Frustration & Anger in Games*

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1 Introduction

Can anger shape economic outcomes in important ways? Consider some examples/questions:

1. In 2006-07 gas prices went up and up. Many folks were quite upset. Did this cause road rage?

2. In 2007 Apple launched its iPhone at $499. Just two months later they introduced a new version, priced at $399, re-pricing the old model at $299. This caused outrage among early adopters. Apple paid back the difference. Did this help long run profit?

3. When local football teams favored to win lose, the police gets more reports of husbands assaulting wives. Did the unexpected loss spur frustration and violence?

4. In 2008 the government bailed out banks through its TARP program. Some voters were infuriated. Did this spur the Tea Party and Occupy Wall Street movements?

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5. Following the European Soveriegn Debt Crises (2009-), many EU countries embarked on austerity programs. Was it because citizens were frustrated with loss of benefits that many cities then experienced riots and unrest?

Traffic safety, profit maximization, domestic violence, political landscapes, ...; the hinted at themes seem important. However, in order to systematically assess relevance and consequences one needs to develop theory connecting anger, decisions, and outcomes. Our goal is to take first steps in that direction.

Economists traditionally paid scant attention to anger. Insights from psychology, however suggest ways that anger has strategic implications. Psychologists refer to the behavioral consequences of emotions as "action tendencies," and the action tendency of anger is aggression. One may imagine that angry players may be willing to forego material gains to punish others, or that a predisposition to behave aggressively when angered may benefit a player by serving as a credible threat, and so on. But while insights of this nature can be gleaned from psychologists' writings, their analysis usually stops with the individual rather than goes on to assess overall economic implications. Our goal is to take the basic insights about anger that psychology has produced as input and inspiration for the theory we develop and apply.[1]

Our reading of the evidence suggests that anger is typically anchorded in frustration, which occurs when someone is unexpectedly denied something they care about.[2] An Archimedean point of our analysis is to assume that people are frustrated when they get less material rewards than they expected beforehand. Examples 1, 3, 5 above furnish illustrations, and indicate the broad applied potential. Nevertheless, our focus is somewhat restrictive, as examples 2 and 4 illustrate. In example 2 an early adapter is frustrated because he regrets he already bought, not because the new information implies that his expected rewards of the purchase already made go down. In example 4 someone may be materially unaffected personally, yet frustrated because of perceived unfairness. Thus, while our analysis will address many subtle

[1] The relevant literature is huge. A good point of entry, and source of insights & inspiration for us, is the recent *International Handbook of Anger* (Potegal, Stemmler & Spielberger, eds., 2010), which offers a cross-disciplinary perspective spanning 32 chapters reflecting "affective neuroscience, business administration, epidemiology, health science, linguistics, political science, psychology, psychophysiology, and sociology" (p. 3, citation from Potegal & Stemmler's opening chapter). We take the non-occurrence of "economics" in the list as an indication our approach is original and needed!

[2] Psychologists often refer to this as "goal-blockage" (cf. p.3 of the (*op.cit.*) *Handbook.*

considerations, there are nevertheless meaningful and nuanced ways to get frustrated that we leave for future research.

How do decision-makers react to frustration and anger? The evidence from psychology, and introspection, suggest a variety of possibilities often centered on the idea that it triggers agression.[3] We present three related approaches, captured by distinct utility functions. While players motivated by simple anger become generally hostile to, those motivated by anger from blaming behavior or by anger from blaming intentions go after others more discriminately with references to who actually caused, or who intended to cause, their dismay.

What are the overall implications when people interact? To answer this questions a solution concept is needed, and there are many seemingly sensible ways to proceed. We develop and compare notions of sequential equilibrium, equilibrium with perceived intentions, and polymorhic sequential equilibrium. We mix & match these solution conceptss with the three aforementioned different utilities, explore properties, and compare predictions.

A player's frustration depends on his beliefs about others' choices. The blame a player attributes to another may depend on his beliefs about others choices or beliefs. For these reasons, all our models find their intellectual home in the framework of so-called psychological game theory; see Geanakoplos, Pearce & Stacchetti (1989) for a pioneering contribution and Battigalli & Dufwenberg (2009) for the framework extension needed to be well-position to understand what tools our current exercise draws on.

Section 2 contains preliminaries on how we represent games and beliefs. Section 3 develops the three key notions of utility that we adress. Section introduces and applies the various solution concepts. Section 5 combines the material of the two preceding sections, and derives/highlight various results and insights. Section 6 concludes. An Appendix contains some details, extensions, and proofs which do not include in the main text.

[3]Baumeister & Bushman (2007, p. 66), e.g., say that "anger is an important and powerful cause of aggression." They define aggression (p. 62) as "any behavior that is intended to harm another person who is motivated to avoid the harm."

2 Preliminaries: games and beliefs

We adopt the framework of Battigalli & Dufwenberg (2012).[4] Agents play a multistage game with simultaneous actions in each stage, perfect recall, and possibly imperfect monitoring of other agents' past actions. At each information set, a player has conditional beliefs about the continuation. Beliefs at different information sets on the same path are related by Bayes' rule whenever possible. A player's beliefs about his own action represent his plan. The multi-stage structure provides a natural time line. Along each path, a player experiences a temporal sequence of beliefs affecting his preferences about actions and consequences.

2.1 Multistage game forms

A multistage game form is given by a set of players, a set of feasible sequences of action profiles, called histories, an information structure and a material payoff function for each player. The material payoff function of a player describes the monetary (or consumption) consequences of interaction, but not his preferences.

Formally, we describe the game tree as a set of sequences of action profiles, with the understanding that, at each stage, a subset of players move simultaneously. Chance is a player with a fixed, fully randomized behavior strategy. To simplify the notation we assume that all players obtain information and take an action at each decision stage, and **inactive** players have only one feasible action. This is not a mere convention when the timing of belief-dependent emotions is relevant. We adopt this specification for simplicity, not because it is without substantial loss of generality.

More formally, a **multistage extensive form** is a tuple

$$(I, A, X, \sigma_c, (H_i)_{i \in I})$$

where

- I is a finite set of **players** with generic element i; the **chance** player is denoted c, with $c \notin I$; the players set including chance is denoted I_c;

- A is a finite set of **actions**;

[4] See also Battigalli *et al.* (2013).

- X is a finite set of **histories**: X contains the **empty history** $\varnothing$, each other element of X is a finite sequence of action profiles $x = (\mathbf{a}_1, ...\mathbf{a}_k)$, with $\mathbf{a}_t = (a_{i,t})_{i \in I_c} \in A^{I_c}$ for each $t \in \{1, ..., k\}$; X with the "prefix of" relation $\prec$ is a tree with root $\varnothing$;

- σ_c is a profile of strictly positive **probability measures**, one for each **chance move**;[5]

- for each $i \in I$, H_i is an **information partition** of X that satisfies *perfect recall,* and it is such that every information set $h \in H_i$ contains histories of the *same length.*

We refer the reader to the Appendix for a more formal definition. Here we stress three features. First, the game has a multistage structure because information tracks time; we often emphasize this writing $h_{i,t}$ for an information set of player i containing histories of length t, hence representing i's information at the beginning of period/stage $t+1$. Second, we specify a player's information also at nodes/histories where he is not active,[6] including terminal histories; the reason is that information affects beliefs and hence emotions, therefore information may be relevant even if it does not have instrumental value. Of course, non-terminal information sets play a special role; the collection of such information sets is denoted by H_i^D. Third, according to our general definition action profiles are written in a somewhat redundant way, that is, we write $\mathbf{a} = (a_i)_{i \in I}$ even if only a strict subset of players $J \subset I$ are active; we can think of the "action" of an inactive player as "waiting" or some compulsory action. The following table summarizes quite standard definitions and notation for derived objects that will be frequently

[5] With the notation of the following table, $\sigma_c = (\sigma_c(\cdot|x))_{x \in X \setminus Z} \in \times_{x \in X \setminus Z} \Delta^\circ(A_c(x))$, where $\Delta^\circ(A_c(x))$ is the relative interior of $\Delta(A_c(x))$.

[6] A player is active at x if he has at least two feasible actions at x.

used in the analysis:

Name	Notation	Definition
Length of x	$\ell(x)$	$\ell(x) = t \Leftrightarrow x \in (A^{I_c})^t$
Terminal histories	Z	$\{x \in X : \nexists y \in X, x \prec y\}$
Terminal successors of x	$Z(x)$	$\{z \in Z : x \prec z\}$
Terminal successors of $h \subseteq X$	$Z(h)$	$\cup_{x\in h} Z(x)$
Non-terminal informations sets	H_i^D	$\{h \in H_i : h \cap Z = \emptyset\}$
Information set of i at x	$H_i(x)$	$H_i(x) = h \Leftrightarrow x \in h \in H_i$
Feasible action profiles at x	$\mathbf{A}(x)$	$\{\mathbf{a} \in A^{I_c} : (x, \mathbf{a}) \in X\}$
Feasible actions of i at x	$A_i(x)$	$\{a_i \in A : \exists \mathbf{a}_{-i} \in A^{I_c\setminus\{i\}}, (a_i, \mathbf{a}_{-i}) \in \mathbf{A}(x)\}$
Feasible actions of i at $h \in H_i$	$A_i(h)$	$\cup_{x\in h} A_i(x)$
Feasible actions of $-i$ at x	$\mathbf{A}_{-i}(x)$	$\times_{j\in I_c\setminus\{i\}} A_j(x)$

Table 1. Extensive form: derived elements

A **multistage game form** is a structure

$$\Gamma = (I, A, X, \sigma_c, (H_i, \pi_i)_{i\in I})$$

where $(I, A, X, \sigma_c, (H_i)_{i\in I})$ is a multistage extensive form and, for each $i \in I$,

$$\pi_i : Z \to \mathbb{R}$$

is the **material payoff function** of player i. Roughly, a multistage game form describes rules of the game that an experimenter can specify in the lab, making them common knowledge among the subjects. These rules include the map from actions to material consequences, that in this paper are represented by the monetary payoff vectors attached to terminal histories, $(\pi_i(z))_{i\in I}$. From now on we will often omit the adjective "multistage."

Game forms with a special structure deserve attention. A game form Γ

- has **observable actions** if each information set is a singleton; hence, each H_i is isomorphic to X;
- has **perfect information** if it has observable actions and at most one player is active at each non-terminal history;
- is a **leader-follower game** if (i) it has perfect information and two stages, (ii) the **leader** is the player who is active in the first stage, (iii) whoever is active in the second stage is a **follower**, a player different from the leader, and (iv) there is at least one action of the leader after which a player is active.

The following game form is a leading example we repeatedly refer to, and it illustrates our notation.

Example 1 *(*Ultimatum Minigame*) Ann and Bob play the following leader-follower game: in the first period Ann can make a fair offer, which is automatically accepted and implemented, or a greedy offer; in the second period Bob can either accept or reject the offer. The actions, timing, and material payoffs are described in Figure A (profiles – here pairs – are written according to players' alphabetical order):*

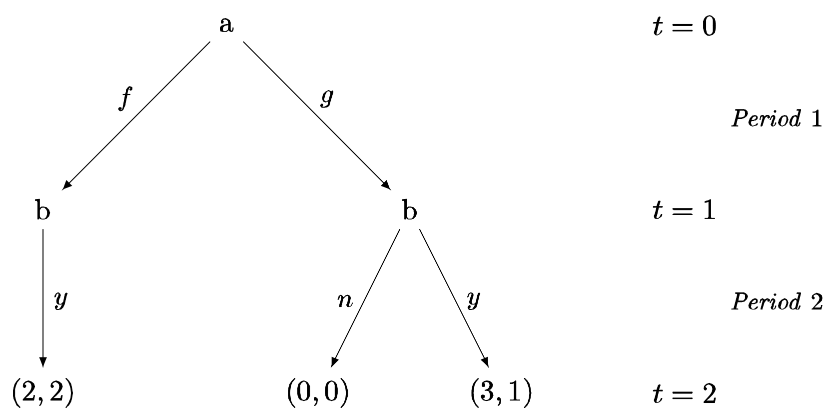


Figure A. *Ultimatum Minigame.*

According to our notation, and suppressing the chance player, we have

$$I=\{a,b\},\ A=\{f,g,n,y,*\},$$

where $*$ *is the pseudo-action of "waiting," and*

$$\begin{aligned}
X &= \{\varnothing,((f,*)),((g,*)),((f,*),(*,y)),((g,*),(*,n))\,((g,*),(*,y))\},\\
Z &= \{((f,*),(*,y)),((g,*),(*,n))\,((g,*),(*,y))\},\\
A_a(\varnothing) &= \{f,g\},\ A_a(x)=\{*\}\ \textit{for each}\ x\in\{((f,*)),((g,*))\},\\
A_b(\varnothing) &= \{*\},\ A_b(((f,*)))=\{y\},\ A_b(((g,*)))=\{y,n\}.
\end{aligned}$$

Hence, Ann is active only at the root $\varnothing$ *and Bob is active only after the greedy offer* g. H_a *and* H_b *are isomorphic to* X, H_a^D *and* H_b^D *are isomorphic to* $X\backslash Z$.

From now on, when we discuss examples, we will not mention pseudo-actions any more. Furthermore, we omit redundant parentheses whenever this does not hurt clarity. For example, we will write g for the history of length one $((g,*))$ containing the action pair $(g,*)$ of the Ultimatum Minigame.

2.2 Information and time

The exact timing of experienced beliefs may be key in games with belief-dependent preferences. We model time by assuming that it corresponds to the stages of the game. Thus, if there is a period in which, given the previous history, no player moves and nothing is learned, we still want to model it as a separate stage. Specifically, we assume the following *time line*: the play unfolds through time periods $t = 1, 2, 3, ...$ (first, second, third ...) of *equal duration*; by convention, period t is the time interval between date $t-1$ and date t (a date is a point in time, a period is an interval). For example, period 1 is the time interval between date 0 and date 1. The information of player i at the *beginning* of period t is given by some information set h formed by histories of length $t-1$. We will sometime write $h_{i,t-1}$ to emphasize that we refer to an information set of player i that consists of histories of length $t-1$. For $t = 1$, we have $h_{i,0} = \{\varnothing\}$: at date 0, the beginning of stage 1, players have no information, which corresponds to the singleton information set containing the only history of length 0.[7]

Whenever player i observes $h_{i,t-1} \in H_i$ at the beginning of stage t and takes action $a_i \in A_i(h_{i,t-1})$ he automatically and immediately observes that he has just taken this action, therefore his information is given by the set of histories

$$(h_{i,t-1}, a_i) := \{(x', \mathbf{a}') \in X : x' \in h_{i,t-1}, a'_i = a_i\}.$$

Then, given that the other players choose $\mathbf{a}_{-i}$ and the play moves to the beginning of stage $t+1$, player i may learn something about the actions taken by the co-players (in stage t or earlier), and his information (h, a_i) is refined to some $h_{i,t} \in H_i$ with $Z(h_{i,t}) \subseteq Z(h_{i,t-1})$ (see the time-line in Figure T). Note that some history in $(h_{i,t-1}, a_i)$ may be terminal.

[7] The multistage information structure implies that the infomation set containing the empty history cannot contain any other history.

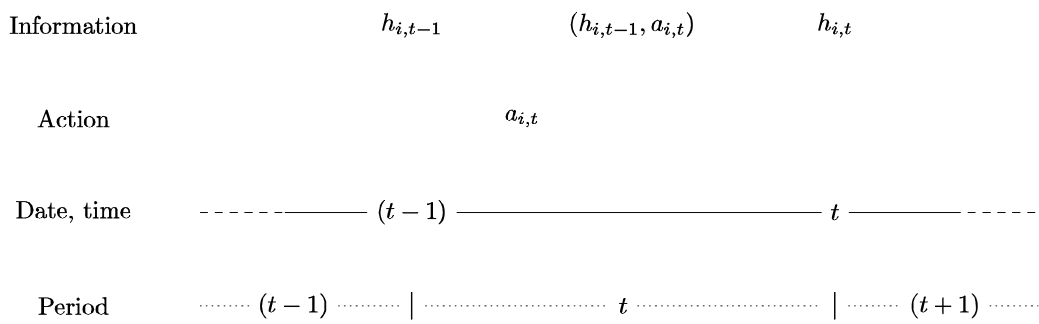


Figure T. Time-line.

Players have to be able to assign a material or psychological value to each action a_i they take at each information set $h \in H_i$, and we want to represent these values as their conditional expected payoff (or psychological utility) given (h, a_i). Therefore, it is convenient to enrich the information structure of each player i, adding to H_i to the "end-of-period information sets" of the form (h, a_i) described above. Thus, we obtain the larger collection

$$\bar{H}_i := H_i \cup \{(h, a_i) : h \in H_i, a_i \in A_i(h)\}.$$

containing also the information sets of the form (h_i, a_i). By perfect recall, one can extend the "prefix of" relation to this collection of information sets: for every $h', h'' \in \bar{H}_i$, let $h' \prec h''$ if and only if for every $x'' \in h''$ there is $x' \in h'$ such that $x' \prec x''$. With this, $(\bar{H}_i, \prec)$ is a tree. (As we said, $\bar{H}_i$ is a set of personal histories of information states and actions of i).

We need a formal notation for the sequence of beliefs experienced by a player along a path, which depends on the sequence of information sets crossed by this path. For any history $x = (\mathbf{a}^1, ...\mathbf{a}^t)$ and stage $k = 1, ..., t$, let $x^{\leq k} = (\mathbf{a}^1, ...\mathbf{a}^k)$ denote the k-prefix of x; by convention $x^{\leq 0} := \varnothing$ is the 0-prefix of every x. The information set of i that obtains at the beginning of stage $k+1$ along path x is therefore $H_i(x^{\leq k})$.

Of course, by perfect recall, $H_i(x^{\leq k}) = H_i(y^{\leq k})$ whenever $x, y \in h_i \in H_i$. Therefore, for each information set h of player i made of histories of length t ($h \in H_i$, $h \subseteq (A^I)^t$) and each $k = 0, ..., t$, it makes sense to write $h^{\leq k} = H_i(x^{\leq k})$, where $x \in h$ is arbitrary.

The following game form illustrates our notation on the information structure.

Example 2 *Ann, Bob and Penny the punisher play the following game form with observable actions, where Ann and Bob move simultaneously in the first period and Penny may move in the second:*

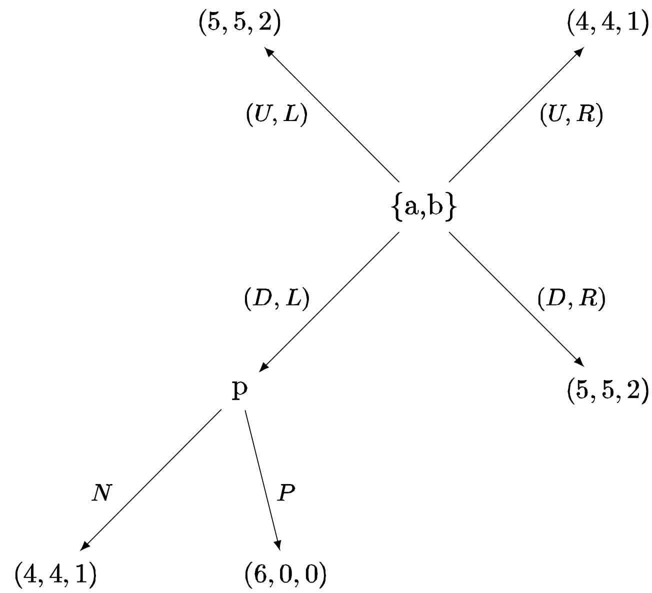


Figure B. *Asymmetric punishment.*

This is a game with asymmetric punishment because, if Penny punishes the coordination failure (D, L)*, she badly hurts Bob and at the same time slightly rewards Ann.*[8] *Since there are observable actions, the information structure of each player is isomorphic to the set of histories* X*. The expanded information structure* $\bar{H}_p$ *of Penny coincides with her information structure* H_p *because when she moves she is the only active player. The action sets of Ann and Bob are, respectively,*

$$A_a(\varnothing) = \{U, D\},\ A_b(\varnothing) = \{L, R\},$$

and their expanded information structures are, respectively,

$$\begin{aligned} \bar{H}_a &= H_a \cup \{\{(U,L),(U,R)\},\{(D,L),(D,R)\}\}\,, \\ \bar{H}_b &= H_b \cup \{\{(U,L),(D,L)\},\{(U,R),(D,R)\}\}\,. \end{aligned}$$

For example, $\{(D, L), (D, R)\}$ *is the information of Ann after she has chosen action* D*. The date-*1 *information set of Penny implied by terminal history*

[8] We will comment on this feature to discuss solution concepts.

$z = ((D, L), N)$ *is*

$$H_p(z^{\leq 1}) = H_p((D, L)) = \{(D, L)\}.$$

2.3 Beliefs and plans

Players have beliefs about the behavior and beliefs of co-players; given such beliefs they form plans; a plan and beliefs about other players yield beliefs about path of play, i.e., a probability measure over the set of terminal histories $z \in Z$. Of course, at each information set $h \in \bar{H}_i$, player i assigns probability one to the set $Z(h)$ of terminal successors of h, and beliefs held at information sets ordered by precedence have to be related according to the rules of conditional probabilities. For example, in a two-person game with observable actions without chance moves, beliefs of player i about j's behavior can be represented by a behavioral strategy of j, that is, an array of probabilities of actions $\sigma_j = (\sigma_j(\cdot|x))_{x \in X \backslash Z} \in \times_{x \in X \backslash Z} \Delta(A_j(x))$; a plan of i can also be represented as a behavioral strategy, viz. σ_i. The pair of behavioral strategies (σ_i, σ_j) yields, for each $x \in X \backslash Z$, the conditional probability of reaching terminal history $z \in Z(x)$ from x, viz. $\mathbb{P}_{\sigma_i, \sigma_j}(z|x)$. Clearly, if $x' \prec x''$ (x' is an initial subsequence, or "prefix" of x''), then

$$\mathbb{P}_{\sigma_i, \sigma_j}(x''|x') > 0 \Rightarrow \mathbb{P}_{\sigma_i, \sigma_j}(z|x'') = \frac{\mathbb{P}_{\sigma_i, \sigma_j}(z|x')}{\mathbb{P}_{\sigma_i, \sigma_j}(x''|x')}.$$

However, player i also has beliefs about what j thinks; what i believes j will do, typically depends on what i believes j thinks. Therefore, observing the previous moves by j in partial history x make i update his beliefs about what j thinks.

Since we have not yet modeled the space of possible beliefs of co-players, we start with an abstract representation of conditional beliefs where a player has beliefs over an uncertainty space of the form $Z \times Y$, an event is a (measurable) subset $E \subseteq Z \times Y$; in particular, the event that information set h is reached is $Z(h) \times Y$. The interpretation of Y is that it represents the set of possible belief systems of i's co-players.

Definition 3 *Fix a compact metrizable space Y. A **conditional probability system** (CPS) on $(Z \times Y, \bar{H}_i)$ is an array of probability measures $\mu = (\mu(\cdot|Z(h) \times Y))_{h \in \bar{H}_i} \in [\Delta(Z \times Y)]^{\bar{H}_i}$ such that*

(1) (knowledge implies probability-one belief) *for every* $h \in \bar{H}_i$,

$$\mu(Z(h) \times Y | Z(h) \times Y) = 1;$$

(2) (chain rule) *for every* $g, h \in \bar{H}_i$ *with* $h \prec g$ *and measurable* $E \subseteq Z(g) \times Y$

$$\mu(E|Z(h) \times Y) = \mu(E|Z(g) \times Y)\mu(Z(g) \times Y|Z(h) \times Y).$$

Note that condition (2) in Definition 3 makes sense because $h \prec g$ implies $Z(g) \subseteq Z(h)$. Hence this condition expresses the rule of conditional probabilities applied to the chain of events $E \subseteq Z(g) \times Y \subseteq Z(h) \times Y$. The set of **first-order** CPSs on $(Z, \bar{H}_i)$ is defined analogously. Therefore we omit a formal definition.[9] A CPS μ on $(Z \times Y, \bar{H}_i)$ induces a first-order CPS α on $(Z, \bar{H}_i)$ in the obvious way: for every $h \in \bar{H}_i$ and $z \in Z(h)$

$$\alpha(z|Z(h)) = \mu(\{z\} \times Y|Z(h) \times Y).$$

To ease notation, we will write $\mu(\cdot|h)$ instead of $\mu(\cdot|Z(h) \times Y)$ whenever this causes no confusion; similarly, for a first-order CPS $\alpha \in \Delta^{\bar{H}_i}(Z)$, we will write $\alpha(x|h)$ and $\alpha(h'|h)$ instead of, respectively, $\alpha(Z(x)|Z(h))$ and $\alpha(Z(h')|Z(h))$.

Definition 3 may represent the system of beliefs of an external observer that obtains the same information as player i. But, arguably, reasonable beliefs of player i should satisfy a further condition: *At each stage* k, *player* i *takes the belief systems of his co-players and their stage-k choices as given; therefore, his beliefs about them are independent of his choice.* We express this property by means of probabilities conditional on histories (rather than information sets), and marginal probabilities of actions, action profiles and events in Y. Therefore, for any CPS μ on $(Z \times Y, \bar{H}_i)$, we introduce a simplified notation for marginal conditional probabilities summarized by the following table, where $h \in \bar{H}_i$, $E_Y \subseteq Y$ is measurable, $x \in h$ is such that

[9] Indeed, $\Delta^{\bar{H}_i}(Z)$ is isomorphic to $\Delta^{\bar{H}_i}(Z \times \{*\})$, where $Y = \{*\}$ is an arbitrary singleton.

$\mu(Z(x) \times Y|h) > 0$, $a_i \in A_i(h)$ and $\mathbf{a}_{-i} \in \mathbf{A}_{-i}(x)$:

Notation	**Definition**
$\mu(x\|h)$	$\mu(Z(x) \times Y\|h)$
$\mu(E_Y\|h)$	$\mu(Z \times E_Y\|h)$
$\mu(a_i\|h)$	$\mu(Z(h, a_i) \times Y\|h)$
$\mu(a_i, \mathbf{a}_{-i}\|x)$	$\mu(Z(x, (a_i, \mathbf{a}_{-i})) \times Y\|h)/\mu(Z(x) \times Y\|h)$
$\mu(a_i\|x)$	$\sum_{\mathbf{b}_{-i} \in \mathbf{A}_{-i}(x)} \mu(a_i, \mathbf{b}_{-i}\|x)$
$\mu(\mathbf{a}_{-i}\|x)$	$\sum_{b_i \in A_i(x)} \mu(b_i, \mathbf{a}_{-i}\|x)$

Table 2. Marginal conditional probabilities

Definition 4 *A CPS μ on $(Z \times Y, \bar{H}_i)$ satisfies* own-action independence *(OAI) if, for every $h \in H_i$, the following conditions hold:*[10]
(1) for every $a_i \in A_i(h)$ and measurable $E_Y \subseteq Y$

$$\mu(E_Y|h) = \mu(E_Y|h, a_i);$$

(2) for every $x \in h$ such that $\mu(x|h) > 0$, $a_i \in A_i(x)$ and $\mathbf{a}_{-i} \in \mathbf{A}_{-i}(x)$,

$$\mu(a_i|x) = \mu(a_i|h),$$

$$\mu((a_i, \mathbf{a}_{-i})|x) = \mu(a_i|x) \times \mu(\mathbf{a}_{-i}|x);$$

The set of CPSs of i satisfying OAI is denoted by $\Delta_i^{\bar{H}_i}(Z \times Y)$. Similarly, the set of first-order CPSs of i satisfying OAI is denoted by $\Delta_i^{\bar{H}_i}(Z)$.

OAI implies that a CPS of player i is made of two independent parts, i's beliefs about his own behavior and his beliefs about the co-players' behavior and beliefs.[11]

[10] Condition (2) holds vacuously for terminal information sets.

[11] These are *minimal* own-action independence properties. For example, they do not imply that if j plays after i without observing anything about i's move then the probability of j's action conditional on i's earlier action a_i is independent of a_i (see the discussion in Battigalli *et al.*, 2013). It is as if i is allowed to believe that j may observe a signal correlated with his action. This is prevented by the OAI condition above only for simultaneous actions. In sum, the OAI property expressed above is just the minimal property required

Remark 5 *A CPS μ on $(Z \times Y, \bar{H}_i)$ satisfies condition (2) of Definition 4 if and only if there is a unique behavioral strategy $\sigma_i = (\sigma_i(\cdot|h))_{h \in H_i^D} \in \Sigma_i := \times_{h \in H_i^D} \Delta(A_i(h))$ such that, for every $h \in H_i^D$, $x \in h$ with $\mu(x|h) > 0$, $a_i \in A_i(x)$ and $\mathbf{a}_{-i} \in \mathbf{A}_{-i}(x)$,*

$$\mu(a_i|x) = \sigma_i(a_i|h),$$

$$\mu((a_i, \mathbf{a}_{-i})|x) = \sigma_i(a_i|h) \times \mu(\mathbf{a}_{-i}|x).$$

In game forms with observable actions (that is, with H_i isomorphic to X) and two players, a CPS μ on $(Z \times Y, \bar{H}_i)$ satisfies condition (2) if and only if there is a unique pair of behavioral strategies (σ_i, σ_{-i}) such that, for every $x \in X \backslash Z$, $a_i \in A_i(x)$ and $a_{-i} \in A_{-i}(x)$,

$$\mu((a_i, a_{-i})|x) = \sigma_i(a_i|x) \times \sigma_{-i}(a_{-i}|x).$$

We interpret the behavioral strategy given by condition (2) of Definition 4 as the **plan** of player i.

As a technical aside, we note that the set of CPS's with the OAI property is compact metrizable, just like Y:[12]

Lemma 6 $\Delta_i^{\bar{H}_i}(S \times Y)$ *is compact metrizable.*

With this result, we can recursively define **beliefs of order** m:[13] Henceforth, $j \neq i$ means $j \in I \backslash \{i\}$. For every $i \in I$ and $m \in \mathbb{N}$, define the set of m-order CPSs of i as

$$Y_i^m := \Delta_i^{\bar{H}_i}(Z \times (\times_{j \neq i} Y_j^{m-1})), \tag{1}$$

to make sense of expected utility maximization. The remaining condition to be added is:

$$x', x'' \in \bigcap_{j \neq i} H_j(x') \cap H_j(x'') \Rightarrow \mu(\mathbf{a}_{-i}|x') = \mu(\mathbf{a}_{-i}|x'').$$

for all $x', x'' \in X \backslash Z$ and $\mathbf{a}_{-i} \in A_{-i}(x')$ such that $\mu(\mathbf{a}_{-i}|x')$ and $\mu(\mathbf{a}_{-i}|x'')$ are well defined. It says that i does not think he can affect the probability of future opponents' actions unless the opponents can observe that somebody (i actually) changed action. The equilibrium concepts introduced later contain consistency conditions that imply OAI and the condition above, and also rule out some other correlations allowed by OAI.

[12] $\Delta(Y)$ is endowed with the weak convergence topology, which has a compatible metric (the Euclidean metric when Y is finite). Product spaces are endowed with the product topology. Subsets of topological spaces are endowed with the relative topology.

[13] This is not the usual construction of belief hierarchies. Since we consider only finitely many orders of belief, we can use a simplified construction.

where

$$Y_i^1 := \Delta_i^{\bar{H}_i}(Z).$$

By Lemma 6, each Y_j^1 ($j \in I$) is compact metrizable, and if each Y_j^{m-1} ($j \in I$) is compact metrizable, then also each $Y_i^m := \Delta_i^{\bar{H}_i}(Z \times (\times_{j\neq i} Y_j^{m-1}))$ ($i \in I$) is compact metrizable; therefore the recursive definition is well-posed.

For every $m \geq 2$, $1 \leq k < m$ and $\mu_i^m \in Y_i^m$, we let $(\mu_i^m)^k$ denote the k-**order CPS derived from** μ_i^m under the assumption that players' hierarchies of beliefs are coherent. Formally, also the definition of such lower-order beliefs is recursive: For all $i \in I$, $m \geq 2$ and $\mu_i^m \in Y_i^m$, let $(\mu_i^m)^1 := (\mathrm{marg}_Z \mu_i^m(\cdot|h))_{h\in\bar{H}_i}$. It is routine to show that $(\mu_i^m)^1 \in Y_i^1$. Now suppose that $(\mu_j^{m-1})^k \in Y_j^k$ has been defined for every j and $1 \leq k < m-1$; then, for all $i \in I$, $\bar{h} \in \bar{H}_i$, $z \in Z(\bar{h})$ and measurable $E_{-i}^k \subseteq \times_{j\neq i} Y_j^k$, let

$$(\mu_i^m)^{k+1}(\{z\}\times E_{-i}^k|\bar{h}) := \mu_i^m \left(\{z\} \times \{(\mu_j^{m-1})_{j\neq i} \in \times_{j\neq i} Y_j^{m-1} : ((\mu_j^{m-1})^k)_{j\neq i} \in E_{-i}^k\}|\bar{h}\right)$$

and then extend by countable additivity to the sigma-algebra on $Z \times Y_{-i}^k$. It is also convenient to define $(\mu_i^m)^m := \mu_i^m$. It can be shown that $k \leq \ell \leq m$ implies $((\mu_i^m)^\ell)^k = (\mu_i^m)^k$.

In this paper we focus on **beliefs** of the **first**, **second** and **third** order, respectively denoted by the first three letters of the Greek alphabet:

$$\begin{aligned}
\alpha_i &\in Y_i^1 := \Delta_i^{\bar{H}_i}(Z), \\
\beta_i &\in Y_i^2 := \Delta_i^{\bar{H}_i}(Z \times (\times_{j\neq i} Y_j^1)), \\
\gamma_i &\in Y_i^3 := \Delta_i^{\bar{H}_i}(Z \times (\times_{j\neq i} Y_j^2)).
\end{aligned}$$

Higher-order beliefs implicitly determine lower-order beliefs: when we consider α_i, β_i and γ_i at the same time, it is implicitly understood that α_i is the first-order belief implied by β_i, that is $\alpha_i = (\beta_i)^1$, and similarly β_i is the second-order belief derived from γ_i, $\beta_i = (\gamma_i)^2$; hence $\alpha_i = ((\gamma_i)^2)^1 = (\gamma_i)^1$. Higher-order beliefs are important in the analysis of anger from blame, not in the analysis of simple anger.

Finally, we introduce notation for **conditional expectations**. Fix a CPS $\mu_i \in \Delta_i^{\bar{H}_i}(Z \times Y)$. For any bounded and measurable, real-valued function $\psi : Z \times Y \to \mathbb{R}$ and any $\bar{h} \in \bar{H}_i$, we let

$$\mathbb{E}[\psi|\bar{h};\mu_i] := \int_{Z(\bar{h})\times Y} \psi(z,y)\mu_i(z \times \mathrm{d}y|\bar{h})$$

denote the expected value of ψ conditional on $\bar{h}$ given CPS μ_i. If ψ depends only on z, and $\alpha_i = (\alpha_i(\cdot|\bar{h}))_{\bar{h}\in\bar{H}_i} = (\text{marg}_Z\mu_i(\cdot|\bar{h}))_{\bar{h}\in\bar{H}_i}$ is the first-order CPS, then we obtain

$$\mathbb{E}[\psi|\bar{h};\mu_i] = \mathbb{E}[\psi|\bar{h};\alpha_i] = \sum_{z\in Z(\bar{h})} \psi(z)\alpha_i(z|\bar{h}).$$

3 Anatomy of belief-dependent anger

In this section we describe action tendencies determined by simple anger (3.1) and anger from blame (3.2). Blame may be caused by observed behavior of co-players (3.2.1) or inferences about their intentions (3.2.2).

3.1 Simple anger

Our first model – which we call *simple anger* – may be seen as a representation of the classic "frustration-aggression hypothesis" of Dollard et al. (1939). A player thus motivated may behave aggressively not only when another person is responsible for some adverse event, but also after frustrating events that are due to third parties or even to nature. It formalizes Frijda's (1993, p. 362) statement that "many experiences or responses of anger...are elicited by events that involve no blameworthy action."

Fix a first-order CPS $\alpha_i \in Y_i^1$ and a date-t information set $\bar{h}_{i,t} \in \bar{H}_i$ ($\bar{h}_t \subseteq (A^I)^t$). The expected material payoff of i at date t (either the end of period t, or the beginning of period $t+1$) conditional on $\bar{h}_{i,t}$ is $\mathbb{E}[\pi_i|\bar{h}_{i,t};\alpha_i]$. Fix $h_{i,t} \in H_i$, the information of player i at the beginning of period $t+1$. If i chooses $a_i \in A_i(h_{i,t})$, his expected material payoff at the end of period $t+1$ is then $\mathbb{E}[\pi_i|(h_{i,t},a_i);\alpha_i]$. Now let

$$\Pi_i^*(h_{i,t};\alpha_i) := \begin{cases} \max_{a_i\in A_i(h_{i,t})} \mathbb{E}[\pi_i|(h_{i,t},a_i);\alpha_i], & \text{if } h_{i,t} \in H_i^D, \\ \mathbb{E}[\pi_i|h_{i,t};\alpha_i], & \text{if } h_{i,t} \in H_i \backslash H_i^D. \end{cases} \quad (2)$$

(The second part of eq. (2) is the relevant expected payoff when $h_{i,t}$ is a terminal information set.)

Fix two consecutive information sets $h_{i,t-1}, h_{i,t} \in H_i$; they give i's information at the beginning of period t and $t+1$ respectively. Note that, by perfect recall, $h_{i,t-1}$ is uniquely determined by $h_{i,t}$. We assume that *the extent of* i's **frustration in period** $t+1$ *is given by the gap, if positive,* *between his expected payoff at date* $t-1$ (beginning of period t) *and the best expected payoff he can choose to obtain at date* $t+1$ (at the end of period $t+1$, before learning new information about the other players):

$$\mathrm{F}_i(h_{i,t};\alpha_i) = \max\{0, \mathbb{E}[\pi_i|h_{i,t-1};\alpha_i] - \Pi_i^*(h_{i,t};\alpha_i)\}. \tag{3}$$

Notice that we only make explicit the dependence of F_i on $h_{i,t}$ because $h_{i,t-1}$ is implicitly determined by $h_{i,t}$: with our notation, we are considering $h_{i,t-1} = h_{i,t}^{\leq t-1}$; also, when $h_{i,t}$ is a terminal information set, $\mathrm{F}_i(h_{i,t};\alpha_i)$ is the frustration experienced at the end of the game, that is, in the period after the last stage of the game. Since there cannot be any frustration in the first period, we adopt the convention

$$\mathrm{F}_i(\{\varnothing\};\alpha_i) = 0.$$

Here is the *intuition* behind this formulation: Frustration is related to lack of full control over gain and losses, in particular, to the inability to eliminate perceived losses. Therefore, in order to define frustration in period $t+1$, we look at the gap between the previously expected material payoff, $\Pi_i(h_{i,t-1};\alpha_i)$, and the expected payoff that can be achieved by choosing a particular action a_i in period $t+1$, $\Pi_i((h_{i,t},a_i);\alpha_i)$ (note that $(h_{i,t},a_i)$ is a date $(t+1)$ information set, just before the uncertainty about other players partially resolves). Decision maker i can partially control this gap, which could be negative, by choosing $a_i \in A_i(h_{i,t})$. The gap, or perceived loss, which is *beyond* i's control is the difference, if positive,

$$\mathbb{E}[\pi_i|h_{i,t-1};\alpha_i] - \max_{a_i \in A_i(h_{i,t})} \mathbb{E}[\pi_i|(h_{i,t},a_i);\alpha_i] =$$

$$= \mathbb{E}[\pi_i|h_{i,t-1};\alpha_i] - \Pi_i^*(h_{i,t};\alpha_i).$$

We can compare this to a simpler formulation: the extent of i's frustration is just given by the difference, if positive, between $\mathbb{E}[\pi_i|h_{i,t-1};\alpha_i]$ and $\mathbb{E}[\pi_i|h_{i,t};\alpha_i]$, which is larger than the difference above. This latter difference simply measures the extent of i's disappointment, or diminished expectations. We propose that *diminished expectations, which depend on the planned action for the current period, do not by themselves cause frustration, it is the unavoidable perceived loss that causes frustration, and the related action tendency of aggression.* This distinction is illustrated by the following example.

Example 7 *Suppose that, in the game form of Figure C, Bob expects L and plans to choose ℓ if R. Such expectation and plan are given by the first-order CPS α_b such that $\alpha_b(L|\varnothing) = 1$ and $\alpha_b(\ell|R) = 1$. Given such beliefs, the disappointment of Bob after he observes R is*

$$\mathbb{E}[\pi_b|\varnothing;\alpha_b] - \mathbb{E}[\pi_b|R;\alpha_b] = 1 - 0 = 1.$$

But Bob can avoid the loss by choosing r:

$$\mathbb{E}[\pi_b|\varnothing;\alpha_b] - \Pi_b^*(R;\alpha_b) = 1 - \max\{\pi_b(R,\ell), \pi_b(R,r)\} = 1 - 2 = -1.$$

According to our assumption, the extent of Bob's frustration in the second stage is therefore zero:

$$F_b(R;\alpha_b) = \max\{0, \mathbb{E}[\pi_b|\varnothing;\alpha_b] - \Pi_b^*(R;\alpha_b)\} = 0.$$

Thus, if anger and aggressive behavior are caused by frustration as defined here, Bob would not be aggressive R. If instead anger were caused by mere disappoinment, Bob could be angry and after R, and choose the aggressive action ℓ.

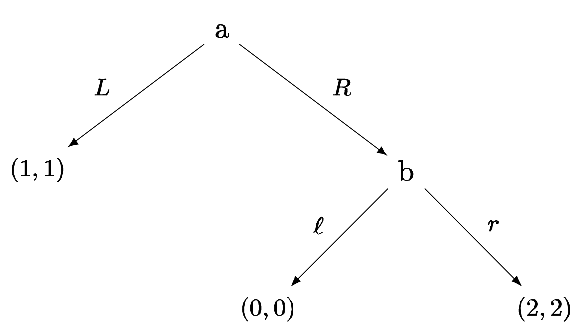


Figure C. *Sequential coordination game.*

Besides this difference between disappointment and our definition of frustration, there is an even more important difference in the assumed impact on behavior. In models of disappointment avoidance, it is the anticipation of disappointment that plays a role, inducing behavior that trades off expected material gains with expected disappointment. When disappointment occurs, it is – in a sense – too late to take a remedial action. The frustration experienced in stage k instead generates the action tendency of aggression in stage k and possibly in later stages as well. In principle, also the anticipation of frustration may play a role, inducing behavior that tends to avoid future frustration. But here neglet this effect for the sake of simplicity.

Our simplest assumption about the action tendency generated by frustration is that, when i is frustrated, he alleviates his frustration, which may have cumulated over time, by taking it on other players and decreasing their material payoff. We call this action tendency **simple anger (SA)**. In order to represent action tendencies, we assume that, at each stage, player i maximizes a "decision utility" function, which combines his expected material payoff and a psychological component. The impact of frustration felt in the distant past is reduced according to an exponentially decreasing weight.

Definition 8 *The **SA-decision utility** of choosing action a_i in period $t+1$ at a non-terminal information set $h_{i,t} \in H_i^D$ given CPS α_i is*

$$u_i^{SA}(h_{i,t}, a_i; \alpha_i) =$$

$$\mathbb{E}[\pi_i|(h_{i,t}, a_i); \alpha_i] - d_i^{SA} \left(\sum_{k=1}^{t} (r_i^{SA})^{t-k} \mathrm{F}_i(h_{i,t}^{\leq k}; \alpha_i) \right) \left(\sum_{j \neq i} \mathbb{E}[\pi_j|(h_{i,t}, a_i); \alpha_i] \right), \tag{4}$$

with $d_i^{SA} \geq 0$, $r_i^{SA} \in [0, 1]$.

Parameter d_i^{SA} is the weight of present action tendencies given by the desire to alleviate frustration by reducing the expected material payoff of other players. Parameter r_i^{SA} gives the exponential rate at which past frustration fades, when $r_i^{SA} = 0$ only present frustration matters, when $r_i^{SA} = 1$ past frustration matters as much as present frustration.

Example 9 *Consider the Ultimatum Minigame of Figure A. If Bob expects f at $t = 0$ (1st-order belief $\alpha_b(f|\varnothing)$ close to one), after g he is frustrated and trades off his material payoff π_b with the desire to hurt Ann. Indeed, if $\alpha_b(f|\varnothing) = 1$, the extent of Bob's frustration after g is*

$$\mathrm{F}_b(g; \alpha_b) = \max\{0, (2 - \max\{0, 1\})\} = 1.$$

Example 10 *Now suppose that Bob has to wait one period before he can respond to Ann's offer, as in Figure D.*

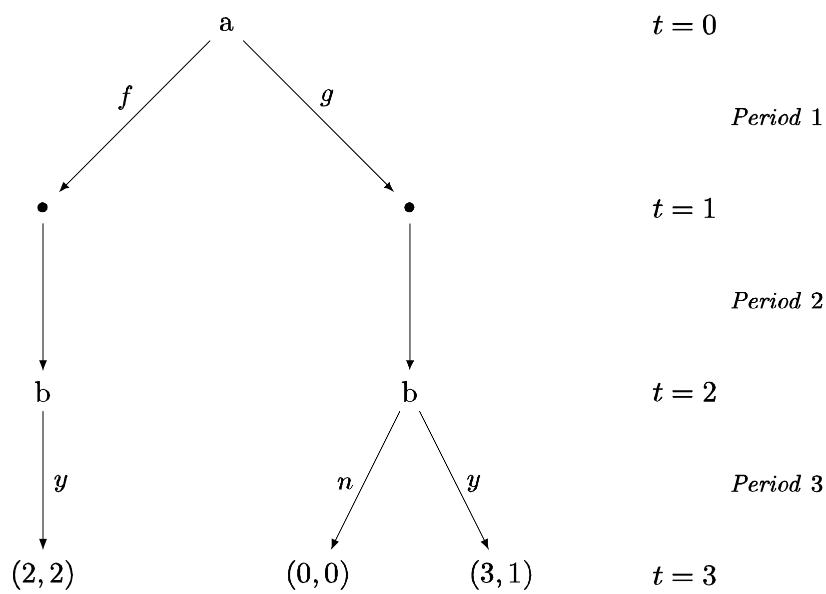


Figure D. *Ultimatum Minigame with delayed reply.*

If Bob initially expects the fair offer f and gets the greedy offer g, he is frustrated at beginning of the second period, but there is no further frustration in the third period because his expected payoff cannot change. Therefore the cumulated frustration at the beginning of the third period is only

$$r_b^{SA} \times \mathrm{F}_b(g; \alpha_b) + 1 \times \mathrm{F}_b((g, *); \alpha_b) = r_b^{SA} + 0 = r_b^{SA}.$$

If Bob "cools off" fast, i.e., if r_b^{SA} is small, then he accepts the greedy offer.

In the Ultimatum Minigame the cause of a player's frustration (given his beliefs) is the other player. But simple anger can also cause aggressive behavior against players who cannot have caused any frustration, e.g., inactive players. Simple anger can even be caused by one's inability to follow his own plan.

Example 11 *(Inspired by Frijda, 1993) Andy the handyman uses a hammer. He has an apprentice, Bob, that has no payoff-relevant action. Chance determines whether it is a good day of a bad day. In a bad day Andy hammers* *his thumb and then can either take it on Bob or not, if he does, he further disrupts the production process. See Figure E.*

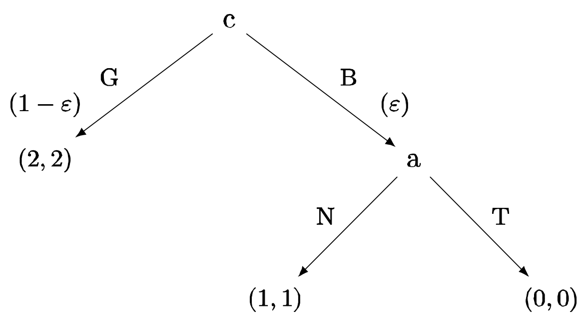


Figure E. *Hammering one's thumb.*

Assuming that $\alpha_a(B|\varnothing) = \varepsilon < \frac{1}{2}$, *the extent of Andy's frustration in a bad day is*

$$\mathrm{F}_a(B;\alpha_a) = \max\{0, (2(1-\varepsilon) + \varepsilon\alpha_a(N|B) - \max\{0,1\})\} = 2(1-\varepsilon)+\varepsilon\alpha_a(N|B)-1 > 0.$$

If Andy is sufficiently prone to simple anger (d_a^{SA} *sufficiently high), then he takes it on Bob in a bad day.*

Example 12 *Andy is the only active player in the game form depicted in Figure F (a modification of Figure E), Bob is an interested observer:*

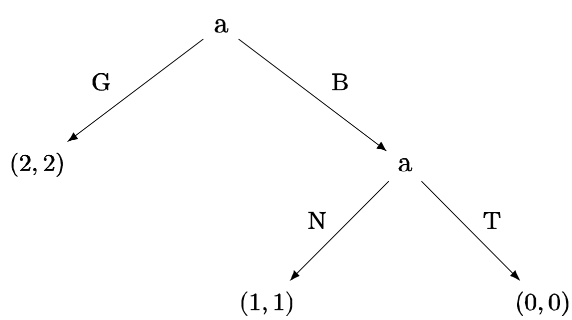


Figure F. *Game with an interested observer.*

The only reasonable action for Andy is the good one, G, hence he plans L ($\alpha_a(G|\varnothing) = 1$*) and initially expects \$2. Choosing B by mistake would frustrate Andy:*

$$\mathrm{F}_a(B;\alpha_a) = \max\{0, (2 - \max\{0,1\})\} = 1.$$

Again, if Andy is sufficiently prone to simple anger ($d_a^{SA} > 1$*), then he would give up \$1 to take it on Bob.*

3.2 Anger from blame

Much of the psychology literature associates anger with other-responsibility or blame for negative events. For example, Averill (1983, p. 1150) says "More than anything else, anger is an attribution of blame."[14] With anger from blame, responsibility matters. Players are never angry at coplayers whom they judge to be blameless. We offer two models which take this up, in different ways.

For each history $x = (\mathbf{a}^1, ..., \mathbf{a}^{t-1}, \mathbf{a}^t)$, let $x/_{t,j}b_j$ denote the modified history obtained by replacing stage-t action of player j with action $b_j \in A_j(x^{\leq t-1})$:

$$x/_{t,j}b_j = (\mathbf{a}^1, ..., \mathbf{a}^{t-1}, (\mathbf{a}^t_{-j}, b_j)).$$

With this, the extent of i's frustration in period $t+1$ to be "apportioned" to j's t-period action in history x is

$$\mathrm{F}_{ji}(x;\alpha_i) := \mathrm{F}_i(x;\alpha_i) - \min_{b_j \in A_j(x^{\leq t-1})} \mathrm{F}_i(x/_{t,j}b_j;\alpha_i) \tag{5}$$

(note that $\mathrm{F}_{ji}(x;\alpha_i) \geq 0$ by definition).

Note, even in a two-person game form it is possible that $\mathrm{F}_{ji}(x;\alpha_i) < \mathrm{F}_i(x;\alpha_i)$ because i's frustration may have been caused by himself or by chance. In particular, when j is a passive player (as in the game forms of Figures D and E) $A_j(x^{\leq t-1})$ is a singleton, hence $\min_{b_j \in A_j(x^{\leq t-1})} \mathrm{F}_i(x/_{t,j}b_j;\alpha_i) = \mathrm{F}_i(x;\alpha_i)$ and $\mathrm{F}_{ji}(x;\alpha_i) = 0$. But in some two-person games $\mathrm{F}_{ji}(x;\alpha_i) = \mathrm{F}_i(x;\alpha_i)$ at each history x where i is active.

Remark 13 *In a leader-follower game, a follower apportions all his frustration on the leader: let j be the leader, then for every $i \neq j$, α_i and $a \in A_j(\varnothing)$, $\mathrm{F}_{ji}(a;\alpha_i) = \mathrm{F}_i(a;\alpha_i)$.*

For example, in an Ultimatum Game (such as the game form of Figure A) the responder's (player i) frustration is entirely due to the proposer's (player j) non-fully expected offer a, hence $\mathrm{F}_{ji}(a;\alpha_i) = \mathrm{F}_i(a;\alpha_i)$.

3.2.1 Blaming behavior

Our simplest theory of anger from blame is that in each period player i blames his co-players for his frustration, apportioning blame according to

[14] See also See also Smith & Ellsworth (1985), Ortony, Clure & Collins (1988), Lazarus (1991), Weiner (1995), and Lerner & Keltner (2000).

eq. (5), and he is angry at co-players that caused his frustration *irrespective of their intentions*. The extent of i's blame on j depends on i's first-order beliefs α_i for two reasons: first, frustration depends on α_i; second, in game forms without observable actions, the likelihood of past imperfectly observed actions of co-players is given by α_i:

$$\mathrm{B}_{ij}(h_{i,t};\alpha_i) := \mathbb{E}[\mathrm{F}_{ji}|h_{i,t};\alpha_i] = \sum_{x\in h_{i,t}} \mathrm{F}_{ji}(x;\alpha_i)\alpha_i(x|h_{i,t})$$

Anger from blame may build up over time. At any point in time, an angry player i has the tendency to decrease the material payoff of the co-players he blame. We capture this action tendency with a decision-utility function that trades off expected material payoff with harm inflicted on blameworthy co-players:

Definition 14 *The decision utility due to anger from blaming behavior, or* ***ABB-decision utility*** *of action a_i at a non-terminal information set $h_{i,t} \in H_i^D$ given first-order CPS α_i is*

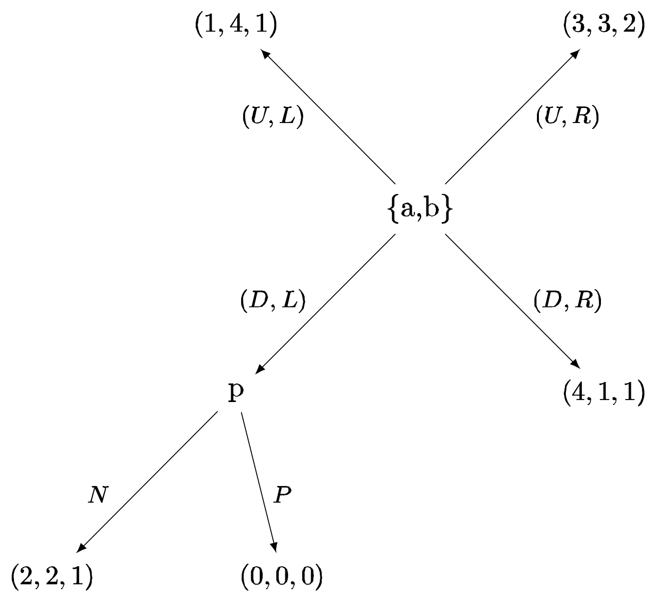
$$u_i^{ABB}(h_{i,t},a_i;\alpha_i) =$$

$$\mathbb{E}[\pi_i|(h_{i,t},a_i);\alpha_i] - d_i^{ABB}\sum_{j\neq i}\left(\sum_{k=1}^{t}(r_i^{ABB})^{t-k}\mathrm{B}_{ij}(h_{i,t}^{\leq k};\alpha_i)\right)\mathbb{E}[\pi_j|(h_{i,t},a_i);\alpha_i], \tag{6}$$

As in eq. (4), parameter d_i^{ABB} is the weight of present action tendencies due to anger from blame. When $d_i^{ABB}=0$, player i maximizes his expected material payoff. As before, parameter r_i^{ABB} gives the exponential rate at which past anger from blame fades, when $r_i^{ABB}=0$ only present frustration matters, when $r_i^{ABB}=1$ past frustration matters as much as present frustration.

For example, consider the game form of Figure B and suppose that Penny initially expects perfect coordination between Ann and Bob (it does not matter whether (U,L) or (D,r)) and then observes the anti-coordination pair (D,L); then eq. (6) implies that Penny blames Ann and Bob equally because each one of them could have unilaterally avoided this unexpected bad outcome, and hence she is equally angry toward both. We will go back to this example in the analysis of equilibrium behavior. In the following example we have just the opposite situation: nobody can be individually blamed:

Example 15 *Ann and Bob play a kind of Prisoners' Dilemma game, Penny stands to gain if and only if they both cooperate (action pair (U, R)) and has the opportunity to punish if and only if they both defect (action pair (D, L), see Figure G).*



if N	*L*	*R*
U	*1,4,1*	*3,3,2*
D	*2,2,1*	*4,1,1*

if P	*L*	*R*
U	*1,4,1*	*3,3,2*
D	*0,0,0*	*4,1,1*

Figure G. *Prisoner's Dilemma with punishment: multistage and strategic form*

According to eq. (6), if Penny expects cooperation and then observes that both defected she is not angry with either Ann or Bob and hence has no tendency to punish.

3.2.2 Blaming intentions

Our second theory of anger from blame is that i's anger toward j is driven by how much j intended to frustrate i, where such intentions are assessed according to i's higher-order beliefs. To model this, we first look at how much player j expects to frustrate i. Using his second-order beliefs $\beta_j \in Y_j^2 := \Delta^{\bar{H}_j}\left(Z \times (\times_{k \neq j} Y_j^1\right)$, player j can compute the expectation $\mathbb{E}[\mathrm{F}_{ji}|h_{j,t-1};\beta_j]$ of the period-$(t+1)$ frustration he is going to inflict on i with his period t

choice conditional on $h_{j,t-1}$:

$$\bar{\mathrm{F}}_{ji}(h_{j,t-1};\beta_j) := \sum_{x\in h_{j,t-1}} \sum_{\mathbf{a}\in\mathbf{A}(x)} \int_{Y_i^1} \mathrm{F}_{ji}((x,\mathbf{a});\alpha_i)\beta_j(Z(x,\mathbf{a})\times \mathrm{d}\alpha_i \times Y^1_{-ji}|h_{j,t-1}). \tag{7}$$

Then, using his third-order beliefs $\gamma_i \in Y_i^3 = \Delta^{\bar{H}_i}(Z \times (\times_{k\neq i} Y_k^2)$, player i can compute the expectation of this expectation (held by j in the previous period) conditional on $h_{i,t}$, which is how much i blames j for the intention to frustrate him

$$\bar{\mathrm{B}}_{ij}(h_{i,t};\gamma_i) := \sum_{x\in h_{i,t}} \int_{Y_j^2} \bar{\mathrm{F}}_{ji}(H_j(x^{\leq t-1});\beta_j)\gamma_i(Z(x)\times \mathrm{d}\beta_j \times Y^2_{-ij}|h_{i,t}). \tag{8}$$

When i is frustrated and thinks that his co-players intended (planned) to frustrate him, he blames them and becomes angry. Such anger from blame may build up over time. As in the case of blaming behavior, an angry player i has the tendency to decrease the material payoff of the co-players he blame, and this action tendency is captured by a decision-utility function that trades off expected experience utility with harm inflicted on blameworthy co-players:

Definition 16 *The decision utility due to anger from blaming intentions, or* ***ABI-decision utility*** *of action a_i at a non-terminal information set $h_{i,t} \in H_i^D$ given third-order CPS γ_i is*

$$u_i^{ABI}(h_{i,t}, a_i;\gamma_i) = \mathbb{E}[\pi_i|(h_{i,t},a_i);\alpha_i] +$$

$$-d_i^{ABI}\sum_{k=1}^{t}(r_i^{ABI})^{t-k}\mathbf{1}_{\{\mathrm{F}_i(h_{i,t}^{\leq k};\alpha_i)>0\}}(h_{i,t}^{\leq k};\alpha_i)\left(\sum_{j\neq i}\bar{\mathrm{B}}_{ij}(h_{i,t}^{\leq k};\gamma_i)\right)\mathbb{E}[\pi_j|(h_{i,t},a_i);\alpha_i], \tag{9}$$

where $\mathbf{1}_{\{\cdot\}}(\cdot)$ is the indicator function, $d_i^{ABI}, r_i^{ABI} \in [0,1]$, and $\alpha_i = \gamma_i^1$ is the first-order belief derived from γ_i.

4 Solution concepts

In this section we define solution concepts for general psychological games with m-order beliefs, which can be applied to games with simple anger and with anger from blame. Recall that $H_i^D := \{h \in H_i : h \cap Z = \emptyset\}$ is the collection of non terminal information sets of i, that is, the information sets containing decision nodes/histories.

Definition 17 *An ℓ-**order psychological game** is a structure*

$$\Gamma^\ell = (I, A, X, \sigma_c, (H_i, u_i^\ell)_{i\in I}),$$

where $(I, A, X, \sigma_c, (H_i)_{i\in I})$ is a multistage extensive form and, for each $i \in I$,

$$u_i^\ell = (u_i^\ell(h, \cdot, \cdot) : A_i(h) \times Y_i^\ell \to \mathbb{R})_{h\in H_i^D}$$

is a profile of decision-utility functions.

For example, if players' actions tendencies are determined by simple anger (eq. 4), or anger from blaming behavior (eq. 6), then we have a 1^{st}-order psychological game; if instead action tendencies are determined by anger from blaming behavior (eq. 9), then we have a 3^{rd}-order psychological game.

We define equilibrium beliefs of order m for psychological games of order $\ell \leq m$. The reason is twofold. First, we want to compare equilibrium beliefs of order m for psychological games of different orders.[15] Second, we introduce restrictions on off-equilibrium-path updated beliefs that require $m > \ell$.

4.1 Sequential equilibrium

For every point y in a metrizable space Y, we let δ_y denote the Dirac measure that assigns probability one to y. Also, for every behavioral strategy profile σ and history x, we let $\mathbb{P}_\sigma(x)$ denote the probability of x given by σ.[16] We say that σ is **fully randomized** if $\sigma_i(a_i|h_i) > 0$ for all $i \in I$, $h_i \in H_i^D$ and $a_i \in A_i(h_i)$. Of course, if σ is fully randomized $\mathbb{P}_\sigma(x) > 0$ for every history x. Finally, recall that we use obvious abbreviations such as $\mu_i^m(x|h)$ for marginal conditional beliefs, and that - for each $k \leq m$ - $(\mu_i^m)^k$ is the k-order belief derived from m-order belief μ_i^m.

We build on Battigalli & Dufwenberg (2009) to extend Kreps & Wilson's (1982) definition of sequential equilibrium assessment to psychological games.

[15] Geanakoplos *et al.* (1989) and Battigalli & Dufwenberg (2009) define equilibria by means of infinite hierarchies of beliefs, i.e. with $m = \infty$. In this more applied paper we can limit our attention to finite-order beliefs.

[16] If $x = (\mathbf{a}^1, ..., \mathbf{a}^\ell)$, then

$$\mathbb{P}_\sigma(x) := \prod_{t=1}^{\ell} \prod_{j\in I} \sigma_j(a_j^t | H_j(x^{\leq t-1})).$$

An m-order **assessment** is a profile $(\sigma, \mu^m) = (\sigma_i, \mu_i^m)_{i \in I} \in \times_{i \in I} \Sigma_i \times Y_i^m$ such that σ_i is the behavioral strategy induced by CPS μ_i^m (recall Remark 5); (σ, μ^m) is fully randomized if σ is fully randomized.

Definition 18 *An m-order assessment (σ, μ^m) is **consistent** if there is a converging sequence $(\sigma_n, \mu_n^m) \to (\sigma, \mu^m)$ of* fully randomized *assessments such that, for every $i \in I$, $h \in H_i$, $x \in h$, $\mathbf{a} \in \mathbf{A}(x)$ and n*
(1)

$$\mu_{i,n}^m(x|h) = \frac{\mathbb{P}_{\sigma_n}(x)}{\sum_{x' \in h} \mathbb{P}_{\sigma_n}(x')},$$

and

$$\mu_{i,n}^m(\mathbf{a}|x) = \sigma_c(a_c|x) \prod_{j \in I} \sigma_{j,n}(a_j|H_j(x));$$

(2) for every $j \neq i$ and $k = 1, ..., m-1$, $\mathrm{marg}_{Y_j^k}(\mu_{i,n}^m)^{k+1}(\cdot|h) = \delta_{(\mu_{j,n}^m)^k}$.

By continuity, conditions (1)-(2) must also hold at the limit (σ, μ^m) (as long as probabilities conditional on histories are well defined). Part (1) of Definition 18 adapts the definition of consistency of Kreps & Wilson (1982). It implies that players share the same first-order beliefs, hence that the beliefs of each player i about the behavior of a given player j coincide with the plan (behavior strategy) of j. Part (2) requires that players have correct conditional beliefs about the co-players' systems of beliefs (see Battigalli & Dufwenberg, 2009). Consistency implies that the behavior strategy profile σ essentially determines the corresponding m-order belief profile μ^m, with possibly some freedom for beliefs about co-players' past moves at information sets off the σ-path.

Remark 19 *For every behavior strategy profile σ there is an ℓ-order consistent assessment (σ, μ^ℓ). If σ is fully randomized or the extensive form has observable actions there is a* unique *ℓ-order consistent assessment (σ, μ^ℓ). For every ℓ-order consistent assessment (σ, μ^ℓ) there is a unique m-order consistent assessment (σ, μ^m) such that $\mu_i^\ell = (\mu_i^m)^\ell$ for every $i \in I$.*

We say that a player plans rationally if the behavior strategy implied by his first-order CPS assigns positive probability only to actions that maximize his decision utility. Formally:

Definition 20 *Let σ_i be the behavior strategy induced by CPS μ_i^m. Player i* ***plans rationally*** *in (σ_i, μ_i^m) if*

$$\mathrm{Supp}\sigma_i(\cdot|h) \subseteq \arg\max_{a_i \in A_i(h)} u_i^\ell(h, a_i, (\mu_i^m)^\ell)$$

for every $h \in H_i^D$.

Fix a game form Γ, a player $i \in I$ and a CPS $\mu_i \in \Delta_i^{H_i}(Z \times Y)$ (where Y is some metrizable space). Assuming that i cares only about his monetary payoff and is risk neutral, μ_i determines a decision tree $\Gamma(\mu_i)$ and a behavior strategy σ_i.[17] By the One-Shot Deviation Principle σ_i is sequentially optimal in $\Gamma(\mu_i)$ if and only if i plans rationally given μ_i. This observation can be extended to a class of psychological games. But for general psychological games the One-Shot Deviation principle does not hold; a CPS where player i plans rationally is an intrapersonal equilibrium (given beliefs about others) that solves a fixed point problem, and the solution may have to be randomized (see Battigalli & Dufwenberg, 2009 and 2012).

Definition 21 *An m-order assessment $(\sigma_i, \mu_i^m)_{i\in I}$ is a* ***sequential equilibrium*** *(SE) of an ℓ-order psychological game Γ^ℓ ($\ell \leq m$) if it is consistent and each player $i \in I$ plans rationally in (σ_i, μ_i^m). An* ***SE outcome*** *is a distribution $\zeta \in \Delta(Z)$ over terminal histories induced by an SE behavior strategy profile.*

The previous observation about the One-Shot Deviation Principle implies that for games with material preferences our definition of SE is equivalent to the definition of Kreps & Wilson (1982).

Comment According to the SE concept, a player never has doubts about the belief systems, and hence the plans of his co-players. If player i observes an action of j that, according to i's higher-order beliefs, j did not plan to choose, this is explained away as a non-intentional choice (a "mistake"); furthermore, i believes that there will be no other un-planned actions in the continuation (cf. Selten, 1975). This is a definition of equilibrium in beliefs; of course, a prediction is obtained by assuming that each player i actually behaves according to his plan σ_i.

[17] Recall that μ_i satisfies own-action independence. Hence we can turn the game in a decision problem where i and $-i$ never play simultaneously and use μ_i to assing probabilitites to "arcs" $\mathbf{a}_{-i}$ in a unique way. The probabilities of decision nodes x conditional on information sets are given by $\mu_i(x|H_i(x))$.

Example 22 *Consider the Ultimatum Minigame of Figure A with the decision utility function (9). Is the strategy pair* (f,n) *a sequential equilibrium? No, because in any third-order consistent assessment* (f,n,γ_a,γ_b) *Bob's would be certain, upon observing the greedy offer* g*, that this was an unintentional choice made by mistake; therefore he would not blame Ann and would rather deviate and accept* (y) *the greedy offer.*

The consistency condition in the definition of SE implies that the set of m-order sequential equilibria of an ℓ-order psychological game ($\ell \leq m$) is essentially independent of m:

Remark 23 *For all* $\ell, k, m \in \mathbb{N}$*, with* $\ell \leq k \leq m$*, and* $(\sigma_i, \mu_i^m)_{i\in I}$*, the following are equivalent:*
(a) $(\sigma_i, \mu_i^m)_{i\in I}$ *is an SE,*
(b) $(\sigma_i, (\mu_i^m)^k)_{i\in I}$ *is an SE and* $(\sigma_i, \mu_i^m)_{i\in I}$ *is consistent*
(c) $(\mu_i^m)_{i\in I}$ *is the unique* m*-order beliefs profile such that* $(\sigma_i, (\mu_i^m)^k)_{i\in I}$ *is an SE.*

Adapting the proof of existence of Battigalli & Dufwenberg (2009) one can show the following:

Theorem 24 *Every* ℓ*-order psychological game with continuous decision-utility functions has an* m*-order sequential equilibrium for each* $m \geq \ell$*.*

4.2 Equilibrium with perceived intentionality

Fix an m-order CPSs $\mu_i^m \in Y_i^m$ of player i, two consecutive information sets of player i, $h_{t-1}, h_t \in H_i$ and an action $a_i \in A_i(h_{t-1})$ so that $(h_{t-1}, a_i) \supseteq h_t$, that is, a_i is the unique[18] action of i at h_{t-1} leading to h_t. If $\mu_i^m(h_t|(h_{t-1}, a_i)) = 0$, it must be the case that i is "surprised" by what he learns at date t about his *co-players'* behavior. As we said, the sequential equilibrium concept postulates that i always holds correct beliefs about the plans of his co-players. If this is the case, then i's surprise must be due to a deviation of some $j \neq i$ from his plan (recall that j's beliefs about his own behavior are equivalent to a behavior strategy of j), and i indeed infers that some co-player j deviated from his plan, but has no doubt about what this plan may be.

[18] By perfect recall.

If instead i interpreted a surprise as the result of an intentional choice, then he would be forced to conclude that some co-player's plan are different from what he thought. We explore a notion of equilibrium whereby each player interprets deviations from the expected path as due to intentional choices. This forces us to weaken the consistency condition of Definition 18. If we assume that players's initial beliefs about their co-players beliefs (hence their plans) are correct and that deviations from the expected path are perceived as intentional, then we must allow beliefs conditional on deviations to be incorrect. In the following definitions we consider deviations by different co-players separately, checking whether a history x has positive probability under j's strategy σ_j for some σ_{-j}; in this case we say that σ_j **allows** x.[19]

Definition 25 *An m-order assessment (σ, μ^m) is **weakly consistent** if there are $|I|$ converging sequences of fully randomized assessments $(\sigma^i_n, \mu^m_{i,n}) \to (\sigma^i, \mu^m_i)$ $(i \in I)$ such that, for every $i \in I$, $h \in H_i$, $x \in h$, $\mathbf{a} \in \mathbf{A}(x)$ and n*
(1)

$$\mu^m_{i,n}(x|h) = \frac{\mathbb{P}_{\sigma^i_n}(x)}{\sum_{x' \in h} \mathbb{P}_{\sigma^i_n}(x')},$$

and

$$\mu^m_{i,n}(\mathbf{a}|x) = \sigma_c(a_c|x) \prod_{j \in I} \sigma^i_{j,n}(a_j|H_j(x)),$$

(2) for every $j \neq i$, $k = 1, ..., m-1$, if σ^i_j allows x, then

$$\sigma^i_j(\cdot|H_j(x)) = \sigma^j_j(\cdot|H_j(x))$$

$$\mu^m_i(x|h) > 0 \Longrightarrow \mathrm{marg}_{Y^k_j}(\mu^m_i)^{k+1}(\cdot|x) = \delta_{(\mu^m_j)^k}.$$

*An m-order assessment (σ, μ^m) is **weak sequential equilibrium** (WSE) of an ℓ-order psychological game ($\ell \leq m$) if it is weakly consistent and each $i \in I$ plans rationally in (σ_i, μ^m_i).*

As the name suggests, WSE weakens the SE concept. In a WSE a player may change his beliefs about the intentions of a co-player after a deviation of the latter. The next step is a theory of how players change their beliefs about

[19] Formally, σ_j **allows** x if $\mathbb{P}_{\sigma_j, \sigma_{-j}}(x) > 0$ for some σ_{-j} (or, equivalently, for every strictly positive σ_{-j}). Two behavioral strategies σ_j and τ_j allow the same histories and behave in the same way at such histories if and only if they are realization-equivalent ($\mathbb{P}_{\sigma_j, \sigma_{-j}}(x) = \mathbb{P}_{\tau_j, \sigma_{-j}}(x)$ for all x and σ_{-j}). See Kuhn (1953).

co-players' intentions. There are several plausible alternatives, including a theory of forward-induction thinking along the lines of the rationalizability analysis of Battigalli & Dufwenberg (2009, Section 5). Here we adopt an approach more in the spirit of the equilibrium refinement literature (see in particular Reny, 1992). The idea is that a deviation by j is explained, if possible, by assuming that j is following a different equilibrium.

For any beliefs profile $(\mu_i^k)_{i\in I}$, we say that player j allows history x within $(\mu_i^k)_{i\in I}$ if the behavior strategy σ_j determined by μ_j^k (the plan of j within μ_j^k) allows x. Let $WSE^k(j,x)$ denote the set of k-order WSE beliefs such that j allows x, and let $WSE_j^k(j,x)$ be the projection of $WSE^k(j,x)$ on Y_j^k. In other words, $WSE_j^k(j,x)$ is the set of CPS's $\mu_j^k \in Y_j^k$ that belong to some WSE where j allows x. First suppose for simplicity that past actions are observable. In this case our refinement of WSE requires that *each player i believes conditional on each history x with $WSE_j^k(j,x) \neq \emptyset$ that j is following a WSE plan contained in an k-order WSE where j allows x.* The following definition applies to more general extensive forms.

Definition 26 *An m-order assessment (σ,μ^m) is an* ***equilibrium with perceived intentionality*** *(EPI) of an ℓ-order psychological game, with $\ell < m$, if it is an m-WSE such for every $i \in I$, $j \neq i$ and $x \in X$ with $WSE_j^{m-1}(j,x) \neq \emptyset$*

$$\mu_i^m(x|H_i(x)) > 0 \Longrightarrow \mathrm{marg}_{Y_j^{m-1}}\mu_i^m(WSE_j^{m-1}(j,x)|x) = 1.$$

An ***EPI outcome*** *is a distribution $\zeta \in \Delta(Z)$ over terminal histories induced by an EPI behavior strategy profile.*

Comment EPI differs from SE in two related ways. First, as explained above, the strong requirement of consistency is replaced by weak consistency: each player i has a plan σ_i^i and $|I|-1$ independent conjectures $(\sigma_j^i)_{i\neq i}$ about co-players' behavior, conjecture σ_j^i and higher-order beliefs about each co-player j are assumed to be correct as long as no deviation by j is detected. Furthermore, at off-equilibrium-path histories players may disagree about who deviated and how. Second, when player i is surprised by co-player j at history x, he assumes – if possible – that j is following some other equilibrium plan allowing x. This implies that at some histories i would change his mind about j's beliefs, whereas in an SE each player has fixed beliefs about the co-players' beliefs.

Example 27 *Consider again the Ultimatum Minigame of Figure A with the decision utility function (9). We claim that, for sufficiently high d_b^{ABI}, the strategy pair (f, n) is part of a fourth-order EPI assessment $(f, n, \delta_a, \delta_b)$.*[20] *First note that (g, y) is part of a third-order SE $(g, y, \gamma_a \gamma_b)$, which by definition is also a third-order WSE: In this equilibrium Bob expects the greedy offer g, which therefore cannot anger him, hence he accepts. With this, we can construct an EPI $(f, n, \delta_a, \delta_b)$ whereby Bob would interpret the deviation to the greedy offer as Ann having in mind the "wrong" equilibrium $(g, y, \gamma_a \gamma_b)$, hence as an intentional choice by Ann. Then Bob would blame Ann and punish her (if d_b^{ABI} is high enough) saying no.*

4.3 Polymorphic equilibria

There is another way to reconcile consistency requirements with non trivial inferences about co-players' beliefs, which is more easily understood in the context of population games. Suppose that the given game is played by agents drawn at random from large populations, one for each player role $i \in I$. Different agents in the same population i may have different plans, hence different first-order beliefs, even if their beliefs agree about the behavior and beliefs of co-players $-i$. In this case we say that the population is "polymorphic." Once an agent playing in role i observes some moves of co-players, he makes inferences about the intentions of the agents playing in the co-players' roles.

Let λ_i be a finite support distribution over $\Sigma_i \times Y_i^m$. We interpret λ_i as a statistical distribution of plans and beliefs in the population of agents playing in role i and for every $(\sigma_{i,k}, \mu_{i,k})$ in the support of λ_i we let $\lambda_{i,k}$ denote the fraction of agents with plan and beliefs $(\sigma_{i,k}, \mu_{i,k})$. With a slight abuse of terminology we call "support of λ_i" the set of indices k_i such that $\lambda_{i,k_i} > 0$ and we will refer to such indices as "types."[21] A type profile is denoted $\mathbf{k} = (k_i)_{i \in I}$. Assuming random matching, a profile of distributions $\lambda = (\lambda_i)_{i \in I}$ determines the probability of reaching each history x:Tthe probability of assessment $(\sigma_{i,k_i}, \mu_{i,k_i})_{i \in I}$ is $\prod_{i \in I} \lambda_{i,k_i}$; the probability of reaching x given that each player

[20] Here δ_i denotes the fourth-order CPS of i, not a Dirac measure (δ is the fourth letter of the Greek alphabet).

[21] The marginal of λ_i on Σ_i is a behavior strategy mixture (see Selten, 1975).

follows his plan in $(\sigma_{i,k_i}, \mu_{i,k_i})_{i\in I}$ is $\mathbb{P}_{(\sigma_{i,k_i})_{i\in I}}(x)$; therefore,

$$\mathbb{P}_\lambda(x) = \sum_{\mathbf{k}\in\times_{i\in I}\mathrm{Supp}\lambda_i} \prod_{i\in I} \lambda_{i,k_i}\mathbb{P}_{(\sigma_{i,k_i})_{i\in I}}(x).$$

Let $\mathbb{P}_{\lambda_{-i}}(x)$ denote the probability obtained from $\mathbb{P}_\lambda(x)$ by replacing λ_i with a degenerate distribution (Dirac measure) δ_{σ_i} where σ_i chooses each action of i in history x with probability one. By perfect recall, if $\mathbb{P}_\lambda(x) > 0$, then

$$\mathbb{P}_\lambda(x|H_i(x)) = \frac{\mathbb{P}_{\lambda_{-i}}(x)}{\sum_{x'\in H_i(x)} \mathbb{P}_{\lambda_{-i}}(x')}$$

and

$$\mathbb{P}_\lambda(\mathbf{a}_{-i}|x) = \sum_{a_i\in A_i(x)} \frac{\mathbb{P}_{\lambda_{-i}}(x,(a_i,\mathbf{a}_{-i}))}{\mathbb{P}_{\lambda_{-i}}(x)}$$

are well defined and independent of λ_i.[22]

Definition 28 *An m-order **polymorphic assessment** is a profile of finite support probability measures $\lambda^m = (\lambda_i^m)_{i\in I} \in \times_{i\in I}\Delta(\Sigma_i \times Y_i^m)$ such that σ_{i,k_i} is the behavior strategy induced by μ^m_{i,k_i} for every $i \in I$ and every k_i in the support of λ_i^m. A polymorphic assessment λ^m is **consistent** if there is a converging sequence of polymorphic assessments $\lambda_n^m \to \lambda^m$ such that, for every $i \in I$, $k_i \in \mathrm{Supp}\lambda_{i,n}$, $x \in X$, $\mathbf{a}_{-i}\in \mathbf{A}_{-i}(x)$ and $n \in \mathbb{N}$*
(1) $\sigma_{i,k_i,n}$ is fully randomized,

$$\mu^m_{i,k_i,n}(x|H_i(x)) = \mathbb{P}_{\lambda_{-i,n}}(x|H_i(x)),$$

and

$$\mu^m_{i,k_i,n}(\mathbf{a}_{-i}|x) = \sum_{a_i'\in A_i(x)} \mathbb{P}_{\lambda_{-i,n}}(\mathbf{a}_{-i}|x);$$

(2) for every $\ell = 1, ..., m-1$, and $\mathbf{k}_{-i} = (k_j)_{j\neq j}$ in the support of $\lambda_{-i,n} = \prod_{j\neq i}\lambda_{j,n}$

$$\mathrm{marg}_{Y^\ell_{-i}}(\mu^m_{i,k_i,n})^{\ell+1}((\mu^m_{j,k_j,n})_{j\neq i}|x) = \frac{\lambda_{\mathbf{k}_{-i},n}\mathbb{P}_{\sigma_{k_{-i}}}(x)}{\mathbb{P}_{\lambda_{-i,n}}(x)}.$$

[22] Indeed, $\mathbb{P}_\lambda(x|H_i(x)) = \frac{\mathbb{P}_{\bar\sigma^\lambda}(x)}{\sum_{x'\in H_i(x')}\mathbb{P}_{\bar\sigma^\lambda}(x')}$ and $\mathbb{P}_\lambda(\mathbf{a}_{-i}|x) = \sigma_c(a_c|x)\prod_{j\neq i}\bar\sigma_j^\lambda(a_j|H_j(x))$, where $\bar\sigma_j^\lambda$ is any behavior strategy of j realization-equivalent to the behavior strategy mixture implied by λ_i, that is, $\mathrm{marg}_{\Sigma_j}\lambda_j$. See selten (1975).

Comments All the conditional probabilities in Definition 28 are well defined. By continuity, all the equalities hold in the limit (as long as conditional probabilities are well defined in the limit). Condition (1) implies that

$$\mu^m_{i,k_i}(z|\{\varnothing\}) = \mathbb{P}_{(\lambda_{-i},\sigma_{i,k_i})}(z)$$

for every $z \in Z$, $i \in I$ and k_i in the support of λ_i. In a mixed consistent assessment different types k_i of player i have different plans, but they agree on the statistics λ and hence on beliefs of the co-players, and at each stage they assume that the co-players will follow their plans (whatever they are) in the continuation of the game. Furthermore, the types of players i and i' agree on the beliefs of any third player j. The only uncertainty concerns the plans implied by co-players beliefs.

Definition 29 *An m-order mixed assessment λ^m is a* ***polymorphic sequential equilibrium*** *if it is consistent and, for every $i \in I$ and k_i in the support of λ_i, i plans rationally in $(\sigma_{i,k_i}, \mu^m_{i,k_i})$.*

Remark 30 *Every SE is a degenerate polymorphic SE. Therefore every ℓ-order psychological game with continuous decision-utility functions has an m-order polymorphic SE for each $m \geq \ell$.*

Example 31 *Consider the Ultimatum Minigame of Figure A and the ABI-decision utility of eq. (9). Let $d^{ABI}_b = 1$ for simplicity. We claim that the following profile (λ_a, λ_b) is a mixed SE: λ_b assigns probability one to just one type of Bob who randomizes with $\sigma_b(y|g) = \frac{2}{3}$, there are two equally likely types of Ann, $\lambda_a = \left(\frac{1}{2}, \frac{1}{2}\right)$, type 1 (resp. 2) has the deterministic plan f (resp. g). First-order beliefs α_i and higher-order beliefs β_i and γ_i are determined by the consistency conditions. Both types of Ann are indifferent given their belief beliefs about Bob, because only material payoffs matter for them (there cannot be any frustration in the first period), and both offers yield \$2 in expectation. Bob is frustrated by the greedy offer*

$$\mathrm{F}_b(g; \alpha_b) = \max\left\{0, 2 \times \frac{1}{2} + \left(1 \times \frac{2}{3} + 0 \times \frac{1}{3}\right) \times \frac{1}{2} - 1\right\} = \frac{1}{3}.$$

Bob infers from observing g that he is facing the type of Ann who planned the greedy offer and he fully blames his frustration on Ann. With this, Bob *is indifferent between accepting and rejecting:*

$$\begin{aligned} u_b^{ABI}(g, y; \gamma_b) &= 1 - \frac{1}{3} \times 3 = 0, \\ u_b^{ABI}(g, n; \gamma_b) &= 0 - \frac{1}{3} \times 0 = 0. \end{aligned}$$

5 Equilibrium analysis of anger

For a given multistage game form Γ (a game with material payoffs) we let $\Gamma^{SA}(\mathbf{d}^{SA}, \mathbf{r}^{SA})$, $\Gamma^{ABB}(\mathbf{d}^{ABB}, \mathbf{r}^{ABB})$ and $\Gamma^{ABI}(\mathbf{d}^{ABI}, \mathbf{r}^{ABI})$ respectively denote the psychological games with simple anger, anger from blaming behavior and anger from blaming intentions obtained from Γ with parameters values $(d_i^{SA}, r_i^{SA})_{i \in I}$, $(d_i^{ABB}, r_i^{ABB})_{i \in I}$ and $(d_i^{ABI}, r_i^{ABI})_{i \in I}$. In statements where the precise parameter values do not matter, we simply write Γ^{SA}, Γ^{ABB} and Γ^{ABI} to denote the psychological games based on game form Γ. The material-preferences equilibria of Γ are the equilibria of $\Gamma^{SA}(\mathbf{0}, \mathbf{0})$ and $\Gamma^{ABB}(\mathbf{0}, \mathbf{0})$ of the same kind (SE, EPI or mixed SE), and are the 1^{st}-order belief projection of the higher-order beliefs equilibria of $\Gamma^{ABI}(\mathbf{0}, \mathbf{0})$. We first establish some general relationships between equilibria of the same kind with different models of anger and equilibria of different kinds with the same model of anger. We compare equilibrium strategies and equilibrium outcomes across different games and/or type of equilibrium.

Proposition 32 *Let Γ be a two-stage game form with two players and (at most) one active player in the second stage. Then every pure SE (resp. EPI) outcome of Γ is also an SE (resp. EPI) outcome of the psychological games Γ^{ABB} and Γ^{ABI}; furthermore, if there are no chance moves every pure SE (resp. EPI) outcome of Γ is also an SE (resp. EPI) outcome of the psychological game Γ^{SA}.*

To understand this, note that players cannot be frustrated at the beginning of the game, hence they cannot be angry and expected material payoff maximization coincides with decision utility maximization in the first stage. The same holds in the second stage on the equilibrium path. If at off-equilibrium-path information sets a player is angry his decision utility maximizing action gives the co-player a lower material payoff compared to the material-payoff equilibrium action. Therefore, if the active player's strategy of the material-payoff equilibrium is modified off the equilibrium path so as to maximize his decision utility, the incentive of the co-player to deviate in the first stage is even lower than in the material-payoff equilibrium.

It is easy to show that some psychological games with simple anger and anger from blaming behavior have more pure SE or EPI outcomes than the material payoff game. For example, it is easy to check that the fair offer f is a pure SE outcome of of the Ultimatum Minigame with simple anger and anger from blaming behavior if Bob is sufficiently prone to anger. The same holds for the EPI outcomes of psychological games with anger from blaming intentions, as shown in Example 31. On the other hand, the SEs of Γ and Γ^{ABI} tend to coincide. The reason is that, as explained above, in an SE players never change their mind about co-players intentions, thus when they are frustrated by unexpected moves they do not blame the co-players.

Proposition 33 *Let Γ be a game form without chance moves. Then the pure (third-order) SEs of Γ and Γ^{ABI} coincide.*

The intuition is quite simple. In a pure equilibrium of a game without chance moves players can feel frustrated only after deviations. But according to the SE equilibrium concept, deviations are always interpreted as unintentional mistakes and players do not blame their frustration on co-players, hence they are not aggressive. This means that in equilibrium players maximize their material payoff at every non-terminal history.

The following proposition concerns leader-follower games, a kind of two-stage games. In two-stage games the rate of decay of anger is irrelevant because players can feel frustrated only in the second and final period. Therefore we omit the vector $\mathbf{r}$ of decay parameters from our notation.

Proposition 34 *In a leader-follower game Γ, every SE outcome of $\Gamma^{ABB}(\mathbf{d}^{ABB})$ where the leader's strategy is fully randomized is also a* polymorphic *SE outcome of $\Gamma^{ABI}(\mathbf{d}^{ABI})$ provided that $\mathbf{d}^{ABB} = \mathbf{d}^{ABI}$.*

The intuition is as follows. Let σ be an SE strategy profile of $\Gamma^{ABB}(\mathbf{d}^{ABB})$ where the leader strategy is fully randomized. First note that since there cannot be any frustration in the first period, the leader just maximizes his expected payoff. The indifference condition for the leader implies that each pure action of the leader, say player 1, is a best reply to σ_{-1}. Then let $\boldsymbol{\lambda} = (\lambda_i)_{i \in I}$ be as follows: for each $a_1 \in A_1(\varnothing)$, there is a corresponding type and $\lambda_{1,a_1} = \sigma_1(a_1|\varnothing)$; for each follower i there is a unique type with plan/strategy σ_i. Beliefs are determined by the consistency conditions because action are observable. By construction, the second-period frustration of followers is the same in both assessments. Upon observing a_1 the active follower infers that he is facing the type who plans precisely a_1, therefore blaming intentions is equivalent to blaming behavior and incentives are the same in both assessments (with the respective decision utilities).

For example, in the Ultimatum Minigame there is a randomized SE of $\Gamma^{ABB}(\mathbf{1})$ where the leader (Ann) randomizes $\frac{1}{2} : \frac{1}{2}$ on the fair and greedy offer and the follower (Bob) accepts the greedy offer with probability $\frac{2}{3}$. As shown in Example 31, this corresponds to a polymorphic SE of $\Gamma^{ABI}(\mathbf{1})$ with the same outcome.

6 Concluding remarks

A series of experimental studies suggest that anger is an important driver of behavior. In the the ultimatum game... using emotion self-reports, Pillutla & Murnighan (1996) find that reported anger predicted rejections better than perceived unfairness ... Schotter & Sopher (2007) measure second-mover expectations and find that unfulfilled expectations drive rejections ... Sanfey (2009) reports that psychology students who are told that a typical offer is \$4-\$5 reject low offers more frequently than students who are told that a typical offer is \$1 - \$2 ... Sanfey *et al.* (2003) find that rejections of low offers are associated with greater activation of the anterior insula, a brain region associated with negative emotional states ... Koenigs & Tranel (2007) find that patients with lesions in the ventromedial prefrontal cortex (vmpfc) – a region of the brain associated with emotion regulation – reject unfair offers more frequently than normal subjects. Patients with vmpfc damage have a tendency towards exaggerated anger and emotional outbursts in social situations involving frustration or provocation, and the results of this study suggest that anger regulation plays a key role in economic behavior ... Cross-cultural studies show considerable variation in behavior (Henrich *et al.* 2006); one explanation for these results is that cultural factors in uence the initial expectations subjects have about play in the game, subjects compare observed play with these expectations, negative deviations from the culturally influenced benchmark lead to emotional reactions, based on frustrations &

anger, which then affect behavior.[23]

To boot, psychologist have argued for years that anger shapes behavior in important ways for years – we have cited this scholarship extensively earlier and will not repeat here.

What has been missing, from an economist's point-of-view, are theoretical models of anger that can be employed in economic analysis. Our goal in this paper has been to take first steps in that direction. We hope that the models we have proposed will prove useful for applied work, and that our paper will also inspire experimental work that can shed light on the empirical, and situational, relevance of the various nuances of anger that our approach has highlighted.

7 Appendix

7.1 Definition of multistage extensive form

Let $\mathbb{N}_0$ denote the set of natural numbers including zero, and fix a range $\mathbf{A}$. For each $\ell \in \mathbb{N}_0$, $\mathbf{A}^\ell$ is the set of sequences of length ℓ of elements of $\mathbf{A}$, with the convention that $\mathbf{A}^0 = \{\varnothing\}$ is the singleton containing the **empty sequence** $\varnothing$. For every $k, \ell \in \mathbb{N}_0$, $x \in \mathbf{A}^k$ is a **prefix** of $y \in \mathbf{A}^\ell$, written $x \prec y$, if $k < \ell$ and y is obtained by concatenating x with a sequence of $\ell - k$ elements of $\mathbf{A}$, that is $x = \varnothing$ ($k = 0$), or $x = (\mathbf{a}^1, ...\mathbf{a}^k)$, $y = (\mathbf{b}^1, ..., \mathbf{b}^k, ..., \mathbf{b}^\ell)$ and $(\mathbf{a}^1, ...\mathbf{a}^k) = (\mathbf{b}^1, ..., \mathbf{b}^k)$. Notice that $\varnothing$ is a prefix of every nonempty sequence. We say that x is a **weak prefix** of y, written $x \preceq y$, if either $x \prec y$ or $x = y$. For any index set J and range A, A^J is the set of **profiles** $(a_j)_{i \in J}$, that is, the set of functions that assign an element of A to each $i \in I$.

[23] While we devote this paragraph mainly to the ultimatum game, we note that there exists relevant evidence also from other games. Fehr & Gachter (2002) conclude that negative emotions such as anger are the proximate cause of costly punishment of free-riders in public goods games. Experiments by Frans van Winden and several coauthors (*e.g.* Bosman & van Winden 2002; Bosman, Sutter & van Winden 2005); Reuben & van Winden 2008) record both emotions and expectations in the power-to-take game. In this game a first-mover can "take" some fraction of a second-mover's endowment. The second-mover, upon seeing the take decision, decides how much of the endowment to destroy before the first-mover gets it (unlike in the ultimatum game, where all-or-nothing gets destroyed). These papers show that second-mover expectations about take-rates are a key factor in the decision to destroy income, and that anger-like emotions are also triggered by the difference between expected and actual take rates.

Definition 35 *A set of sequences*

$$X \subseteq \bigcup_{\ell \in \mathbb{N}_0} \mathbf{A}^\ell$$

is a ***tree*** *if the following holds: for each* $x, y \in \bigcup_{\ell \in \mathbb{N}_0} \mathbf{A}^\ell$*, if* $y \in X$ *and* $x \prec y$*, then* $x \in X$*. A sequence* $z \in X$ *is* ***terminal*** *(in* X*) if* z *is not the prefix of any other element of* X*; the set of terminal sequences in* X *is denoted by* Z*.*

Definition 36 *A* ***multistage extensive form*** *is a structure* $(I, A, X, \sigma_c, (H_i)_{i \in I})$ *whose elements are specified as follows:*
(1) I *is a finite set of* ***players;***
(2) A *is a finite set of* ***actions;***
(3) $X \subseteq \bigcup_{\ell \in \mathbb{N}_0} (A^{I_c})^\ell$ *is a finite tree of sequences of action profiles* $\mathbf{a} = (a_i)_{i \in I_c} \in A^{I_c}$*, called* ***histories;***
(4) $\sigma_c = (\sigma_c(\cdot|x))_{x \in X \setminus Z} \in \times_{x \in X \setminus Z} \Delta^o(A_c(x))$ *(where* $A_c(x) = \{a_c \in A : \exists \mathbf{a}_{-c} \in A^I, (x, (a_c, \mathbf{a})) \in X\}$*) is the probability function of chance moves;*
(5) for every $i \in I$*,* $H_i \subseteq 2^X$ *is a partition of* X*, called the* ***information partition*** *of* i*;*
The tree X *has the following property:*
(X, independent choices*) for every* $i \in I$*,* $x \in X$*,* $a_i, b_i \in A$*,* $\mathbf{a}_{-i}, \mathbf{b}_{-i} \in A^{I_c \setminus \{i\}}$*, if* $(x, (a_i, \mathbf{a}_{-i})), (x, (b_i, \mathbf{b}_{-i})) \in X$ *then* $(x, (a_i, \mathbf{b}_{-i})) \in X$*.*
The information partitions $(H_i)_{i \in I}$ *have the following properties:*
(H.1, knowledge of stage*) for every* $i \in I$*,* $h \in H_i$*,* $x, y \in h$*,* x *and* y *have the same length;*
(H.2, knowledge of feasible actions*) for every* $i \in I$*,* $h \in H_i$*,* $x, y \in h$*,* $a_i \in A$*,* $\mathbf{a}_{-i} \in A^{I_c \setminus \{i\}}$*, if* $(x, (a_i, \mathbf{a}_{-i})) \in X$ *then there is some* $\mathbf{b}_{-i} \in A^{I_c \setminus \{i\}}$ *such that* $(y, (a_i, \mathbf{b}_{-i})) \in X$*;*
(H.3, perfect recall*) for every* $i \in I$*,* $h', h'' \in H_i$*,* $x'', y'' \in h''$*, if* $x' \prec x''$ *for some history* $x' \in h$ *then there are a history* $y' \prec y''$*, an action* $a_i \in A$ *and action profiles* $\mathbf{a}_{-i}, \mathbf{b}_{-i} \in A^{I_c \setminus \{i\}}$ *such that* $y' \in h'$*,* $(x', (a_i, \mathbf{a}_{-i})) \preceq x''$ *and* $(y', (a_i, \mathbf{b}_{-i})) \preceq y''$*.*

Property (X) says that the set of action profiles that are feasible at a given history is a cross-product, $\mathbf{A}(x) = \times_{j \in I_c} A_j(x)$, which means that what i can choose is independent of what the co-players and chance choose (if choices are truly simultaneous and fully controlled by players it cannot be otherwise). Property (H.1) says that the game form has a multistage structure: players always know how many action profiles have already been chosen. This implies that the singleton containing the empty history is an information set of each player: for every $i \in I$, $\{\varnothing\} \in H_i$.[24] Property (H.2) says that a player knows his set of feasible actions: for every $x \in h \in H_i$, $A_i(x) = A_i(h)$. Property (H.3) says that players have perfect recall and this is captured by the information partitions.[25] Therefore, each information set $h \in H_i$ is identified by the unique personal history recording the information states and actions of i that obtains for every $x \in h$. In a game form with **observable actions**, each $h \in H_i$ is a singleton and H_i is isomorphic to the set of histories X. By convention, at a non terminal history $x \in X \backslash Z$, each player i (including chance) has a nonempty set $A_i(x)$ of feasible actions; if he is **active** $|A_i(x)| \geq 2$, if he is **inactive** $|A_i(x)| = 1$. By definition, a history x is terminal if and only if $A_i(x) = \emptyset$ for each $i \in I$. If a game form with observable actions has only one active player at each non-terminal history, then it has **perfect information.** Notice that when the game ends players learn that it has ended[26] and obtain information about the path of play.

7.2 Impact of positive surprises on frustration

In eq. (4) we are excluding the possibility that positive surprises can partially offset frustration. To account for this possibility, we can introduce a less extreme functional form where positive surprises have a positive impact. Let

$$f(\Delta) = \begin{cases} \Delta, & \text{if } \Delta \geq 0 \\ g\Delta, & \text{if } \pi < 0 \end{cases}, \ (0 \leq g \leq 1), \tag{10}$$

$$\hat{\mathrm{F}}_i(h_{i,t};\alpha_i) = \begin{cases} f(\mathbb{E}[\pi_i|h_{i,t-1};\alpha_i] - \Pi_i^*(h_{i,t};\alpha_i)), & \text{if } h_{i,t} \in H_i^D \\ f(\mathbb{E}[\pi_i|h_{i,t-1};\alpha_i] - \Pi_i(h_{i,t};\alpha_i)), & \text{if } h_{i,t} \subseteq H_i \backslash H_i^D \end{cases} \tag{11}$$

(the second part of eq. (11) takes into account that if $h_{i,t}$ is terminal i cannot affect any more his expected payoff). Parameter g is the weight of a gain

[24] Since H_i is a partition of X and $\varnothing \in X$, there is some $h \in H_i$ such that $\varnothing \in h$. Let $x \in X$ be a history of length $\ell > 0$; then $x \notin h$, otherwise h would contain two histories of different length, thus violating property (H.1).

[25] Information partitions capture the information structure given by the rules of the game as well as assumptions about the cognitive abilities of players, such positive introspection, negative introspection and perfect recall.

[26] If an information set $h \in H_i$ contained a terminal history x and a non-terminal history y, then property (H.2) would be violated: $A_i(x) = \emptyset \neq A_i(y)$.

over lagged expectations. When $g = 0$, $f(\Delta) = \max\{0, \Delta\}$ and we obtain the expression of eq. (3), $\hat{\mathrm{F}}_i(h_{i,t}; \alpha_i) = \mathrm{F}_i(h_{i,t}; \alpha_i)$; when $g = 1$, $f(x) = x$ and we get $\hat{\mathrm{F}}_i(h_{i,t}; \alpha_i) = \Pi_{i,t-1}(h_{i,t-1}; \alpha_i) - \Pi_i^*(h_{i,t}; \alpha_i)$. Then a plausible specification of decision utilities could be the following:

$$\hat{u}_i^F(h_{i,t}, a_i; \alpha_i) =$$

$$\mathbb{E}[\pi_i|(h_{i,t}, a_i); \alpha_i] - d_i^{SA} \max\left\{0, \sum_{k=1}^{t} (r_i^F)^{t-k} \hat{\mathrm{F}}_i(h_{i,t}^{\leq k}; \alpha_i)\right\} \cdot \left(\sum_{j \neq i} \mathbb{E}[\pi_j|(h_{i,t}, a_i); \alpha_i]\right). \tag{12}$$

When $g = 0$, eq.s (**??**) and (12) give back, respectively eq.s (**??**) and (4), that is, $\hat{v}_i^F(z, \alpha_i) = v_i^F(z, \alpha_i)$ and $\hat{u}_i^F(h_{i,t}, a_i; \alpha_i) = u_i^F(h_{i,t}, a_i; \alpha_i)$. When $g = 1$ positive surprises completely cancel out negative surprises of the same size.

7.3 Impact of positive surprises on anger from blame

Also in the case of anger from blaming behavior we can assume that positive surprises in some periods partially or totally offset negative surprises in other periods and hence take off some anger, a possibility excluded by eq. (6).[27] To account for this possibility, we can introduce a less extreme functional form where positive surprises have a positive impact. Fix a history x of length t, then

$$\mathrm{F}_i(x; \alpha_i) := f\left(\mathbb{E}[\pi_i|H_i(x^{\leq t-1}); \alpha_i] - \Pi_i^*(H_i(x); \alpha_i)\right) \tag{13}$$

where f is the loss-gain function defined in eq. (10) parametrized by the weight g of a gain over lagged expectations. Then $\hat{\mathrm{F}}_{ji}(x; \alpha_i)$ and $\hat{\mathrm{B}}_{ij}(h_{i,t}; \alpha_i)$ can be derived in much the same way as $\mathrm{F}_{ji}(x; \alpha_i)$ and $\mathrm{B}_{ij}(h_{i,t}; \alpha_i)$:

$$\hat{\mathrm{F}}_{ji}(x; \alpha_i) := \hat{\mathrm{F}}_i(x; \alpha_i) - \min_{b_j \in A_j(x^{\leq t-1})} \hat{\mathrm{F}}_i(x|_{t,j} b_j; \alpha_i),$$

$$\hat{\mathrm{B}}_{ij}(h_{i,t}; \alpha_i) := \mathbb{E}[\hat{\mathrm{F}}_{ji}|h_{i,t}; \alpha_i].$$

The following is a plausible specification of decision utility:

$$\hat{u}_i^{ABB}(h_{i,t}, a_i; \alpha_i) =$$

[27] The required modifications for the case of blaming intentions are similar.

$$\mathbb{E}[\pi_i|(h_{i,t},a_i);\alpha_i] - d_i^{ABB}\sum_{j\neq i}\left(\sum_{k=1}^{t}(r_i^{ABB})^{t-k}\hat{\mathrm{B}}_{ij}(h_{i,t}^{\leq k};\alpha_i)\right)\mathbb{E}[\pi_j|(h_{i,t},a_i);\alpha_i], \tag{14}$$

where $\hat{v}_i$ is the modified experience utility function defined in eq. (**??**) and $d_i^{ABB}, r_i^{ABB} \in [0,1]$. When $g = 0$, eq.s (**??**) and (14) give back, respectively, eq.s (**??**) and (6), that is, $\hat{v}_i(z,\alpha_i) = v_i(z,\alpha_i)$ and $\hat{u}_i^{ABB}(h_{i,t},a_i;\alpha_i) = u_i^{ABB}(h_{i,t},a_i;\alpha_i)$. When $g = 1$ positive surprises completely cancel out negative surprises of the same size.

7.4 Proofs

7.4.1 Proof of Lemma 6

Battigalli & Siniscalchi (1999) shows that the set of CPS's on $(Z,\bar{H}_i)$ is a closed subset of $[\Delta(Z\times Y)]^{\bar{H}_i}$. We only have to show that $\Delta_i^{\bar{H}_i}(S\times Y)$ is a closed subset of the set of CPS's. Fix an arbitrary converging sequence $\mu^n \to \mu^0$ of CPS's that satisfy conditions (1)-(2) of Definition 4. Then, $\mu^n(\cdot|h)$ converges weakly to $\mu^0(\cdot|h)$ for each $h\in\bar{H}_i$. Fix some $h\in H_i$, $x\in h$, $a_i\in A_i(x)$ and $\mathbf{a}_{-i}\in\mathbf{A}_{-i}(x)$. Function $\mathrm{marg}_Y(\cdot):\Delta(Z\times Y)\to\Delta(Y)$ is continuous, therefore $\mu^n(\cdot|h)\to\mu^0(\cdot|h)$, $\mu^n(\cdot|h,a_i)\to\mu^0(\cdot|h,a_i)$ and $\mathrm{marg}_Y\mu^n(\cdot|h) = \mathrm{marg}_Y\mu^n(\cdot|h,a_i)$ imply $\mathrm{marg}_Y\mu^0(\cdot|h) = \mathrm{marg}_Y\mu^0(\cdot|h,a_i)$. Thus, μ^0 satisfies condition (1). Suppose that $\mu^0(Z(x)\times Y|h) > 0$. Then

$$\mu^n((a_i,\mathbf{a}_{-i})|x) = \frac{\mu^n(Z(x,(a_i,\mathbf{a}_{-i}))\times Y|h)}{\mu^n(Z(x)\times Y|h)}$$

is well defined, not only for $n=0$, but also for each n large enough, so that $\mu^n(Z(x)\times Y|h) > 0$. $Z(x,(a_i,\mathbf{a}_{-i}))\times Y$ and $Z(x)\times Y$ are both closed and open in the product topology given by the discrete topology of Z and the topology of Y, therefore $\mu^n(\cdot|h)\to\mu^0(\cdot|h)$ implies

$$\begin{aligned}
\mu^n(Z(x)\times Y|h) &\to \mu^0(Z(x)\times Y|h),\\
\mu^n(Z(x,(a_i,\mathbf{a}_{-i}))\times Y|h) &\to \mu^0(Z(x,(a_i,\mathbf{a}_{-i}))\times Y|h),\\
\mu^n((a_i,\mathbf{a}_{-i}|x) &\to \mu^0((a_i,\mathbf{a}_{-i}|x),
\end{aligned}$$

so that condition (2) must hold in the limit for μ^0. ■

7.4.2 Proof of Proposition 32

(...) ■

7.4.3 Proof of Proposition 33

(...) ■

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