Games and Economic Behavior 79 (2013) 201–222

Contents lists available at SciVerse ScienceDirect

Games and Economic Behavior

www.elsevier.com/locate/geb

Information sharing between vertical hierarchies ☆

Salvatore Piccolo [a,*], Marco Pagnozzi [b]

[a] Università Cattolica del Sacro Cuore, Department of Economics, Via Necchi 5, 20123 Milano, Italy
[b] Università di Napoli Federico II and CSEF, Department of Economics, Via Cintia (Monte S. Angelo), 80126 Napoli, Italy

ARTICLE INFO

Article history:
Received 31 March 2012
Available online 27 February 2013

JEL classification:
D43
D82
L14

Keywords:
Communication
Information sharing
Adverse selection
Vertical hierarchies

ABSTRACT

When do principals independently choose to share the information obtained from their privately informed agents? Information sharing affects contracting within competing organizations and induces agents' strategies to be correlated through the distortions imposed by principals to obtain information. We show that the incentives to share information depend on the nature of upstream externalities between principals and the correlation of agents' information. With small externalities, principals share information when externalities and correlation have opposite signs, and do not share information when externalities and correlation have the same sign. In this second case, principals face a prisoners' dilemma since they obtain higher profits by sharing information.

1. Introduction

We analyze the incentive of competing principals to share the information that they privately obtain from their exclusive agents. Consider, for example competing manufacturers selling through exclusive outlets who are privately informed about their marginal costs, which may be correlated. Manufacturers learn outlets' information through their interaction with them, and may commit to share this information if they expect to increase their profit by doing so.

Information sharing agreements between competing principals who contract with privately informed agents are widespread in real life: banks exchange information about borrowers; sellers share with competitors information about their customers' demand; corporations disclose information about their management's performance.

In oligopolistic markets, information sharing can emerge both to increase efficiency and to reduce competition (see, e.g., Novshek and Sonnenschein, 1982; Clarke, 1983; Vives, 1984; Raith, 1996). Moreover, Pagano and Jappelli (1993) show that lenders exchange information to screen investment projects or discipline borrowers, Lizzeri (1999) and Gromb and Martimort (2007) analyze the role of experts who acquire and disclose information to trading counterparts, Taylor (2004) and Acquisti and Varian (2005) show how sellers can use information on consumers' purchasing history to implement price discrimination.

But these papers do not consider the source of the information shared by players, and model communicators as *black boxes*. Hence, they are silent on the interplay between information exchange and agency conflicts within organizations, when

☆ For valuable comments and discussions, we would like to thank Alessandro Bonatti, Michela Cella, Piero Gottardi, David Martimort, Johannes Muthers, Alessandro Pavan, Francesca Stroffolini, the editor and the referees, as well as seminar participants at the University of Turin, the University of Verona, the 2012 EARIE conference in Rome, and the workshop on "The Economics of Coordination and Communication" in Ravello. A previous version of this paper was circulated under the title "Communicating Vertical Hierarchies: The Adverse Selection Case". We gratefully acknowledge financial support by MIUR ("Futuro in Ricerca" RBFR087PSB).

* Corresponding author.
E-mail addresses: salvapiccolo@gmail.com (S. Piccolo), pagnozzi@unina.it (M. Pagnozzi).

0899-8256/$ – see front matter
http://dx.doi.org/10.1016/j.geb.2013.02.005

organizations have to obtain information from their privately informed members. A notable exception is Calzolari and Pavan (2006), who consider a sequential common agency model in which two principals may share the information they obtain by contracting with a common agent.[1] While Calzolari and Pavan (2006) focus on non-exclusive contracting, we analyze the effects of communication when contracts are exclusive.

Exclusivity clauses are common in many real markets (Caillaud et al., 1995). Several employment relationships are, by their own nature, exclusive (e.g., because of labor natural indivisibility); supply and franchising contracts in the manufacturing industry are often exclusive (e.g., when retailers cannot distribute competing brands); many procurement, regulatory and financial contracts feature forms of exclusivity. And information sharing agreements are common in markets with exclusive deals (Connor, 2001).[2]

What are the drivers of information sharing decisions in these contexts? How does information sharing between organizations interact with rent extraction within organizations and horizontal externalities across organizations? To answer these questions, we analyze a model with two independent principals who exert production externalities on each other and delegate production to exclusive agents.[3] Agents are privately informed about their marginal costs of production, which may be positively or negatively correlated.[4] Hence, agents' information must be obtained by principals through the design of incentive-compatible contracts. Before contracting with agents, each principal simultaneously and non-cooperatively chooses whether to commit to share this information.[5]

Information sharing between principals has a first-order effect on their profits, which is absent when agents have no private information. The incentive of a principal to share information depends on the impact that this decision has on the opponent principal's contract, and hence on outputs. With adverse selection within organizations, information sharing induces players' strategies to be correlated, because the information revealed by one principal affects the other principal's incentive to distort her agent's output in order to reduce the information rent. And because principals want to reduce these distortions and also affect their competitor's output, the benefits of communication depend on the interaction between the nature of upstream externalities and the correlation of agents' costs.

As a result, if externalities are small, there is always a unique equilibrium in dominant strategies in the information sharing game between principals. When upstream externalities and cost correlation have the same sign — i.e., they are either both positive or both negative — principals do not share information; when upstream externalities and cost correlation have opposite signs, both principals share information.

To see why, first suppose that costs are negatively correlated. Since principals choose higher distortions (that reduce agents' information rents) in the less likely states, by revealing her agent's cost a principal induces the rival to reduce the output of her high-cost agent relatively more when the first principal's agent has a high cost (and hence produces less), because this is less likely. With negative externalities, this reduces the first principal's profit because reaction functions are downward sloping and, hence, a principal prefers to produce more when her rival produces less. By contrast, with positive externalities this increases the first principal's profit because reaction functions are upward sloping and, hence, a principal prefers to produce less when her rival produces less.

Second, suppose that costs are positively correlated. By revealing her agent's cost, a principal induces the rival to reduce the output of her high-cost agent relatively more when the first principal's agent has a low cost (and hence produces more), because this is less likely. With positive externalities, this reduces the first principal's profit because she prefers to produce more when her rival produces more. By contrast, with negative externalities this increases the first principal's profit because she prefers to produce less when her rival produces more.

We assume that externalities between principals are small in order to analytically compare principals' expected profits with and without information sharing. Even an arbitrarily small externality has a first-order effect on outputs, since principals' choices to share information depend on the sign of the externality. We also analyze the effects of stronger externalities by numerical simulations and show that, if externalities are not too large, the effect of information sharing on principals' profits through the distortions due to agents' information rents is stronger than the standard effects of information sharing, that arise even when agents have no private information (e.g., Shapiro, 1986).

[1] The authors show how the information disclosed by one principal affects the contractual relationships between all other players and derive conditions under which the upstream principal wants to offer full privacy to the agent. See also Bennardo et al. (2010) and Maier and Ottaviani (2009) for models with moral hazard.

[2] For instance, according to Briley et al. (1994), the mandatory disclosure of franchising contracts required by the Federal Trade Commission since 1979 allows firms to have free access to some of their rivals' information. Moreover, the spread of information intensive systems in the manufacturing industry (like telecommunication and satellite linkages, bar coding and electronic scanning, etc.) facilitates the dissemination of information between competing organizations (Stern et al., 1996).

[3] The production externalities between principals reflect the type of strategic interaction between them: negative externalities arise when production decisions are strategic substitutes, positive externalities arise when production decisions are strategic complements.

[4] Costs are negatively correlated if, for example, agents compete in an R&D race: the agent who wins the race reduces his own cost of production, while the other agent continues to produce at the original higher cost. Positive correlation between costs, instead, arises when agents use common inputs, so that variations in the prices of these inputs affect symmetrically their production costs.

[5] We assume that a principal cannot manipulate the information transmitted to the other principal. This is a standard assumption in the information sharing literature and is consistent with various types of communication — e.g., when the information concerns certifiable contractual agreements. For example, Connor (2001) reports that, in the citric acid market, an international auditing firm was hired to independently audit the reports made by firms participating in an information sharing agreement.

With complete information within organizations, principals independently choose to share information if and only if this maximizes their joint profits. By contrast, principals may face a *prisoners' dilemma* when they have to obtain information from agents, since they may not share information even if their joint profits are always higher with information sharing. In fact, when agents' information is correlated, information sharing reduces agents' rent because it makes an agent's contract depend on the rival agent's type, and a *relative performance evaluation* relaxes the incentive compatibility constraints.[6] This effect is stronger than the strategic effect due to correlation among distortions. Therefore, principals would always choose to jointly sign an information sharing agreement, but our results suggest that this agreement may not be stable because it may be vulnerable to unilateral deviations by principals.

The literature on information sharing in oligopolistic markets shows that firms' incentives to share information about their common demand function (Novshek and Sonnenschein, 1982; Clarke, 1983; Vives, 1984; Gal-Or, 1985) or about their private costs of production (Fried, 1984; Gal-Or, 1986; Shapiro, 1986) depend on the nature of competition between them (Bertrand or Cournot). Raith (1996) rationalizes the results of this vast literature in a unified framework. We focus on information about production costs and complement this literature by allowing costs to be negatively correlated and by assuming that firms have to obtain information from the privately informed agents with whom they contract, before sharing this information. This allows us both to extend previous results on firms' incentives to share exogenous information (Section 3), and to analyze new incentives that emerge when firms have to provide information rents to their privately informed agents (Sections 4 and 5). Jansen (2008) and Ganuza and Jansen (forthcoming) also endogenize information acquisition in a model of information sharing, but by assuming that firms have to acquire a costly signal to obtain the information that they may share.[7]

We also explore the robustness of our results to the possibility of implicit collusion among agents. Because with information sharing an agent's expected utility depends on his opponent's report, agents may have an incentive to (implicitly) coordinate on an equilibrium in which they both misreport their type. However, we show that, if side payments across agents are not possible, principals can always design transfers that induce a collusion-proof equilibrium with information sharing and truthful reports by agents.

Although we develop our arguments in a principal–agent framework, the scope of our analysis is broader. The results apply to any situation involving horizontal externalities between competing organizations, where principals deal with exclusive and privately informed agents, like procurement contracting, manufacturer–retailer relations, executive compensations, patent licensing, and insurance or credit relationships.

The rest of the paper is organized as follows. Section 2 describes the model and Section 3 analyzes the case of complete information within organizations. Section 4 introduces asymmetric information and characterizes the equilibrium outputs when no principal shares information, when both principals share information, and when only one principal shares information. Principals' decisions to share information are analyzed in Section 5. Section 6 considers large externalities and Section 7 collusion among agents. Finally, Section 8 concludes. All proofs are in Appendix A.

2. The model

Players and payoffs There are two (female) principals, P_1 and P_2, and two (male) exclusive agents, A_1 and A_2, who produce outputs q_1 and q_2, respectively. All players are risk neutral. P_i's utility is

$$V_i(q_i, q_j, t_i) = S(q_i, q_j) - t_i, \quad i, j = 1, 2,$$

where t_i is the monetary transfer paid by P_i to A_i. We assume that

$$S(q_i, q_j) = \kappa + \beta q_i - q_i^2 + \delta q_i q_j.$$

This quadratic surplus function suits various standard oligopoly models, including the Cournot model with differentiated goods and linear demand, and is commonly used in the literature on information sharing (e.g., Gal-Or, 1985; Raith, 1996; Vives, 2000) since it allows to obtain closed-form solutions. Hence, P_i's surplus is strictly concave in q_i.

The parameter δ measures the magnitude of strategic complementarity ($\delta > 0$) or substitutability ($\delta < 0$) between outputs. Since $\frac{\partial^2 S(q_i, q_j)}{\partial q_i \partial q_j} = \delta$, a positive δ implies that principals' reaction functions are upward sloping and a negative δ implies that principals' reaction functions are downward sloping. We assume that δ is small in order to compare principals' expected profits with and without information sharing through Taylor approximations around $\delta = 0$ (in Propositions 1 and 2, and in Section 5), and we show that even a small δ has a significant effect on equilibrium outputs when agents have private information, because the *sign* of δ determines principals' choice to share information. Moreover, in Section 6 we consider the effects of a larger δ on principals' choices by numerical simulations and show that all our results hold if δ is not too large.

[6] See, e.g., Crémer and McLean (1988) and Riordan and Sappington (1988), who show how principals can extract all agents' surplus in this context. Full surplus extraction by principals is not possible in our model because, by assumption, agents have limited liability.

[7] Specifically, Jansen (2008) analyzes firms' incentive to share information when they can acquire a costly signal about demand and when competitors cannot observe whether a firm is informed or not, while Ganuza and Jansen (forthcoming) analyze the effects of information sharing on consumers' surplus when firms can acquire a costly signal about their marginal cost.

A_i's utility is

$$U_i(t_i, q_i, \theta_i) = t_i - \theta_i q_i, \quad i = 1, 2,$$

where θ_i is A_i's marginal cost of production. We assume that agents have limited liability — i.e., that

$$U_i(t_i, q_i, \theta_i) \geqslant 0 \quad \forall (t_i, q_i, \theta_i).$$

This standard hypothesis of the screening literature implies that P_i cannot use information about θ_j to extract all surplus from A_i and leave him with no information rent, as in the mechanism by Crémer and McLean (1988) that imposes negative utility to agents in some states of word.[8]

Information The parameter $\theta_i \in \Theta \equiv \{\underline{\theta}, \bar{\theta}\}$, $i = 1, 2$, is private information to A_i; it can be learned by P_i only through a revelation mechanism, and by P_j and A_j only if P_i chooses to share information.

The vector of random variables (θ_1, θ_2) is drawn from a joint cumulative distribution function with:

- $\Pr(\underline{\theta}, \underline{\theta}) = \nu^2 + \alpha$;
- $\Pr(\underline{\theta}, \bar{\theta}) = \Pr(\bar{\theta}, \underline{\theta}) = \nu(1 - \nu) - \alpha$;
- $\Pr(\bar{\theta}, \bar{\theta}) = (1 - \nu)^2 + \alpha$.

The parameter α measures the correlation between θ_1 and θ_2 since $\Pr(\underline{\theta}, \underline{\theta}) \Pr(\bar{\theta}, \bar{\theta}) - \Pr(\underline{\theta}, \bar{\theta})^2 = \alpha$. Hence, $\alpha > 0$ (resp. $<$) indicates positive (resp. negative) correlation between agents' marginal costs, while agents' costs are independent when $\alpha = 0$. For example, negative correlation may arise when agents compete in R&D races, while positive correlation may arise when agents use common inputs. It follows that $\Pr(\underline{\theta}) = \nu$, $\Pr(\bar{\theta}) = 1 - \nu$ and, using the Bayes rule, $\Pr(\underline{\theta}|\underline{\theta}) = \nu + \frac{\alpha}{\nu}$, $\Pr(\underline{\theta}|\bar{\theta}) = \nu - \frac{\alpha}{1-\nu}$, $\Pr(\bar{\theta}|\underline{\theta}) = 1 - \nu - \frac{\alpha}{\nu}$, and $\Pr(\bar{\theta}|\bar{\theta}) = 1 - \nu + \frac{\alpha}{1-\nu}$.

To ensure that probabilities are not negative, we assume that: (i) $\nu(1 - \nu) \geqslant \alpha$ if $\alpha \geqslant 0$, and (ii) $\min\{(1 - \nu), \nu\} \geqslant \sqrt{|\alpha|}$ if $\alpha < 0$. We also assume that $\beta > \bar{\theta} > \underline{\theta} > 0$ and that $\Delta\theta \equiv \bar{\theta} - \underline{\theta}$ is small to ensure that outputs are positive in all states of the world.

Communication Before contracting with agents, principals simultaneously and independently choose whether to share the information obtained from their agents. Hence, we allow for asymmetric information sharing decisions by principals, in order to obtain insights on the stability of information sharing agreements between them. Notice that, since principals' decisions on whether to share information are in dominant strategies, our results do not hinge on the assumption that principals make simultaneous choices. At the end of Section 5, we also consider the case in which principals can jointly sign information sharing contracts. Following Vives (1984) and Raith (1996), we assume that principals follow an "all-or-none" sharing rule: they either fully commit to disclose their agents' costs, or they keep this information secret.[9]

We also assume that, once a principal commits to an information sharing decision, she cannot renegotiate this decision after learning her agent's costs — see, e.g., Vives (2000, Ch. 8) and Raith (1996) for a similar approach. Commitment requires, for example, the presence of a third party (e.g., an independent auditing intermediary) that controls communication between vertical hierarchies. Moreover, the information transmitted by a principal is verifiable, so that a report made by A_i to P_i can be credibly shared with P_j, and then transmitted by P_j to A_j — i.e., there is no moral hazard since principals cannot transmit misleading information. (Ziv, 1993, analyzes information sharing in a standard oligopoly model in which information is not verifiable.)

Contracts Principals design contracts to obtain information from their agents and, based on this, determine how much to produce. Contracts are private: P_j and A_j cannot observe the contract offered by P_i to A_i. Of course, to make information sharing between vertical hierarchies meaningful, the information obtained by a principal from her agent cannot be observed by the other principal. Notice, however, that our results do not hinge on the assumption of private contracts, since they also hold with public contracts, if a principal's contract cannot be contingent on her rival's contract.[10] The reason is that an agent has no incentive to change his action depending on the contract offered by the rival principal, since the agent's utility only depends on his own principal's contract (and not on the quantity produced by the other hierarchy).

Given that principals commit to deterministic disclosure policies before contracting with agents, we can use the *Revelation Principle* and consider direct deterministic mechanisms in which A_i sends a private message $m_i \in \Theta$ about his cost to P_i. When P_j does not share her information about θ_j — i.e., A_j's report m_j — P_i offers to A_i a mechanism

[8] See Bertoletti and Poletti (1997) for an application of the mechanism to competing vertical hierarchies.

[9] In our model, there is no scope for (deterministic) type-contingent disclosure policies; e.g., when a principal commits to only revealing her agent's type when the agent has a low cost. This is because, with only two types, an unraveling argument implies that this policy is equivalent to full disclosure of the agent's cost.

[10] These types of contracts are usually banned by antitrust authorities to prevent collusion. Moreover, a theoretical analysis of these contracts is problematic because the Revelation Principle may not apply.

$$\mathcal{M}_i^N \equiv \{t_i(m_i), q_i(m_i)\}_{m_i \in \Theta},$$

which maps m_i into a monetary transfer $t_i(m_i)$ and an output $q_i(m_i)$. When, instead, P_j shares her information about θ_j, P_i offers a mechanism

$$\mathcal{M}_i^I \equiv \{t_i(m_i, m_j), q_i(m_i, m_j)\}_{(m_i, m_j) \in \Theta^2},$$

in which the transfer and the output are also contingent on m_j.

Because of limited liability: if P_j does not share information, A_i's utility must be non-negative for all m_i; if P_j shares information, A_i's utility must be non-negative for all (m_i, m_j).

Timing The timing of the game is as follows:

1. Principals publicly choose whether to commit to share information.
2. Agents privately observe their costs.
3. Principals contract with agents.
4. Each principal shares information if and only if she has committed to do so.
5. Agents produce and payments are made.

Equilibrium concept The equilibrium concept is Perfect Bayesian Equilibrium (PBE). Following most of the literature on private contracts (e.g., Caillaud et al., 1995, and Martimort, 1996), we assume that agents have *passive beliefs* — i.e., that an agent's belief about the private contract offered to the other agent is not affected by the contract offered to the first agent. This assumption captures the idea that, since principals are independent and act simultaneously, a principal cannot signal to her agent information that she does not posses about the other principal's contract. With passive beliefs, when an agent is offered a contract different from the one he expects in equilibrium, he still believes that the other agent is offered the equilibrium contract.[11]

3. Complete information within hierarchies

First assume that costs are common knowledge within each hierarchy — i.e., each principal observes her agent's cost, but not the rival agent's cost. In this case, regardless of principals' communication choices, agents obtain no rent. Hence, the two hierarchies act as vertically integrated firms and our model is analogous to the one in Shapiro (1986), who analyzes information sharing between firms competing *à la* Cournot (but does not consider negative correlation between firms' information).

We denote by $q^*(\theta_i)$ the equilibrium output of A_i when both principals do not share information; by $q^*(\theta_i, \theta_j)$ the equilibrium output of A_i when both principals share information; and by $q_i^*(\theta_i)$ and $q_j^*(\theta_j, \theta_i)$ the equilibrium outputs of A_i and A_j, respectively, when P_i does not share information and P_j shares information.

Let s_i be the information upon which P_i conditions the contract offered to A_i. Hence, $s_i = \theta_i$ if P_j does not share information, and $s_i = (\theta_1, \theta_2)$ if P_j shares information. Abusing notation, let $\tilde{q}_i(s_i)$ be P_i's equilibrium output, given all possible communication decisions.[12]

Lemma 1. *Regardless of principals' communication decisions, P_i's expected profit is*

$$V_i^* = \kappa + \underbrace{\left(\mathbb{E}_{s_i}\left[\tilde{q}_i(s_i)|\theta_i\right]\right)^2}_{\text{average } \tilde{q}_i(s_i)} + \underbrace{\mathbb{E}_{s_i}\left[\tilde{q}_i(s_i) - \mathbb{E}_{s_i}\left[\tilde{q}_i(s_i)|\theta_i\right]\right]^2}_{\text{variance of } \tilde{q}_i(s_i)},$$

and the expected output is $q^* \equiv \frac{\beta-\theta}{2-\delta} - \frac{1-\nu}{2-\delta}\Delta\theta$.

Hence, principals obtain higher profit if production increases or becomes more volatile, because the indirect profit function is convex. Moreover, since outputs are linear in costs, information sharing decisions do not affect expected output. (See, e.g., Shapiro, 1986, and Vives, 2000.)

Therefore, when choosing whether to share information, each principal simply maximizes the volatility of her own output (given the other principal's communication choice). The reason is that, by allowing P_j to learn θ_i, P_i can influence the distribution of P_j's output, and therefore her own output volatility, because reaction functions are linear. *Ceteris paribus*, if $\tilde{q}_j(s_j)$ becomes more volatile, the variance of $\tilde{q}_i(s_i)$ increases too.

---

[11] See Pagnozzi and Piccolo (2012) for a discussion of the role of beliefs with private contracts.

[12] Hence, $\tilde{q}_i(s_i) = q^*(\theta_i)$ if both principals do not share information; $\tilde{q}_i(s_i) = q^*(\theta_i, \theta_j)$ if both principals share information; and $\tilde{q}_i(s_i) = q_i^*(\theta_i)$ and $\tilde{q}_j(s_j) = q_j^*(\theta_j, \theta_i)$ if P_i does not share information and P_j shares information.

Proposition 1. *Suppose that $\delta \neq 0$ and small. With complete information (within each hierarchy): if $\alpha > -\nu(1-\nu)$, there is a unique equilibrium in dominant strategies in which both principals share information; if $\alpha < -\nu(1-\nu)$, there is a unique equilibrium in dominant strategies in which no principal shares information.*

If $\delta = 0$ or $\Pr(\bar{\theta}, \underline{\theta}) = 0$, with complete information (within each hierarchy) principals obtain the same payoff regardless of whether they share information or not.

When principals do not have to obtain information from agents, they share information only when agents' costs are positively or not too negatively correlated; while production externalities do not affect principals' choice to share information. To see this, suppose that P_j commits to disclose θ_j, and consider P_i's incentive to reveal θ_i. Sharing information has both a *direct* and an *indirect* effect on the equilibrium distribution of outputs. Revealing θ_i to P_j has a positive direct effect because it expands the set of contingencies upon which A_j's output can be conditioned, thus increasing the volatility of output.

But revealing θ_i also has an indirect effect because it affects the correlation between the outputs produced by the two hierarchies. If $\delta > 0$, a positive α increases the variance of outputs, and hence profits, with information sharing, because outputs are very high in state $(\underline{\theta}, \underline{\theta})$ and very low in state $(\bar{\theta}, \bar{\theta})$. By contrast, a negative α reduces the variance of outputs with information sharing, because outputs are more similar in states $(\bar{\theta}, \underline{\theta})$ and $(\underline{\theta}, \bar{\theta})$. If instead $\delta < 0$, a negative α increases the variance of outputs, and hence profits, without information sharing, because it increases the distortion of $q^*(\bar{\theta})$ with respect to $q^*(\underline{\theta})$;[13] while a positive α reduces the variance of outputs without information sharing. Hence, if costs are negatively correlated, the indirect effect of information sharing reduces volatility.

On balance, with positive or not too negative correlation between costs, principals share information because the direct effect dominates the indirect effect. By contrast, the indirect effect is stronger than the direct effect with negative and large correlation between costs. Of course, with no externality ($\delta = 0$) P_i's payoff does not depend on P_j's output, and learning θ_j does not affect P_i's strategy. Similarly, when costs are perfectly correlated — i.e., when $\Pr(\bar{\theta}, \underline{\theta}) = 0$ — output volatility is the same irrespective of whether principals share information or not.

The next proposition shows that total principals' profits are higher with information sharing than without information sharing if and only if each principal has an incentive to unilaterally share information.

Proposition 2. *With complete information (within each hierarchy), principals' information sharing decisions always maximize total principals' profit.*

Hence, when they are informed about their agents' costs, principals independently share information only when it is jointly beneficial for them to do so. As we will show in the next sections, this is not necessarily the case when principals are uninformed about their agents' costs.

Our results with complete information within hierarchies are consistent with the general analysis of information sharing in oligopoly by Raith (1996). In addition, they complement the results of the previous literature that only considers positive correlation between costs and shows, for example, that sharing information always increases principals' profits with strategic substitutes (e.g., Shapiro, 1986). Specifically, Propositions 1 and 2 show that this is not necessarily the case with negative correlation between costs, if α is negative and large in absolute value.

4. Asymmetric information

Suppose now that agents are privately informed about their costs. Before sharing information, principals must learn their agents' costs through contracting and, hence, they must give agents an information rent in order to induce them to truthfully report their costs. To minimize this rent, principals distort outputs away from efficiency. Of course, distortions depend on whether principals share information, which in turn affects the strategic interaction between principals and agents and, therefore, the value of communication.

Since principals' decisions to share information are public, before analyzing whether they prefer to share information or not in Section 5, we first characterize the equilibrium contracts in the following three subgames: no communication — i.e., when principals do not share information; bilateral information sharing — i.e., when both principals share information; and unilateral information sharing — i.e., when only one principal shares information.

4.1. No communication

Suppose that principals do not share information. In a separating equilibrium, P_i offers a contract that satisfies the following participation and incentive compatibility constraints:

$$\begin{cases} U_i(\theta_i) \equiv t_i(\theta_i) - \theta_i q_i(\theta_i) \geqslant 0 \quad \forall \theta_i \in \Theta, \\ U_i(\theta_i) \geqslant t_i(m_i) - \theta_i q_i(m_i) \quad \forall (\theta_i, m_i) \in \Theta^2. \end{cases}$$

[13] See the proof of Proposition 1.

As usual, only the incentive constraint of the efficient type and the participation constraint of the inefficient type matter (see, e.g., Laffont and Martimort, 2002). Hence, letting $q^e(\theta_j)$ be A_j's output in a (symmetric) separating equilibrium, in order to maximize her profit P_i solves[14]

$$\max_{\{q_i(.),t_i(.)\}} \sum_{\theta_i} \Pr(\theta_i)\left[\sum_{\theta_j} \Pr(\theta_j|\theta_i) S\big(q_i(\theta_i), q^e(\theta_j)\big) - t_i(\theta_i)\right],$$

subject to

$$\begin{cases} U_i(\bar\theta) \equiv t_i(\bar\theta) - \bar\theta q_i(\bar\theta) \geqslant 0, \\ U_i(\underline\theta) \equiv t_i(\underline\theta) - \underline\theta q_i(\underline\theta) \geqslant U_i(\bar\theta) + \Delta\theta q_i(\bar\theta). \end{cases}$$

Since at the optimum both constraints bind, P_i's problem is

$$\max_{q_i(.)}\left\{\sum_{\theta_i} \Pr(\theta_i)\left[\sum_{\theta_j} \Pr(\theta_j|\theta_i) S\big(q_i(\theta_i), q^e(\theta_j)\big) - \theta_i q_i(\theta_i)\right] - \nu\Delta\theta q_i(\bar\theta)\right\}.$$

Unlike in the complete information case, P_i must grant an information rent $\Delta\theta q_i(\bar\theta)$ to an agent with a low cost, in order to induce him to reveal his information, and this rent is increasing in the quantity produced by an agent with a high cost.

The necessary and sufficient first-order conditions for P_i's problem are[15]

$$\sum_{\theta_j} \Pr(\theta_j|\underline\theta) S_1\big(q^e(\underline\theta), q^e(\theta_j)\big) = \underline\theta, \tag{1}$$

and

$$\sum_{\theta_j} \Pr(\theta_j|\bar\theta) S_1\big(q^e(\bar\theta), q^e(\theta_j)\big) = \bar\theta + \frac{\nu}{1-\nu}\Delta\theta. \tag{2}$$

Therefore, a low-cost agent produces the "efficient" output that equalizes the (expected) marginal benefit for the principal to the marginal cost, while a high-cost agent produces an inefficiently low output to reduce the information rent.

Recall from Section 3 that $q^*(\theta_i)$ is agent A_i's efficient output when principals know their agents' cost and do not share information.

Proposition 3. *Suppose that principals do not share information. In the unique symmetric separating PBE, outputs are*

$$q^e(\underline\theta) = q^*(\underline\theta) - \frac{\delta\nu(\nu(1-\nu)-\alpha)}{(2-\delta)(2\nu(1-\nu)-\alpha\delta)}\Delta\theta, \qquad q^e(\bar\theta) = q^*(\bar\theta) - \frac{2\nu-\delta(\nu^2+\alpha)}{(2-\delta)(2\nu(1-\nu)-\alpha\delta)}\Delta\theta.$$

Moreover,

- $q^e(\underline\theta) > q^e(\bar\theta)$ *for* $\Delta\theta \neq 0$ *and* $\nu \neq 0$;
- *Expected output is downward distorted — i.e.,* $q^e \equiv \sum_{\theta_i} \Pr(\theta_i) q^e(\theta_i) < q^*$.

Because of the production externality, the output of a low-cost agent is distorted with respect to the output with complete information: $q^e(\underline\theta) > q^*(\underline\theta)$ if $\delta < 0$, and $q^e(\underline\theta) < q^*(\underline\theta)$ if $\delta > 0$. The reason is that a principal expects the rival's output to be distorted downward because of information rents, and this distortion affects the choice of her own output: since A_j produces a lower output when he has a high cost, if goods are substitutes (resp. complements) P_i responds by producing a higher (resp. lower) output than with complete information when A_i has a low cost. This *two-way* distortion has also been analyzed by Cella and Etro (2012) in similar model (that does not consider information sharing) in which two competing principals have to induce their privately informed agents to undertake costly effort.[16] By contrast, when there is no externality ($\delta = 0$) the output of a low-cost agent is efficient. Finally, expected output is lower when agents are privately informed (than with complete information), because the output of a high-cost agent is distorted downward.

Consider now the equilibrium relationship between δ and α.

Lemma 2. *sign* $\frac{\partial[q^e(\underline\theta)-q^e(\bar\theta)]}{\partial\alpha} =$ *sign* δ.

[14] Because agents' costs are correlated, P_i's beliefs about θ_j depend on A_i's report.

[15] We denote by $S_1(q_i, q_j)$ the partial derivative of $S(q_i, q_j)$ with respect to q_i.

[16] See also Cella and Etro (forthcoming) that considers a model with many firms and a continuous of types.

The effect of an increase in cost correlation on $q^e(\underline{\theta}) - q^e(\bar{\theta})$ — the output difference between a low-cost and a high-cost agent — depends on the sign of δ. The reason is that a higher α implies that, when one agent's cost is high, his opponent's cost is more likely to be high too. Hence, if $\delta < 0$, a principal prefers to increase the production of her high-cost agent (and hence reduce $q^e(\underline{\theta}) - q^e(\bar{\theta})$), because with strategic substitutes she wants to produce more when her rival has a high cost and produces less. By contrast, if $\delta > 0$, a higher α increases $q^e(\underline{\theta}) - q^e(\bar{\theta})$ because, with strategic complements, a principal prefers to reduce the production of her high-cost agent when her rival has a high cost.

4.2. Bilateral information sharing

Suppose now that both principals share information. Consider a pure-strategy, symmetric, separating equilibrium in which agents truthfully report their types to principals, who then share this information. Since an agent does not know his rival's cost when he reports his own cost, the incentive and participation constraints are

$$\begin{cases} \sum_{\theta_j} \Pr(\theta_j|\theta_i) U_i(\theta_i, \theta_j) \geqslant \sum_{\theta_j} \Pr(\theta_j|\theta_i)[t_i(m_i, \theta_j) - \theta_i q_i(m_i, \theta_j)] & \forall (m_i, \theta_i) \in \Theta^2, \\ \sum_{\theta_j} \Pr(\theta_j|\theta_i) U_i(\theta_i, \theta_j) \geqslant 0 \quad \forall \theta_i \in \Theta, \end{cases}$$

while the limited liability constraint is

$$U_i(\theta_i, \theta_j) \equiv t_i(\theta_i, \theta_j) - \theta_i q_i(\theta_i, \theta_j) \geqslant 0 \quad \forall (\theta_i, \theta_j) \in \Theta^2.$$

Clearly, when this last constraint is satisfied, the participation constraint is also satisfied.

As usual, the relevant limited liability constraint is that of the high-cost type

$$U_i(\bar{\theta}, \theta_j) \geqslant 0 \quad \forall \theta_j \in \Theta, \tag{3}$$

while the relevant incentive constraint is that of the low-cost type

$$\sum_{\theta_j} \Pr(\theta_j|\underline{\theta}) U_i(\underline{\theta}, \theta_j) \geqslant \sum_{\theta_j} \Pr(\theta_j|\underline{\theta}) \big[t_i(\bar{\theta}, \theta_j) - \underline{\theta} q_i(\bar{\theta}, \theta_j)\big]$$

$$\Leftrightarrow \quad \sum_{\theta_j} \Pr(\theta_j|\underline{\theta}) U_i(\underline{\theta}, \theta_j) \geqslant \sum_{\theta_j} \Pr(\theta_j|\underline{\theta}) U_i(\bar{\theta}, \theta_j) + \Delta\theta \sum_{\theta_j} \Pr(\theta_j|\underline{\theta}) q_i(\bar{\theta}, \theta_j). \tag{4}$$

Hence, P_i's conditional expectation about A_j's marginal cost affects the output that P_i chooses for her high-cost agent in order to minimize her low-cost agent's information rent. Precisely, *ceteris paribus*, P_i wants to reduce $q_i(\bar{\theta}, \theta_j)$ when $\Pr(\theta_j|\underline{\theta})$ is high (since this makes it more likely that she has to pay the information rent to her low-cost agent) — i.e., when $\theta_j = \underline{\theta}$ if costs are positively correlated or when $\theta_j = \bar{\theta}$ if costs are negatively correlated.

Therefore, letting $q^e(\theta_i, \theta_j)$ denote the equilibrium output, P_i solves

$$\max_{\{q_i(.,.), t_i(.,.)\}} \sum_{\theta_i} \Pr(\theta_i) \sum_{\theta_j} \Pr(\theta_j|\theta_i) \big[S\big(q_i(\theta_i, \theta_j), q^e(\theta_j, \theta_i)\big) - t_i(\theta_i, \theta_j)\big],$$

subject to constraints (3) and (4). Since at the optimum the transfer paid to a high-cost agent $t_i(\bar{\theta}, \theta_j)$ is such that he obtains no rent regardless of his opponent's cost and since the incentive constraint (4) is binding, P_i's optimization problem can be simplified to[17]

$$\max_{q_i(.,.)} \Big\{ \sum_{\theta_i} \Pr(\theta_i) \sum_{\theta_j} \Pr(\theta_j|\theta_i) \big[S\big(q_i(\theta_i, \theta_j), q^e(\theta_j, \theta_i)\big) - \theta_i q_i(\theta_i, \theta_j)\big] - \nu \Delta\theta \sum_{\theta_j} \Pr(\theta_j|\underline{\theta}) q_i(\bar{\theta}, \theta_j) \Big\}.$$

Hence, with information sharing, the low-cost agent's information rent also depends on θ_j.

The symmetric equilibrium outputs are determined by the following necessary and sufficient first-order conditions

$$S_1\big(q^e(\underline{\theta}, \theta_j), q^e(\theta_j, \underline{\theta})\big) = \underline{\theta} \quad \forall \theta_j \in \Theta, \tag{5}$$

and

$$S_1\big(q^e(\bar{\theta}, \theta_j), q^e(\theta_j, \bar{\theta})\big) = \bar{\theta} + \frac{\nu}{1-\nu} \frac{\Pr(\theta_j|\underline{\theta})}{\Pr(\theta_j|\bar{\theta})} \Delta\theta \quad \forall \theta_j \in \Theta. \tag{6}$$

As when principals do not share information, the output of a low-cost agent is chosen efficiently to equalize marginal benefit to marginal cost, while the output of a high-cost agent is distorted downward to induce a low-cost agent to reveal

[17] Notice that, although each principal can condition her contract on her opponent's cost, agents still earn an information rent because they must obtain a non-negative utility in every contractible state due to limited liability.

his type. By condition (6), the distortion chosen by P_i when A_j has cost θ_j is proportional to $\frac{\Pr(\theta_j|\underline{\theta})}{\Pr(\theta_j|\bar{\theta})}$ (an index of the informativeness of A_i's marginal costs on the rival agent's cost) since: (i) $\Pr(\theta_j|\underline{\theta})$ measures how often P_i has to pay the information rent to her low-cost agent; (ii) $\Pr(\theta_j|\bar{\theta})$ measures how often her high-cost agent produces an inefficient output.

Essentially, a principal imposes a higher distortion on the output of a high-cost agent when the cost of the rival agent is the conditionally less likely one. In fact,

$$\frac{\Pr(\underline{\theta}|\underline{\theta})}{\Pr(\underline{\theta}|\bar{\theta})} > \frac{\Pr(\bar{\theta}|\underline{\theta})}{\Pr(\bar{\theta}|\bar{\theta})} \quad \Leftrightarrow \quad \frac{\alpha}{\Pr(\underline{\theta}|\bar{\theta})\Pr(\bar{\theta}|\bar{\theta})} > 0.$$

If costs are positively correlated, the distortion of a high-cost agent's output is larger when his opponent has a low rather than a high cost, because a principal whose agent has a high cost expects the opponent's agent to have a high cost too, and therefore imposes a higher distortion when the opponent has a low cost, which is less likely. By contrast, if costs are negatively correlated, the distortion of a high-cost agent's output is larger when his opponent has a high cost, because this is less likely.

Moreover, fixing the output of A_j (i.e., letting $q_j(\bar{\theta},\bar{\theta}) = q_j(\underline{\theta},\bar{\theta})$), A_i's output when he has a high cost only depends on the distortion imposed by P_i to reduce the information rent, so that $q_i(\bar{\theta},\underline{\theta}) < q_i(\bar{\theta},\bar{\theta}) \Leftrightarrow \alpha > 0$. But, of course, A_i's output also depends on the strategic effect of his production on the rival agent's production, as shown in the next proposition.

Recall from Section 3 that $q^*(\theta_i,\theta_j)$ is agent A_i's efficient output when principals know their agent's cost and share information.

Proposition 4. *Suppose that both principals share information. In the unique symmetric PBE, outputs are*

$$q^e(\underline{\theta},\underline{\theta}) = q^*(\underline{\theta},\underline{\theta}), \qquad q^e(\underline{\theta},\bar{\theta}) = q^*(\underline{\theta},\bar{\theta}) - \frac{\delta(\nu^2+\alpha)}{(4-\delta^2)(\nu(1-\nu)-\alpha)}\Delta\theta,$$

$$q^e(\bar{\theta},\underline{\theta}) = q^*(\bar{\theta},\underline{\theta}) - \frac{2(\nu^2+\alpha)}{(4-\delta^2)(\nu(1-\nu)-\alpha)}\Delta\theta, \qquad q^e(\bar{\theta},\bar{\theta}) = q^*(\bar{\theta},\bar{\theta}) - \frac{\nu(1-\nu)-\alpha}{(2-\delta)((1-\nu)^2+\alpha)}\Delta\theta.$$

Moreover,

- $q^e(\underline{\theta},\bar{\theta}) > q^*(\underline{\theta},\bar{\theta})$ *if* $\delta < 0$, *and* $q^e(\underline{\theta},\bar{\theta}) < q^*(\underline{\theta},\bar{\theta})$ *if* $\delta > 0$;
- *Expected output is the same when both principals share information and when they do not communicate — i.e.,* $\sum_{\theta_i}\Pr(\theta_i) \times \sum_{\theta_j}\Pr(\theta_j|\theta_i)q^e(\theta_i,\theta_j) = q^e$.

The output produced when both agents have a low cost is efficient; while the output produced by a high-cost agent is inefficiently low to reduce information rents — i.e., $q^e(\bar{\theta},\theta_j) < q^*(\bar{\theta},\theta_j)\ \forall\theta_j \in \Theta$.

This induces principals to also distort the quantity produced by a low-cost agent when the rival agent has a high cost, because of the production externality. Hence, information sharing creates a strategic linkage between a principal's output and the rival agent's cost. More precisely, $q^e(\underline{\theta},\bar{\theta})$ is either higher or lower than the quantity produced with complete information depending on the sign of δ. If $\delta < 0$, P_i induces her low-cost agent to overproduce when $\theta_j = \bar{\theta}$, because outputs are strategic substitutes and a principal wants to increase production when the rival produces less; if $\delta > 0$, P_i induces her low-cost agent to underproduce when $\theta_j = \bar{\theta}$, because outputs are strategic complements and a principal wants to reduce production when the rival produces less.

Finally, expected outputs are the same when there is no communication and when both principals share information because of the linearity of outputs with respect to costs.

4.3. Unilateral information sharing

Suppose now that only one principal, say P_i, commits to share information, while P_j does not. Let $q_i^e(\theta_i)$ and $q_j^e(\theta_j,\theta_i)$ be the equilibrium outputs. In this case, P_i's optimization problem is

$$\max_{q_i(.)}\left\{\sum_{\theta_i}\Pr(\theta_i)\left[\sum_{\theta_j}\Pr(\theta_j|\theta_i)S\big(q_i(\theta_i), q_j^e(\theta_j,\theta_i)\big) - \theta_i q_i(\theta_i)\right] - \nu\Delta\theta q_i(\bar{\theta})\right\},$$

while P_j's optimization problem is

$$\max_{q_j(.,.)}\left\{\sum_{\theta_j}\Pr(\theta_j)\sum_{\theta_i}\Pr(\theta_i|\theta_j)\big[S\big(q_j(\theta_j,\theta_i), q_i^e(\theta_i)\big) - \theta_j q_j(\theta_j,\theta_i)\big] - \nu\Delta\theta\sum_{\theta_i}\Pr(\theta_i|\underline{\theta})q_j(\bar{\theta},\theta_i)\right\}.$$

Notice that P_i's contract and her agent's information rent only depend on her agent's report, while P_j's contract and her agent's information rent also depend on A_i's report.

The necessary and sufficient first-order conditions of P_i's program are

$$\sum_{\theta_j} \Pr(\theta_j|\underline{\theta}) S_1\big(q_i^e(\underline{\theta}), q_j^e(\theta_j, \underline{\theta})\big) = \underline{\theta}, \tag{7}$$

and

$$\sum_{\theta_j} \Pr(\theta_j|\bar{\theta}) S_1\big(q_i^e(\bar{\theta}), q_j^e(\theta_j, \bar{\theta})\big) = \bar{\theta} + \frac{\nu}{1-\nu}\Delta\theta, \tag{8}$$

while the necessary and sufficient first-order conditions of P_j's program are

$$S_1\big(q_j^e(\underline{\theta}, \theta_i), q_i^e(\theta_i)\big) = \underline{\theta} \quad \forall \theta_i \in \Theta, \tag{9}$$

and

$$S_1\big(q_j^e(\bar{\theta}, \theta_i), q_i^e(\theta_i)\big) = \bar{\theta} + \frac{\nu}{1-\nu}\frac{\Pr(\theta_i|\underline{\theta})}{\Pr(\theta_i|\bar{\theta})}\Delta\theta \quad \forall \theta_i \in \Theta. \tag{10}$$

Therefore, low-cost agents' outputs are chosen efficiently; while both principals induce a high-cost agent to produce an inefficiently low output to reduce information rents. The interpretation of this distortion is analogous to the interpretation of condition (6) in Section 4.2.

Proposition 5. *Suppose that P_i shares information while P_j does not. In the unique symmetric PBE, outputs are*

$$q_i^e(\underline{\theta}) = q^*(\underline{\theta}, \underline{\theta}) - \frac{\delta}{4-\delta^2}\Delta\theta, \qquad q_i^e(\bar{\theta}) = q_i^e(\underline{\theta}) - \frac{2}{(1-\nu)(4-\delta^2)}\Delta\theta,$$
$$q_j^e(\underline{\theta}, \underline{\theta}) = q^*(\underline{\theta}, \underline{\theta}) - \frac{\delta^2}{2(4-\delta^2)}\Delta\theta, \qquad q_j^e(\underline{\theta}, \bar{\theta}) = q_j^e(\underline{\theta}, \underline{\theta}) - \frac{\delta}{(1-\nu)(4-\delta^2)}\Delta\theta,$$
$$q_j^e(\bar{\theta}, \underline{\theta}) = q_j^e(\underline{\theta}, \underline{\theta}) - \frac{\nu}{2(\nu(1-\nu)-\alpha)}\Delta\theta, \qquad q_j^e(\bar{\theta}, \bar{\theta}) = q_j^e(\underline{\theta}, \bar{\theta}) - \frac{1-\nu}{2(\alpha+(1-\nu)^2)}\Delta\theta.$$

Moreover,

- $q_j^e(\underline{\theta}, \underline{\theta}) > q_j^e(\underline{\theta}, \bar{\theta})$ *if* $\delta > 0$, *and* $q_j^e(\underline{\theta}, \underline{\theta}) < q_j^e(\underline{\theta}, \bar{\theta})$ *if* $\delta < 0$;
- *Expected output is the same for both hierarchies and it is equal to the expected output without communication and when both principals share information — i.e.,* $\sum_{\theta_i} \Pr(\theta_i) q_i^e(\theta_i) = \sum_{\theta_j} \Pr(\theta_j) \sum_{\theta_i} \Pr(\theta_i|\theta_j) q_j^e(\theta_j, \theta_i) = q^e$.

The intuitions for the distortion imposed by principals to the outputs of low-cost and high-cost agents are the same as the ones discussed after Propositions 3 and 4. Since expected outputs are always the same regardless of principals' communication decisions, sharing information only induces principals to reallocate output distortions across states.

Since P_j is able to choose outputs as a function of both agents' costs, she has an advantage relative to P_i because she can impose a higher distortion in the states that are conditionally less likely. However, as shown in Section 5, this does not necessarily harm P_i.

5. Do principals share information?

Consider now principals' decision to share information. Principals' profits when they choose to share information (I) or not to share information (N) are

		P_2: I	P_2: N
P_1	I	V_I (P1), V_I (P2)	$V_{I,N}$ (P1), $V_{N,I}$ (P2)
	N	$V_{N,I}$ (P1), $V_{I,N}$ (P2)	V_N (P1), V_N (P2)

where V_I and V_N are principals' profits when they both share information and when they do not share information, respectively; $V_{I,N}$ is a principal's profit when she shares information but her opponent does not; and $V_{N,I}$ is a principal's profit when she does not share information but her opponent does. An equilibrium where both principals share information exists if and only if $V_I \geqslant V_{N,I}$, an equilibrium with no communication exists if and only if $V_N \geqslant V_{I,N}$.

The incentive for a principal to disclose her agent's cost depends on how this information affects the rival's behavior. By Proposition 4, with information sharing a principal's output depends on the opponent agent's cost; hence information sharing induces principals to coordinate distortions for strategic reasons: a *correlated distortions* effect. This increases a principal's profit if it softens competition, while it reduces a principal's profit if it increases competition. Our main result is that, when δ is small, principals' decision to share information is uniquely determined by the sign of $\delta \times \alpha$.

Proposition 6. *Assume that δ is small. When agents are privately informed about their marginal costs and principals obtain this information through contracting*:

- *If $\delta \neq 0$ and $\delta\alpha \leqslant 0$, there is a unique symmetric equilibrium in dominant strategies in which both principals share information.*
- *If $\delta\alpha > 0$, there is a unique symmetric equilibrium in dominant strategies in which no principal shares information.*
- *If $\delta = 0$, principals are indifferent between sharing information or not.*

When $\delta = 0$ communication has no effect because there is no strategic interaction between principals; hence, even if by disclosing information a principal affects a rival's output, this does not affect the principal's own output and, hence, profit. By contrast, if goods are strategic substitutes (resp. complements) and sharing information induces a rival to reduce (resp. increase) her output in the most likely states, then each principal prefers to share information about her agent's cost. Hence, the impact of the correlated distortions effect on the incentive to share information depends on the signs of δ and α.

P_i prefers to share information about θ_i if and only if $\delta\alpha < 0$. The reason is as follows. Suppose first that $\alpha > 0$ — i.e., costs are positively correlated. When P_i reveals information about θ_i, she induces P_j to distort the output of her high-cost agent relatively more (i.e., to produce less) when A_i's cost is low (so that A_i produces more) and relatively less (i.e., to produce more) when A_i's cost is high (so that A_i produces less), because the first case is less likely when costs are positively correlated. This increases P_i's profits when $\delta < 0$ — i.e., with strategic substitutes — because P_i prefers to produce less (resp. more) when her rival produces more (resp. less); while it reduces P_i's profit when $\delta > 0$ — i.e., with strategic complements — because principals prefer to produce positively correlated outputs.

Suppose now that $\alpha < 0$ — i.e., costs are negatively correlated. By revealing θ_i, P_i induces P_j to distort the output of her high-cost agent relatively more (i.e., to produce less) when A_i's cost is high (so that A_i produces less) and relatively less (i.e., to produce more) when A_i's cost is low (so that A_i produces more), because the first case is less likely. This increases P_i's profit when $\delta > 0$ — i.e., with strategic complements — because principals prefer to produce positively correlated outputs; while it reduces P_i's profits when $\delta < 0$ — i.e., with strategic substitutes — because P_i prefers to produce less (resp. more) when her rival produces more (resp. less).

The equilibria characterized in Proposition 6 are in dominant strategies. When $\delta\alpha < 0$, a principal strictly prefers to share information regardless of what her competitor does since both $V_I > V_{N,I}$ and $V_{I,N} > V_N$; when $\delta\alpha > 0$, a principal strictly prefers not to share information regardless of what her competitor does since both $V_{N,I} > V_I$ and $V_N > V_{I,N}$. Hence, there is no equilibrium in mixed strategies where principals randomize between sharing and not sharing information.

Notice that the correlated distortion effect is of first-order magnitude relative to the effects of information sharing with complete information between principals and agents, where only the sign and magnitude of α, and not δ, affect the value of communication. Hence, even if δ is small, the presence of externalities between principals has a considerable strategic effect on their choice of outputs with asymmetric information, because the *sign* of δ affects principals' decisions to share information. This is true even if the effect of δ on the distortions generated by asymmetric information between principals and agents may be small (when δ is small).

Proposition 6 extends the results of the literature on information sharing in oligopolistic markets that assumes that firms do not have to acquire the information that they may share with competitors. For example, while Shapiro (1986) shows that in a Cournot model firms always share information about their costs, we show that this is not necessarily the case when principals have to obtain information from their privately informed agents. Specifically, principals do not share information in this context when costs are negatively correlated.

The next proposition compares equilibrium expected profits when both principals share information and when they both do not share information.

Proposition 7. *If δ is small, principals' expected profits are higher when they both share information than with no communication, while agents' expected rents are higher with no communication than when both principals share information.*

Hence, when agents are privately informed about their costs, principals and agents have opposing preferences regarding information sharing. The reason is that, since costs are correlated, communication creates a positive information externality for principals that reduces agents' information rents. Specifically, when an agent's contract depends on the rival's type, information correlation generates a *relative performance evaluation* effect that relaxes the incentive compatibility constraints, thus making information acquisition less costly for principals. For δ small this effect is stronger than the strategic effect due to correlated distortions, because upstream externalities are negligible relative to the cost of information acquisition.[18]

[18] In Section 6, however, we show that this is true even when δ is large.

An implication of Propositions 6 and 7 is that, while under complete information principals' decisions regarding information sharing always maximizes their total profit, principals who need to obtain information from agents may choose not to share this information, even though they would obtain higher total profit by doing so.

Corollary 8. *Principals' decision not to share information when* $\delta\alpha > 0$ *does not maximize their joint profits.*

Hence, when cost correlation and production externalities have the same sign, principals face a prisoners' dilemma, since they independently choose not to share information, even if they would jointly benefit from coordinating on information sharing. Of course, principals would share information if they could choose their strategies cooperatively — e.g., if each principal could commit to share information if and only if the other principal does the same or if principals could sign contracts that binds them to share information (see the "contractual approach" in Gal-Or, 1985, and Raith, 1996). But our analysis suggests that information sharing agreements between principals may not be stable, because they may be vulnerable to unilateral deviations by agents.

Finally, one may wonder whether a coalition of a principal and her agent would choose to share information or not. As shown in the next lemma, if a principal and her agent can jointly decide whether to share information (e.g., by committing to a system of *ex ante* transfers within the hierarchy), the two hierarchies prefer to both share information, rather than both not share information.[19]

Lemma 3. *If* δ *is small, the joint surplus of a hierarchy — i.e., the sum of the principal's profit and the agent's rent — is higher if no principal shares information, than if both principals share information.*

The intuition for this result is that, when a hierarchy aims to maximize its joint surplus, information rents are simple transfers within the hierarchy and the benefits of information sharing for relative performance evaluation become irrelevant. Therefore, the main effect of not sharing information is to make outputs more volatile, which increases a hierarchy's joint surplus because indirect joint profit functions are convex with respect to output.

6. Large externalities

In this section, we consider principals' decision to share information when production externalities are not necessarily small, and we show that all the results of Section 5 hold as long as $|\delta|$ is not too large. Hence, if production externalities are not too strong, the correlated distortions effect of information sharing (which arises because of asymmetric information within hierarchies) is stronger than the standard effects of information sharing discussed in the literature (which arise even when agents have no private information). By contrast, if externalities are sufficiently strong, these second effects may dominate the correlated distortions effect.

First notice that, when $\alpha \to 0$, there is a unique equilibrium in which both principals share information since[20]

$$\lim_{\alpha\to 0}(V_I - V_{N,I}) = -\lim_{\alpha\to 0}(V_N - V_{I,N}) > 0.$$

The reason is that, when α is arbitrarily small, the correlated distortion effect discussed in Section 5 vanishes. But for arbitrary α and δ, principals' expected profits with and without information sharing are not comparable analytically. Hence, in order to determine whether principals share information in equilibrium, we use numerical simulations.[21] Specifically, we assume that $\nu = \frac{1}{2}$ — i.e., that high and low costs are equally likely *ex ante* — and we analyze how principals' profits depend on δ and α.

First assume that $\alpha < 0$. The first graph of Fig. 1 represents $V_N - V_{I,N}$ as a function of δ, for different values of α.[22] When $V_N - V_{I,N}$ is positive, there is an equilibrium without information sharing; when it is negative, there is no equilibrium without information sharing. The second graph of Fig. 1 represents $V_I - V_{N,I}$ as a function of δ, for different values of α. When $V_I - V_{N,I}$ is positive, there is an equilibrium with information sharing; when it is negative, there is no equilibrium with information sharing.

Fig. 1 confirms the result of Proposition 6 for $\alpha < 0$ and $\delta > 0$: there is a unique equilibrium with information sharing in this case. When $\alpha < 0$ and $\delta < 0$, there is a unique equilibrium without information sharing if negative externalities are not too large, which is consistent with Proposition 6. However, if both δ and α are sufficiently small, there may be an equilibrium with information sharing, and there may be no equilibrium without information sharing. Hence, when $\alpha < 0$, all results in Section 5 hold as long as negative externalities are not too large or costs are sufficiently correlated.

Assume now that $\alpha > 0$. Fig. 2 represents $V_N - V_{I,N}$ and $V_I - V_{N,I}$ as functions of δ, for different values of α. When $\alpha > 0$ and $\delta < 0$, Fig. 2 confirms the result of Proposition 6: there is a unique equilibrium with information sharing in

[19] Of course, in contrast to the case of complete information within hierarchies, in this case the outputs produced take into account the distortions due to agents' private information.

[20] All the functions used in this section are derived in Appendix A.

[21] All numerical simulations and figures are produced using MuPAD and Scientific WorkPlace.

[22] In all figures, we normalize $\Delta\theta = 1$ without loss of generality, since this does not affect the comparison of principals' profits.

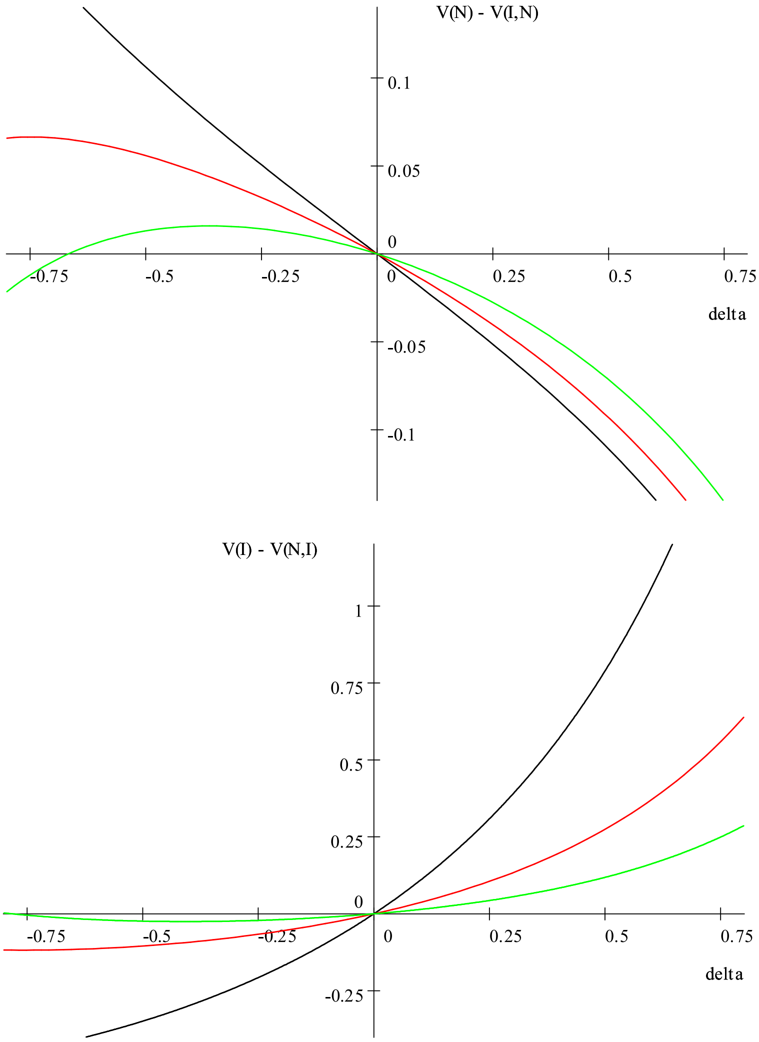

Fig. 1. Existence of equilibria with and without information sharing when $\alpha < 0$ (black line: $\alpha = -\frac{1}{5}$, red line: $\alpha = -\frac{1}{7}$, green line: $\alpha = -\frac{1}{12}$). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

this case. When $\alpha > 0$ and $\delta > 0$, there is a unique equilibrium without information sharing if positive externalities are not too strong, which is consistent with Proposition 6. However, if δ is sufficiently large there may also be an equilibrium with information sharing; and if δ is sufficiently large and α is sufficiently small, there may not be an equilibrium without information sharing. Hence, when $\alpha > 0$, all results in Section 5 hold as long as positive externalities are not too large.

The numerical simulations show that, when $|\delta|$ is sufficiently large, principals' incentive to share information in order to coordinate their outputs — by producing less (more) with negative (positive) externalities when the rival produces more (e.g., Shapiro, 1986) — may dominate the correlated distortions effect of information sharing discussed in Section 5. This is especially true when costs are positively correlated, and may result in information sharing always being an equilibrium. Moreover, when δ is large, there may be multiple equilibria, with and without information sharing.

Finally, numerical simulations also show that $V_I - V_N$ is positive even when $|\delta|$ is large, so that principals' expected profits are always higher when they both share information. This generalizes the result of Proposition 7 and confirms that principals face a prisoners' dilemma when they independently choose not to share information.

7. Agents' collusion

In our analysis, we have assumed that, when an agent contracts with his principal, he believes that the other agent makes a truthful report to his own principal. With information sharing, however, the expected utility of an agent is affected by his opponent's report, because his principal's payoff depends on it. In this context, agents may have an incentive to coordinate on an equilibrium in which they both misreport their type in order to obtain higher rents at the expense of principals.

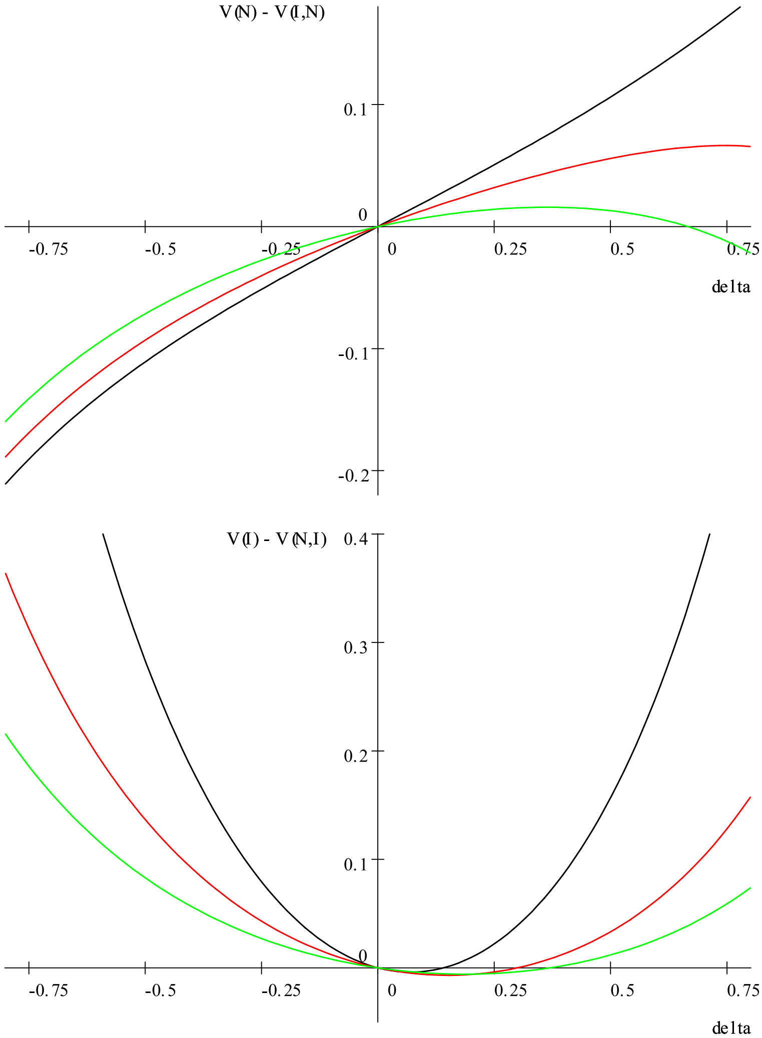

Fig. 2. Existence of equilibria with and without information sharing when $\alpha > 0$ (black line: $\alpha = \frac{1}{5}$, red line: $\alpha = \frac{1}{7}$, green line: $\alpha = \frac{1}{12}$). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Hence, a natural question that arises in our model is whether the equilibrium with information sharing in which each agent truthfully reports his type expecting the rival to do the same is robust to collusion among agents.

In order to address this issue, in this section we consider the possibility of implicit collusion among agents, assuming that agents cannot make side payments.[23] Let $t^e(\theta_i, \theta_j)$ be the equilibrium transfer paid to A_i when both principals share information. In order for agents to truthfully report their costs, a low-cost agent must prefer to truthfully report his cost, rather than lie, when his low-cost rival lies and reports a high cost — i.e.,

$$t^e(\underline{\theta}, \bar{\theta}) - \underline{\theta} q^e(\underline{\theta}, \bar{\theta}) > t^e(\bar{\theta}, \bar{\theta}) - \underline{\theta} q^e(\bar{\theta}, \bar{\theta}). \tag{11}$$

But the equilibrium transfers with information sharing are indeterminate in some states, because agents make their reports before learning the rival's type. Hence, the number of constraints that bind in a truthful equilibrium is smaller than the number of instruments available to principals.

Notice that the limited liability constraints of a high-cost agent imply that $t^e(\bar{\theta}, \theta_j) = \bar{\theta} q^e(\bar{\theta}, \theta_j)$, for all θ_j. Hence, (11) is equivalent to

$$t^e(\underline{\theta}, \bar{\theta}) - \underline{\theta} q^e(\underline{\theta}, \bar{\theta}) > \Delta\theta q^e(\bar{\theta}, \bar{\theta}). \tag{12}$$

[23] When agents' collusion can be enforced through side transfers, there is no reason to exclude side transfers among principals. In this case, the analysis is equivalent to Laffont and Martimort (2000), where a single principal — i.e., the coalition formed by P_1 and P_2 — contracts with two colluding and privately informed agents, with correlated types.

This implies that agents have no incentive to collude and misreport their types if principals — actually even only one of them — implement a transfer $t^e(\underline{\theta}, \bar{\theta})$ such that: (i) agents tell the truth when rivals are expected to do so; (ii) limited liability constraints are satisfied in all states; (iii) inequality (12) is satisfied.

Proposition 9. *The equilibrium characterized in Proposition 4 is robust to the threat of implicit collusion between agents when transfers are*

$$t^e(\bar{\theta}, \theta_j) = \bar{\theta} q^e(\bar{\theta}, \theta_j) \quad \forall \theta_j \in \Theta, \qquad t^e(\underline{\theta}, \underline{\theta}) = \underline{\theta} q^e(\underline{\theta}, \underline{\theta}),$$
$$t^e(\underline{\theta}, \bar{\theta}) = \underline{\theta} q^e(\underline{\theta}, \bar{\theta}) + \Delta\theta \frac{\Pr(\underline{\theta}|\underline{\theta})}{\Pr(\bar{\theta}|\underline{\theta})} q^e(\bar{\theta}, \underline{\theta}) + \Delta\theta q^e(\bar{\theta}, \bar{\theta}).$$

Therefore, our results are robust to the possibility of implicit collusion between agents, since principals can choose transfers that induce an equilibrium in which they choose to share information and agents truthfully report their private information.[24]

8. Conclusions

In order to explore the effects of information sharing between complex organizations, we have considered two principals who produce externalities on each other, and independently choose whether to commit to share the information they obtain when they contract with exclusive and privately informed agents. In this context, we have highlighted a novel effect of information sharing, that may be stronger than the effects discussed in the previous literature. Specifically, principals' incentive to share information depends on the effects on their profits, through the production externalities, of the output distortions generated to induce agents to reveal their information, because information sharing makes these distortions correlated.

When externalities between principals are not too large, principals choose to share information if and only if externalities have an opposite sign than the correlation of agents' private information. In contrast to the case of complete information within organizations, principals may face a *prisoners' dilemma* when agents have private information, because they may independently choose not to share information, even though an information sharing agreement would increase their profits. Our results are robust to the threat of (implicit) collusion among agents.

Appendix A

Proof of Lemma 1. With complete information, principals fully extract their agents' rents. We characterize the equilibrium outputs in the three possible cases: (i) both principals share information; (ii) no principal shares information; (iii) only one principal shares information.

No information sharing When principals do not share information, the (symmetric) equilibrium output is

$$q^*(\theta_i) = \arg\max_{q_i(.)} \sum_{\theta_j} \Pr(\theta_j|\theta_i)\big[S\big(q_i(\theta_i), q^*(\theta_j)\big) - \theta_i q_i(\theta_i)\big] \quad \forall \theta_i \in \Theta,$$

and the equilibrium transfer, $t^*(\theta_i)$, is such that

$$U_i(\theta_i) = 0 \quad \Rightarrow \quad t^*(\theta_i) = \theta_i q^*(\theta_i) \quad \forall \theta_i \in \Theta.$$

Hence, a symmetric equilibrium satisfies the following necessary and sufficient first-order conditions

$$\sum_{\theta_j} \Pr(\theta_j|\theta_i) S_1\big(q^*(\theta_i), q^*(\theta_j)\big) = \theta_i \quad \forall \theta_i \in \Theta, \tag{A.1}$$

where $S_1(.)$ denotes the partial derivative of $S(q_i, q_j)$ with respect to q_i. Solving these conditions yields

$$q^*(\underline{\theta}) = \frac{\beta - \underline{\theta}}{2-\delta} - \frac{\delta(1-\nu)(\nu(1-\nu)-\alpha)}{(2-\delta)(2\nu(1-\nu)-\alpha\delta)} \Delta\theta, \qquad q^*(\bar{\theta}) = q^*(\underline{\theta}) - \frac{\nu(1-\nu)}{2\nu(1-\nu)-\alpha\delta} \Delta\theta.$$

[24] Of course, this depends on the assumption that there are no production externalities across agents, otherwise information sharing may also affect agents' incentives to jointly misreport their types (see, e.g., Martimort, 1996).

Bilateral information sharing When both principals share information, the equilibrium output is

$$q^*(\theta_i,\theta_j) = \arg\max_{q_i(.,.)} \big[S\big(q_i(\theta_i,\theta_j), q^*(\theta_j,\theta_i)\big) - \theta_i q_i(\theta_i,\theta_j)\big] \quad \forall(\theta_i,\theta_j)\in\Theta^2,$$

and the equilibrium transfer, $t_i^*(\theta_i,\theta_j)$, is such that

$$U_i(\theta_i,\theta_j)=0 \quad \Rightarrow \quad t_i^*(\theta_i,\theta_j)=\theta_i q_i^*(\theta_i,\theta_j) \quad \forall(\theta_i,\theta_j)\in\Theta^2.$$

The first-order necessary and sufficient conditions are

$$S_1\big(q^*(\theta_i,\theta_j), q^*(\theta_j,\theta_i)\big)=\theta_i \quad \forall(\theta_i,\theta_j)\in\Theta^2. \tag{A.2}$$

Solving (A.2), outputs in the unique equilibrium are

$$q^*(\underline{\theta},\underline{\theta})=\frac{\beta-\underline{\theta}}{2-\delta}, \qquad q^*(\underline{\theta},\bar{\theta})=q^*(\underline{\theta},\underline{\theta})-\frac{\delta}{4-\delta^2}\Delta\theta,$$
$$q^*(\bar{\theta},\underline{\theta})=q^*(\underline{\theta},\underline{\theta})-\frac{2}{4-\delta^2}\Delta\theta, \qquad q^*(\bar{\theta},\bar{\theta})=q^*(\underline{\theta},\underline{\theta})-\frac{1}{2-\delta}\Delta\theta.$$

Unilateral information sharing Finally, suppose that one principal, say P_i, discloses her agent's cost, while P_j does not share information. For each θ_i, P_i's optimization program is

$$\max_{q_i(.)} \sum_{\theta_j} \Pr(\theta_j|\theta_i) S\big(q_i(\theta_i), q_j^*(\theta_j,\theta_i)\big) - \theta_i q_i(\theta_i),$$

and, for each (θ_i,θ_j), P_j's optimization program is

$$\max_{q_j(.,.)} \big[S\big(q_j(\theta_j,\theta_i), q_i^*(\theta_i)\big) - \theta_j q_j(\theta_j,\theta_i)\big].$$

The first-order necessary and sufficient conditions are

$$\sum_{\theta_j} \Pr(\theta_j|\theta_i) S_1\big(q_i^*(\theta_i), q_j^*(\theta_j,\theta_i)\big)=\theta_i \quad \forall\theta_i\in\Theta, \tag{A.3}$$

$$S_1\big(q_j^*(\theta_j,\theta_i), q_i^*(\theta_i)\big)=\theta_j \quad \forall(\theta_i,\theta_j)\in\Theta^2, \tag{A.4}$$

yielding the equilibrium outputs

$$q_i^*(\underline{\theta})=\frac{\beta-\underline{\theta}}{2-\delta}-\frac{\delta(\nu(1-\nu)-\alpha)}{\nu(4-\delta^2)}\Delta\theta, \qquad q_i^*(\bar{\theta})=q_i^*(\underline{\theta})-\frac{2\nu(1-\nu)+\alpha\delta}{\nu(1-\nu)(4-\delta^2)}\Delta\theta,$$
$$q_j^*(\underline{\theta},\underline{\theta})=q^*(\underline{\theta},\underline{\theta})-\frac{(\nu(1-\nu)-\alpha)\delta^2}{2\nu(4-\delta^2)}\Delta\theta, \qquad q_j^*(\underline{\theta},\bar{\theta})=q^*(\underline{\theta},\bar{\theta})-\frac{(\alpha+(1-\nu)^2)\delta^2}{2(1-\nu)(4-\delta^2)}\Delta\theta,$$
$$q_j^*(\bar{\theta},\underline{\theta})=q^*(\bar{\theta},\underline{\theta})+\frac{(\nu^2+\alpha)\delta^2}{2\nu(4-\delta^2)}\Delta\theta, \qquad q_j^*(\bar{\theta},\bar{\theta})=q^*(\bar{\theta},\bar{\theta})+\frac{(\nu(1-\nu)-\alpha)\delta^2}{2(1-\nu)(4-\delta^2)}\Delta\theta.$$

Expected outputs and profits Consider now expected outputs in the three cases. Letting $q^* \equiv q^*(\underline{\theta},\underline{\theta})-\frac{1-\nu}{2-\delta}\Delta\theta$, it follows that

$$\begin{aligned} q^* &= \sum_{\theta_i}\Pr(\theta_i)q_i^*(\theta_i)=\sum_{\theta_i}\Pr(\theta_i)\sum_{\theta_j}\Pr(\theta_j|\theta_i)q_i^*(\theta_i,\theta_j)\\ &=\sum_{\theta_i}\Pr(\theta_i)q^*(\theta_i)=\sum_{\theta_i}\Pr(\theta_i)\sum_{\theta_j}\Pr(\theta_j|\theta_i)q^*(\theta_i,\theta_j)\\ &=\sum_{\theta_i}\Pr(\theta_i)q_i^*(\theta_i)=\sum_{\theta_j}\Pr(\theta_j)\sum_{\theta_i}\Pr(\theta_i|\theta_j)q_j^*(\theta_j,\theta_i). \end{aligned}$$

Using conditions (A.1), (A.2), (A.3) and (A.4), principals' expected profits are

$$\begin{aligned} V_i^* &= \kappa + \sum_{\theta_i}\Pr(\theta_i)\sum_{\theta_j}\Pr(\theta_j|\theta_i)q_i^*(s_i)^2\\ &= \kappa + \underbrace{\Big[\sum_{\theta_i}\Pr(\theta_i)\sum_{\theta_j}\Pr(\theta_j|\theta_i)\tilde{q}_i(s_i)\Big]^2}_{\text{average } \tilde{q}_i(s_i)} + \underbrace{\sum_{\theta_i}\Pr(\theta_i)\sum_{\theta_j}\Pr(\theta_j|\theta_i)\Big[\tilde{q}_i(s_i)-\sum_{\theta_i}\Pr(\theta_i)\sum_{\theta_j}\Pr(\theta_j|\theta_i)\tilde{q}_i(s_i)\Big]^2}_{\text{variance of } \tilde{q}_i(s_i)}, \end{aligned}$$

where the second equality follows because $\mathbb{E}[x^2]=(\mathbb{E}[x])^2+\mathbb{E}[x-\mathbb{E}[x]]^2$. □

Proof of Proposition 1. Let V_I^* and V_N^* be principals' expected profits when they both share information and when they do not share information, respectively. Let $V_{N,I}^*$ be P_i's profit and $V_{I,N}^*$ be P_j's profit when P_i does not share information while P_j shares information.

A symmetric equilibrium where both principals share information exists if and only if $V_I^*\geqslant V_{N,I}^*$. Assuming that δ is small but different from 0 and using a second-order Taylor approximation around $\delta=0$, we have

$$\begin{aligned}V_{N,I}^* &\approx \kappa+\lim_{\delta\to 0}\sum_{\theta_i}\Pr(\theta_i)\sum_{\theta_j}\Pr(\theta_j|\theta_i)q_i^*(\theta_i,\theta_j)^2\\ &\quad+2\delta\lim_{\delta\to 0}\sum_{\theta_i}\Pr(\theta_i)\sum_{\theta_j}\Pr(\theta_j|\theta_i)q_i^*(\theta_i,\theta_j)\frac{\partial q_i^*(\theta_i,\theta_j)}{\partial\delta}\\ &\quad+\delta^2\lim_{\delta\to 0}\sum_{\theta_i}\Pr(\theta_i)\sum_{\theta_j}\Pr(\theta_j|\theta_i)\left[q_i^*(\theta_i,\theta_j)\frac{\partial^2 q_i^*(\theta_i,\theta_j)}{\partial\delta^2}+\left(\frac{\partial q_i^*(\theta_i,\theta_j)}{\partial\delta}\right)^2\right],\end{aligned}$$

and

$$\begin{aligned}V_I^* &\approx \kappa+\lim_{\delta\to 0}\sum_{\theta_i}\Pr(\theta_i)\sum_{\theta_j}\Pr(\theta_j|\theta_i)q^*(\theta_i,\theta_j)^2\\ &\quad+2\delta\lim_{\delta\to 0}\sum_{\theta_i}\Pr(\theta_i)\sum_{\theta_j}\Pr(\theta_j|\theta_i)q^*(\theta_i,\theta_j)\frac{\partial q^*(\theta_i,\theta_j)}{\partial\delta}\\ &\quad+\delta^2\lim_{\delta\to 0}\sum_{\theta_i}\Pr(\theta_i)\sum_{\theta_j}\Pr(\theta_j|\theta_i)\left[q^*(\theta_i,\theta_j)\frac{\partial^2 q^*(\theta_i,\theta_j)}{\partial\delta^2}+\left(\frac{\partial q^*(\theta_i,\theta_j)}{\partial\delta}\right)^2\right].\end{aligned}$$

Using the equilibrium outputs from Lemma 1, we have

$$\lim_{\delta\to 0}\sum_{\theta_i}\Pr(\theta_i)\sum_{\theta_j}\Pr(\theta_j|\theta_i)q_i^*(\theta_i,\theta_j)^2=\lim_{\delta\to 0}\sum_{\theta_i}\Pr(\theta_i)\sum_{\theta_j}\Pr(\theta_j|\theta_i)q^*(\theta_i,\theta_j)^2,$$

and

$$\lim_{\delta\to 0}\sum_{\theta_i}\Pr(\theta_i)\sum_{\theta_j}\Pr(\theta_j|\theta_i)q_i^*(\theta_i,\theta_j)\frac{\partial q_i^*(\theta_i,\theta_j)}{\partial\delta}=\lim_{\delta\to 0}\sum_{\theta_i}\Pr(\theta_i)\sum_{\theta_j}\Pr(\theta_j|\theta_i)q^*(\theta_i,\theta_j)\frac{\partial q^*(\theta_i,\theta_j)}{\partial\delta}.$$

Hence,

$$\begin{aligned}V_I^*-V_{N,I}^* &\approx \delta^2\Bigg\{\lim_{\delta\to 0}\sum_{\theta_i}\Pr(\theta_i)\sum_{\theta_j}\Pr(\theta_j|\theta_i)q^*(\theta_i,\theta_j)\frac{\partial^2 q^*(\theta_i,\theta_j)}{\partial\delta^2}\\ &\quad-\lim_{\delta\to 0}\sum_{\theta_i}\Pr(\theta_i)\sum_{\theta_j}\Pr(\theta_j|\theta_i)q_i^*(\theta_i,\theta_j)\frac{\partial^2 q_i^*(\theta_i,\theta_j)}{\partial\delta^2}+\lim_{\delta\to 0}\sum_{\theta_i}\Pr(\theta_i)\sum_{\theta_j}\Pr(\theta_j|\theta_i)\left(\frac{\partial q^*(\theta_i,\theta_j)}{\partial\delta}\right)^2\\ &\quad-\lim_{\delta\to 0}\sum_{\theta_i}\Pr(\theta_i)\sum_{\theta_j}\Pr(\theta_j|\theta_i)\left(\frac{\partial q_i^*(\theta_i,\theta_j)}{\partial\delta}\right)^2\Bigg\}\approx\frac{(\nu(1-\nu)+\alpha)\Pr(\bar\theta,\underline\theta)\delta^2\Delta\theta^2}{8\nu(1-\nu)}.\end{aligned}\tag{A.5}$$

Therefore, there is a symmetric equilibrium where both principals share information if and only if $\nu(1-\nu)+\alpha\geqslant 0$ — i.e., if $\alpha\geqslant 0$ or if $\alpha<0$ and $|\alpha|\leqslant\nu(1-\nu)$.

A symmetric equilibrium where principals do not share information exists if and only if $V_N^*\geqslant V_{I,N}^*$. Assuming that δ is small but different from 0 and using a second-order Taylor approximation around $\delta=0$, we have

$$\begin{aligned}V_{I,N}^* &\approx \kappa+\lim_{\delta\to 0}\sum_{\theta_j}\Pr(\theta_j)q_j^*(\theta_j)^2+2\delta\lim_{\delta\to 0}\sum_{\theta_j}\Pr(\theta_j)q_j^*(\theta_j)^2\frac{\partial q_j^*(\theta_j)}{\partial\delta}\\ &\quad+\delta^2\lim_{\delta\to 0}\sum_{\theta_j}\Pr(\theta_j)\left[q_j^*(\theta_j)^2\frac{\partial^2 q_j^*(\theta_j)}{\partial\delta^2}+\left(\frac{\partial q_j^*(\theta_j)}{\partial\delta}\right)^2\right],\end{aligned}$$

and

$$V_N^* \approx \kappa + \lim_{\delta\to 0}\sum_{\theta_i}\Pr(\theta_i)q^*(\theta_i)^2 + 2\delta\lim_{\delta\to 0}\sum_{\theta_i}\Pr(\theta_i)q^*(\theta_i)\frac{\partial q^*(\theta_i)}{\partial\delta} + \delta^2\lim_{\delta\to 0}\sum_{\theta_i}\Pr(\theta_i)\left[q^*(\theta_i)\frac{\partial^2 q^*(\theta_i)}{\partial\delta^2} + \left(\frac{\partial q^*(\theta_i)}{\partial\delta}\right)^2\right].$$

Using the equilibrium outputs from Lemma 1, we have

$$\lim_{\delta\to 0}\sum_{\theta_j}\Pr(\theta_j)q_j^*(\theta_j)^2 = \lim_{\delta\to 0}\sum_{\theta_j}\Pr(\theta_j)q^*(\theta_j)^2,$$

and

$$\lim_{\delta\to 0}\sum_{\theta_j}\Pr(\theta_j)q_j^*(\theta_j)^2\frac{\partial q_j^*(\theta_j)}{\partial\delta} = \lim_{\delta\to 0}\sum_{\theta_j}\Pr(\theta_j)q^*(\theta_j)^2\frac{\partial q^*(\theta_j)}{\partial\delta}.$$

Hence,

$$V_N^* - V_{I,N}^* \approx \delta^2\lim_{\delta\to 0}\left[\sum_{\theta_j}\Pr(\theta_j)q^*(\theta_j)^2\frac{\partial^2 q^*(\theta_j)}{\partial\delta^2} - \sum_{\theta_j}\Pr(\theta_j)q_j^*(\theta_j)^2\frac{\partial^2 q_j^*(\theta_j)}{\partial\delta^2} + \sum_{\theta_j}\Pr(\theta_j)\left(\frac{\partial q^*(\theta_j)}{\partial\delta}\right)^2 - \sum_{\theta_j}\Pr(\theta_j)\left(\frac{\partial q_j^*(\theta_j)}{\partial\delta}\right)^2\right] \approx -\frac{(\nu(1-\nu)+\alpha)\Pr(\bar\theta,\underline\theta)\delta^2\Delta\theta^2}{8\nu(1-\nu)}. \tag{A.6}$$

Therefore, there is a symmetric equilibrium where principals do not share information if and only if α is negative and sufficiently low — i.e., if $\alpha < -\nu(1-\nu)$.

Because the sign of (A.5) is always opposite to the sign of (A.6), the equilibria are in dominant strategies for $\delta \neq 0$. Finally, if $\delta = 0$ or $\Pr(\bar\theta,\underline\theta) = 0$, information sharing has no impact on principals' profits — i.e., $V_N^* = V_{I,N}^* = V_I^* = V_{N,I}^*$. Hence, principals are indifferent between sharing information or not. □

Proof of Proposition 2. We compare principals' equilibrium profit when they both share information, V_I^*, with principals' equilibrium profit when they do not share information, V_N^*. By Lemma 1, $V_I^* = V_N^*$ for $\delta = 0$ and

$$V_I^* - V_N^* = \frac{\nu(1-\nu)(12-\delta^2)\delta^2\Delta\theta^2}{4(2+\delta)^2(2-\delta)^2} > 0$$

for $\alpha = 0$. Suppose that $\delta \neq 0$. Using the Taylor approximations for V_I^* and V_N^* in the proof of Proposition 1,

$$V_I^* - V_N^* \approx \delta^2\lim_{\delta\to 0}\left[\sum_{\theta_i}\Pr(\theta_i)\sum_{\theta_j}\Pr(\theta_j|\theta_i)q^*(\theta_i,\theta_j)\frac{\partial^2 q^*(\theta_i,\theta_j)}{\partial\delta^2} - \sum_{\theta_i}\Pr(\theta_i)q^*(\theta_i)\frac{\partial^2 q^*(\theta_i)}{\partial\delta^2} + \sum_{\theta_i}\Pr(\theta_i)\sum_{\theta_j}\Pr(\theta_j|\theta_i)\left(\frac{\partial q^*(\theta_i,\theta_j)}{\partial\delta}\right)^2 - \sum_{\theta_i}\Pr(\theta_i)\left(\frac{\partial q^*(\theta_i)}{\partial\delta}\right)^2\right] \approx \frac{3\Pr(\bar\theta,\underline\theta)(\nu(1-\nu)+\alpha)\delta^2\Delta\theta^2}{16\nu(1-\nu)}.$$

This is positive if and only if $\nu(1-\nu)+\alpha > 0$ — i.e., if and only if each principal strictly prefers to share information. □

Proof of Proposition 3. Equilibrium outputs are computed by solving the system of first-order conditions (1) and (2). Moreover, $\sum_{\theta_i}\Pr(\theta_i)q^e(\theta_i) = \frac{\beta-\bar\theta}{2-\delta}$ and $q^e(\underline\theta) - q^e(\bar\theta) = \frac{\nu\Delta\theta}{2\nu(1-\nu)-\alpha\delta}$. □

Proof of Lemma 2. Differentiating $q^e(\underline\theta) - q^e(\bar\theta)$ with respect to α,

$$sign\frac{\partial[q^e(\underline\theta) - q^e(\bar\theta)]}{\partial\alpha} = sign\frac{\delta\nu}{(2\nu(1-\nu)-\alpha\delta)^2}. \quad \square$$

Proof of Proposition 4. Solving the system of first-order conditions (5) and (6) yields the equilibrium outputs with bilateral information sharing. Therefore,

$$\sum_{\theta_i}\Pr(\theta_i)\sum_{\theta_j}\Pr(\theta_j|\theta_i)q^e(\theta_i,\theta_j)=\nu\left[\left(\nu+\frac{\alpha}{\nu}\right)q^e(\underline{\theta},\underline{\theta})+\left(1-\nu-\frac{\alpha}{\nu}\right)q^e(\underline{\theta},\bar{\theta})\right]$$
$$+(1-\nu)\left[\left(\nu-\frac{\alpha}{1-\nu}\right)q^e(\bar{\theta},\underline{\theta})+\left(1-\nu+\frac{\alpha}{1-\nu}\right)q^e(\bar{\theta},\bar{\theta})\right]$$
$$=\frac{\beta-\bar{\theta}}{2-\delta},$$

and

$$\sum_{\theta_i}\Pr(\theta_i)q^e(\theta_i)=\nu q^e(\underline{\theta})+(1-\nu)q^e(\bar{\theta})=\frac{\beta-\bar{\theta}}{2-\delta}.$$

Hence, $\sum_{\theta_i}\Pr(\theta_i)\sum_{\theta_j}\Pr(\theta_j|\theta_i)q^e(\theta_i,\theta_j)=\sum_{\theta_i}\Pr(\theta_i)q^e(\theta_i)$. Moreover,

$$\sum_{\theta_i}\Pr(\theta_i)q^e(\theta_i)-\sum_{\theta_i}\Pr(\theta_i)q^*(\theta_i)=\frac{\beta-\bar{\theta}}{2-\delta}-\frac{\beta-\underline{\theta}-(1-\nu)\Delta\theta}{2-\delta}=-\frac{\nu\Delta\theta}{2-\delta}<0.$$

The rest of the proof is straightforward. □

Proof of Proposition 5. Solving the system of first-order conditions (7), (8), (9) and (10) yields the equilibrium outputs with unilateral information sharing. Moreover, the expected output of both principals is

$$\sum_{\theta_i}\Pr(\theta_i)q_i^e(\theta_i)=\sum_{\theta_j}\Pr(\theta_j)\sum_{\theta_i}\Pr(\theta_i|\theta_j)q_j^e(\theta_j,\theta_i)=\frac{\beta-\bar{\theta}}{2-\delta},$$

which is equal to the expected output when both principals share information. □

Proof of Proposition 6. Suppose first that both α and δ are different from 0. A symmetric equilibrium where both principals share information exists if and only if $V_I\geqslant V_{N,I}$. Using a second-order Taylor approximation around $\delta=0$,

$$V_I\approx\kappa+\lim_{\delta\to 0}\sum_{\theta_i}\Pr(\theta_i)\sum_{\theta_j}\Pr(\theta_j|\theta_i)q^e(\theta_i,\theta_j)^2+2\delta\lim_{\delta\to 0}\sum_{\theta_i}\Pr(\theta_i)\sum_{\theta_j}\Pr(\theta_j|\theta_i)q^e(\theta_i,\theta_j)\frac{\partial q^e(\theta_i,\theta_j)}{\partial\delta},\tag{A.7}$$

and

$$V_{N,I}\approx\kappa+\lim_{\delta\to 0}\sum_{\theta_i}\Pr(\theta_i)q_i^e(\theta_i)^2+2\delta\lim_{\delta\to 0}\sum_{\theta_i}\Pr(\theta_i)q_i^e(\theta_i)\frac{\partial q_i^e(\theta_i)}{\partial\delta}.$$

Hence,

$$V_I-V_{N,I}\approx\lim_{\delta\to 0}\left\{\sum_{\theta_i}\Pr(\theta_i)\left[\sum_{\theta_j}\Pr(\theta_j|\theta_i)q^e(\theta_i,\theta_j)^2-q_i^e(\theta_i)^2\right]\right\}$$
$$+2\delta\lim_{\delta\to 0}\left\{\sum_{\theta_i}\Pr(\theta_i)\left[\sum_{\theta_j}\Pr(\theta_j|\theta_i)q^e(\theta_i,\theta_j)\frac{\partial q^e(\theta_i,\theta_j)}{\partial\delta}-q_i^e(\theta_i)\frac{\partial q_i^e(\theta_i)}{\partial\delta}\right]\right\}.$$

And, using the outputs characterized in Propositions 4 and 5,

$$V_I-V_{N,I}\approx-\frac{\alpha\delta\Delta\theta^2}{4(\alpha+(1-\nu)^2)}.\tag{A.8}$$

Therefore, there is a symmetric equilibrium where both principals share information if and only if $\alpha\delta<0$.

A symmetric equilibrium where principals do not share information exists if and only if $V_N\geqslant V_{I,N}$. Using a second-order Taylor approximation around $\delta=0$,

$$V_N\approx\kappa+\lim_{\delta\to 0}\sum_{\theta_i}\Pr(\theta_i)q^e(\theta_i)^2+2\delta\lim_{\delta\to 0}\sum_{\theta_i}\Pr(\theta_i)q^e(\theta_i)\frac{\partial q^e(\theta_i)}{\partial\delta},\tag{A.9}$$

and

$$V_{I,N} \approx \kappa + \lim_{\delta\to 0}\sum_{\theta_i}\Pr(\theta_i)q_i^e(\theta_i)^2 + 2\delta\lim_{\delta\to 0}\sum_{\theta_i}\Pr(\theta_i)q_i^e(\theta_i)\frac{\partial q_i^e(\theta_i)}{\partial\delta}.$$

Hence,

$$V_N - V_{I,N} \approx \lim_{\delta\to 0}\left[\sum_{\theta_i}\Pr(\theta_i)q^e(\theta_i)^2 - \sum_{\theta_i}\Pr(\theta_i)q_i^e(\theta_i)^2\right] + 2\delta\lim_{\delta\to 0}\left[\sum_{\theta_i}\Pr(\theta_i)q^e(\theta_i)\frac{\partial q^e(\theta_i)}{\partial\delta} - \sum_{\theta_i}\Pr(\theta_i)q_i^e(\theta_i)\frac{\partial q_i^e(\theta_i)}{\partial\delta}\right].$$

And, using the outputs characterized in Propositions 3 and 5,

$$V_N - V_{I,N} \approx \frac{\delta\alpha\Delta\theta^2}{4(1-\nu)^2}. \tag{A.10}$$

Therefore, there is a symmetric equilibrium where both principals do not share information if and only if $\delta\alpha > 0$.

Clearly, $\lim_{\delta\to 0}(V_I - V_{N,I}) = \lim_{\delta\to 0}(V_N - V_{I,N}) = 0$. While

$$\lim_{\alpha\to 0}(V_I - V_{N,I}) = \frac{\nu(8-\delta^2)\delta^2\Delta\theta^2}{4(1-\nu)(2+\delta)^2(2-\delta)} > 0$$

and

$$\lim_{\alpha\to 0}(V_N - V_{I,N}) = -\frac{\nu(8-\delta^2)\delta^2\Delta\theta^2}{4(1-\nu)(2+\delta)^2(2-\delta)} < 0.$$

Because the sign of (A.8) is always opposite to the sign of (A.10), the equilibria are in dominant strategies. □

Proof of Proposition 7. Suppose that both δ and α are different from 0. Using (A.7), (A.9), and the equilibrium outputs in Propositions 3 and 4,

$$\begin{aligned} V_I - V_N &\approx \lim_{\delta\to 0}\left\{\sum_{\theta_i}\Pr(\theta_i)\sum_{\theta_j}\Pr(\theta_j|\theta_i)q^e(\theta_i,\theta_j)^2 - \sum_{\theta_i}\Pr(\theta_i)q^e(\theta_i)^2\right\} \\ &\quad + 2\delta\lim_{\delta\to 0}\left\{\sum_{\theta_i}\Pr(\theta_i)\sum_{\theta_j}\Pr(\theta_j|\theta_i)q^e(\theta_i,\theta_j)\frac{\partial q^e(\theta_i,\theta_j)}{\partial\delta} - \sum_{\theta_i}\Pr(\theta_i)q^e(\theta_i)\frac{\partial q^e(\theta_i)}{\partial\delta}\right\} \\ &\approx \frac{\Delta\theta^2}{4(1-\nu)((1-\nu)^2+\alpha)}\left[\frac{\alpha^2}{\nu(1-\nu)-\alpha} - \frac{\alpha\delta(\alpha+2(1-\nu)^2)}{1-\nu}\right], \end{aligned} \tag{A.11}$$

which is strictly positive for δ small and different than 0. Moreover,

$$\lim_{\alpha\to 0}(V_I - V_N) = \frac{(12-\delta^2)\Delta\theta^2\delta^2\nu}{4(1-\nu)(2+\delta)^2(2-\delta)^2} > 0.$$

Consider now agents' expected rents and let $U^e(s_i)$ be A_i's equilibrium utility when the information upon which P_i conditions her contract is s_i, $s_i \in \{\theta_i, (\theta_1,\theta_2)\}$. Without information sharing,

$$\sum_{\theta_i}\Pr(\theta_i)U^e(\theta_i) = \nu\Delta\theta q^e(\bar\theta). \tag{A.12}$$

When instead both principals share information,

$$\sum_{\theta_i}\Pr(\theta_i)\sum_{\theta_j}\Pr(\theta_j)U^e(\theta_i,\theta_j) = \nu\Delta\theta\sum_{\theta_j}\Pr(\theta_j|\underline{\theta})q^e(\bar\theta,\theta_j). \tag{A.13}$$

The difference between (A.13) and (A.12) is

$$\nu\Delta\theta\left[\sum_{\theta_j}\Pr(\theta_j|\underline{\theta})q^e(\bar\theta,\theta_j) - q^e(\bar\theta)\right].$$

First, notice that

$$\nu\Delta\theta \lim_{\alpha\to 0}\Bigg[\sum_{\theta_j}\Pr(\theta_j|\underline{\theta})q^e(\bar{\theta},\theta_j)-q^e(\bar{\theta})\Bigg]=-\frac{\nu^2\delta^2\Delta\theta^2}{2(1-\nu)(2-\delta)(2+\delta)}<0.$$

Suppose now that $\alpha \neq 0$ and that δ is small. Using a first-order Taylor approximation,

$$\nu\Delta\theta\Bigg[\sum_{\theta_j}\Pr(\theta_j|\underline{\theta})q^e(\bar{\theta},\theta_j)-q^e(\bar{\theta})\Bigg]\approx-\frac{\Delta\theta^2}{2(1-\nu)(\alpha+(1-\nu)^2)}\Bigg[\frac{\alpha^2}{\nu(1-\nu)-\alpha}-\frac{\delta\alpha(\alpha\nu+(1-\nu)^2(1+\nu))}{2(1-\nu)}\Bigg],$$

which is negative for δ small. □

Proof of Lemma 3. Let $W_i = S(q_i,q_j)-\theta_i q_i$ be the joint profit of a hierarchy. When both principals do not share information, a first-order Taylor approximation of the equilibrium joint profit yields

$$W_N \approx \kappa + \lim_{\delta\to 0}\sum_{\theta_i}\Pr(\theta_i)\Bigg[\beta - q^e(\theta_i)+\delta\sum_{\theta_j}\Pr(\theta_j|\theta_i)q^e(\theta_j)-\theta_i\Bigg]q^e(\theta_i)$$
$$+\,\delta\lim_{\delta\to 0}\sum_{\theta_i}\Pr(\theta_i)\Bigg\{\Bigg[\beta-2q^e(\theta_i)+\delta\sum_{\theta_j}\Pr(\theta_j|\theta_i)q^e(\theta_j)-\theta_i\Bigg]\frac{\partial q^e(\theta_i)}{\partial\delta}+\delta q^e(\theta_i)\sum_{\theta_j}\Pr(\theta_j|\theta_i)\frac{\partial q^e(\theta_j)}{\partial\delta}\Bigg\},$$

where $q^e(\theta_i)$ is defined in Proposition 3. When instead both principals share information, a first-order Taylor approximation of the equilibrium joint profit yields

$$W_I \approx \kappa + \lim_{\delta\to 0}\sum_{\theta_i}\Pr(\theta_i)\sum_{\theta_j}\Pr(\theta_j|\theta_i)\big[\beta-q^e(\theta_i,\theta_j)+\delta q^e(\theta_j,\theta_i)-\theta_i\big]q^e(\theta_i,\theta_j)$$
$$+\,\delta\lim_{\delta\to 0}\sum_{\theta_i}\Pr(\theta_i)\sum_{\theta_j}\Pr(\theta_j|\theta_i)\Bigg\{\big[\beta-2q^e(\theta_i,\theta_j)+\delta q^e(\theta_j,\theta_i)-\theta_i\big]\frac{\partial q^e(\theta_i,\theta_j)}{\partial\delta}+\delta q^e(\theta_i,\theta_j)\frac{\partial q^e(\theta_j,\theta_i)}{\partial\delta}\Bigg\},$$

where $q^e(\theta_i,\theta_j)$ is defined in Proposition 4. It can be shown that

$$W_I - W_N \approx -\frac{\Delta\theta^2}{4(1-\nu)}\Bigg[\frac{\alpha^2}{(\nu(1-\nu)-\alpha)((1-\nu)^2+\alpha)}+\nu\alpha\delta\Bigg],$$

which is negative for δ small. □

Principals' profits for δ large. We derive the functions used and plotted in Section 6. When no principal shares information, the first-order conditions (1) and (2) imply that

$$V_N=\sum_{\theta_i}\Pr(\theta_i)q^e(\theta_i)^2.$$

When both principals share information, the first-order conditions (5) and (6) imply that

$$V_I=\sum_{\theta_i}\Pr(\theta_i)\sum_{\theta_j}\Pr(\theta_j|\theta_i)q^e(\theta_i,\theta_j)^2.$$

When only P_i shares information, the first-order conditions (7) and (8) imply that

$$V_{I,N}=\sum_{\theta_i}\Pr(\theta_i)q_i^e(\theta_i)^2.$$

When only P_j shares information, the first-order conditions (9) and (10) imply that

$$V_{N,I}=\sum_{\theta_i}\Pr(\theta_i)\sum_{\theta_j}\Pr(\theta_j|\theta_i)q_i^e(\theta_i,\theta_j)^2.$$

The proof of Proposition 6 shows that $\lim_{\alpha\to 0}(V_I - V_{N,I}) = -\lim_{\alpha\to 0}(V_N - V_{I,N}) > 0$. When $\nu=\frac{1}{2}$, using the outputs in Propositions 3, 4 and 5, it can be shown that

$$V_I - V_{N,I}=\frac{[\delta^2(8-\delta^2)-16\alpha\delta(4+\alpha\delta^3-16\alpha(1+\delta))]\Delta\theta^2}{4(1-16\alpha^2)(4-\delta^2)^2},$$

$$V_N - V_{I,N}=\frac{(\delta-8\alpha)(8\alpha\delta+\delta^2-8)\delta\Delta\theta^2}{4(1-2\alpha\delta)^2(4-\delta^2)^2},$$

and

$$V_I - V_N = \frac{\Delta\theta^2}{(1-2\alpha\delta)^2(1-16\alpha^2)(4-\delta^2)^2}\Big[128\alpha^2 - 32\delta\alpha\big(1-2\alpha(1-4\alpha)\big) - 4\delta^3\alpha\big(1+32\alpha^2(1-2\alpha)\big) + \delta^2\big(3+64\alpha^2(1-2\alpha)^2\big) - \delta^4\left(\frac{1}{4} - 8\alpha^2\big(8\alpha^2+1\big)\right)\Big].$$

(Of course, these expressions are consistent with (A.8), (A.10) and (A.11).) Notice that $\Delta\theta$ is just a scalar that does not affect the sign of all these expressions. Hence, normalizing $\Delta\theta = 1$ is with no loss of generality.

Proof of Proposition 9. At the optimum, the high-cost type obtains no rent regardless of his opponent's cost — i.e., $U_i(\bar{\theta}, \theta_j) = 0 \Leftrightarrow t^e(\bar{\theta}, \theta_j) = \bar{\theta} q^e(\bar{\theta}, \theta_j)\ \forall \theta_j$ — and the incentive compatibility constraint (4) is binding — i.e.,

$$\sum_{\theta_j} \Pr(\theta_j|\underline{\theta}) U_i(\underline{\theta}, \theta_j) = \Delta\theta \sum_{\theta_j} \Pr(\theta_j|\underline{\theta}) q^e(\bar{\theta}, \theta_j).$$

Hence, the maximal transfer in state $(\underline{\theta}, \bar{\theta})$ that is compatible with the incentive compatibility constraint is such that

$$t^e(\underline{\theta}, \bar{\theta}) = \underline{\theta} q^e(\underline{\theta}, \bar{\theta}) - \frac{\Pr(\underline{\theta}|\underline{\theta})}{\Pr(\bar{\theta}|\underline{\theta})}\big(t^e(\underline{\theta}, \underline{\theta}) - \underline{\theta} q^e(\underline{\theta}, \underline{\theta})\big) + \Delta\theta \frac{\Pr(\underline{\theta}|\underline{\theta})}{\Pr(\bar{\theta}|\underline{\theta})} q^e(\bar{\theta}, \underline{\theta}) + \Delta\theta q^e(\bar{\theta}, \bar{\theta})$$

$$\Rightarrow \quad \hat{t}^e(\underline{\theta}, \bar{\theta}) = \max_{t^e(\underline{\theta}, \underline{\theta})} \big\{ t^e(\underline{\theta}, \bar{\theta}) \colon t^e(\underline{\theta}, \underline{\theta}) - \underline{\theta} q^e(\underline{\theta}, \underline{\theta}) \geqslant 0 \big\} = \underline{\theta} q^e(\underline{\theta}, \bar{\theta}) + \Delta\theta \frac{\Pr(\underline{\theta}|\underline{\theta})}{\Pr(\bar{\theta}|\underline{\theta})} q^e(\bar{\theta}, \underline{\theta}) + \Delta\theta q^e(\bar{\theta}, \bar{\theta}).$$

This also implies that $t^e(\underline{\theta}, \underline{\theta}) = \underline{\theta} q^e(\underline{\theta}, \underline{\theta})$.

The transfer $\hat{t}^e(\underline{\theta}, \bar{\theta})$ satisfies the agent's limited liability constraint in state $(\underline{\theta}, \bar{\theta})$. Moreover, substituting this transfer into condition (12) yields $\frac{\Pr(\underline{\theta}|\underline{\theta})}{\Pr(\bar{\theta}|\underline{\theta})} q^e(\bar{\theta}, \underline{\theta}) > 0$. Hence, agents have no incentive to collude when they receive the transfer $\hat{t}^e(\underline{\theta}, \bar{\theta})$. □

References

Acquisti, A., Varian, H., 2005. Conditioning prices on purchase history. Mark. Sci. 24, 367–381.
Bennardo, A., Pagano, M., Piccolo, S., 2010. Multiple-bank lending, creditor rights and information sharing. Working paper.
Bertoletti, P., Poletti, C., 1997. X-inefficiency, competition and market information. J. Ind. Econ. 45, 359–375.
Briley, M., Homchick, R., Singer, T., 1994. Information Sharing Among Health Care Providers: Antitrust Analysis and Practical Guide. American Bar Association, Chicago.
Caillaud, B., Jullien, B., Picard, P., 1995. Competing vertical structures: precommitment and renegotiation. Econometrica 63, 621–646.
Calzolari, G., Pavan, A., 2006. On the optimality of privacy in sequential contracting. J. Econ. Theory 130, 168–204.
Cella, M., Etro, F., 2012. Contract competition and oligopolistic screening. Working paper.
Cella, M., Etro, F., forthcoming. Equilibrium principal–agent contracts. J. Econ. Manag. Strategy.
Clarke, R., 1983. Collusion and the incentives for information sharing. Bell J. Econ. 14, 383–394.
Connor, J.M., 2001. Global Price Fixing: Our Customers Are the Enemy. Kluwer, Boston.
Crémer, J., McLean, R., 1988. Full extraction of the surplus in Bayesian and dominant strategy auctions. Econometrica 56, 1247–1257.
Fried, D., 1984. Incentives for information production and disclosure in a duopolistic environment. Q. J. Econ. 99, 367–381.
Gal-Or, E., 1985. Information sharing in oligopoly. Econometrica 53, 329–343.
Gal-Or, E., 1986. Information transmission — Cournot and Bertrand equilibria. Rev. Econ. Stud. 53, 85–92.
Ganuza, J.-J., Jansen, J., forthcoming. Too much information sharing? Welfare effects of sharing acquired cost information in oligopoly. J. Ind. Econ.
Gromb, D., Martimort, D., 2007. Collusion and the organization of delegated expertise. J. Econ. Theory 37, 271–299.
Jansen, J., 2008. Information acquisition and strategic disclosure in oligopoly. J. Econ. Manag. Strategy 17, 113–148.
Laffont, J.J., Martimort, D., 2000. Mechanism design with collusion and correlation. Econometrica 68, 309–342.
Laffont, J.J., Martimort, D., 2002. The Theory of Incentives: The Principal–Agent Model. Princeton University Press, Princeton.
Lizzeri, A., 1999. Information revelation and certification intermediaries. RAND J. Econ. 30, 214–231.
Maier, N., Ottaviani, M., 2009. Information sharing in common agency: when is transparency good? J. Eur. Econ. Assoc. 7, 162–187.
Martimort, D., 1996. Exclusive dealing, common agency, and multiprincipals incentive theory. RAND J. Econ. 27, 1–31.
Novshek, W., Sonnenschein, H., 1982. Fulfilled expectations Cournot duopoly with information acquisition and release. Bell J. Econ. 13, 214–218.
Pagano, M., Jappelli, T., 1993. Information sharing in credit markets. J. Finance 48, 1693–1718.
Pagnozzi, M., Piccolo, S., 2012. Vertical separation with private contracts. Econ. J. 122, 173–207.
Raith, M., 1996. A general model of information sharing in oligopoly. J. Econ. Theory 71, 260–288.
Riordan, M., Sappington, D., 1988. Optimal contracts with public ex post information. J. Econ. Theory 45, 189–199.
Shapiro, C., 1986. Exchange of cost information in oligopoly. Rev. Econ. Stud. 53, 433–446.
Stern, L., El-Ansary, A., Coughlan, A., 1996. Marketing Channels. Prentice Hall, Upper Saddle River.
Taylor, C., 2004. Consumer privacy and the market for customer information. RAND J. Econ. 35, 631–650.
Vives, X., 1984. Duopoly information equilibrium: Cournot and Bertrand. J. Econ. Theory 34, 71–94.
Vives, X., 2000. Oligopoly Pricing. MIT Press, Cambridge.
Ziv, A., 1993. Information sharing in oligopoly: the truth-telling problem. RAND J. Econ. 24, 455–465.