Entrepreneurial Risk and Diversification through Trade

Federico Esposito*
Tufts University

March 2017

Abstract

Firms face considerable uncertainty about consumers' demand, arising from the existence of random shocks. In presence of incomplete financial markets or liquidity constraints, entrepreneurs may not be able to perfectly insure against unexpected demand fluctuations. The key insight of my paper is that firms can reduce demand risk through geographical diversification. I first develop a general equilibrium trade model with monopolistic competition, characterized by stochastic demand and risk-averse entrepreneurs, who exploit the imperfect correlation of demand across countries to lower the variance of their total sales, in the spirit of modern portfolio analysis. The model predicts that both entry and trade flows to a market are affected by its risk-return profile. Moreover, welfare gains from trade can be significantly higher than the gains predicted by standard models which neglect firm level risk. After a trade liberalization, risk-averse firms boost exports to countries that offer better diversification benefits. Hence, in these markets foreign competition becomes stronger, increasing average productivity and lowering the price level more. Therefore, countries with better risk-return profiles gain more from international trade. I then look at the data using Portuguese firm-level trade flows from 1995 to 2005 and provide evidence that exporters behave in a way consistent with my model's predictions. Finally, I estimate the parameters of the model with the Simulated Method of Moments to perform a number of counterfactual exercises. The main policy counterfactual reveals that, for the median country, the risk diversification channel increases welfare gains from trade by 13% relative to models with risk neutrality.

*Department of Economics, Tufts University, 8 Upper Campus Road, Somerville, 02144, MA, USA. Email: federico.esposito@tufts.edu. I am extremely grateful to my advisor Costas Arkolakis, and to Lorenzo Caliendo, Samuel Kortum and Peter Schott for their continue guidance as part of my dissertation committee at Yale University. I thank the hospitality of the Economic and Research Department of Banco de Portugal where part of this research was conducted. I have benefited from discussions with Treb Allen, Mary Amiti, David Atkin, Andrew Bernard, Kirill Borusyak, Arnaud Costinot, Penny Goldberg, Gene Grossman, Tim Kehoe, William Kerr, Giovanni Maggi, Matteo Maggiori, Monica Morlacco, Peter Neary, Luca Opromolla, Emanuel Ornelas, Michael Peters, Tommaso Porzio, Vincent Rebeyrol, Steve Redding, Joe Shapiro, Robert Staiger, James Tybout as well as seminar participants at Yale University, Tufts University, SUNY Albany, Federal Reserve Board, University of Florida, Yale SOM, Bank of Italy, SED Toulouse, World Bank, AEA Meetings 2017. Finally, I thank Siyuan He and Guangbin Hong for excellent research assistance. All errors are my own.

1 Introduction

Firms face substantial uncertainty about consumers' demand. Recent empirical evidence has shown that demand-side shocks explain a large fraction of the total variation of firm sales (see Fitzgerald et al. (2016)), Hottman et al. (2015), Kramarz et al. (2014), Munch and Nguyen (2014), Eaton et al. (2011)).[1] The role of demand uncertainty is particularly important when firms must undertake costly irreversible investments, such as producing a new good or selling in a new market. However, in presence of incomplete financial markets or credit constraints, firms may not be able to perfectly insure against unexpected demand fluctuations.[2]

The key idea I put forward in this paper is that firms can hedge demand risk through geographical diversification. The intuition is that selling to markets with imperfectly correlated demand can hedge against idiosyncratic shocks hitting sales. Although this simple insight has always been at the core of the financial economics literature, starting from the seminal works by Markowitz (1952) and Sharpe (1964), the trade literature has so far overlooked the risk diversification potential that international trade has for firms.[3]

The main contribution of this work is to highlight, both theoretically and empirically, the relevance of demand risk for firms' exporting decisions, and to quantify the risk diversification benefits that international trade has for firms and for the aggregate economy. The main finding of the paper is that the welfare gains from trade can be much higher than the ones predicted by traditional models neglecting firm level risk. These additional gains arise from the fact that firms use international trade not only to increase profits, as in standard models, but also to globally diversify risk. Therefore when trade barriers go down, firms export more

[1]Hottman et al. (2015) have shown that 50-70 percent of the variance in firm sales can be attributed to differences in firm appeal. Eaton et al. (2011) and Kramarz et al. (2014) with French data and Munch and Nguyen (2014) with Danish data have instead estimated that firm-destination idiosyncratic shocks drive around 40-45% percent of sales variation. di Giovanni et al. (2014) show that the firm-specific component accounts for the overwhelming majority of the variation in sales growth rates across firms (the remaining is sectoral and aggregate shocks). In addition, about half of the variation in the firm-specific component is explained by variation in that component across destinations, which can be interpreted as destination-specific demand shocks in our conceptual framework. Using the same metric, Haltiwanger (1997) and Castro et al. (2011) find that idiosyncratic shocks account for more than 90% of the variation in firm growth rates in the U.S. Census Longitudinal Research Database.

[2]This may be the case especially in less developed countries (see Jacoby and Skoufias (1997), Greenwood and Smith (1997) and Knight (1998)), and for small-medium firms (see Gertler and Gilchrist (1994) and Hoffmann and Shcherbakova-Stewen (2011)).

[3]There are some recent exceptions, as Fillat and Garetto (2015) and Riaño (2011). See the discussion below.

to countries which are a good hedge against demand risk, i.e. markets with either a stable demand or whose demand is negatively correlated with the rest of the world. This increases the entry of foreign firms, which in turn increases the level of competition among firms, lowering prices and leading to higher welfare gains from trade. Once I calibrate the model parameters using firm-level data from Portugal, I quantify this general equilibrium effect of the risk diversification to be up to 30% of total welfare gains.

In the first tier of my analysis, I develop a general equilibrium trade model with monopolistic competition, as in Melitz (2003), and Pareto distributed firm productivity, as in Chaney (2008) and Arkolakis et al. (2008). The model is characterized by two new elements. First, consumers have a Constant Elasticity of Substitution utility over a continuum of varieties, and demand is subject to country-variety random shocks. In addition, for each variety these demand shocks are imperfectly correlated across countries. Second, firms are owned by risk-averse entrepreneurs who have mean variance preferences over business profits. This assumption reflects the evidence, discussed in Section 2, that most firms across several countries are owned by entrepreneurs whose wealth is not perfectly diversified and whose main source of income are their firm's profits, therefore exposing their income to demand fluctuations.[4] In addition, even for multinational or public listed firms, stock-based compensation exposes their managers to firm-specific risk, who therefore attempt to minimize such risks (see Ross (2004), Parrino et al. (2005) and Panousi and Papanikolaou (2012)).[5]

The entrepreneurs' problem consists of two stages. In the first stage, the entrepreneurs know only the moments of the demand shocks but not their realization. Firms make an irreversible investment: they choose in which countries to operate, and in these markets perform costly marketing and distributional activities. After the investment in marketing costs, firms learn the realized demand. Then, after uncertainty is resolved, entrepreneurs finally produce, using a production function linear in labor.[6]

The fact that demand is correlated across countries implies that, in the first stage, en-

[4]See Moskowitz and Vissing-Jorgensen (2002), Lyandres et al. (2013) and Herranz et al. (2015).

[5]I assume that financial markets are absent. This assumption captures in an extreme way the incompleteness of financial markets. Even if there were some financial assets available in the economy, as long as capital markets are incomplete firms would always be subject to a certain degree of demand risk. Shutting down financial markets therefore allows to focus only on international trade as a mechanism firms can use to stabilize their sales.

[6]The fact that companies cannot change the number of consumers reached after observing the shocks has an intuitive explanation. Investing in marketing activities is an irreversible activity, and thus very costly to adjust after observing the realization of the shocks. An alternative interpretation of this irreversibility is that firms sign contracts with buyers before the actual demand is known, and the contracts cannot be renegotiated.

trepreneurs face a combinatorial problem. Indeed, both the extensive margin (whether to export to a market) and the intensive margin (how much to export) decisions are intertwined across markets: any decision taken in a market affects the outcome in the others. Then, for a given number of potential countries N, the choice set includes 2^N elements, and computing the indirect utility function corresponding to each of its elements would be computationally unfeasible.[7]

I deal with this computational challenge by assuming that firms send costly ads in each country where they want to sell. These activities allow firms to reach a fraction n of the consumers in each location, as in Arkolakis (2010). This implies that the firm's choice variable is continuous rather than discrete, and thus firms simultaneously choose *where to sell* (if n is optimally zero) and *how much to sell* (firms can choose to sell to some or all consumers). In addition, the concavity of the firm's objective function, arising from the mean-variance specification, implies that the optimal solution is unique.[8]

Therefore, the firm's extensive and intensive margin decisions are not taken market by market, but rather by performing a *global* diversification strategy. Entrepreneurs trade off the expected global profits with their variance, the exact slope being governed by the risk aversion, along the lines of the "portfolio analysis" pioneered by Markowitz (1952) and Sharpe (1964).[9]

I show that both the probability of entering a market and the intensity of trade flows are increasing in the market's "Sharpe Ratio". This variable measures the diversification benefits that a market can provide to firms exporting there. If demand in a country is relatively stable and negatively/mildly correlated with the rest of the world, then firms optimally choose, *ceteribus paribus*, to export more there to hedge their business risk. Therefore, my model suggests that neither the demand volatility in a market, nor the bilateral covariance of demand with the domestic market, are sufficient to predict the direction of trade. Instead, what determines trade patterns is the multilateral covariance: how much demand in a market is correlated with all other countries.

Furthermore, in a two country version of the model, I show that the welfare gains from

[7]Other works in trade, such as Antras et al. (2014), Blaum et al. (2015) and Morales et al. (2014), deal with similar combinatorial problems, but in different contexts.

[8]In particular, to numerically solve the firm's problem I use standard methods (such as the active set method) employed in quadratic programming problems with bounds. This is way faster than evaluating all the possible combinations of extensive/intensive margin decisions.

[9]The firms' problem, however, is more involved than a standard portfolio problem, because it is subject to bounds: the number of consumers reached in a destination can neither be negative nor greater than the size of the population.

international trade are increasing in the Sharpe Ratio.[10] The intuition is simple: if the Sharpe Ratio is high, firms can hedge their domestic demand risk by exporting to the foreign country. This implies tougher competition among firms, and thus an increase in the average productivity of surviving firms, which in general equilibrium leads to lower prices and higher welfare gains.

In the second tier of my analysis, I rely on a panel dataset of Portuguese manufacturing firms' exports, from 1995 to 2005, to test the model's predictions and to calibrate the model. Portugal is a small and export-intensive country, being at the 72nd percentile worldwide for exports per capita, and therefore can be considered a good laboratory to analyze the implications of my model. Furthermore, 70% of Portuguese exporters in 2005 were small firms, for which the exposure to demand risk is likely to be a first-order concern.

I first estimate the cross-country covariance matrix of demand, Σ, using the firm-level data on exports from 1995 to 2004. Given the static nature of the model, Σ can be interpreted as a *long-run* covariance matrix that firms take as given when they choose their risk diversification strategy. However, there is evidence that, in the short run, firms sequentially enter different markets to learn their demand behavior (see Albornoz et al. (2012) among others). In the data, this behavior may confound the pure risk diversification behavior of exporters predicted by my model, affecting the estimation of Σ. Therefore, I consider only sales by "established" firm-destination pairs, i.e. exporters selling to a certain market for at least 5 years. In this way, my estimates capture only the long run covariance of demand, rather than picking also some short-run noise due to the firms' learning process.

Moreover, I estimate the risk aversion by matching the observed (positive) gradient of the relationship between the mean and the variance of firms' profits, as suggested by the firm's first order conditions. The reasoning is straightforward: if firms are risk-averse, they want to be compensated for taking additional risk, and thus higher sales variance must be associated with higher expected revenues.[11] Interestingly, the results suggest that a modest amount of risk aversion is sufficient to rationalize the magnitudes in the data.[12] Finally, I calibrate the remaining parameters, such as marketing and iceberg trade costs, with the Simulated Method of Moments, as in Eaton et al. (2011).[13]

From the estimated covariance matrix, I easily recover the Sharpe Ratios, the country

[10]Given the complexity of the model, I can explicitly derive an expression for the welfare gains only in the case of two symmetric countries.

[11]Allen and Atkin (2016) use a similar approach to estimate the risk aversion of Indian farmers.

[12]In addition, my estimate is close to the ones found by Allen and Atkin (2016) and Herranz et al. (2015).

[13]In particular, I match the observed i) bilateral manufacturing trade shares; ii) normalized number of Portuguese exporters to each destination; iii) mean and dispersion of export shares.

level measure of diversification benefits. Then I test the prediction that firms' probability of entry and trade flows to a market are increasing in the market's Sharpe Ratio, using the Portuguese firm-level trade data for 2005. The findings confirm that, controlling for destination characteristics and barriers to trade, firms are more likely to enter in countries with a high Sharpe Ratio, i.e. countries that provide good diversification benefits. Moreover, conditional on entering a destination, firms export more to countries where they can better hedge their demand risk.

Finally, I perform a number of counterfactual simulations to quantify the risk diversification benefits that international trade has for aggregate welfare. The main policy experiment is to compute the welfare gains from international trade, i.e. from a reduction in trade barriers. My results illustrate that countries providing better risk-return trade-offs to foreign firms, i.e. countries with a high Sharpe Ratio, benefit more from opening up to trade. The rationale is that firms exploit a trade liberalization not only to increase their profits, but also to diversify their demand risk. This implies that they optimally increase trade flows toward markets that provide better diversification benefits. Consequently, the increase in foreign competition is stronger in these countries, thereby lowering more the price level. Therefore, "safer" countries gain more from trade.[14]

In addition, I compare the gains in my model with those predicted by traditional trade models that neglect risk, as in Arkolakis et al. (2012) (ACR henceforth).[15] My results show that gains from trade are, for the median country, 13% higher than in ACR, and up to 30% higher. While safer countries reap higher welfare gains than in ACR, markets with a worse risk-return profile have lower gains than in ACR, because the competition from foreign firms is weaker.

This paper relates to the growing literature studying the importance of second order moments for international trade.[16] Allen and Atkin (2015) use a portfolio approach to study the crop choice of Indian farmers under uncertainty. They show that greater trade openness increases farmers' revenues volatility, leading farmers to switch to safer crops, which in turn increases their welfare. Similarly, in my model a trade liberalization induces firms to export

[14]These findings are robust to the specification used for the entrepreneurs' utility. In particular, I show that having a decreasing rather than constant absolute risk aversion does not affect substantially the welfare results.

[15]The models considered in ACR are characterized by (i) Dixit-Stiglitz preferences; (ii) one factor of production; (iii) linear cost functions; and (iv) perfect or monopolistic competition. Among them, there are the seminal papers by Eaton and Kortum (2002), Melitz (2003) and Chaney (2008).

[16]For earlier works, see Helpman and Razin (1978), Kihlstrom and Laffont (1979), Newbery and Stiglitz (1984) and Eaton and Grossman (1985).

more to less risky countries, which increases welfare gains through a general equilibrium force. Fillat and Garetto (2015) argue that multinational firms, due to the large sunk costs of accessing foreign markets, are the most exposed to foreign demand risk, and therefore are riskier than firms selling domestically, especially in presence of persistent disaster risk. While they focus on the link between a company's international status and its stock return, I argue that international trade provides relevant risk diversification benefits to exporters, especially small and medium ones. De Sousa et al. (2015) use a partial equilibrium model with risk averse firms to rationalize the empirical finding that volatility and skewness of demand affect the firms' exporting decision. My contribution relative to these papers is i) to establish that the cross-country covariance of demand is a key driver of trade patterns, and ii) to quantify the welfare benefits of risk diversification by means of a novel general equilibrium framework.

Other recent works exploring the link between uncertainty and exporters' behavior are Koren (2003), Rob and Vettas (2003), Di Giovanni and Levchenko (2010), Riaño (2011), Nguyen (2012), Impullitti et al. (2013), Vannoorenberghe (2012), Ramondo et al. (2013), Vannoorenberghe et al. (2014), Novy and Taylor (2014), and Gervais (2016).

Previous models of firms' export decision have studied a simple binary exporting decision (Roberts and Tybout (1997); Das et al. (2007)) or have assumed exporters make independent entry decisions for each destination market (Helpman et al. (2008); Arkolakis (2010); Eaton et al. (2011)). In contrast, in my model entry in a given market depends on the global diversification strategy of the firm. Another trade model where the entry decision is interrelated across markets is Morales et al. (2015), in which the firm's export decision depends on its previous export history. Similarly, Berman et al. (2015) show that there are strong complementarities between exports and domestic sales.

My paper also complements the strand of literature that studies the connection between openness to trade and macroeconomic volatility. Di Giovanni et al. (2014) investigate how idiosyncratic shocks to large firms directly contribute to aggregate fluctuations, through input-output linkages across the economy. Caselli et al. (2012) show that openness to international trade can lower GDP volatility by reducing exposure to domestic shocks and allowing countries to diversify the sources of demand and supply across countries. My paper, in contrast, investigates the implications of firm-level demand risk for international trade patterns and aggregate welfare.

Finally, my paper connects to the literature that studies the implications of incomplete financial markets for entrepreneurial risk and firms' behavior and performance. Herranz et al. (2015) show, using data on ownership of US small firms, that entrepreneurs are risk-averse and hedge business risk by adjusting the firm's capital structure and scale of production. Other notable contributions to this literature are Kihlstrom and Laffont (1979), Heaton and Lucas (2000), Moskowitz and Vissing-Jorgensen (2002), Roussanov (2010), Luo et al. (2010), Chen et al. (2010), Hoffmann (2014) and Jones and Pratap (2015).

The remainder of the paper is organized as follows. Section 2 presents some stylized facts that corroborate the main assumptions used in the model, presented in Section 3. In Section 4, I estimate the model and empirically test its implications. In Section 5, I perform a number of counterfactual exercises. Section 6 concludes.

2 Motivating evidence

Compared to standard trade models, such as Melitz (2003), the main novelty of my framework is that entrepreneurs are risk averse. There is recent evidence supporting this assumption. Cucculelli et al. (2012) survey several Italian entrepreneurs in the manufacturing sector and show that 76.4% of interviewed decision makers are risk averse. Interestingly, larger firms tend to be managed by decision makers with lower risk aversion.[17] A survey promoted by the consulting firm Capgemini reveals that, among 300 managers/CEO of leading companies across several countries, 40% of them believes that market/demand volatility is the most important challenge for their firm.[18] Further evidence that entrepreneurs are risk averse has been recently provided by Herranz et al. (2015), De Sousa et al. (2015) and Allen and Atkin (2016).

It is important to note that risk aversion is a factor affecting the behavior of large firms/multinationals as well, not just small-medium enterprises. Indeed, risk aversion arises if corporate management seeks to avoid default risk and the costs of financial distress, where these costs rise with the variability of the net cash flows of the firm (see Froot et al. (1993) and Allayannis et al. (2008)). Moreover, stock-based compensation exposes managers to firm-specific risk (see Petersen and Thiagarajan (2000), Ross (2004),Parrino et al. (2005) and Panousi and Papanikolaou (2012)). Thus, in making economic decisions such as investment and production, managers reasonably attempt to minimize their risk exposure.

Two objections could be raised to the risk aversion assumption. The first is that en-

[17]I will take into account for these differences in risk aversion in an extension of the model.

[18]This survey was conducted in 2011 among 300 companies from Europe (59%), the US and Canada (25%), Asia-Pacific (10%) and Latin America (6%). The survey can be found here: https://www.capgemini-consulting.com/resource-file-access/resource/pdf/The_2011_Global_Supply_Chain_Agenda.pdf.

trepreneurs could invest their wealth across several assets, diversifying away business risk. In reality, however, the majority of firms around the globe are controlled by *imperfectly diversified* owners. Using a dataset about ownership of 162,688 firms in 34 European countries, Lyandres et al. (2013) show that entrepreneurs' holdings are far from being well-diversified.[19] The median entrepreneur in their sample owns shares of only two firms, and the Herfindhal Index of his holdings is 0.67, a number indicating high concentration of wealth.[20] According to the Survey of Small Business Firms (2003), a large fraction of US small firms' owners invest substantial personal net-worth in their firms: half of them have 20% or more of their net worth invested in one firm, and 87% of them work at their company.[21] Moreover, Moskowitz and Vissing-Jorgensen (2002) estimate that US households with entrepreneurial equity invest on average more than 70 percent of their private holdings in a single private company in which they have an active management interest.[22]

The second objection that could be raised is that firms can hedge demand risk on financial and credit markets. However, often small firms (which account for the vast majority of existing firms) have a limited access to capital markets (see Gertler and Gilchrist (1994), Hoffmann and Shcherbakova-Stewen (2011)), and even large firms under-invest in financial instruments (see Guay and Kothari (2003)) and, when they do, such instruments often do not successfully reduce risks (see Hentschel and Kothari (2001)).[23] In addition, notice that financial derivatives can be used to hedge interest rate, exchange rate, and commodity price risks, rather than demand risk, which is the focus of this paper.

Moreover, the model features country-variety demand shocks. Recent empirical evidence has shown that demand shocks explain a large fraction of the total variation of firm sales.

[19] 96% of firms in their sample are privately-held. They use three measures of diversification of entrepreneurs' holdings: i) total number of firms in which the owner holds shares, directly or indirectly; ii) Herfindhal index of firm owner's holdings; iii) the correlation between the mean stock return of public firms in the firm's industry and the shareholder's overall portfolio return.

[20] There is a growing body of theoretical literature that explains this concentration of entrepreneurs' portfolios and thus their exceptional role as owners of equity. See Carroll (2002), Roussanov (2010), Luo et al. (2010) and Chen et al. (2010).

[21] This Survey, administered by Federal Reserve System and the U.S. Small Business Administration, is a cross sectional stratified random sample of about 4,000 non-farm, non-financial, non-real estate small businesses that represent about 5 million firms.

[22] Similar evidence that companies are controlled by imperfectly diversified owners has been provided by Benartzi and Thaler (2001), Agnew et al. (2003), Heaton and Lucas (2000), Faccio et al. (2011) and Herranz et al. (2013).

[23] Hentschel and Kothari (2001), using data from financial statements of 425 large US corporations find that many firms manage their exposures with large derivatives positions. Nonetheless, compared to firms that do not use financial derivatives, firms that use derivatives display few, if any, measurable differences in risk that are associated with the use of derivatives.

Hottman et al. (2015) have shown that 50-70 percent of the variance in firm sales can be attributed to differences in firm appeal. Eaton et al. (2011) and Kramarz et al. (2014) with French data and Munch and Nguyen (2014) with Danish data have instead estimated that firm-destination idiosyncratic shocks drive around 40-45% percent of sales variation. Recent contributions also include Bricongne et al. (2012), Nguyen (2012), Munch and Nguyen (2014), Berman et al. (2015) and Armenter and Koren (2015).

The insight of this paper is that risk averse entrepreneurs optimally hedge these idiosyncratic demand shocks by exporting to markets with *imperfectly correlated* shocks.[24] I now describe the theoretical framework, where I introduce entrepreneurs' risk aversion and correlated demand shocks in a general equilibrium trade model, and show their implications trade patterns and welfare gains from trade.

3 A trade model with risk-averse entrepreneurs

I consider a static trade model with N asymmetric countries. The importing market is denoted by j, and the exporting market by i, where $i, j = 1, ..., N$. Each country j is populated by a continuum of workers of measure $\tilde{L}_j$, and a continuum of *risk-averse* entrepreneurs of measure M_j. Each entrepreneur owns a non-transferable technology to produce, with productivity z, a differentiated variety under monopolistic competition, as in Melitz (2003) and Chaney (2008). The productivity z is drawn from a known distribution, independently across countries and firms, and its realization is known by the entrepreneurs at the time of production. Since there is a one-to-one mapping from the productivity z to the variety produced, throughout the rest of the paper I will always use z to identify both. Finally, I assume that financial markets are absent.[25]

[24] In the empirical analysis I estimate the cross-country correlation of these demand shocks.

[25] This assumption captures in an extreme way the incompleteness of financial markets. Even if there were some financial assets available in the economy, as long as capital markets are incomplete firms would always be subject to a certain degree of demand risk. Shutting down financial markets therefore allows to focus only on international trade as a mechanism firms can use to stabilize their sales. See also Riaño (2011) and Limão and Maggi (2013).

3.1 Consumption side

Both workers and entrepreneurs have access to a potentially different set of goods Ω_{ij}. Each agent v chooses consumption by maximizing a CES aggregator of a continuum number of varieties, indexed with z:

$$\max U_j(v) = \left(\sum_i \int_{\Omega_{ij}} \alpha_j(z)^{\frac{1}{\sigma}} q_j(z,v)^{\frac{\sigma-1}{\sigma}} dz \right)^{\frac{\sigma}{\sigma-1}} \tag{1}$$

$$\text{s.to} \sum_i \int_{\Omega_{ij}} p_j(z) q_j(z,v) dz \leq y(v) \tag{2}$$

where $y(v)$ is agent v's income, and $\sigma > 1$ is the elasticity of substitution across varieties. Although the consumption decision, given income $y(v)$, is the same for workers and entrepreneurs, their incomes differ. In particular, workers earn labor income by working (inelastically) for the entrepreneurs. I assume that there is perfect and frictionless mobility of workers across firms, and therefore they *all* earn the same *non-stochastic* wage w. In contrast, entrepreneurs' only source of income are the profits they reap from operating their firm. Entrepreneurs, therefore, own a technology to maximize their income, but they incur in business risk, as it will be clearer in the next subsection.

The term $\alpha_j(z)$ reflects an exogenous demand shock specific to good z in market j, similarly to Eaton et al. (2011), Nguyen (2012) and Di Giovanni et al. (2014). This is the only source of uncertainty in this economy. Define $\alpha(z) \equiv \alpha_1(z), ...\alpha_N(z)$ to be the vector of realizations of the demand shock for variety z. I assume that:

Assumption 1. $\alpha(z) \sim G(\bar{\alpha}, \Sigma)$, i.i.d. across z

Assumption 1 states that the demand shocks are drawn, independently across varieties, from a multivariate distribution characterized by an N-dimensional vector of means $\bar{\alpha}$ and an $N \times N$ variance-covariance matrix Σ. Given the interpretation of $\alpha_j(z)$ as a consumption shifter, I assume that the distribution has support over $\mathbb{R}^+$.

Few comments are in order. First, by simply specifying a generic covariance matrix Σ, I am not making any restrictions on the cross-country correlations of demand, which therefore can range from -1 to 1. Second, I assume that these shocks are variety specific. Therefore I am ruling out, for the moment, any aggregate shocks that would affect the demand for all varieties. Third, for simplicity I assume that the moments of the shocks are the same for all varieties, but it would be fairly easy to extend the model to have $G(\bar{\alpha}, \Sigma)$ varying across sectors.

The maximization problem implies that the agent υ's demand for variety z is:

$$q_j(z,\upsilon) = \alpha_j(z)\frac{p_j(z)^{-\sigma}}{P_j^{1-\sigma}}y_j(\upsilon), \tag{3}$$

where $p_j(z)$ is the price of variety z in j, and P_j is the standard Dixit-Stiglitz price index. In equation 3, the demand shifter $\alpha_j(z)$ can reflect shocks to preferences, climatic conditions, consumers confidence, regulation, firm reputation, etc. (see also De Sousa et al. (2015)).

3.2 Production side

Entrepreneurs are the only owners and managers of their firms, and their only source of income are their firm's profits.[26] This assumption captures, in an extreme way, the evidence shown earlier that the majority of entrepreneurs around the globe do not have a well-diversified wealth. They choose how to operate their firm z in country i by maximizing the following indirect utility in real income:

$$\max V\left(\frac{y_i(z)}{P_i}\right) = E\left(\frac{y_i(z)}{P_i}\right) - \frac{\gamma}{2}Var\left(\frac{y_i(z)}{P_i}\right) \tag{4}$$

where $y_i(z)$ equals net profits. The mean-variance specification above can be derived assuming that the entrepreneurs maximize an expected CARA utility in real income (see Eeckhoudt et al. (2005)).[27] The CARA utility has been widely used in the portfolio allocation literature (see, for example, Markowitz (1952), Sharpe (1964) and Ingersoll (1987)), and has the advantage of having a *constant* absolute risk aversion, given by the parameter $\gamma > 0$, which gives a lot of tractability to the model. One shortcoming of the CARA utility is that the absolute risk aversion is independent from wealth. In Section 7.1, I will consider a variation of the model where the entrepreneurs have a CRRA utility, and thus a decreasing absolute risk aversion, and show that the overall implications do not change substantially.

[26] Alternatively, we can think of them as the majority shareholders of their firm, with complete power over the firm's production choices.

[27] If the entrepreneurs have a CARA utility with parameter γ, a second-order Taylor approximation of the expected utility leads to the expression in 4 (see Eeckhoudt et al. (2005) and De Sousa et al. (2015) for a standard proof). If the demand shocks are normally distributed, the expression in 4 is exact (see Ingersoll (1987)).

The production problem consists of two stages. In the first, firms know only the distribution of the demand shocks, $G(\alpha)$, but not their realization. Under uncertainty about future demand, firms make an irreversible investment: they choose in which countries to operate, and in these markets perform costly marketing and distributional activities. After the investment in marketing costs, firms learn the realized demand. Then, entrepreneurs produce using a production function linear in labor, and allocate their real income to different consumption goods, according to the sub-utility function in 1.[28]

I assume that the first stage decision cannot be changed after the demand is observed. This assumption captures the idea that marketing activities present irreversibilities that make reallocation costly after the shocks are realized.[29] An alternative interpretation of this irreversibility is that firms sign contracts with buyers before the actual demand is known, and the contracts cannot be renegotiated.

The fact that demand is correlated across countries implies that, in the first stage, entrepreneurs face a combinatorial problem. Indeed, both the extensive margin (whether to export to a market) and the intensive margin (how much to export) decisions are intertwined across markets: any decision taken in a market affects the outcome in the others. Then, for a given number of potential countries N, the choice set includes 2^N elements, and computing the indirect utility function corresponding to each of its elements would be computationally unfeasible.[30]

I deal with such computational challenge by assuming that firms send costly ads in each country where they want to sell. These activities allow firms to reach a fraction $n_{ij}(z)$ of consumers in location j, as in Arkolakis (2010).[31] This implies that the firm's choice variable is continuous rather than discrete, and thus firms simultaneously choose *where to sell* (if $n_{ij}(z)$ is optimally zero, firm z does not sell in country j) and *how much to sell* (firms can choose to sell to some or all consumers). In addition, the concavity of the firm's objective function, arising from the mean-variance specification, implies that the optimal solution is unique, as I prove in Proposition 1 below.

The fact that the ads are sent independently across firms and destinations, and the existence of a continuum number of consumers, imply that the total demand for variety z in

[28]See Koren (2003) for a similar configuration of the production structure.

[29]For a similar assumption, but in different settings, see Ramondo et al. (2013), Albornoz et al. (2012) and Conconi et al. (2016).

[30]Other works in trade, such as Antras et al. (2014), Blaum et al. (2015) and Morales et al. (2014), deal with similar combinatorial problems, but in different contexts.

[31]Estimates of marketing costs (see Barwise and Styler (2003), Butt and Howe (2006) and Arkolakis (2010)) indicate that the amount of marketing spending in a certain market is between 4 to 7.7% of GDP.

country j is:

$$q_{ij}(z) = \alpha_j(z)\frac{p_{ij}(z)^{-\sigma}}{P_j^{1-\sigma}}n_{ij}(z)Y_j, \tag{5}$$

where Y_j is the total income spent by consumers in j, and P_j is the Dixit-Stiglitz price index:

$$P_j^{1-\sigma} \equiv \sum_i \int_{\Omega_{ij}} n_{ij}(z)\alpha_j(z)\left(p_{ij}(z)\right)^{1-\sigma} dz. \tag{6}$$

Therefore, the first stage problem is to choose $n_{ij}(z)$ to maximize the following:

$$\max_{\{n_{ij}\}} \sum_j E\left(\frac{\pi_{ij}(z)}{P_i}\right) - \frac{\gamma}{2}\sum_j\sum_s Cov\left(\frac{\pi_{ij}(z)}{P_i}, \frac{\pi_{is}(z)}{P_i}\right) \tag{7}$$

$$\text{s. to } 1 \geq n_{ij}(z) \geq 0 \tag{8}$$

where $\pi_{ij}(z)$ are net profits from destination j:

$$\pi_{ij}(z) = q_{ij}(n_{ij}(z))p_{ij}(z) - q_{ij}(n_{ij}(z))\frac{\tau_{ij}w_i}{z} - f_{ij}(z), \tag{9}$$

and $\tau_{ij} \geq 1$ are iceberg trade costs and f_{ij} are marketing costs.[32] In particular, I assume that there is a non-stochastic cost, $f_j > 0$, to reach each consumer in country j, and that this cost is paid in both domestic and foreign labor, as in Arkolakis (2010).[33] Thus, total marketing costs are:

$$f_{ij}(z) = w_i^{\beta}w_j^{1-\beta}f_jL_jn_{ij}(z). \tag{10}$$

where $L_j \equiv \tilde{L}_j + M_j$ is the total measure of consumers in country j, and $\beta > 0$.[34]

The bounds on $n_{ij}(z)$ in equation (8) are a resource constraint: the number of consumers reached by a firm cannot be negative and cannot exceed the total size of the population.

[32] I normalize domestic trade barriers to $\tau_{ii} = 1$, and I further assume $\tau_{ij} \leq \tau_{iv}\tau_{vj}$ for all i, j, v to exclude the possibility of transportation arbitrage.

[33] Sanford and Maddox (1999) provide evidence that exporters use foreign advertising agencies, and Leonidou et al. (2002) review some direct evidence of the use of domestic labor for foreign advertising.

[34] In accordance with Arkolakis (2010), I will make specific assumptions on f_j in the calibration section. However, the fact that f_j does not depend on $n_{ij}(z)$ means that the marginal cost of reaching an additional consumer is constant, which is a special case of Arkolakis (2010).

Using finance jargon, a firm cannot "short" consumers ($n_{ij}(z) < 0$) or "borrow" them from other countries ($n_{ij}(z) > 1$). This makes the maximization problem in (7) quite challenging, because it is subject to $2N$ *inequality* constraints. In finance, it is well known that there is no closed form solution for a portfolio optimization problem with lower and upper bounds (see Jagannathan and Ma (2002) and Ingersoll (1987)).

Notice that the variance of global real profits is the sum of the variances of the profits reaped in all potential destinations. In turn, these variances are the sum of the covariances of the profits from j with all markets, including itself. If the demand shocks were not correlated across countries, then the objective function would simply be the sum of the expected profits minus the variances.

The assumption that the shocks are independent across a continuum of varieties implies that aggregate variables w_j and P_j are non-stochastic. Therefore, plugging into $\pi_{ij}(z)$ the optimal consumers' demand from equation (5), I can write expected profits more compactly as:

$$E\left(\pi_{ij}(z)\right) = \bar{\alpha}_j n_{ij}(z) r_{ij}(z) - \frac{1}{P_i} f_{ij}(z), \tag{11}$$

where $\bar{\alpha_j}$ is the expected value of the demand shock in destination j, and

$$r_{ij}(z) \equiv \frac{1}{P_i} \frac{Y_j p_{ij}(z)^{-\sigma}}{P_j^{1-\sigma}} \left(p_{ij}(z) - \frac{\tau_{ij} w_i}{z}\right). \tag{12}$$

Note that $n_{ij}(z)r_{ij}(z)$ are real gross profits in j. Similarly, the covariance between $\pi_{ij}(z)$ and $\pi_{is}(z)$ is simply:

$$Cov\left(\frac{\pi_{ij}(z)}{P_i}, \frac{\pi_{is}(z)}{P_i}\right) = n_{ij}(z) r_{ij}(z) n_{is}(z) r_{is}(z) Cov(\alpha_j, \alpha_s), \tag{13}$$

where $Cov(\alpha_j, \alpha_s)$ is the covariance between the shock in country j and in country s.[35]

Although there is no *analytical* solution to the first stage problem, because of the presence of inequality constraints, we can take a look at the firm's *interior* first order condition:

[35]The covariance does not depend on the marketing costs because these are non-stochastic.

$$\underbrace{r_{ij}(z)\bar{\alpha}_j - \gamma r_{ij}(z)\sum_s n_{is}(z)r_{is}(z)Cov(\alpha_j,\alpha_s)}_{\text{marginal benefit}} = \underbrace{\frac{1}{P_i}w_i^{\beta}w_j^{1-\beta}f_jL_j}_{\text{marginal cost}}. \tag{14}$$

Equation (14) equates the real marginal benefit of adding one consumer to its real marginal cost. While the marginal cost is constant, the marginal benefit is decreasing in $n_{ij}(z)$. In particular, it is equal to the marginal revenues minus a "penalty" for risk, given by the sum of the covariances that destination j has with all other countries (including itself). The higher the covariance of market j with the rest of the world, the smaller the diversification benefit the market provides to a firm exporting from country i.

An additional interpretation is that a market with a high covariance with the rest of the world must have high average real profits to compensate the firm for the additional risk taken: this trade-off between risk and return is determined by the degree of risk aversion. I will indeed use this intuition to calibrate the risk aversion parameter in the data.

Note the difference in the optimality condition with Arkolakis (2010). In his paper, the marginal benefit of reaching an additional consumer is constant, while the marginal penetration cost is increasing in $n_{ij}(z)$. In my setting, instead, the marginal benefit of adding a consumer is decreasing in $n_{ij}(z)$, due to the concavity of the utility function of the entrepreneur, while the marginal cost is constant.

To find the general solution for n_{ij} and p_{ij}, I only need to make the following assumption, which I assume will hold throughout the paper:

Assumption 2. $det(\Sigma) > 0$

Assumption 2 is a necessary and sufficient condition to have uniqueness of the optimal solution. Since Σ is a covariance matrix, which by definition always has a non-negative determinant, this assumption simply rules out the knife-edge case of a zero determinant.[36] In the Appendix, I prove that (dropping the subscripts i and z for simplicity):

[36] A zero determinant would happen only in the case where all pairwise correlations are exactly 1.

Proposition 1. *For firm z from country i, the unique vector of optimal* $\mathbf{n}$ *satisfies:*

$$\mathbf{n} = \frac{1}{\gamma}\tilde{\Sigma}^{-1}\left[\pi - \mu + \lambda\right], \tag{15}$$

where $\tilde{\Sigma}$ *is firm z's matrix of profits covariances,* π *is the vector of expected net profits,* μ *and* λ *are the vectors of Lagrange multipliers associated with the bounds.*

Moreover, the optimal price charged in destination j is a constant markup over the marginal cost:

$$p_{ij}(z) = \frac{\sigma}{\sigma - 1}\frac{\tau_{ij} w_i}{z} \tag{16}$$

Proposition 1 shows that the optimal solution, as expected, resembles the standard mean-variance optimal rule, which dictates that the fraction of wealth allocated to each asset is proportional to the inverse of the covariance matrix times the vector of expected excess returns (see Ingersoll (1987) and Campbell and Viceira (2002)). The novelty of this paper is that such diversification concept is applied to the problem of the firm. The entrepreneurs, rather than solving a maximization problem country by country, as in traditional trade models, perform a *global* diversification strategy: they trade off the expected global profits with their variance, the exact slope being governed by the absolute degree of risk aversion $\gamma > 0$.

Note that the firm's entry decision in a market (that is, whether $n > 0$) does not depend on a market-specific entry cutoff, but rather on the global diversification strategy of the firm. Therefore, the fact that a firm with productivity z_1 enters market j, i.e. $n_{ij}(z_1) > 0$, does not *necessarily* imply that a firm with productivity $z_2 > z_1$ will enter j as well. For example, a small firm may enter market j because it provides a good hedge from risk, while a larger firm does *not* enter j since it prefers to diversify risk by selling to other markets, where the small firm is not able to export. This is a novel feature of my model, and it differs from traditional trade models with fixed costs, such as Melitz (2003) and Chaney (2008), where the exporting decision is strictly hierarchical. Recent empirical evidence (see Bernard et al. (2003), Eaton et al. (2011) and Armenter and Koren (2015)) suggests instead that, although exporters are more productive than non-exporters in general, there are firms which are more productive than exporters but that still only serve the domestic market.

Finally, since the pricing decision is made after the uncertainty is resolved, and for a given $n_{ij}(z)$, the optimal price follows a standard constant markup rule over the marginal cost, shown in equation 16. Therefore, the realization of the shock in market j only shifts upward or downward the demand curve, without changing its slope.

A limit case. It is worth looking at the optimal solution in the special case of risk neutrality, i.e.$\gamma = 0$. In the Appendix I show that, in this case, a firm sells to country j only if its productivity exceeds an entry cutoff:

$$(\bar{z}_{ij})^{\sigma-1} = \frac{w_i^{\beta} w_j^{1-\beta} f_j L_j P_j^{1-\sigma} \sigma}{\bar{\alpha}_j \left(\frac{\sigma}{\sigma-1}\tau_{ij} w_i\right)^{1-\sigma} Y_j}, \tag{17}$$

and that, whenever the firm enters a market, it sells to all consumers, so that $n_{ij}(z) = 1$. This case is isomorphic (with $\bar{\alpha}_j = 1$) to the firm's optimal behavior in trade models with risk-neutrality and fixed entry costs, such as Melitz (2003) and Chaney (2008). In these models, firms enter all profitable locations, i.e. the markets where the revenues are higher than the fixed costs of production, and upon entry they serve all consumers.[37] The case of $\gamma = 0$ constitutes an important benchmark, as I will compare the welfare impact of counterfactual policies in my model with a positive risk aversion versus a model with $\gamma = 0$, i.e. the canonical trade models by Melitz (2003), Chaney (2008).

3.2.1 Trade patterns

To gain more intuition from Proposition 1, let us ignore for a moment the inequality constraints in the firm problem. Then, equation (15) becomes:

$$n_{ij}(z) = \frac{S_j}{r_{ij}(z)\gamma} - \frac{\sum_k C_{jk} \frac{w_i f_k L_k}{r_{ik}(z)}}{r_{ij}(z)\gamma}, \tag{18}$$

where S_j is the Sharpe Ratio of country j:

$$S_j = \sum_k C_{jk} \bar{\alpha}_j \tag{19}$$

and C_{jk} is the $j - k$ cofacor of the covariance matrix of demand Σ.[38] The Sharpe Ratio

[37] Even in models with endogenous marketing costs, such as Arkolakis (2010), firms may not reach all consumers in a destination, but they enter only if the producitivty is larger than an entry cutoff.

[38] The cofactor is defined as $C_{kj} \equiv (-1)^{k+j} M_{kj}$, where M_{kj} is the (k, j) minor of Σ. The minor of a matrix is the determinant of the sub-matrix formed by deleting the k-th row and j-th column.

in equation (19) is an (inverse) measure of country risk. For example, with two symmetric countries, S_j equals:

$$S = \frac{\bar{\alpha}}{\sigma^2(1+\rho)}, \tag{20}$$

where σ^2 and $\bar{\alpha}$ denote the variance and the mean of the demand shocks, respectively, and ρ is the cross-country correlation. Equation (20) shows that the Sharpe Ratio is decreasing in the volatility of the shocks, and decreasing in the correlation of demand with the other country.[39] In the general case of N countries, i.e. equation (19), it is easily verifiable that S_j is decreasing in the variance of demand in market j and in the correlation of demand in j with the *rest of the world*. The intuition is that the more volatile demand in market j, relative to its mean, or the more demand is correlated with the rest the world, the riskier is country j, and the *lower* S_j. Therefore the Sharpe Ratio summarizes the diversification benefits that a country provides to firms, since it is inversely proportional to the overall riskiness of its demand.

Then, equation (18) implies that both the probability of exporting to a country and the number of consumers reached are increasing in the Sharpe Ratio, holding constant wages and prices.[40] Thus, a firm is more likely to enter a market with a higher Sharpe Ratio, i.e. a market that provides good diversification benefits, conditional on trade barriers and market specific characteristics. In addition, conditional on entering a destination, the amount exported is larger in markets with high Sharpe Ratio. The intuition is that, if a market is "safe", then firms optimally choose to be more exposed there to hedge their business risk, and thus export more intensely to that market.

In the Appendix, I prove that this result holds also in the general case where some inequality constraints are binding, i.e. the firm does not enter all markets:

Proposition 2. *Define A a matrix whose $i-j$ element equals $A_{ij} = -\sum_{k\neq 1} C_{ik} Cov(\alpha_k, \alpha_j)$ for $i \neq j$, and $A_{ij} = 1$ for $i = j$. If A is a M-matrix, then the probability of exporting and the amount exported to a market are increasing in its Sharpe Ratio.*

Proposition 2 suggests that neither the demand volatility in a market, nor the bilateral

[39] Recall that the Sharpe Ratio of a stochastic variable is defined as the ratio of its expected mean (or sometimes its "excess" expected return over the risk-free rate) over its standard deviation (or sometimes the variance).

[40] Note that if the Sharpe Ratio of a country changes because of a shock to the covariance matrix, that will have also a general equilibrium effect on wages and prices. In Proposition 2, I focus on the partial equilibrium effect of the Sharpe Ratio on the firm decision. The prediction, however, holds true also in general equilibrium, as I show in the counterfactual analysis in Section 5.

covariance of demand with the domestic market, are sufficient to predict the direction of trade. Instead, what determines trade patterns is the multilateral covariance, i.e. how much the demand in a market covariates with demand in all other countries. The sufficient, but not necessary, condition to have a positive effect of the Sharpe Ratio on $n_{ij}(z)$ is that the matrix A is a M-matrix, i.e. all off-diagonal elements are negative. It is easy to verify that A is a M-matrix whenever *some* demand correlations are negative.[41]

Propositions 1 and 2 also suggest how my model can reconcile the positive relationship between firm entry and market size with the existence of many small exporters in each destination, as shown by Eaton et al. (2011) and Arkolakis (2010). On one hand, upon entry firms can extract higher profits in larger markets. Therefore, more companies enter markets with larger population size. On the other hand, the firms' global diversification strategy may induce them to optimally reach only few consumers, and thus export small amounts. In contrast, the standard fixed cost models, such as Melitz (2003) and Chaney (2008), require *large* fixed costs to explain firm entry patterns, which contradict the existence of many small exporters. In the empirical section, I will use this feature to test the model's goodness of fit in the data.

Having characterized the exporting behavior of risk averse firms, I now define the world equilibrium and discuss its properties.

3.3 Trade equilibrium

I now describe the equations that define the trade equilibrium of the model. Following Helpman et al. (2004), Chaney (2008) and Arkolakis et al. (2008), I assume that the productivities are drawn, independently across firms and countries, from a Pareto distribution with density:

$$g(z) = \theta z^{-\theta-1}, \;\; z \geq \underline{z}, \tag{21}$$

where $\underline{z} > 0$. The price index is:

$$P_i^{1-\sigma} = \sum_j M_j \int_{\underline{z}}^{\infty} \bar{\alpha}_i n_{ji}(z) p_{ji}(z)^{1-\sigma} g(z) dz, \tag{22}$$

[41] This can be seen, for example, for the case $N = 4$, where a typical element of the matrix A looks like:

$$A_{21} = \rho_{12}\sigma_1^3\sigma_2\sigma_3^2\sigma_4^2(1 - \rho_{13}^2 - \rho_{14}^2 - \rho_{34}^2 + 2\rho_{13}\rho_{14}\rho_{34}).$$

Then, to have $A_{21} < 0$, at least one correlation needs to be negative.

where $n_{ji}(z)$ and $p_{ji}(z)$ are given in Proposition 1.[42] Since the optimal fraction of consumers reached, $n_{ij}(z)$, is bounded between 0 and 1, a sufficient condition to have a finite integral is that $\theta > \sigma - 1$. As in Chaney (2008), the number of firms is fixed to M_i, implying that in equilibrium there are profits, which equal:

$$\Pi_i = M_i \sum_j \left(\frac{1}{\sigma} \int_{\underline{z}}^{\infty} \bar{\alpha}_j q_{ij}(z) p_{ij}(z) g(z) dz - \int_{\underline{z}}^{\infty} f_{ij}(z) g(z) dz \right). \tag{23}$$

I impose a balanced current account, thus the sum of labor income and business profits must equal the total income spent in the economy:

$$Y_i = w_i \tilde{L}_i + \Pi_i. \tag{24}$$

Finally, the labor market clearing condition states that in each country the supply of labor must equal the amount of labor used for production and marketing:

$$M_i \sum_j \int_{\underline{z}}^{\infty} \frac{\tau_{ij}}{z} \bar{\alpha}_j q_{ij}(z) g(z) dz + M_i \sum_j \int_{\underline{z}}^{\infty} f_j n_{ij}(z) L_j g(z) dz = \tilde{L}_i, \tag{25}$$

Therefore the trade equilibrium in this economy is characterized by a vector of wages $\{w_i\}$, price indexes $\{P_i\}$ and income $\{Y_i\}$ that solve the system of equations (22), (24), (25), where n_{ij} is given by equation (15). It is worth noting that the realization of the demand shocks does not affect the equilibrium wages and prices, because on aggregate the idiosyncratic shocks average out by the Law of Large Numbers.[43]

Proposition 1 implies that the sales of firm z to country j are given by:

$$x_{ij}(z) = p_{ij}(z) q_{ij}(z) = \alpha_j(z) \left(\frac{\sigma}{\sigma - 1} \frac{\tau_{ij} w_i}{z} \right)^{1-\sigma} \frac{Y_j}{P_j^{1-\sigma}} n_{ij}(z) \tag{26}$$

where $n_{ij}(z)$ satisfies equation (15). From equation (26), aggregate trade flows from i to j are:

[42] The assumptions that the demand shocks are i.i.d. across a continuum of varieties, and that the mean of the shocks is the same for all z, imply that in the expression for the price index there is simply $\bar{\alpha}_i = \bar{\alpha}_i(z) \equiv \int_0^{\infty} \alpha_i(z) g_i(\alpha) d\alpha$, where $g_i(\alpha)$ is the marginal density function of the demand shock in destination i.

[43] This happens because shocks are i.i.d. across a continuum number of varieties. Also, labor markets are frictionless, and thus workers can freely (and instantaneously) reallocate from a firm hit by a bad shock to another firm. Note that my model is not isomorphic to an economy with country-specific shocks because, in that case, the idiosyncratic shocks would not average out since the number of countries is finite.

$$X_{ij} = M_i \int_{\underline{z}}^{\infty} \bar{\alpha}_j \left(\frac{\sigma}{\sigma - 1} \frac{\tau_{ij} w_i}{z} \right)^{1-\sigma} \frac{Y_j}{P_j^{1-\sigma}} n_{ij}(z) \theta z^{-\theta-1} dz. \tag{27}$$

Proposition 2 then implies that aggregate trade flows X_{ij} are increasing in S_j, the measure of diversification benefits that destination j provides to exporters. I will test this prediction in the data.

3.4 Welfare gains from trade

I define welfare in country i as the equally-weighted sum of the welfare of workers and entrepreneurs:

$$W_i = U_i^w \tilde{L}_i + M_i \int_{\underline{z}}^{\infty} U_i^e(z)\, dG(z), \tag{28}$$

where U_i^w is the indirect utility of each worker (which is the same for all workers), while $U^e(z)$ is the indirect utility of each entrepreneur (which differs depending on the productivity z). Since workers simply maximize a CES utility, their welfare is simply the real wage $\frac{w_i}{P_i}$. In contrast, the entrepreneurs maximize a stochastic utility, and thus the correct *money-metric* measure of their welfare is the Certainty Equivalent (see Pratt (1964) and Pope et al. (1983)). The Certainty Equivalent is simply the *certain* level of wealth for which the decision-maker is indifferent with respect to the uncertain alternative. The assumption of CARA utility implies that the Certainty Equivalent is, for entrepreneur z:[44]

$$U_i^e(z) = E\left(\frac{\pi_i(z)}{P_i}\right) - \frac{\gamma}{2} Var\left(\frac{\pi_i(z)}{P_i}\right). \tag{29}$$

Then, aggregate welfare is:

$$W_i = \frac{w_i \tilde{L}_i}{P_i} + \frac{\Pi_i}{P_i} - R_i, \tag{30}$$

where $R_i \equiv M_i \int_{\underline{z}}^{\infty} \frac{\gamma}{2} Var\left(\frac{\pi_i(z)}{P_i}\right) dG(z)$ is the aggregate "risk premium". Note that when the risk aversion equals zero, or when there is no uncertainty, total welfare simply equals the real

[44] As explained earlier, this is true up to a second-order Taylor approximation.

income produced in the economy, as in canonical trade models (see Chaney (2008), Arkolakis (2010)).

Welfare gains from trade. I now characterize the percentage change in the aggregate certainty equivalent associated with a change in trade costs from τ_{ij} to $\tau'_{ij} < \tau_{ij}$. As common in the welfare economics literature, welfare changes are measured with the compensating variation CV, defined as:

$$CV_i \equiv W_i(\tau'_{ij}) - W_i(\tau_{ij}). \tag{31}$$

Thus, CV_i is the ex-ante sum of money which, if paid in the counterfactual equilibrium, makes all consumers indifferent to a change in trade costs. For small changes in trade costs, the welfare gains are, from equation (30):

$$d\ln W_i = \underbrace{\frac{w_i \tilde{L}_i / P_i}{W_i} d\ln\left(\frac{w_i}{P_i}\right)}_{\text{workers' gains}} + \underbrace{\underbrace{\frac{\Pi_i / P_i}{W_i} d\ln\left(\frac{\Pi_i}{P_i}\right)}_{\text{profit effect}} - \underbrace{\frac{R_i}{W_i} d\ln R_i}_{\text{risk effect}}}_{\text{entrepreneurs' gains}}. \tag{32}$$

The first term reflects the gains that are accrued by workers, since their welfare is simply given by the real wage. The second term in 32 represents the entrepreneurs' welfare gains, which are the sum of a profit effect and a risk effect. The first effect is the change in real profits after the trade shock, weighted by the share of real profits in total welfare. Note that in models with risk neutrality and Pareto distributed productivities, such as Chaney (2008) and Arkolakis et al. (2008), profits are a constant share of total income. Consequently, the sum of workers' gains and the profits effect simply equals $-d\ln P_i$ (taking the wage as numeraire). In my model, in contrast, profits are no longer a constant share of Y_i, as can be gleaned from equation 24.

The third term in 32 is the percentage change in the aggregate risk premium. Note that, a priori, it is ambiguous whether this term increases or decreases after a trade liberalization. Indeed, lower trade barriers imply that firms can better diversify their risk across markets, and thus the volatility of their profits goes down. However, lower trade costs imply higher profits and, mechanically, also higher variance. In the case of two symmetric countries, as well as in empirical analysis, I show that the first effect dominates and the overall variance decreases after a trade liberalization.

A limit case. As shown earlier, when the risk aversion is zero the firm optimal behavior is the same as in standard monopolistic competition models as in Melitz. It is easy to show that, in the special case of $\gamma = 0$, the welfare gains after a reduction in trade costs are given by:

$$d\text{ln}W_i|_{\gamma=0} = -d\text{ln}P_i = -\frac{1}{\theta}d\text{ln}\lambda_{ii} \tag{33}$$

where λ_{ii} denotes domestic trade shares and θ equals the trade elasticity. As shown by ACR, several trade models predict the welfare gains from trade to be equal to equation (33). Therefore, in the following section and in the quantitative analysis the case of $\gamma = 0$ will be an important benchmark for the welfare gains from trade in my model.

In the following section I analytically solve the model in the special case of two symmetric countries, and derive an analytical expression for the welfare gains from trade directly as a function of the Sharpe Ratio.

3.4.1 Two symmetric countries

To illustrate some properties of the model and to obtain a closed-form expression for the welfare gains from trade, I study the special case where there are two perfectly symmetric countries, home and foreign. Define $\bar{\alpha}$ to be the expected value of the demand shock, $Var(\alpha)$ its variance and ρ the cross-country correlation of shocks. For simplicity, I assume that $\bar{\alpha} = Var(\alpha) = 1$. I consider two opposite equilibria: one in which there is autarky, and one in which there is free trade, so $\tau_{ij} = 1$ for all i and j.[45]

Under autarky, the Sharpe Ratio is simply the ratio between the mean and the variance of the demand shocks:

$$S_A = \frac{\bar{\alpha}}{Var(\alpha)} = 1. \tag{34}$$

Instead, under free trade the Sharpe Ratio is

$$S = \frac{\bar{\alpha}}{Var(\alpha)\,(1+\rho)} = \frac{1}{1+\rho}. \tag{35}$$

Notice that the Sharpe Ratio is decreasing in the cross-country correlation of demand: the larger this correlation, then the smaller the diversification benefits from selling abroad.

[45] Throughout this section, I will set $\underline{z} = 1$.

In the Appendix, I show that in both equilibria the firm's optimal solution is:[46]

$$n(z) = 0 \text{ if } z \leq z^*$$
$$0 < n(z) < 1 \text{ if } z > z^*$$

where $n(z)$ is given by:

$$n(z) = \frac{S}{\gamma} \frac{\left(1 - \left(\frac{z^*}{z}\right)^{\sigma-1}\right)}{r(z)}, \tag{36}$$

where $r(z)$ are real gross profits, as in equation (12), and the entry cutoff is:

$$z^* = \left(\left(\frac{\sigma}{\sigma - 1}\right)^{\sigma-1} \frac{fP^{1-\sigma}\sigma}{\bar{\alpha}Y}\right)^{\frac{1}{\sigma-1}}. \tag{37}$$

Notice that the entrepreneur's optimal decision under free trade is the same as in autarky, except that the Sharpe Ratio under free trade reflects the cross-country correlation of demand.[47] The more correlated is demand with the foreign country, the "riskier" the world and thus the lower the number of consumers reached. Finally, the existence of a single entry cutoff means that there is strict sorting of firms into markets. However, that happens only because of the perfect symmetry between the two countries, which implies that $n(z)$ is not affected by the Lagrange multipliers of the other location. In the general case of N asymmetric countries, firms do not strictly sort into foreign markets, as explained in the previous section.

I now investigate the welfare impact of going from autarky to free trade, and study how the Sharpe Ratio plays a role in determining the welfare gains from trade. Recall from the previous section, equation (30), that welfare can be written as total real income minus the aggregate risk premium. In the Appendix I prove the following result:

[46]I assume that $\gamma > \tilde{\gamma}$ (where $\tilde{\gamma}$ depends only on parameters), so that $n(z) < 1$ always for all z. This allows me to get rid of the multiplier of the upper bound. The intuition is that the entrepreneurs are sufficiently risk averse so that they always prefer to *not* reach all consumers. See Appendix for more details.

[47]The perfect symmetry and the absence of trade costs imply that any firm will choose the same $n(z)$ in both the domestic and foreign market. This means that either a firm enters in both countries, or in neither of the two. This feature is the reason why perfect symmetry and free trade is the only case in which I can derive an analytical expression for $n(z)$. If there were trade costs $\tau_{ij} > 1$, the optimal $n(z)$ would still depend on the Lagrange multiplier of the other destination.

Proposition 3. *Welfare gains of going from autarky to free trade are given by:*

$$\hat{W} = \frac{W_{FT}}{W_A} - 1 = S^{\frac{1}{\theta+1}}\xi - 1 \tag{38}$$

where $\xi > 1$ is a function of θ and σ. Moreover, welfare gains are higher than ACR only if $\bar{\rho} > \rho$, where $\bar{\rho} < 1$ is a function of parameters.

Proposition 3 states that the welfare gains of moving from autarky to free trade are increasing in the Sharpe Ratio, or equivalently, are decreasing in ρ, the cross-country correlation of demand. The intuition is simple: if the correlation is low, or even negative, it means that firms can hedge their domestic demand risk by exporting to the foreign country. This implies tougher competition among firms, and thus an increase in the average productivity of surviving firms, which leads to lower prices. If instead the correlation is high, and closer to 1, demand in the foreign market moves in the same direction as the domestic demand, and thus firms cannot fully hedge risk by exporting abroad. This implies a lower competitive pressure, and a smaller decrease in the price index. It is easy to verify that, as long as $\theta > \sigma - 1$, the expression in 32 is always positive, and thus there are always gains from trade.[48]

It is worth noting that the total number of varieties available does not change between autarky and free trade.[49] The (unbounded) Pareto assumption implies that the additional number of foreign varieties is exactly offset by the lower number of domestic varieties. Therefore the gains from trade arise from the selection of more efficient firms, which increases the average productivity and lowers prices.[50] The higher the Sharpe Ratio, the larger the increase in average productivity.

Furthermore, my model with risk averse firms predicts larger welfare gains from trade than standard models with risk neutral firms, as long as the correlation of demand is not too high.[51] The intuition is that when the correlation is low, or even negative, in my model there

[48]Note that welfare gains do not depend on neither the risk aversion, nor the mean/variance ratio. The reason is simply that countries are perfectly symmetric, and thus the only variable that affects the gains from trade is the demand correlation, which is a cross-country force.

[49]See Analytical Appendix for a proof.

[50]See Melitz and Redding (2014) and Feenstra (2016) for a discussion about the implications of assuming an unbounded Pareto distribution of productivities.

[51]It is easy to verify that, when the risk aversion is zero, the gains of moving from autarky to free trade are, using the ACR formula:

$$\hat{W}|_{\gamma=0} = \left(\frac{1}{2}\right)^{-\frac{1}{\theta}} - 1$$

is more entry of foreign firms, because they want to diversify their demand risk by selling to the other country. This implies tougher competition and lower prices, and this price decrease is stronger than in a model with risk neutral firms, where firms use international trade only to increase profits, not to decreases their variance. The additional gains from the risk diversification strategy of the firms raises aggregate welfare gains compared to ACR. When instead the correlation is too high, firms rely less on international trade to diversify risk, implying less competition among firms compared to a model with risk neutral firms, and thus welfare gains from trade are lower.

Decomposition of welfare gains. As suggested by equation (32), I can decompose the welfare gains from trade in workers' gains and entrepreneurs' gains. In the Analytical Appendix I show that both workers and entrepreneurs gains are given by:

$$\hat{W}_{\tilde{L}} = \hat{W}_M = \left(\frac{S}{2}\right)^{\frac{1}{\theta+1}} - 1 \tag{39}$$

Workers' and entrepreneurs' gains are always positive and decreasing in the cross-country correlation of demand. Notice that for the workers the welfare gains are simply the percentage change in the real wage, and thus they can only gain from trade, since prices go down. For some entrepreneurs, instead, gains from trade could be negative: on one hand nominal profits are higher because firms can sell also to the foreign market, but on the other hand they are lower because of the competition from foreign firms. On aggregate, however, these two effects offset each other, due to the Pareto assumption, and thus *nominal* profits stay constant. Since prices go down with free trade, aggregate *real* profits increase. In addition, aggregate variance of real profits goes up, because prices go down and because, if ρ is sufficiently high, the total variance of nominal profits is higher than the variance under autarky. Equation 39 states that the increase in aggregate real profits dominates over the increase in the variance, and thus aggregate entrepreneurial gains are positive.

4 Quantitative implications

I use the general equilibrium model laid out in the previous section as a guide through the data. I first use aggregate and firm-level data to estimate the relevant parameters, and then I test the empirical implications of the model.

4.1 Data

The analysis mostly relies on a panel dataset on international sales of Portuguese firms to 210 countries, between 1995 and 2005.[52] These data come from Statistics Portugal and roughly aggregate to the official total exports of Portugal. I merged this dataset with data on some firm characteristics, such as number of employees, total sales and equity, which I extracted from a matched employer–employee panel dataset called Quadros de Pessoal.[53] I also merge the trade data with another dataset, called Central de Balancos, containing balance sheet information, such as net profits, for all Portuguese firms from 1995 to 2005. I describe these datasets in more detail in the Appendix. Finally, in the calibration I use data on manufacturing trade flows in 2005 from the UN Comtrade database as the empirical counterpart of aggregate bilateral trade in the model, and data on manufacturing production from WIOD and UNIDO.[54]

From the Portuguese trade dataset, I consider the 10,934 manufacturing firms that, between 1995 to 2005, were selling domestically and exporting to at least one of the top 34 destinations served by Portugal.[55] Trade flows to these countries accounted for 90.56% of total manufacturing exports from Portugal in 2005. I exclude from the analysis foreign firms' affiliates, i.e. firms operating in Portugal but owned by foreign owners, since their exporting decision is most likely affected by their parent's optimal strategy. The universe of Portuguese manufacturing exporters is comprised of mostly small firms and fewer large players. The median number of destinations served is 3, and the average export share is 30%. Other empirical studies have revealed similar statistics using data from other countries, such as Bernard et al. (2003) and Eaton et al. (2011).

4.2 Parameters estimation

The year in which I estimate the model and test its predictions is 2005, in which I assume the world equilibrium reached its steady state. The estimation approach is tightly connected

[52] I focus on sales at the firm-level, rather than at the plant-level, both for the domestic and foreign markets. This choice allows me to look at firm statistics on sales across different destinations and is consistent with the monpolistic competition model shown in the previous section.

[53] I thank the Economic and Research Department of Banco de Portugal for giving me access to these datasets.

[54] I use data from the INDSTAT 4 2016 dataset. See Dietzenbacher et al. (2013) for details about the WIOD database.

[55] I first select the top 45 destinations from Portugal by value of exports, and then I keep the countries for which there is data on manufacturing production, in order to construct bilateral trade flows. See the list of countries in Table 6 in the Data Appendix.

to the model, and consists of two main stages. In the first, I use data on international sales from 1995 and 2004 to estimate the moments of the demand distribution, $G(\bar{\alpha}, \Sigma)$, as well as the risk aversion parameter γ. To implement the first stage, I do not need to solve for the general equilibrium model. In the second stage, taking as given $G(\bar{\alpha}, \Sigma)$ and γ, I calibrate the remaining parameters with the Simulated Method of Moments, using data for 2005.

4.2.1 Estimation of Σ

Given the static nature of the model, Σ is a *long-run* covariance matrix that firms i) know and ii) take as given when they choose their risk diversification strategy. However, there is evidence that, in the short run, firms sequentially enter different markets to learn their demand behavior (see Albornoz et al. (2012) and Ruhl and Willis (2014) among others). In the data, this behavior may confound the pure risk diversification behavior of exporters predicted by my model, affecting the estimation of Σ. For this reason, I estimate the covariance matrix considering only "established" firm-destination pairs, i.e. exporters selling to a certain market for at least 5 years. For these exporters, the learning process is most likely over, and therefore the estimates of the covariance matrix are less affected by the noisy learning process.

I make the following parametric assumption:

Assumption 3. $\log\alpha(z,t) \sim N\left(\mathbf{0}, \hat{\Sigma}\right)$, i.i.d. across z and across t

where z and t stand for firm and year, respectively. Assumption 3 states that the demand shocks are drawn from a multivariate log-normal distribution with vector of means 0 and covariance matrix $\hat{\Sigma}$, and that the shocks are drawn independently across firms and time. In other words, the log of demand shocks follow a Standard Brownian Motion.[56] This assumption allows to exploit *both* cross-sectional and time-series variation in trade flows to estimate the country-level covariance matrix.[57]

The estimation of Σ entails several steps.

Step 1. To identify the demand shocks, I assume that the parameters of the model stay constant during the estimation period. This implies, from equation (26), that any variation over time of x_{Pjz}, i.e. the exports of firm z from Portugal to destination j, is due solely to the demand shock α_{jz}. However, in the estimation I control for other types of shocks as well.

[56] Arkolakis (2016) has a similar assumption for productivity shocks, which can be reintepreted as demand shocks. See discussion in footnote 28 of Arkolakis (2016).

[57] The data supports this assumption: most of the firm-destinations pairs do not have strongly serially correlated demand shocks, according to Durbin-Watson tests not reported here.

Specifically, I run the following regression (omitting the source subscript):

$$\Delta\tilde{x}_{jzt} = f_{jt} + f_{zt} + \varepsilon_{jzt} \tag{40}$$

where $\Delta\tilde{x}_{jzt} \equiv log\left(x_{jzt}\right) - log\left(x_{jzt-1}\right)$ is the growth rate of firm z' s exports to destination j at time t. f_{jt} is a destination-time fixed effect, which controls for any aggregate shock affecting all products in market j at time t; f_{zt} is a firm-time fixed effect, which controls for any shock, like productivity, affecting sales of firm z to all destinations.[58] The residual from the above regression, ε_{jzt}, is the change in the log of the demand shock for firm z in market j, $\Delta\tilde{\alpha}_{jzt}$. A similar approach, i.e. using annual sales growth rates to identify firm-specific shocks as deviations from country-specific trends, has been adopted by Di Giovanni et al. (2014), Gabaix (2011) and Castro et al. (2010).

Step 2. Assumption 3 implies that I can stack the residuals $\Delta\tilde{\alpha}_{jzt}$ and compute the NxN covariance matrix $\Sigma_\Delta$of the *change* of the *log* shocks, which are normally distributed with mean 0.[59]

Step 3. From Σ_Δ, estimated in Step 2, I easily obtain, using Assumption 3, the long run covariance matrix of the *level* of the shocks, Σ.[60]

Results. Using the estimated covariance matrix Σ, I compute the country-level Sharpe Ratios, using equation (19).[61] Table 6 in the Data Appendix lists the estimated Sharpe Ratios for the destinations in the sample, together with their standard errors, computed with a bootstrap tecnique.[62] We can see that the standard errors are small relative to the point estimates, suggesting that the Sharpe Ratios are quite precisely estimated.

Recall that the Sharpe Ratio summarizes the multilateral covariance of a country's demand with the rest of the world, and therefore is affected by both its variance and the correlation with the other countries. Figure 1 plots the estimated Sharpe Ratios against the estimated demand variance (top figure), as well as the average demand correlation with the other countries (bottom figure). As expected, in both panels there is a negative relationship:

[58] Controlling for destination, time or firm fixed effects has a marginal impact on the estimates.

[59] An alternative would be to compute a covariance matrix for each year and take the average $\bar{\Sigma}_\Delta = \frac{1}{T}\sum_t \Sigma^t_\Delta$. In the Appendix I prove that, since the mean of $\Delta\tilde{\alpha}_{jzt}$ is zero, this leads to exactly the same covariance matrix.

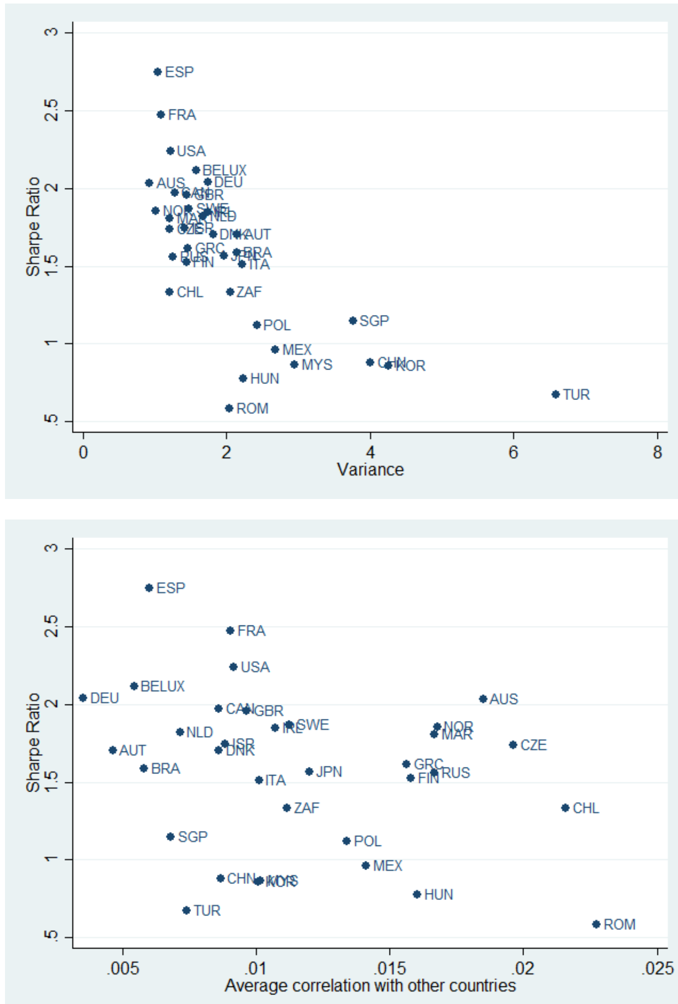
[60] See the analytical Appendix for a formal derivation.

[61] For simplicity I set $\bar{\alpha} = \mathbf{1}$, as in Eaton et al. (2011).

[62] For the bootstrap, I repeat the estimation process 1,000 times, replacing the original data with a random sample, drawn with replacement, of the original firms in the dataset. The bootstrapped standard errors are not centered.

the higher the volatility of demand, or the larger is the average correlation with the other countries, the smaller the risk diversification benefits and thus the lower the Sharpe Ratio.

Figure 1: Sharpe Ratios and their components



Notes: The figure at the top plots the estimated Sharpe Ratio of the destinations in the sample against the corresponding demand variance. The figure at the bottom, instead, plots the Sharpe Ratios against the corresponding average correlation of demand with all other countries.

4.2.2 Estimation of risk aversion

To estimate the firms' risk aversion, I follow Allen and Atkin (2016) and directly use the firms' first order conditions. For simplicity, I assume that marketing costs are sufficiently high so that there is no Portuguese firm selling to the totality of consumers in any country (given the size of the median Portuguese firm, this seems a reasonable assumption). This implies that $\mu_j(z) = 0$ for all j and z.[63] For each destination j where firm z is selling to, the FOC is (omitting the source subscript, since all firms are from Portugal):

$$\bar{\alpha_j} r_j(z) - w^{\beta} w_j^{1-\beta} f_j L_j / P - \gamma \sum_s r_j(z) n_s(z) r_s(z) Cov(\alpha_j, \alpha_s) = 0$$

where I set $\lambda_j(z) = 0$ as well, since $n_j(z) > 0$. Multiplying and dividing by $n_j(z)$, and summing over j, the above can be rewritten as:

$$E[\pi(z)] = \gamma Var(\pi(z)) \tag{41}$$

where $E[\pi(z)] \equiv \sum_j E[\pi_j(z)]$ are expected net profits and $Var(\pi(z)) \equiv \sum_j \sum_s Cov(\pi_j(z), \pi_s(z))$ is the variance of total net profits.[64] The intuition behind equation (41) is that the risk aversion regulates the slope of the relationship between the mean of profits and their variance. The higher γ, the more firms want to be compensated for taking additional risk, and thus higher variance of profits must be associated with higher expected profits.

To estimate equation (41), I use Portuguese data on firms' total net profits from 1995 to 2004, available from Inquérito Anual, and for each firm I compute the average and variance of profits.[65] Table 1 shows that there is a positive and statistically significant relationship between the average profits and their variance, with a risk-aversion parameter of 0.0046. The reason for such a small number is that equation (41) is in *levels*, and the variance is proportional to the square of the mean. If instead I were to estimate equation (41) in *logs*, I would obtain a risk aversion of 0.707, very close to the estimate of 1 in Allen and Atkin (2016), which use the log returns of crops to estimate Indian farmers' risk aversion.[66]

[63] I verify that this condition holds also when I simulate the model in the counterfactual exercises run below.

[64] Since marketing costs are non-stochastic, we have that $Cov(x_j(z), x_s(z)) = Cov(\pi_j(z), \pi_s(z))$.

[65] Note that I only observe each firm's total net profits, not firm-destination profits. I consider only Portuguese firms active for at least 5 years during the sample period.

[66] One additional reason for the risk aversion being lower than in Allen and Atkin (2016) is that they correct for measurement error downward bias by instrumenting the variance of crop returns with the variance of

Table 1: Estimation of risk aversion

Dep. Variable	Average profits
Variance of profits	0.0046***
	(0.0001)
Observations	1,316
R-squared	0.5468

Notes: The table regresses the the average profits of Portuguese exporters on their variance. Both statistics are computed using yearly data from 1995 to 2004 for firms exporting for more than 5 years. Robust standard errors are shown in parenthesis (*** p<0.01, ** p<0.05, * p<0.1).

It is worth noting that estimating equation (41) may not exactly identify the risk aversion parameter, because some firms in the sample may actively hedge profits fluctuations by means of financial derivatives. If such derivatives hedging was effective, then some firms could reduce the volatility of their cash-flows, which means that I would overestimate the true risk aversion. However, this concern is mitigated by the evidence that hedging practices are not widespread among Portuguese firms (see Iyer et al. (2014)), and by the fact that the sample is composed mostly by small firms, whose access to financial markets is more limited (see Gertler and Gilchrist (1994), Hoffmann and Shcherbakova-Stewen (2011)).

4.2.3 Simulated Method of Moments

Given the estimated covariance matrix Σ and risk aversion γ, the remaining parameters are calibrated with the Simulated Method of Moments, so that endogenous outcomes from the model match salient features of the data. I calibrate the parameters using data for 2005.

Some parameters are directly observable in the data, and thus, I directly assigned values to them. The elasticity of substitution σ directly regulates the markup that firms charge. Estimates for the average mark-up for the manufacturing sector range from 20 percent (Martins et al. (1996)) to 37 percent (Domowitz et al. (1988) and Christopoulou and Vermeulen (2012)). Since the model needs to satisfy the restriction $\theta > \sigma - 1$, I set $\sigma = 4$, implying a markup of 33 percent.[67] I proxy $\tilde{L}_j$ with the total number of workers in the manufacturing

rainfall-predicted returns. Unfortunately data limitations prevent me to address such downward bias.

[67] This is also consistent with the estimates using plant-level U.S. manufacturing data in Bernard et al. (2003).

sector, while M_j is the total number of manufacturing firms.[68]

To reduce the dimensionality of the problem, I assume, similarly to Tintelnot (2016), that trade costs have the following functional form:

$$\ln\tau_{ij} = \kappa_0 + \kappa_1 \ln(dist_{ij}) + \kappa_2 cont_{ij} + \kappa_3 lang_{ij} + \kappa_4 RTA_{ij}, \quad i \neq j, \tag{42}$$

where $dist_{ij}$ is the geographical distance between countries i and j, $cont_{ij}$ is a dummy equal to 1 if the two countries share a border, $lang_{ij}$ is a dummy equal to 1 if the two countries share the same language, and RTA_{ij} is a dummy equal to 1 if the two countries have a regional trade agreement.[69]

I follow Arkolakis (2010) and assume that per-consumer marketing costs f_j are given by:

$$f_j = \tilde{f}\,(L_j)^{\chi-1} \tag{43}$$

where $\tilde{f} > 0$. This functional form can be micro-founded as each firm sending costly ads that reach consumers in j, and the number of consumers who see each ad is given by $L_j^{1-\chi}$.[70] Assuming that the labor requirement for each ad is $\tilde{f}$, the amount of labor required to reach a fraction $n_{ij}(z)$ of consumers in a market of size L_j is equal to $f_{ij} = w_i^{\beta} w_j^{1-\beta} f_j n_{ij}(z) L_j$.[71] I follow Arkolakis (2010) and set $\beta = 0.71$. Finally, I normalize the lower bound of the Pareto distribution to 1.

The calibration algorithm is as follows:
1) Guess a vector $\Theta = \left\{\theta, \kappa_0, \kappa_1, \kappa_2, \kappa_3, \kappa_4, \chi, \tilde{f}\right\}$.
2) Solve the trade equilibrium using the system of equations (15), (22), (24) and (25).[72]
3) Produce 3 sets of moments:

[68] See the Data Appendix for details.

[69] These "gravity" variables were downloaded from the CEPII website. See Head et al. (2010) and Head and Mayer (2013).

[70] The parameter χ is expected to be between 0 and 1, given the empirical evidence that the cost to reach a certain number of consumers is lower in markets with a larger population (see Mathewson (1972) and Arkolakis (2010)).

[71] Notice that this formulation corresponds to the special case in Arkolakis (2010) where the marginal cost of reaching an additional consumer is constant.

[72] Note that the firm problem has to be solved numerically. Therefore, I simulate a discrete number of firms, each with a given productivity, and compute the optimal $n_{ij}(z)$, $\forall i, j, z$. Since the firm maximization problem is a quadratic problem with bounds, it can be quickly solved in Matlab, for example, using the function quadprog.m. Finally, to solve for the general equilibrium, I normalize world GDP to a constant, as in Allen et al. (2014).

- **Moment 1**. Aggregate trade shares, $\lambda_{ij} \equiv \frac{X_{ij}}{\sum_k X_{kj}}$, for $i \neq j$, where X_{ij} are total trade flows from i to j, as shown in equation (27). I stack these trade shares in a $N(N-1)$-element vector $\hat{m}(1;\Theta)$ and compute the analogous moment in the data, $m^{data}(1)$, using manufacturing trade data in 2005.[73] This moment is used to calibrate the trade costs parameters.

- **Moment 2.** Number of Portuguese exporters M_{Pj} to destination $j \neq P$, normalized by trade shares λ_{Pj}.[74] Stack all M_{Pj}/λ_{Pj} in a $(N-1)$-element vector $\hat{m}(2;\Theta)$, and compute the analogous moment in the data, $m^{data}(2)$, using the Portuguese data in 2005. This moment is used to calibrate the marketing costs parameters.

- **Moment 3**. Median and standard deviation of export shares of Portuguese exporters, computed as the ratio between total exports and total sales. Compute the analogous moment in the data, $m^{data}(3)$, using the Portuguese data in 2005. This moment is used to calibrate the technology parameter θ, since it regulates the dispersion of productivities, and thus export shares, across firms (see Gaubert and Itskhoki (2015)).

4) I stack the differences between observed and simulated moments into a vector of length 1,226, $y(\Theta) \equiv m^{data} - \hat{m}(\Theta)$. I iterate over Θ such that the following moment condition holds:

$$E[y(\Theta_0)] = 0$$

where Θ_0 is the true value of Θ. In particular, I seek a $\hat{\Theta}$ that achieves:

$$\hat{\Theta} = \text{argmin}_{\Theta} g(\Theta) \equiv y(\Theta)'\mathbf{W}y(\Theta)$$

where $\mathbf{W}$ is a positive semi-definite weighting matrix. Ideally I would use $\mathbf{W} = \mathbf{V}^{-1}$ where $\mathbf{V}$ is the variance-covariance matrix of the moments. Since the true matrix is unknown, I follow Eaton et al. (2011) and Arkolakis et al. (2015) and use its empirical analogue:

$$\hat{\mathbf{V}} = \frac{1}{T^{sample}} \sum_{t=1}^{T} \left(m^{data} - m_t^{sample}\right)\left(m^{data} - m_t^{sample}\right)'$$

[73] To construct trade shares, I use bilateral trade data from WIOD and Comtrade, and production data from UNIDO.

[74] I normalize by trade shares to control for distance from Portugal and other "gravity" forces that, besides the marketing costs, may affect the number of exporters to a destination.

where m_t^{sample} are the moments from a random sample drawn with replacement of the original firms in the dataset and $T^{sample} = 1,000$ is the number of those draws. To find $\hat{\Theta}$, I use the derivative-free Nelder-Mead downhill simplex search method.[75]

Results. The best fit is achieved with the values shown in Table 2:

Table 2: Calibrated parameters

Parameter	θ	χ	$\tilde{f}$	γ	κ_0	κ_1	κ_2	κ_3	κ_4
Value	6.2	0.43	0.073	0.5	0.18	0.14	-0.1	-0.1	-0.2

The calibrated parameters are consistent with previous estimates in the trade literature. In particular, the technology parameter θ is equal to 6.2, which is in line with the results obtained using different methodologies (see Eaton and Kortum (2002), Bernard et al. (2003), Simonovska and Waugh (2014), Costinot et al. (2012)). Both the elasticity of marketing costs with respect to the size of the market, χ, and the cost of each ad, $\tilde{f}$, correspond with the values estimated in Arkolakis (2010). Using equation (24), these estimates indicate that, in the median country, marketing costs dissipate 40% of gross profits.[76]

Once I estimate the parameters of the model, I investigate how well the model matches other important features of the data. Specifically, in the Appendix I show how the model outperforms risk neutral models in predicting entry patterns of firms into markets, as well as in matching the distribution of exports in a given destination.

4.3 Testing the model predictions

In this section I test the predictions of the model. I rely only on the estimates of the covariance matrix Σ and thus of the Sharpe Ratios.

Extensive margin and risk. Proposition 2 states that the probability of entering a market is increasing in the market's Sharpe Ratio.[77] I test this prediction in the data with the following regression:

[75] Numerical simulations suggest that the rank condition needed for identification, $\nabla_{\Theta} g(\Theta) = dim(\Theta)$, holds, and therefore the objective function has a unique local minimizer (see Hayashi (2000)).

[76] Eaton et al. (2011) estimate this fraction to be 59 percent.

[77] The complexity of the firm problem, being subject to 2*N inequality constraints, does not allow to explicitly write the firm-level trade flows as a log-linear function of the Sharpe Ratio. Therefore, one can interpret equation (44) as a "reduced-form" test of Proposition 2.

$$Pr(x_{jz} > 0) = \delta_0 + \delta_1 \ln(S_j) + \delta_2 \Gamma_j + \kappa_z + \varepsilon_{jz} \tag{44}$$

where x_{jz} are trade flows of Portuguese firm z to market j in 2005, S_j is the Sharpe Ratio of country j, computed using the estimated covariance matrix from the previous section, and Γ_j is a vector of country-level controls. Specifically, I include standard variables used in gravity regressions, such as distance from Portugal, dummies for trade agreement with Portugal, contiguity, common language, colonial links, common currency, WTO membership. Since I cannot control for destination fixed effects, given the presence of S_j in the regression, I additionally control for the log of GDP, log of openness (trade/GDP), export and import duties as a fraction of trade, and an index of the remoteness of the country to further proxy for trade costs (as in Bravo-Ortega and Giovanni (2006) and Frankel and Romer (1999)). Finally, κ_z controls for firm fixed effects.

Columns 1 and 2 in Table 4.2.3 show the results from a linear probability model and from a Probit model, respectively.[78] We can see that the coefficient of S_j is positive and statistically significant, as predicted by Proposition 2. When the Sharpe Ratio is high, the market provides good diversification benefits to the firms exporting there, and as a result the probability that a firm enters there is higher, controlling for barriers to trade and to market specific characteristics. This result holds also if the dependent variable is the probabilty to enter *for the first time* a destination in 2005, as shown in Table 7.3 in Appendix 7.3.

Intensive margin and risk. Proposition 2 states that firm-level trade flows to a market are increasing in the market's Sharpe Ratio. I test this prediction with the same specification as above:

$$\ln(x_{jz}) = \delta_0 + \delta_1 \ln(S_j) + \delta_2 \Gamma_j + \kappa_z + \varepsilon_{jz} \tag{45}$$

where the dependent variable is the log of trade flows of firm z from Portugal to country j, in 2005. As before, we expect risk averse firms to export more to locations with a higher Sharpe Ratio, conditional on entering there. Column 3 in Table 4.2.3 shows the result of a least square regression, indicating that the effect of the Sharpe Ratio on trade flows is positive and statistically significant, as predicted by Proposition 2.[79] The results are robust

[78] To control for firm fixed effects, I estimate the entry equation (44) with a linear probability model, which avoids the incidental parameter problem that arises with a Probit regression.

[79] The findings are also robust to heteroskedasticity, as it is revealed by a Poisson Pseudo-Maximum Likelihood estimation (as in Silva and Tenreyro (2006) and Martin and Pham (2015)). Results are not reported to save space but are available upon request.

also to selection bias, as it can be seen from Column 4, where I use a two stages Heckman procedure to correct for the selection of firms into exporting, using the entry equation (44).[80]

Table 3: Firm-level trade patterns and risk

	(1)	(2)	(3)	(4)
Dep. Variable	Prob. of entering	Prob. of entering	Log of trade flows	Log of trade flows
Method	Least Squares	Probit	Least Squares	Heckman
Log of Sharpe Ratio	0.102***	0.563***	1.130***	0.892***
	(0.005)	(0.033)	(0.139)	(0.165)
Log of GDP	0.074***	0.263***	0.648***	0.631***
	(0.002)	(0.010)	(0.039)	(0.061)
Log of Distance	-0.048***	0.293***	-0.273*	-0.285*
	(0.004)	(0.032)	(0.143)	(0.158)
Firm fixed effects	YES	NO	YES	NO
# of add. controls	11	11	11	10
Observations	125,346	125,346	15,369	15,369
R-squared	0.124	0.145	0.103	0.100

Notes: In Columns 1 and 2 the dependent variable is an indicator equals to 1 if a firm from Portugal enters market j, and equal 0 otherwise. In Columns 3 and 4 the dependent variable is the log of sales of a Portuguese firm to market j. All data are for 2005. Additional not reported controls are: dummies for trade agreement with Portugal, contiguity, common language, colonial links, common currency, common legal origins, WTO membership, log of openness (trade/GDP), export and import duties as a fraction of trade, remoteness. Column 4 reports only the second stage of a Heckman 2SLS procedure, where the excluded variable is the dummy for common language. Clustered standard errors are shown in parenthesis (*** p<0.01, ** p<0.05, * p<0.1).

Proposition 2 and equation (27) suggest that the Sharpe Ratio positively affect trade also at the aggregate level. I test this implication of the model with the following specification:

$$\ln\left(X_{ij}\right) = \delta_0 + \kappa_i + \delta_1 \ln\left(S_j\right) + \delta_2 \Gamma_{ij} + \varepsilon_{ij}$$

where the dependent variable is the log of bilateral manufacturing trade flows for the 35 countries in the sample, for 2005, κ_i is a source fixed effect, and Γ_{ij} is a vector of bilateral gravity variables, such as log of bilateral distance, dummies for bilateral trade agreement,

[80]I follow Helpman et al. (2008) and use the dummy for common language to provide the needed exclusion restriction for identification of the second stage trade equation.

contiguity, common language, colonial links, common currency, WTO membership. I also include, as before, the log of GDP, log of openness (trade/GDP), export and import duties as a fraction of trade, and remoteness.

Table 4: Aggregate trade patterns and risk

	(1)	(2)
Dep. Variable	Log of bilateral trade flows	Bilateral trade flows
Method	Least Squares	PPML
Log of Sharpe Ratio	0.255**	0.362***
	(0.093)	(0.099)
Log of GDP	1.123***	1.123***
	(0.032)	(0.038)
Log of Distance	-0.964*	-0.697***
	(0.051)	(0.065)
Source fixed effects	YES	YES
Number of add. controls	11	11
Observations	1,225	1,225
R-squared	0.9039	0.1034

Notes: In Columns 1 and 2 the dependent variable is the log of bilateral sales between from country i to j. Data is for the 35 countries in the sample, for 2005, from Comtrade and WIOD. Additional not reported controls are: dummies for bilateral trade agreement, contiguity, common language, colonial links, common currency, common legal origins, WTO membership, as well as log of openness (trade/GDP), export and import duties as a fraction of trade, remoteness of destination j. Clustered standard errors are shown in parenthesis (*** p<0.01, ** p<0.05, * p<0.1).

Column 1 in Table 4.2.3 shows that aggregate bilateral trade flows are increasing in the Sharpe Ratio of the destination country, controlling for trade barriers and other country characteristics, lending support to the model prediction. The results are robust to heteroskedasticity, as shown in Column 2, where I estimate the equation in levels with a Poisson Pseudo-Maximum Likelihood (as in Silva and Tenreyro (2006) and Martin and Pham (2015)).

Finally, I further investigate the relationship between the Sharpe Ratio and trade patterns. Recall that the Sharpe Ratio is a measure that summarizes the *multilateral* covariance of a country's demand with the rest of the world. Thus, the effect of the Sharpe Ratio on extensive and intensive margins can be decomposed into a variance and a covariance components. Table 4.2.3 reports the results of regressions similar to (44) and (45), where I control, rather than for the Sharpe Ratio, for the variance of demand and the simple average covariance with the other countries in the sample. The table suggests that both components have a significant impact on trade patterns.

Table 5: Firm-level trade patterns and risk, II

	(1)	(2)	(3)	(4)
Dep. Variable	Prob. of entering	Prob. of entering	Log of trade flows	Log of trade flows
Method	Least Squares	Least Squares	Heckman	Heckman
Variance	-0.04***		-0.333***	
	(0.001)		(0.059)	
Average covariance		-0.021***		-0.14***
		(0.000)		(0.029)
Firm fixed effects	YES	YES	NO	NO
Number of controls	13	13	12	12
Observations	125,346	125,346	15,369	15,369
R-squared	0.125	0.125	0.11	0.12

Notes: In Columns 1-2 the dependent variable is an indicator equals to 1 if a firm from Portugal enters market j, and equal 0 otherwise. In Columns 3-4 the dependent variable is the log of sales of a Portuguese firm to market j. All data are for 2005. Additional not reported controls are: log of GDP, log of distance from Portugal, dummies for trade agreement with Portugal, contiguity, common language, colonial links, common currency, common legal origins, WTO membership, log of openness (trade/GDP), export and import duties as a fraction of trade, remoteness. Columns 3-4 report only the second stage of a Heckman 2SLS procedure, where the excluded variable is the dummy for common language. Clustered standard errors are shown in parenthesis (*** p<0.01, ** p<0.05, * p<0.1).

5 Counterfactual analysis

In this section I use the calibrated parameters to conduct a number of counterfactual simulations in order to study the aggregate effects of firms' risk-hedging behavior.

To perform the counterfactual experiments, I add three elements to the model, following Caliendo and Parro (2014) and Arkolakis and Muendler (2010). (i) I introduce a non-tradeable good produced, under perfect competition, with labor and unitary productivity.

Consumers spend a constant share ξ of their income on the manufacturing tradeable goods, and a share $1-\xi$ on the non-tradeable good.[81] I set $\xi = 0.23$, which is the median value, across several countries, of the consumption shares on manufacturing estimated by Caliendo and Parro (2014). (ii) I introduce intermediate inputs. In particular, I assume that the production of each variety uses a Cobb-Douglas aggregate of labor, a composite of all manufactured tradeable products, and the non-tradeable good. Therefore the total variable input cost is:

$$c_i = w_i^{\gamma_i^w} \left(P_i^T\right)^{\gamma_i^T} \left(P_i^N\right)^{\gamma_i^N}$$

where P_i^T is the price index of tradeables, P_i^N is the price index of non-tradeables, and $\gamma_i^w + \gamma_i^T + \gamma_i^N = 1$. I compute these shares using data from UNIDO and WIOD in 2005.[82] (iii) I allow for a manufacturing trade deficit D_i. The deficits are assumed to be exogenous and set to their observed levels in 2005, using data from UN Comtrade.

5.1 Welfare gains from trade

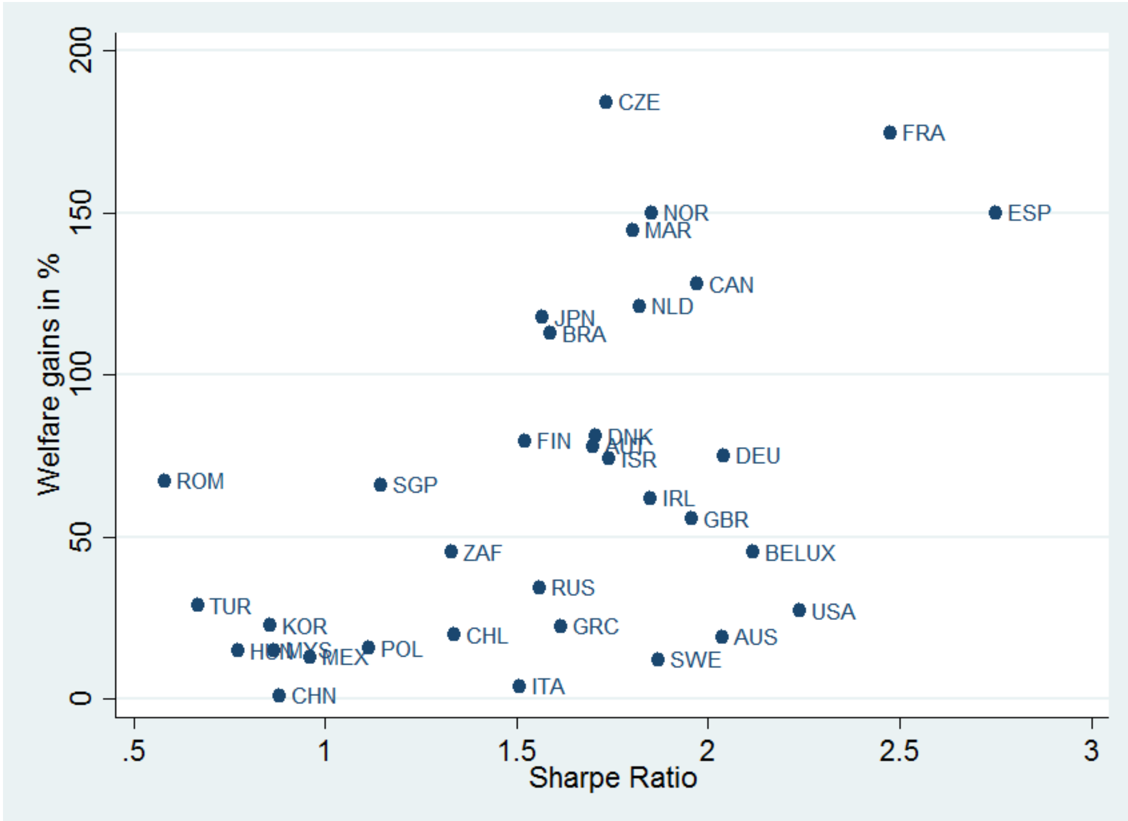
I first focus on an important counterfactual exercise: moving to autarky. Formally, starting from the calibrated trade equilibrium in 2005, I assume that variable trade costs in the new equilibrium are such that $\tau_{ij} = +\infty$ for any pair of countries $i \neq j$. All other structural parameters are the same as in the initial equilibrium. Once I solve the equilibrium under autarky, I compute the welfare gains associated with moving from autarky to the observed equilibrium (similarly to ACR and Costinot and Rodriguez-Clare (2013)).

Figure 2 illustrates the welfare gains for the 35 countries in the sample, as a function of their measure of risk-return, S_j. We can see that the total gains are increasing in S_j: countries that provide a better risk-return trade-off to foreign firms benefit more from opening up to trade. Firms exploit a trade liberalization not only to increase their profits, but also to diversify their demand risk. This implies that they optimally increase trade flows toward markets that provide better diversification benefits, as shown in the previous section. This also implies that the increase in foreign competition is stronger in these countries, additionally lowering the price level and increasing the average productivity of the surviving firms. Consequently, "safer" countries gain more from trade. Importantly, this selection effect, i.e. foreign competition crowding out inefficient domestic firms, is novel compared to existing trade models, because it arises from the diversification strategy of foreign firms.

[81] I assume that demand for the non-tradeable is non stochastic.

[82] I exclude agriculture and mining sectors. For countries for which I do not have this information, I set the shares equal to the median value of the other countries.

Figure 2: Welfare gains from trade

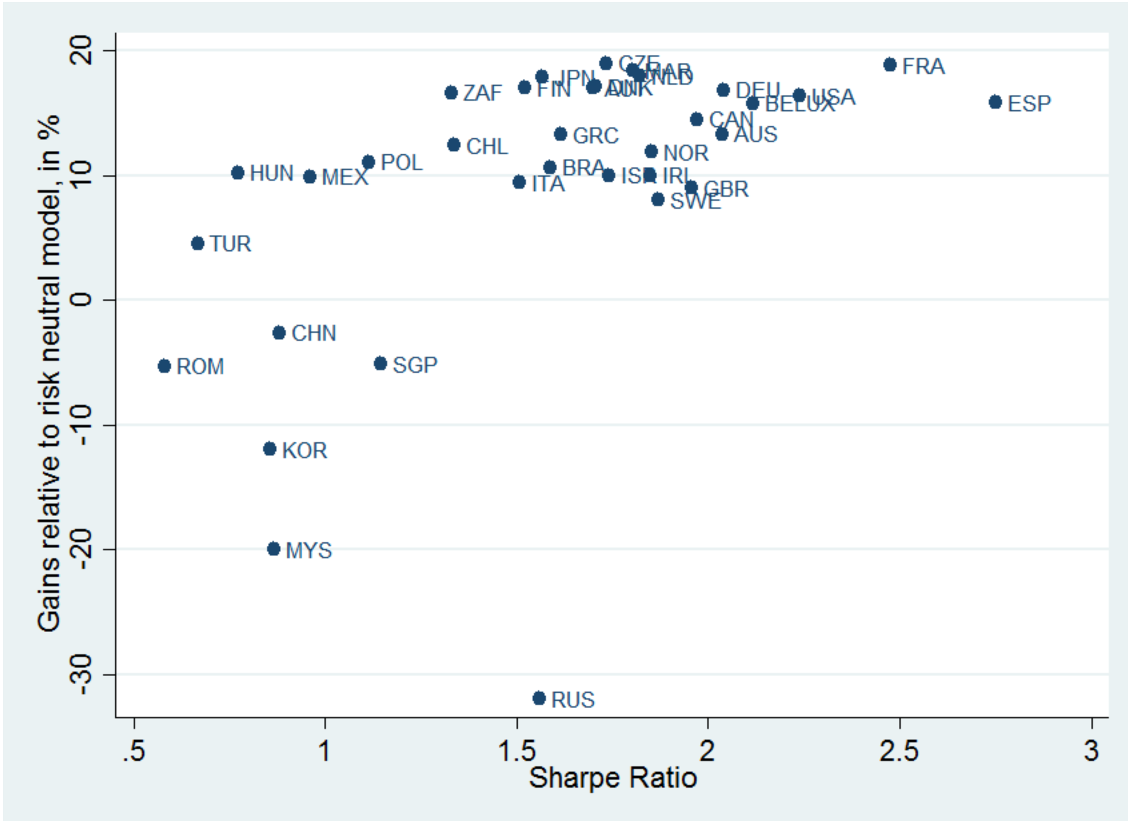


Notes: The figure plots the percentage change in welfare after going to autarky. The variable on the x-axis is the Sharpe Ratio, the country-level measure of risk-return, shown in equation (19).

In addition, I compare the welfare gains in my model with those predicted by models without risk aversion. As shown earlier, if the risk aversion is 0, welfare gains from trade are the same as the ones predicted by the ACR formula, and therefore can be written only as a function of the change in domestic trade shares and θ. Since in autarky domestic trade shares are by construction equal to 1, it suffices to know the domestic trade shares in the initial calibrated equilibrium to compute the welfare gains under risk neutrality.

Figure 3 plots the percentage deviations of the welfare gains in my model against those in ACR, as a function of S_j. As expected, the gains from trade in "safer" countries are higher than the gains in ACR, while the opposite happens for "riskier" markets. For the median country, gains from risk diversification are 13% of the total welfare gains from trade.

Figure 3: Welfare gains from trade vs ACR



Notes: The figure plots the difference between the welfare gains predicted by my model and those predicted by ACR, after moving to autarky. The variable on the x-axis is ψ, the country-level measure of risk-return, shown in equation (19).

5.2 Shock to volatility

[...]

6 Concluding remarks

In this paper, I characterize the link between demand risk, firms' exporting decisions, and welfare gains from trade. The proposed framework is sufficiently tractable to be estimated using the Method of Moments. Overall, an important message emerges from my analysis: welfare gains from trade significantly differ from trade models that neglect firms' risk aversion. In addition, I stress the importance of the cross-country covariance of demand in amplifying the impact of a change in trade costs through a simple variety effect.

The main conclusion is that how much a country gains from international trade hinges crucially on its ability to attract foreign firms looking for risk diversification benefits. Policy makers should implement policies that stabilize a country's demand, in order to improve its risk-return profile.

Interesting avenues for future research emerge from my study. For example, it would be instructive to extend my model to a dynamic setting, where firms are able to re-optimize their portfolio of destinations over time. Another interesting extension would be to introduce the possibility of mergers and acquisitions among firms or the possibility of holding shares from different companies, as alternative ways to diversify business risk.

References

Agnew, J., Balduzzi, P., Sunden, A., 2003. Portfolio choice and trading in a large 401 (k) plan. American Economic Review, 193–215.

Albornoz, F., Pardo, H. F. C., Corcos, G., Ornelas, E., 2012. Sequential exporting. Journal of International Economics 88 (1), 17–31.

Allayannis, G., Rountree, B., Weston, J. P., 2008. Do investors value smooth performance? Journal of Financial Economics 90 (3), 237–251.

Allen, T., Arkolakis, C., Takahashi, Y., 2014. Universal gravity. Tech. rep., Mimeo Northwestern and Yale University.

Allen, T., Atkin, D., 2015. Volatility, insurance, and the gains from trade. mimeo.

Allen, T., Atkin, D., 2016. Volatility and the gains from trade. Tech. rep., National Bureau of Economic Research.

Antras, P., Fort, T. C., Tintelnot, F., 2014. The margins of global sourcing: theory and evidence from us firms. Tech. rep., National Bureau of Economic Research.

Arkolakis, C., 2010. Market penetration costs and the new consumers margin in international trade. Journal of Political Economy 118 (6), 1151–1199.

Arkolakis, C., 2016. A unified theory of firm selection and growth. The Quarterly Journal of Economics 131 (1), 89–155.

Arkolakis, C., Costinot, A., Rodríguez-Clare, A., 2012. New trade models, same old gains? American Economic Review 102 (1), 94–130.

Arkolakis, C., Ganapati, S., Muendler, M.-A., 2015. The extensive margin of exporting products: A firm-level analysis.

Arkolakis, C., Muendler, M.-A., 2010. The extensive margin of exporting products: A firm-level analysis. Tech. rep., National Bureau of Economic Research.

Arkolakis, K., Klenow, P., Demidova, S., Rodriguez-Clare, A., 2008. The gains from trade with endogenous variety. In: American Economic Review Papers and Proceedings. Vol. 98. pp. 444–450.

Armenter, R., Koren, M., 2015. Economies of scale and the size of exporters. Journal of the European Economic Association 13 (3), 482–511.

Barwise, P., Styler, A., 2003. The marketing expenditure trends report 2003. London Business School report.

Benartzi, S., Thaler, R. H., 2001. Naive diversification strategies in defined contribution saving plans. American economic review, 79–98.

Bento, P., Restuccia, D., 2016. Misallocation, establishment size, and productivity. Tech. rep., National Bureau of Economic Research.

Berman, N., Berthou, A., Héricourt, J., 2015. Export dynamics and sales at home. Journal of International Economics.

Bernard, A. B., Eaton, J., Jensen, J. B., Kortum, S., 2003. Plants and productivity in international trade. American Economic Review 93 (4), 1268–1290.

Blaum, J., Lelarge, C., Peters, M., 2015. The gains from input trade in firm-based models of importing.

Bravo-Ortega, C., Giovanni, J. D., 2006. Remoteness and real exchange rate volatility. IMF Staff Papers, 115–132.

Bricongne, J.-C., Fontagné, L., Gaulier, G., Taglioni, D., Vicard, V., 2012. Firms and the global crisis: French exports in the turmoil. Journal of international Economics 87 (1), 134–146.

Butt, J., Howe, C., 2006. Sizing US marketing 2006. Blackfriar corporation research report.

Caliendo, L., Parro, F., 2014. Estimates of the trade and welfare effects of nafta. The Review of Economic Studies, rdu035.

Campbell, J. Y., Viceira, L. M., 2002. Strategic asset allocation: portfolio choice for long-term investors. Oxford University Press.

Cardoso, A. R., Portugal, P., 2005. Contractual wages and the wage cushion under different bargaining settings. Journal of Labor economics 23 (4), 875–902.

Carroll, C. D., 2002. Portfolios of the rich in household portfo" lios: Theory and evidence.

Caselli, F., Koren, M., Lisicky, M., Tenreyro, S., 2012. Diversification through trade. In: Society for Economic Dynamics Meeting Papers. No. 539.

Castro, R., Clementi, G. L., Lee, Y., 2010. Cross–sectoral variation in firm–level idiosyncratic risk.

Chaney, T., 2008. Distorted gravity: The intensive and extensive margins of international trade. American Economic Review 98 (4), 1707–1721.

Chen, H., Miao, J., Wang, N., 2010. Entrepreneurial finance and nondiversifiable risk. Review of Financial Studies, hhq122.

Christopoulou, R., Vermeulen, P., 2012. Markups in the euro area and the us over the period 1981–2004: a comparison of 50 sectors. Empirical Economics 42 (1), 53–77.

Conconi, P., Sapir, A., Zanardi, M., 2016. The internationalization process of firms: from exports to fdi. Journal of International Economics 99, 16–30.

Costinot, A., Donaldson, D., Komunjer, I., 2012. What goods do countries trade? a quantitative exploration of ricardo's ideas. The Review of Economic Studies 79 (2), 581–608.

Costinot, A., Rodriguez-Clare, A., 2013. Trade theory with numbers: Quantifying the consequences of globalization. Tech. rep., National Bureau of Economic Research.

Cucculelli, M., Ermini, B., et al., 2012. Individual risk attitude, product innovation and firm performance. evidence from survey data. Economics Bulletin 32 (4), 3197–3212.

Das, S., Roberts, M. J., Tybout, J. R., 2007. Market entry costs, producer heterogeneity, and export dynamics. Econometrica 75 (3), 837–873.

De Sousa, J., Disdier, A.-C., Gaigné, C., 2015. Export decision under risk.

Di Giovanni, J., Levchenko, A. A., 2010. The risk content of exports: a portfolio view of international trade. Tech. rep., National Bureau of Economic Research.

Di Giovanni, J., Levchenko, A. A., Méjean, I., 2014. Firms, destinations, and aggregate fluctuations. Econometrica 82 (4), 1303–1340.

Dietzenbacher, E., Los, B., Stehrer, R., Timmer, M., de Vries, G., 2013. The construction of world input–output tables in the wiod project. Economic Systems Research 25 (1), 71–98.

Domowitz, I., Hubbard, R. G., Petersen, B. C., 1988. Market structure and cyclical fluctuations in us manufacturing. Review of Economics and Statistics 70 (1), 55–66.

Eaton, J., Grossman, G. M., 1985. Tariffs as insurance: Optimal commercial policy when domestic markets are incomplete. Canadian Journal of Economics, 258–272.

Eaton, J., Kortum, S., 2002. Technology, geography and trade. Econometrica 70 (5), 1741–1779.

Eaton, J., Kortum, S., Kramarz, F., 2011. An anatomy of international trade: Evidence from french firms. Econometrica 79 (5), 1453–1498.

Eeckhoudt, L., Gollier, C., Schlesinger, H., 2005. Economic and financial decisions under risk. Princeton University Press.

Faccio, M., Marchica, M.-T., Mura, R., 2011. Large shareholder diversification and corporate risk-taking. Review of Financial Studies, hhr065.

Feenstra, R. C., 2016. Gains from trade under monopolistic competition. Pacific Economic Review 21 (1), 35–44.

Fernandes, A. M., Freund, C., Pierola, M. D., 2016. Exporter behavior, country size and stage of development: Evidence from the exporter dynamics database. Journal of Development Economics 119, 121–137.

Fillat, J., Garetto, S., 2015. Risk, return and multinational production. The Quarterly Journal Economics 130 (4), 2027–2073.

Fitzgerald, D., Haller, S., Yedid-Levi, Y., 2016. How exporters grow. Tech. rep., National Bureau of Economic Research.

Frankel, J. A., Romer, D., 1999. Does trade cause growth? American economic review, 379–399.

Froot, K. A., Scharfstein, D. S., Stein, J. C., 1993. Risk management: Coordinating corporate investment and financing policies. the Journal of Finance 48 (5), 1629–1658.

Gabaix, X., 2011. The granular origins of aggregate fluctuations. Econometrica 79 (3), 733–772.

Gaubert, C., Itskhoki, O., 2015. Granular comparative advantage. Tech. rep., UC Berkeley, mimeo.

Gertler, M., Gilchrist, S., 1994. Monetary policy, business cycles, and the behavior of small manufacturing firms. The Quarterly Journal of Economics, 309–340.

Gervais, A., 2016. Uncertainty, risk aversion and international trade. Browser Download This Paper.

Greenwood, J., Smith, B. D., 1997. Financial markets in development, and the development of financial markets. Journal of Economic Dynamics and Control 21 (1), 145–181.

Guay, W., Kothari, S. P., 2003. How much do firms hedge with derivatives? Journal of Financial Economics 70 (3), 423–461.

Hayashi, F., 2000. Econometrics. 2000. Princeton University Press. Section 1, 60–69.

Head, K., Mayer, T., 2013. Gravity equations: Workhorse, toolkit, and cookbook. Centre for Economic Policy Research.

Head, K., Mayer, T., Ries, J., 2010. The erosion of colonial trade linkages after independence. Journal of international Economics 81 (1), 1–14.

Heaton, J., Lucas, D., 2000. Portfolio choice and asset prices: The importance of entrepreneurial risk. The journal of finance 55 (3), 1163–1198.

Helpman, E., Melitz, M., Rubinstein, Y., 2008. Estimating trade flows: Trading partners and trading volumes. Quarterly Journal of Economics 2 (5), 441–487.

Helpman, E., Melitz, M. J., Yeaple, S. R., 2004. Export versus FDI with heterogeneous firms. American Economic Review 94 (1), 300–316.

Helpman, E., Razin, A., 1978. Uncertainty and international trade in the presence of stock markets. The Review of Economic Studies, 239–250.

Hentschel, L., Kothari, S. P., 2001. Are corporations reducing or taking risks with derivatives? Journal of Financial and Quantitative Analysis 36 (01), 93–118.

Herranz, N., Krasa, S., Villamil, A., 2015. Entrepreneurs, risk aversion, and dynamic firms. The Journal of Political Economy 123 (5), 1133–1176.

Herranz, N., Krasa, S., Villamil, A. P., 2013. Entrepreneurs, risk aversion and dynamic firms. Tech. rep., Economics, The Univeristy of Manchester.

Hoffmann, M., 2014. The consumption–income ratio, entrepreneurial risk, and the us stock market. Journal of Money, Credit and Banking 46 (6), 1259–1292.

Hoffmann, M., Shcherbakova-Stewen, I., 2011. Consumption risk sharing over the business cycle: the role of small firms' access to credit markets. Review of Economics and Statistics 93 (4), 1403–1416.

Hottman, C., Redding, S. J., Weinstein, D. E., 2015. Quantifying the sources of firm heterogeneity. The Quarterly Journal of Economics.

Impullitti, G., Irarrazabal, A. A., Opromolla, L. D., 2013. A theory of entry into and exit from export markets. Journal of International Economics 90 (1), 75–90.

Ingersoll, J. E., 1987. Theory of financial decision making. Vol. 3. Rowman & Littlefield.

Iyer, R., Peydró, J.-L., da Rocha-Lopes, S., Schoar, A., 2014. Interbank liquidity crunch and the firm credit crunch: Evidence from the 2007–2009 crisis. Review of Financial studies 27 (1), 347–372.

Jacoby, H. G., Skoufias, E., 1997. Risk, financial markets, and human capital in a developing country. The Review of Economic Studies 64 (3), 311–335.

Jagannathan, R., Ma, T., 2002. Risk reduction in large portfolios: Why imposing the wrong constraints helps. Tech. rep., National Bureau of Economic Research.

Jones, J. B., Pratap, S., 2015. An estimated structural model of entrepreneurial behavior.

Kihlstrom, R. E., Laffont, J.-J., 1979. A general equilibrium entrepreneurial theory of firm formation based on risk aversion. The Journal of Political Economy, 719–748.

Knight, M., 1998. Developing countries and the globalization of financial markets. World Development 26 (7), 1185–1200.

Koren, M., 2003. Financial globalization, portfolio diversification, and the pattern of international trade. No. 3-233. International Monetary Fund.

Kramarz, F., Martin, J., Mejean, I., 2014. Diversification in the small and in the large: Evidence from trade networks. Tech. rep., working paper, CREST.

Kramarz, F., Martin, J., Mejean, I., 2015. Volatility in the small and in the large: Diversification in trade networks. Tech. rep., mimeo.

Leonidou, L. C., Katsikeas, C. S., Samiee, S., 2002. Marketing strategy determinants of export performance: A meta analysis. Journal of Business Research 55 (1), 51–67.

Limão, N., Maggi, G., 2013. Uncertainty and trade agreements. Tech. rep., National Bureau of Economic Research.

Luo, Y., Gong, L., Zou, H.-f., 2010. A note on entrepreneurial risk, capital market imperfections, and heterogeneity. Macroeconomic Dynamics 14 (02), 269–284.

Lyandres, E., Marchica, M.-T., Michaely, R., Mura, R., 2013. The effects of owners' portfolio diversification on firm strategies: Theory and evidence from private and public firms. Johnson School Research Paper Series (18-2013).

Markowitz, H., 1952. Portfolio selection*. The journal of finance 7 (1), 77–91.

Martin, W., Pham, C. S., 2015. Estimating the gravity model when zero trade flows are frequent and economically determined. World Bank Policy Research Working Paper (7308).

Martins, J. O., Scarpetta, S., Pilat, D., 1996. Mark-up ratios in manufacturing industries.

Mathewson, G., 1972. A note on the price effects of market power in the canadian newspaper industry. The Canadian Journal of Economics/Revue canadienne d'Economique 5 (2), 298–301.

Melitz, M. J., 2003. The impact of trade on intra-industry reallocations and aggregate industry productivity. Econometrica 71 (6), 1695–1725.

Melitz, M. J., Redding, S. J., 2014. Missing gains from trade? The American Economic Review 104 (5), 317–321.

Mion, G., Opromolla, L. D., 2014. Managers' mobility, trade performance, and wages. Journal of International Economics 94 (1), 85–101.

Morales, E., Sheu, G., Zahler, A., 2014. Gravity and extended gravity: Using moment inequalities to estimate a model of export entry. Tech. rep., National Bureau of Economic Research.

Morales, E., Sheu, G., Zahler, A., 2015. Extended gravity. NBER Working Paper 21351.

Moskowitz, T. J., Vissing-Jorgensen, A., 2002. The returns to entrepreneurial investment: A private equity premium puzzle? Tech. rep., National Bureau of Economic Research.

Munch, J. R., Nguyen, D. X., 2014. Decomposing firm-level sales variation. Journal of Economic Behavior & Organization 106, 317–334.

Newbery, D. M., Stiglitz, J. E., 1984. Pareto inferior trade. The Review of Economic Studies, 1–12.

Nguyen, D. X., 2012. Demand uncertainty: Exporting delays and exporting failures. Journal of International Economics 86 (2), 336–344.

Novy, D., Taylor, A. M., 2014. Trade and uncertainty. Tech. rep., National Bureau of Economic Research.

Panousi, V., Papanikolaou, D., 2012. Investment, idiosyncratic risk, and ownership. The Journal of Finance 67 (3), 1113–1148.

Parrino, R., Poteshman, A. M., Weisbach, M. S., 2005. Measuring investment distortions when risk-averse managers decide whether to undertake risky projects. Financial Management 34 (1), 21–60.

Pena, J. M., 1995. M-matrices whose inverses are totally positive. Linear algebra and its applications 221, 189–193.

Petersen, M. A., Thiagarajan, S. R., 2000. Risk measurement and hedging: With and without derivatives. Financial Management, 5–29.

Pope, R., Chavas, J.-P., Just, R. E., 1983. Economic welfare evaluations for producers under uncertainty. American Journal of Agricultural Economics 65 (1), 98–107.

Pratt, J. W., 1964. Risk aversion in the small and in the large. Econometrica: Journal of the Econometric Society, 122–136.

Ramondo, N., Rappoport, V., Ruhl, K. J., 2013. The proximity-concentration tradeoff under uncertainty. The Review of Economic Studies, rdt018.

Riaño, A., 2011. Exports, investment and firm-level sales volatility. Review of World Economics 147 (4), 643–663.

Rob, R., Vettas, N., 2003. Foreign direct investment and exports with growing demand. The Review of Economic Studies 70 (3), 629–648.

Roberts, M. J., Tybout, J. R., 1997. The decision to export in colombia: An empirical model of entry with sunk costs. American Economic Review 87 (4), 545–564.

Ross, S. A., 2004. Compensation, incentives, and the duality of risk aversion and riskiness. The Journal of Finance 59 (1), 207–225.

Roussanov, N., 2010. Diversification and its discontents: Idiosyncratic and entrepreneurial risk in the quest for social status. The Journal of Finance 65 (5), 1755–1788.

Ruhl, K. J., Willis, J. L., 2014. New exporter dynamics.

Sanford, D. M., Maddox, L., 1999. Advertising agency management of domestic and international accounts. International Marketing Review 16 (6), 504–517.

Sharpe, W. F., 1964. Capital asset prices: A theory of market equilibrium under conditions of risk*. The journal of finance 19 (3), 425–442.

Silva, J. M. C. S., Tenreyro, S., 2006. The log of gravity. Review of Economics and Statistics 88 (4), 641–658.

Simonovska, I., Waugh, M. E., 2014. The elasticity of trade: Estimates and evidence. Journal of International Economics 92 (1), 34–50.

Tintelnot, F., 2016. Global production with export platforms. Tech. rep., National Bureau of Economic Research.

Vannoorenberghe, G., 2012. Firm-level volatility and exports. Journal of International Economics 86 (1), 57–67.

Vannoorenberghe, G., Wang, Z., Yu, Z., 2014. Volatility and diversification of exports: Firm-level theory and evidence.

7 Appendix

7.1 Robustness of the results

The results presented in the paper rely on specific assumptions made for tractability. In this section I explore the robustness of the general results to these assumptions.

7.1.1 Approximation of expected utility

If the entrepreneurs have a CARA utility with parameter γ, a second-order Taylor approximation of the expected utility leads to the expression in 4 (see Eeckhoudt et al. (2005) and De Sousa et al. (2015) for a standard proof). While this approximation has been used for the tractability, it implies that the results derived in the paper hold only locally, i.e. we can interpret them for the case of "small risks" (see also De Sousa et al. (2015)). In this subsection I study how my findings are affected by this approximation.

[...]

7.1.2 CRRA Utility

In the baseline model, I assume that entrepreneurs maximize an expected CARA utility in real income. One shortcoming of the CARA utility is that the absolute risk aversion is independent from wealth. This implies that large firms display the same risk aversion as small firms, which may be too restrictive. In this subsection, I consider an extension of the model where the entrepreneurs have a CRRA utility, and thus a decreasing absolute risk aversion. In particular, the owners now maximize the following utility:

$$\max E\left[\frac{1}{1-\rho}\left(\frac{y_i(z)}{P_i}\right)^{1-\rho}\right] \tag{46}$$

which, by means of the same Taylor expansion used before, can be approximated as:[83]

[83]Take a second-order expansion of $E\left[\frac{1}{1-\rho}z^{1-\rho}\right]$ around $\bar{z} \equiv E\left(\frac{y_i(z)}{P_i}\right)$:

$$E\left[\frac{1}{1-\rho}z^{1-\rho}\right] \approx E\left[\frac{1}{1-\rho}\bar{z}^{1-\rho} + \bar{z}^{-\rho}\left(z-\bar{z}\right) + \frac{(-\rho)}{2}\bar{z}^{-\rho-1}\left(z-\bar{z}\right)^2\right] =$$

$$= \frac{1}{1-\rho}\bar{z}^{1-\rho} + \bar{z}^{-\rho}E\left(z-\bar{z}\right) - \frac{\rho}{2}\bar{z}^{-\rho-1}E\left(z-\bar{z}\right)^2 =$$

$$= \frac{1}{1-\rho}\bar{z}^{1-\rho} - \frac{\rho}{2}\bar{z}^{-\rho-1}Var\left(z\right) =$$

$$\max \frac{1}{1-\rho}\left(E\left[\frac{y_i(z)}{P_i}\right]\right)^{1-\rho} - \frac{\rho}{2}\left(E\left[\frac{y_i(z)}{P_i}\right]\right)^{-1-\rho} Var\left(\frac{y_i(z)}{P_i}\right). \tag{47}$$

In this case, ρ is the coefficient of *relative* risk aversion, while the coefficient of absolute risk aversion is decreasing in the size of the firm, i.e. $E\left(\frac{y_i(z)}{P_i}\right)$. I calibrate the parameters of the model with this different specification, and then run the same counterfactual as in section 5.

[...]

7.2 Data Appendix

Trade data. Statistics Portugal collects data on export and import transactions by firms that are located in Portugal on a monthly basis. These data include the value and quantity of internationally traded goods (i) between Portugal and other Member States of the EU (intra-EU trade) and (ii) by Portugal with non-EU countries (extra-EU trade). Data on extra-EU trade are collected from customs declarations, while data on intra-EU trade are collected through the Intrastat system, which, in 1993, replaced customs declarations as the source of trade statistics within the EU. The same information is used for official statistics and, besides small adjustments, the merchandise trade transactions in our dataset aggregate to the official total exports and imports of Portugal. Each transaction record includes, among other information, the firm's tax identifier, an eight-digit Combined Nomenclature product code, the destination/origin country, the value of the transaction in euros, the quantity (in kilos and, in some case, additional product-specific measuring units) of transacted goods, and the relevant international commercial term (FOB, CIF, FAS, etc.). I use data on export transactions only, aggregated at the firm-destination-year level.

Data on firm characteristics. The second main data source, Quadros de Pessoal, is a longitudinal dataset matching virtually all firms and workers based in Portugal. Currently, the data set collects data on about 350,000 firms and 3 million employees. As for the trade data, I was able to gain access to information from 1995 to 2005. The data is made available by the Ministry of Employment, drawing on a compulsory annual census of all firms in Portugal that employ at least one worker. Each year, every firm with wage earners is legally obliged to fill in a standardized questionnaire. Reported data cover the firm itself, each of its plants, and each of its workers. Variables available in the dataset include the firm's location,

---

$$= \frac{1}{1-\rho}\left(E\left[\frac{y_i(z)}{P_i}\right]\right)^{1-\rho} - \frac{\rho}{2}\left(E\left[\frac{y_i(z)}{P_i}\right]\right)^{-1-\rho} Var\left(\frac{y_i(z)}{P_i}\right)$$

industry (at 5 digits of NACE rev. 1), total employment, sales, ownership structure (equity breakdown among domestic private, public or foreign), and legal setting. Each firm entering the database is assigned a unique, time-invariant identifying number which I use to follow it over time.

The two datasets are merged by means of the firm identifier. As in Mion and Opromolla (2014) and Cardoso and Portugal (2005), I account for sectoral and geographical specificities of Portugal by restricting the sample to include only firms based in continental Portugal while excluding agriculture and fishery (Nace rev.1, 2-digit industries 1, 2, and 5) as well as minor service activities and extra-territorial activities (Nace rev.1, 2-digit industries 95, 96, 97, and 99). The analysis focuses on manufacturing firms only (Nace rev.1 codes 15 to 37) because of the closer relationship between the export of goods and the industrial activity of the firm. The location of the firm is measured according to the NUTS 3 regional disaggregation. In the trade dataset, I restrict the sample to transactions registered as sales as opposed to returns, transfers of goods without transfer of ownership, and work done. I I neglect the sales of firms that produce in Portugal but are owned by foreign firms.

Data on $\tilde{L}_j$ and M. $\tilde{L}_j$ is the total number of workers in the manufacturing sector in 2005, obtained from UNIDO.[84] From UNIDO, I also observe the number of establishments in the manufacturing sector. To compute the number of firms, M_j, I divide the number of establishments in each country by the ratio between number of firms and number of establishments in Portugal, which is 0.32. I obtain the number of manufacturing firms in Portugal, $M_P = 27,970$, from Quadros de Pessoal. For the countries for which I do not have data on number of establishments, I set $M_j = 0.021\tilde{L}_j$, where 0.021 is the median ratio of workers to firms in the other countries. Setting the number of firms to be proportional to the working population of a country has been shown to be a good approximation of the data (see Bento and Restuccia (2016) and Fernandes et al. (2016)).

Data on profits. I obtain data on net profits from Central de Balanços, a repository of yearly balance sheet data for non financial firms in Portugal.

List of countries.

The countries in the sample are the top destinations of Portuguese exporters for which there is available data, from WIOD or UNIDO, to construct manufacturing trade shares. The final list of destinations is:

[84]For some countries, I do not observe $\tilde{L}_j$, and thus I set it proportional to the population in country j. In particular, I compute $\tilde{L}_j = L_j/r$, where r is the average ratio of population over manufacturing workers in the other countries.

Table 6: List of destinations in the sample

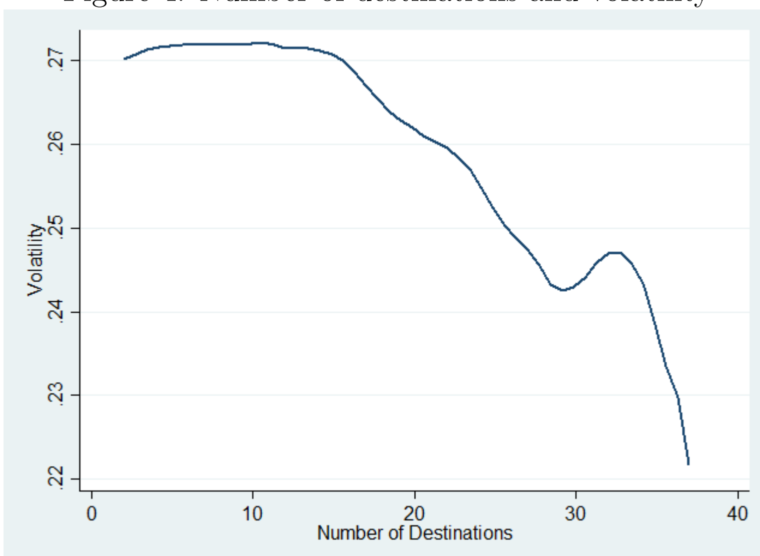
Country	**Fraction of Port. exports in 2005**	**Number of Port. exporters in 2005**	**Sharpe Ratio**
Australia	.44 %	266	2.04 (0.33)
Austria	.52 %	367	1.7 (0.19)
Belgium-Lux.	2.64 %	949	2.12 (0.2)
Brazil	.71 %	302	1.59 (0.3)
Canada	.61 %	533	1.97 (0.24)
Chile	.27 %	74	1.34 (0.57)
China	.41 %	184	.88 (0.24)
Czech Republic	.18 %	211	1.74 (0.42)
Denmark	.96 %	572	1.71 (0.18)
Finland	.68 %	366	1.52 (0.22)
France	13.83 %	1971	2.48 (0.18)
Germany	7.9 %	1283	2.04 (0.16)
Greece	.6 %	386	1.61 (0.19)
Hungary	.25 %	189	.77 (0.44)
Ireland	.83 %	436	1.85 (0.31)
Israel	.3 %	213	1.74 (0.37)
Italy	3.83 %	897	1.51 (0.16)
Japan	.31 %	300	1.57 (0.23)
Rep. of Korea	.1 %	112	.87 (0.26)
Malaysia	.02 %	55	.86 (0.49)
Mexico	.21 %	187	.96 (0.31)
Morocco	.65 %	286	1.80 (0.4)
Netherlands	4.82 %	954	1.82 (0.17)
Norway	.34 %	370	1.85 (0.28)
Poland	.48 %	241	1.12 (0.23)
Romania	.24 %	167	.58 (0.44)
Russia	.34 %	164	1.56 (0.7)
Singapore	.12 %	100	1.12 (0.25)
South Africa	.4 %	195	1.33 (0.25)
Spain	29 %	2420	2.75 (0.21)
Sweden	1.19 %	597	1.87 (0.22)
Turkey	.69 %	221	.67 (0.18)
United Kingdom	9.90 %	1294	1.96 (0.15)
United States	6.89 %	931	2.24 (0.23)
Total	90.56 %	4,821	

Notes: The fourth column reports the estimated Sharpe Ratios, with the standard errors in parenthesis.

7.3 Additional empirical results

Geographical diversification and volatility. The fundamental mechanism of the model is that the imperfect correlation of demand across markets implies that geographical diversification reduces the volatility of firms' total sales. The estimate of the covariance matrix in the previous section suggests that the cross-country correlations are heterogeneous and far from being equal to 1, indeed suggesting the potential for diversification through trade. Figure 7.3 lends support to this hypothesis. It shows that Portuguese firms exporting to more markets, over the course of 10 years, tended to have less volatile total sales.[85]

Figure 4: Number of destinations and volatility



Notes: The figure shows the volatility of Portuguese firms' total sales against the number of destinations to which they were selling. The volatility is measured as the standard deviation of total sales, computed using sales between 1995 and 2005, rescaled by the average total sales over the same period (to take into account for the size of the firms). The number of destinations is the average number of destinations across 1995-2005. I only consider firms exporting for at least 5 years. The plot is obtained by means of an Epanechnikov Kernel-weighted local polynomial smoothing, with parameters: degree = 0, bandwidth = 3.74.

Entry of firms. The global diversification strategy of the firms implies that there is no "strict sorting" of firms into markets: a large firm may decide not to enter a market even though a smaller firm does. An implication of such non-hierarchical structure of the exporting decision is related to the number of entrants to a certain location. First, recall that models

[85] Results are similar if I measure diversification with 1 minus the Herfindhal index. This is result is consistent with Kramarz et al. (2015).

characterized by fixed costs and absence of risk, such as Melitz (2003) and Chaney (2008), imply that firms obey a hierarchy: any firm selling to the $k+1$st most popular destination necessarily sells to the kk-th most popular destination as well.[86] The data however shows a different picture.[87] Following Eaton et al. (2011), I list in Table 7 each of the strings of top-seven destinations from Portugal that obey a hierarchical structure, together with the number of Portuguese firms selling to each string (irrespective of their export activity outside the top 7). It can be seen that only 28% of Portuguese exporters were obeying a hierarchical structure in their exporting status. While classical trade models with fixed costs and risk neutrality would predict that all exporters follow a strict sorting into exporting, my model with risk averse firms instead is able to predict fairly well the number of exporters selling to each string of destinations.

Table 7: Firms exporting to strings of top 7 destinations

Export string	Number of exporters, data	Number of exporters, model
ES	675	725
ES-FR	318	401
ES-FR-GE	143	181
ES-FR-GE-UK	141	159
ES-FR-GE-UK-AO	18	56
ES-FR-GE-UK-AO-BE	49	74
ES-FR-GE-UK-AO-BE-US	92	104
Total	1436	1700

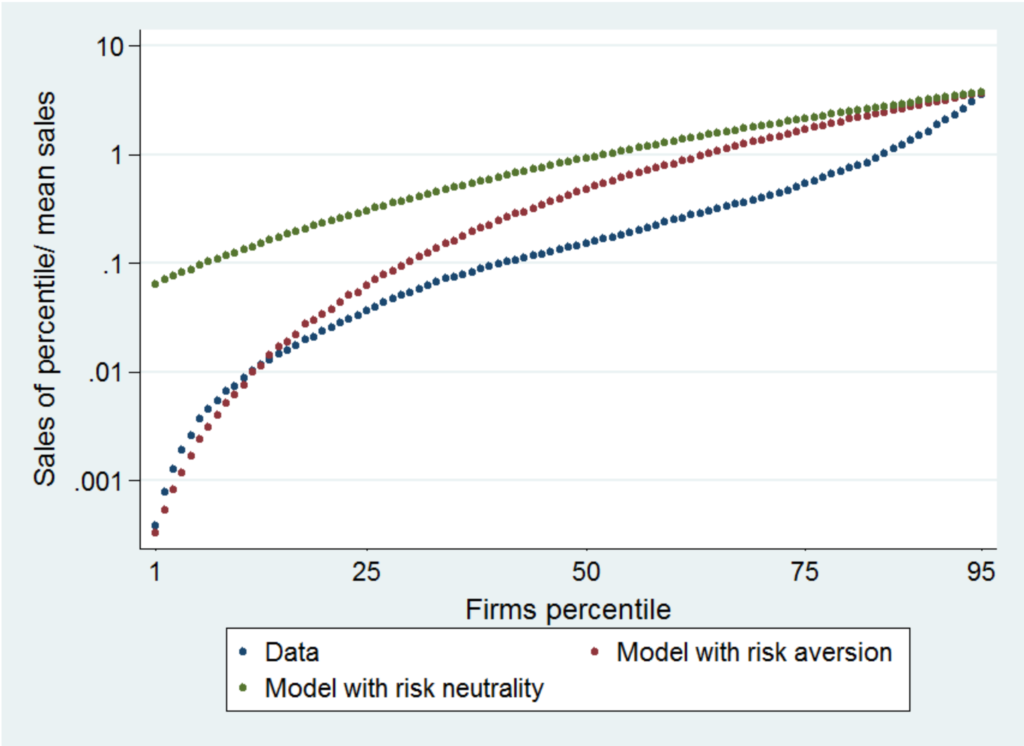
Distribution of firm-level trade flows. I compare the observed distribution of firm-level exports to a certain destination with the one predicted by my calibrated model. Figure 7.3 plots these distributions for all Portuguese firms exporting to Spain, the top destination.[88] The graph also plots the distribution predicted when I set the risk aversion to zero, which corresponds to the Melitz-Chaney model.

[86] This is because all firms with $z > z^*_{ij}$ will enter j.

[87] Evidence that exporters and non-exporters are not strictly sorted has been shown also by Eaton et al. (2011) and Armenter and Koren (2015), among others.

[88] Results look very similar for other destinations.

Figure 5: Distribution of sales relative to mean sales in calibrated model and in the data



Notes: The figure shows the distribution of sales relative to mean sales from Portugal to Spain in the calibrated model with risk aversion, in the data for 2005, and in the calibrated model with risk neutrality.

We can see that while both models successfully predict the right tail of the distribution, my model outperforms the risk-neutral model in matching the left tail of the distribution. The reason is that some firms, when they are risk averse, optimally choose to reach a small number of consumers in a certain destination, rather than the whole market, and therefore export small amounts of their goods. In the Melitz-Chaney framework, instead, the presence of fixed costs are not compatible with the existence of small exporters, and thus over-predicts their size by many orders of magnitude.[89]

Extensive margin and risk

[89]The model in Arkolakis (2010) also successfully predicts the distribution of firm-level sales, by assuming that the marketing costs are convex in the number of consumers.

Table 8: Firm-level trade patterns and risk

	(1)	(2)
Dep. Variable	Prob. of entering for the first time	Prob. of entering for the first time
Method	Least Squares	Probit
Log of Sharpe Ratio	0.021***	0.196***
	(0.003)	(0.044)
Log of GDP	0.023***	0.186***
	(0.001)	(0.015)
Log of Distance	-0.025***	0.368***
	(0.003)	(0.045)
Firm fixed effects	YES	NO
# of add. controls	11	11
Observations	114,272	114,272
R-squared	0.0281	0.015

Notes: In Columns 1 and 2 the dependent variable is an indicator equals to 1 if a firm from Portugal enters market j for the first time in 2005, and equal 0 otherwise. Additional not reported controls are: dummies for trade agreement with Portugal, contiguity, common language, colonial links, common currency, common legal origins, WTO membership, log of openness (trade/GDP), export and import duties as a fraction of trade, remoteness. All data are for 2005. Clustered standard errors are shown in parenthesis (*** p<0.01, ** p<0.05, * p<0.1).

7.4 Analytical appendix

7.4.1 Proof of Proposition 1

Since the firm decides the optimal price after the realization of the shock, in the first stage it chooses the optimal fraction of consumers to reach in each market based on the expectation of what the price will be in the second stage. I solve the optimal problem of the firm by backward induction, so starting from the second stage. Since at this stage the shocks are known, any element of uncertainty is eliminated and the firm then can choose the optimal pricing policy that maximizes profits, *given* the optimal $n_{ij}(z, E[p_{ij}(z)])$ decided in the previous stage:

$$\max_{\{p_{ij}\}} \sum_j \alpha_j(z) \frac{p_{ij}(z)^{-\sigma}}{P_j^{1-\sigma}} n_{ij}(z, E[p_{ij}(z)]) Y_j \left(p_{ij}(z) - \frac{\tau_{ij} w_i}{z} \right).$$

It is easy to see that this leads to the standard constant markup over marginal cost:

$$p_{ij}(z) = \frac{\sigma}{\sigma - 1} \frac{\tau_{ij} w_i}{z}. \tag{48}$$

Notice that, given the linearity of profits in $n_{ij}(z, E[p_{ij}(z)])$ and $\alpha_j(z)$, due to the assumptions of CES demand and constant returns to scale in labor, the optimal price does not depend on neither $n_{ij}(z, E[p_{ij}(z)])$ nor α_j. By backward induction, in the first stage the firm can take as given the pricing rule in (48), independently from the realization of the shock, and thus the optimal quantity produced is:

$$q_{ij}(z) = \alpha_j(z) \left(\frac{\sigma}{\sigma - 1} \frac{\tau_{ij} w_i}{z} \right)^{-\sigma} \frac{n_{ij}(z, p_{ij}(z)) Y_j}{P_j^{1-\sigma}}.$$

I now solve the firm problem in the first stage, when there is uncertainty. The maximization problem of firm z is:

$$\max_{\{n_{ij}\}} \sum_j \bar{\alpha}_j n_{ij}(z) r_{ij}(z) - \frac{\gamma}{2} \sum_j \sum_s n_{ij}(z) r_{ij}(z) n_{is}(z) r_{is}(z) Cov(\alpha_j, \alpha_s) - \sum_j w_i^\beta w_j^{1-\beta} n_{ij}(z) f_j L_j$$

$$\text{s. to } 1 \geq n_{ij}(z) \geq 0$$

where $r_{ij}(z) \equiv \frac{1}{P_i} \frac{p_{ij}(z)^{-\sigma} Y_j}{P_j^{1-\sigma}} \left(p_{ij}(z) - \frac{\tau_{ij} w_i}{z} \right)$. Given the optimal price in (48), this simplifies to:

$$r_{ij}(z) = \frac{1}{P_i} \left(\frac{\sigma}{\sigma - 1} \frac{\tau_{ij} w_i}{z} \right)^{1-\sigma} \frac{Y_j}{P_j^{1-\sigma} \sigma}$$

The Lagrangian is, omitting the z for simplicity:

$$L = \sum_j \bar{\alpha}_j n_{ij} r_{ij} - \frac{\gamma}{2} \sum_j \sum_s n_{ij} r_{ij} n_{is} r_{is} Cov(\alpha_j, \alpha_s) - \sum_j w_i^\beta w_j^{1-\beta} n_{ij}(z) f_j L_j - \sum_j \mu_{ij} g(n_{ij})$$

where $g(n_{ij}) = n_{ij} - 1$. The necessary KT conditions are:

$$\begin{aligned} &\frac{\partial L}{\partial n_{ij}} = \frac{\partial U}{\partial n_{ij}} - \mu_{ij}\frac{\partial g(n_{ij})}{\partial n_{ij}} \leq 0 \quad \frac{\partial L}{\partial n_{ij}} n_{ij} = 0 \\ &\frac{\partial L}{\partial \mu_{ij}} \geq 0 \quad \frac{\partial L}{\partial \mu_j}\mu_{ij} = 0 \end{aligned}$$

A more compact way of writing the above conditions is to introduce the auxiliary variable λ_{ij}, which is such that

$$\frac{\partial U}{\partial n_{ij}} - \mu_{ij}\frac{\partial g(n_{ij})}{\partial n_{ij}} + \lambda_{ij} = 0$$

and thus $\lambda_{ij} = 0$ if $n_{ij} > 0$, while $\lambda_{ij} > 0$ if $n_{ij} = 0$. Then the first order condition for n_{ij} is:

$$\bar{\alpha_j} r_{ij} - \gamma \sum_s r_{ij} n_{is} r_{is} Cov(\alpha_j, \alpha_s) - w_i^{\beta} w_j^{1-\beta} f_j L_j / P_i - \mu_{ij} + \lambda_{ij} = 0$$

I can write the solution for $n_{ij}(z)$ in matricial form as:

$$\mathbf{n}_i = \frac{1}{\gamma}\left(\tilde{\Sigma}_i\right)^{-1} \mathbf{r}_i, \tag{49}$$

where each element of the $N-$dimensional vector $\mathbf{r}_i$ equals:

$$\mathbf{r}_i^j \equiv r_{ij}\bar{\alpha}_j - w_i^{\beta} w_j^{1-\beta} f_j L_j / P_i - \mu_{ij} + \lambda_{ij}, \tag{50}$$

and $\tilde{\Sigma}_i$ is a NxN covariance matrix, whose k, j element is, from equation (13):

$$\tilde{\Sigma}_{i,kj} = r_{ij} r_{ik}(z) Cov(\alpha_j, \alpha_k).$$

The inverse of $\tilde{\Sigma}_i$ is, by the Cramer's rule:

$$\left(\tilde{\Sigma}_i\right)^{-1} = r_i \frac{1}{det(\Sigma)} C_i r_i, \tag{51}$$

where r_i is the inverse of a diagonal matrix whose $j - th$ element is r_{ij}, and C_i is the (symmetric) matrix of cofactors of Σ.[90] Since $r_{ij} > 0$ for all i and j, then

$$det(\Sigma) \neq 0$$

[90] The cofactor is defined as $C_{kj} \equiv (-1)^{k+j} M_{kj}$, where M_{kj} is the (k, j) minor of Σ. The minor of a matrix is the determinant of the sub-matrix formed by deleting the k-th row and j-th column.

is a sufficient condition to have invertibility of $\tilde{\sum}_i$. This is Assumption 2 in the main text.[91] Replacing equations (51) and (50) into (49), the optimal n_{ij} is:

$$n_{ij} = \frac{\sum_k \frac{C_{jk}}{r_{ik}} \left(r_{ik}\bar{\alpha}_k - w_i^{\beta} w_k^{1-\beta} f_k L_k / P_k - \mu_{ik} + \lambda_{ik} \right)}{\gamma r_{ij}},$$

where C_{jk} is the j, k cofactor of Σ, rescaled by $det(\Sigma)$. Finally, the solution above is a global maximum if i) the constraints are quasi convex and ii) the objective function is concave. The constraints are obviously quasi convex since their are linear. The Hessian matrix of the objective function is:

$$H(z) = \begin{bmatrix} \frac{\partial^2 U}{\partial^2 n_{ij}} & & \frac{\partial^2 U}{\partial n_{ij} \partial n_{iN}} \\ . & . & \\ . & . & \\ \frac{\partial^2 U}{\partial n_{iN} \partial n_{ij}} & & \frac{\partial^2 U}{\partial^2 n_{iN}} \end{bmatrix},$$

where, for all pairs j, k:

$$\frac{\partial^2 U}{\partial n_{ij} \partial n_{ik}} = \frac{\partial^2 U}{\partial n_{ik} \partial n_{ij}} = -\gamma \delta_{ij} \delta_{ik} Cov(\alpha_j, \alpha_k) < 0$$

Given that $\frac{\partial^2 U}{\partial^2 n_{ij}} < 0$, the Hessian is negative semi-definite if and only if its determinant is positive. It is easy to see that the determinant of the Hessian can be written as:

$$det(H) = \prod_{j=1}^{N} \gamma \delta_{ij}(z)^2 det(\Sigma),$$

which is always positive if

$$det(\Sigma) > 0.$$

Therefore the function is concave and the solution is a global maximum, given the price index P, income Y and wage w.■

[91] Since Σ is a covariance matrix, its determinant is always non-negative, but**??** rules out the possibility that all the correlations are $|1|$.

7.4.2 Proof of Proposition 2

From Proposition 1, the optimal solution can be written as (again omitting the z to simplify notation):

$$n_{ij} = \frac{\sum_k \frac{C_{jk}}{r_{ik}} \left(r_{ik}\bar{\alpha}_k - w_i^{\beta} w_k^{1-\beta} f_k L_k / P_k - \mu_{ik} + \lambda_{ik} \right)}{\gamma r_{ij}} =$$

$$= \frac{S_j}{\gamma r_{ij}} - \frac{\sum_k \frac{C_{jk}}{r_{ik}} \left(w_i^{\beta} w_k^{1-\beta} f_k L_k / P_k \right)}{\gamma r_{ij}} + \frac{\sum_k \frac{C_{jk}}{r_{ik}} \left(\lambda_{ik} - \mu_{ik} \right)}{\gamma r_{ij}} \tag{52}$$

where $S_j = \sum_k C_{jk}\bar{\alpha}_k$ is the Sharpe Ratio of destination j. In the case of an interior solution, we have that:

$$n_{ij}(z) = \frac{S_j}{\gamma r_{ij}} - \frac{\sum_k \frac{C_{jk}}{r_{ik}} \left(w_i^{\beta} w_k^{1-\beta} f_k L_k / P_k \right)}{\gamma r_{ij}} \tag{53}$$

and therefore both the probability of entering j (i.e. the probability that $n_{ij}(z) > 0$) and the level of exports to j,

$$x_{ij}(z) = \alpha_j(z) \left(\frac{\sigma}{\sigma - 1} \frac{\tau_{ij} w_i}{z} \right)^{1-\sigma} \frac{Y_j}{P_j^{1-\sigma}} n_{ij}(z) \tag{54}$$

are increasing in S_j.[92] When instead there is at least one binding constraint (either the firm sets $n_{ik}(z) = 0$ or $n_{ik}(z) = 1$ for at least one k), then the corresponding Lagrange multiplier will be positive. Therefore:

$$\frac{\partial n_{ij}(z)}{\partial S_j} = \underbrace{\frac{1}{\gamma r_{ij}}}_{\text{direct effect}} + \underbrace{\frac{1}{\gamma r_{ij}} \left[\sum_{k \neq j} \frac{C_{jk}}{r_{ik}} \frac{\partial \lambda_{ik}}{\partial S_j} - \sum_{k \neq j} \frac{C_{jk}}{r_{ik}} \frac{\partial \mu_{ik}}{\partial S_j} \right]}_{\text{indirect effect}} \tag{55}$$

Note that λ_{ik} is zero if $n_{ik}(z) > 0$, otherwise it equals:

$$\lambda_{ik} = -\bar{\alpha}_k r_{ik} + \gamma r_{ik} \sum_{s \neq j} n_{is} r_{is} Cov(\alpha_k, \alpha_s) + w_i^{\beta} w_k^{1-\beta} f_k L_k / P_k$$

and therefore

$$\frac{\partial \lambda_{ik}}{\partial S_j} = \gamma r_{ik} \sum_{s \neq j} \frac{\partial n_{is}(z)}{\partial S_j} r_{is} Cov(\alpha_k, \alpha_s) \tag{56}$$

[92] To obtain the result, I am implicitly assuming that each firm neglects the general equilibrium effect of S_j on aggregate variables, such as wages. Numerical simulations of the calibrated model show that these partial equilibrium result holds also when taking into account the GE effects.

Similarly for the other Lagrange multiplier:

$$\mu_{ik} = \bar{\alpha}_k r_{ik} - \gamma r_{ik} \sum_{s \neq j} n_{is} r_{is} Cov(\alpha_k, \alpha_s) - \gamma r_{ik}^2 Var(\alpha_k) - w_i^{\beta} w_k^{1-\beta} f_k L_k / P_k$$

and thus:

$$\frac{\partial \mu_{ik}}{\partial S_j} = -\gamma r_{ik} \sum_{s \neq j} \frac{\partial n_{is}(z)}{\partial S_j} r_{is} Cov(\alpha_k, \alpha_s) = -\frac{\partial \lambda_{ik}}{\partial S_j} \tag{57}$$

Now notice that either $\mu_{ik} > 0$ and $\lambda_{ik} = 0$, or $\lambda_{ik} > 0$ and $\mu_{ik} = 0$. Combining this fact with equations 56 and 57, equation 55 becomes:

$$\frac{\partial n_{ij}(z)}{\partial S_j} = \frac{1}{\gamma r_{ij}} \left[1 + \gamma \sum_{k \neq j} C_{jk} \sum_{s \neq j} \frac{\partial n_{is}(z)}{\partial S_j} r_{is} Cov(\alpha_k, \alpha_s) \right]$$

Define $x_j \equiv \frac{\partial n_{ij}(z)}{\partial S_j} \gamma r_{ij}$. Then the above can be written as:

$$x_j = 1 + \sum_{k \neq j} C_{jk} \sum_{s \neq j} x_s Cov(\alpha_k, \alpha_s)$$

This is a linear system of N equations in N unknowns, x_j. We can rewrite it as $AX = B$, where A is a NxN matrix:

$$A = \left[\begin{array}{cccc} 1 & -\sum_{k \neq 1} C_{1k} Cov(\alpha_k, \alpha_2) & ... & -\sum_{k \neq 1} C_{1k} Cov(\alpha_k, \alpha_N) \\ -\sum_{k \neq 2} C_{2k} Cov(\alpha_k, \alpha_1) & 1 & ... & -\sum_{k \neq 2} C_{2k} Cov(\alpha_k, \alpha_N) \\ ... & ... & ... & ... \\ -\sum_{k \neq N} C_{Nk} Cov(\alpha_k, \alpha_1) & -\sum_{k \neq N} C_{2k} Cov(\alpha_k, \alpha_2) & ... & 1 \end{array} \right],$$

that is

$$A_{ij} = \begin{cases} -\sum_{k \neq i} C_{ik} Cov(\alpha_k, \alpha_j) \,, & i \neq j \\ 1 \qquad\qquad\qquad\qquad , & i = j \end{cases}.$$

and B is a $Nx1$ vector of ones. It follows that

$$X = A^{-1} B.$$

Since B is a positive vector, in order to have X positive, it is sufficient to have A^{-1} totally positive. By theorem 2.2. in Pena (1995), a necessary and sufficient condition for A^{-1} to be totally positive is A being a M-matrix, i.e. all off-diagonal elements are negative. It is easy to verify that A is a M-matrix whenever at least one, but not all, demand correlation is negative.[93] ■

[93] For example, this can be seen for the case $N = 4$, where a typical element of the matrix A looks like:

$$A_{21} = \rho_{12} \sigma_1^3 \sigma_2 \sigma_3^2 \sigma_4^2 (1 - \rho_{13}^2 - \rho_{14}^2 - \rho_{34}^2 + 2 \rho_{13} \rho_{14} \rho_{34}).$$

7.4.3 Model with risk neutrality

With risk neutrality, the objective function is:

$$\max_{\{n_{ij}\}} \sum_j \bar{\alpha}_j n_{ij}(z) r_{ij}(z) - \sum_j w_i^{\beta} w_j^{1-\beta} n_{ij}(z) f_j L_j / P_j$$

Notice that the above is simply linear in $n_{ij}(z)$, and therefore it is always optimal, upon entry, to set $n_{ij}(z) = 1$. Therefore the firm's problem boils down to a standard entry decision, as in Melitz (2003), which implies that the firm enters a market j only if expected profits are positive. This in turn implies the existence of an entry cutoff, given by:

$$(\bar{z}_{ij})^{\sigma-1} = \frac{w_i^{\beta} w_j^{1-\beta} f_j L_j P_j^{1-\sigma} \sigma}{\bar{\alpha}_j \left(\frac{\sigma}{\sigma-1} \tau_{ij} w_i\right)^{1-\sigma} Y_j} \tag{58}$$

To find the welfare gains from trade in the case of $\gamma = 0$, I first write the equation for trade shares

$$\lambda_{ij} = \frac{M_i \int_{\bar{z}_{ij}}^{\infty} \bar{\alpha}_j p_{ij}(z) q_{ij}(z) g_i(z) dz}{w_j L_j} = \frac{M_i \int_{\bar{z}_{ij}}^{\infty} \bar{\alpha}_j p_{ij}(z)^{1-\sigma} g_i(z) dz}{P_j^{1-\sigma}} \tag{59}$$

Inverting the above:

$$\frac{M_i \gamma (\tau_{ij} w_i)^{1-\sigma} (\bar{z}_{ij})^{\sigma-\theta-1}}{\lambda_{ij}} = P_j^{1-\sigma}. \tag{60}$$

Substituting for the cutoff, and using the fact that when $\gamma = 0$ profits are a constant share of total income (see ACR), I can write the real wage as a function of trade shares:

$$\left(\frac{w_j}{P_j}\right) = \vartheta \lambda_{jj}^{-\frac{1}{\theta}}, \tag{61}$$

where ϑ is a constant. Since the risk aversion is zero, and profits are a constant share of total income, the percentage change in welfare is simply:

$$d\ln W_j = -d\ln P_j \tag{62}$$

where I have also set the wage as the numeraire. Substituting 61 into 62, we get:

$$d\ln W_j = -\frac{1}{\theta} d\ln \lambda_{jj}$$

Lastly, from the equation for trade share it is to verify that $-\theta$ equals the trade elasticity.

7.4.4 Model with autarky

Lemma 1. Assume that $\gamma > \left(\chi \tilde{L}\right)^{\frac{\theta+1-\sigma}{(1-\sigma)\theta}} \left(\bar{\alpha} M S_A \sigma \left(\left(\frac{\sigma}{\sigma-1}\right)^{\sigma-1} \frac{\sigma f}{\bar{\alpha}}\right)^{\frac{\theta}{1-\sigma}} \left(\frac{\sigma-1}{\theta+\sigma-1}\right)\right)^{-\frac{1}{\theta}} \left(\frac{S_A \bar{\alpha}}{4f}\right)^{\frac{1+\theta}{\theta}}$.
Then the optimal solution is:
- $n(z) = 0$ if $z \leq z^*$
- $0 < n(z) < 1$ if $z > z^*$, where:

$$n(z) = \frac{S_A}{\gamma} \frac{\left(1 - \left(\frac{z^*}{z}\right)^{\sigma-1}\right)}{r(z)}$$

and the cutoff is given by:

$$z^* = \left(\left(\frac{\sigma}{\sigma-1}\right)^{\sigma-1} \frac{f P^{1-\sigma} \sigma}{\bar{\alpha} Y}\right)^{\frac{1}{\sigma-1}}$$

Proof. As in Proposition 1, the optimal price is a constant markup over marginal cost:

$$p = \frac{\sigma}{\sigma-1} \frac{1}{z}$$

and thus total gross profits are:

$$r(z) = \frac{1}{P} \left(\frac{\sigma}{\sigma-1} \frac{1}{z}\right)^{1-\sigma} \frac{Y_j}{P_j^{1-\sigma} \sigma}$$

The Lagrangian is:

$$\mathcal{L}_i(z) = \bar{\alpha} n(z) r(z) - \frac{\gamma}{2} Var(\alpha) n^2(z) r^2(z) - n(z) f + \lambda n(z) + \mu(1 - n(z))$$

and the FOCs are:

$$\bar{\alpha} r(z) - f/P - \gamma n(z) r^2(z) Var(\alpha) + \lambda - \mu = 0$$

Thus $n(z)$ becomes:

$$n(z) = \frac{\bar{\alpha} r(z) - f/P + \lambda - \mu}{r^2(z) Var(\alpha) \gamma}$$

To get rid of the upper bound multiplier μ, I now find a restriction on parameters such that it is always optimal to choose $n(z) < 1$. When the optimal solution is $n = 0$, then this holds trivially. If instead $n > 0$, and thus $\lambda = 0$, then it must hold that:

$$n(z) = \frac{\bar{\alpha} r(z) - f/P}{r^2(z) Var(\alpha) \gamma} < 1$$

Rearranging:

$$\gamma > \frac{\bar{\alpha}r(z) - f/P}{r^2(z)Var(\alpha)} \tag{63}$$

The RHS of the above inequality is a function of the productivity z. For the inequality to hold for any z, it suffices to hold for the productivity z that maximizes the RHS. It is easy to verify that such z is:

$$z_{max} = \left(\frac{2f}{\bar{\alpha}\tilde{u}}\right)^{\frac{1}{\sigma-1}} \tag{64}$$

where $\tilde{u} = \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} \frac{Y}{P^{1-\sigma}\sigma}$. Therefore a sufficient condition to have 63 is:

$$\gamma > \frac{\bar{\alpha}\frac{\tilde{u}}{P}\frac{2f}{\bar{\alpha}\tilde{u}} - f/P}{\left(\frac{\tilde{u}}{P}\frac{2f}{\bar{\alpha}\tilde{u}}\right)^2 Var(\alpha)} = P\frac{\bar{\alpha}^2}{f4Var(\alpha)} \tag{65}$$

In what follows (see equation (71), I show that if the above inequality holds, the optimal price index is given by:

$$P = \left(\chi\tilde{L}\right)^{\frac{\theta+1-\sigma}{(1-\sigma)(1+\theta)}} (\kappa_2)^{-\frac{1}{\theta+1}} \tag{66}$$

where χ depends only on σ and θ, and where $\kappa_2 \equiv \bar{\alpha}M\frac{S_A\sigma}{\gamma}(x)^{\frac{\theta}{1-\sigma}}\left(\frac{\sigma-1}{\theta+\sigma-1}\right)$ and $x \equiv \left(\frac{\sigma}{\sigma-1}\right)^{\sigma-1}\frac{\sigma f}{\bar{\alpha}}$. Plugging equation (71) into the above inequality implies that:

$$\gamma > \left(\chi\tilde{L}\right)^{\frac{\theta+1-\sigma}{(1-\sigma)(1+\theta)}} \left(\bar{\alpha}MS_A\sigma\frac{1}{\gamma}(x)^{\frac{\theta}{1-\sigma}}\left(\frac{\sigma-1}{\theta+\sigma-1}\right)\right)^{-\frac{1}{\theta+1}} \frac{S_A\bar{\alpha}}{f4}$$

Rearranging:

$$\gamma > \left(\chi\tilde{L}\right)^{\frac{\theta+1-\sigma}{(1-\sigma)\theta}} \left(\bar{\alpha}MS_A\sigma(x)^{\frac{\theta}{1-\sigma}}\left(\frac{\sigma-1}{\theta+\sigma-1}\right)\right)^{-\frac{1}{\theta}} \left(\frac{S_A\bar{\alpha}}{f4}\right)^{\frac{1+\theta}{\theta}} \tag{67}$$

If 67 holds, then any firm will always choose to set $n_{ij}(z) < 1$. Then, the FOC becomes:

$$\bar{\alpha}r(z) - f/P - \gamma n(z)r^2(z)Var(\alpha) + \lambda = 0$$

I now guess and verify that the optimal $n(z)$ is such that: if $z > z^*$ then $n(z) > 0$, otherwise $n(z) = 0$. First I find such cutoff by solving $n(z^*) = 0$:

$$z^* = \left(\left(\frac{\sigma}{\sigma-1}\right)^{\sigma-1}\frac{fP^{1-\sigma}\sigma}{\bar{\alpha}Y}\right)^{\frac{1}{\sigma-1}}$$

and the corresponding optimal $n(z)$ is:

$$n(z) = \frac{1}{\gamma}\frac{\bar{\alpha}}{Var(\alpha)}\frac{\left(1-\left(\frac{z^*}{z}\right)^{\sigma-1}\right)}{r(z)}$$

If the guess is correct, then it must be that, when $z < z^*$, the FOC is satisfied with a positive λ and thus $n(z) = 0$. Indeed, notice that setting $n(z) = 0$ gives:

$$\bar{\alpha} r(z) - f + \lambda = 0$$

and so the multiplier is:

$$\lambda = f - \bar{\alpha} r(z)$$

which is positive only if $f > \bar{\alpha} r(z)$, that is, when $z < z^*$. Therefore the guess is verified. Lastly, the optimal solution can be written more compactly as:

$$n(z) = \frac{S_A}{\gamma} \frac{\left(1 - \left(\frac{z^*}{z}\right)^{\sigma-1}\right)}{r(z)}$$

where $S_A \equiv \frac{\bar{\alpha}}{Var(\alpha)}$ is the Sharpe Ratio.■

Equilibrium. Assuming that $\theta > \sigma - 1$, and normalizing the wage to 1, current account balance implies that total income is:

$$Y_A = w_i \tilde{L}_i + \Pi_i = \tilde{L} + \kappa_1 P^{1+\theta} Y_A^{\frac{\theta}{\sigma-1}} \tag{68}$$

where $\kappa_1 \equiv \frac{MS_A}{\gamma} (x)^{\frac{\theta}{1-\sigma}} \bar{\alpha} \left[\frac{\sigma-1-\theta}{\theta+\sigma-1} + \frac{\theta}{\theta+2\sigma-2}\right]$ and where $x \equiv \left(\frac{\sigma}{\sigma-1}\right)^{\sigma-1} \frac{\sigma f}{\bar{\alpha}}$.
The price index equation is:

$$P_i^{1-\sigma} = \bar{\alpha} M \int_{z^*}^{\infty} n_{ji}(z) p_{ji}(z)^{1-\sigma} \theta z^{-\theta-1} dz =$$

$$= Y_A^{\frac{-\theta-1+\sigma}{1-\sigma}} P^{2-\sigma+\theta} \kappa_2$$

where $\kappa_2 \equiv \bar{\alpha} M \frac{S_A \sigma}{\gamma} (x)^{\frac{\theta}{1-\sigma}} \left(\frac{\sigma-1}{\theta+\sigma-1}\right)$. Rearranging:

$$Y_A^{\frac{\theta+1-\sigma}{1-\sigma}} / \kappa_2 = P^{1+\theta} \tag{69}$$

Plug equation 69 into equation 68:

$$Y_A = \tilde{L} + \kappa_1 P^{1+\theta} Y_A^{\frac{\theta}{\sigma-1}} =$$

$$= \tilde{L} + \frac{\kappa_1}{\kappa_2} Y_A^{\frac{\theta+1-\sigma}{1-\sigma}} Y_A^{\frac{\theta}{\sigma-1}} = \tilde{L} + \frac{\kappa_1}{\kappa_2} Y_A$$

and therefore total income is:

$$Y_A = \chi \tilde{L} \tag{70}$$

where $\chi \equiv \frac{\sigma\left(\frac{\sigma-1}{\theta+\sigma-1}\right)}{\sigma\left(\frac{\sigma-1}{\theta+\sigma-1}\right) - \left[\frac{\sigma-1-\theta}{\theta+\sigma-1} + \frac{\theta}{\theta+2\sigma-2}\right]}$, and the price index is:

$$P_A = \left(\chi \tilde{L}\right)^{\frac{\theta+1-\sigma}{(1-\sigma)(1+\theta)}} (\kappa_2)^{-\frac{1}{\theta+1}} \tag{71}$$

7.4.5 Model with two symmetric countries and free trade

Lemma 2. Assume countries are perfectly symmetric and there is free trade. Assume that $\gamma > \left(\chi\tilde{L}\right)^{\frac{\theta+1-\sigma}{(1-\sigma)\theta}} \left(\bar{\alpha}2MS_{FT}\sigma\left(\left(\frac{\sigma}{\sigma-1}\right)^{\sigma-1}\frac{\sigma f}{\bar{\alpha}}\right)^{\frac{\theta}{1-\sigma}}\left(\frac{\sigma-1}{\theta+\sigma-1}\right)\right)^{-\frac{1}{\theta}}\left(\frac{S\bar{\alpha}}{4f}\right)^{\frac{\theta+1}{\theta}}$. Then the optimal solution is:
- $n_{ij} = 0$ if $z \leq z^*$
- $0 < n(z) < 1$ if $z > z^*$, where:

$$n(z) = \frac{S_{FT}}{\gamma}\frac{\left(1-\left(\frac{z^*}{z}\right)^{\sigma-1}\right)}{r(z)}$$

and the cutoff is given by:

$$z^* = \left(\left(\frac{\sigma}{\sigma-1}\right)^{\sigma-1}\frac{fP^{1-\sigma}\sigma}{\bar{\alpha}Y}\right)^{\frac{1}{\sigma-1}}$$

Proof: As in Proposition 1, the optimal price is a constant markup over marginal cost:

$$p = \frac{\sigma}{\sigma-1}\frac{1}{z}$$

and thus total gross profits are:

$$r_{ij}(z) = \frac{1}{P}\left(\frac{\sigma}{\sigma-1}\frac{1}{z}\right)^{1-\sigma}\frac{Y_j}{P_j^{1-\sigma}\sigma}$$

In the first stage, the FOCs are:

$$\bar{\alpha}r_{ih}(z) - f/P - \gamma\left(n_{ih}r_{ih}^2(z)Var(\alpha_h) + r_{ih}(z)n_{if}(z)r_{if}(z)Cov(\alpha_h,\alpha_f)\right) + \lambda_h - \mu_h = 0$$

$$\bar{\alpha}r_{if}(z) - f/P - \gamma\left(n_{if}r_{if}^2(z)Var(\alpha_f) + r_{if}(z)n_{ih}(z)r_{ih}(z)Cov(\alpha_h,\alpha_f)\right) + \lambda_f - \mu_f = 0$$

From the above we have that:

$$n_{ih} = \frac{d_h r_{if}(z) - d_f r_{ih}(z)\rho + r_{if}(z)\left(\lambda_h-\mu_h\right) - r_{ih}(z)\rho\left(\lambda_f-\mu_f\right)}{\gamma Var(\alpha)r_{ih}^2(z)r_{if}(z)\left(1-\rho^2\right)}$$

$$n_{if} = \frac{d_f r_{ih}(z) - d_h r_{if}(z)\rho + r_{ih}(z)\left(\lambda_f-\mu_f\right) - r_{if}(z)\rho\left(\lambda_h-\mu_h\right)}{\gamma Var(\alpha)r_{if}^2(z)r_{ih}(z)\left(1-\rho^2\right)}$$

where

$$d_j \equiv \bar{\alpha}r_{ij}(z) - f/P$$

To get rid of the upper bound multipliers μ_h and μ_f, I now find a restriction on parameters such that it is always optimal to choose $n_{ij}(z) < 1$. When the optimal solution is $n_{ij} = 0$, then this holds trivially. If instead $n_{ij} > 0$, and thus $\lambda_j = 0$, then it must hold that:

$$n_{ij} = \frac{d_j r_{ik}(z) - d_k r_{ij}(z)\rho}{\gamma Var(\alpha) r_{ij}^2(z) r_{ik}(z)\left(1-\rho^2\right)} < 1$$

for all j, where $k \neq j$. For the home country, this becomes:

$$\left(\bar{\alpha} r_{ih}(z) - f/P\right) r_{if}(z) - \left(\bar{\alpha} r_{if}(z) - f/P\right) r_{ih}(z)\rho < \gamma Var(\alpha) r_{ih}^2(z) r_{if}(z)\left(1-\rho^2\right)$$

Invoking symmetry:

$$\left(\bar{\alpha} u z^{\sigma-1} - f/P\right) u z^{\sigma-1} - \left(\bar{\alpha} u z^{\sigma-1} - f/P\right) u z^{\sigma-1}\rho < \gamma Var(\alpha) u^2 z^{2(\sigma-1)} u z^{\sigma-1}\left(1-\rho^2\right)$$

$$\left(\bar{\alpha} u z^{\sigma-1} - f/P\right)(1-\rho) < \gamma Var(\alpha) u^2 z^{2(\sigma-1)}\left(1-\rho^2\right)$$

$$\left(\bar{\alpha} u z^{\sigma-1} - f/P\right) < \gamma Var(\alpha) u^2 z^{2(\sigma-1)}(1+\rho)$$

where $u = \frac{1}{P}\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\frac{Y}{P^{1-\sigma}\sigma}$. Rearranging:

$$\gamma > \frac{1}{Var(\alpha) u z^{\sigma-1}(1+\rho)}\left(\bar{\alpha} - \frac{f/P}{z^{\sigma-1}u}\right) \tag{72}$$

The RHS of the above inequality is a function of the productivity z. For the inequality to hold for any z, it suffices to hold for the productivity z that maximizes the RHS. It is easy to verify that such z is:

$$z_{max} = \left(\frac{2f}{\bar{\alpha}\tilde{u}}\right)^{\frac{1}{\sigma-1}} \tag{73}$$

where $\tilde{u} = \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\frac{Y}{P^{1-\sigma}\sigma}$. Therefore a sufficient condition to have 72 is:

$$\gamma > \frac{1}{Var(\alpha) u \frac{2f}{\bar{\alpha}\tilde{u}}(1+\rho)}\left(\bar{\alpha} - \frac{f}{\frac{2f}{\bar{\alpha}\tilde{u}}\tilde{u}}\right) = P\frac{\bar{\alpha}^2}{Var(\alpha) 4f(1+\rho)} \tag{74}$$

In what follows, I show that if the above inequality holds, the optimal price index is given by:

$$P = \left(\chi\tilde{L}\right)^{\frac{\theta+1-\sigma}{(1-\sigma)(1+\theta)}}(\kappa_3)^{-\frac{1}{\theta+1}} \tag{75}$$

where χ depends only on σ and θ, and $\kappa_3 \equiv \bar{\alpha} 2M\frac{S\sigma}{\gamma}\left(\left(\frac{\sigma}{\sigma-1}\right)^{\sigma-1}\frac{\sigma f}{\bar{\alpha}}\right)^{\frac{\theta}{1-\sigma}}\left(\frac{\sigma-1}{\theta+\sigma-1}\right)$. Therefore the risk aversion has to satisfy:

$$\gamma > \left(\chi\tilde{L}\right)^{\frac{\theta+1-\sigma}{(1-\sigma)(1+\theta)}}(\kappa_3)^{-\frac{1}{\theta+1}}\frac{\bar{\alpha}^2}{Var(\alpha) 4f(1+\rho)}$$

Rearranging:

$$\gamma > \left(\chi\tilde{L}\right)^{\frac{\theta+1-\sigma}{(1-\sigma)\theta}} \left(\bar{\alpha}2MS_{FT}\sigma\left(\left(\left(\frac{\sigma}{\sigma-1}\right)^{\sigma-1}\frac{\sigma f}{\bar{\alpha}}\right)^{\frac{\theta}{1-\sigma}}\left(\frac{\sigma-1}{\theta+\sigma-1}\right)\right)\right)^{-\frac{1}{\theta}}\left(\frac{S\bar{\alpha}}{4f}\right)^{\frac{\theta+1}{\theta}} \tag{76}$$

where the right hand side is only function of parameters.

If 76 holds, then any firm will always choose to set $n_{ij}(z) < 1$. Then, given the symmetry of the economy, each firm will either sell to both the domestic and foreign market, or to none. This implies that the FOC becomes:

$$\bar{\alpha}r(z) - f/P - \gamma n_{ih}(z)r^2(z)Var(\alpha_h)\left(1+\rho\right) + \lambda_h = 0$$

I now guess and verify that the optimal $n_{ih}(z)$ is such that: if $z > z^*$ then $n_{ih}(z) > 0$, otherwise $n_{ih}(z) = 0$. First I find such cutoff by solving $n_{ih}(z^*) = 0$:

$$z^* = \left(\left(\frac{\sigma}{\sigma-1}\right)^{\sigma-1}\frac{fP^{1-\sigma}\sigma}{\bar{\alpha}Y}\right)^{\frac{1}{\sigma-1}}$$

and the corresponding optimal $n(z)$ is:

$$n(z) = \frac{1}{\gamma}\frac{\bar{\alpha}}{Var(\alpha)\left(1+\rho\right)}\frac{\left(1-\left(\frac{z^*}{z}\right)^{\sigma-1}\right)}{r(z)}$$

If the guess is correct, then it must be that, when $z < z^*$, the FOC is satisfied with a positive λ_h and thus $n(z) = 0$. Indeed, notice that setting $n(z) = 0$ gives:

$$\bar{\alpha}r(z) - f + \lambda_h = 0$$

and so the multiplier is:

$$\lambda_h = f - \bar{\alpha}r(z)$$

which is positive only if $f > \bar{\alpha}r(z)$, that is, when $z < z^*$. Therefore the guess is verified. Lastly, the optimal solution can be written as:

$$n(z) = \frac{S_{FT}}{\gamma}\frac{\left(1-\left(\frac{z^*}{z}\right)^{\sigma-1}\right)}{r(z)}$$

where $S_{FT} \equiv \frac{\bar{\alpha}}{Var(\alpha)(1+\rho)}$ is the Sharpe Ratio.■

The intuition is that the risk aversion must be high enough to avoid the firm choosing to sell to all consumers in a certain destination. In a sense, the firm always wants to diversify risk by selling a little to multiple countries, rather than being exposed a lot to only one country. Instead, when $\gamma = 0$, as in standard trade models, it is optimal to always set $n_{ij} = 1$, upon entry. As entrepreneurs become more risk averse, they will choose a lower n_{ij} and diversify their sales across countries.

Equilibrium with free trade. Assuming as before that $\theta > \sigma - 1$, and normalizing the wage to 1, current account balance implies that total income is:

$$Y_{FT} = w_i \tilde{L}_i + \Pi_i = \tilde{L} + \kappa_4 P_{FT}^{1+\theta} Y^{\frac{\theta}{\sigma-1}} \tag{77}$$

where $\kappa_4 \equiv \frac{2MS_{FT}}{\gamma} (x)^{\frac{\theta}{1-\sigma}} \bar{\alpha} \left[\frac{\sigma-1-\theta}{\theta+\sigma-1} + \frac{\theta}{\theta+2\sigma-2}\right]$ and where $x \equiv \left(\frac{\sigma}{\sigma-1}\right)^{\sigma-1} \frac{\sigma f}{\bar{\alpha}}$.
The price index equation is:

$$P_{FT}^{1-\sigma} = \bar{\alpha} 2M \int_{z^*}^{\infty} n_{ji}(z) p_{ji}(z)^{1-\sigma} \theta z^{-\theta-1} dz =$$

$$= Y_{FT}^{\frac{-\theta-1+\sigma}{1-\sigma}} P_{FT}^{2-\sigma+\theta} \kappa_5$$

where $\kappa_5 \equiv \bar{\alpha} 2M \frac{S_{FT}\sigma}{\gamma} (x)^{\frac{\theta}{1-\sigma}} \left(\frac{\sigma-1}{\theta+\sigma-1}\right)$. Rearranging:

$$Y_{FT}^{\frac{\theta+1-\sigma}{1-\sigma}} / \kappa_5 = P_{FT}^{1+\theta} \tag{78}$$

Plug equation 78 into equation 77:

$$Y_{FT} = \tilde{L} + \kappa_4 P_{FT}^{1+\theta} Y_{FT}^{\frac{\theta}{\sigma-1}} =$$

$$= \tilde{L} + \frac{\kappa_4}{\kappa_5} Y_{FT}^{\frac{\theta+1-\sigma}{1-\sigma}} Y_{FT}^{\frac{\theta}{\sigma-1}} = \tilde{L} + \frac{\kappa_4}{\kappa_5} Y_{FT}$$

and therefore total income is:

$$Y_{FT} = \chi \tilde{L} \tag{79}$$

where $\chi \equiv \frac{\sigma\left(\frac{\sigma-1}{\theta+\sigma-1}\right)}{\sigma\left(\frac{\sigma-1}{\theta+\sigma-1}\right) - \left[\frac{\sigma-1-\theta}{\theta+\sigma-1} + \frac{\theta}{\theta+2\sigma-2}\right]}$, and the price index is:

$$P_{FT} = \left(\chi \tilde{L}\right)^{\frac{\theta+1-\sigma}{(1-\sigma)(1+\theta)}} (\kappa_5)^{-\frac{1}{\theta+1}} \tag{80}$$

7.4.6 Proof of Proposition 3

Welfare under autarky is:

$$W_A = \frac{Y_A}{P_A} - M \int_{z^*} \frac{\gamma}{2} Var\left(\frac{\pi(z)}{P_A}\right) \theta z^{-\theta-1} dz =$$

$$= \frac{Y_A}{P_A} - M \int_{z^*} \frac{\gamma}{2} Var(\alpha) n^2(z) r^2(z) \theta z^{-\theta-1} dz$$

since marketing costs are non-stochastic. Then

$$W_A = \frac{Y_A}{P_A} - \frac{M}{2} Var(\alpha) \frac{S^2}{\gamma} \int_{z^*} \left(1 - \left(\frac{z^*}{z}\right)^{\sigma-1}\right)^2 \theta z^{-\theta-1} dz =$$

$$= \frac{Y_A}{P_A} - \frac{M}{2} Var(\alpha) \frac{S^2}{\gamma} \int_{z^*} \left(1 + \left(\frac{z^*}{z}\right)^{2(\sigma-1)} - 2\left(\frac{z^*}{z}\right)^{\sigma-1}\right) \theta z^{-\theta-1} dz =$$

$$= \frac{Y_A}{P_A} - \frac{M}{2} Var(\alpha) \frac{S^2}{\gamma} \left((z^*)^{-\theta} + (z^*)^{-\theta} \frac{\theta}{\theta+2-2\sigma} - 2 (z^*)^{-\theta} \frac{\theta}{\theta+\sigma-1} \right) =$$

$$= \frac{Y_A}{P_A} - \frac{M}{2} Var(\alpha) \frac{S^2}{\gamma} (z^*)^{-\theta} \left(\frac{\sigma-1-\theta}{\theta+\sigma-1} + \frac{\theta}{\theta+2-2\sigma} \right) =$$

$$= \frac{Y_A}{P_A} - \frac{M}{2} Var(\alpha) \frac{S^2}{\gamma} \left(\left(\frac{\sigma}{\sigma-1}\right)^{\sigma-1} \frac{f P^{1-\sigma} \sigma}{\bar{\alpha} Y} \right)^{\frac{-\theta}{\sigma-1}} \left(\frac{\sigma-1-\theta}{\theta+\sigma-1} + \frac{\theta}{\theta+2-2\sigma} \right) =$$

$$= \frac{Y_A}{P_A} - P_A^{\theta} Y_A^{\frac{\theta}{\sigma-1}} \frac{M}{2\gamma} S \bar{\alpha} x^{\frac{\theta}{1-\sigma}} \left(\frac{\theta}{\theta-2+2\sigma} + \frac{\sigma-1-\theta}{\theta+\sigma-1} \right) =$$

$$= \frac{Y_A}{P_A} - \kappa_7 P_A^{\theta} Y_A^{\frac{\theta}{\sigma-1}} \tag{81}$$

where $\kappa_7 = M \frac{S_A \bar{\alpha}}{2\gamma} (x)^{\frac{\theta}{1-\sigma}} \left[\frac{\sigma-1-\theta}{\theta+\sigma-1} + \frac{\theta}{\theta+2\sigma-2}\right]$. Let's further simplify the above:

$$W_A = \left(\chi \tilde{L}\right)^{\frac{\sigma\theta}{(\sigma-1)(1+\theta)}} (\kappa_2)^{\frac{1}{\theta+1}} - \kappa_7 \left(\chi \tilde{L}\right)^{\frac{\theta\sigma}{(\sigma-1)(1+\theta)}} (\kappa_2)^{-\frac{\theta}{\theta+1}} =$$

$$= \left(\chi \tilde{L}\right)^{\frac{\sigma\theta}{(\sigma-1)(1+\theta)}} \left[(\kappa_2)^{\frac{1}{\theta+1}} - \kappa_7 (\kappa_2)^{-\frac{\theta}{\theta+1}} \right] \tag{82}$$

Note that $W_A > 0$ always, since $\theta > \sigma - 1$. Welfare under free trade is:

$$W_{FT} = \frac{Y}{P} - M \int_{z^*}^{\infty} \frac{\gamma}{2} Var\left(\frac{\pi(z)}{P}\right) \theta z^{-\theta-1} dz =$$

$$= \frac{Y}{P} - M\frac{\gamma}{2}\int_{z^*}^{\infty}\left(Var\left(\frac{\pi_{HH}(z)}{P}\right) + Var\left(\frac{\pi_{HF}(z)}{P}\right) + 2Cov\left(\frac{\pi_H(z)}{P}, \frac{\pi_F(z)}{P}\right)\right)\theta z^{-\theta-1}dz =$$

$$= \frac{Y}{P} - M\int_{z^*}^{\infty}\frac{\gamma}{2}\left(Var(\alpha)\left(\frac{\pi_{HH}(z)}{P}\right)^2 + Var(\alpha)\left(\frac{\pi_{HF}(z)}{P}\right)^2 + 2\frac{\pi_{HF}(z)}{P}\frac{\pi_{HH}(z)}{P}Cov\left(\alpha_H, \alpha_F\right)\right)\theta z^{-\theta-1}dz$$

where $\pi_{ij}$are gross profits (since marketing costs are non-stochastic). By symmetry (and by absence of trade costs):

$$W_{FT} = \frac{Y}{P} - M\int_{z^*}^{\infty}\frac{\gamma}{2}\left(Var(\alpha)\left(\frac{\pi(z)}{P}\right)^2 + Var(\alpha)\left(\frac{\pi(z)}{P}\right)^2 + 2\left(\frac{\pi(z)}{P}\right)^2 Cov\left(\alpha_H, \alpha_F\right)\right)\theta z^{-\theta-1}dz =$$

$$= \frac{Y}{P} - M\gamma Var(\alpha)\left(1+\rho\right)\int_{z^*}^{\infty}\left(\frac{\pi(z)}{P}\right)^2\theta z^{-\theta-1}dz =$$

$$= \frac{Y}{P} - MVar(\alpha)\left(1+\rho\right)\frac{S^2}{\gamma}\int_{z^*}^{\infty}\left(1 - \left(\frac{z^*}{z}\right)^{\sigma-1}\right)^2\theta z^{-\theta-1}dz =$$

$$= \frac{Y}{P} - MVar(\alpha)\left(1+\rho\right)\frac{S^2}{\gamma}\left(z^*\right)^{-\theta}\left(\frac{\sigma-1-\theta}{\theta+\sigma-1} + \frac{\theta}{\theta+2-2\sigma}\right) =$$

$$= \frac{Y}{P} - MVar(\alpha)\left(1+\rho\right)\frac{S^2}{\gamma}\left(\left(\frac{\sigma}{\sigma-1}\right)^{\sigma-1}\frac{fP^{1-\sigma}\sigma}{\bar{\alpha}Y}\right)^{-\frac{\theta}{\sigma-1}}\left(\frac{\sigma-1-\theta}{\theta+\sigma-1} + \frac{\theta}{\theta+2-2\sigma}\right) =$$

$$= \frac{Y}{P} - \kappa_8 P^{\theta} Y^{\frac{\theta}{\sigma-1}} \tag{83}$$

where $\kappa_8 = M\frac{1}{\gamma}\bar{\alpha}S_{FT}\left(x\right)^{\frac{\theta}{1-\sigma}}\left[\frac{\sigma-1-\theta}{\theta+\sigma-1} + \frac{\theta}{\theta+2\sigma-2}\right]$. Further simplify:

$$W_{FT} = \left(\chi\tilde{L}\right)^{1-\frac{\theta+1-\sigma}{(1-\sigma)(1+\theta)}}\left(\kappa_5\right)^{\frac{1}{\theta+1}} - \kappa_8 P^{\theta} Y^{\frac{\theta}{\sigma-1}} =$$

$$= \left(\chi\tilde{L}\right)^{\frac{\sigma\theta}{(\sigma-1)(1+\theta)}}\left[\left(\kappa_5\right)^{\frac{1}{\theta+1}} - \kappa_8\left(\kappa_5\right)^{-\frac{\theta}{\theta+1}}\right] \tag{84}$$

Using equations 82 and 84, welfare gains are:

$$\hat{W} = \frac{W_{FT}}{W_A} - 1 =$$

$$= \frac{\left(\chi\tilde{L}\right)^{\frac{\sigma\theta}{(\sigma-1)(1+\theta)}}\left[\left(\kappa_5\right)^{\frac{1}{\theta+1}} - \kappa_8\left(\kappa_5\right)^{-\frac{\theta}{\theta+1}}\right]}{\left(\chi\tilde{L}\right)^{\frac{\sigma\theta}{(\sigma-1)(1+\theta)}}\left[\left(\kappa_2\right)^{\frac{1}{\theta+1}} - \kappa_7\left(\kappa_2\right)^{-\frac{\theta}{\theta+1}}\right]} - 1 =$$

$$= \frac{\left(\kappa_5\right)^{\frac{1}{\theta+1}} - \kappa_8\left(\kappa_5\right)^{-\frac{\theta}{\theta+1}}}{\left(\kappa_2\right)^{\frac{1}{\theta+1}} - \kappa_7\left(\kappa_2\right)^{-\frac{\theta}{\theta+1}}} - 1 =$$

$$= \left(\frac{S_{FT}}{S_A}\right)^{\frac{1}{\theta+1}}\xi - 1 =$$

$$= \left(S_{FT}\right)^{-\frac{1}{\theta+1}}\xi - 1 \tag{85}$$

since I set $Var(\alpha) = \bar{\alpha} = 1$, and where $\xi \equiv \frac{\left(2\sigma\left(\frac{\sigma-1}{\theta+\sigma-1}\right)\right)^{\frac{1}{\theta+1}} - \left[\frac{\sigma-1-\theta}{\theta+\sigma-1}+\frac{\theta}{\theta+2\sigma-2}\right]\left(2\sigma\left(\frac{\sigma-1}{\theta+\sigma-1}\right)\right)^{-\frac{\theta}{\theta+1}}}{\left(\sigma\left(\frac{\sigma-1}{\theta+\sigma-1}\right)\right)^{\frac{1}{\theta+1}} - \frac{1}{2}\left[\frac{\sigma-1-\theta}{\theta+\sigma-1}+\frac{\theta}{\theta+2\sigma-2}\right]\left(\sigma\left(\frac{\sigma-1}{\theta+\sigma-1}\right)\right)^{-\frac{\theta}{\theta+1}}} > 1$.

For the second part of the proposition, consider trade shares:

$$\lambda_{ij} = \frac{M_i\bar{\alpha}\int_{z^*}^{\infty} q_{ij}(z)p_{ij}(z)\theta z^{-\theta-1}dz}{w\tilde{L}+\Pi} = \kappa_6 P_{FT}^{1+\theta} Y_{FT}^{\frac{\theta-\sigma+1}{\sigma-1}} \tag{86}$$

where $\kappa_6^{FT} = M\bar{\alpha}\frac{S_{FT}}{\gamma}\sigma\frac{\sigma-1}{\theta+\sigma-1}\left(x\right)^{\frac{\theta}{1-\sigma}}$. Note that $\kappa_6^{A} = M\bar{\alpha}\frac{S_{A}}{\gamma}\sigma\frac{\sigma-1}{\theta+\sigma-1}\left(x\right)^{\frac{\theta}{1-\sigma}}$. Substitute for Y and rearrange for $j = i$:

$$P = \left(\frac{\lambda_{jj}}{\kappa_9}\right)^{\frac{1}{\theta+1}} \tag{87}$$

where $\kappa_9 \equiv \kappa_6\left(\chi\tilde{L}\right)^{\frac{\theta-\sigma+1}{1-\sigma}}$. Substitute this equation into welfare:

$$W_{FT} = \chi\tilde{L}\left(\frac{\lambda_{jj}}{\kappa_9}\right)^{-\frac{1}{\theta+1}} - \kappa_8\left(\frac{\lambda_{jj}}{\kappa_9}\right)^{\frac{\theta}{\theta+1}}\left(\chi\tilde{L}\right)^{\frac{\theta}{\sigma-1}} =$$

$$= \left(\chi\tilde{L}\right)^{\frac{-\sigma\theta+2(1+\theta-\sigma)}{(1-\sigma)(1+\theta)}}\left(\lambda_{jj}\right)^{-\frac{1}{\theta+1}}\left(\kappa_6\right)^{\frac{1}{\theta+1}} - \kappa_8\left(\lambda_{jj}\right)^{\frac{\theta}{\theta+1}}\left(\chi\tilde{L}\right)^{\frac{2\theta(\theta+1)-\sigma\theta}{(\sigma-1)(1+\theta)}}\left(\kappa_6\right)^{-\frac{\theta}{\theta+1}} \tag{88}$$

Similarly under autarky:

$$W_A = \frac{Y_A}{P_A} - \kappa_7 P_A^{\theta} Y_A^{\frac{\theta}{\sigma-1}} =$$

$$= \chi\tilde{L}\left(\frac{\lambda_{jj}}{\kappa_9}\right)^{-\frac{1}{\theta+1}} - \kappa_7\left(\frac{\lambda_{jj}}{\kappa_9}\right)^{\frac{\theta}{\theta+1}}\left(\chi\tilde{L}\right)^{\frac{\theta}{\sigma-1}} =$$

$$= \chi\tilde{L}\left(\lambda_{jj}\right)^{-\frac{1}{\theta+1}}\left(\kappa_6\left(\chi\tilde{L}\right)^{\frac{\theta-\sigma+1}{1-\sigma}}\right)^{\frac{1}{\theta+1}} - \kappa_7\left(\lambda_{jj}\right)^{\frac{\theta}{\theta+1}}\left(\kappa_6\left(\chi\tilde{L}\right)^{\frac{\theta-\sigma+1}{1-\sigma}}\right)^{-\frac{\theta}{\theta+1}}\left(\chi\tilde{L}\right)^{\frac{\theta}{\sigma-1}} =$$

$$= \left(\chi\tilde{L}\right)^{\frac{-\sigma\theta+2(1+\theta-\sigma)}{(1-\sigma)(1+\theta)}} \left(\lambda_{jj}\right)^{-\frac{1}{\theta+1}} \left(\kappa_6\right)^{\frac{1}{\theta+1}} - \kappa_7 \left(\lambda_{jj}\right)^{\frac{\theta}{\theta+1}} \left(\kappa_6\right)^{-\frac{\theta}{\theta+1}} \left(\chi\tilde{L}\right)^{\frac{2\theta(\theta+1)-\sigma\theta}{(\sigma-1)(1+\theta)}} \tag{89}$$

Given the symmetry, with free trade $\lambda_{jj} = \frac{1}{2}$ in both models. In autarky instead, $\lambda_{jj} = 1$. Therefore the change in trade shares is the same across models, and we can use the ACR formula to compare welfare gains:

$$\tilde{W}_{ACR} = \left(\lambda_{jj}\right)^{-\frac{1}{\theta}} - 1 = \left(\frac{1}{2}\right)^{-\frac{1}{\theta}} - 1 \tag{90}$$

In my model instead welfare gains are:

$$\hat{W} = \frac{\left(\chi\tilde{L}\right)^{\frac{-\sigma\theta+2(1+\theta-\sigma)}{(1-\sigma)(1+\theta)}} \left(\frac{1}{2}\right)^{-\frac{1}{\theta+1}} \left(\kappa_6\right)^{\frac{1}{\theta+1}} - \kappa_8 \left(\frac{1}{2}\right)^{\frac{\theta}{\theta+1}} \left(\chi\tilde{L}\right)^{\frac{2\theta(\theta+1)-\sigma\theta}{(\sigma-1)(1+\theta)}} \left(\kappa_6\right)^{-\frac{\theta}{\theta+1}}}{\left(\chi\tilde{L}\right)^{\frac{-\sigma\theta+2(1+\theta-\sigma)}{(1-\sigma)(1+\theta)}} \left(\kappa_6\right)^{\frac{1}{\theta+1}} - \kappa_7 \left(\kappa_6\right)^{-\frac{\theta}{\theta+1}} \left(\chi\tilde{L}\right)^{\frac{2\theta(\theta+1)-\sigma\theta}{(\sigma-1)(1+\theta)}}} - 1$$

The welfare gains are higher in my model than in ACR as long as:

$$\frac{\left(\chi\tilde{L}\right)^{\frac{-\sigma\theta+2(1+\theta-\sigma)}{(1-\sigma)(1+\theta)}} \left(\frac{1}{2}\right)^{-\frac{1}{\theta+1}} \left(\kappa_6^{FT}\right)^{\frac{1}{\theta+1}} - \kappa_8 \left(\frac{1}{2}\right)^{\frac{\theta}{\theta+1}} \left(\chi\tilde{L}\right)^{\frac{2\theta(\theta+1)-\sigma\theta}{(\sigma-1)(1+\theta)}} \left(\kappa_6^{FT}\right)^{-\frac{\theta}{\theta+1}}}{\left(\chi\tilde{L}\right)^{\frac{-\sigma\theta+2(1+\theta-\sigma)}{(1-\sigma)(1+\theta)}} \left(\kappa_6^{A}\right)^{\frac{1}{\theta+1}} - \kappa_7 \left(\kappa_6^{A}\right)^{-\frac{\theta}{\theta+1}} \left(\chi\tilde{L}\right)^{\frac{2\theta(\theta+1)-\sigma\theta}{(\sigma-1)(1+\theta)}}} > \left(\frac{1}{2}\right)^{-\frac{1}{\theta}}$$

$$\left(\chi\tilde{L}\right)^{\frac{-\sigma\theta+2(1+\theta-\sigma)}{(1-\sigma)(1+\theta)}} \left[\left(\kappa_6^{FT}\right)^{\frac{1}{\theta+1}} \left(\frac{1}{2}\right)^{-\frac{1}{\theta+1}} - \left(\kappa_6^{A}\right)^{\frac{1}{\theta+1}} \left(\frac{1}{2}\right)^{-\frac{1}{\theta}}\right] > \left(\chi\tilde{L}\right)^{\frac{2\theta(\theta+1)-\sigma\theta}{(\sigma-1)(1+\theta)}} \left[\left(\kappa_6^{FT}\right)^{-\frac{\theta}{\theta+1}} \kappa_8 \left(\frac{1}{2}\right)^{\frac{\theta}{\theta+1}} - \left(\kappa_6^{A}\right)\right.$$

$$\phi \left[\left(S_{FT}\right)^{\frac{1}{\theta+1}} \left(\frac{1}{2}\right)^{-\frac{1}{\theta+1}} - \left(S_A\right)^{\frac{1}{\theta+1}} \left(\frac{1}{2}\right)^{-\frac{1}{\theta}}\right] > \left[\left(S_{FT}\right)^{\frac{1}{\theta+1}} \left(\frac{1}{2}\right)^{\frac{\theta}{\theta+1}} - \left(S_A\right)^{\frac{1}{\theta+1}} \left(\frac{1}{2}\right)^{\frac{\theta-1}{\theta}}\right]$$

$$\phi \left(\frac{1}{(1+\rho)}\right)^{\frac{1}{\theta+1}} \left(\frac{1}{2}\right)^{-\frac{1}{\theta+1}} - \phi \left(\frac{1}{2}\right)^{-\frac{1}{\theta}} > \left(\frac{1}{(1+\rho)}\right)^{\frac{1}{\theta+1}} \left(\frac{1}{2}\right)^{\frac{\theta}{\theta+1}} - \left(\frac{1}{2}\right)^{\frac{\theta-1}{\theta}}$$

$$\frac{\left(\frac{1}{2}\right) \left[\phi - \left(\frac{1}{2}\right)^{\frac{1}{\theta}}\right]^{\theta+1}}{\left[\phi - \left(\frac{1}{2}\right)\right]^{\theta+1}} - 1 > \rho \tag{91}$$

where $\phi = \frac{\left(\chi\tilde{L}\right)^{2\left(1+\frac{\theta}{(1-\sigma)}\right)} \left(\sigma \frac{\sigma-1}{\theta+\sigma-1}\right)}{\left[\frac{\sigma-1-\theta}{\theta+\sigma-1} + \frac{\theta}{\theta+2\sigma-2}\right]}$. ■

7.4.7 Effect of trade liberalization on number of varieties

The number of varieties sold from i to j is:

$$V_{ij} = M_i Pr\left\{n_{ij}(z) > 0\right\} = M_i \int_{z^*}^{\infty} n_{ij}(z)\theta z^{-\theta-1} dz$$

With free trade and two symmetric countries, there exists a unique entry cutoff. Then:

$$V_{FT} = M \int_{z^*}^{\infty} \frac{S_{FT}}{\gamma} \frac{\left(1 - \left(\frac{z^*}{z}\right)^{\sigma-1}\right)}{\frac{1}{P}\left(\frac{\sigma}{\sigma-1}\frac{1}{z}\right)^{1-\sigma} \frac{Y}{P^{1-\sigma}\sigma}} \theta z^{-\theta-1} dz =$$

$$= M \frac{1}{\gamma} \frac{S_{FT}}{\frac{1}{P}\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} \frac{Y}{P^{1-\sigma}\sigma}} \int_{z^*}^{\infty} \left(1 - \left(\frac{z^*}{z}\right)^{\sigma-1}\right) \theta z^{-\theta-\sigma} dz =$$

$$= M \frac{1}{\gamma} \frac{S_{FT}}{\frac{1}{P}\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} \frac{Y}{P^{1-\sigma}\sigma}} \left(z^*\right)^{-\theta-\sigma+1} \left(\frac{\theta}{\theta+\sigma-1} - \frac{\theta}{-2+\theta+2\sigma}\right) =$$

$$= M \frac{1}{\gamma} S_{FT} P^{1+\theta} \left(Y\right)^{\frac{\theta}{\sigma-1}} \left(\frac{\theta}{\theta+\sigma-1} - \frac{\theta}{-2+\theta+2\sigma}\right) \left(\frac{f}{\bar{\alpha}}\right)^{\frac{-\theta-\sigma+1}{\sigma-1}} \left(\left(\frac{\sigma}{\sigma-1}\right)^{\sigma-1} \sigma\right)^{\frac{-\theta}{\sigma-1}}$$

Given symmetry, the total number of varieties available in the home country is $2V_{FT}$. Under autarky the number of varieties is:

$$V_A = M \frac{1}{\gamma} S_A P_A^{1+\theta} \left(Y_A\right)^{\frac{\theta}{\sigma-1}} \left(\frac{\theta}{\theta+\sigma-1} - \frac{\theta}{-2+\theta+2\sigma}\right) \left(\frac{f}{\bar{\alpha}}\right)^{\frac{-\theta-\sigma+1}{\sigma-1}} \left(\left(\frac{\sigma}{\sigma-1}\right)^{\sigma-1} \sigma\right)^{\frac{-\theta}{\sigma-1}}$$

The change in the number of varieties is:

$$\hat{V} = \frac{2V_{FT}}{V_A} - 1 = \frac{2 S_{FT} P^{1+\theta} \left(Y\right)^{\frac{\theta}{\sigma-1}}}{S_A P_A^{1+\theta} \left(Y_A\right)^{\frac{\theta}{\sigma-1}}} - 1 =$$

$$= \frac{2\kappa_A}{(1+\rho)\kappa_{FT}} - 1 = \frac{2}{2} - 1 = 0$$

Therefore the total number of varieties available does not change. This is a result of the Pareto assumption.

7.4.8 Decomposition of welfare gains from trade

The welfare gains from trade for workers are simply given by the change in the real wage:

$$\hat{W}_{\tilde{L}} = \frac{\frac{1}{P_{FT}}}{\frac{1}{P_A}} - 1 = \frac{P_A}{P_{FT}} - 1 = \frac{\left(\chi \tilde{L}\right)^{\frac{\theta+1-\sigma}{(1-\sigma)(1+\theta)}} (\kappa_2)^{-\frac{1}{\theta+1}}}{\left(\chi \tilde{L}\right)^{\frac{\theta+1-\sigma}{(1-\sigma)(1+\theta)}} (\kappa_5)^{-\frac{1}{\theta+1}}} - 1 =$$

$$= \frac{\left(\bar{\alpha} M \frac{S_A \sigma}{\gamma} (x)^{\frac{\theta}{1-\sigma}} \left(\frac{\sigma-1}{\theta+\sigma-1}\right)\right)^{-\frac{1}{\theta+1}}}{\left(\bar{\alpha} 2M \frac{S_{FT} \sigma}{\gamma} (x)^{\frac{\theta}{1-\sigma}} \left(\frac{\sigma-1}{\theta+\sigma-1}\right)\right)^{-\frac{1}{\theta+1}}} - 1 =$$

$$= \left(\frac{1+\rho}{2}\right)^{-\frac{1}{\theta+1}} - 1$$

Instead, the welfare gains for the entrepreneurs are:

$$\hat{W}_M = \frac{\Pi_{FT}/P_{FT} - R_{FT}}{\Pi_A/P_A - R_A} - 1 =$$

$$= \frac{\frac{\left[\frac{\sigma-1-\theta}{\theta+\sigma-1} + \frac{\theta}{\theta+2\sigma-2}\right]}{\sigma\left(\frac{\sigma-1}{\theta+\sigma-1}\right)} (\kappa_5)^{\frac{1}{\theta+1}} - \kappa_8 (\kappa_5)^{-\frac{\theta}{\theta+1}}}{\frac{\left[\frac{\sigma-1-\theta}{\theta+\sigma-1} + \frac{\theta}{\theta+2\sigma-2}\right]}{\sigma\left(\frac{\sigma-1}{\theta+\sigma-1}\right)} (\kappa_2)^{\frac{1}{\theta+1}} - \kappa_7 (\kappa_2)^{-\frac{\theta}{\theta+1}}} - 1 =$$

$$= \left(\frac{S_{FT}}{S_A}\right)^{\frac{1}{\theta+1}} \left((2)^{\frac{\theta+2}{\theta+1}} - (2)^{\frac{1}{\theta+1}}\right) - 1 =$$

$$= \left(\frac{1+\rho}{2}\right)^{-\frac{1}{\theta+1}} - 1$$

7.4.9 Covariance estimation

I first prove that, if the shocks are i.i.d. over time and their mean is zero, computing the covariance stacking together all observations for products p and time t is equivalent to computing a covariance across products for each year t and taking the average across the years.

To save notation, define $X \equiv \Delta\tilde{\alpha}_x$ and $Y \equiv \Delta\tilde{\alpha}_y$, where x and y are any two destinations. The covariance between X and Y, computed stacking together the observed $\Delta^t \tilde{\alpha}_{xp}$, is:

$$Cov(X,Y) = \frac{1}{T \cdot P} \sum_{k=1}^{T \cdot P} (y_k - \bar{y})(x_k - \bar{x}) \tag{92}$$

where x_k (y_k) is the observed change in the log of the shock in destination x (y) for k, where k is a pair of product p and year t. Since $\bar{x} \equiv E[\Delta\tilde{\alpha}_x] = 0$ and $\bar{y} \equiv E[\Delta\tilde{\alpha}_p] = 0$, the above becomes:

$$Cov(X,Y) = \frac{1}{T \cdot P} \sum_{k=1}^{T \cdot P} y_k x_k \tag{93}$$

If instead I compute the covariance for each year, this equals:

$$Cov(X^t, Y^t) = \frac{1}{P} \sum_{p=1}^{P} y_p^t x_p^t \tag{94}$$

where x_p^t (y_p^t) is the observed change in the log of the shock in destination x (y) in year t and product p. The average across years of this covariance is simply:

$$\frac{1}{T} \sum_{t=1}^{T} Cov(X^t, Y^t) = \frac{1}{T} \sum_{t=1}^{T} \frac{1}{P} \sum_{p=1}^{P} y_p^t x_p^t =$$

$$= \frac{1}{T \cdot P} \sum_{t=1}^{T} \sum_{p=1}^{P} y_p^t x_p^t = \frac{1}{T \cdot P} \sum_{k=1}^{T \cdot P} y_k x_k \tag{95}$$

by the associative property. Therefore, equation 93 is equivalent to equation 95.

Given an estimate of the covariance matrix of the *log-changes* of the shocks, I first recover the covariance matrix of the log of the shocks, using the fact that, for all j and i:

$$\begin{aligned} Cov\left(\Delta\tilde{\alpha}_j, \Delta\tilde{\alpha}_i\right) &= Cov\left(\tilde{\alpha}_{jt} - \tilde{\alpha}_{jt-1}, \tilde{\alpha}_{it} - \tilde{\alpha}_{it-1}\right) \\ &= Cov\left(\tilde{\alpha}_{jt}, \tilde{\alpha}_{it}\right) - Cov\left(\tilde{\alpha}_{jt}, \tilde{\alpha}_{it-1}\right) - Cov\left(\tilde{\alpha}_{jt-1}, \tilde{\alpha}_{it}\right) + Cov\left(\tilde{\alpha}_{jt-1}, \tilde{\alpha}_{it-1}\right) \\ &= 2Cov\left(\tilde{\alpha}_j, \tilde{\alpha}_i\right) \end{aligned}$$

where the last inequality is implied by the i.i.d. assumption, i.e. $Cov\left(\tilde{\alpha}_{jt-1}, \tilde{\alpha}_{it}\right) = 0$ for all i and j.

Given a covariance matrix of the *log* of the shocks, I can recover the covariance matrix of the *level* of the shocks as follows. For any pair of destinations $X \equiv \tilde{\alpha}_x$ and $Y \equiv \tilde{\alpha}_y$, the pairwise covariance is:

$$Cov\left(X,Y\right)=Cov\left(e^{\tilde{X}},e^{\tilde{Y}}\right)=E\left[e^{\tilde{X}}e^{\tilde{Y}}\right]-E[e^{\tilde{X}}]E[e^{\tilde{Y}}]=$$

$$=E\left[e^{\tilde{Z}}\right]-E[e^{\tilde{X}}]E[e^{\tilde{Y}}]$$

where $\tilde{Z}=\tilde{X}+\tilde{Y}$ is the sum of two normally distributed variables, and has mean $E[\tilde{Z}]=E[\tilde{X}]+E[\tilde{Y}]=0$ and variance $Var(\tilde{Z})=Var(\tilde{X})+Var(\tilde{Y})+2Cov(\tilde{X},\tilde{Y})$. Note that I have already obtained $Var(\tilde{X})$, $Var(\tilde{Y})$ and $Cov(\tilde{X},\tilde{Y})$ in the previous step. Then, by the moment generating function of the normal distribution:

$$E\left[e^{j}\right]=e^{E[j]+\frac{1}{2}Var(j)}$$

for $j=\tilde{Z},\tilde{X},\tilde{Y}$. Plugging these back I can derive the covariance of the level of the shocks:

$$Cov\left(X,Y\right)=e^{\frac{1}{2}Var(\tilde{Z})}-e^{\frac{1}{2}Var(\tilde{X})+\frac{1}{2}Var(\tilde{Y})}=$$

$$=e^{\frac{1}{2}\left(Var(\tilde{X})+Var(\tilde{Y})+2Cov(\tilde{X},\tilde{Y})\right)}-e^{\frac{1}{2}\left(Var(\tilde{X})+Var(\tilde{Y})\right)}$$