance of the noise trader, as he could not trade negatively. This also impact on the expectation of his order flow.

This linear pricing function is not optimal for the Market Maker, as he would be better off by using

[9] Notice that the opposite will be true if there was a buying constraint. (32) will be exactly the same but with a positive sign this time. Now if before the type ν was buying is now willing to non-sell, meaning that all else equal for this type, the expected price should be higher.

[10] From Diamond and Verrecchia (1987): “Constraints eliminate some informative trades, but do not bias prices upward. Prohibiting traders from shorting reduces the adjustment speed of prices to private information, especially to bad news”.

Bayes' rule. I now study the asymptotic behavior of the function that would result from Bayes' rule if the insider were to use a linear strategy in his order flow.

3.2.4. *Asymptotic behavior of the real conditional expectation*

In order to break even the Market Maker should set a price that is indeed different from the linear price schedule used in this section:

$$p(q) = E(v|q) \tag{35}$$

Now suppose that the insider is using the following strategy:

$$x(v) = \begin{cases} 0 & \text{if } v < \nu \\ \\ f(v) & \text{if } v \geq \nu \end{cases} \tag{36}$$

with $f(\nu) = 0$ and $f'(v) \geq 0$, that is continuous and non-decreasing strategy. W.l.g., assume $[v,n] \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right)$), I have the following:

$$p(q) = \frac{\int_{-\infty}^{\nu} v * \phi(v)\phi(q)dv + \int_{\nu}^{+\infty} v * \phi(v)\phi(q - f(v))dv}{\int_{-\infty}^{\nu} \phi(v)\phi(q)dv + \int_{\nu}^{+\infty} \phi(v)\phi(q - f(v))dv} \tag{37}$$

Now, I notice that the first terms in the numerator and denominator depend on the strategy of the insider only for ν, and it is constant to the conditional expectation/probability of v being lower than ν observing q. The first term in the numerator will be always equal to:

$$\int_{-\infty}^{\nu} v * \phi(v)\phi(q)dv = -\frac{e^{-\frac{\nu^2}{2} - \frac{q^2}{2}}}{2\pi} \tag{38}$$

This term, first of all is negative, due to the fact that the expectation is taken on the negative part of the distribution of v. In general this will be lower than the unconditional expectation of v.

Let us focus our attention to the following set of strategy for the insider. The insider is using a

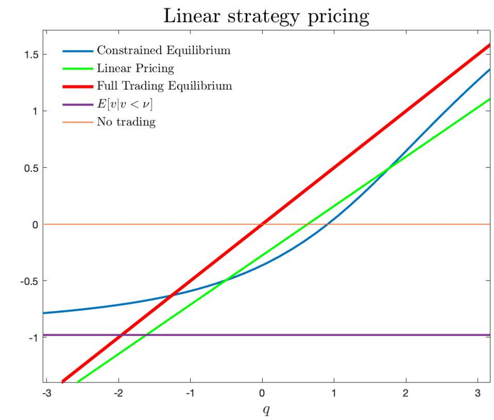

Figure 2. Asymptotic Behavior of conditional expectation

linear strategy such as:

$$x(v)=\begin{cases}0 & v<\nu\\ \beta(v-\nu) & v\geq\nu\end{cases}\tag{39}$$

with $\beta>0$. This makes the expression in 37 closed form:

$$p(q)=\frac{\beta e^{\frac{q^2}{2}}\left((\beta\nu+q)\Phi\left(-\frac{\nu-\beta q}{\sqrt{\beta^2+1}}\right)-\beta\sqrt{\beta^2+1}\frac{e^{-\frac{(\nu-\beta q)^2}{2(\beta^2+1)}}}{\sqrt{2\pi}}\right)}{(\beta^2+1)\left(e^{\frac{q^2}{2}}\Phi\left(\frac{\beta q-\nu}{\sqrt{\beta^2+1}}\right)+\sqrt{\beta^2+1}\Phi(\nu)e^{\frac{(\beta\nu+q)^2}{2(\beta^2+1)}}\right)}\tag{40}$$

I can show that

$$\lim_{q\to-\infty}p(q)=\frac{\beta}{1+\beta^2}\frac{e^{-\frac{\nu^2}{2}}}{\Phi(\nu)}\left(0-\frac{\sqrt{\frac{2}{\pi}}}{\beta}-\sqrt{\frac{2}{\pi}}\beta\right)=\frac{-\sqrt{\frac{2}{\pi}}e^{-\frac{\nu^2}{2}}}{\Phi(\nu)}=E[v|v<\nu]\tag{41}$$

and more importantly

$$\lim_{q\to+\infty}p'(q)=\frac{\beta}{1+\beta^2}\tag{42}$$

I plot the above pricing function in Figure 2, given the result of the analysis with a market maker using OLS, that were $\nu=-0.27$ and $\beta=1.13$.

If one considers the standard normal distribution and no short-selling constraint equilibrium, the term in (42) is exactly the coefficient of the Market Maker strategy. This result is generalizable to a situation with different mean and variance.

3.2.5. *Goodness to fit*

From Figure 2, it is clear that a linear approximation fails to capture the curvature in the conditional expectation that a selling constraint will induce. In order to understand the goodness to fit of the above function, given the distribution of order flows, I simulate order flows and price reactions according to equations (25) and (26). In order to smooth out noise in conditional expectation, I numerically solve for the Bayes' rule price function and interpolate and weight the observation. I consider different values for both σ_n and σ_v, while w.l.g. I fix v_0 at zero. A regression of the fitted price given OLS versus the Bayes' rule pricing function results in a R-squared of 93% on average, a insignificant intercept and a slope not significantly different from 1. Average pricing error is close to zero. These results are encouraging, but the shape of the conditional expectation function suggests some curvature in the pricing function, thus the need for some polynomial approximation for it.

3.3. *Quadratic approximation of Market Maker strategy*

I now guess that the strategy used by the market maker is:

$$p(q) = \mu + \lambda_1 q + \lambda_2 q^2 \tag{43}$$

Using OLS, this predictor should still satisfy the assumption of minimum variance. Notice that the function is no longer monotone and has a minimum (or a maximum) at

$$q^{min} = -\frac{\lambda_1}{2\lambda_2} \tag{44}$$

$$p^{min} = \mu - \frac{\lambda_1^2}{4\lambda_2} \tag{45}$$

I claim the point with the above coordinates is the minimum of the pricing function, which simply means that $\lambda_2 > 0$. I should be careful here, as I should interpret the minimum price as the price that the market maker should be setting for any order that is indeed less than q^{min}.

3.3.1. Insider Strategy and market maker response

Under the above price function, assuming a positive λ_1, the insider chooses $x(v)$ to solve the following problem:

$$\max_{x \geq 0} \quad x(v - \mu - \lambda_2\sigma_n^2 - \lambda_1 x - \lambda_2 x^2)$$

The solution for the FOC is[11]:

$$x(v) = \begin{cases} \sqrt{\frac{1}{3\lambda_2}v - \frac{\mu+\lambda_2\sigma_n^2}{3\lambda_2} + \left(\frac{\lambda_1}{3\lambda_2}\right)^2} - \frac{\lambda_1}{3\lambda_2} & if \quad v \geq \mu + \lambda_2\sigma_n^2 \\ 0 & otherwise \end{cases}$$

I need to make sure that $x(v)$ is positive and real. Thus, one need v to be big enough. Notice that x now follows a half normal. Given the not close form of the Chi-distribution mean and variance, I simulate the model.

Defining the following:

$$\begin{aligned} \beta =& \frac{1}{3\lambda_2} \\ \alpha_1 =& -\frac{\mu + \lambda_2\sigma_n^2}{3\lambda_2} \\ \alpha_2 =& -\frac{\lambda_1}{3\lambda_2} \\ \nu =& \mu + \lambda_2\sigma_n^2 \end{aligned}$$

[11]For the problem to have a solution, one need to impose some restriction on the parameters. In particular, the positive solution to the FOC will guarantee a maximum for both λs positive. While for λ_2 negative, I need λ_1 to be high enough. The solution of the unconstrained problem gives only local minimum and maximum, centered around the value of -$\lambda_1/3\lambda_2$ while imposing x non-negative, one can pick only one solution.

The Market Maker observes q and uses OLS according to (43).

3.3.2. Simulations

In order to close the model, I perform a convergence loop. I generate normal distribution observation for v and n, both with mean zero, w.l.g. I used a starting point for the coefficient in x. I then compute x and q and I performed a Multivariate OLS to fit the price. The usual formulas for MOLS apply here. After computed the lambdas I used the functional form of x to compute the coefficients and generated the new x. I compute the difference in norm on the coefficients for the OLS regression and retired the process until the absolute value of it will converge to zero. I performed the simulation on a varying sample from 1000 to 1000000 observations and with multiple starting points. The estimation is not sensible to sample size, but starting points must be far enough from zero, otherwise the resulting matrix for OLS will be singular and estimation will be biased.

No matter the starting point, lambdas are always positive, that guarantees a maximum in the insider problem and a minimum price.

The market maker strategy can be approximated by by the following coefficient, when $\bar{x} = 0$

$$\mu = -0.3\sigma_n$$
$$\lambda_1 = 0.35\frac{\sigma_v}{\sigma_n}$$
$$\lambda_2 = 0.054\frac{\sigma_v}{\sigma_n^2}$$

The strategy of the insider is thus:

$$\beta = 6.12\frac{\sigma_n^2}{\sigma_v}$$
$$\alpha_1 = 1.5\sigma_n^2$$
$$\alpha_2 = -2.13\sigma_n$$
$$\nu = -0.244\sigma_v$$

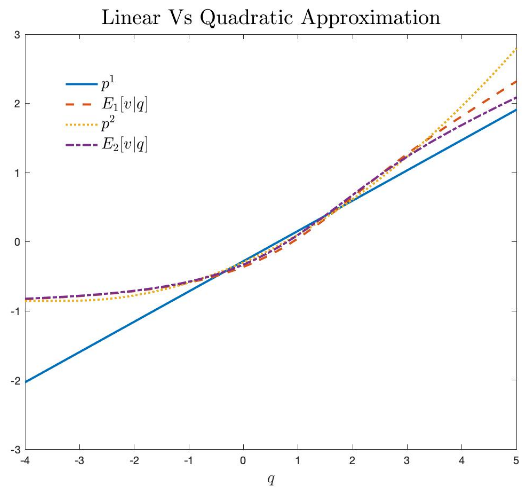

Figure 3. Quadratic Approximation and Comparison

3.3.3. Some Graphic analysis for asymptotic behavior

The major problem with a quadratic approximation is the impossibility to have a closed form of the integral for the conditional expectation, differently under the assumption of a linear strategy.

I numerically solve for the strategy of the Market Maker assuming that $f(v) = \sqrt{\beta v + \gamma} + \alpha$. The above strategy results from the a conjecture of the insider to use a quadratic price approximation, given some sign conditions on $\{\beta, \gamma, \alpha\}$. At least numerically, the function does not differs so much to the conditional expectation given a linear f(v). In particular, one could plot the numerical approximation and see that the conditional expectation is very similar to the one under the linear and the root strategy.

With the “solution” parameter one could plot the above function again and notice the very close resemblance to the one assuming a linear strategy. Moreover, the majority of the errors comes from values of q that are for more than two standard deviation from the average, so that at least 95% of the time the Market Maker is indeed pricing according to Zero profit condition. The second order approximation is quite accurate in the lower part of the distribution of order flows (where the probability of facing an insider is low, so order flow lack informativeness). However, the Market Maker overvalues positive order flows. To further investigate the goodness of approximation, I solve the integral numerically and interpolate the conditional expectation for some simulated order flows. The OLS regression

/Users/giococco/Desktop/3.jpg

Figure 4. Square strategy and Conditional Expectations

between the approximated price and conditional expectation shows an R-squared of 99%, with a slope coefficient of 1, very high significance, and an intercept of zero.

Although the approximation is still somewhat far to perfectly match the function suggested by Bayes' rule, the shape is still similar. This would suggest that the not only the behavior of the market maker is converging to some sort of curved function, potentially linear for $q \to +\infty$, but also the behavior of the insider is somewhat converging to a concave function.

3.4. Analysis of the distribution of the posterior price

Without trading from an insider, the market maker has no means to infer the value of the company from the order flow. As a consequence, the price of the company will be the expected value of the prior of the distribution of cashflow.

On the other hand, allowing full possibility of trading convey some information to the market, as the insider will shed his order flow according to Proposition 2, and the posterior distribution of the price will be characterized by half of the prior variance. This because the market maker is capable infer the part of noise in the order flow coming form the noise trader. In that situation, the insider is hiding his order flows behind the noise trader, by mimicking the same expected value and variance.

With trading constraints, posterior expected value of price is still matching the prior expected value, however the variance of price is now further decreased. This is because the market maker understands that only a fractions of order now contains information, as in Diamond and Verrecchia (1987). More negative order flows convey less informations, as the probability of the insider to be present there is low. In our approximation the variance of price is about 26% the variance of the cashflow, while the variance of the order flow from insider is only 45% of the variance of the noise trader, and they cannot use the whole support of the distribution of the noise trader order flow. The inability to not being able to perfectly camouflage the order flow for the insider has thus two different effect on price. The first one in on their level. Prices will converge to a lower bound that is in line with the expected value of the company under selling constraint to the insider. Moreover, the ex-post distribution is less disperse, given this bound.

3.5. Trading constraints

To simplify the discussions, I analyzed the trading constraints as selling constraints. However, the shareholder could set any sort of $\bar{x}$ as a constraint in the model. While it is easier to interpret a selling constraints ($\bar{x} \leq 0$) as an initial share grant at t=-1 and a subsequent short-selling constraint imposition, which will result in a minimum trade of $\bar{x}$, a buying constraint ($\bar{x} > 0$) can be interpret as stock vesting with the cliff at t=1.

I can specify equilibrium for different trading constraints. I define the following:

Definition 5 (Constrained Trading Equilibrium)**.** An equilibrium in the constrained trading game is such that:

- The insider chooses $x(v) \geq \bar{x}$ to maximize his expected utility,
- The market maker sets price $p^C(q)$ according to Zero Profit condition, considering the constraint $\bar{x}$
- The Shareholder sets $\bar{x}$ such that $w(p,x,v) = -\infty \ \forall x < \bar{x}$ and to maximize her utility

Definition 6 (No Trading equilibrium)**.** has $w(p,x,v) = -\infty \ \forall x \neq 0$, $x(v) = 0 \forall v$. and $p^{NT}(q) = v_0 \forall q$.

Definition 7 (Full Trading equilibrium)**.** has $w(p,x,v) = 0 \ \forall x$, and $x(v)$ and $p^T(q)$ as in Proposition 2

Definition 5 is a specific restatement of Definition 1, where I specify the constrained trading strategy and the amount of punishment. The No Trading Equilibrium is intuitive: the insider is not allowed to trade, thus all variability in order flows comes from the noise trader, the price will not reveal any sort of information. In the Full Trading Equilibrium, the insider is capable to perfectly mimic the noise trader, thus only half of the information coming from order flows is incorporated into price, as the classical model ofKyle (1985). In a constrained equilibrium, the shareholder can pick $\bar{x}$. The resulting equilibrium will thus be characterized in between the other two, as the insider will be only able to mimic partially the strategy of the noise trader. By Bayes' rule, the market maker will always match the expected price with the prior of the distribution, independently on the imposed constraints and the variance of the price is decreasing in the amount of constraint imposed. Lastly, as $\bar{x} \to +\infty$, the price process of the constrained equilibrium converges in distribution to the No Trading Equilibrium, and, as $\bar{x} \to -\infty$, the price process of the constrained equilibrium converges in distribution to the Full Trading Equilibrium. My tie-breaking assumption is that if $\bar{x}$ approaches $-\infty$, the Full Trading is the preferred equilibrium (being both the noise and the cash flow normally distributed, I will consider this cut-off to be 3 standard deviation around the mean of order flows), while as $\bar{x}$ approaches $+\infty$, I pick the No Trading equilibirum.

3.6. Price efficiency and liquidity

In order to study the impact of the constraints on standard measures of market functioning, the second order approximation methodology highlighted in the previous subsection allows for a visual comparison of the Trading game equilibrium characteristics.

As the insider is now constrained, he can only mimic the behavior of the noise trader (as in the unconstrained equilibrium), up to some fraction of the variance of the noise trader, and as the constraints becomes more stringent, his ability to match moments of the noise trader distribution decreases even further. As a consequence, the market maker realize that less order flows now are not carrying all the information, so the average impact of the order flow, measured as the inverse of Kyle's lambda, is increasing. Kyle's lambda measures however the first order impact of order flows on price. As order flows convey less information when the constraint tights, the market maker will reduce the average sensitivity of order flows.

If I look at Q as one of the standard measures of price efficiency:

$$Q = Var(v|p)^{-1}$$

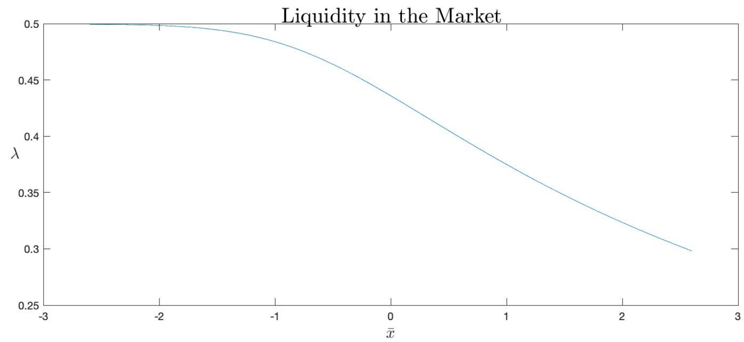

Figure 5. Liquidity

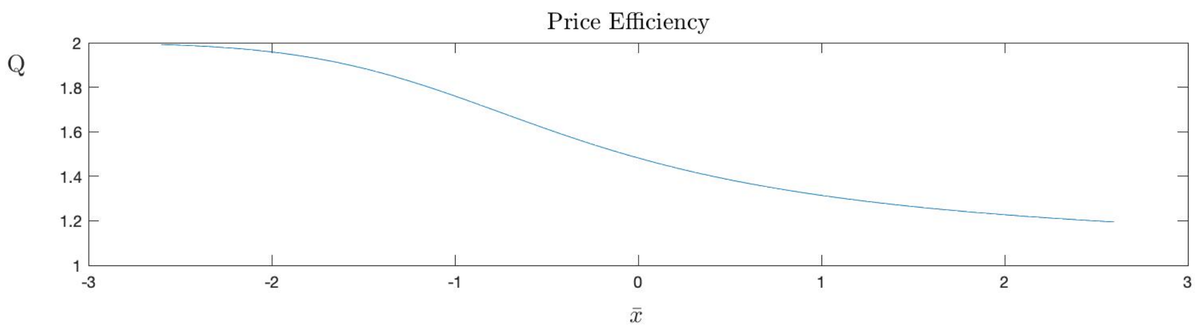

Figure 6. Price Efficiency

this measures prices's average predictive power. Not surprising, when the market becomes more and more liquid, less information is embedded into price, thus the price's capability to reflect fundamentals worsens.

However, I can compute the following Absolute Pricing Error $APE(\bar{x},v)$

$$APE(\bar{x},v) = E_{\bar{x}}[|v - p(q)||v]$$

this measures consider the absolute distance from v of $p(q)$ given the imposition of a trading constraint under $\bar{x}$. Notice, that without constraint

$$APE(FT,v) = \frac{|v - v_0|}{2}$$

,while for a contract that grants no trading rights

$$APE(NT,v) = |v - v_0|$$

Notice that for v and $-v$ the two measures are equivalent. This means that in this two trading regimes the APE and thus market efficiency (and the pricing error) is linear in the absolute value of the state. This is not the case if I look instead at the pricing functions resulting for the imposition of the constraints. While on average, APE is above the one in the full trading equilibrium, there is a tremendous disparity between good or bad states of the world. In fact, as v goes to $-\infty$ the APE converges to the one under the No Trading Regime, when the state of the world is good, the APE is concave, and for extreme value of v, the APE is indeed lower than the one under Full Trading. This difference highlights the major result of the paper. Under unilateral trading constraints, the pricing function has two main characteristics: the sensitivity to order flows is non constant, and the price efficiency is non-linearly state dependent. In bad states of the world, fewer informations are incorporated into prices due to a binding trading constraints, thus prices do not reflect fundamentals. Instead, in good states of the world, the prices maintain a similar level of informativeness as under the Full Trading regimes, while they become even more informative for extremely good states of the world. This result is in line with empirical patters that speaks about the ability of insiders' purchases to predict positive abnormal returns, while confirming that sales have less predictive power, not because they are associated to liquidity shocks, or the desire to disinvest, but because of the implicit constraints on trading. In bad states of the world, insiders will not trade, and as a result the price will not adjust to negative information. This model has no room for liquidity trading from the insider. However, I could expect that the same results will hold as long as the firm can observe the insiders transaction, with the caveat that now the contract will specify both region of the state space and order flows where the insider is allowed to trade. This result will be further accentuated if, on top of liquidity trading, there are effects on cashflows following the punishment of the insider.

The next section will characterize the optimal level of $\bar{x}$, under different specification of the share-

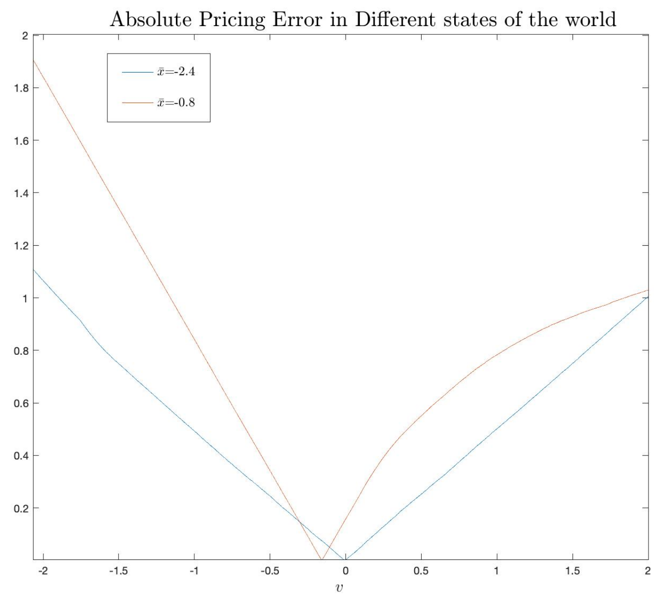

Figure 7. Absolute Pricing Error

holder's utility.

4. Shareholder's Problem

In this section, I am going to characterize various utility specification for the shareholder, and when the different regimes of trading will emerge in equilibrium. Recall that if shareholders do not allow insider to trade, order flows will not contain any information, the price of the company will be equal to its expected value. Price will not reveal any information, and cannot be used by stakeholders to make further decisions about the company. The distribution of the company price will collapse to a degenerated point. [12]

Recall that the contract specifying full trading allows for each order flow to carry some information. As a consequence, price is somewhat impacted by the order flow, and the variance of the

[12] Not allowing for the possibility of the insider to trade comes with a complete separation between ownership and control, and shareholders may see the value of the company impacted by agency frictions. Indirect costs may be associated with having a completely uninformative price, that cannot be useful to stakeholder of the company. Other direct costs can be cost associated to actual compensation costs, or litigation fees for imposing the punishment. While I acknowledge these consideration, the modeling of them is left for future research as they are not the main objective of this paper.

posterior is half of the variance of the prior. However, in this case, the price could reveal bad information in a timely manner, and if the shareholders care about the price of the firm, this contract will not be beneficial for them.

A contract generated by trading constraints will allow shareholder to benefit from good information while limiting the downside risk, by imposing a lower bound to the price distribution. Price on average will be lower, for the upper part of the distribution of cash flows, but much higher for the lower part of it. By reducing the trading the shareholders buy protection against too low value of the company. Low prices are thus less informative of the actual value of the company.

4.1. Risk-neutral shareholder

A risk neutral shareholder has preferences given by the utility function:

$$U^{\mathscr{W}}(p) = E_{\mathscr{W}}[p]$$

Given that the market maker uses Bayes' rule to update prices after observing order flow q, the ex-ante expectation over prices is not different from the prior of the distribution, thus for all order flow q and all types of trading constraints[13]

$$E[p(q)] = v_0$$

Thus the shareholder is indifferent between enforcing or not trading constraints. This should not be surprising as a risk-neutral shareholder act as if she can diversify away her holding in the company. However, when I interpret the shareholder as the company itself, with the objective to minimize its cost of capital, the argument in favor of a risk neutral shareholder is faulty and cannot be applied generally.

[13] In models à la Kyle (1985), the price process is a martingale with respect to the previous period order flow. In a two period model the expectation after trading in the first period is simply the realized price, thus a short-term oriented shareholder could potentially impact the price by allowing the insider to trade a total amount over the time of the game, as long as the market maker is unable to perfectly anticipate the trading patterns (thus with the addition of noise) of the insider. Result could differs, but I leave the discussion of a multi-period or continuous model for future research.

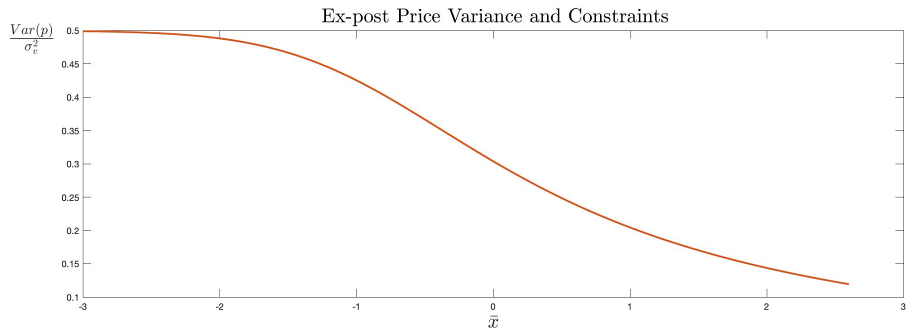

Figure 8. Associated Decrease In Variance as $\bar{x} \to \infty$

4.2. *Risk averse shareholder*

In a situation where the shareholder has Mean variance preferences with a given degree of risk aversion, the problem will simply result in the minimization of the price variance. Figure 8 plots the variance of price, normalized by the variance of the cash flow, as the constraint gets tighter and tighter. As I can clearly see, the shareholder will prefer to fully restrict trading (recall that $p^C \xrightarrow{d} p^{NT}$, as $\bar{x} \to +\infty$). In this situation, the shareholder has symmetric preferences for both the negative and positive realization of the price. What this means is that the shareholder suffers as much as bad information is licked into the order flow, as well as if good information arrive on the market. In the appendix, I analyze alternative specification of the utility function of the shareholder that all point to the same conclusion. The constraint is reducing the price variance or the lower bound to price, thus alternative specification will push for a No Trading Equilibrium. What this analysis seems suggesting is that whenever lower than average price are given either more or equal importance than higher than average price, shareholder should impose a No Trading Equilibrium.

4.3. *Asymmetric utility*

It is not surprising that a risk averse shareholder will prefer no trading at all compared to a restricted equilibrium where only some trades are allowed. Given that the market maker applies Bayes' rule, any price process can be seen as a mean preserving spread, thus the lowest variance (No Trading) equilibrium will always second order stochastically dominate the Constrained Trading Equilibrium.

However, while preferences characterized by risk aversion are useful and captures those dislike for risk that an individual may feel, companies will appreciate upswing movements, and fear negative swings in their market price. To capture these preferences for upside potentials, I am going to use preferences that have the following characteristics

Definition 8 (Short-term price utility function)**.** An expected utility function

$$U^{\mathscr{W}}(p) = E_{\bar{x}}[g(p)[(1-\omega)\mathbb{1}_{\{p-v_0\geq 0\}} + \omega\mathbb{1}_{\{p-v_0<0\}}]]$$

is a short-term utility function if:

- $g : \mathbb{R} \to \mathbb{R}$, $g(v_0) = 0$ and $g'(p) \geq 0$
- $\omega \in [0,1]$

The short-term price utility function is characterized by a non-decreasing function that is centered around the prior of the cash flow distribution v_0, and a parameter ω capturing the preferences of the shareholder for downside risk. This parameter governs the expected utility by assigning relative weights to realization of prices. However, I do not restrict the function to have a specific concavity or convexity. Examples of this kind of functions are loss aversion functions or exogenous reference point functions. v_0 can be interpret as a rescaling factor. What this implies is that all realizations of p north of the expected value of the company carry more utility of realization south of the expected value. Relative to a classical mean-variance or risk averse preferences, I believe this preferences can better describe firms' attitudes towards their price. Companies will fear a price depreciation and enjoy the benefits coming from a better market price (think for example to better cost of capital, or a more profitable exit strategy for a large shareholder such as a fund). As this class of utility function has a asymmetric treatment of losses compared to gains, it encompasses prospect theory, as well as other non-classical preferences, but at the same time collapse to more standard preferences such as risk-neutrality ($g(p) = p - v_0$) or mean-variance ($g(p) = (p-v_0) - \gamma(p-v_0)^2$) for $\omega = \frac{1}{2}$. The expectation is taken by considering then imposed constrained $\bar{x}$. Notice that the utility associated to the no-trading equilibrium is zero for all values of ω and all function $g(p)$. Thus, the constrained or full trading equilibrium is preferred to the unconstrained one whenever

$$\begin{aligned}
U^{\mathscr{W}}(p) \geq & U^{NT}(p) \\
Prob_{\bar{x}}(p \geq v_0)(1-\omega)E_{\bar{x}}[g(p)|p \geq v_0] + (1-Prob_{\bar{x}}(p \geq v_0))\omega E_{\bar{x}}[g(p)|p \leq v_0] \geq & 0 \\
-\frac{(1-\omega)E_{\bar{x}}[g(p)|p \geq v_0]}{\omega E_{\bar{x}}[g(p)|p \leq v_0]} \geq & \frac{1-Prob_{\bar{x}}(p \geq v_0)}{Prob_{\bar{x}}(p \geq v_0)} \\
\frac{(1-\omega)E_{\bar{x}}[g(p)|p \geq v_0]}{\omega |E_{\bar{x}}[g(p)|p \leq v_0]|} \geq & \frac{1-Prob_{\bar{x}}(p \geq v_0)}{Prob_{\bar{x}}(p \geq v_0)}
\end{aligned} \tag{46}$$

Equation (46)is self explanatory. The insider will prefer to allow for some trading if the ratio between the positive and negative utility is at least as good as the inverse odds ratio of the price distribution. Given normality, the odds ratio for the full trading equilibrium is equal to 1 w.p. 1, while is not defined for the No Trading Equilibrium. The more constraints the shareholder adds, the higher is the odd ratio. The left hand side of the equation is mostly dominated by the behavior of the denominator. As $\bar{x}$ increases, prices result more and more bounded downward, and the denominator of the LHS decreases (and with the negative sign, thus the term is increasing). The effect of the constraints on the denominator is two-fold: on one side, imposing more and more constraints makes the price to be more sensitive to order flows and thus increase more, but the probability of observing some unconstrained order flows decreases, decreasing the expectation. The final effect will however depend on the concavity of the function $g(\cdot)$. The magnifying effect is given by ω. A lower ω in fact increases the LHS, making the constrained equilibrium always preferred to the No trading. Notice that I may happen that the term of the LHS and RHS of the equations may meet only at $\bar{x} \to +\infty$, as $\omega \to 1$.

When $g(\cdot)$ is a symmetric function, for the unconstrained equilibrium (Full trading), the LHS is simply $\frac{1-\omega}{\omega}$, implying that the full trading equilibrium is preferred to the non trading equilibrium any time $\omega \in [0, \frac{1}{2})$.

Before further developing property of the equilibrium with the following utility function, let's consider some examples.

Example 9. Consider the following function

$$g(p) = (p - v_0)$$

For any ω=$\frac{1}{2}$, the utility is simply a rescaling of risk neutrality. This utility form allows us to compute:

$$\begin{aligned} U^{NT}(p) =& 0 \\ U^{T}(p) =& \frac{\sigma_v(1-2w)}{2\sqrt{\pi}} \end{aligned}$$

Easy to see that the Full Trading Equilibrium is preferred to the No Trading Equilibrium whenever $\omega < \frac{1}{2}$. Given the linearity of the function, which is kinked for $p = v_0$, the solution is not smooth, and maximizing over the $\bar{x}$ space results in the Full Trading equilibrium for all $\omega < \frac{1}{2}$, while as soon as $\omega > \frac{1}{2}$, the preferred contract has No Trading Rights. This happens because, when the contract restrict trading, the constrained price function lies below the unconstrained one in some area of the positive quadrant and above it in some area of the negative quadrant. As a consequence a "upside risk lover" shareholder ($\omega < 1/2$), doesn't want to sacrifice upside potentials. Whenever the loss matters more than the gains ($\omega > 1/2$), the shareholder picks the No Trading Equilibrium, as it will ensure a higher price in state of the world she cares the most.

Example 10. Now instead consider

$$g(p) = (p - v_0)^3$$

Again

$$\begin{aligned} U^{NT}(p) =& 0 \\ U^{T}(p) =& \frac{\sigma_v^3(1-2w)}{2\sqrt{\pi}} \end{aligned}$$

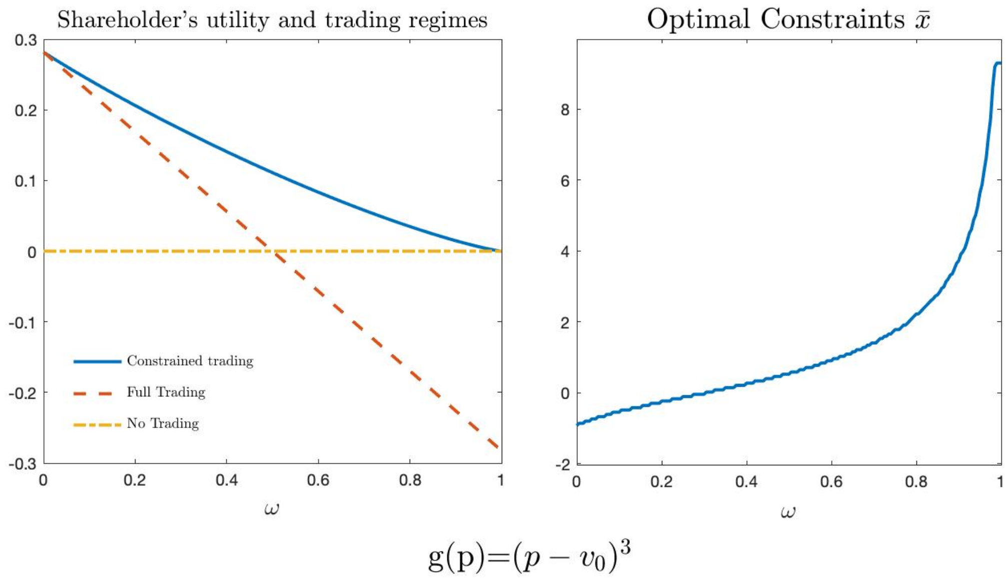

Figure 9. Optimal Constraints

This time however, I can show that a constrained equilibrium is indeed always preferred to the unconstrained one, and converges to the full constrained equilibrium for $\omega \to 1$. This utility specification is capturing the asymmetric behavior of prices around its mean. This insider will always prefer a right skewed distribution to a left skewed one. This utility specification seems to best capture the behavior of the company that wishes to maintain a substantial level of price. This utility specification, moreover, makes the shareholder willing to propose more and more restricted contract as $\omega \to 1$, and the full trading contract is always dominated by the constrained trading contact. Figure 9 plots the optimal constraints with the above utility specification. Not surprising is that a very downside risk shareholder will pose a lot of constraints. However, even the most upside risk lover shareholder ($\omega = 0$) prefers to impose some constraint. This because the shareholder wants to ensure price to remain somewhat "high" in negative state of the world, and at the same time allow the insider to trade freely (and transfer information into price) in good state of the world.

Example 11. To conclude, consider now

$$g(p) = 1 - e^{-(p-v_0)}$$

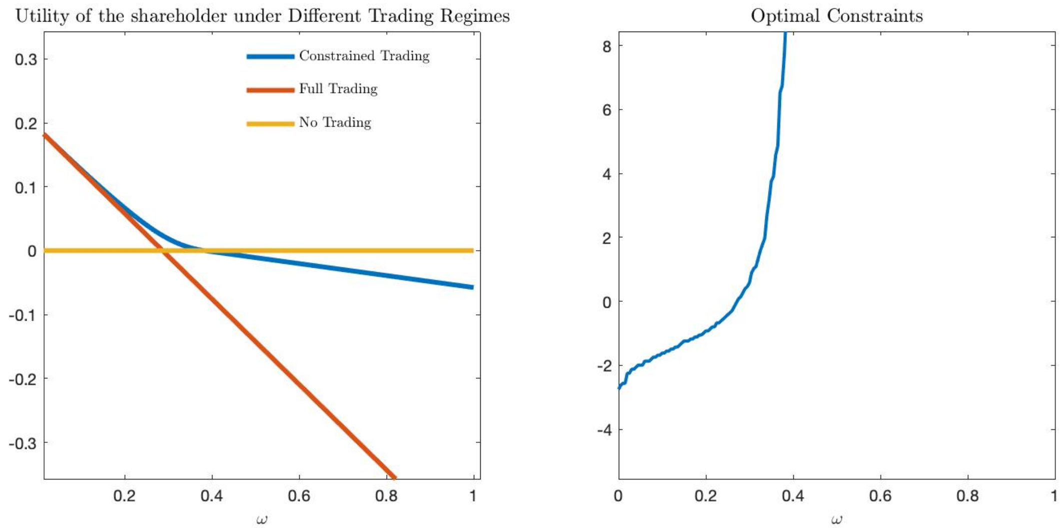

Figure 10. CARA Short-term Utility

With constant absolute risk aversion (of 1)

$$
\begin{aligned}
U^{NT}(p) &= 0 \\
U^{T}(p) &= \frac{1}{2}\left(e^{\frac{\sigma_v^2}{4}}\left((1-2w)\left(2\Phi\left(\frac{\sigma_v}{\sqrt{2}}\right)-1\right)-1\right)+1\right)
\end{aligned}
$$

Thus for $w \leq \frac{1-2e^{\frac{\sigma_v^2}{4}}\left(1-\Phi\left(\frac{\sigma_v}{\sqrt{2}}\right)\right)}{2e^{\frac{\sigma_v^2}{4}}\left(2\Phi\left(\frac{\sigma_v}{\sqrt{2}}\right)-1\right)}$, the Full Trading regimes dominates the Non Trading regime. It is easy to see that as σ_v is associated with a decrease in this cut-off. The more the asset is volatile, the "sooner" the shareholder wants to shut down full trading. However, it may still have some value to impose some constraints without limiting trading itself. In fact, reverting to simulations, there exists level of ω for which the shareholder prefers to impose some constraints on the insider. [14] Recall that, given CARA, and thus risk-aversion, all the price processes induced by the constraint are mean-preserving spread of the price process associated to $\bar{x} = +\infty$ (that is equal in distribution to the non-trading equilibrium). From the figure I can see there are 2 regions in the ω space. In the first region the constraint moves from finite values of $\bar{x}$, where the constrained equilibrium is unique and always dominates the Full

[14]The above figure shows the behavior of the optimal constraint. Unfortunately, the simulation does not allow for convergence above some level of the constraints, as the matrix will be highly collinear.

Trading . As $\omega \to 0.5$, $\bar{x} \to +\infty$, and for the tie-breaking condition, for ω >0.5, the only equilibrium is the No Trading Equilibrium.

The above examples illustrate a clear feature of the model. Curvature in the utility function is fundamental to have existence of an equilibrium in the constrained space. In fact, without such curvature, the shareholder will prefer either the Full Trading or the No Trading equilibrium. As the utility gets some curvature, the shareholder finds optimal to impose some constraints. Notice that, a relatively high constraint is needed as soon as some curvature is imposed on the insider strategy. Moreover, as the shareholder becomes more and more downside averse, $\bar{x}$ increases, but while there exists some ω for which it is better to completely shut down trading if the utility captures risk aversion, this is not the case for preferences that instead look to skewness.

Lemma 12. *If $g(\cdot)$ is concave (the shareholder is risk averse), the No-Trading equilibrium dominates the Constrained Trading equilibrium for $\omega \geq \frac{1}{2}$. That is $U^{\mathscr{W}}(p) \leq U^{NT}(p)$ for all $\omega \geq \frac{1}{2}$.*

Proof. See Appendix A.2 □

Corollary 13 (Comparative Statics on the optimal constraints)**.** *The optimal amount of constraint $\bar{x}$ chosen by the shareholder is:*

- is increasing increasing in ω
- is increasing in σ_v if $g(\cdot)$ shows risk-aversion characteristics
- is increasing (decreasing) in σ_n for $\omega > (<)\omega_0$, where ω_0 is the level of ω for which $\bar{x} = 0$ is the solution of the problem

Proof. See Appendix A.3 □

5. Conclusions

In this paper, I analyze the role of myopic preferences of the shareholders, and how these can impact manager's incentives via the imposition of selling constraints. While shareholder should be focused in developing contracts that incentivize the long-term value of the company, when stakeholder of the company can make decisions based on market price, shareholders would like to postpone revelation of negative information in the future. Imposing constraints on insiders' trading activity may help in achieving a less volatile and higher price.

Imposing selling constraints on one side reduces informativeness of the order flows, but at the same time also reduces the price variance. The resulting price function is characterized by lower limit where all lower types are pooled together. For very high order flows the price becomes more reactive to them. For intermediate order flow, the behavior of the price is convex, with the objective of reverting the concavity of order flows induced by the selling constraint. As a consequence, the informativeness of price depends on their level. Higher price are very informative, and as a consequence, markets are an efficient mechanism to communicate quality of the firm to stakeholders. On the opposite side, low quality firm will be pooled together, and the resulting price conveys worse information, as now stakeholders cannot perfectly infer the quality of the firm. If those stakeholders are provider of capitals, the term they will impose is the same, and some firms will benefit from being pooled together.

Trading constraints may arise in equilibrium as a result of the variability of the cash-flow of the company. The more the company cashflow is variable, the higher is the probability of lower cashflows. By imposing more stringent trading constraints, the company market price will not drop so much. At the same time, the firm is able to protect it self against swings in the market conditions.

I am abstracting from compensation oriented situations, imposing trading restrictions on the manager, he should demand extra compensation to account for the forgone profit from trading. Empirical evidence in this directions are provided by Roulstone (2003). Moreover, trading restrictions may on one side reduce the wedge in objectives between insiders and shareholders, but at the same time comes with indirect costs, and thus there exists a tradeoff between value maximizing and short-term price behavior.

Although not explicitly, this paper speaks to managerial compensation and incentives. Fixing the volatility of cashflow, a more short-term oriented company should provide managers with less share grants, so that short-selling constraints are more stringent, or alternatively, should have more temporal restriction on trading. Moreover, the benefit of introducing selling constraints should increase as the company's cashflows become more volatile. In conclusion, granted trading rights should depend on cash flow volatility, as well as on the short-term investment opportunity of the firm.

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Appendix A. Proofs

A.1. Proof of Lemma 3

Proof.

$$
\begin{aligned}
&\lim_{q\to-\infty}\frac{\phi\left(\frac{q-\bar{x}}{\sigma_n}\right)\int_{-\infty}^{\nu} v\phi\left(\frac{v-v_0}{\sigma_v}\right)\mathrm{d}v+\int_{\nu}^{+\infty} v\phi\left(\frac{v-v_0}{\sigma_v}\right)\phi\left(\frac{q-f(v)}{\sigma_n}\right)\mathrm{d}v}{\phi\left(\frac{q-\bar{x}}{\sigma_n}\right)\Phi\left(\frac{\nu-v_0}{\sigma_v}\right)+\int_{\nu}^{+\infty}\phi\left(\frac{v'-v_0}{\sigma_v}\right)\phi\left(\frac{q-f(v')}{\sigma_n}\right)\mathrm{d}v'} &&=\\
&\lim_{q\to-\infty}\frac{\int_{-\infty}^{\nu} v\phi\left(\frac{v-v_0}{\sigma_v}\right)\mathrm{d}v+\int_{\nu}^{+\infty} v\phi\left(\frac{v-v_0}{\sigma_v}\right)\frac{\phi\left(\frac{q-f(v)}{\sigma_n}\right)}{\phi\left(\frac{q-\bar{x}}{\sigma_n}\right)}\mathrm{d}v}{\Phi\left(\frac{\nu-v_0}{\sigma_v}\right)+\int_{\nu}^{+\infty}\phi\left(\frac{v'-v_0}{\sigma_v}\right)\frac{\phi\left(\frac{q-f(v)}{\sigma_n}\right)}{\phi\left(\frac{q-\bar{x}}{\sigma_n}\right)}\mathrm{d}v'} &&=\\
&\frac{\int_{-\infty}^{\nu} v\phi\left(\frac{v-v_0}{\sigma_v}\right)\mathrm{d}v+\lim_{q\to-\infty}\int_{\nu}^{+\infty} v\phi\left(\frac{v-v_0}{\sigma_v}\right)\frac{\phi\left(\frac{q-f(v)}{\sigma_n}\right)}{\phi\left(\frac{q-\bar{x}}{\sigma_n}\right)}\mathrm{d}v}{\Phi\left(\frac{\nu-v_0}{\sigma_v}\right)+\lim_{q\to-\infty}\int_{\nu}^{+\infty}\phi\left(\frac{v'-v_0}{\sigma_v}\right)\frac{\phi\left(\frac{q-f(v)}{\sigma_n}\right)}{\phi\left(\frac{q-\bar{x}}{\sigma_n}\right)}\mathrm{d}v'} &&=\\
&\frac{\int_{-\infty}^{\nu} v\phi\left(\frac{v-v_0}{\sigma_v}\right)\mathrm{d}v+\int_{\nu}^{+\infty} v\phi\left(\frac{v-v_0}{\sigma_v}\right)\lim_{q\to-\infty}\frac{\phi\left(\frac{q-f(v)}{\sigma_n}\right)}{\phi\left(\frac{q-\bar{x}}{\sigma_n}\right)}\mathrm{d}v}{\Phi\left(\frac{\nu-v_0}{\sigma_v}\right)+\int_{\nu}^{+\infty}\phi\left(\frac{v'-v_0}{\sigma_v}\right)\lim_{q\to-\infty}\frac{\phi\left(\frac{q-f(v)}{\sigma_n}\right)}{\phi\left(\frac{q-\bar{x}}{\sigma_n}\right)}\mathrm{d}v'} &&=\\
&\frac{\int_{-\infty}^{\nu} v\phi\left(\frac{v-v_0}{\sigma_v}\right)\mathrm{d}v}{\Phi\left(\frac{\nu-v_0}{\sigma_v}\right)} &&=\\
&E[v|v<\nu]
\end{aligned}
$$

the second to last equality hold true for Dominated Convergence Theorem, while the last equality is coming from applying the limit and recalling that $f(v)\geq\bar{x}$. □

A.2. Proof of Lemma 12

Proof. For $\omega = 0.5$, Suppose Not. There exists $\bar{x}$ such that $U^{\bar{x}}(p) > U^{NT}(p)$, or

$$\begin{aligned} E(g(p)) &> 0 \\ g(E(p)) \geq E(g(p)) > & \quad 0 \quad \text{by Jensen Inequality} \\ 0 \geq E(g(p)) > & \quad 0 \quad \text{by definition of g}(\cdot) \text{ and Bayes rule} \end{aligned}$$

thus contradicting the statement. The proof for $\omega > 0.5$ follows from the fact that the optimal constraint is always increasing in ω from equation (46) and the tie-breaking condition. □

A.3. Proof of Corollary 13

Proof. I can rewrite Equation (46) as:

$$\frac{Prob_{\bar{x}}(p \geq v_0)}{1 - Prob_{\bar{x}}(p \geq v_0)} \frac{E_{\bar{x}}[g(p)|p \geq v_0]}{|E_{\bar{x}}[g(p)|p \leq v_0]|} \geq \frac{\omega}{1 - \omega} \tag{47}$$

The odd ratio of the price distribution $\frac{Prob_{\bar{x}}(p \geq v_0)}{1 - Prob_{\bar{x}}(p \geq v_0)}$ is strictly decreasing in $\bar{x}$, as the more constrained is the insider, the less information will be given embedded in trading, thus lowering the probability of meeting price above the prior. At the same time, the RHS of the equation is increasing in ω. By construction, as long as $g(\cdot)$ is increasing, the ratio of expectations is increasing (the denominator decreases faster than the numerator due to normality and a Bayesian Market maker). This would imply that, for the inequality to be satisfied, $\bar{x} \to +\infty$, as $\omega \to 1$.

Unfortunately, the impossibility of having a close form solution for both the pricing function and the generality imposed on the utility function makes it difficult to characterize the behavior of the optimal constraint. However, if $g(\cdot)$ is characterized by risk aversion, an increase in σ_v, the cash flows' standard deviation, will, all else equal, be characterized by a decrease in utility, and as a consequence, an increase in the constraints for all level of ω, as long $\bar{x}$ is finite. This results is immediate if we consider Equation (47) and the impact on the utility of the variance.

The last parameter if the model is σ_n, the standard deviation of noise. Again, in the following specification, we cannot fully characterize the behavior of the company, at least formally, but a common pattern seems to emerge. In particular, as long as the solution for $\bar{x}$ is internal for level of ω, the behavior of the optimal constraint varies if the ω is below or above a cut-off ω_0, associated with the optimal level of the constraint to be $\bar{x} = 0$. In particular for level of $\omega < \omega_0$, the shareholder will find beneficial to relax the constraint, in order to bet with the market for an increase in price. For level of $\omega > \omega_0$, instead, the shareholder will impose further constraints as σ_n increases, suggesting now a desire to protect the price from unexpected swings in the market. □

Appendix B. Asymptotic behavior of the pricing function in 40

I check the behavior of the function in 40 for q that goes to -∞ and +∞. Rewriting the above equation as

$$
\begin{aligned}
p(q) = \frac{\beta}{1+\beta^2} & \frac{\left((\beta\nu+q) - \beta\frac{\sqrt{\beta^2+1}}{\sqrt{2\pi}} e^{-\frac{(\nu-\beta q)^2}{2(\beta^2+1)}} \Phi\left(-\frac{\nu-\beta q}{\sqrt{\beta^2+1}}\right)^{-1}\right)}{1+\sqrt{\beta^2+1}\Phi(\nu)e^{\frac{(\beta\nu+q)^2}{2(\beta^2+1)}-\frac{q^2}{2}} \Phi\left(-\frac{\nu-\beta q}{\sqrt{\beta^2+1}}\right)^{-1}} \\
=& \frac{\beta}{1+\beta^2} \frac{(\beta\nu+q) - \beta\sqrt{\beta^2+1}\phi\left(-\frac{\nu-\beta q}{\sqrt{\beta^2+1}}\right)\Phi\left(-\frac{\nu-\beta q}{\sqrt{\beta^2+1}}\right)^{-1}}{1+\Phi(\nu)e^{\frac{\nu^2}{2}}\beta\sqrt{\beta^2+1}e^{-\frac{(\nu-\beta q)^2}{2(\beta^2+1)}}\Phi\left(-\frac{\nu-\beta q}{\sqrt{\beta^2+1}}\right)^{-1}} \\
=& \frac{\beta}{1+\beta^2} \frac{e^{-\frac{\nu^2}{2}}}{\Phi(\nu)} \frac{\beta\nu+q-\beta\sqrt{\beta^2+1}\phi\left(-\frac{\nu-\beta q}{\sqrt{\beta^2+1}}\right)\Phi\left(-\frac{\nu-\beta q}{\sqrt{\beta^2+1}}\right)^{-1}}{\frac{e^{-\frac{\nu^2}{2}}}{\Phi(\nu)}+\beta\sqrt{\beta^2+1}e^{-\frac{(\nu-\beta q)^2}{2(\beta^2+1)}}\Phi\left(-\frac{\nu-\beta q}{\sqrt{\beta^2+1}}\right)^{-1}}
\end{aligned}
\tag{48}
$$

Now recall that $e^{\left(-\frac{(\nu-\beta q)^2}{2(\beta+1)}\right)}\Phi\left(-\frac{\nu-\beta q}{\sqrt{\beta^2+1}}\right)^{-1} \to +\infty$ as $q \to -\infty$. As a consequence, if I expand the equation and take the limit for q to minus infinity, the first term is equal to zero. Thus, by taking the limit of the whole expression I have that:

$$
\begin{aligned}
\lim_{q\to-\infty} p(q) &= \frac{\beta}{1+\beta^2}\frac{e^{-\frac{\nu^2}{2}}}{\Phi(\nu)}\left(0-\frac{\sqrt{\frac{2}{\pi}}}{\beta}-\sqrt{\frac{2}{\pi}}\beta\right) \\
&= \frac{-\sqrt{\frac{2}{\pi}}e^{-\frac{\nu^2}{2}}}{\Phi(\nu)} = E[v|v<\nu] \qquad (49)
\end{aligned}
$$

Equation (49) is a constant and it is equal to the expectation of v to be lower than ν. The function has an horizontal asymptote at the above value. I now consider the part of the expression that depends on q of (48):

$$
g(q) = \frac{\beta\nu + q - \beta\sqrt{\beta^2+1}\phi\left(-\frac{\nu-\beta q}{\sqrt{\beta^2+1}}\right)\Phi\left(-\frac{\nu-\beta q}{\sqrt{\beta^2+1}}\right)^{-1}}{\frac{e^{-\frac{\nu^2}{2}}}{\Phi(\nu)}+\beta\sqrt{\beta^2+1}e^{-\frac{(\nu-\beta q)^2}{2(\beta^2+1)}}\Phi\left(-\frac{\nu-\beta q}{\sqrt{\beta^2+1}}\right)^{-1}} \qquad (50)
$$

One can characterize the derivatives of this term at $q\to+\infty$. This derivatives is

$$
\begin{aligned}
g'(q) =& \frac{\beta^2}{4e^{-\frac{\nu^2}{2}}\Phi(\nu)^{-1}\pi\left(\sqrt{\beta^2+1}e^{\frac{\nu^2}{2}}\Phi(\nu)+e^{\frac{(\nu-\beta q)^2}{2(\beta^2+1)}}\Phi\left(-\frac{\nu-\beta q}{\sqrt{2}\sqrt{\beta^2+1}}\right)\right)^2} \\
&+\frac{\sqrt{2\pi}\beta^2 e^{\frac{(\nu-\beta q)^2}{2(\beta^2+1)}}(\beta q-\nu)\Phi\left(-\frac{\nu-\beta q}{\sqrt{2}\sqrt{\beta^2+1}}\right)+\pi\sqrt{\beta^2+1}e^{\frac{(\nu-\beta q)^2}{\beta^2+1}}2\Phi\left(-\frac{\nu-\beta q}{\sqrt{2}\sqrt{\beta^2+1}}\right)^2}{4e^{-\frac{\nu^2}{2}}\Phi(\nu)^{-1}\pi\sqrt{\beta^2+1}\left(\sqrt{\beta^2+1}e^{\frac{\nu^2}{2}}\Phi(\nu)+e^{\frac{(\nu-\beta q)^2}{2(\beta^2+1)}}\Phi\left(-\frac{\nu-\beta q}{\sqrt{2}\sqrt{\beta^2+1}}\right)\right)^2} \\
&+\frac{\pi e^{\frac{(\nu-\beta q)^2}{2(\beta^2+1)}}\left(\beta\left(-\beta\nu^2+\beta\left(q^2+1\right)+\left(\beta^2-1\right)\nu q\right)+1\right)2\Phi\left(-\frac{\nu-\beta q}{\sqrt{2}\sqrt{\beta^2+1}}\right)+\sqrt{2\pi}\beta\sqrt{\beta^2+1}(\beta\nu+q)}{4\pi\sqrt{\beta^2+1}\left(\sqrt{\beta^2+1}e^{\frac{\nu^2}{2}}\Phi(\nu)+e^{\frac{(\nu-\beta q)^2}{2(\beta^2+1)}}\Phi\left(-\frac{\nu-\beta q}{\sqrt{2}\sqrt{\beta^2+1}}\right)\right)^2}
\end{aligned}
$$

I show that

$$
\lim_{q\to-\infty} g'(q) = 0
$$

and more interestingly:

$$\lim_{q \to +\infty} g'(q) = e^{\frac{\nu^2}{2}} \Phi(\nu) \tag{51}$$

Combining this with the expression for p(q) I get a result:

$$\lim_{q \to +\infty} p'(q) = \frac{\beta}{1+\beta^2} \tag{52}$$