Delayed Capital Reallocation

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Introduction
Motivation

- Less restructuring in recessions
  (1) Capital reallocation is sizeable
  (2) Capital stock reallocation across firms 🔽

[Data]
Motivation

- Less restructuring in recessions
  (1) Capital reallocation is sizeable
  (2) Capital stock reallocation across firms

- Significantly slow down recovery
Motivation

- Less restructuring in recessions
  (1) Capital reallocation is sizeable
  (2) Capital stock reallocation across firms ↓

- Significantly slow down recovery

- What frictions and shocks in a (heterogenous firms) model?
  - Generate less capital reallocation in recessions
  - Tractable for backing out shocks
### Goals

- Idiosyncratic productivity risks
Goals

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- Costs in reallocation:
  - Partial irreversible investment + financing constraints
  - Dynamics after aggregate productivity shocks / credit crunches

- A simple idea
  - Selling delay and the delay is prolonged in recessions
Goals

- Idiosyncratic productivity risks

- Costs in reallocation:
  - Partial irreversible investment + financing constraints
  - Dynamics after aggregate productivity shocks / credit crunches

- A simple idea
  - Selling delay and the delay is prolonged in recessions

- But complex issues
  - Difficulties: distribution of firms with different status
  - Buying assets, holding, selling, waiting to come back
Financing frictions: Kiyotaki & Moore (1997), Bernanke et al. (1999), Brunnermeier & Sannikov (2011)...


Uncertainty shocks: Bloom (2009), Gilchrist et al. (2010), Christiano et al. (2014)...


DSGE Estimation
The Model

1. Households and firms (run by entrepreneurs)
2. Households’ problem
3. Entrepreneurs’ problem
4. The stationary equilibrium
Households (measure 1) and firms run by entrepreneurs (measure 1)

The representative household solves

$$\max \sum_{s=t}^{\infty} \beta_h^{s-t} \left[ \frac{c_{h,s}^{1-\gamma} - 1}{1 - \gamma} - \frac{\kappa (l_{h,s})^{1+\nu}}{1 + \nu} \right],$$

subject to

$$c_{h,t} + b_{h,t} = w_t l_{h,t} + R_t b_{h,t}.$$
Entrepreneurs

- Entrepreneur $j$’s preferences:

$$E_0 \sum_{t=0}^{\infty} \beta^t [\log(c_{jt}) + \eta(1 - h_{jt})]$$

- $\eta$ : fixed costs of running the firm
- $j$ chooses whether to operate ($h_{jt} = 1$) or not ($h_{jt} = 0$)
Entrepreneurs

- Entrepreneur $j$’s preferences:

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  - $\eta$: fixed costs of running the firm
  - $j$ chooses whether to operate ($h_{jt} = 1$) or not ($h_{jt} = 0$)

- $j$’s production technology:

$$y_{jt} = (z_{jt} k_{jt})^\alpha (A_t l_{jt})^{1-\alpha}, \alpha \in (0, 1)$$

  - $z_{jt}$ is idiosyncratic. $z^h > z^l$ with $p^{hl} + p^{lh} < 1$:

$$P = \begin{bmatrix} p^{hh} & p^{hl} \\
p^{lh} & p^{ll} \end{bmatrix}$$

- Who will operate is endogenous (aggregate TFP is endogenous)
Resale discounts and borrowing constraints

- Capital adjustment cost function \( \psi(k_{jt+1}, k_{jt}) \)

\[
\psi(k_{jt+1}, k_{jt}) = \begin{cases} 
  k_{jt+1} - (1 - \delta)k_{jt} & \text{if } k_{jt+1} > (1 - \delta)k_{jt}, \\
  0 & \text{if } k_{jt+1} = (1 - \delta)k_{jt}, \\
  - (1 - d)((1 - \delta)k_{jt} - k_{jt+1}) & \text{if } k_{jt+1} < (1 - \delta)k_{jt}.
\end{cases}
\]
Resale discounts and borrowing constraints

- Capital adjustment cost function \( \psi(k_{jt+1}, k_{jt}) \)

\[
\begin{align*}
\psi(k_{jt+1}, k_{jt}) &= \begin{cases} 
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  0 & \text{if } k_{jt+1} = (1 - \delta)k_{jt}, \\
  -(1 - d)[(1 - \delta)k_{jt} - k_{jt+1}] & \text{if } k_{jt+1} < (1 - \delta)k_{jt}.
\end{cases}
\end{align*}
\]

- Borrowing constraint: \( \theta \geq 0 \)

\[
Rb_{jt+1} \geq -\theta (1 - \delta) (1 - d) k_{jt+1}
\]
Resale discounts and borrowing constraints

- Capital adjustment cost function \( \psi(k_{jt+1}, k_{jt}) \)

\[
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\end{cases}
\]

- Borrowing constraint: \( \theta \geq 0 \)

\[
Rb_{jt+1} \geq -\theta (1 - \delta)(1 - d)k_{jt+1}
\]

- Budget constraint:

\[
c_{jt} + b_{jt+1} + \psi(k_{jt+1}, k_{jt}) = y_{jt} - w_t l_{jt} + Rb_{jt} = z_{jt} \pi k_{jt} + Rb_{jt}
\]
An entrepreneur’s problem
In Steady State

\[ V(k, b, z) = \max \{ W^1(k, b, a), W^0(k, b, z) \} \]

\[ W^1(k, b, z) = \max_{k' > 0, b'} \{ \log(c^1) + \beta \mathbb{E}_z[V(k', b', z')] \} \]

\[ W^0(k, b, z) = \max_{b'} \{ \log(c^0) + \eta + \beta \mathbb{E}_z[V(0, b', z')] \} \]

where

\[ c^1 = z \pi k + Rb - \psi(k', k) - b' \]

\[ c^0 = z \pi k + Rb + (1 - \delta)(1 - d)k - b' \]

\( W^1 \) and \( W^0 \) denote running and not running a firm and \( R'b' \geq -\theta(1 - d)(1 - \delta)k' \)
Stationary equilibrium

**Definition**

The equilibrium is consists of policy functions \( l = g^l(k, b, z) \), \( k' = g^k(k, b, z) \), \( b' = g^b(k, b, z) \) and pricing functions \((\pi, R')\) such that:

1. \( c_h, l_h, \) and \( b_h \) solve the household’s problem, given \( w \) and \( R' \)
2. \( l, k' \) and \( b' \) solve the entrepreneur’s problem, given \( w, R', \) and \( \pi = \alpha \left[ \frac{(1-\alpha)A}{w} \right]^{\frac{1-\alpha}{\alpha}} \)

(2). Markets for labor and bonds clear

\[
\int l_j dj = l_h, \int b'_j dj + b_h = 0
\]

**Remark** When there are aggregate shocks, we need aggregate state variable \( X = (\theta, A, \Gamma) \) where \( \Gamma(k, b, z) \) is the joint CDF.
Model Solution

1. Policy functions
2. Option value of capital
3. Exact aggregation
Dynamics of \((k, b)\): only the ratio matters

**Lemma**

\[
V(\gamma k, \gamma b, z) = V(k, b, z) + \frac{\log \gamma}{1-\beta}
\]
Dynamics of \((k, b)\): only the ratio matters

Intuition: Without \(d\), low \(z\) firms sell immediately to pay off debt. With \(d\), hold on and gradually pay off debt.
Inaction region and action boundary
Inaction region and action boundary
The option value of capital 

Option value of capital $q(k, b, z)$ satisfies

$$V_k(k, b, z) = u'(c)[z\pi + q(k, b, z)(1 - \delta)]$$
The option value of capital

Option value of capital \( q(k, b, z) \) satisfies

\[
V_k(k, b, z) = u'(c)[z\pi + q(k, b, z)(1 - \delta)]
\]

- Buying: \( q(k, b, z) = 1 \). Selling: \( q(k, b, z) = 1 - d \)
The option value of capital

- Option value of capital $q(k, b, z)$ satisfies

$$V_k(k, b, z) = u'(c)[z\pi + q(k, b, z)(1 - \delta)]$$

- Buying: $q(k, b, z) = 1$. Selling: $q(k, b, z) = 1 - d$

- The inaction region:

$$1 - d < q(k, b, z) < 1$$
The option value of capital

- Option value of capital $q(k, b, z)$ satisfies

$$V_k(k, b, z) = u'(c)[z\pi + q(k, b, z)(1 - \delta)]$$

- Buying: $q(k, b, z) = 1$. Selling: $q(k, b, z) = 1 - d$

- The inaction region:

$$1 - d < q(k, b, z) < 1$$

- To characterize $q$. Homogeneity $\rightarrow q(k, b, z) = q\left(\frac{k}{k+b}, z\right)$

some derivation
Asset Pricing Formula

- FOC (multipliers $\mu$) + envelope $\Rightarrow E[m' r'|I] = 1$

$$E_z \left[ \frac{\beta u'(c') z' \pi'}{u'(c)} \left( 1 - \delta \right) q \left( \frac{k'}{k'+b'}, z' \right) \right] + \mu(k, b, z) = 1$$
Asset Pricing Formula

- FOC (multipliers $\mu$) + envelope $\Rightarrow E[m' r'|I] = 1$

$$E_z \left[ \frac{\beta u'(c')}{u'(c)} z' \pi' + (1 - \delta) q\left(\frac{k'}{k'+b'}, z'\right) \right] + \mu(k, b, z) = 1$$

Proposition (Policy functions for $k' > 0$)

$$c = (1 - \beta)(z\pi k + (1 - \delta)qk + Rb)$$

$$k' = \phi \beta (z\pi k + (1 - \delta)qk + Rb)$$

$$b' = (1 - \phi) \beta (z\pi k + (1 - \delta)qk + Rb)$$

where $\phi$ satisfies the asset pricing equation.
Liquidation

- When to liquidate?

\[ \frac{(1 - \beta)\eta}{\beta} = p^{lh}E_X \left[ \log \left( 1 + (1 - \delta) \frac{z^h\pi' + (1 - \delta)(1 - d)R'}{\beta(z^l\pi + (1 - \delta)(1 - d) + R\frac{1 - \lambda}{\lambda})R'} \right) \right] + p^{ll}E_X \left[ \log \left( 1 + (1 - \delta) \frac{z^l\pi' + (1 - \delta)(1 - d) - (1 - d)R'}{\beta(z^l\pi + (1 - \delta)(1 - d) + R\frac{1 - \lambda}{\lambda})R'} \right) \right] \]
Liquidation

- When to liquidate?

\[
\frac{(1 - \beta)\eta}{\beta} = p^h \mathbb{E}_X \left[ \log \left( 1 + (1 - \delta) \frac{z'\pi' + (1 - \delta)(1 - d)R'}{\beta(z'\pi + (1 - \delta)(1 - d) + R'\frac{1 - \lambda}{\lambda})R'} \right) \right]
\]

\[
+ p'^{ll} \mathbb{E}_X \left[ \log \left( 1 + (1 - \delta) \frac{z'\pi' + (1 - \delta)(1 - d) - (1 - d)R'}{\beta(z'\pi + (1 - \delta)(1 - d) + R'\frac{1 - \lambda}{\lambda})R'} \right) \right]
\]

- The drop of \( \pi \) and \( R' \) delays liquidation
Liquidation

- When to liquidate?

\[
\frac{(1 - \beta)\eta}{\beta} = p^{h\Gamma X} \left[ \log \left( 1 + (1 - \delta) \frac{z^h\pi' + (1-\delta)(1-d)R'}{\beta(z^l\pi + (1-\delta)(1-d)+R\frac{1-\lambda}{\lambda})R'} \right) \right]
\]

\[
+ p^{ll\Gamma X} \left[ \log \left( 1 + (1 - \delta) \frac{z^l\pi' + (1-\delta)(1-d)(1-d)R'}{\beta(z^l\pi + (1-\delta)(1-d)+R\frac{1-\lambda}{\lambda})R'} \right) \right]
\]

- The drop of \( \pi \) and \( R' \) delays liquidation

- Uncertainty shocks alone may not delay liquidation decisions
  - Importance of credit market in response to uncertainty shocks
  - Gilchrist et al. (2010) and Christiano et al. (2014)
Exact aggregation

Figure: Evolution of the Distribution

Recursive equilibrium

$p^0_{\text{h}}$

$z^0 = z^h, \lambda_0$

Invest

$p^1_{\text{h}}$

$z^1 = z^l, \lambda_1$

Wait

$p^1_{\text{t}}$

$z^1 = z^l, \lambda_1$

Wait

$p^0_{\text{t}}$

$z^0 = z^h, \lambda_0'$

$p^1_{\text{t}}$

$z^2 = z^l, \lambda_2'$

Wait

$p^N_{\text{h}}$

$z^N = z^l, \lambda_N$

$p^{N+1}_{\text{h}}$

$z^{N+1} = z^l, \lambda_{N+1}$

Wait

$p^{N+1}_{\text{t}}$

$z^{N+1} = z^l, \lambda_{N+1}$

$p^{N+2}_{\text{h}}$

$z^{N+2} = z^l, 0$

Sell

$p^{N+2}_{\text{t}}$

$z^{N+2} = z^l, 0$

Indifferent

$\lambda_0 = \lambda_0'$

$\lambda_1 = \lambda_1'$

$\lambda_N = \lambda_N'$

$\lambda_{N+1} = \lambda_{N+1}'$

$\lambda_2 = \lambda_2'$

$\lambda_{N+2} = \lambda_{N+2}'$

$p_{\text{h}}$

$p_{\text{t}}$

$p_{\text{t}} = (1-f)p_{\text{NI}}$

$fp_{\text{NI}}$

$\lambda_0 = \lambda_0'$

$\lambda_1 = \lambda_1'$

$\lambda_N = \lambda_N'$

$\lambda_{N+1} = \lambda_{N+1}'$

$\lambda_2 = \lambda_2'$

$\lambda_{N+2} = \lambda_{N+2}'$
## Results

1. Calibrate the model  
2. Comparative statics  
3. Shocks and estimation
### Some Calibration

<table>
<thead>
<tr>
<th>Preferences</th>
<th>Value</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>Household discount factor $\beta_h$</td>
<td>0.9900</td>
<td>annual interest rate 4%</td>
</tr>
<tr>
<td>Relative risk aversion $\gamma$</td>
<td>2</td>
<td>exogenous</td>
</tr>
<tr>
<td>Inverse Frisch elasticity $\nu$</td>
<td>0.3300</td>
<td>exogenous</td>
</tr>
<tr>
<td>Utility weight on leisure $\kappa$</td>
<td>8.9682</td>
<td>working time: 33%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Production Technology</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Depreciation rate of capital $\delta$</td>
<td>0.0252</td>
<td>capital-to-GDP ratio: 6.0</td>
</tr>
<tr>
<td>Capital share of output $\alpha$</td>
<td>0.2471</td>
<td>investment-to-GDP ratio: 16.0%</td>
</tr>
<tr>
<td>Entrepreneurs discount factor $\beta$</td>
<td>0.9890</td>
<td>exogenous</td>
</tr>
<tr>
<td>Fixed costs $\eta$</td>
<td>1.0590</td>
<td>waiting periods: 12.0</td>
</tr>
<tr>
<td>Transition probability $p^{hh} = p^{ll}$</td>
<td>0.9375</td>
<td>expected 4 year turn-over</td>
</tr>
<tr>
<td>log high productivity $\Delta$</td>
<td>0.0570</td>
<td>cross-sectional std 5.70%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Financial and Resale Frictions</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Financing Constraint $\theta$</td>
<td>0.4135</td>
<td>average debt/asset = 0.325</td>
</tr>
<tr>
<td>Resale Discount $d$</td>
<td>0.0971</td>
<td>reallocation/capital expenditure</td>
</tr>
</tbody>
</table>
Interactions and TFP Losses

Comparative Statics
Financial shocks and aggregate productivity shocks

- **Reallocation**: The graph shows the percentage changes in reallocation over time. The red line represents financial shocks, and the blue dashed line represents aggregate productivity shocks.

- **Investment**: The graph displays the percentage changes in investment. The red line indicates financial shocks, while the blue dashed line represents aggregate productivity shocks.

- **Output**: This graph illustrates the percentage changes in output over time. The red line is for financial shocks, and the blue dashed line is for aggregate productivity shocks.

- **Aggregate TFP**: The graph shows the percentage changes in aggregate TFP. The red line represents financial shocks, and the blue dashed line represents aggregate productivity shocks.

- **Interest Rate**: The graph depicts the percentage changes in the interest rate over time. The red line is for financial shocks, and the blue dashed line represents aggregate productivity shocks.

- **Turn Over**: The graph shows the percentage changes in turn over. The red line is for financial shocks, and the blue dashed line is for aggregate productivity shocks.
Why financial shocks?

- Financial shocks: lower labor costs and lower interest rate
  - Less competition from the productive firms
  - Holding onto assets are more attractive
- Productivity shocks
  - Reduce everyone’s incentive to stay in business
  - Note

\[
\pi = \alpha \left[ \frac{(1 - \alpha)A}{W} \right]^{\frac{1-\alpha}{\alpha}}
\]
**Procyclical reallocation?**

<table>
<thead>
<tr>
<th>Volatility</th>
<th>Co-movement</th>
</tr>
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<tbody>
<tr>
<td><strong>Standard deviation</strong></td>
<td><strong>Correlation with output</strong></td>
</tr>
<tr>
<td>Output</td>
<td>Reallocation</td>
</tr>
<tr>
<td>Data:</td>
<td>1.42%</td>
</tr>
<tr>
<td>Model:</td>
<td></td>
</tr>
<tr>
<td>Only financial shocks</td>
<td>1.38%</td>
</tr>
<tr>
<td>Only aggregate TFP shocks</td>
<td>1.31%</td>
</tr>
</tbody>
</table>

**Table: Only One Type of Shocks**
Smoothed Shocks

\( \epsilon^\theta \)

\( \epsilon^A \)
Final Remark

1. Summary and Extension
2. Takeaways
Conclusion

- Partial irreversible and financing constraints
  - Capital reallocation delay and prolonged delay in recessions
  - But aggregate productivity shocks shorten the delay

- Complicated inaction region can still be solved easily

- Policy implication: rethink interest rate policy?

- Implication on labor reallocation.
Hypothesis: Firms that allow wide swings in their leverage ratios, i.e., firms with large leverage ratio ranges, have tighter financial constraints when they are investing.
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Data
- Randomly selected firms over a period
- For each firm, compute the difference between maximum and minimum leverage ratio
- Group firms into different financial constrained categories
Hypothesis

- Hypothesis: Firms that allow wide swings in their leverage ratios, i.e., firms with large leverage ratio ranges, have tighter financial constraints when they are investing.

- Data
  - Randomly selected firms over a period
  - For each firm, compute the difference between maximum and minimum leverage ratio
  - Group firms into different financial constrained categories

- Test
  - Under null hypothesis, the degree of financial constraints does not have impacts on the leverage difference
Capital Reallocation Decreases in Recessions
Sales of Property, Plants and Equipment / Acquisition in 2005 dollars: definition from Eisfeldt & Rampini (2006)

Correlation: 0.85
Benefits to Reallocation Increase in Recessions

Idiosyncratic TFP dispersion: gap between 75% quantile and 25% quantile from Bloom et.al (2012)
Data Source

- COMPUSTAT / SDC data
  - For those who has assets acquired once in 2000-2012
  - Leverage before selling
- Sell immediately when profits are bad?
  - 5174 cases of selling
  - With about 60% selling all their assets.
  - 2071 * 20 firm-quarter observations, after merged with COMPUSTAT (adjusting missing value in debt for consecutive 20 quarters)
Firm-level Data: Debt/Asset ratio
deleveraging before selling assets

Debt/Asset Ratio

![Graph showing the Debt/Asset Ratio over time. The ratio decreases sharply before stabilizing.]
Asset Pricing Formula

- FOC (multipliers $\mu$) + envelope $\Rightarrow E[m'r'|I] = 1$

$$E_z \left[ \frac{\beta u'(c') z' \pi'}{u'(c)} \frac{z' q(k'+b', z')}{q(k+k', z)} \right] + \mu(k, b, z) = 1$$
Asset Pricing Formula

- FOC (multipliers $\mu$) + envelope $\Rightarrow E[m' r'|I] = 1$

$$E_z \left[ \frac{\beta u'(c')}{u'(c)} \frac{z' \pi'}{(1 - \delta)q\left(\frac{k'}{k' + b'}, z'\right)} \right] + \mu(k, b, z) = 1$$

Proposition (Policy functions for $k' > 0$)

- $c = (1 - \beta)(z \pi k + (1 - \delta)qk + Rb)$
- $k' = \phi \beta(z \pi k + (1 - \delta)qk + Rb)$
- $b' = (1 - \phi)\beta(z \pi k + (1 - \delta)qk + Rb)$

where $\phi$ satisfies the asset pricing equation.
Stopping Criteria and Inaction Boundary

Liquidation gains (safe) = Liquidation costs (risky)

**Proposition**

Let \( n = z^l \pi + (1 - \delta)(1 - d) + R \frac{1 - \lambda}{\lambda} \). Suppose \( \lambda \in [0, \bar{\lambda}] \) solves

\[
\eta = p^{lh} Value(n, z^h) + p^{ll} Value(n, z^l)
\]

\( z^l \) entrepreneurs liquidate the assets when \( \frac{k}{k+b} \leq \lambda \).

**Corollary**

Inaction region \( \bar{\lambda} - \lambda \) is larger when \( \eta \) is higher, \( d \) is higher, and \( \theta \) is lower.
Adjustment Cost Function

\[ \psi(k', k) \]

- **Slope** = 1
- **Slope** \( \in [1 - d, 1] \)
- **Inaction**
  \[ k' = (1 - \delta)k \]
- **Slope** = 1 - d

**Buy** --- **Sell**
To normalize capital to be 1. Continuation value for selling,
\( n = z^l \pi + (1 - \delta)(1 - d) + R \tilde{b} \):

\[
V^{out} = \log((1 - \beta)n) + \eta \\
+ \beta p^{lh} \left[ A^0 + \frac{\log(\beta nR)}{1 - \beta} \right] \\
+ \beta p^{ll} \left[ A^{N+1} + \frac{\log(\beta nR)}{1 - \beta} \right]
\]

Continuation value with one-shot inactive deviation

\[
V^{in} = \log((1 - \beta)n) \\
+ \beta p^{lh} \left[ A^0 + \frac{\log \left( (z^l \pi + (1 - \delta))\tilde{k} + R \left( \beta n - (1 - d) \tilde{k} \right) \right)}{1 - \beta} \right] \\
+ \beta p^{ll} \left[ A^{N+1} + \frac{\log \left( (z^l \pi + (1 - \delta)(1 - d))\tilde{k} + R \left( \beta n - (1 - d) \tilde{k} \right) \right)}{1 - \beta} \right]
\]
Optimal Stopping Time Rule - Proof Sketch

The difference of the two value is $V^{out} - V^{in}$

\[ \eta + \frac{\beta \log (\beta R)}{1 - \beta} \]
\[ - \left[ \frac{\beta}{1 - \beta} p^{\text{IH}} \log \left( \beta R + \tilde{k} z^l \pi + (1 - \delta) - (1 - d) R \right) \right] \]
\[ + \frac{\beta}{1 - \beta} p^{\text{II}} \log \left( \beta R + \tilde{k} z^l \pi + (1 - \delta) (1 - d) - (1 - d) R \right) \]

As $b/k$ goes to infinity, the difference goes to $\eta > 0$. Meanwhile, the term in the bracket is an increasing function of $m$ (and $b/k$). Thus, there is possible crossing of $V^{out}$ and $V^{in}$. 
Optimal Stopping Time Rule - A graph
### Key Statistics

**Table**: Key statistics in the data and in the model

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<td></td>
<td>Standard deviation to that of output</td>
<td>Consumption</td>
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<td><strong>Output</strong></td>
<td></td>
<td></td>
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<tr>
<td>Data: 1.42%</td>
<td>0.55</td>
<td>0.95</td>
</tr>
<tr>
<td>Model: 1.35%</td>
<td>0.61</td>
<td>0.84</td>
</tr>
</tbody>
</table>
Smoothed Shocks

\[ \varepsilon^\theta \]


\[ \varepsilon^A \]

Liquidation Smoothing

- Bring closer to the data may need large shocks

- Extension: fixed costs $\eta$ is drawn from an uniform distribution with support $[\underline{\eta}, \bar{\eta}]$

- Some entrepreneurs in each vintage will liquidate, because of high fixed costs

- The cut-off of fixed costs move in response to shocks
Liquidation Costs and Financing Constraints

In financial firms?

- Similar problem in financial institution

- Which assets to sell when borrowing is tougher?
  - Liquid assets first
  - Leaving illiquid assets later

- Systematic risks accumulate if only illiquid assets are left economy wide


