Robust Bayes Inference for Non-identified SVARs

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Capri, 26/6/14

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  - Frequentist inference in partially identified models unappealing because: 1) relies on asymptotic approximation; 2) computationally cumbersome; 3) validity not proven for general equality and inequality restrictions
  - Bayesian inference attractive but sensitive to prior choice: effect does not disappear asymptotically under partial identification (Poirier, 1998; Baumeister and Hamilton, 2013)

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  - Limit attention to convex sets because of computational ease and asymptotic properties
  - Give easy to verify conditions that guarantee convexity
- Use robust Bayesian method that overcomes sensitivity to prior choice
  - Show that our estimated set converges asymptotically to true identified set, unlike current Bayesian methods

Partially identified SVARs

- n-variable SVAR with orthogonal shocks:

$$A_0 y_t = a + \sum_{j=1}^{p} A_j y_{t-j} + \epsilon_t, \quad \epsilon_t \sim \mathcal{N}(0, I_n),$$

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- Reduced-form VAR

$$y_t = b + \sum_{j=1}^{p} B_j y_{t-j} + u_t, \quad u_t \sim \mathcal{N}(0, \Sigma)$$

where $\Sigma = A_0^{-1} \left(A_0^{-1}\right)'$

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- Reduced-form parameters are $\phi = (B, \Sigma)$

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- Impulse-response function at horizon h is $IR^h = C_h(B) A_0^{-1}$

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- Following analysis in Uhlig (2005), without identifying restrictions, knowledge of ϕ corresponds to a set of observationally equivalent $A_0's$

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- This corresponds to a set of observationally equivalent impulse-responses

$$IR^h \equiv C_h(B)\Sigma_{tr}Q$$

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  2. Estimate Bayesian reduced form VAR and obtain posterior for ϕ
  3. Draw ϕ's from the posterior and Q's from the **uniform prior**
  4. Retain the draws of $IR^h = C_h(B)\Sigma_{tr}Q$ that satisfy the sign restrictions (+ sign normalizations)

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- Challenge:
    - Difficult to specify plausible informative prior for Q
    - Uninformative (uniform) prior for Q can give informative prior for $IR^h \longrightarrow$ can affect posterior inference for impulse-responses

Illustration of prior sensitivity

- SVAR from Aruoba & Schorfheide (2011) with sign restrictions.

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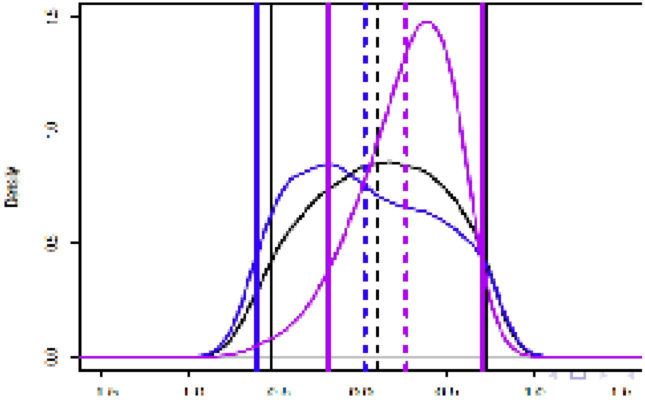
- SVAR from Aruoba & Schorfheide (2011) with sign restrictions.
- 3 different priors for Q (uniform for each shock but different correlation among shocks)



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- One could choose prior for Q that reflects true "agnostic" belief about $IR^h \longrightarrow$ might be doable in special cases but challenging in general
- We take different approach: from agnostic belief to "ambiguous" belief $\longrightarrow$ multiple prior for non-identified aspects of the model (here Q)
- Robust Bayes approach to inference (Berger & Berliner, 1986, Wasserman, 1990)

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- $\rightarrow$ this makes the inference robust to the prior for drawing $Q's$
- $\rightarrow$ our estimated set ("posterior bounds") converges asymptotically to true identified set

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$$F(\phi, Q) \equiv \begin{pmatrix} F_1(\phi)q_1 \\ \vdots \\ F_j(\phi)q_j \\ \vdots \end{pmatrix} = 0, \quad F_j(\phi)\colon\ f_j \times n.$$

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- Without loss of generality, order the variables in such way that $f_1 \geq f_2 \geq \cdots \geq f_n \geq 0$.

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- $IR^h[i,j]=0$. Since $IR^h = C_h(B)\Sigma_{tr}Q$, restriction is $c_{ih}\left(\phi\right)' q_j = 0$
- $IR^\infty[i,j]=0$. Since $IR^\infty = (I-\sum_{l=1}^{p} B_l)^{-1}\Sigma_{tr}Q$, restriction is $c_{i\infty}\left(\phi\right)' q_j = 0$

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- Stack the sign restrictions over multiple shocks, and represent them by $S(\phi, Q) \geq 0$.

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- The identified set for r :

$$IS_r = \{r(\phi, Q) : Q \in \mathcal{Q}(\phi|F,S)\}$$

Results on convexity of identified set

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3. Sign restrictions on multiple shocks can yield non-convex identified sets

Constructing the agnostic prior class

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- Choose single prior for ϕ but multiple prior for $Q|\phi$
- The posterior of the impulse response r given the data Y can be written as

$$\pi_{r|Y}(A) = \int_{\phi} \pi_{r|\phi}(A) d\pi_{\phi|Y}$$

where $\pi_{r|\phi}(A) = \pi_{Q|\phi}(r(\phi, Q) \in A)$

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- Consider the class of all conditional priors $\pi_{Q|\phi}$ that assign probability one to the set $\mathcal{Q}(\phi|F,S)$:

$$\Pi_{Q|\phi} = \left\{ \pi_{Q|\phi} : \pi_{Q|\phi} \left(\mathcal{Q}(\phi|F,S) \right) = 1 \text{ for all } \phi \right\}$$

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- Then the agnostic class of joint priors is

$$\Pi_{\phi Q} = \{ \pi_{Q|\phi} \pi_\phi : \pi_{Q|\phi} \in \Pi_{Q|\phi} \}$$

Posterior inference with the agnostic prior class

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Posterior bounds in theory

- For a convex identified set $IS_r = [l(\phi), u(\phi)]$, we propose reporting the posterior mean bounds

$$\left[E_{\phi|Y} l(\phi), E_{\phi|Y} u(\phi)\right]$$

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- Restricting attention to convex sets makes computation of posterior mean bounds simple, once we show how to draw $Q's$ that are compatible with equality and inequality restrictions

Posterior bounds in practice

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- If $\mathcal{Q}(\phi|F,S)$ is nonempty, solve

$$\begin{aligned} & max/min_Q r(\phi, Q) \\ \text{s.t. } & Q'Q = I_n, \; F(\phi, Q) = 0, \; S(\phi, Q) \geq 0 \end{aligned}$$

to obtain $l(\phi)$ and $u(\phi)$

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  to obtain $l(\phi)$ and $u(\phi)$
- Repeat Step 2 and 3 to obtain many draws of $(l(\phi), u(\phi))$
- Report means of draws $\left[\widehat{E}_{\phi|Y} l(\phi), \widehat{E}_{\phi|Y} u(\phi)\right]$

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- **Posterior robustified credible region** with credibility α

$$C_{\alpha *} \equiv argmin_C \text{length}(C), \text{ s.t. } \inf_{\pi_{\phi Q|Y} \in \Pi_{\phi Q|Y}} \pi_{r|Y}(C) \geq \alpha$$

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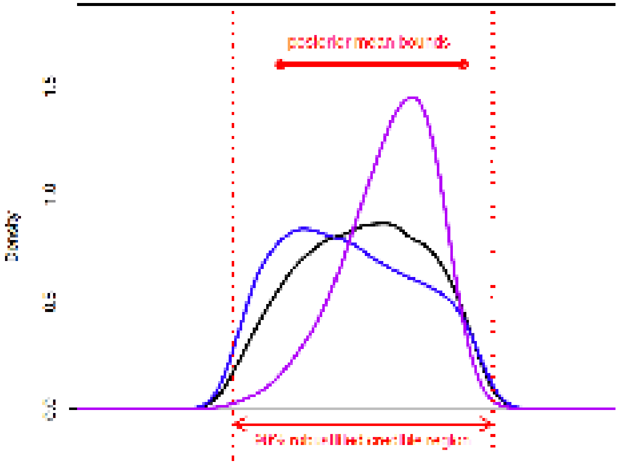
- In words: "the shortest interval to which all the posteriors assign probability at least α"

The output of our procedure

- Consider again Aruoba Schorfheide (2011) example of SVAR with sign restrictions



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- Similar diagnostic tool not available in frequentist inference

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- Multiple prior ensure consistency, whereas estimated identified set using single prior is not consistent in the presence of partial identification

Real data example

- Aruoba & Schorfheide (2011) SVAR

$$A_0 \begin{pmatrix} i_t \\ \Delta y_t \\ \pi_t \\ m_t \end{pmatrix} = c + A(L) \begin{pmatrix} i_t \\ \Delta y_t \\ \pi_t \\ m_t \end{pmatrix} + \begin{pmatrix} \epsilon_{it} \\ \epsilon_{yt} \\ \epsilon_{\pi t} \\ \epsilon_{mt} \end{pmatrix}$$

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- Compare to single prior approach (uniform prior)

Results

- Consider different combinations of restrictions

Table 1: Models and Posterior Plausibility

Restrictions	I	II	III	IV	V	VI	VII
(i) $a_{12} = 0$	-	x	-	-	x	x	-
(ii) $IR^0(y, i) = 0$	-	-	x	-	x	-	x
(iii) $IR^\infty(y, i) = 0$	-	-	-	x	-	x	x
(iv) sign restrict	x	x	x	x	x	x	x
$\Pr\left(IS_r \text{ non-empty}\right)$	1	1	1	1	.99	.93	.98

Note: "x" indicates the restriction is imposed

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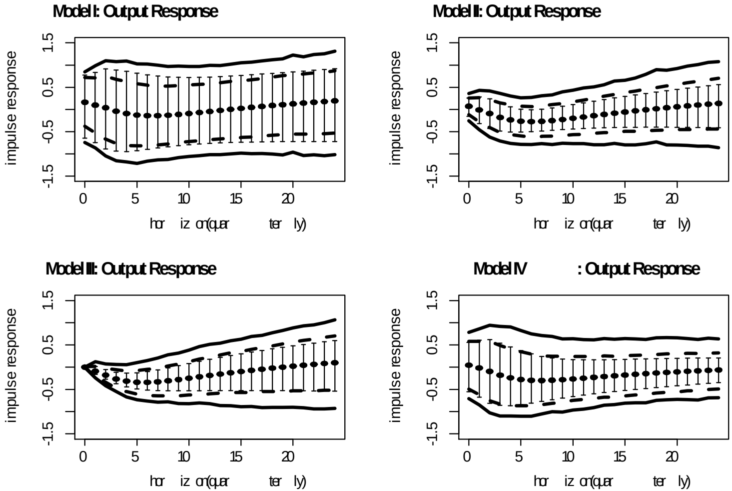
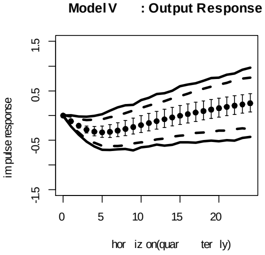
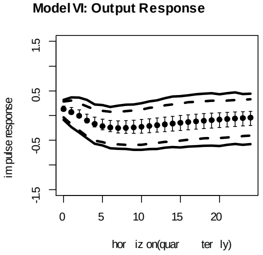
- Report output response to monetary policy shock

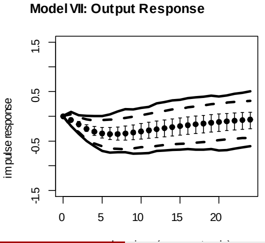
Results - Models I-IV



Results - Models V-VII







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- Single prior and robust prior approaches can lead to different conclusions

Conclusion and extensions

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- Inference not sensitive to choice of prior $\longrightarrow$ posterior bounds converge to true identified set asymptotically
- Uncertainty captured by posterior robustified credible region $\longrightarrow$ pointwise validity
- Extension: joint credible regions (probably too wide to be useful....)