Optimal Contracting and the Organization of Knowledge

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Matching problems with expertise

- A key problem: using available knowledge optimally
- Specifically: superior knowledge of experts must be conserved for the 'right' questions
- A solution: a 'knowledge-based hierarchy' which allows experts to leverage their expertise by having less expensive workers deal with routine tasks (Garicano, 2000, Garicano and Rossi Hansberg, 2006)
  - Firms and other organizations often structure these hierarchies by placing agents in different positions according to their expertise
- But: informational asymmetries

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Consequences:

- Too little or too much trade
- Inefficient matching between problems and experts

Our question

How should markets / organizations mediate hierarchical (vertical) specialization in the presence of incomplete information?

- Are there market arrangements / contracts that support an efficient allocation of talent despite these informational asymmetries?
- If so, what form must such contracts take?

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- One sided asymmetries can be circumvented by having the appropriate market arrangement
- Show limits of ex post (spot) contracting – whenever consultant time is scarce, cannot achieve first best
  - excess entry of intermediate agents (lemons) who are neither good enough to help nor offer valuable enough problems
- Ex ante "firm-like" contracts uniquely implement an efficient allocation of talent
- The optimal firm-like contract involves an endogenous principal who is the full residual claimant and pays a bonus
  - An Alchian Demsetz flavor to results (endogenous principal), but originating from screening rather than monitoring

Implications for Organizations

Results in line with the organization of firms and the structure of compensation in the knowledge intensive sector:

- Firms in that sector are structured as hierarchies, with fewer partners than associates
- Partners retain the entire equity in the firm and earn all residual income
- High-powered incentives for associates, with larger bonuses for associates with higher skills

Related work

- Moral hazard issues involved in the provision of expert services, i.e. over- and under-treatment
  - Demski and Sappington (1987), Wolisnky (1993), Pesendorfer and Wolinsky (2003), Taylor (1995), Garicano and Santos (2004) (moral hazard and one-sided adverse selection). Unlike literature, we study double-sided informational asymmetries
- Trade with bilateral asymmetric information
  - Myerson and Satterthwaite (1983), Lu and Robert, 2001) – matching is not relevant. Gale (2001), double sided matching, but no endogenous participation in market and no heterogeneous outside option
- Management-worker sorting literature
  - Garicano and Rossi-Hansberg (2004 and 2006), but here adverse selection. Worker/manager sorting models have been studied and generalized by Eeckhout and Kircher (2011)
- Crowdsourcing/optimal allocation of problems to workers
  - Acemoglu, Mostagir, and Ozdaglar (2014), exogenous principle and no knowledge hierarchies

The model (1): Agents and knowledge

- Continuum of risk-neutral agents who differ in their knowledge $z \in [0, 1]$
- Knowledge levels are private information and distributed according to pdf $f(z) > 0$
- Each agent endowed with 1 unit of time
  - Agents sort into two occupations: producers (specialize in production) and consultants (specialize in offering advice)

The model (2): Occupations and production

Producers spend their unit of time generating a productive opportunity

- Each opportunity has a random (unknown) level of difficulty $x \in [0, 1]$, without loss distributed uniformly
- Absent advice from a consultant, producer z overcomes a difficulty of level x and succeeds if and only if $z \geq x$
- If producer fails, she may or may not seek advice

The model (2'): Occupations and production

Consultants spend their unit of time helping producers with unsolved problems

- Helping each producer consumes h ("help") units of time, with $h \in (0, 1)$
- Receiving advice consumes no additional producer time
- With advice from consultant of type m, producer solves problem x if and only if $m \geq x$
- A consultant can help $\frac{1}{h}$ producers

A solved problem is worth \$1, an unsolved problem is worth \$0

The model (3): Teams and expected output

Ex-post team is composed of one consultant and $\frac{1}{h}$ producers with unsolved problems

Ex-post expected output (when consultant has type m and all producers have type z):

$$\pi(m,z) = \frac{1}{h}\Pr[x < m | x > z] = \frac{1}{h}\frac{m-z}{1-z}$$

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- π is decreasing in z: higher skilled producers pose more difficult problems
- π is supermodular ($\pi_{mz} > 0$): a higher consultant skill is more valuable when applied to harder problems

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- Π is increasing in z: more talented producers ask for less help and team can be larger
- Π is supermodular ($\Pi_{mz} > 0$): a higher consultant skill is more valuable when applied to producers who ask less frequent, harder problems

The model (4): Bilateral contracts

Ex-post (i.e. spot) contract

- Consultant pays each producer a fixed fee w
- Consultant keeps share $\alpha \in [0, 1]$ of output

Ex-ante contract

- Consultant pays each producer a fixed fee ω up front
- If producer solves problem on her own, producer keeps share $\beta \in [0, 1]$ of output
  ($1 - \beta$ may be viewed as a "tax" imposed on producer)
- If producer asks for advice, consultant keeps share $\alpha \in [0, 1]$ of output

Ex-post contract is special case of ex-ante contract with restriction $1 - \beta = \omega = w$

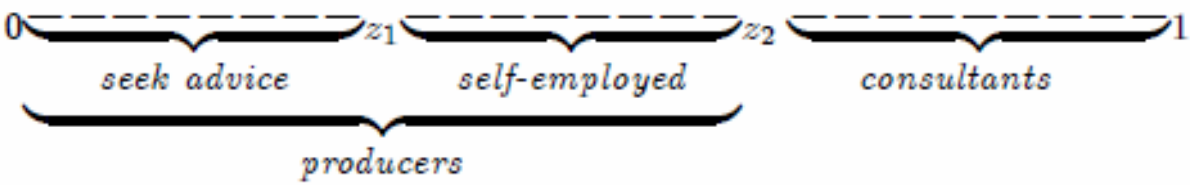
Roadmap

1. First Best / Full Information Benchmark
2. Two-sided Adverse Selection Case
3. Extensions

Planner's problem (1): First best

Remark. In the (a.e. unique) first-best allocation:

1. Agents $z \in [0, z_1]$ are "matched" producers
2. Agents $z \in (z_1, z_2)$ are "self-employed" producers
3. Agents $m \in [z_2, 1]$ are consultants
4. Matching is positive assortative



Planner's problem (1'): First best

Let $M(z)$ be the consultant assigned to producer z

Let $Z(m)$ be the producers assigned to consultant m

Time constraint:

$$h \int_{z}^{z'} (1-t) dF(t) = \int_{M(z)}^{M(z')} dF(t) \text{ for all } z, z' \leq z_1$$

Solving for the first best (1)

$$\max_{z_1} \int_0^{z_1} M(z; z_1) dF(z) + \int_{z_1}^{z_2} z dF(z)$$

$$s.t.$$

$$z_1 \leq \underbrace{M(0; z_1)}_{z_2}$$

Solving for the first best (2)

Lemma 0. The first-best allocation takes one of two forms, depending on value of h:

1. $z_1 < z_2$ and
$$\frac{1}{h} - z_2 = \int_{z_2}^{1} n\left(Z(m)\right) dm.$$

2. $z_1 = z_2$ and
$$\frac{1}{h} - z_2 \geq \int_{z_2}^{1} n\left(Z(m)\right) dm.$$

In what follows, unless otherwise noted, we focus on case 1

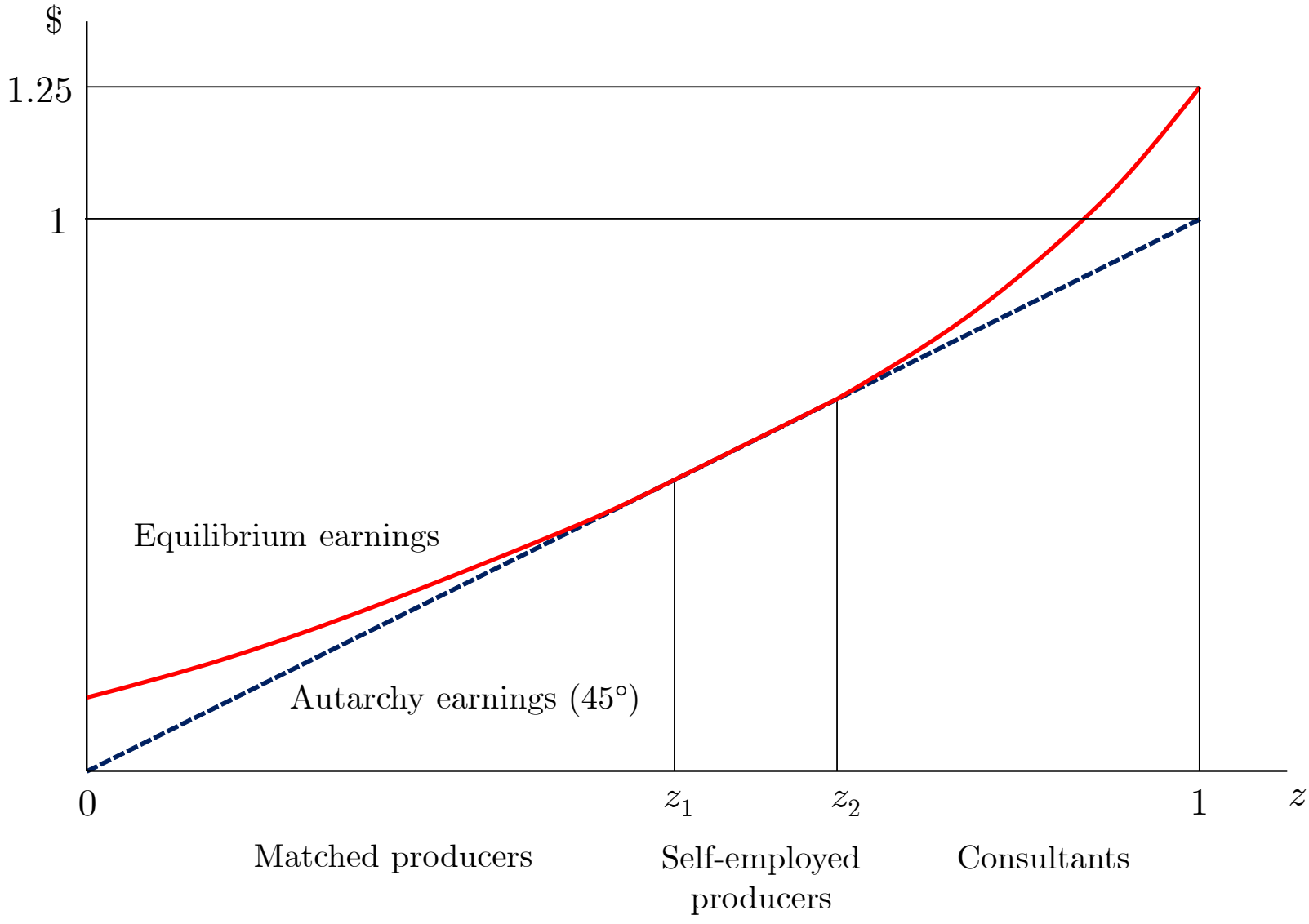
First best: observation

The *FOC* with respect to z_1 is:

$$\frac{1}{h} - z_2 = \int_{z_2}^{1} n(Z(m))dm$$

- Consider, as a benchmark, a full-information competitive equilibrium in which consultants earn a fixed wage $W(m)$
- The L.H.S. is the difference in equilibrium payoffs between the most skilled and least skilled consultant
- The integrand on the R.H.S. is $W'(m)$, the full marginal contribution of a consultant's talent to the output of her team

Full-information competitive equilibrium (uniform F)



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- The decentralization with a consulting market is robust to private information on the difficulty of the problems.
- A referral market where there is a price to buy unsolved opportunities is another possible decentralization. This decentralization is robust to private information on the quality of the consultant.
- Both arrangements make the party with private information the full residual claimant.
- Interestingly, although the "intrinsic" trade is the same, markets could exhibit very different arrangements if their information structures differ.

Two-sided adverse selection: A key building block.

For both ex-ante or ex-post contracts, when there are

self-employed all consultants must be full residual claimants to the output for the problems they must handle.

Lemma 2. If $z_1 < z_2$ a family of contracts implements the first best **only** if $\alpha(z) = 1$ for all producers z.

A key building block (cont.)

Proof:

(1) Planner's FOC:

$$\frac{1}{h} - z_2 = \int_{z_2}^{1} \underbrace{n(Z(m))}_{\text{Social marginal value of knowledge}} \, dm$$

(2) Consultant incentive constraint:

$$\frac{1}{h} - z_2 = \int_{z_2}^{1} \underbrace{\alpha(Z(m))n(Z(m))}_{\text{Private marginal value of knowledge}} \, dm$$

It follows that:

$$\alpha(z) = 1 \text{ for all } z$$

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- With $\alpha(z) = 1$ it must follow that separation should come from different "prices" $w(z)$.
- But, there cannot be separation on $w(z)$ alone. All producers would claim the z that maximizes $w(z)$.
- In equilibrium there is too much trade. In the ex-post market the agents that should be self-employed cannot be kept out of the market.

How can ex-ante contracting help?

- Suppose we could observe the z of the producers.
- With $\alpha(z) = 1$ we can solve the truthful revelation problem for the consultants if they face the correct schedule of expected payments to the producers they choose to match with.
- With ex-post contracts this cannot be done since there is only one instrument $w(z)$.
- With ex-ante contracts we have an additional key instrument, the share of the output the producer retains when he solves the problem without the consultants help. This is crucial to keep the self-employed from wanting to participate in the market.

Two-sided adverse selection

We seek a family of contracts $\langle \omega(z), \alpha(z), \beta(z) \rangle_{z \in [0, z_1]}$ that implements the first best

Payoffs are as follows

Self-employed producer z: obtains payoff z

Matched producer z:

$$\begin{aligned} R(z) &= \omega(z) + z\beta(z) + (1-z)\left[\frac{M(z)-z}{1-z}\right](1-\alpha(z)) \\ R'(z) &= \beta(z) - [1-\alpha(z)] \end{aligned}$$

Consultant m who matches with producers $z = Z(m)$:

$$\begin{aligned} S(m) &= n(z)\left[-\omega(z) + z\left(1-\beta(z)\right) + (1-z)\frac{m-z}{1-z}\alpha(z)\right] \\ S'(m) &= \alpha(z)n(z) = \alpha(z)\Pi_m(m,z) \end{aligned}$$

Incentive constraints

Using standard envelope arguments:

Lemma 1. A family of contracts $\langle \omega(z), \alpha(z), \beta(z) \rangle$ implements the first best if and only if:

(i) Incentive constraints within each occupation are met

$$R(z) = R(0) + \int_0^z \underbrace{[\beta(z) - [1 - \alpha(z)]]}_{R'(t)} dt \qquad (E_R)$$

$$\beta(z) - [1 - \alpha(z)] \text{ is nondecreasing in } z \qquad (M_R)$$

$$S(m) = z_2 + \int_{z_2}^m \underbrace{\alpha(Z(t))n(Z(t))}_{S'(t)} dt \qquad (E_S)$$

$$\alpha(Z(m))n(Z(m)) \text{ is nondecreasing in } m \qquad (M_S)$$

Incentive constraints (cont.)

(ii) Occupational choice constraints:

$$R(z_1) = z_1, \ S(z_2) = z_2 \qquad (PC)$$

$$R'(z_1-) \leq 1, \ S'(z_2+) \geq 1 \qquad (DD)$$

Ex-ante contracts achieve the first best

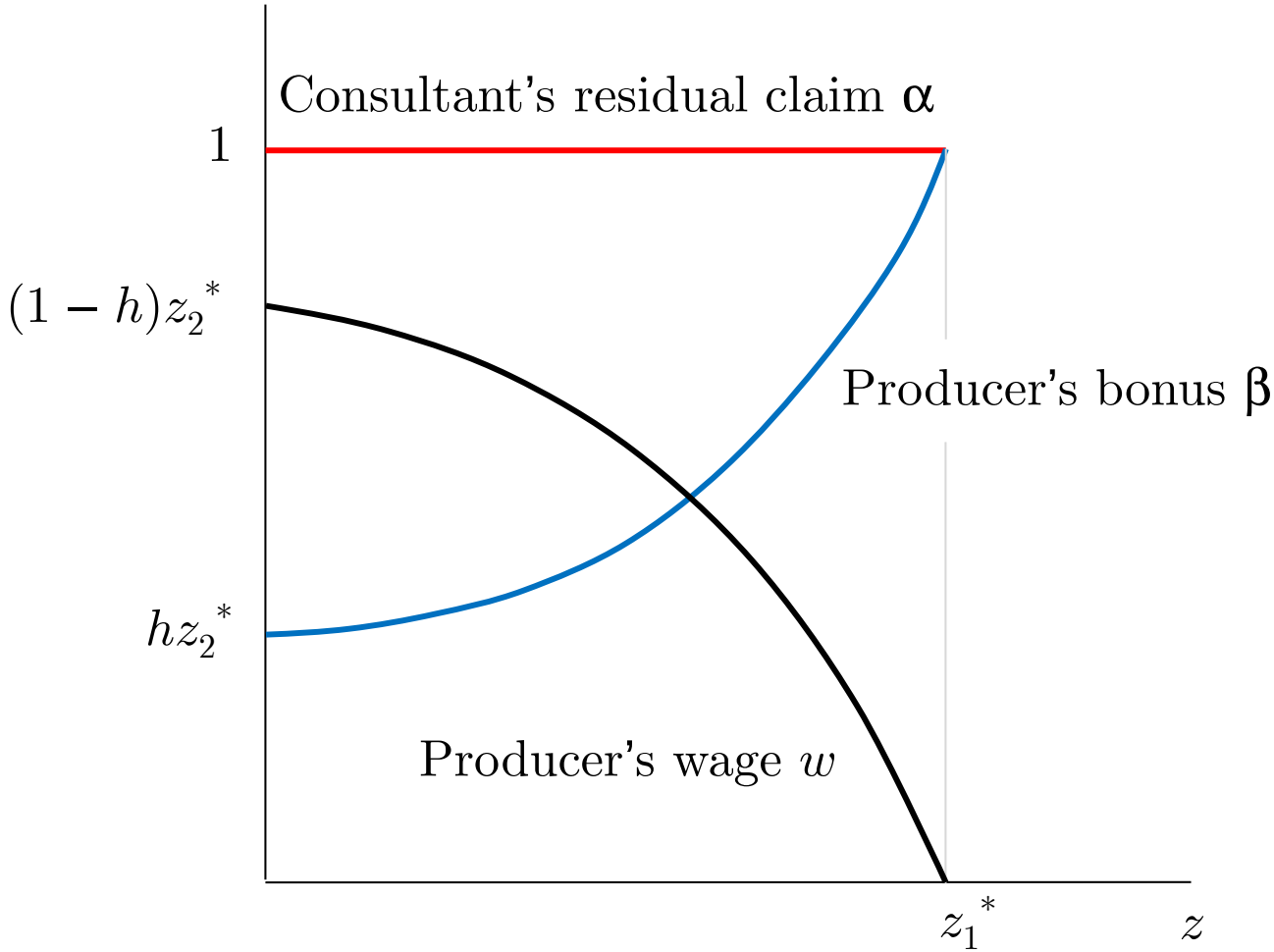
Theorem

The following ex-ante contract implements the first best allocation:

1. $\alpha(z) = 1$
2. $\underbrace{\beta(z)}_{\text{bonus}} = hS(M(z))$
3. $\underbrace{\omega(z)}_{\text{wage}} = M(z) - hS(M(z))$

When $z_1 < z_2$ this is the unique efficient contract.

Ex-ante contracts achieve the first best



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- Self-employed producers do not become consultants because although they would have more problems they have to pay for them too much given their probability of succeeding.

No self-employment case. Ex-post

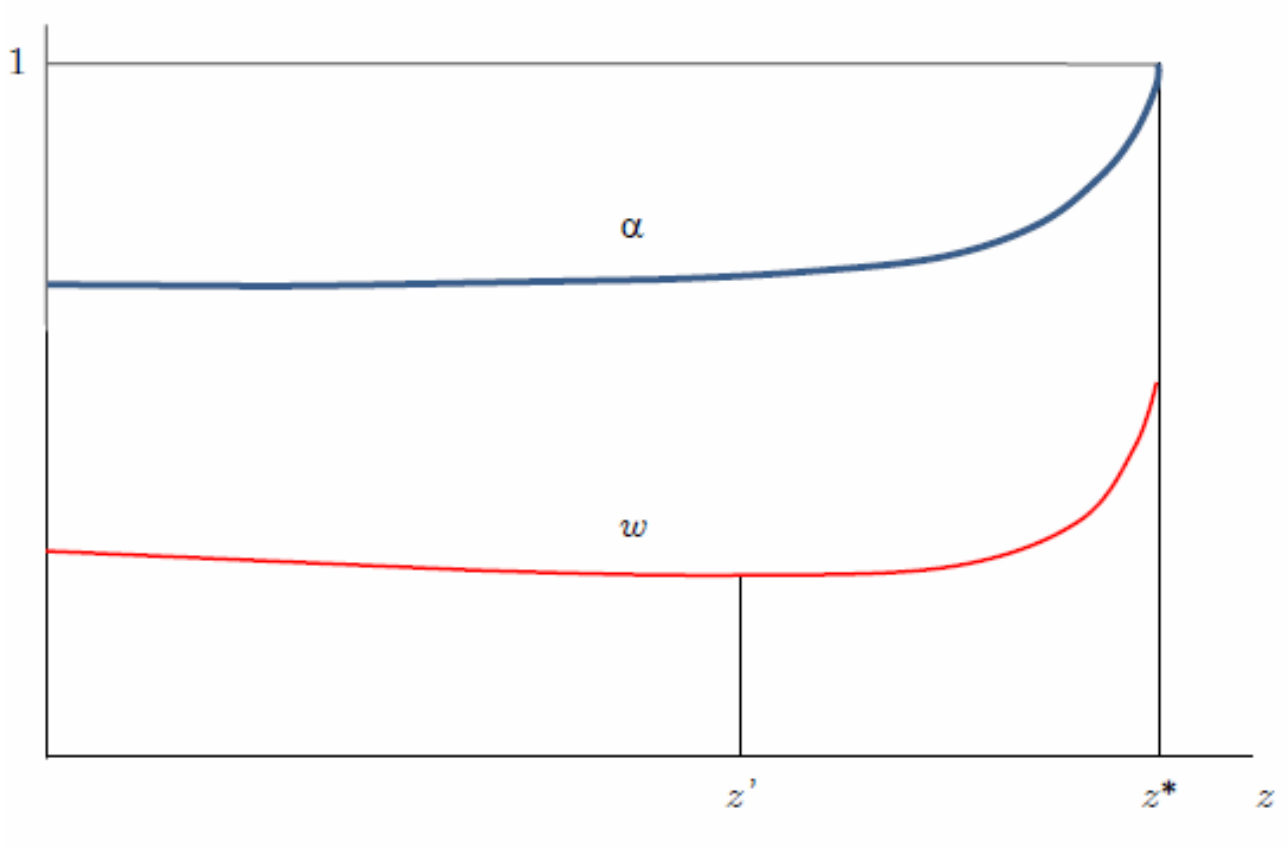
Ex-post contracting may or may not implement the first best, depending on parameter values

Proposition

When F is uniform, the following ex-post contract uniquely implements the first best:

1. $\alpha(z) = \frac{h(1-z)^2}{h(1-z)^2+(1-m(z))}$, which is increasing in z with $\alpha(z_1) = 1$.

2. $w(z) = h\left[\alpha(z)\pi(M(z), z) - \int_{z_1^*}^{M(z)} \alpha(Z(t))n(Z(t))\,dt - C\right]$, which is u-shaped.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Extensions

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2. Contracting absent contingent pay for consultants.

Result: Market for knowledge breaks down completely, regardless of potential gains from trade

Conclusion

- When efficiency calls for self-employed agents, spot contracting is inefficient
- Firm-like contracts implement an efficient allocation of talent – uniquely so when efficiency calls for self-employed agents
- Suggest rationale for organization of firms and structure of compensation in the knowledge intensive sector:
  - Firms in that sector are structured as knowledge-based hierarchies, with fewer partners than associates
  - Partners are more knowledgeable than associates
  - Partners retain the entire equity in the firm and earn the residual income
- Ex-ante contracting/bonus structure deters lemons from entering the market