Valuation Risk and Asset Pricing

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Correlation puzzle

Classic asset pricing models

- Expected return for holding an asset reflects covariance between asset’s payoff and agent’s SDF.

Important challenge facing these models:

- Covariance and correlation between stock returns and measurable fundamentals, especially consumption, is weak at 1, 5, and 10 year horizons.

- Lettau and Ludvigson (2011): shock that accounts for vast majority of asset-price fluctuations is uncorrelated with consumption at virtually all horizons.
This fact underlies many important asset-pricing puzzles.

Equity premium puzzle, Hansen-Singleton-style rejection of asset pricing models, etc.

High estimates of risk aversion, correspondingly large amounts that agents would pay for early resolution of uncertainty in LRR models.
Conventional view: variation in asset returns is overwhelmingly due to variation in discount factors (Cochrane (2011)).

How should we model that variation?

Classic asset-pricing models: all SDF variation comes from shocks to supply-side of economy.
  - Stochastic process for endowment in Lucas-tree models.
  - Stochastic process for productivity in production economies.

Not surprising that these models can’t simultaneously account for equity premium, correlation puzzles.
Introduce shocks to the demand for assets

- Demand shocks arise from stochastic changes in agents’ rate of time preference.

- Parsimonious way of modeling variation in discount factors.

- High frequency changes in household savings behavior emphasized in macro.
  - ZLB literature, Eggertsson and Woodford (2003), Hall (2014).

- Simple, tractable way to capture notion that fluctuations in market sentiment contribute to volatility of asset prices
  - Noise trader literature.
Disciplining the analysis in two versions of our model

- **Benchmark model**
  - Designed to highlight role played by time-preference shocks per se.
  - Consumption, dividends modeled as random walks with conditionally homoscedastic shocks.
  - Very useful for expositional purposes, but suffers from some clear empirical shortcomings.

- **Extended model**
  - Shocks to consumption, dividend process are conditionally heteroskedastic.

- Law of motion for preference shocks must be consistent with time-series properties of variables like price-dividend ratio, equity returns and bond returns.
Estimation Strategy

- Estimate model using GMM implemented with annual data for the period 1929 to 2011.

- Agents make decisions on a monthly basis, deduce the model’s implications for annual data.

- For large set of parameter values, model implies GMM estimators suffer from substantial small-sample bias.
  - Modify GMM procedure to focus on plim of model-implied small-sample moments rather than plim of moments themselves.
The correlation puzzle: U.S. data, 1929-2011

<table>
<thead>
<tr>
<th>Horizon</th>
<th>Consumption</th>
<th>Output</th>
<th>Dividends</th>
<th>Earnings</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 year</td>
<td>$-0.05^{(0.12)}$</td>
<td>$0.05^{(0.10)}$</td>
<td>$0.05^{(0.11)}$</td>
<td>$0.10^{(0.10)}$</td>
</tr>
<tr>
<td>5 years</td>
<td>$0.002^{(0.14)}$</td>
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<tr>
<td>10 years</td>
<td>$-0.11^{(0.20)}$</td>
<td>$-0.09^{(0.14)}$</td>
<td>$0.59^{(0.14)}$</td>
<td>$0.30^{(0.11)}$</td>
</tr>
</tbody>
</table>

- See paper for alternative data set, 1871-2006, NIPA data.
Covariance versus correlation

  - Needs a risk aversion coefficient of 379 to account for equity premium.

- There’s a larger covariance between current stock returns, cumulative consumption growth over next 12 quarters.

- He also uses this larger covariance in his calculations.
  - Still needs a risk aversion coefficient of 38 to rationalize equity premium.
Correlation puzzle: a challenge for pure ‘supply-side’ models

- Lucas-style CRRA or standard Epstein-Zin type models.
- Habit-formation model (internal or external).
- Long-run risk models.
- Rare-disaster models: all shocks, disaster or not, are to supply side of the model.
- In principle, model with time-varying disaster probability could account for correlation puzzle as small sample phenomenon.
  - But correlation puzzle holds even in long sample 1870 - 2006.
A model with time-preference shocks

- Epstein-Zin preferences

\[ U_t = \max_{C_t} \left[ \lambda_t C_t^{1-1/\psi} + \delta (U_{t+1}^*)^{1-1/\psi} \right]^{1/(1-1/\psi)} \]

\[ U_{t+1}^* = \left[ E_t \left( U_{t+1}^{1-\gamma} \right) \right]^{1/(1-\gamma)} \]

- \( \lambda_{t+1}/\lambda_t \) determines how agents trade off current versus future utility, isomorphic to a time-preference shock.

- \( \psi \) is elasticity of intertemporal substitution, \( \gamma \) is coefficient of risk aversion.

- Normandin and St. Amour (1998) first proposed this specification but solved the model incorrectly, obtain very strange results.
The benchmark model

- Consumption follows a random walk:

\[
\log(C_{t+1}) = \log(C_t) + \mu + \sigma_c \varepsilon_{c,t+1} \\
\varepsilon_{c,t+1} \sim N(0, 1)
\]

- Process for dividends and preference shock:

\[
\log(D_{t+1}) = \log(D_t) + \mu + \pi_{dc} \varepsilon_{c,t+1} + \sigma_d \varepsilon_{d,t+1} \\
\log(\lambda_{t+1}/\lambda_t) = \rho \log(\lambda_t/\lambda_{t-1}) + \sigma_\lambda \varepsilon_{\lambda,t+1}
\]

- $\varepsilon_{c,t+1}, \varepsilon_{d,t+1}, \varepsilon_{\lambda,t+1}$ are uncorrelated.
When $\gamma = 1/\psi$, preferences reduce to CRRA with a time-varying rate of time preference.

$$V_t = E_t \sum_{i=0}^{\infty} \delta^i \lambda_{t+i} C_{t+i}^{1-\gamma},$$

where $V_t = U_{t}^{1-\gamma}$.

This case was considered by Garber and King (1983), Campbell (1986), Pavlova and Rigobon (2007).
Suppose $\gamma = 1/\psi$.

Unconditional equity premium is proportional to risk-free rate:

$$E (R_{c,t+1} - R_{f,t+1}) = E (R_{f,t+1}) \left[ \exp (\gamma \sigma_c^2) - 1 \right].$$

Average risk-free rate ($E (R_{f,t+1})$) and volatility of consumption ($\sigma_c^2$) are small in the data.

Constant of proportionality $\exp (\gamma \sigma_c^2) - 1$, is independent of $\rho$ and $\sigma_\lambda$.

So time-preference shocks don’t help to resolve equity premium puzzle without having counter-factual implications for $E (R_{f,t+1})$. 
Equity premium and valuation risk

\[ \theta = \frac{1 - \gamma}{1 - 1/\psi}. \]

- Given our simple consumption process, equity premium is constant.

- Compensation for \textit{valuation risk}: part of one-period expected excess return to asset that’s due to \( \sigma^2_\lambda \).

- Compensation for \textit{conventional risk}: part of expected excess return due to volatility of consumption and dividends.

- For valuation risk to help explain equity premium, we need \( \theta < 1 \).

- Same condition plays key role in generating high equity premium in LRR models.
  - Long-run risks are resolved in distant future, they’re more heavily penalized than current risks.
Valuation risk, intuition

- Suppose you buy stock today.

- At some point in future, you may get a preference shock and want to consume more (compared to today).

- You’ll sell stock at same time as everyone else, so price will fall just when discounted value of consumption is high.

- Since stocks are infinitely-lived compared to one-period bond, they’re more exposed this source of risk.

- So equity-premium will be high.
Valuation risk vs conventional risk

- Say there’s no risk associated with physical payoff of assets like stocks.
  - Standard models imply equity premium is zero.
  - In our model, there’s a positive equity premium because bonds, stocks have different exposure to valuation risk.

- Agents are uncertain about how much they’ll value future dividend payments.

- The longer the maturity of an asset, the higher is its exposure to time-preference shocks and the larger is the valuation risk.
Valuation risk vs conventional risk

- Say there are supply-side shocks to the economy but agents are risk neutral ($\gamma = 0$).

- Component of equity premium due to valuation risk is positive as long as $\psi$ is less than one.

- Stocks are long-lived assets whose payoffs can induce unwanted variation in the period utility of representative agent, $\lambda_t C_t^{1-1/\psi}$.

- Even when agents are risk neutral, they must be compensated for risk of this unwanted variation.
Relation to long-run risk models

- Our model and long-run-risk model pioneered by BY (2004) emphasize low-frequency shocks that induce large, persistent changes in SDF.

- Re-write representative agent’s utility function

  \[ U_t = \left[ \tilde{C}_t^{1-1/\psi} + \delta (U_{t+1}^*)^{1-1/\psi} \right]^{1/(1-1/\psi)} \]

  where

  \[ \tilde{C}_t = \lambda_t^{1/(1-1/\psi)} C_t. \]

- Taking logarithms of this expression we obtain:

  \[ \log (\tilde{C}_t) = 1/ (1 - 1/\psi) \log(\lambda_t) + \log (C_t) \]
Relation to long-run risk models

\[
\log (\tilde{C}_t) = 1/ (1 - 1/\psi) \log(\lambda_t) + \log (C_t)
\]

- BY (2004) introduce highly persistent component in \( \log(C_t) \), which is source of long-run risk.

- We introduce highly persistent component into \( \log(\tilde{C}_t) \) via our specification of time-preference shocks.

- Both specifications can induce large, persistent movements in \( m_{t+1} \).

- Two models are not observationally equivalent.
  - Different implications for correlation between \( \log(C_{t+1}/C_t) \) and asset returns.
  - Very different implications for average return to long-term bonds, and term structure of interest rates.
Benchmark model

- Useful to highlight role of time-preference shocks.
- Clear empirical shortcomings.
- Since consumption is a martingale, only state variable that’s relevant for asset returns is $\lambda_{t+1}/\lambda_t$.
  - All asset returns, price-dividend ratio are highly correlated with each other.
  - Model displays constant risk premia, can’t address evidence on predictability of excess returns.
Extended model

- Stochastic processes for consumption and dividend growth

\[
\log(C_{t+1} / C_t) = \mu + \alpha_c (\sigma^2_{t+1} - \sigma^2) + \pi_c \lambda \varepsilon_{t+1}^c + \sigma_t \varepsilon_{t+1}^c
\]

\[
\log(D_{t+1} / D_t) = \mu + \alpha_d (\sigma^2_{t+1} - \sigma^2) + \sigma_d \sigma_t \varepsilon_{t+1}^d + \pi_d \lambda \varepsilon_{t+1}^d + \pi_d c \sigma_t \varepsilon_{t+1}^c
\]

\[
\sigma^2_{t+1} = \sigma^2 + \nu (\sigma^2_t - \sigma^2) + \sigma_w \omega_{t+1}
\]

- Conditional heteroskedasticity in consumption generates time-varying risk premia
  - When volatility is high, stock is risky, price of equity is low, expected return is high.
  - High volatility leads to higher precautionary savings motive so that the risk-free rate falls, reinforcing rise in risk premium.
Allow for correlation between time-preference shocks, growth rate of consumption and dividends.

In production economy, time-preference shocks induces changes in aggregate output, consumption.

Taken literally, endowment economy doesn’t allow for such co-movements.
Extended model: additional extensions

- Benchmark model: price-dividend ratio, risk-free rate driven by single state variable, so they have same degree of persistence.

- Extended model: assume $\lambda_{t+1}/\lambda_t$ is sum of a persistent shock and an i.i.d. shock:

  \[
  \log(\lambda_{t+1}/\lambda_t) = x_t + \sigma_\eta \eta_{t+1}, \\
  x_{t+1} = \rho x_t + \sigma_\lambda \varepsilon_{t+1}^\lambda.
  \]

  - $x_t$: low-frequency changes in growth rate of discount rate.
  - $\eta_{t+1}$: high-frequency changes in investor sentiment that affect demand for assets.
Estimate model parameters using GMM

- Find parameter vector $\hat{\Phi}$ that minimizes distance between empirical, $\Psi_D$, and model population moments, $\Psi(\hat{\Phi})$,

$$L(\hat{\Phi}) = \min_{\Phi} [\Psi(\Phi) - \Psi_D]' \Omega_D^{-1} [\Psi(\Phi) - \Psi_D].$$

- We found GMM estimator is subject to small sample bias, especially predictability of excess returns.

- Focus on plim of model-implied small-sample moments when constructing $\Psi(\Phi)$, rather than plim of moments.
  
  - For given $\Phi$, create 500 synthetic time series, each of length equal to our sample size.
  - On each sample, calculate sample moments of interest.
  - Vector $\Psi(\Phi)$ that enters criterion function is average value of sample moments across synthetic time series.
Temporal Aggregation Issues

- We assume that agents make decisions at a monthly frequency.

- Derive model’s implications for variables computed at an annual frequency.
We use realized real stock returns.

As in Mehra and Prescott (1985), we measure risk free rate using realized real returns on nominal, one-year Treasury Bills.

This measure is far from perfect because there’s inflation risk, which can be substantial.

Alternative: use time-varying VARs to bridge very different monetary regimes (D’Agostino and Surico, 2013, Luo 2014.).

More about this approach when we discuss the term-structure of bonds.
Parameter estimates

- Coefficient of risk aversion is quite low (1.6 and 1.2) in benchmark, extended models.

- For both models, IES is somewhat larger than one (about 1.4).

- For both models, point estimates easily satisfy necessary condition for valuation risk to be positive ($\theta < 1$).

- Parameter $\rho$ (governs serial correlation of $\lambda_{t+1}/\lambda_t$) is estimated to be high in both models (0.991 and 0.997).

- Parameter $\nu$, which governs persistence of consumption volatility in extended model, is also quite high (0.962).

- High degree of persistence in $\lambda_{t+1}/\lambda_t$ and volatility shock: root cause of small-sample biases in standard GMM estimators.
# Equity-premium statistics

<table>
<thead>
<tr>
<th>Moments</th>
<th>Data</th>
<th>Data</th>
<th>Model</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Constrained</td>
<td>Unconstrained</td>
<td>Benchmark</td>
<td>Extended</td>
</tr>
<tr>
<td>$E(r_{d,t})$</td>
<td>7.55 \ (1.74)</td>
<td>6.20 \ (1.87)</td>
<td>6.11</td>
<td>3.63</td>
</tr>
<tr>
<td>$E(r_{d,t}) - E(r_{f,t})$</td>
<td>7.19 \ (1.77)</td>
<td>6.13 \ (1.84)</td>
<td>5.75</td>
<td>3.24</td>
</tr>
</tbody>
</table>

- Taking sampling uncertainty into account, models account for equity premium.
- Result holds even though estimated degree of risk aversion is moderate in both models.
- In contrast, LRR models require high degree of risk aversion to match equity premium.
For valuation risk to contribute to equity premium, we need $\theta < 1$.

- Estimated value of $\theta$ is $-2.00$ (0.23) and $-0.74$ (0.10) in benchmark and extended model.

Taking sampling uncertainty into account

- Benchmark model easily accounts for equity premium.
- Extended model does so marginally.

Easily reject null hypothesis of $\theta = 1$, CRA case.
<table>
<thead>
<tr>
<th>Risk-Free Rate Moments</th>
<th>Data Constrained</th>
<th>Benchmark Model</th>
<th>Extended Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average risk free rate</td>
<td>0.36 (0.81)</td>
<td>0.36</td>
<td>0.387</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>3.19 (0.80)</td>
<td>3.99</td>
<td>3.48</td>
</tr>
<tr>
<td>First order serial correlation</td>
<td>0.60 (0.08)</td>
<td>0.90</td>
<td>0.62</td>
</tr>
</tbody>
</table>
The correlation puzzle

- Benchmark model has to produce correlations that are essentially invariant across horizon.
  - Consumption, dividends follow random walk.
  - Estimated process for growth rate of $\lambda_{t+1}/\lambda_t$ is close to random walk.

- In extended model
  - Persistent changes in variance of growth rate of consumption, dividends can induce persistent changes in conditional means.
  - So model produces correlations that vary across different horizons.
The correlation puzzle: benchmark model

- Benchmark model does well at matching correlation between stock returns, consumption growth
  - In data, this correlation is similar at all horizons.

- Empirical correlation between stock returns, dividend growth increases with horizon.

- Estimation procedure chooses to match long-horizon correlations, does less well at matching yearly correlation.
  - Hard for model to capture yearly correlation because dividend growth rate enters directly into equation for stock returns.
### Implications for the correlation puzzle

<table>
<thead>
<tr>
<th>Moments</th>
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<tbody>
<tr>
<td>1-year correlation between equity returns and consumption growth</td>
<td>-0.03 (0.12)</td>
<td>-0.05 (0.12)</td>
<td>0.047</td>
<td>0.062</td>
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<td>5-year correlation between equity returns and consumption growth</td>
<td>0.07 (0.17)</td>
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<td>1-year correlation between equity returns and dividend growth</td>
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<td>0.345</td>
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<td>0.325</td>
<td>0.024</td>
</tr>
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<td>10-year correlation between equity returns and dividend growth</td>
<td>0.51 (0.22)</td>
<td>0.59 (0.14)</td>
<td>0.386</td>
<td>0.100</td>
</tr>
</tbody>
</table>
Extended model generates upward profile

- Increase in \((\sigma^2_{t+1} - \sigma^2)\) decreases \(\log(D_{t+1}/D_t)\).
- When volatility is high, returns to equity are high.
- So one-year correlation between dividend growth, equity returns is negative.
- Variance of shock to dividend growth rate is mean reverting so this effect becomes weaker as horizon extends.
- Direct positive effect of dividend growth on equity returns eventually dominates.

Implied correlations are consistent with data, taking sampling uncertainty into account.
The correlation puzzle: extended model

- Estimation algorithm chooses parameters to allow model to do better at matching 1, 5 year correlations.

- Model does less well ten-year correlation.

- Choice reflects greater precision relative precision with which correlations are estimated.

- Extended model matches correlation between stock returns, consumption growth, taking sampling uncertainty into account.
## Implications for the correlation puzzle

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## Matching the equity premium

<table>
<thead>
<tr>
<th></th>
<th>Data Constrained</th>
<th>Extended Model Match equity premium</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \gamma )</td>
<td>-</td>
<td>1.205 (0.029)</td>
</tr>
<tr>
<td>( \psi )</td>
<td>-</td>
<td>1.382 (0.004)</td>
</tr>
<tr>
<td>( E(r_{d,t}) )</td>
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<td>3.62</td>
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</tr>
<tr>
<td>( \theta )</td>
<td>-0.74 (0.10)</td>
<td></td>
</tr>
</tbody>
</table>
Model continues to produce low correlations between stock returns, consumption growth.

But one-year correlation between stock returns, dividend growth implied by model is much higher than in data.

One-year correlation between stock returns, dividend growth is estimated much more precisely than equity premium.

Estimation algorithm chooses parameters that imply lower equity premium to match one-year correlation between stock returns.
### Matching the equity premium

<table>
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<th>Data Constrained</th>
<th>Extended Model Match equity premium</th>
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<tbody>
<tr>
<td>1 year</td>
<td>0.08 (0.12)</td>
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<td>5 year</td>
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<tr>
<td>10 year</td>
<td>0.51 (0.22)</td>
<td>0.10</td>
</tr>
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</table>

0.64

0.56

0.58
Comparison with LRR model

• BKY (2012).
  • Correlation between stock returns, consumption growth are 0.66, 0.88, and 0.92 at 1, 5, 10 year horizons.
  • Correlations between stock returns, dividend growth are 0.66, 0.90, and 0.93 at 1, 5, 10 year horizons.

• Both sets of correlations are counterfactually high.

• Source of problem: all uncertainty in LRR model stems from endowment process.
Both benchmark, extended models match average of price-dividend ratio very well.

Benchmark model somewhat under predicts persistence, volatility of price-dividend ratio.
- Risk-free and price-dividend ratio have same persistence.
- Estimation algorithm splits the difference.

Extended model does much better at matching those moments.
- Moments implied by this model are within two standard errors of sample counterparts.
## Price-dividend ratio moments

<table>
<thead>
<tr>
<th>Price-dividend Moments</th>
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<th>Extended Model</th>
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<tbody>
<tr>
<td>Average price-dividend ratio</td>
<td>3.38 (0.15)</td>
<td>3.16</td>
<td>3.57</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.45 (0.08)</td>
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<td>0.49</td>
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<td>First order serial correlation</td>
<td>0.95 (0.03)</td>
<td>0.84</td>
<td>0.92</td>
</tr>
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</table>
Predictability of excess returns

- LHS: cumulative excess returns, $k$ periods, $k = 1, 3, 5$

\[ \sum_{j=1}^{k} (r_{dt+j} - r_{ft+j}) = a_0 + a_{1k} \left( \frac{P_t}{D_t} \right) + \varepsilon_{t+j} \]

- Evidence that $a_{1k} < 0$.

- Benchmark model, consumption is a martingale with conditionally homoscedastic innovations.
  - By construction excess returns are unpredictable in population.

- Stambaugh (1999), Boudoukh et al. (2008): predictability of excess returns may be artifact of small-sample bias and persistence in the price-dividend ratio.

- Our results are consistent with this hypothesis.
Predictability of excess returns by price dividend ratio

<table>
<thead>
<tr>
<th></th>
<th>Data (median)</th>
<th>Model (median)</th>
<th>Model (plim)</th>
<th>Data (median)</th>
<th>Model (median)</th>
<th>Model (plim)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Slope Coefficient</strong></td>
<td></td>
<td></td>
<td></td>
<td>R-square (% of values larger than R-square in data)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 year</td>
<td>-0.09</td>
<td>-0.05</td>
<td>0.005</td>
<td>0.04</td>
<td>0.01</td>
<td>0.0001</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.03)</td>
<td></td>
<td>(0.12)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 years</td>
<td>-0.26</td>
<td>-0.14</td>
<td>0.021</td>
<td>0.13</td>
<td>0.03</td>
<td>0.0006</td>
</tr>
<tr>
<td></td>
<td>(0.07)</td>
<td>(0.07)</td>
<td></td>
<td>(0.09)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5 years</td>
<td>-0.39</td>
<td>-0.21</td>
<td>0.025</td>
<td>0.23</td>
<td>0.04</td>
<td>0.0006</td>
</tr>
<tr>
<td></td>
<td>(0.11)</td>
<td>(0.11)</td>
<td></td>
<td>(0.06)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Predictability of excess returns by price dividend ratio

<table>
<thead>
<tr>
<th></th>
<th>Data (median)</th>
<th>Model (median)</th>
<th>Model (plim)</th>
<th>Data (median)</th>
<th>Model (median)</th>
<th>Model (plim)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Slope Coefficient</td>
<td>R-square (% of values larger than R-square in data)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 year</td>
<td>-0.09 (0.03)</td>
<td>-0.05 (0.03)</td>
<td>-0.01 (0.03)</td>
<td>0.04 (0.23)</td>
<td>0.02 (0.23)</td>
<td>0.001 (0.23)</td>
</tr>
<tr>
<td>3 years</td>
<td>-0.26 (0.07)</td>
<td>-0.14 (0.07)</td>
<td>-0.02 (0.07)</td>
<td>0.13 (0.19)</td>
<td>0.05 (0.19)</td>
<td>0.002 (0.19)</td>
</tr>
<tr>
<td>5 years</td>
<td>-0.39 (0.11)</td>
<td>-0.22 (0.11)</td>
<td>-0.03 (0.11)</td>
<td>0.23 (0.16)</td>
<td>0.08 (0.16)</td>
<td>0.004 (0.16)</td>
</tr>
</tbody>
</table>
Implications for the bond term premium

- In models that stress LRR, long-term bonds command a negative risk premium.
  - This negative premium reflects fact that long-term bonds are a hedge against long-run risk (Piazzesi and Schneider (2006)).
  - BKY model implies a 10-year yield of -0.43 percent and 20-year yield of -0.88.

- Standard rare-disaster models also imply downward sloping term structure for real bonds and negative real yield on long-term bonds.

- Our model implies long-term bonds receive a positive premium, upwards sloping term structure.
Following table presents key statistics for ex ante, ex-post real returns to short-term, intermediate-term long-term government bonds (1-year, 5 year, 20 year).

Luo (2014) constructs alternative models of expected inflation for one, five and ten-year horizons.

- Sample period: 1870 - 2011.

Random walk model better job at forecasting one-year inflation than:

- time-varying VAR methods (Primiceri (2005))
- Bayesian VARs (MN priors).

Bayesian VARs do best forecasting inflation at five, ten year horizons.
### Implications for the term premium, levels

<table>
<thead>
<tr>
<th>Bond Type</th>
<th>Ex post Unconstrained</th>
<th>Ex ante Unconstrained</th>
<th>Model Benchmark</th>
<th>Model Extended Model</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Long-term bond</td>
<td>1.32 (1.01)</td>
<td>2.90 (0.84)</td>
<td>5.14</td>
<td>2.82</td>
</tr>
<tr>
<td>Int.-term bond</td>
<td>1.39 (0.91)</td>
<td>1.93 (0.99)</td>
<td>2.29</td>
<td>1.39</td>
</tr>
<tr>
<td>One-year bond</td>
<td>0.42 (0.80)</td>
<td>0.46 (0.78)</td>
<td>0.36</td>
<td>0.39</td>
</tr>
<tr>
<td>(r_{d,t}-)long-term yield</td>
<td>4.16 (2.39)</td>
<td>2.54 (2.09)</td>
<td>1.07</td>
<td>0.78</td>
</tr>
</tbody>
</table>
Implications for the term premium, volatility

<table>
<thead>
<tr>
<th></th>
<th>Ex post Unconstrained</th>
<th>Ex ante Unconstrained</th>
<th>Model Benchmark</th>
<th>Model Extended Model</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Standard deviation</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Long-term</td>
<td>3.02 (0.65)</td>
<td>2.59 (0.52)</td>
<td>1.73</td>
<td>2.11</td>
</tr>
<tr>
<td>Intermediate-term</td>
<td>3.29 (0.53)</td>
<td>3.14 (0.57)</td>
<td>3.19</td>
<td>2.64</td>
</tr>
<tr>
<td>One year</td>
<td>3.87 (0.77)</td>
<td>3.85 (0.77)</td>
<td>3.97</td>
<td>3.48</td>
</tr>
<tr>
<td>$r_{d,t} - $long-term yield</td>
<td>20.2 (2.47)</td>
<td>20.09 (1.96)</td>
<td>15.61</td>
<td>18.08</td>
</tr>
</tbody>
</table>
Bond results

- Real yield on long-term bond are positive, statistically significant from zero.
  - Consistent with Campbell, Shiller and Viceira (2009): real yield on long-term TIPS has always been positive, usually above 2%.

- Yield curve is upward sloping.
  - Consistent with Alvarez and Jermann (2005).

- Taking sampling uncertainty into account, extended model is consistent with our data.
Equity and Term Premia

- Equity premium in our model isn’t solely driven by term premium.

- Regressing equity premium on two alternative measures of excess bond yields.
  - Difference between yields on bonds of 20 year and 1 year maturities.
  - Difference between yields on bonds of 5 year and 1 year maturities.

- Table 9 reports our results.
  - For both models, slope coefficients are quite close to point estimates.
  - Both models are consistent with fact that $R^2$ in these regressions are quite low.
Regressions of Excess Stock Returns on Long Term Bond Yields in Excess of Short Rate

<table>
<thead>
<tr>
<th>Data</th>
<th>Data</th>
<th>Benchmark Model</th>
<th>Extended Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>1929-2011</td>
<td>1939-2011</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Long Term Gov. Bond (20 years)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>R-square (%) 0.04</td>
<td>0.04</td>
<td>0.13 (0.96)</td>
<td>0.02 (0.27)</td>
</tr>
<tr>
<td>Slope 3.49 (1.52)</td>
<td>2.83 (1.72)</td>
<td>3.44</td>
<td>1.16</td>
</tr>
<tr>
<td>Constant -0.72 (3.67)</td>
<td>1.63 (3.49)</td>
<td>-0.05</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Intermediate Term Gov. Bond (5 years)

| R-square (%) 0.02 | 0.03 | 0.07 (0.79) | 0.02 (0.30) |
| Slope 3.89 (2.41) | 3.85 (2.91) | 3.33 | 1.07 |
| Constant 0.87 (3.93) | 2.29 (4.18) | -0.01 | 0.02 |
Model Shortcomings

- Model overstates negative correlation between price-dividend ratio and risk free rate
  - Positive in data, not statistically different from zero.
  - Sharp negative in our models.

- Model understates correlation between stock returns and future consumption growth at one year horizon.
  - Does better at five and ten horizons.

- If you estimate the model, dropping contemporaneous correlations between stock returns and consumption, dividend growth, you do much better on these moments.

- Highlights importance of the correlation puzzle.
Conclusion

- We propose a simple model of asset pricing with valuation risk that accounts for level, volatility of the equity premium and of the risk free rate.

- The model is broadly consistent with the correlations between stock market returns and fundamentals, consumption and dividend growth.
  - The model accounts for these with low levels of risk aversion.

- Key features of the model
  - Consumption and dividends follow random walks; EZ utility; stochastic rate of time preference.
  - Shocks to demand for assets matter.

- Valuation risk is by far the most important determinant of the equity premium and the bond term premia.