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IMPERFECT COMPETITION, UNDEREMPLOYMENT EQUILIBRIA AND FISCAL POLICY*

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A number of recent contributions to macroeconomics have centred on the idea that market economies can be stuck at an inefficiently low level of employment because of 'coordination failure' among market participants. Despite their apparent diversity, all these models share two common features (Cooper and John, 1988); *first*, the presence of externalities or 'spillovers', meaning that individual choices affect the welfare of others directly, rather than via the market (eg. 'if I buy a fax machine, those who need to contact me are better off'); *second*, the existence of 'strategic complementarities', in the sense that the choice rules of individuals produce a positive correlation between their equilibrium decisions ('if I buy a fax machine, those who need to contact me may decide to buy one'). If both features are present, a model can generate multiple equilibria, characterised by different levels of activity[1] – or employment, if the focus is on the labour market. A key property of these equilibria is their welfare ranking: high-employment equilibria Pareto-dominate low-employment ones, often called 'underemployment' equilibria.

The differences between coordination failure models derive from the specific externality they focus on. As suggested by Cooper and John (1988) and Drazen (1987), the main distinction is between models where the externality arises from joint production and those where it stems from demand spillovers and imperfect competition. Joint production is taken here in the broad sense that some economic 'good' can be produced only by the joint activity of several agents.[2] Demand spillovers, instead, produce reciprocal externalities if firms behave non-competitively, i.e. react to quantity as well as to price signals.[3]

The model presented here belongs to the 'demand spillover' variety. As the model of underemployment by Weitzman (1982) and Solow (1984), it builds upon the spatial analysis of monopolistic competition proposed by Salop

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[1] In the example, different sizes of the market for fax machines.

[2] In some of these models the jointly-produced good is a commodity (Bryant, 1983), while in others it takes the form of an increase in trade opportunities (Diamond, 1982, 1984; Howitt and McAfee 1985), a reduction in uncertainty (Chatterjee, 1988 and Pagano, 1989*a*) or an increase in market liquidity (Pagano, 1989*b*).

[3] Demand spillovers may occur between product markets (Cooper and John, 1988; Chatterjee and Cooper, 1988; Heller, 1986; Roberts, 1988; Weitzman, 1982) or between product and factor markets (Drazen, 1985). Moreover, imperfect competition has been modelled in various ways: in Hart (1982) firms and unions behave as Cournot quantity-setters, while in the other models firms behave as price-setting monopolistic competitors.

(1979).[4] The present paper, however, adds two novel features to the Weitzman–Solow framework: it treats explicitly both the consumption-saving decisions and the labour supply choices of households (Section I).

Each of these two features has important implications. By modelling saving behaviour explicitly within an overlapping generations framework, I can analyse the intertemporal effects of fiscal policy, without assuming that the government's actions reduce to changes in the supply of a non-produced good to consumers (as in Hart (1982) and in Blanchard and Kiyotaki (1987)). Introducing labour supply in the analysis serves for its part the purpose of closing the Weitzman-Solow model. The key result of that model – that appears also in this paper – is that the demand for labour is positively related to the real wage across steady-state zero-profit equilibria. Weitzman and Solow then proceed by assuming an exogenous unemployment rate and argue informally that, since any such rate is stable, it is also an equilibrium in a dynamic sense. Being closed with a micro-based labour supply curve and a competitive labour market, instead, the present model can be solved explicitly for equilibrium.

The model produces two key results. *First*, the economy can have multiple steady-state, zero-profit equilibria: those with higher employment feature also a higher real wage and welfare level (Section II), and stable equilibria alternate with unstable ones (Section III). *Second*, fiscal policies that lower national saving, such as a balanced-budget or debt-financed increase in expenditure, decrease the steady-state level of welfare (Section IV).

To understanding heuristically how in this model multiple equilibria can result from the demand externality between firms, consider the following thought experiment. Assume that initially the economy is at a stable steady-state equilibrium with zero profits, and suppose that there is an increase in employment. Due to the imperfectly competitive nature of the equilibrium, profits are an increasing function of aggregate demand and thus of real labour income. Thus the increase in employment tends to raise profits, and thereby to promote entry of new firms. The increased competition associated with entry, compressing the rate of markup, lowers prices, and so increases the real wage, for any level of the real interest rate. With an elastic labour supply, this increase in the real wage raises employment. If this increase in employment equals that postulated at the beginning, also the new employment level is an equilibrium. However, if the initial equilibrium is stable, no firm has the incentive to start the process on its own, either by hiring additional workers (if it is already operating) or by entering the market (if it is not). As we shall see below, the

[4] Rather than using Salop's spatial model, monopolistic competition can be formalised by using the Dixit and Stiglitz (1977) assumption of constant elasticities of substitution in consumption and production, as in the demand spillover models by Drazen (1985) and Blanchard and Kiyotaki (1987). In these models, each consumer demands a little of every commodity, whereas in the models based on Salop's setup (like the model in this paper) each consumer buys only her favourite brand. This difference should not be overstated, however: Salop's analysis can be reinterpreted so as to make 'everyone a generalist in consumption', as in the Dixit and Stiglitz framework. According to Weitzman (1982), in fact, in Salop's model a consumer can be reinterpreted as 'composite of random preference atoms distributed uniformly around the attribute circle'. Thus, despite their formal difference, these two classes of models are more closely related than they appear on the surface.

multiplicity of equilibria crucially depends on a sufficiently large elasticity of the aggregate labour supply.[5]

Obviously, with a competitive labour market, underemployment equilibria are positions of full employment at inefficiently low level or low-participation rate equilibria, as Drazen (1987) calls them. All unemployment is voluntary. However, workers who are unemployed in a low-level equilibrium are happy to work in a high-level one. If it can be eliminated by government policies, such unemployment, though voluntary, is socially wasteful.

Like the multiplicity of equilibria, also the effects of fiscal policy can be given an intuitive interpretation: policies that lower national saving lead to higher real interest rates, which depress profits and cause firms to exit the market. This triggers the process just described in reverse: exit leads to less competition, higher prices, lower real wages, employment and welfare. The fiscal shock can have two types of effects: it can either cause a local shift of the initial equilibrium or push the system to a different, inferior equilibrium. In the latter case, the economy will remain permanently trapped at the new equilibrium also after the policy is discontinued.

Although these results are obtained within a rather specific model, its implications are of more general interest for models of underemployment equilibria. The existence of such equilibria in highly stylised models has often led to the presumption that an aggregate demand expansion would be beneficial. This paper shows that, insofar as fiscal policy is concerned, this is not necessarily true. In fact, in this model, that blends the reciprocal externality among imperfectly competitive firms with a competitive model of the capital and labour market, one obtains a complete reversal of the Keynesian prescriptions for a depressed economy. On one hand, this highlights the importance of fully specifying the model of the macroeconomy before drawing policy conclusions about underemployment equilibria. On the other hand, it suggests that demand externalities due to imperfect competition, though capable of producing underemployment results, do not appear *per se* to hold great promise as foundation for Keynesian-type fiscal policy actions. For this, one may need additional deviations from the competitive, market-clearing standard in modelling the labour or capital market.

I. THE MODEL

The economy can be described as a version of Salop's spatial model of monopolistic competition embedded within an overlapping generation framework.[6] At each date t, there are m_t firms (indexed by i), n young households and n old households (indexed by j). Consumers are uniformly distributed around

[5] Interestingly, a large labour supply elasticity is critical for demand externalities to have large effects on output and employment also in the models of Blanchard and Kiyotaki (1987) and Kiyotaki (1988).

[6] As is well-known, these models may exhibit multiple equilibria by their very structure, but here I want to focus on the multiplicity of equilibria due to demand spillovers among imperfect competitors. Thus I choose functional forms for preferences and technology that ensure a unique steady state equilibrium under perfect competition. For the same reason, I also disregard the potential for indeterminacy of equilibrium studied by Geanakoplos and Polemarchakis (1986), that stems from the expectation that markets may not clear in the future (I assume that markets are expected to clear in all future periods).

a circumference of length H, that I standardise by setting $H = 2n$ (so that there is one consumer in each unit interval). A consumer located at a certain point on the circumference prefers the brand corresponding to that point to other brands. Firms can settle anywhere along the circumference: by settling at a point on the circumference a firm selects its brand type. The distance between consumer j and firm i at time t (denoted by $\delta_{ijt} \equiv |j-i|$) measures the distaste of consumer j for brand i relative to his most preferred one.

After locating, each firm i produces an amount q_{it} of the corresponding brand employing capital and labour. In equilibrium, the demand of all the households who select that brand equals this production level, i.e. $\Sigma_j q^d_{ijt} = q_{it}$. Beside the m_t markets corresponding to the various brands, there is a competitive market for labour, that clears at a nominal wage W_t, and one for capital, that sets the equilibrium nominal interest rate i_t. The rest of this section specifies the behaviour of households and firms in greater detail, and sketches the solutions to their respective choice problems and to the computation of equilibrium in the $m_t + 2$ markets of the economy. Actual derivations of the equations in the text are confined to the Appendix.

(A) *Households*

People live for two periods. In the first period, they take three decisions: they choose if they want to work or not, how much they want to save for the next period and which brand they want to buy in the current period. In their second period, they simply spend their savings, possibly on a different brand.

Their first-period income consists of the after-tax wage $W_t(1-\tau)$ if they work and of the after-tax unemployment benefit $B_t(1-\tau)$ if they do not. The decision to work is a discrete choice: everyone has an indivisible unit of labour time that can sell at the going wage. Unemployment benefits are equal to the taxes levied on the income of the young cohort, i.e. $B_t(n-L_t) = [B_t(n-L_t) + W_t L_t]\,\tau$, where n is the total workforce and L_t the number of employed workers. For the moment, I assume that all tax revenue is spent on unemployment benefits: government spending and debt will enter the picture in Section IV.

Households allocate part of their first-period income to consumption of their favourite brand, and save the rest. Saving takes the form of storage of physical output, that in the subsequent period is made available to firms as capital: storage by households effectively 'transforms' the heterogeneous brands purchased in period $t-1$ into the homogeneous capital demanded by firms in period t. Each young household is assumed to save by purchasing for storage goods of the same brand that it buys for consumption,[7] so that it ends up spending all of its first period disposable income on its favourite brand.

In the second period of its life, household j earns interest $I_{t+1} = 1 + i_{t+1}$ on

[7] This assumption is made for analytical convenience: it makes the elasticity of demand for each brand depend only on the preferences of consumers. If instead firms could borrow from households and buy directly the output to be used subsequently as capital, different brands would have to enter as imperfect substitutes also in production, not just in consumption, and the elasticity of demand for each brand would depend on technological parameters as well. A reasonable way to motivate the assumption that households save by storing the same brand that they choose for consumption is to imagine that there is a (possibly small) fixed cost in moving to a different location to switch brand.

each dollar of its savings, plus a share λ_j of total profits $\Pi^T_{t+1} = \Sigma_i \Pi_{it+1}$ (the household can be thought of as holding a fraction λ_j of the aggregate equity portfolio).[8] I employ two assumptions about the distribution of profits across households, i.e. about the λ_js, that will prove analytically convenient. First, profits are distributed uniformly across locations, so that there is no concentration of demand on any particular brand due to unequal distribution of profit income across different market areas. Second, workers – labour force participants – do not own shares and thus do not earn profits (for them, $\lambda_j = 0$): this simplifies the labour–leisure decision rule that generates the aggregate labour supply curve. This assumption is obviously unnecessary in the analysis of equilibria with zero profits (Section II), but it simplifies the dynamics around these equilibria (Section III), where profits are non-zero – essentially preventing interactions between the dynamics of profits and labour supply choices. Together, the two assumptions imply that capitalists distribute their consumption uniformly across brands, and appropriate all the profits.[9]

To keep things simple, the preferences of households are modelled so that their three choices – saving, brand quality and work – can be analysed in stages rather than simultaneously. The utility of consumer j born at time t is

$$U(x_{jt}, x_{jt+1}, l_{jt}) = \ln x_{jt} + \beta \ln x_{jt+1} - \gamma_j l_{jt}, \quad \beta > 1, \gamma_j > 0, \tag{1}$$

where $$x_{jt} = \sum_{i=1}^{m_t} c_{ijt} e^{-a\delta_{ijt}}, \quad \text{and} \quad l_{jt} = \begin{cases} 1 & \text{if consumer } j \text{ chooses to work,} \\ 0 & \text{otherwise.} \end{cases}$$

The sub-utility function x_{jt} attaches to the consumption of each brand i (c_{ijt}) a weight inversely related to its 'distance' from consumer j's favourite variety (δ_{ijt}) and to the consumer's 'loyalty parameter' (a). The parameter β is the discount factor, and γ_j is the disutility of work effort of individual j. The γ_js are assumed to be uniformly distributed across locations, so that households' labour supply choices (determined by the γ_js) are independent from their brand choices (determined by their location). As we shall see, this ensures that unemployment, and thus income and demand, are uniformly spread across all the locations on the circumference (brands).

Due to the additive form in which brands enter the sub-utility function x_{jt}, the marginal rate of substitution among them is constant. Thus each household demands only one brand (or is indifferent between two adjacent brands). Denoting by i and h the two brands that household j buys in the two periods of its life, one can rewrite the utility function (1) as

$$V(c_{ijt}, c_{hjt+1}, l_{jt}) = \ln c_{ijt} - a\delta_{ijt} + \beta(\ln c_{hjt+1} - a\delta_{hjt+1}) - \gamma_j l_{jt}. \tag{2}$$

[8] There is no stock market in this economy. Equities entitle the owner to receive the firm's profits (and oblige him to cover its losses), but cannot be traded. We can think of them as being bequeathed by each generation to the next one.

[9] Alternatively, one can assume that for the recipients of profit income the disutility of work effort, γ_j, and thus the reservation wage, is zero (this is not implausible if capitalists are owner-managers, since they may derive utility from looking after their business). This assumption is functionally equivalent to that in the text, because it implies that, if the wage is positive, the labour–leisure choice involves only people who earn no profits.

The relevant budget constraint is then

$$p_{ht+1}\, c_{hjt+1} = (Y_{jt} - p_{it}\, c_{ijt})\, I_{t+1} + \lambda_j \prod_{t+1}^{T} \quad (3)$$

where $$Y_{jt} = \begin{cases} W_t(1-\tau) & \text{if consumer } j \text{ is unemployed,} \\ B_t(1-\tau) & \text{otherwise.} \end{cases}$$

The household's decision problem then decomposes in three sequential steps:

(i) *inter-temporal allocation of income*: maximising utility (2) subject to the budget constraint (3), one gets household j's demand for current and future consumption (c_{ijt} and c_{hjt+1}), and, summing across households, one obtains the demand for the each brand and the aggregate level of consumption and saving;

(ii) *brand choice*: choosing the value of the index i (and h) so as to maximise indirect utility, each household j selects its preferred brand, that – not surprisingly – turns out to be that closest to its location;

(iii) *labour supply choice*: each individual selects a reservation wage,[10] again by maximising his indirect utility function; ranking people by ascending values of their work disutility γ_j, and thus of their reservation wage, one obtains the aggregate (inverse) labour supply function, whose shape depends on the distribution of γ_j in the population. This distribution, to be denoted by $\gamma(L_t)$, relates each value of the disutility of effort to the number of workers L_t with disutility lower than (or equal to) that value.

(B) *Firms*

Each firm takes two decisions:

(i) the *pricing decision*: it sets the profit-maximising price p_{it} (and the corresponding supply q_{it}), taking its competitors' prices as given and knowing that the number of its customers is a decreasing function of its price;

(ii) the *choice of technique*: it selects the profit-maximising combination of labour and capital to be used in production.

For convenience, I assume that capital and labour are transformed into output according to a Cobb-Douglas function, but that there is a critical scale level f below which no output is obtained:

$$q_{it} = K_{it}^{\alpha}\, L_{it}^{1-\alpha} - f. \quad (4)$$

The assumption of a setup cost (that is equivalent to that of increasing returns) creates an entry barrier that endogenously pins down the number of firms operating at zero-profit equilibria. In its absence, at zero profits the number of firms would be unbounded and the imperfectly competitive feature of the economy would vanish (see also Weitzman (1982) on this point).

[10] The labour supply decision turns out to depend only on the real wage, the level of the unemployment subsidy and on preference parameters, such as the disutility of work effort. If the leisure term were not restricted to be additive in the utility function, the labour-leisure choice would depend also on the real interest rate and the variety of available goods. This gain in generality would make the analysis considerably more intricate, obscuring the presentation of the results. In particular, if labour supply L_t were to depend also on the real interest rate, the dynamic representation of the economy (see equations 8–12 below) would be quite different: it would turn to be a system of second-order difference equations, rather than a first-order system (as in equations 19–20 below). I thank a referee for raising this point.

Each firm i maximises its profits Π_{it}:

$$\Pi_{it} = p_{it}(K_{it}^{\alpha} L_{it}^{1-\alpha} - f) - W_t L_{it} - I_t p_{t-1}^{k} K_{it}, \tag{5}$$

where p_{t-1}^{k} is the price of a unit of capital at time $t-1$. The optimality conditions of firm j, i.e. the familiar equalities between marginal revenue products and factor prices, yield the firm's demand for labour and capital.

(C) *Equilibrium*

In a symmetric Nash equilibrium, all firms charge the same price p_t, and are equally spaced along the circumference, each with a market segment H/m_t and $2n/m_t$ customers. The mark-up above marginal cost is aH/m_t, i.e. it is increasing in the 'loyalty parameter' a and decreasing in the number of competitors m_t.

Symmetry implies also that each firm employs the same number of workers L_t/m_t and the same amount of capital K_t/m_t. Exploiting this, one can easily turn the optimality conditions of individual firms into an expression for the aggregate demand for capital, K_t^d:

$$K_t^d = \frac{\alpha}{1-\alpha}\frac{w_t L_t}{R_t}. \tag{6}$$

where w_t is the real wage W_t/p_t and R_t is the gross real interest $I_t p_{t-1}/p_t$, i.e. 1 *plus* the real rate of interest.

The supply of capital at time t, K_t^s, equals the real resources saved at $t-1$ by those who were young at that time, i.e. the aggregate saving at time $t-1$. The latter obviously depends on the lifetime income of the generation born at time $t-1$, namely on their total labour income $w_{t-1}L_{t-1}$ and on their discounted profit income $m_t \pi_t / R_t$ (where π_t is the real profit per firm):

$$K_t^s = \frac{\beta}{1+\beta} w_{t-1} L_{t-1} - \frac{1}{1+\beta}\frac{m_t \pi_t}{R_t}. \tag{7}$$

Equating the demand for capital (6) with the supply of saving (7) yields capital market equilibrium

$$\frac{\alpha}{1-\alpha}\frac{w_t L_t}{R_t} = \frac{\beta}{1+\beta} w_{t-1} L_{t-1} - \frac{1}{1+\beta}\frac{m_t \pi_t}{R_t}. \tag{8}$$

Given past labour income $w_{t-1}L_{t-1}$, this equilibrium condition determines the real interest R_t as a function of current labour income $w_t L_t$ and profits $m_t \pi_t$.

Exploiting the firms' optimality conditions, one obtains another relationship tying the real interest R_t to the contemporaneous real wage w_t:

$$w_t = \alpha^{\frac{\alpha}{1-\alpha}}(1-\alpha)\left(1+\frac{aH}{m_t}\right)^{-\frac{1}{1-\alpha}} R_t^{-\frac{\alpha}{1-\alpha}}. \tag{9}$$

This relationship describes a 'modified factor-price frontier': under perfect competition, the factor-price frontier would involve only the real wage w_t and interest R_t, and not the markup rate aH/m_t. In fact, the usual factor-price frontier follows from (9) as a special case, letting $m_t \to \infty$. The presence of the markup rate is crucial in the model: a higher number of firms m_t implies a lower markup, i.e. a lower price level, and thus makes a higher real wage w_t consistent with the same real interest R_t.

Using the equilibrium value of R_t in equation (9), one obtains the equilibrium real wage w_t as a function of m_t. The real wage can then be fed in the aggregate supply of labour to solve for employment L_t. As explained at the end of Section I. A, the inverse labour supply schedule is a function of the distribution of the disutility of effort in the population, $\gamma(L_t)$:

$$\ln w_t = \ln b + \gamma(L_t)/(1+\beta) \rightarrow L_t = L(w_t),\ L'(\cdot) \geqslant 0, \tag{10}$$

where $b = B_t/p_t$, the real unemployment subsidy, is supposed to be constant over time and the specific functional form of the labour supply function $L(w_t)$ hinges on the distribution of effort disutility $\gamma(L_t)$. For instance, if $\gamma(L_t)$ is a logarithmic function, then the labour supply function is isoelastic.

At this point, one has imposed equilibrium in the capital and the labour market: equations (8), (9) and (10) pin down the market-clearing values of R_t, w_t and L_t. But there is still one degree of freedom left in the model: the number of firms m_t. This is determined endogenously by the entry decisions of firms in response to profits. I suppose that profits (losses) induce gradual entry (exit) according to the simple rule:

$$m_t = m_{t-1} + \theta\pi_t, \quad \theta > 0. \tag{11}$$

The final step is to relate real profits per firm π_t to the other macroeconomic variables. Evaluated at the optimum, real profits π_t turn out to be negatively related to the number of firms and positively related to real labour income:

$$\pi_t = \frac{aH}{(1-\alpha)\, m_t^2} w_t L_t - f. \tag{12}$$

Equation (12) shows that the sensitivity of profits to real labour income $w_t L_t$ depends on the imperfectly competitive nature of the equilibrium. In a competitive setup, the markup rate aH/m_t is zero (as $m_t \rightarrow \infty$) and profits do not respond to aggregate income and demand.

The equilibrium condition in the capital market (8), the modified factor-price frontier (9) and the supply of labour (10), complemented by the assumption on entry in (11) and the expression for profits (12), form a dynamic system of five independent equations. The system determines the five unknowns w_t, R_t, L_t, m_t and π_t, for given values of the predetermined variables w_{t-1} and L_{t-1}. These five equations are the building blocks in the analysis of zero-profit steady-state equilibria and in that of their stability.

II. ZERO-PROFIT STEADY-STATE EQUILIBRIA

Consider an equilibrium position where all variables are at their steady state values (and thus are unchanging over time) and profits are zero:

$$\pi_t = \frac{aH}{(1-\alpha)\, m_t^2} w_t L_t - f = 0. \tag{13}$$

The equilibrium condition in the capital market (8) then reduces to:

$$R_t = \frac{\alpha}{1-\alpha}\frac{1+\beta}{\beta}. \tag{14}$$

With zero profits, the equilibrium real interest rate is a constant. Substituting this constant into the factor-price frontier (9), one obtains a positive relationship between the real wage w_t and the number of firms m_t. The number of firms is determined by the zero-profits condition: using (13) in the factor-price frontier to substitute out m_t, one gets an equation involving only the real wage w_t and the employment level L_t:

$$w_\tau^{1-\alpha}\left\{1+\left[\frac{f(1-\alpha)aH}{w_t L_t}\right]^{\frac{1}{2}}\right\} = (1-\alpha)\left(\frac{\beta}{1+\beta}\right)^{\alpha}. \tag{15}$$

This equation has been derived by imposing equilibrium in all markets except one – the labour market. In fact, it embodies only the demand side of the labour market, having been obtained from the relationship between demand for labour and capital by firms (the factor-price frontier). Thus, the employment–wage locus implied by (15) can be seen as the demand for labour that is consistent with equilibrium in all the other markets (at zero-profit steady states). To clear the labour market also, one must use the labour supply equation (10). Graphically, equilibrium obtains where the 'labour demand' locus defined by (15) – labelled LL – crosses the labour supply curve $L(w_t)$ in (10).

The potential for multiple equilibria becomes apparent upon observing that the 'labour demand' locus LL is upward-sloping (at least over part of its domain): since labour supply is non-decreasing by construction, the two curves can have multiple intersections, as shown in Fig. 1*a–c*. Total differentiation of (15) in fact shows that the slope of the LL locus is positive everywhere if $1-\alpha > \frac{1}{2}$ (i.e. labour has a larger share than capital in factor income),[11] with a finite asymptote at real wage w^*:

$$\left.\frac{dL_t}{dw_t}\frac{w_t}{L_t}\right|_{LL} = 2(1-\alpha)\left\{1+\left[\frac{w_t L_t}{f(1-\alpha)\,aH}\right]^{\frac{1}{2}}\right\}-1 = 2(1-\alpha)\left(1+\frac{m_t}{aH}\right)-1. \tag{16}$$

The fact that in this economy a larger real wage is consistent with higher employment can be explained heuristically as follows. More employment raises demand, output and profits, inducing entry. The latter leads to lower prices, implying that the initial increase in employment is consistent with a higher real wage. Hence the positive slope of the 'labour demand' locus LL, which is the analogue, in this model, of the positive relationship between employment and

[11] If instead $1-\alpha < \frac{1}{2}$, the locus has a C-shape: its slope is negative for low values of w_t, corresponding to low real income $(w_t L_t)$, low number of firms and high markup rate (aH/m_t), and eventually turns positive for higher values of w_t. This case is worth mentioning because it can produce *two equilibria even if labour supply is totally inelastic*: since the LL has a C-shape, a vertical $L(w_t)$ line can intersect it at two different levels of the real wage. This is in contrast with what happens in the monotone increasing case $1-\alpha > \frac{1}{2}$ (see text, below). Since realism suggests that labour has the larger share in factor income, in the text I concentrate on the monotone case $1-\alpha > \frac{1}{2}$. In either case, the LL locus has an upper finite asymptote at $w^* = (1-\alpha)^{1/1-\alpha}(\beta/1+\beta)^{\alpha/1-\alpha}$

real wage across zero-profit equilibria found by Weitzman (1982) and Solow (1984) in their one-period model. In fact, since LL is a zero-profit locus, the points above it are wage-employment combinations that imply *losses* (the wage is too high for firms to break even at that employment level), whereas the points below it are associated with *profits*.[12]

What pins down the number of equilibria is the shape of the labour supply curve $L(w_t)$ – the missing equation of the Weitzman–Solow model. For multiple equilibria to arise, the aggregate supply of labour must be *elastic*, at least if the LL locus is upward-sloping everywhere (i.e. if $1-\alpha > \frac{1}{2}$).[13] The isoelastic case ($L_t = Aw_t^{\epsilon}$) is shown in Fig. 1*a*. Here there can be up to three equilibria, one of which is at zero activity level, provided the elasticity parameter is large enough ($\epsilon > 2(1-\alpha)-1$). If the elasticity changes over the domain of the labour supply function, the number of equilibria can be greater, as shown by Fig. 1*b*, where labour supply is still continuous, and by Fig. 1*c*, where it is stepwise, implying an elasticity switching discretely between 0 and infinity. The latter occurs when workers cluster in groups with different reservation wages (all members of a group having the same work disutility γ_j).

Beside changes in the elasticity of labour supply, another factor that can increase the number of equilibria is the existence of firms with different fixed costs, as in Chatterjee and Cooper (1988).[14] So far all firms have been assumed to face the same cost f to start production. Assume instead that they fall in K 'cost classes', each formed by m_k firms and ranked by increasing values of the fixed cost f_k (i.e. $f_k > f_{k-1}$, for $k = 1, 2, \dots K$). This creates a discontinuity in the LL locus, as shown in Fig. 1*d*: the locus has a stepwise drop every time a new class of firms enters the market, because these firms, having higher costs, make losses at (w_t, L_t) combinations that imply zero profits for existing firms. For each value of L_t, the new entrants require a lower real wage w_t in order to break even. If the labour supply function $L(w_t)$ crosses the LL locus at a point of discontinuity, the equilibrium real wage is such that firms in the next cost class have no incentive to enter, whereas existing firms make profits. This point could obviously be generalised to a continuous distribution of costs across firms. Via this route one introduces an entire new set of potential equilibria.

Each equilibrium can be characterised in terms of the elasticities of the LL and $L(w_t)$ curves (where these are differentiable): when the $L(w_t)$ locus crosses the LL from below, i.e. is steeper at the intersection, the elasticity in (16) exceeds the elasticity of labour supply ϵ_{lw}:

$$2(1-\alpha)\left(1+\frac{m_t}{aH}\right)-1 > \epsilon_{lw}, \quad \text{where} \quad \epsilon_{lw} \equiv L'(w_t)\, w_t / L(w_t) \tag{17}$$

and *viceversa*. This condition coincides with that for local stability, as will be seen in the next section.

[12] One can show that profits π_t are decreasing in the real wage w_t (for given employment L_t) by total differentiation of (8), (9) and (12): the derivative $d\pi_t/dw_t$, evaluated at $\pi_t = 0$, is unambiguously negative, provided $1-\alpha > \frac{1}{2}$.

[13] For the case where $1-\alpha < 0$, see footnote 11.

[14] Also in that model, the participation externality associated with entry (due to the implied fall in the markup), jointly with different fixed costs, gives rise to multiple, Pareto-ranked equilibria.

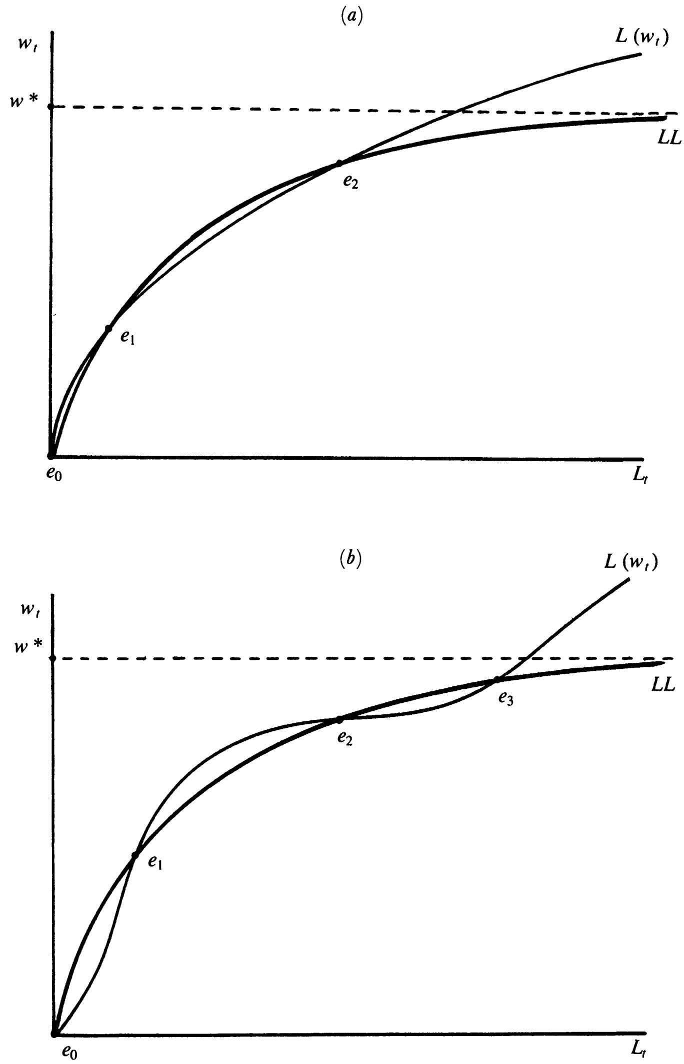
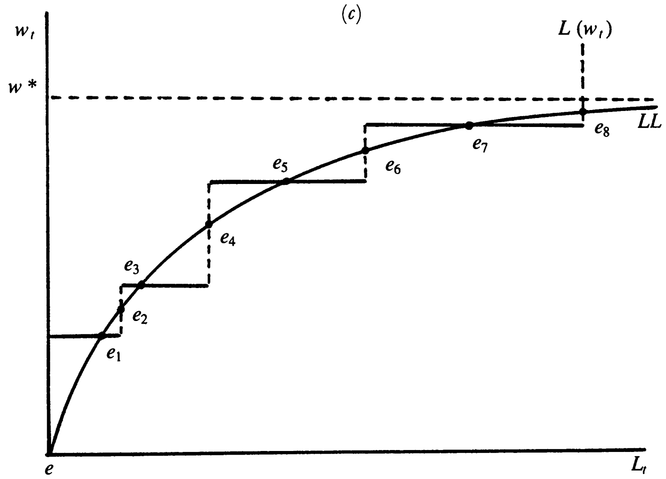


Fig. 1. (*a–b*). For legend see opposite.

The analysis presented so far, by centring on the labour market, reveals most graphically the relationship of this model to that by Weitzman and Solow: the LL locus brings out the same positive relationship between real wage and employment that is present in their model, while the labour supply $L(w_t)$, absent from their model, pins down the equilibria. Fiscal policy and dynamics are, however, much easier to analyse by focussing on the capital market, rather than on the labour market. To this purpose, one can recast the analysis so far performed in (L_t, w_t) space in (R_t, m_t) space, and present it in terms of two relationships between the real rate of interest and the number of firms. The first of these relationships is that of capital market equilibrium, already derived above in (14), and displayed in Fig. 2*a* as the horizontal locus



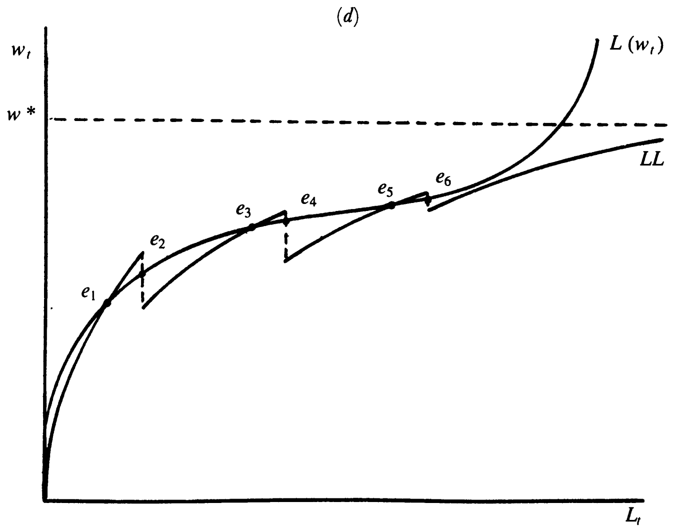


Fig. 1. Multiple steady-state, zero-profit equilibria in (w_t, L_t) space, various cases: (*a*) with an isoelastic labour supply; (*b*) with smooth changes in labour supply elasticity; (*c*) with a stepwise labour supply; (*d*) with firms facing different fixed costs.

RR. The second is obtained using the factor-price frontier (9) and the labour supply (10) into the zero-profit condition (13). The slope of this zero-profit locus, denoted as *ZZ*, is found by implicit differentiation:

$$\left.\frac{dR_t}{dm_t}\frac{m_t}{R_t}\right|_{ZZ} = \frac{\dfrac{1+\epsilon_{lw}}{1-\alpha}\dfrac{aH}{aH+m_t}-2}{\dfrac{\alpha}{1-\alpha}(1+\epsilon_{lw})}. \tag{18}$$

This expression is negative if condition (17) holds, and positive otherwise (provided $1-\alpha > \frac{1}{2}$). Thus, for high values of the elasticity of labour supply ϵ_{lw} and low values of the number of firms m_t, the *ZZ* locus is rising, and in the

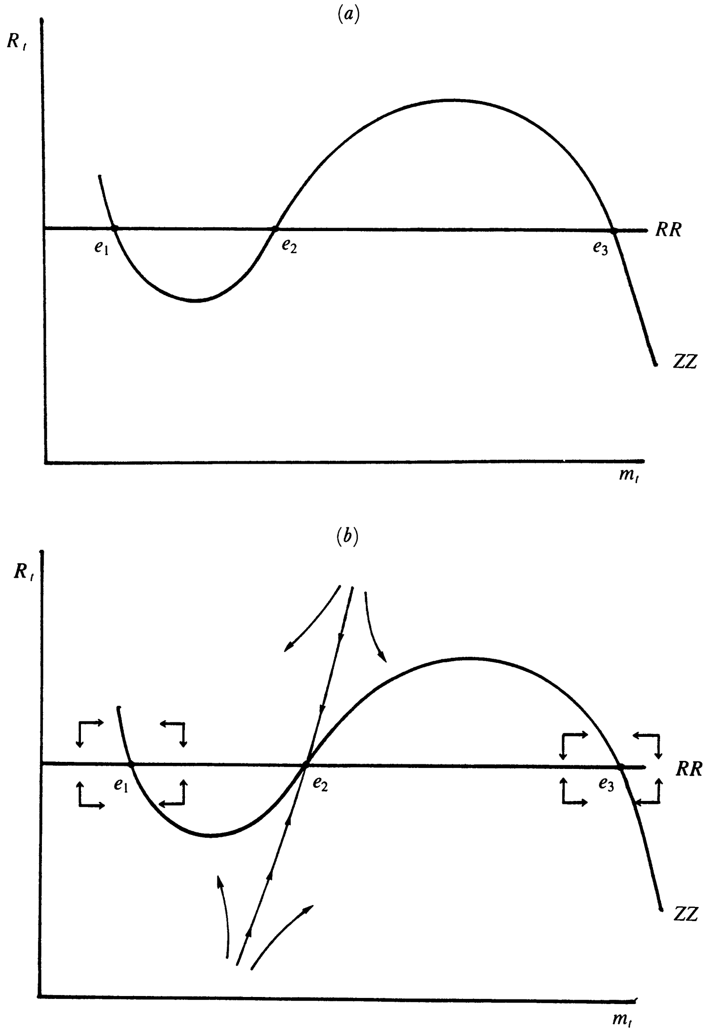


Fig. 2. Multiple steady-state, zero-profit equilibria in (R_t, m_t) space: (a) possible configuration of equilibria; (b) dynamics.

opposite case it is decreasing. As m_t becomes large, the *ZZ* locus becomes eventually downward-sloping (if the elasticity of labour supply is finite).[15]

The points of intersection between the *ZZ* and the *RR* locus are zero-profit, steady-state equilibria: higher activity equilibria feature a larger number of

[15] Intuitively, the reason why the slope of the zero-profit locus is ambiguous can be put as follows. Above this curve profits are positive, and below it firms incur losses, because an increase in the real interest R_t for given m_t lowers profits π_t – from (9), (10) and (12). A higher number of firms m_t, instead, has two opposite effects on profits (for given R_t). It lowers them by reducing the markup rate, and it boosts them by raising real labour income and thus demand. This increase in labour income comes about as the combination of an increase in the real wage (due to the fall in prices) and of its effect on employment. This is where the elasticity of labour supply comes in. The larger it is, the stronger is the expansion of employment and labour income, and the more likely it is that the second effect will prevail. If so, the increase in m_t tends to increase profits, and must be balanced by a higher interest R_t for profits to stay zero, implying a rising *ZZ* locus. Vice-versa, if the elasticity of labour supply is low, the first effect prevails, and *ZZ* slopes downwards.

firms and the same interest rate. This is illustrated in Fig. 2*a*, that shows the same case that Fig. 1*b* displayed in (w_t, L_t) space. Now condition (17) is satisfied at the equilibria where the ZZ locus crosses the RR line from above, such as e_3, and is violated otherwise, i.e. in points like e_2.

The welfare ranking of equilibria is immediate in this model. Consider moving from a low to a high-level equilibrium. The latter features a higher real wage, a higher employment level and the same real interest rate. For workers who were employed also in the low-level equilibrium, lifetime resources increase and the disutility of work does not change, so there is an unambiguous welfare increase. For newly employed workers, the real wage exceeds the disutility of working and of losing the unemployment subsidy, or they would not supply their labour. So they are better off too. For the others, who stay unemployed also in the higher-level equilibrium, welfare is unchanged. In conclusion, the higher-level equilibrium Pareto-dominates the other.

The implication is that policies capable of shifting the economy to such an equilibrium are welfare-improving. To show that such policies are needed, however, one must first show that at least some of the low-level equilibria are stable, so that market forces will not themselves promote the transition to a superior equilibrium. I turn to this issue in the next section.

III. STABILITY

Outside of steady-state, zero-profit equilibria, the existence of profits (or losses) causes firms to enter (or exit) the market. The entry process (11) thus mingles with the dynamics intrinsic in the capital market equilibrium condition (8). The entry equation (11) can be seen as a simple approximation to a more complex game-theoretic story by which entrants, attracted by profits, fight a price war with existing firms until these accept to move aside in the product spectrum. The costs involved in this adjustment process suggests that entry tends to occur gradually over time, rather than as a one-shot response.

For simplicity, I assume that, around the relevant equilibrium point, labour supply is isoelastic ($L_t = Aw_t^{\epsilon}$). This does not affect the generality of the results, since these are obtained by linearising around steady-state values. With some manipulations, the system formed by (8)–(12) can be reduced to two non-linear first-order difference equations in m_t and R_t:

$$\frac{\alpha}{1-\alpha} = \frac{\beta}{1+\beta}\left(\frac{R_t}{R_{t-1}}\right)^{\frac{\alpha(1+\epsilon)}{1-\alpha}} R_t\left(\frac{1+\dfrac{aH}{m_t}}{1+\dfrac{aH}{m_{t-1}}}\right)^{\frac{1+\epsilon}{1-\alpha}} - \frac{1}{(1+\beta)\,\theta A}\left[\frac{\left(1+\dfrac{aH}{m_t}\right)R_t^{\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}\right]^{\frac{1+\epsilon}{1-\alpha}} m_t(m_t-m_{t-1}), \tag{19}$$

$$m_t = m_{t-1}+\theta\left\{\frac{1}{1-\alpha}\frac{aH}{m_t^2}\left[\frac{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}{\left(1+\dfrac{aH}{m_t}\right)R_t^{\alpha}}\right]^{\frac{1+\epsilon}{1-\alpha}} - f\right\}. \tag{20}$$

Differentiating, linearising around steady-state, zero-profit values and rearranging, one can show that this system is locally stable if condition (17) is satisfied – that is, graphically, if the ZZ locus intersects the RR line from above (or, equivalently, if the labour supply curve $L(w_t)$ crosses the LL locus from below); when instead condition (17) fails, the system can display saddle path stability (see Appendix). Fig. 2*b* shows the laws of motion of the economy for this case: stable and unstable equilibria alternate, the saddle path of the latter acting as boundary between stable regions.

IV. FISCAL POLICY

Now let us introduce fiscal policy into the picture. The only redistributive scheme considered so far is the transfer from employed to unemployed workers. That scheme is neutral with respect to saving choices, because it redistributes income among the young, who have the same propensity to consume. In this section, instead, I analyse the non-neutral fiscal actions: a balanced-budget and a debt-financed increase in public spending.

Consider first the case of a *balanced-budget rise in spending*. Besides levying taxes at rate τ to pay for unemployment benefits, the government starts a new spending scheme and finances it with an additional tax, levied at rate t. I suppose that the spending scheme consists of transfers to the old generation. This avoids the complications that would arise if the government did the spending itself (thus generally altering the elasticity of demand for individual brands), while capturing the essential point: since the propensity to consume of the old equals 1, the transfer produces an increase in spending.

Taxes are levied on the income of the young – an innocuous assumption, since any revenue collected from the old would be eventually rebated to them anyway. I suppose that taxes are paid at the same rate out of wage income or unemployment compensation (so as to avoid feedbacks on labour supply choices), and that each old household receives a transfer equal to the taxes paid when young. The taxes paid for the unemployment subsidy are deductible from the base of the new tax. Under these assumptions, it is easy to show that the after-tax real income of the young is simply $(1-t)\,w_t L_t$.[16] In other words, the transfer from employed to unemployed leaves the aggregate income of the young unaffected, and only the new tax (levied at rate t) lowers the disposable income of the young as a group.

The tax unambiguously decreases saving, as it lowers the disposable income of the young and raises by the same amount that of the old. This is clearly seen by rewriting the capital market equilibrium condition (8):

$$\frac{\alpha}{1-\alpha}\frac{w_t L_t}{R_t} = \frac{\beta}{1+\beta}(1-t)\,w_{t-1}L_{t-1} - \frac{1}{1+\beta}\left[\frac{m_t \pi_t + t w_{t-1} L_{t-1}}{R_t}\right], \tag{21}$$

where saving is on the right-hand side. As the supply of capital decreases, the

[16] To show that the after-tax income of the young $(1-t)(1-\tau)[b_t(n-L_t)+w_t L_t] = (1-t)\,w_t L_t$, recall that all the revenue from the tax levied at rate τ is paid as subsidy to the unemployed, i.e. $\tau[b_t(n-L_t)+w_t L_t] = b_t(n-L_t)$.

equilibrium real interest rate must rise. In fact, rewriting (21) in conditions of steady-state, zero-profit equilibrium, one finds that the real rate of interest is higher the larger the tax rate t, i.e. the larger the balanced budget expansion:

$$R_t = \frac{\alpha}{1-\alpha}\frac{1+\beta}{\beta}\frac{1}{1-t}+\frac{1}{\beta}\frac{t}{1-t}. \tag{22}$$

As one would expect, for $t = 0$ this expression for the equilibrium real interest reverts to (14). Graphically, a balanced budget expansion shifts up the RR line, and leaves the ZZ locus unchanged, as shown in Fig. 3*a*.

Thus, if the economy is stuck at the low-level equilibrium e_1, increasing aggregate demand via a balanced-budget expansion moves it to point e_1', and thus reduces output, employment and the real wage, that are all positively related to m_t.[17] Intuitively, the fall in aggregate saving, by raising the real rate of interest and thus the cost of capital, leads to lower profits and exit, and thus to higher markups and lower wages, as well as to lower employment.

Welfare will be unambiguously lower at the new steady-state position if the initial value of the real interest rate is non-negative, i.e. $R_t > 1$, and may be lower even if this condition is not met (see proof in the Appendix).[18] Since the balanced budget expansion reduces saving and thus the aggregate capital stock, it is not surprising to find that it lowers welfare if the real interest rate is positive. It is known since Diamond's (1965) classic work that in overlapping generation models capital decumulation reduces welfare when the rate of interest exceeds the rate of population growth, that in this instance is zero. What is novel here is that this condition is just sufficient, rather than necessary and sufficient: even if the real interest rate is negative, the capital decumulation due to the fiscal expansion can lower welfare, rather than raise it. This is because in this model imperfect competition introduces an additional imperfection relative to standard overlapping generations models, reinforcing the welfare loss due to capital decumulation. Here a smaller capital stock means a lower number of firms and thus a larger departure from the competitive outcome, as well as less product variety for consumers.

The type of experiment just considered is a policy change with 'local' effects. In a model with multiple equilibria, however, changes in policy variables can also cause the economy to pass through a critical point. In this case the policy change is said to have 'catastrophic' effects, in the sense that it moves the economy to a totally different set of equilibria (see Cooper 1987 for an explicit game-theoretic formulation of this distinction). In our context, a catastrophic effect takes place if the government engineers a balanced-budget *reduction* such that the RR' locus passes below the critical point p^* (see Fig. 3*b*). In this case the tax and expenditure reduction is so large that the low-level

[17] These results are reversed for the equilibrium e_2. However, since this is an unstable point, a policy shock of the type considered would inevitably move the economy towards the new point e_1'. For this reason the analysis in the text focusses on the stable equilibria.

[18] Obviously welfare results can be different along the trajectory towards the new steady state, due to intergenerational redistributions: for instance, those who are old when the policy is introduced gain unambiguously.

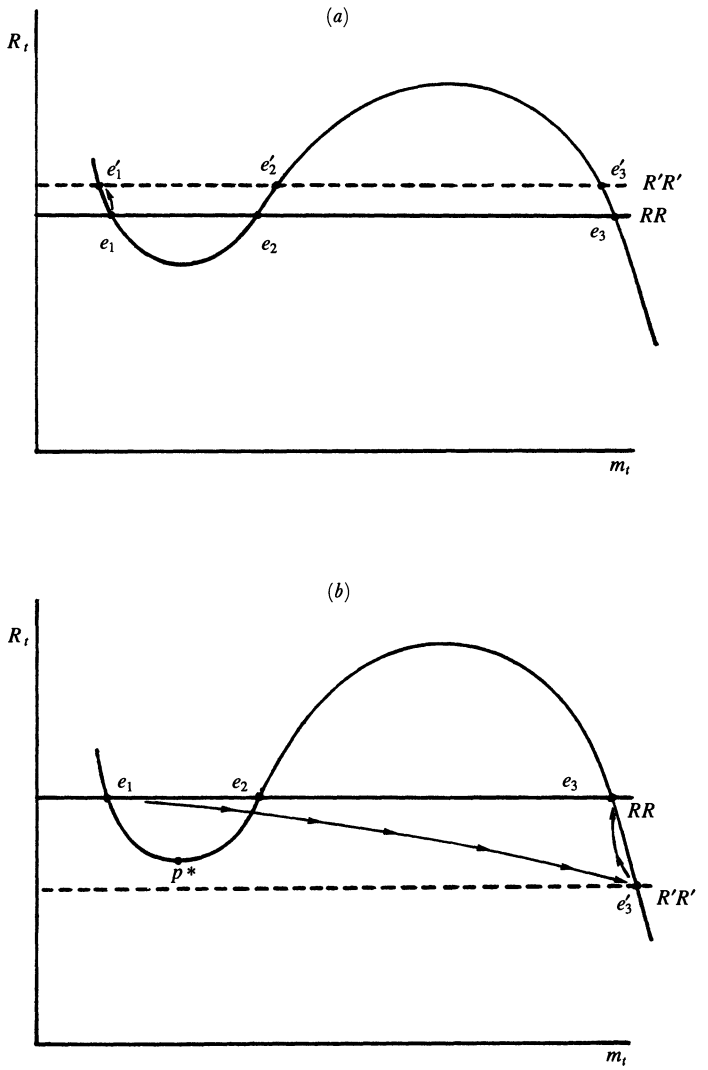


Fig. 3. A balanced-budget rise in spending: (*a*) with 'local' effects; (*b*) with 'catastrophic' effects.

equilibrium actually disappears from the map, and the economy moves towards the stable equilibrium e'_3.[19]

This implies that a policy-maker can move the economy permanently to a superior equilibrium *via* a temporary balanced-budget restriction. If then the government resumes the initial policy stance, the economy will proceed towards point e_3. From Section II we know that equilibrium e_3 *always* Pareto-dominates e_1, as it features a higher real wage and the same real interest rate.

[19] Clearly, the fiscal policy change can have catastrophic rather than local effects even if it is small, if the initial equilibrium is close to the critical point. For a related example, see Kiyotaki (1988). It should be noticed that the experiment described in the text would really require a global stability analysis, rather than the local analysis of Section IV. However, its results are supported by several numerical simulations of the model.

Thus a temporary fiscal restriction can secure a permanent welfare gain. The converse is also true, unfortunately. A temporary fiscal expansion can produce a permanent move to an inferior equilibrium.

Not surprisingly, the analysis of a *deficit-financed increase in public expenditure* runs along similar lines. Assume that the government decides a one-time expenditure G and finances it *via* the issue of debt ($G = D$), so that in the capital market equilibrium condition (21) the left-hand side must be modified by adding public debt D to the demand for capital by firms. In steady state, the government budget constraint implies that debt servicing must be met by taxes, that for convenience I assume again to fall on the young. Thus the tax rate t has to satisfy the condition $tw_t L_t = (R_t - 1) D$. Using this condition and simplifying, the capital market equilibrium condition becomes

$$\frac{\alpha}{1-\alpha}\frac{1}{R_t} + \frac{t}{R_t - 1} = \frac{\beta}{1+\beta}(1-t) \tag{24}$$

in a zero-profit steady state. With debt-financing, the increase of the real interest rate results not only from the reduction of the supply of saving due to taxes but also from the crowding out of private capital by public debt D. Graphically, the debt-financed expansion has the same qualitative effect as a balanced-budget expansion. It shifts the RR line up and leaves the ZZ locus unaffected, leading to a fall in output, employment and the number of firms, and also in welfare if the real interest rate is positive. Conversely, just as a balanced-budget reduction, a policy that uses budget surpluses to foster capital accumulation (ranging from retirement of public debt to tax-financed public investment) can shift the economy to a higher-level equilibrium.

V. CONCLUSION

Until recently, the prevailing views of unemployment have been the Walrasian view, that explains it as the result of intertemporal substitution of leisure or misperceptions of nominal shocks, and the Keynesian view, that attributes unemployment to the slow adjustment of nominal wages and prices. As noted by Cooper and John (1988), coordination failure models have lately emerged as a potential third contender in the field. The point made by these models is that reciprocal externalities and strategic complementarities between economic agents can generate multiple equilibria characterised by different levels of unemployment, and that in the absence of adequate policy intervention the economy can be persistently stuck at equilibria with high unemployment.

The implications of coordination failure models for macroeconomic policy are still largely unexplored. Though in general supportive of the idea that policy intervention can increase welfare, these models are rarely used to study which policy scheme is required, probably because they are still too stylised for this type of analysis. The natural step to take is to enrich their structure, so as to allow the comparison with mainstream macroeconomic models and their policy prescriptions.

This paper moves precisely in this direction. It essentially embodies the reciprocal externality deriving from imperfect competition in a quite orthodox model with intertemporal substitution and labour-leisure choice – an overlapping generations model with elastic labour supply. By modelling saving choices explicitly, I can study the effects of fiscal policy, and particularly their ability to shift the economy from a low-level equilibrium to a high-level one. It turns out that policies that increase aggregate demand (such as a balanced-budget expansion or a debt-financed, one-time increase in public spending) are generally counterproductive, and that instead policies aimed at raising national saving can promote the transition to superior equilibria.

Thus in this model imperfect competition does not provide a sufficient basis for the claim that expansionary fiscal policies can get the economy out of a low-employment trap – the opposite is indeed true. On the one hand, this suggests that in coordination failure models of unemployment the analysis should go beyond the statement that government policy can be beneficial, and address the issue of which policies are to be pursued. On the other hand, it highlights the point that the demand externality due to imperfect competition does not appear to be a promising foundation for Keynesian fiscal policy prescriptions, unless it is supplemented by additional deviations from the competitive standard in the capital or the labour market.

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Appendix

Household Optimisation

(i) *Intertemporal allocation of income*: maximising (2) subject to (3) with respect to c_{ijt}, one obtains the current and future consumption of household j:

$$c_{ijt} = \frac{1}{1+\beta} \frac{Y_{jt} + \lambda_j \Pi^T_{t+1}/I_{t+1}}{p_{it}}, \tag{A 1}$$

$$c_{hjt+1} = \frac{\beta I_{t+1}}{1+\beta} \frac{Y_{jt} + \lambda_j \Pi^T_{t+1}/I_{t+1}}{p_{ht+1}}. \tag{A 2}$$

Substitution of (A 1) and (A 2) in the utility function (2) yields the indirect utility function, that is (up to some additive constants):

$$V(c_{ijt}, c_{hjt+1}, l_{jt}) = (1+\beta)\ln(Y_{jt} + \lambda_j \Pi^T_{t+1}/I_{t+1}) + \beta \ln I_{t+1} \\ - (\ln p_{it} + a\delta_{ijt}) - \beta(\ln p_{ht+1} + a\delta_{hjt+1}) - \gamma_j l_{jt}. \tag{A 3}$$

(ii) *Brand choice*: household j chooses the brand, i.e. the value of the index i, so as to maximise indirect utility (A 3). Clearly, the optimal rule requires choosing the value of i such that

$$\ln p_{it} + a\delta_{ijt} \leqslant \ln p_{kt} + a\delta_{kjt}, \ \forall k \in (0, H]. \tag{A 4}$$

Since below I solve for symmetric equilibria where $p_{it} = p_{kt} = p_t$, rule (A 4) requires household j to select the closest brand ($\min_i \delta_{ijt}$).

(iii) *Labour supply choice*: from (A 3), recalling the definition of Y_{jt}, it is clear that the optimal rule is to work ($l_{jt} = 1$) if

$$(1+\beta)\ln[W_t(1-\tau)+\lambda_j \Pi^T_{t+1}/I_{t+1}] \geq (1+\beta)\ln[B_t(1-\tau)+\lambda_j \Pi^T_{t+1}/I_{t+1}]+\gamma_j, \tag{A 5}$$

and not to work ($l_{jt} = 0$) otherwise. As labour force participants do not receive profits (i.e. $\lambda_j = 0$ for them), the labour supply rule (A 5) reduces to

$$\ln W_t \geq \ln B_t + \gamma_j/(1+\beta), \tag{A 6}$$

that for the marginal worker holds with equality. Let $\gamma(L_t)$ denote a function that relates each value of γ_i to the number of workers L_t with work disutility not greater than that value. Then for the marginal worker (A6) becomes:

$$\ln W_t = \ln B_t + \gamma(L_t)/(1+\beta). \tag{A 7}$$

The inverse labour supply that appears as equation (10) in the text can be obtained subtracting $\ln p_t$ from both sides, and expressing (A 7) in terms of the real wage $w_t = W_t/p_t$ and of the real subsidy $b = B_t/p_t$.

Firm Optimisation

Consider two adjacent firms, i and $i+1$, and denote by D the distance between them and by δ_{it} the distance between firm i and the marginal consumer, who is just indifferent between the two brands (at time t). From (A 4) we get:

$$\ln p_{it} + a\delta_{\text{it}} = \ln p_{i+1t} + a(D-\delta_{it}). \tag{A 8}$$

In a symmetric equilibrium, firm i's market extends over a segment of length $2\delta_{it}$, and, since there is one consumer in each unit interval, it comprises $2\delta_{it}$ customers, δ_{it} young and δ_{it} old. If neighbouring brands sell for p_t, the price p_{it} charged by firm i bears the following relationship to its market area $2\delta_{it}$:

$$\ln p_{it} = \ln p_t + aD - 2a\delta_{it}. \tag{A 9}$$

To derive the demand function for firm i's product, q^d_{it}, recall that the demand of the jth young household equals its disposable income Y_{jt} (see Section I A) whereas that of jth old household equals its consumption c_{ijt} (obtained by lagging expression A 2 once). Aggregating these individual demand functions over the set S_i of $2\delta_{it}$ consumers that select brand i, one obtains the demand function for brand i:

$$q^d_{it} = \frac{1}{p_{it}} \sum_{j\in S_i} \left[Y_{jt} + \frac{\beta I_t}{1+\beta}(Y_{jt-1} + \lambda_j \Pi^T_t/I_t)\right]. \tag{A 10}$$

Since the total first-period income of these customers equals their pre-tax labour income ($\Sigma_j Y_{jt} = \delta_{it} W_t L_t/n$, for $j \in S_i$)[20] and their total profit income is

[20] In the aggregate, unemployment benefits equal taxes levied on the first-period income of the young cohort. Thus, for the cohort as a whole, income is just the gross wage W_t multiplied by the number of employed workers L_t. Since the unemployment rate is the same everywhere along the circumference, unemployment benefits balance out with taxes also within the market area of each firm. It follows that the total first-period income of the households in the market area of firm i is just the gross wage W_t multiplied by the number of employed workers 'resident' in that area ($\delta_{it} L_t/n$).

a fraction δ_{it}/n of aggregate profits ($\Sigma_j \lambda_j \Pi^T_{t+1} = \delta_{it} \Pi^T_{t+1}/n$, for $j \in S_i$),[21] the demand for brand i from (A 10) is

$$q^d_{it} = \frac{\delta_{it}}{p_{it}}\left(\frac{W_t L_t}{n} + \frac{\beta I_t}{1+\beta} \frac{W_{t-1} L_{t-1} + \Pi^T_t / I_t}{n}\right) \equiv \frac{\delta_{it}}{p_{it}} E_t, \tag{A 10$'$}$$

where E_t is a short-hand for the term in the large brackets. Substituting in (A 10′) for δ_{it} from (A 9), one finds the marginal revenue of firm i:

$$\frac{dR_{it}}{dq_{it}} = \frac{p_{it}}{1+2a\delta_{it}}. \tag{A 11}$$

Since at the optimum marginal revenue equals marginal cost, (A 11) states that in equilibrium the rate of markup over marginal cost is $2a\delta_{it}$.

Maximising Π_{it} from equation (5) in the text with respect to the two inputs K_{it} and L_{it}, and using expression (A 11) for marginal revenue, one obtains the two first-order conditions of firm i:

$$\frac{dR_{it}}{dq_{it}} \frac{\partial q_{it}}{\partial L_{it}} = \frac{p_{it}}{1+2a\delta_{it}} (1-\alpha) \left(\frac{K_{it}}{L_{it}}\right)^{\alpha} = W_t \tag{A 12}$$

$$\frac{dR_{it}}{dq_{it}} \frac{\partial q_{it}}{\partial K_{it}} = \frac{p_{it}}{1+2a\delta_{it}} \alpha \left(\frac{K_{it}}{L_{it}}\right)^{\alpha-1} = I_t p^k_{t-1}. \tag{A 13}$$

At the optimum, firm i's profits are:

$$\Pi_{it} = \frac{2a\delta_{it}}{1-\alpha} W_t L_{it} - p_{it} f. \tag{A 14}$$

Equilibrium

In a symmetric Nash equilibrium, all firms charge the same price ($p_{it} = p_t, \forall i$) and have equal market segments $2\delta_{it} = H/m_t$, so that the markup rate is:

$$2a\delta_{it} = aH/m_t, \ \forall i. \tag{A 15}$$

Also, each firm employs the same amount of labour and capital:

$$L_{it} = L_t/m_t, \quad K_{it} = K_t/m_t, \ \forall i. \tag{A 16}$$

The first order conditions (A 12) and (A 13) can be rewritten as:

$$\frac{1}{1+aH/m_t} (1-\alpha) \left(\frac{K_t}{L_t}\right)^{\alpha} = \frac{W_t}{p_t} \equiv w_t, \tag{A 17}$$

$$\frac{1}{1+aH/m_t} \alpha \left(\frac{K_t}{L_t}\right)^{\alpha-1} = \frac{I_t p_{t-1}}{p_t} \equiv R_t, \tag{A 18}$$

where the price of capital has been set equal to that of consumption ($p^k_t = p_t$). Rearranging so as to substitute out the term in m_t, equations (A 17) and (A 18) yield the aggregate demand for capital – equation (6) in the text. Eliminating instead K_t, they yield the factor-price frontier – equation (9) in the text.

[21] Since both profits and consumers have been assumed to be distributed uniformly across locations, the proportion of the aggregate flow of profit accruing to the households of the i-th market area is simply the number of residents of that area (δ_{it}) divided by the total number of households (n).

The aggregate supply of saving K_t^s of equation (7) in the text, instead, is obtained by subtracting the aggregate consumption of those who were young at time $t-1$ $(\Sigma_i \Sigma_j p_{t-1} c_{ijt-1})$ from their aggregate income $(\Sigma_j Y_{jt-1})$ and dividing by the price level p_{t-1}:

$$K_t^s = \sum_{j=1}^{n} \frac{Y_{jt-1}}{p_{t-1}} - \sum_{i=1}^{m_t} \sum_{j=1}^{n} c_{ijt-1} = \frac{\beta}{1+\beta} w_{t-1} L_{t-1} - \frac{1}{1+\beta} \frac{m_t \pi_t}{R_t}. \tag{A 19}$$

In the second step of (A 19), I have substituted for c_{ijt-1} from (A 1), lagged once. I have also used the fact that aggregate first-period real income is equal to the real gross wage bill $\Sigma_j Y_{jt-1}/p_{t-1} = w_{t-1} L_{t-1}$, for $j = 1 \dots n)$[22] and that aggregate real profits received by households are equal to the real profit per firm π_t multiplied by the number of firms m_t $(\Sigma_j \lambda_j \Pi_t^T / p_t = \Pi_t^T / p_t = m_t \pi_t$, for $j = 1, \dots n)$.

Stability Analysis

Linearising around steady-state zero-profit values (marked with a star) and rearranging, the system (20)–(21) becomes:

$$\begin{bmatrix} R_t - R^* \\ m_t - m^* \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} \alpha(1+\epsilon)(1-\theta f \sigma) & \xi \theta f \sigma \dfrac{1+\beta}{\beta} \\ -\alpha \theta f (1+\epsilon)^2 \dfrac{\beta}{1+\beta} & 1 + \alpha\epsilon + \xi \theta f (1+\epsilon) \end{bmatrix} \begin{bmatrix} R_{t-1} - R^* \\ m_{t-1} - m^* \end{bmatrix}, \tag{A 20}$$

where
$$\xi \equiv \frac{\alpha(1+\epsilon)}{1-\alpha} \frac{aH}{m^*(aH+m^*)} + \frac{1}{(1+\beta)\theta f} \frac{aH}{m^*},$$

$$\sigma \equiv \frac{1}{m^*} \left(\frac{1+\epsilon}{1-\alpha} \frac{aH}{aH+m^*} - 2 \right),$$

and
$$\Delta \equiv (1+\alpha\epsilon)(1-\theta f \sigma) + \xi \theta f (1+\epsilon).$$

The necessary and sufficient conditions for stability are:

(i) $\det = \alpha(1+\epsilon)/\Delta < 1$,

(ii) $1 + \det + \mathrm{tr} = -\theta f \sigma \, (1-\alpha)/\Delta > 0$,

(iii) $1 + \det - \mathrm{tr} = [(1+\alpha+2\alpha\epsilon)\,(2-\theta f \sigma) + 2\xi\theta f\,(1+\epsilon)]/\Delta > 0$.

As one can verify easily, for $\sigma < 0$ (which is the same as saying that condition (17) in the text is satisfied) all three inequalities hold. For $0 < \sigma < 1/\theta f$, only (i) and (iii) hold, but (ii) does not. It can be shown that in this case the system displays saddle path stability (with both roots positive).

Welfare Analysis

Consider employed worker j in steady-state, zero-profit equilibrium, after the new tax levied at rate t has been imposed. Besides unemployment contributions, he pays taxes $W_t\,(1-\tau)\,t$ when young and receives a transfer of the same

[22] See footnote 20.

amount when old. Since profit income is zero, his discounted lifetime disposable income is $W_t(1-\tau)(1-t)+W_t(1-\tau)t/I_{t+1}$. To evaluate his utility, substitute the following values in equation (A 3): $Y_{jt} = W_t(1-\tau)(1-t+t/I_{t+1})$, $\Pi^T_{t+1} = 0$, $p_{it} = p_{ht+1} = p_t$, $\delta_{ijt} = H/m_t$, $l_{jt} = 1$, $w_t \equiv W_t/p_t$ and $R_t = R_{t+1} \equiv I_{t+1}p_t/p_{t+1}$:

$$V(\cdot) = (1+\beta)\ln\left[w_t(1-\tau)\left(1-t+\frac{t}{R_{t+1}}\right)\right]+\beta\ln R_{t+1}-(1+\beta)\frac{aH}{m_t}-\gamma_j. \tag{A 21}$$

Using equation (9) to substitute out w_t and rearranging, one obtains (up to some constants involving α, β and τ):

$$V(\cdot) = (1+\beta)\left[\ln\left(1-t+\frac{t}{R_t}\right)+\left(\frac{\alpha}{1-\alpha}\frac{1+\beta}{\beta}-1\right)\frac{\beta}{1+\beta}\ln R_t - \frac{1}{1-\alpha}\ln\left(1+\frac{aH}{m_t}\right)-\frac{aH}{m_t}\right]-\gamma_j. \tag{A 22}$$

If $R_t > 1$, an increase in t causes the *first* term to decrease. This is a *wealth* effect: the discounted value of the transfer is then less than the tax, so that a higher t lowers discounted lifetime income. The *second* term shows how a higher t affects welfare via *capital accumulation*. Recall that in the zero-tax equilibrium $R_t = \alpha(1+\beta)/(1-\alpha)\beta$ (see (14)), and that R_t is increasing in the tax rate t. Thus this term decreases if the zero-tax R_t exceeds 1, i.e. if we are in the dynamically efficient case of a positive interest *rate*. Conversely, the contributions of the first and second term become positive in the dynamically inefficient case of a negative interest rate. The *third* and the *fourth* term show respectively how policy affects welfare via changes in the *degree of monopoly* and in *product variety*: both are increasing in the number of firms m_t, that in turn is decreasing in the tax rate t. Thus, if the initial interest rate is positive, the balanced budget expansion (a larger t) unambiguously lowers welfare. If the initial interest rate is zero or negative, the overall effect is ambiguous: it can still be negative if the welfare increase due to the higher R_t (1st and 2nd terms) is outweighed by the decrease due to the lower m_t (3rd and 4th terms).

References

Blanchard, Olivier and Kiyotaki, Nobuhiro (1987). 'Monopolistic competition and the effects of aggregate demand.' *American Economic Review*, vol. 77, September, pp. 647–66.

Bryant, John (1983). 'A simple rational expectations Keynes-type model.' *Quarterly Journal of Economics*, vol. 98, August, pp. 525–9.

Chatterjee, Satyajit (1988). 'Participation externality as a source of coordination failure in a competitive model with centralised markets.' mimeo, University of Iowa, July.

—— and Cooper, Russell (1988). 'Multiplicity of equilibria and fluctuations in an imperfectly competitive economy with entry and exit.' mimeo, University of Iowa, June.

Cooper, Russell (1987). 'Dynamic behaviour of imperfectly competitive economies with multiple equilibria.' NBER Working Paper No. 2388, September.

—— and John, Andrew (1988). 'Coordinating coordination failures in Keynesian models.' *Quarterly Journal of Economics*, vol. 53, August, pp. 441–63.

Diamond, Peter (1965). 'National debt in a neoclassical growth model.' *American Economic Review*, vol. 55, December, pp. 1126–50.

Diamond, Peter (1982). 'Aggregate demand management in search equilibrium.' *Journal of Political Economy*, vol. 90, October, pp. 881–94.

—— (1984). 'Money in search equilibrium.' *Econometrica*, vol. 52, January, pp. 1–20.

Dixit, Avinash and Stiglitz, Joseph (1977). 'Monopolistic competition and optimum product diversity.' *American Economic Review*, vol. 67, June, pp. 297–308.

Drazen, Allen (1985). 'A rational expectations Keynesian model with price flexibility.' Working Paper No. 2–85, Foerder Institute for Economic Research, Tel-Aviv University, January.

—— (1987). 'Reciprocal externality models of low employment.' *European Economic Review Proceedings*, vol. 31, February–March), pp. 436–43.

Geanakoplos, J. D. and Polemarchakis, H. M. (1986). 'Walrasian indeterminacy and Keynesian macroeconomics.' *Review of Economic Studies*, vol. 53, October, pp. 755–79.

Hart, Oliver (1982). 'A model of imperfect competition with Keynesian features.' *Quarterly Journal of Economics*, vol. 97, February, pp. 109–38.

Heller, Walter P. (1986). 'Coordination failure under complete markets with applications to effective demand.' In *Equilibrium Analysis: Essays in Honor of Kenneth J. Arrow*: vol. II, (ed. Walter P. Heller, Ross, M. Starr, and David A. Starrett), Cambridge: Cambridge University Press, pp. 155–75.

Howitt, Peter and McAfee, R. Preston. (1985). 'Costly search and recruiting.' *International Economic Review*, vol. 75, pp. 88–100.

Kiyotaki, Nobuhiro (1988). 'Multiple expectational equilibria under monopolistic competition.' *Quarterly Journal of Economics*, vol. 103, November, pp. 695–713.

Pagano, Marco (1989*a*). 'Endogenous market thinness and stock price volatility.' *Review of Economic Studies*, vol. 56, April, pp. 269–88.

—— (1989*b*). 'Trading volume and asset liquidity.' *Quarterly Journal of Economics*, vol. 104, May, pp. 255–74.

Roberts, John (1988). 'Involuntary unemployment and imperfect competition: a game theoretic macro model.' *The Economics of Imperfect Competition and Employment: Joan Robinson and Beyond* (ed. George Feiwel), London: Macmillan.

Salop, Steven C. (1979). 'Monopolistic competition with outside goods.' *The Bell Journal of Economics*, vol. 10, pp. 141–56.

Solow, Robert (1984). 'Monopolistic competition and the multiplier.' mimeo, MIT.

Weitzman, Martin (1982). 'Increasing returns and the foundations of unemployment theory.' ECONOMIC JOURNAL, vol. 92, December, pp. 787–804.