Delayed Capital Reallocation

Wei Cui

University College London

Introduction

Motivation

- Less restructuring in recessions
  (1) Capital reallocation is sizeable
  (2) Capital stock reallocation across firms ↓ Data

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  (1) Capital reallocation is sizeable
  (2) Capital stock reallocation across firms $\downarrow$ Data
- Significantly slow down recovery
- What frictions and shocks in a (heterogenous firms) model?
  - Generate less capital reallocation in recessions
  - Tractable for backing out shocks

Goals

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- Costs in reallocation:
  - Partial irreversible investment + financing constraints
  - Dynamics after aggregate productivity shocks / credit crunches
- A simple idea
  - Selling delay and the delay is prolonged in recessions

Goals

- Idiosyncratic productivity risks
- Costs in reallocation:
  - Partial irreversible investment + financing constraints
  - Dynamics after aggregate productivity shocks / credit crunches
- A simple idea
  - Selling delay and the delay is prolonged in recessions
- But complex issues
  - Difficulties: distribution of firms with different status
  - Buying assets, holding, selling, waiting to come back

Literature

- Financing frictions: Kiyotaki & Moore (1997), Bernanke et al. (1999), Brunnermeier & Sannikov (2011)...
- Resale problem: Kurlat (2011), Shleifer & Vishny (1992), Ramey & Shapiro (2001), Eisfeldt & Rampini (2006, 2007), Maksimovic & Phillips (1998, 2001),Khan & Thomas (2011)
- Uncertainty shocks: Bloom (2009), Gilchrist et al. (2010), Christiano et al. (2014)...
- Solution of heterogeneous agents model: Angeletos (2007), Kiyotaki & Moore (2011), Buera & Moll (2012)...
- DSGE Estimation

The Model

1. Households and firms (run by entrepreneurs)
2. Households' problem
3. Entrepreneurs' problem
4. The stationary equilibrium

Environment

- Households (measure 1) and firms run by entrepreneurs (measure 1)
- The representative household solves

$$\max \quad \mathbb{E}_t \sum_{s=t}^{\infty} \beta_h^{s-t} [\frac{c_{h,s}^{1-\gamma}-1}{1-\gamma} - \frac{\kappa\left(l_{h,s}\right)^{1+\nu}}{1+\nu}],$$

$$s.t. \quad c_{h,t} + b_{h,t} = w_t l_{h,t} + R_t b_{h,t}.$$

- Optimal solution:

$$\kappa c_{h,t}^{\gamma} l_{h,t}^{\nu} = w_t, \quad \mathbb{E}_t \frac{\beta_h \left(c_{h,t+1}\right)^{-\gamma}}{\left(c_{h,t}\right)^{-\gamma}} R_{t+1} = 1.$$

Entrepreneurs

- Entrepreneur j's preferences:

$$E_0 \sum_{t=0}^{\infty} \beta^t [\log(c_{jt}) + \eta(1 - h_{jt}))]$$

- η : fixed costs of running the firm
- j chooses whether to operate ($h_{jt} = 1$) or not ($h_{jt} = 0$)

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- j's production technology:

$$y_{jt} = (z_{jt} k_{jt})^{\alpha} (A_t l_{jt})^{1-\alpha}, \ \alpha \in (0, 1)$$

- z_{jt} is *idiosyncratic*. $z^h > z^l$ with $p^{hl} + p^{lh} < 1$:

$$P = \begin{bmatrix} p^{hh} & p^{hl} \\ p^{lh} & p^{ll} \end{bmatrix}$$

- Who will operate is endogenous (aggregate TFP is endogenous)

Resale discounts and borrowing constraints

- Capital adjustment cost function $\psi(k_{jt+1}, k_{jt})$

$$= \begin{cases} k_{jt+1} - (1-\delta)k_{jt} & \text{if } k_{jt+1} > (1-\delta)k_{jt}, \\ 0 & \text{if } k_{jt+1} = (1-\delta)k_{jt}, \\ -(1-d)[(1-\delta)k_{jt} - k_{jt+1}] & \text{if } k_{jt+1} < (1-\delta)k_{jt}. \end{cases}$$

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- Borrowing constraint: $\theta \geq 0$

$$Rb_{jt+1} \geq -\theta\,(1-\delta)\,(1-d)\,k_{jt+1}$$

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- Budget constraint:

$$c_{jt} + b_{jt+1} + \psi(k_{jt+1}, k_{jt}) = y_{jt} - w_t l_{jt} + Rb_{jt} = z_{jt}\pi k_{jt} + Rb_{jt}$$

An entrepreneur's problem

In Steady State

$$V(k,b,z) = \max\{W^1(k,b,a), W^0(k,b,z)\}$$

$$W^1(k,b,z) = \max_{k'>0,b'} \{\log(c^1) + \beta\mathbb{E}_z[V(k',b',z')]\}$$

$$W^0(k,b,z) = \max_{b'}\{\log(c^0) + \eta + \beta\mathbb{E}_z[V(0,b',z')]\}$$

where

$$c^1 = z\pi k + Rb - \psi(k',k) - b'$$

$$c^0 = z\pi k + Rb + (1-\delta)(1-d)k - b'$$

W^1 and W^0 denote running and not running a firm and $R'b' \geq -\theta(1-d)(1-\delta)k'$

Stationary equilibrium

Definition

The equilibrium is consists of policy functions $l = g^l(k, b, z)$, $k' = g^k(k, b, z)$, $b' = g^b(k, b, z)$ *and pricing functions* (π, R') *such that:*

(1). c_h, l_h, *and* b_h *solve the household's problem, given* w *and* R'

(1). l, k' *and* b' *solve the entrepreneur's problem, given* w, R', *and* $\pi = \alpha \left[\frac{(1-\alpha)A}{w}\right]^{\frac{1-\alpha}{\alpha}}$

(2). Markets for labor and bonds clear

$$\int l_j dj = l_h, \int b_j' dj + b_h = 0$$

Remark When there are aggregate shocks, we need aggregate state variable $X = (\theta, A, \Gamma)$ where $\Gamma(k, b, z)$ is the joint CDF.

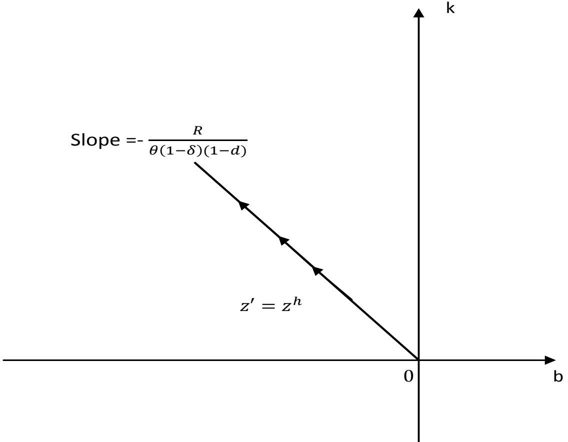
Model Solution

1. Policy functions
2. Option value of capital
3. Exact aggregation

Dynamics of (k, b): only the ratio matters

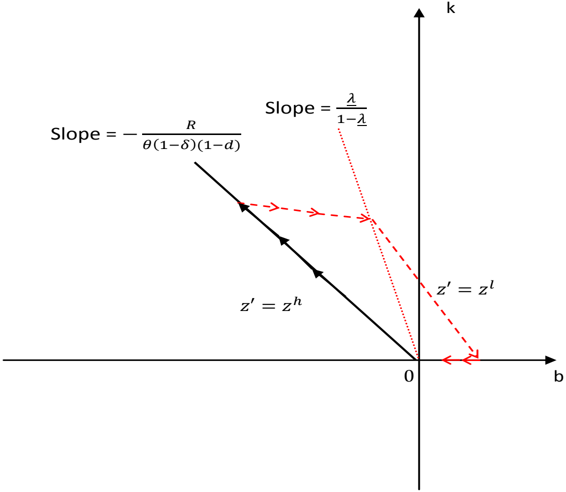
Lemma

$$V(\gamma k, \gamma b, z) = V(k, b, z) + \frac{\log \gamma}{1-\beta}$$

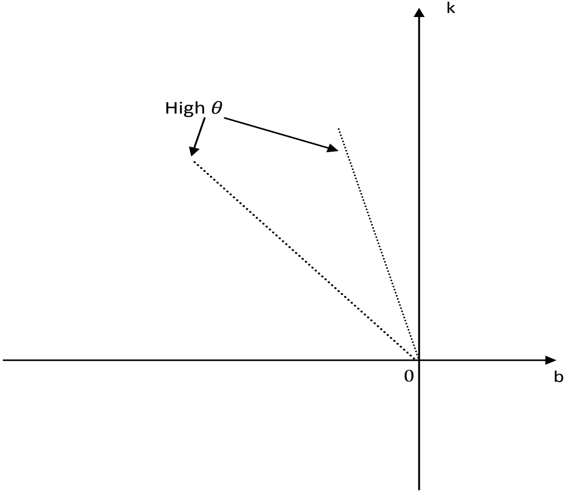


Dynamics of (k, b): only the ratio matters

Intuition: Without d, low z firms sell immediately to pay off debt. With d, hold on and gradually pay off debt.



Inaction region and action boundary



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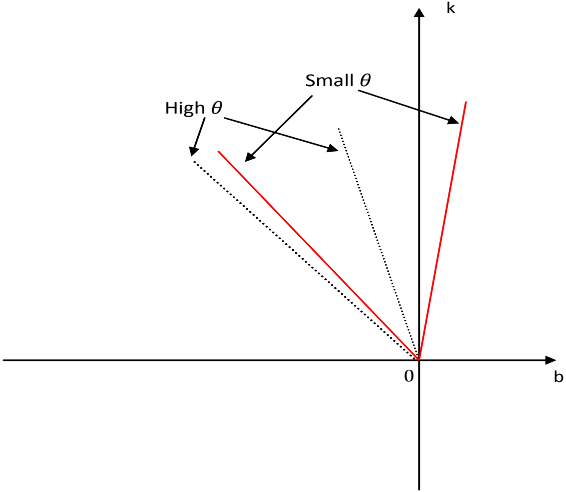

The option value of capital

- Option value of capital $q(k, b, z)$ satisfies

$$V_k(k, b, z) = u'(c)[z\pi + q(k, b, z)(1 - \delta)]$$

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- The inaction region:

$$1 - d < q(k, b, z) < 1$$

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- To characterize q. Homogeneity $\rightarrow q(k,b,z) = q(\frac{k}{k+b}, z)$

some derivation

Asset Pricing Formula

- FOC (multipliers μ) + envelope $\Rightarrow E[m'r'|\mathcal{I}] = 1$

$$E_z\left[\frac{\beta u'(c')}{u'(c)}\frac{z'\pi' + (1-\delta)q(\frac{k'}{k'+b'}, z')}{q(\frac{k}{k+b}, z)}\right] + \mu(k, b, z) = 1$$

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Proposition (Policy functions for $k' > 0$)

$$\begin{aligned} c &= (1-\beta)(z\pi k + (1-\delta)qk + Rb) \\ k' &= \phi\beta(z\pi k + (1-\delta)qk + Rb) \\ b' &= (1-\phi)\beta(z\pi k + (1-\delta)qk + Rb) \end{aligned}$$

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Liquidation

- When to liquidate?

$$
\begin{aligned}
\frac{(1-\beta)\eta}{\beta} = \quad & p^{lh}\mathbb{E}_X\left[\log\left(1+(1-\delta)\frac{z^h\pi'+(1-\delta)-(1-d)R'}{\beta(z^l\pi+(1-\delta)(1-d)+R\frac{1-\lambda}{\underline{\lambda}})R'}\right)\right] \\
+ \quad & p^{ll}\mathbb{E}_X\left[\log\left(1+(1-\delta)\frac{z^l\pi'+(1-\delta)(1-d)-(1-d)R'}{\beta(z^l\pi+(1-\delta)(1-d)+R\frac{1-\lambda}{\underline{\lambda}})R'}\right)\right]
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Liquidation

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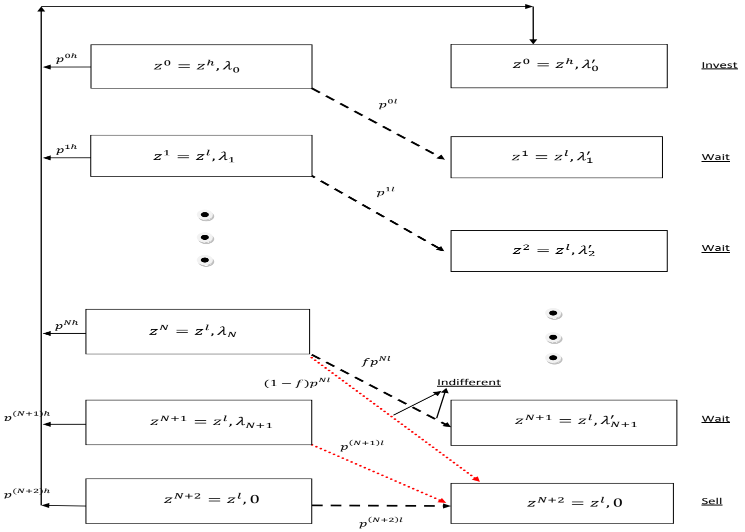
- The drop of π and R' delays liquidation
- Uncertainty shocks alone may *not* delay liquidation decisions
  - Importance of credit market in response to uncertainty shocks
  - Gilchrist et al. (2010) and Christiano et al. (2014)

Exact aggregation



Results

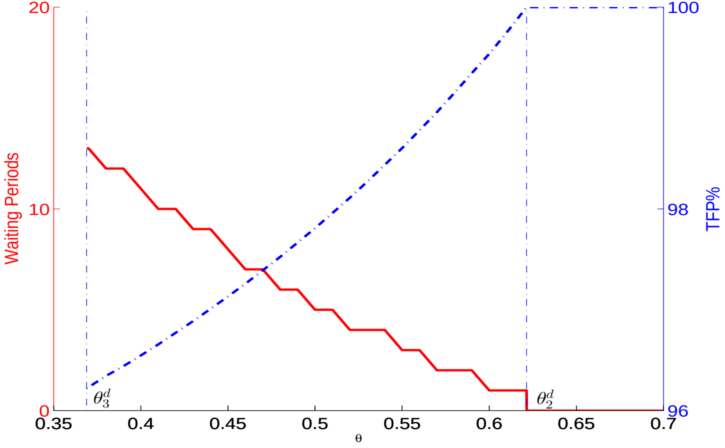
1. Calibrate the model
2. Comparative statics
3. Shocks and estimation

Some Calibration

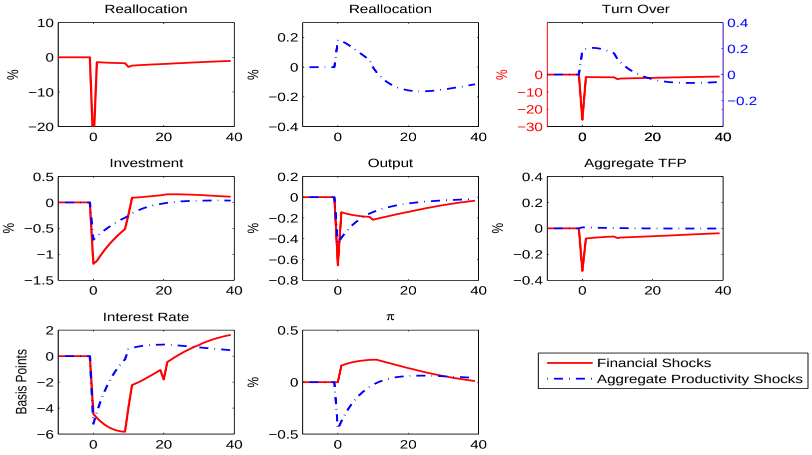
		Value	Target
Preferences			
Household discount factor	β_h	0.9900	annual interest rate 4%
Relative risk aversion	γ	2	exogenous
Inverse Frisch elasiciticity	ν	0.3300	exogenous
Utility weight on leisure	κ	8.9682	working time: 33%
Production Technology			
Depreciation rate of capital	δ	0.0252	capital-to-GDP ratio: 6.0
Capital share of output	α	0.2471	investment-to-GDP ratio: 16.0%
Entrepreneurs discount factor	β	0.9890	exogenous
Fixed costs	η	1.0590	waiting periods: 12.0
Transition probability	$p^{hh} = p^{ll}$	0.9375	expected 4 year turn-over
log high productivity	Δ	0.0570	cross-sectional std 5.70%
Financial and Resale Frictions			
Financing Constraint	θ	0.4135	average debt/asset = 0.325
Resale Discount	d	0.0971	reallocation/capital expenditure

Interactions and TFP Losses

Comparative Statics



Financial shocks and aggregate productivity shocks



Why financial shocks?

- Financial shocks: lower labor costs and lower interest rate
  - Less competition from the productive firms
  - Holding onto assets are more attractive
- Productivity shocks
  - Reduce everyone's incentive to stay in business
  - Note

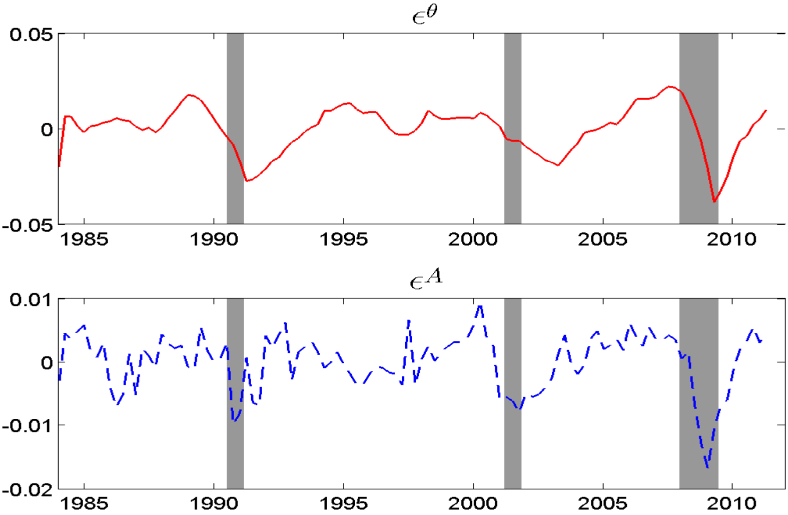
$$\pi = \alpha \left[\frac{(1-\alpha)A}{w} \right]^{\frac{1-\alpha}{\alpha}}$$

Procyclical reallocation?

Table : **Only One Type of Shocks**

	Volatility		Co-movement	
	Standard deviation	Standard deviation to that of output	Correlation with output	
	Output	Reallocation	Reallocation	Reallocation Turn-over
Data:	1.42%	10.91	0.85	0.79
Model:				
Only financial shocks	1.38%	11.03	0.83	0.71
Only aggregate TFP shocks	1.31%	1.77	0.18	-0.33

Smoothed Shocks



Final Remark

1. Summary and Extension
2. Takeaways

Conclusion

- Partial irreversible and financing constraints
  - Capital reallocation delay and prolonged delay in recessions
  - But aggregate productivity shocks shorten the delay
- Complicated inaction region can still be solved easily
- Policy implication: rethink interest rate policy?
- Implication on labor reallocation.

Hypothesis

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  - For each firm, compute the difference between maximum and minimum leverage ratio
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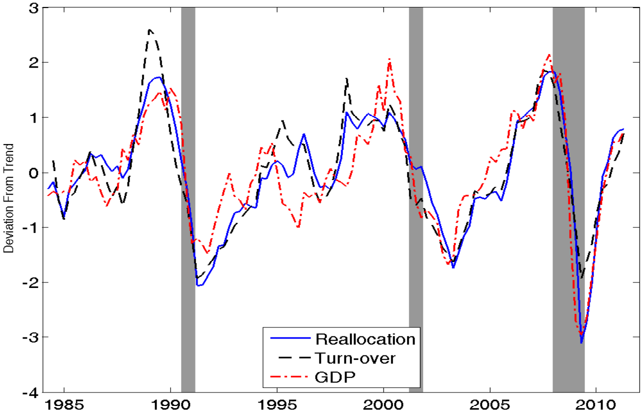
- Data
  - Randomly selected firms over a period
  - For each firm, compute the difference between maximum and minimum leverage ratio
  - Group firms into different financial constrained categories
- Test
  - Under null hypothesis, the degree of financial constraints does not have impacts on the leverage difference

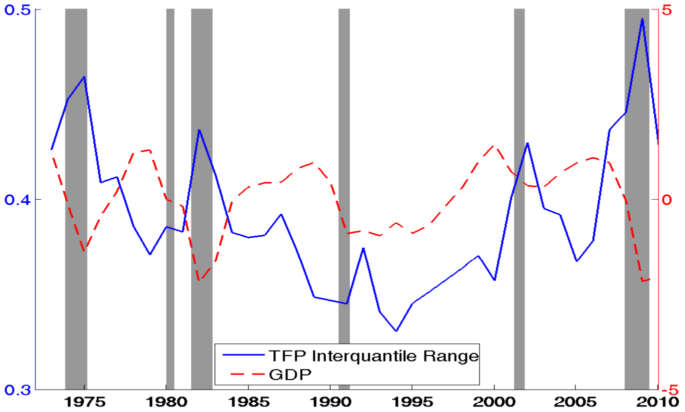
Capital Reallocation Decreases in Recessions

Sales of Property, Plants and Equipment / Acquisition in 2005 dollars: definition from Eisfeldt & Rampini (2006)



Correlation: 0.85 back

Benefits to Reallocation Increase in Recessions



Idiosyncratic TFP dispersion: gap between 75% quantile and 25% quantile from Bloom et.al (2012) back

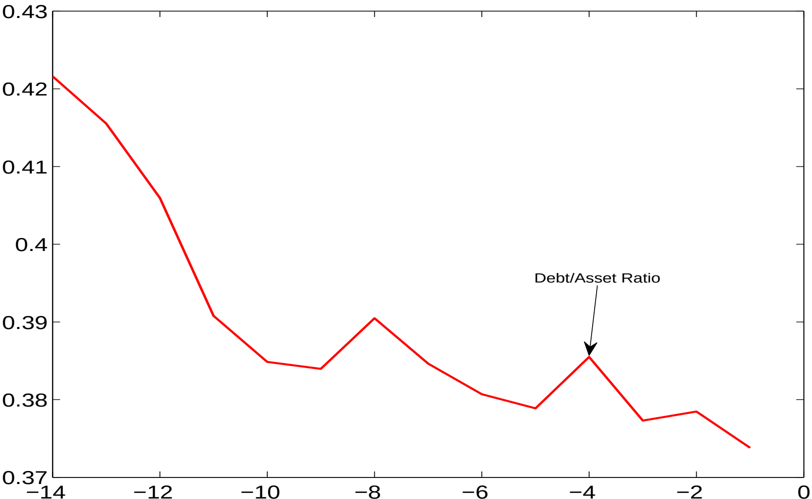
Data Source

- COMPUTSTAT / SDC data
  - For those who has assets acquired once in 2000-2012
  - Leverage before selling
- Sell immediately when profits are bad?
  - 5174 cases of selling
  - With about 60% selling all their assets.
  - 2071 * 20 firm-quarter observations, after merged with COMPUSTAT (adjusting missing value in debt for consecutive 20 quarters)

Firm-level Data: Debt/Asset ratio

deleverage before selling assets

Debt/Asset Ratio ▸ back



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Stopping Criteria and Inaction Boundary

Liquidation gains (safe) = Liquidation costs (risky) Proof

Proposition

Let $n = z^l\pi + (1-\delta)(1-d) + R\frac{1-\lambda}{\underline{\lambda}}$. *Suppose* $\underline{\lambda} \in [0, \bar{\lambda}]$ *solves*

$$\eta = p^{lh} Value(n, z^h) + p^{ll} Value(n, z^l)$$

z^l *entrepreneurs liquidate the assets when* $\frac{k}{k+b} \leq \underline{\lambda}$.

Corollary

Inaction region $\bar{\lambda} - \underline{\lambda}$ *is larger when* η *is higher,* d *is higher, and* θ *is lower.*

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Adjustment Cost Function

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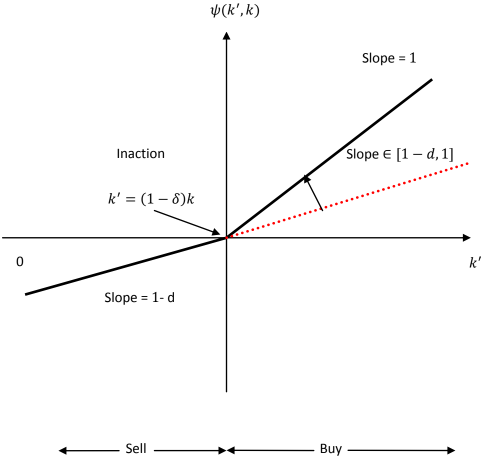


Optimal Stopping Time Rule - Proof Sketch

To normalize capital to be 1. Continuation value for selling, $n = z^l\pi + (1-\delta)(1-d) + R\tilde{b}$:

$$V^{out} = log((1-\beta)n) + \eta \\ + \beta p^{lh}\left[A^0 + \frac{log\left(\beta nR\right)}{1-\beta}\right] + \beta p^{ll}\left[A^{N+1} + \frac{log\left(\beta nR\right)}{1-\beta}\right]$$

Continuation value with one-shot inactive deviation

$$V^{in} = log((1-\beta)n) \\ + \beta p^{lh}\left[A^0 + \frac{log\left((z^l\pi + (1-\delta))\tilde{k} + R\left(\beta n - (1-d)\,\tilde{k}\right)\right)}{1-\beta}\right] \\ + \beta p^{ll}\left[A^{N+1} + \frac{log\left((z^l\pi + (1-\delta)\,(1-d))\tilde{k} + R\left(\beta n - (1-d)\right.\right.}{1-\beta}\right.$$

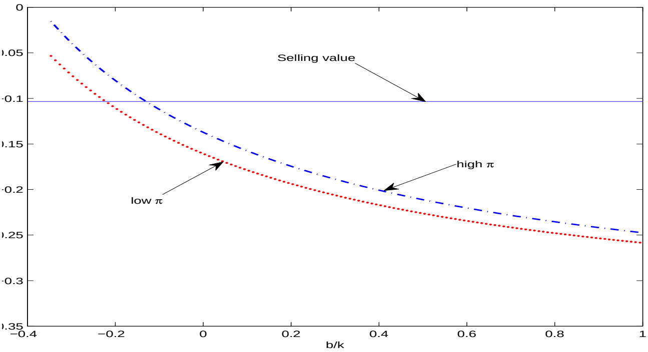
Optimal Stopping Time Rule - Proof Sketch

The difference of the two value is $V^{out} - V^{in}$

$$\eta + \frac{\beta log\left(\beta R\right)}{1-\beta}$$
$$- [\frac{\beta}{1-\beta} p^{lh} log\left(\beta R + \tilde{k}\frac{z^l \pi + (1-\delta) - (1-d) R}{m}\right)$$
$$+ \frac{\beta}{1-\beta} p^{ll} log\left(\beta R + \tilde{k}\frac{z^l \pi + (1-\delta)(1-d) - (1-d) R}{m}\right)]$$

As b/k goes to infinity, the difference goes to $\eta > 0$. Meanwhile, the term in the bracket is an increasing function of m (and b/k). Thus, there is possible crossing of V^{out} and V^{in}.

Optimal Stopping Time Rule - A graph



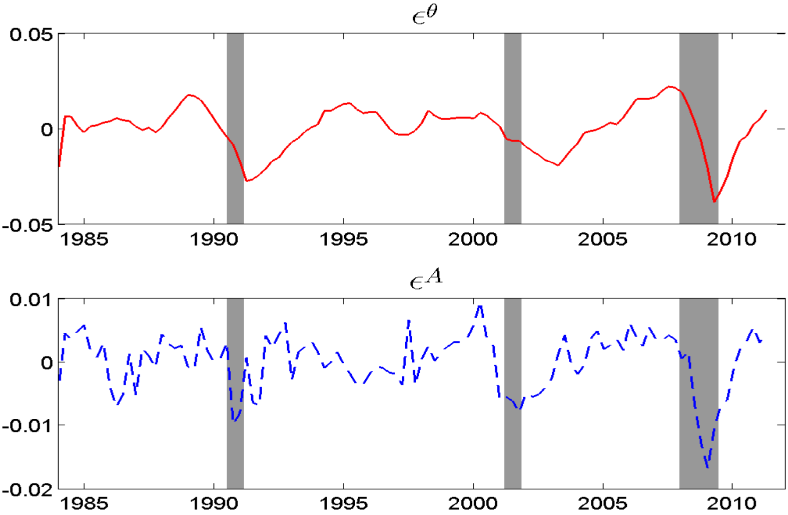
Back

Key Statistics

Table : **Key statistics in the data and in the model**

	Volatility				Co-movement			
	Standard deviation	Standard deviation to that of output			Correlation with Output			
	Output	Consumption	Investment	Reallocation	Consumption	Investment	Reallocation	TFP dispersion
Data:	1.42%	0.55	3.86	10.91	0.95	0.96	0.85	-0.42
Model:	1.35%	0.61	4.01	11.05	0.84	0.91	0.61	-0.37

Smoothed Shocks



Liquidation Smoothing

- Bring closer to the data may need large shocks
- Extension: fixed costs η is drawn from an uniform distribution with support $[\underline{\eta}, \bar{\eta}]$
- Some entrepreneurs in each vintage will liquidate, because of high fixed costs
- The cut-off of fixed costs move in response to shocks

Liquidation Costs and Financing Constraints

In financial firms?

- Similar problem in financial institution
- Which assets to sell when borrowing is tougher?
  - Liquid assets first
  - Leaving illiquid assets later
- Systematic risks accumulate if only illiquid assets are left economy wide back

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