Duration Dependence in Unemployment: A Structural Approach*

Ioannis Kospentaris†

November 10, 2017

JOB MARKET PAPER

Abstract

This paper builds a directed search model of the labor market to quantify the importance of three common explanations for why the job-finding rate strongly falls with unemployment duration: (i) unobserved worker heterogeneity, (ii) skill loss, and (iii) job-search effort decline. I utilize the properties of the model, together with data on reemployment wages and search effort, to identify the contribution of each mechanism to duration dependence. The model predicts a job-finding profile over the unemployment spell very close to US data, even though job-finding rates are not among the calibration targets. Counterfactual simulations lead to two novel results regarding the role of each mechanism for duration dependence: first, the bulk of the effect of unobserved heterogeneity is concentrated in the first six months of the unemployment spell; the drop in job-finding rates observed at longer spells is mostly a result of skill loss and lower search effort. Second, skill loss has a vastly greater impact on job-finding than the decline in search effort. These results have two clear implications for labor market policy: (i) the impact of active labor market programs is expected to be larger for the long-term unemployed; (ii) job-training programs are expected to be more effective than job-search assistance policies at reducing long-term unemployment.

*I am deeply grateful to Gary Hansen, Lee Ohanian, Till von Wachter and Pierre-Olivier Weill for invaluable advice and support. I am also indebted to Andy Atkeson, Carolina Arteaga, Pablo Fajgelbaum, Manolis Galenianos, Andreas Gulyas, Manolis Hatzikonstantinou, Adriana Llerras Muney, Christos Makridis, Musa Orak, Joe Ostroy, Manisha Padi, Liyan Shi, Chad Stecher, and most importantly Ben Smith and Richard Domurat, as well as seminar participants in the Macro Proseminar at UCLA, for many fruitful comments and discussions. I am solely responsible for any errors or omissions.

†University of California, Los Angeles; **email:** `ikospentaris@g.ucla.edu`

1 Introduction

Unemployment features *duration dependence*: recently unemployed workers have a significantly better chance of finding a job than the long-term unemployed (Kaitz, 1970; Van den Berg, 2001; Alvarez et al., 2016; among others).[1] This reflects a combination of several mechanisms including: loss of skills during unemployment, decline of job-search effort, and deterioration of the composition of unobserved worker qualities—that is, worker characteristics that are not captured by the available data. For policy purposes, it is important to know how much each mechanism contributes to the differential job-finding rates among the unemployed. If duration dependence is largely driven by unobserved worker differences, traditional labor market policies will not do much to help the long-term unemployed. On the other hand, if the impact of skill loss and declining search effort is quantitatively significant, then job-training and job-search assistance programs can improve the job-finding prospects of long-term unemployed workers.

This paper develops a directed search model of the labor market that features all of these mechanisms: (i) unobserved worker heterogeneity, (ii) skill loss, and (iii) search effort choice. I exploit the properties of the model to show that each mechanism has different testable implications regarding the effects of unemployment duration on job-finding, wages, and search effort. In the data, workers' job-search effort exhibits a modest decline over the unemployment spell (Krueger and Mueller, 2011; Faberman and Kudlyak, 2014), while reemployment wages are only mildly sensitive to unemployment duration (Schmieder et al., 2016; Ortego-Marti, 2017; Fernández-Blanco and Preugschat, 2016). I employ these empirical patterns to calibrate the model, and use it to quantify the contribution of each mechanism to the differences in job-finding rates among workers at different stages of the unemployment spell.

The mechanisms in the model operate as follows. First, unobserved differences among workers are modeled as differences in suitability for available jobs; that is, each worker is able to produce positive output only in a fraction of the jobs at hand. Workers can be either of broad or limited suitability; the former can perform a strictly greater share of jobs than the latter. Unobserved heterogeneity results in duration dependence due to dynamic selection. That is, as the unemployment spell evolves, broad-suitability workers find jobs faster, leaving more limited-suitability workers in the unemployment pool. Second, skill

[1] This is true even after taking into account the age, education, industry, and other relevant observable worker characteristics. See Machin and Manning (1999), Kroft et al. (2016), Elsby and Hobijn (2010), and Krueger et al. (2014).

loss is captured by depreciation in workers' on-the-job productivity while in unemployment. Consequently, the long-term unemployed have lost a significant part of their productivity and are less attractive to firms. This creates duration dependence for each individual worker. Third, as the unemployment spell evolves, the returns to job-search decrease, due to dynamic selection and skill loss. Workers' search effort, which depends on the returns to job-search, follows that decline. This mechanism amplifies the effects of the other two, resulting in even stronger duration dependence.

To see how the model identifies the effects of unobserved heterogeneity and skill loss, consider the predictions of the following simpler model variants. First, in a model with unobserved heterogeneity alone, the probability of locating a suitable worker is higher in the pool of short- than long-term unemployed workers. However, the long-term unemployed perform equally well while working as those workers who are unemployed for shorter periods. As a result, the long-term unemployed who manage to find a job incur tiny wage losses, equal to only a tenth of the decline observed in the data. This suggests that a model with unobserved heterogeneity alone cannot rationalize the empirical patterns of both job-finding rates and wages. Second, in a model with skill loss alone, the level of skills at each duration stage determines both the probability of getting hired and on-the-job productivity. Consequently, the model predicts a drop of similar magnitude in both job-finding rates and reemployment wages; yet in the data, job-finding drops significantly more than wages. This implies that a model with skill loss alone cannot rationalize the empirical behavior of both variables.

In principle, a model with both unobserved heterogeneity and skill loss would be able to match the observed patterns of job-finding rates and reemployment wages. However, two extensive strands of literature on search theory (e.g. Pissarides, 2000; Mukoyama et al., 2014; among others) and the effects of unemployment benefits (e.g. Nekoei and Weber, 2017; Schmieder et al., 2016; among others) consider job-search effort to be an important determinant of job-finding. Moreover, the data on search effort indicate a significant decline over the unemployment spell. Therefore, it is important to include search effort in the model, otherwise its effect on job-finding would be attributed to either skill loss or unobserved heterogeneity. Since job-search effort amplifies the effects of the other two mechanisms, its omission would bias the quantitative results, and their policy implications. To make this amplification empirically plausible, I calibrate the search effort parameters such that workers in the model participate in job-search activities with the same frequency as in the data.

The effect of each mechanism in the model is associated with a distinct set of parameters, which are calibrated using different data sources. First, high-quality measurements of the effect of unemployment duration on reemployment wages, which are available in the literature, discipline the extent of skill loss. Second, the results from the influential audit study of Kroft et al. (2013) are used to inform unobserved worker heterogeneity. Finally, I use weekly data from a weekly survey of unemployed workers, conducted by Krueger and Mueller (2011) to discipline search effort. To evaluate the quantitative significance of each mechanism for duration dependence, I use the model to compute counterfactual job-finding profiles, shutting down one mechanism at a time. The contribution of each mechanism is measured as the difference between two job-finding profiles, one predicted by the full model and one predicted by the version that excludes this mechanism.

The results of this exercise make two novel contributions to the duration dependence literature. First, according to the model, the bulk of the effect of unobserved worker heterogeneity is concentrated in the first six months of the unemployment spell. As the spell evolves, skill loss and search effort become quantitatively more important, accounting for almost 50% of the observed job-finding differences at durations longer than nine months. Second, skill depreciation and declining search effort affect the job-finding rate in different ways. Search effort has a minor impact on job-finding, accounting for less than 10% of duration dependence among workers unemployed longer than nine months. On the other hand, the effects of skill depreciation are much larger, accounting for almost 40% of job-finding differences among the long-term unemployed. These results illustrate that the importance of each mechanism for the observed duration dependence significantly varies with the stage of the unemployment spell.

To put these findings in perspective, consider the following two comparisons. First, in the US, a newly unemployed worker has a 30% greater chance of finding a job, compared to an observationally similar worker who is jobless for three months. According to my model, 85% of this disparity can be attributed to unobserved differences between the average newly unemployed and the average worker who is unemployed for three months, while skill loss and search effort account for a modest 15%. Second, when comparing a worker unemployed for six months with a worker unemployed for a year or more, the former has a 12% greater chance of finding a job. The model attributes 50% of that disparity to unobserved worker differences, 42% to skill decay, and only 8% to lower search effort exhibited by workers who are unemployed for a year or more.

Overall, these results have two important implications for labor market policy. First, the impact of active labor market programs is expected to be larger for long-term than short-term unemployed workers. Skill loss and search effort account for around 15% of duration dependence among the short-term unemployed, hence one would expect job-search assistance and job-training programs to have modest positive effects for that group. Among the long-term unemployed, however, these two mechanisms account for over 40% of duration dependence, therefore active labor market programs are expected to have sizable positive effects for workers unemployed longer than six months. This result is fully consistent with the meta-analysis of actual labor market programs conducted by Card et al. (2016). They find that real labor marker policies helped the long-term unemployed more; the model developed here sheds light on why this is the case. Second, in quantitative terms, the model predicts that job-training programs are expected to have greater impact than job-search assistance policies. Both policies have significant positive effects, yet the model goes a step further. It implies that job-training programs should have a larger effect on reducing long-term unemployment compared to job-search assistance policies.

To my knowledge, there is no other paper that studies the role of unobserved heterogeneity, skill loss, and search effort for duration dependence with an equilibrium search model, together with data on wages and search effort.[2] Most papers in the literature use data on job-finding rates and observable worker characteristics only. There are few exceptions that also consider reemployment wages: Fernández-Blanco and Preugschat (2016), Flemming (2016), and Doppelt (2014). Fernández-Blanco and Preugschat (2016) were the first to contrast the large decline in job-finding with the mild drop in wages over the unemployment spell. Nevertheless, they only use wages as a non-targeted moment for model validation, and not to calibrate a mechanism contributing to duration dependence, as this paper does. This is an important difference, since it allows me to quantify the role of skill loss for duration dependence, which remains unexplored in Fernández-Blanco and Preugschat (2016).

My work is complementary to Flemming (2016), who also uses wage losses to calibrate skill loss but in a model with home production. In contrast, I employ a model with unobserved heterogeneity to analyze duration dependence. Unobserved heterogeneity is critical because it makes the job-finding rate in the model drop very fast in the first months in unemployment, as in the data. In Flemming's (2016) home production model though, the drop in job-finding in the first months of the spell is slow. As a result, her model predicts a concave drop in job-finding rates, while in the data this drop is convex; unobserved heterogeneity in my model resolves that issue. The most closely related paper to this one is Doppelt (2014), who also builds a model of unobserved heterogeneity to analyze duration dependence. He

[2] See Section 6 for an extensive literature review.

has a model in which inference about worker quality takes place over multiple unemployment spells, while this paper focuses on inference from the last unemployment spell only. Because of that, skill loss mitigates duration dependence in Doppelt's (2014) model, since it lowers the informational value of unemployment. In my model, however, skill loss worsens job-finding prospects, which is consistent with the significant positive impact of actual job-training programs found by Card et al. (2016). Moreover, Doppelt (2014) does not use the observed drop in reemployment wages; skill loss is exogenously set in his approach. Finally, all these papers use the observed job-finding profile to calibrate model parameters, while the model in this paper predicts a realistic job-finding profile without including it in the calibration targets.

It is difficult to obtain results of the type presented here without (i) the use of a structural framework that (ii) includes all relevant mechanisms. First, to identify the magnitude of all three mechanisms with a reduced-form approach, one needs multiple designs with exogenous variation in each mechanism, fixing the rest at different values to control for all potential interactions. Given the unusually extensive data requirements, using a structural framework to make progress seems to be a natural choice. Second, as I will show later, it is the *interaction* of unobserved heterogeneity with skill loss and search effort that drives the predictions of the model. Intuitively, failing to find a job reveals a lot about the quality of the newly unemployed because these workers are evaluated often by firms due to their high skill levels and search effort. The long-term unemployed have lower skill levels and exhibit lower search effort, thus they are rarely evaluated by firms. An extra period in unemployment is not very informative about the unobserved quality of the long-term unemployed and, as a result, the impact of unobserved heterogeneity on job-finding becomes less important at long durations.

The paper proceeds as follows. Section 2 describes the model environment, defines an equilibrium, and analytically establishes equilibrium existence and characterization. In Section 3, I present the empirical evidence that informs the model. Section 4 discusses the identification strategy of the model, and the calibration procedure. In Section 5, I present the quantitative results. Section 6 contains a discussion of the relevant literature, and Section 7 concludes. Finally, Appendix I contains all proofs, and Appendix II extra material regarding the quantitative analysis of the model.

2 Model

This section introduces a tractable equilibrium model of the labor market that contains three important channels of duration dependence: (i) unobserved worker heterogeneity, (ii) human capital depreciation, and (iii) search effort decline. The model builds on the directed search approach of Moen (1997), Acemoglu and Shimer (1999a,b), and Gonzalez and Shi (2010). I begin with a simplified version of the model that incorporates only skill depreciation and unobserved heterogeneity of the unemployment pool, without job separations. I prove existence of equilibrium and characterize its basic properties. In the last part I present a richer version of the model with endogenous participation decision and exogenous separation shocks. This richer version will be used for the quantitative analysis of Sections 4 and 5. All theoretical properties proved for the simple model go through in the full model, albeit with more cumbersome notation.

2.1 The Basic Environment

Time is discrete and runs forever. All agents are risk neutral and discount the future with the same factor $\beta \in (0, 1)$. There is a unit measure of workers, divided between the states of employment and unemployment. There is, also, a positive measure of one-worker firms, which will be endogenously determined by free entry. In this section there is no separation of workers from jobs: if a worker gets hired, she keeps this job forever. The only source of separation is an exit shock ν that forces a worker (employed or unemployed) out of the market. Workers who have exited are replaced by a measure ν of newly unemployed workers.

Workers. Workers' human capital has two components. First, they are of either broad (H) or limited (L) suitability. Suitability is the likelihood of fulfilling the requirements of a job. In other words, suitability captures the probability of a worker producing positive output at a job. If a worker is not suitable for a given job, the match yields zero output. There is a mass $\pi \in (0, 1)$ of broad-suitability workers and $1 - \pi$ of limited-suitability workers. A type-i worker turns out to be suitable for a given job with probability a^i (with $a^H > a^L$). That is, broad-suitability workers have a higher probability of being productive in a given job than limited-suitability workers. This notion of suitability can be thought of as an extreme form of a match-specific shock, which depends on worker's type. Notice that even broad-suitability workers will be unsuitable for some jobs. Importantly, this part of workers' human capital is unobservable to both worker and firms.

Second, workers differ in productivity on-the-job. A job-seeker who is unemployed for $\tau \in \{1, 2, ..., T\}$ periods and turns out to be suitable for a given job, will produce y_τ, with $y_\tau > y_{\tau+1}$, up to the final period T. All workers with unemployment duration greater or equal to T form a homogeneous group. The output of a worker-firm match depends solely on worker's productivity level at the time of the match. Unemployment duration and productivity are observable to both worker and firms. The deterioration of worker's productivity over the unemployment spell captures skill loss. Notice that the effect of skill depreciation affects both broad- and limited-suitability workers in the same way.

The fact that broad-suitability workers can produce positive output in more jobs creates duration dependence due to dynamic selection. At longer durations the unemployment pool contains a larger fraction of low-suitability workers, hence long-term unemployed have worse job-finding prospects. The fact that workers' productivity decreases with unemployment duration creates within-worker duration dependence. This mechanism also worsens the job-finding prospects of long-term unemployed workers. Finally, I do not address search effort at this stage but I incorporate it in the quantitative Section 2.3.

Labor Market. Firms are homogeneous. Each firm opens one vacancy and posts a wage aimed at workers with specific characteristics at cost κ. Meeting workers is subject to matching frictions. Moreover, firms have access to a simple testing technology: after meeting a worker, a firm observes a private, match-specific signal, which perfectly identifies unsuitable workers. Unsuitable candidates are disregarded and only suitable workers are hired.[3] The testing expenses are included in the vacancy creation cost. Neither workers nor other firms learn the match-specific signals generated by the testing process; they only observe the hiring decision. A worker who fails to find a job does not know whether her application has been considered by a firm and found unsuitable or it was not considered at all due to matching frictions.

The labor market consists of many different submarkets, indexed by the unemployment duration and the expected suitability of workers who search for jobs in the submarket. Firms are free to enter any submarket and post any wage they want to attract workers of a specific unemployment duration and expected suitability. Search is directed in the sense that workers of different characteristics search in different submarkets. Hence, when firms post wages and vacancies in a submarket, they calculate the expected profit with workers of only one

[3] As it will be seen later, the matching function will reflect that feature of the model: it will determine the number of productive matches rather than the number of meetings. The process of receiving applications and the process of evaluating applicants are combined in this model.

unemployment duration in mind.[4]

Information Structure. A worker's suitability is unobservable to both the worker herself and potential employers: there is symmetric incomplete information in the model, as in Gonzalez and Shi (2010), Fernández-Blanco and Preugschat (2016) and Doppelt (2014). On the other hand, worker's unemployment duration, and thus her productivity in suitable matches, is public information. In other words, all firms know the output of a successful match with a worker of specific unemployment duration. Due to the fact that lack of information regarding a worker's type is symmetric, the worker and the "labor market" (i.e. all firms and other workers) share the same belief about the probability a worker of a given duration be suitable for a job. Hence, workers of the same unemployment duration are observationally equivalent and a worker's unemployment duration is a sufficient statistic for the probability the worker forming a successful match.

This information structure is based on Gonzalez and Shi (2010); it buys the model a lot of tractability for two reasons. First, it allows me to avoid the complexities arising in the case of adverse selection, analyzed in Guerrieri et al. (2010). Second, when this hiring protocol is combined with a constant returns to scale matching function, it implies that the ratio of suitable workers to vacancies is a summary statistic for all relevant information in a submarket. Hence, the only relevant state variable for workers and firms in a given submarket is the queue length of the submarket. As will be shown shortly, this is crucial for making the model block recursive, in the sense of Menzio and Shi (2011).

Matching. In each submarket the number of matches is given by a Cobb-Douglas matching function. The inputs of this function are the number of vacancies, v, posted in the submarket, as well as the total units of suitable workers searching in this submarket: $u^E = a^H u^H + a^L u^L$, where u^i denotes the measure of unemployed workers of type i searching in the submarket. The matching function for a specific submarket is:

$$m = (u^E)^\alpha (v)^{1-\alpha} \tag{1}$$

[4]Assuming this market structure is without any loss of generality: it is a standard result in directed search models with heterogeneous workers, homogeneous firms and bilateral meetings that labor market participants endogenously choose to search in different submarkets (see Moen (1997), Acemoglu and Shimer (1999a), Mortensen and Wright (2002), Menzio and Shi (2010), Gonzalez and Shi (2010), Guerrieri et al. (2010)). In other words, even if it was assumed that firms are free to post wages for any worker types they want, they would endogenously choose to post a wage directed to workers with a specific unemployment duration and expected suitability. Several papers postulate that firms commit to hire workers of only one type in each submarket; see Doppelt (2014) and Flemming (2016), among others.

When a firm is deciding in which submarket to post a wage, the only relevant piece of information is the vacancy filling probability in each submarket. Due to the constant returns to scale in the matching function, this probability depends only on the ratio of the effective units of search over the posted vacancies in each submarket, q:

$$\lambda = \frac{m}{v} = \lambda(q) = q^{\alpha} \tag{2}$$

where $q = \frac{u^E}{v}$ will be referred to as the *queue length* of the submarket. Moreover, the only relevant pieces of information for a worker is the average job-finding probability for suitable workers, x, as well as her belief about her expected suitability, μ. It is straightforward to repeat the calculation in (2) to show that:

$$x = \frac{m}{u^E} = x(q) = q^{\alpha-1} \tag{3}$$

In the next section I will show that a worker's updated belief about her expected suitability is a function of exogenous parameters and the job-finding probability of the submarket she was looking for a job in the previous period.

To summarize, given the queue length in a submarket (which will be determined in equilibrium), an agent's expected payoff is independent of the level and the composition of workers and firms in the submarket. Free entry of firms ensures that the wage in each submarket in a function of exogenous parameters and the submarket's queue length only. This property of the model is known in the literature as *block recursivity* because it allows the calculation of the equilibrium queues and wages without keeping track of the distribution of worker types in different submarkets. The property of block recursivity crucially rests on the hiring protocol of Gonzalez and Shi (2010), the fact that search is directed, and the assumption of constant returns to scale in matching.

Learning from Unemployment Duration. While in unemployment a worker learns about her a, the probability she will be productive in a randomly selected job.[5] I define the worker's expectation of a to be her belief and denote it as μ. For every worker who enters the labor market as newly unemployed, her initial belief about her expected suitability is;

$$\mu_0 = \pi a^H + (1-\pi)a^L \tag{4}$$

[5]It is important to stress again that every other participant in the labor market would have the same belief regarding a worker's a as the worker herself. It will be shown shortly that the beliefs are functions of publicly observable information, hence the update is symmetric for every participant in the market.

The updating of beliefs depends on the queue length of the submarket into which the worker was searching in the last period. Applying Bayes rule yields:

$$\mu' \equiv H(\mu, x) = a^H - \frac{(a^H - \mu)(1 - xa^L)}{1 - x\mu} \tag{5}$$

Notice that $H(x, \mu)$ is decreasing in x: the higher the job-finding rate in a submarket, the stronger the signal that the worker did not get match because of her limited suitability.

Timing. Each period of the model consists of four stages:

1. Exit of workers and entry of newly unemployed
2. Wage-posting
3. Matching
4. Production

Value Functions. To determine the optimal wage-posting policies by firms, I follow Acemoglu and Shimer (1999a,b) and rely on Bellman's Principle of Optimality to compute the value of one-period deviations. Consider a firm evaluating the prospect of posting wage w aimed at workers of duration τ and expected belief μ.[6] In directed search models, workers adjust their behavior in response to different wages posted by firms. In this sense, when a firm posts wage w for workers (τ, μ), it anticipates a queue length q, which is a function of the posted wage: $q = Q_{\tau,\mu}(w)$. The function $Q_{\tau,\mu}(\cdot)$ represents the firms' rational expectations about the equilibrium relationship between posted wages to queue length. It is defined for any wage w, not only for the wage that will be posted in equilibrium. It is an endogenous object to be determined in equilibrium under a rational expectations condition, which will be articulated in the next section.

The value of posting a vacancy with wage w for workers of unemployment duration τ and expected suitability μ is given by:

$$V_{\tau,\mu}(w) = -\kappa + \Big[\lambda(Q_{\tau,\mu}(w))J_{\tau,\mu}(w) + \big(1 - \lambda(Q_{\tau,\mu}(w))\big)V^*_{\tau,\mu}\Big] \tag{6}$$

[6]An equivalent way to express that is to say that the firm creates a new submarket for workers of unemployment duration τ and expected suitability μ, posting a vacancy paying wage w.

where $V^*_{\tau,\mu} = \max_w V_{\tau,\mu}(w)$. This expression captures the fact that the firm receives the maximum value of looking for workers (τ, μ) after the current period. The firm pays a cost κ to post the vacancy, which is the same for all submarkets. Of course, the probability the vacancy is filled is a function of the expected queue length: $\lambda(w) = \lambda(Q_{\tau,\mu}(w))$ and $Q_{\tau,\mu}(w)$ will be determined in equilibrium. It denotes the queue length a firm anticipates when posts a vacancy paying wage w for workers (τ, μ).

Following the same argument, the value of filling a vacancy with an unemployed of duration τ is given by:

$$J_{\tau,\mu}(w) = y_\tau - w + \beta(1-\nu)J_{\tau,\mu}(w) \tag{7}$$

The worker produces y_τ units of produce and is paid the posted wage w; when the exit shock hits, the vacancy is destroyed.

Turning to workers, the value of being unemployed for τ periods with expected suitability μ and applying to a vacancy paying w with queue length q is:

$$U_{\tau,\mu}(w,q) = \max_s \left\{ b - c(s) + \beta(1-\nu)\Big[s\mu x(q)\big(E_{\tau,\mu}(w) - U^*_{\tau+1,\mu'}\big) + U^*_{\tau+1,\mu'}\right\} \tag{8}$$

where $x(q) = q^{\alpha-1}$, $\mu' = H(x(q), \mu)$ and $U^*_{\tau+1,\mu'} = \max_{w,q} U_{\tau+1,\mu'}(w,q)$. A worker receives b while unemployed, with $b < y_T$. The job-finding probability is the product of the aggregate job-finding probability given that the worker is suitable, $x(q)$, as well as the probability the worker being suitable for the job. The worker does not know that probability, so she uses her beliefs μ to calculate the value of unemployment; if she fails to find a job, she updates her beliefs following Bayes rule in equation (5). Finally, the workers get their maximum value of unemployment after the current period.

Similarly, the value of employment can be computed as:

$$E_{\tau,\mu}(w) = w + \beta\big(1-\nu\big)E_{\tau,\mu}(w) \tag{9}$$

As long as the worker is employed, she receives the wage posted in the submarket she was searching when hired. When the exit shock hits, the worker exits the labor market. In Section 2.3 standard separations shocks, sending workers back to unemployment, are introduced in the model.

2.2 Equilibrium

Equilibrium Queue Lengths. Recall that the queue length function $Q_{\tau,\mu}(w)$ represents a firm's rational expectations about the queue of workers it would face if it posted the wage w directed to unemployed workers of duration τ and expected suitability μ. The idea is that in equilibrium these expectations should be pinned down by subgame perfection: $Q_{\tau,\mu}(w))$ would be the queue length faced by the firm in the subgame where it posts w but all other firms post the equilibrium wage aimed at workers of duration τ.[7]

Following a common practice in the directed search literature, I do not explicitly study the game-theoretic formulation of the model. Rather, I impose the following equilibrium condition on queue lengths to capture the spirit of subgame perfection, which needs to hold for all τ and μ:

$$Q_{\tau,\mu}(w) = \begin{cases} 0, & U_{\tau,\mu}(w,0) < U^*_{\tau,\mu} \\ \in (0,\infty), & U_{\tau,\mu}(w, Q_{\tau,\mu}(w)) = U^*_{\tau,\mu} \\ \infty, & U_{\tau,\mu}(w,\infty) > U^*_{\tau,\mu} \end{cases} \tag{10}$$

When the firm posts wage w there are three possible outcomes. First, if the wage is very low (or $U^*_{\tau,\mu}$ is very high), then the firm attracts no workers and $Q_{\tau,\mu}(w) = 0$. Moreover, workers must find it strictly suboptimal to apply to his job (since the wage is too low) even there are no other workers competing for that vacancy and, as a result, $U_{\tau,\mu}(w,0) < U^*_{\tau,\mu}$. Second, if the wage is very high (or $U^*_{\tau,\mu}$ is very low), then the firm attracts all workers and $Q_{\tau,\mu}(w) = \infty$. A worker must find it strictly optimal to come to apply to this firm, even when she has to compete with all other workers for the vacancy. Third, if the wage is in an intermediate range, then workers will apply to this vacancy until they are indifferent between applying to this job (receiving the value $U_{\tau,\mu}(w, Q_{\tau,\mu}(w)))$ or to any other vacancy (receiving the value $U^*_{\tau,\mu}$). That is, the queue length $Q_{\tau,\mu}(w)$ should solve the equation $U_\tau(w, Q_{\tau,\mu}(w)) = U^*_{\tau,\mu}$.

Notice that, as argued in Shi (2002, 2006), the third case is impossible to take place: if the queue length is infinite, the probability a worker gets a job is zero, hence her expected utility from searching in this submarket is zero, which is less than $U^*_{\tau,\mu}$, a contradiction of

[7] The game-theoretic foundations of the equilibrium queue lengths condition (10) are masterly analyzed in Burdett et al. (2001) and Galenianos and Kircher (2012).

the requirement. Hence, the equilibrium queue length condition can be simplified as:

$$Q_{\tau,\mu}(w) = \begin{cases} 0, & U_{\tau,\mu}(w,0) < U^*_{\tau,\mu} \\ \in (0,\infty), & U_{\tau,\mu}(w, Q_{\tau,\mu}(w)) = U^*_{\tau,\mu} \end{cases} \tag{11}$$

Finally, I show in Lemma 3 that the first case will never be observed in equilibrium. However, condition (11) is important because it pins down the out-of-equilibrium firms' beliefs about workers' responses to wage offers that are not observed in equilibrium.

Definition of Equilibrium. A competitive search equilibrium is a set of wages offered by firms $\mathbb{W}^*_{\tau,\mu}$, a set of queue length functions $\{Q^*_{\tau,\mu}\}$, a function of workers' utility levels $\mathbf{U}^*$, a belief function μ and a set of value functions $\{J_{\tau,\mu}, V_{\tau,\mu}, E_{\tau,\mu}, U_{\tau,\mu}\}$, with the following properties:

1. **Optimal Application**. $U^*_{\tau,\mu} = \sup_{w_{\tau,\mu} \in \mathbb{W}^*_{\tau,\mu}} U_{\tau,\mu}(w_{\tau,\mu}, Q^*_{\tau,\mu}(w_{\tau,\mu}))$, for all τ and μ.
2. **Profit Maximization and Free Entry**. $V^*_{\tau,\mu} = V_{\tau,\mu}(w_{\tau,\mu}) = 0 \geq V_{\tau,\mu}(w)$, for any w, for all $w_{\tau,\mu} \in \mathbb{W}^*_{\tau,\mu}$ and for all τ and μ.
3. **Rational Expectations**. $Q^*_{\tau,\mu}(w_{\tau,\mu})$ satisfies the equilibrium queue lengths condition (11), for all τ and μ and for all $w_{\tau,\mu} \in \mathbb{W}^*_{\tau,\mu}$.
4. **Beliefs Updating**. A worker with beliefs μ uses Bayes rule to update her beliefs: $\mu' = H(x(Q^*_{\tau,\mu}(w_{\tau,\mu})), \mu)$, if she fails to find a job.

Equilibrium as a Solution to an Auxiliary Maximization Problem. An important result, due to Moen (1997) and Acemoglu and Shimer (1999a,b), is that the equilibrium can be characterized as the solution to an auxiliary constrained maximization problem. The objective function of the auxiliary problem is the value function of the agents on one side of the market; the constraint is that the agents on the other side of the marker receive their optimal values. I extend this equivalence result to a framework with skill depreciation and a declining expected suitability of the unemployment pool.

Consider the following constrained maximization problem:

$$V^*_\tau = \max_{w_\tau, q_\tau} -\kappa + \lambda(q_\tau)\frac{y_\tau - w_\tau}{1-\beta(1-\nu)}, \quad \forall \tau \leq T \tag{12}$$

subject to the constraints:

$$U_\tau^* \leq b + \beta(1-\nu)\left[\mu_\tau x(q_\tau)\left(\frac{w_\tau}{1-\beta(1-\nu)} - U_{\tau+1}^*\right) + U_{\tau+1}^*\right] \tag{13}$$

and $q_\tau \geq 0$ with complementary slackness

$$V_\tau^* = 0, \quad \forall \tau \leq T \tag{14}$$

$$\mu_{\tau+1} = H(x(q_\tau), \mu_\tau) \tag{15}$$

In this auxiliary problem the firm takes the optimal values of workers as given. Solving this problem yields the optimal w_τ^* and q_τ^* as functions of U_τ^*, for all τ. The sequence of beliefs is constructed following equation (15) based on the sequence $\{q_\tau^*\}_{\tau \leq T}$ The market values of workers are pinned down by solving equation (14) for all τ.

Suppose for now that this problem has a solution (not necessarily unique): $\{w_\tau^*, q_\tau^*\}_{\tau \leq T}$. Then, the equivalence of the competitive search equilibrium with the solution to the auxiliary optimization problem is obtained through the following lemmas.

Lemma 1 (**Equilibrium $\mapsto$ Auxiliary Problem**). *Let $w_\tau^* \in \mathbb{W}_{\tau,\mu}^*$ and $q_\tau^* = Q_{\tau,\mu}^*(w_\tau^*)$, where $\{\mathbb{W}_{\tau,\mu}^*, \{Q_{\tau,\mu}^*\}_{\tau \leq T}, \mathbf{U}^*\}$ be an equilibrium allocation; then $\{w_\tau^*, q_\tau^*\}_{\tau \leq T}$ solve problem (12) under constraints (13), (14) and (15), with $U_{\tau,\mu}(w_\tau^*, q_\tau^*) = U_{\tau,\mu}^*$ if $q_\tau^* > 0$.*

Lemma 2 (**Auxiliary Problem $\mapsto$ Equilibrium**). *If some $\{w_\tau^*, q_\tau^*\}_{\tau \leq T}$ solve problem (12) under constraints (13), (14) and (15), then there exists an equilibrium $\{\mathbb{W}_{\tau,\mu}^*, \{Q_{\tau,\mu}^*\}_{\tau \leq T}, \mathbf{U}^*\}$ such that $w_\tau^* \in \mathbb{W}_{\tau,\mu}^*$ and $q_\tau^* = Q_{\tau,\mu}^*(w_\tau^*)$, $\forall \tau \leq T$.*

Equilibrium Existence and Characterization. The usefulness of Lemmas 1 and 2 is that they enable me to characterize equilibrium as the solution to the auxiliary profit maximization problem (12) under constraints (13), (14) and (15). A standard assumption, satisfied by my preferred Cobb-Douglas specification, is that $\lambda(\cdot)$ is a strictly concave function. This guarantees the existence of an equilibrium in which workers of different unemployment durations search in different labor markets.

Proposition 1. *There exists an equilibrium in which the labor market is segmented by unemployment duration.*

The proof of existence is based on a fixed-point argument that uses Brouwer's theorem. The strategy of the proof also suggests a computational strategy for how to compute the equilibrium, which is analyzed in Appendix II. It is important to highlight that the algorithm fully exploits the block recursivity of the model: all endogenous variables are computed independently of the distribution of workers across states. Computing the masses of workers across different states becomes a matter of accounting.

An appealing feature of this model is its tractability. Indeed, one can analytically show that workers face declining job-finding probabilities and reemployment wages over a spell of unemployment. The tractability of the model is a result of the Gonzalez and Shi (2010) hiring protocol, as well as of the equivalence of competitive search equilibrium with the auxiliary problem. Extending the machinery of Moen (1997) and Acemoglu and Shimer (1999a, 1999b) to the current environment enables me to exploit the firms' FOCs, as shown explicitly in Section 2.3, and analytically prove the following set of results.

Proposition 2. *In any equilibrium in which the labor market is segmented by unemployment duration, q_τ is increasing and w_τ is decreasing in τ; also, the difference $y_\tau - w_\tau$ is decreasing in τ. Hence, the value of a filled vacancy, $J(w_\tau)$, is decreasing in τ.*

Other papers in the recent macroeconomic literature on duration dependence feature some troubling implications. For example, for a given cohort of unemployed workers, the model of Gonzalez and Shi (2010) predicts job-finding rates that increase with the duration of unemployment for all workers.[8] The model of Doppelt (2014) makes the same prediction but for a minority of workers. I prove that job-finding rates unambiguously decline for all workers following a specific cohort of unemployed. Finally, in the model of Fernández-Blanco and Preugschat (2016) reemployment wages may increase with unemployment duration. Likewise, reemployment wages in Doppelt (2014) are also non-monotone in unemployment duration. On the other hand, I prove that reemployment wages unambiguously fall over the spell of unemployment, as in the data.

It is worth underscoring that skill depreciation is the primary factor supporting these results, not the declining quality of unemployment pool. In other words, in a model with skill depreciation only, Proposition 2 would still hold. On the other hand, in a model which unobserved worker heterogeneity is the only source of duration dependence, Proposition 2

[8] This result is also important because it shows that negative duration dependence is not a trivial outcome when the duration of unemployment provides a signal of worker quality. Learning dynamics may lead workers to target jobs with lower queue lengths to increase the probability of getting hired. If this effect is strong enough, exit rates from unemployment will be increasing in unemployment duration.

would not be unambiguously true. Actually, queue length would be decreasing over the unemployment spell in this model. This echoes the counterfactual findings described above in papers that feature only unobserved worker quality. This shows that skill depreciation is necessary in order the model to deliver all features of Proposition 2. Unobserved heterogeneity is necessary for the model to deliver convex job-finding rates, as explained in Section 4.2. Finally, these features of the model suggest a calibration strategy, since they demonstrate which mechanism accounts for each observable prediction of the model.

To close this section, I state two technical results, along with a more substantive one. It is common in directed search models that all submarkets open in equilibrium feature positive queue lengths (otherwise, firms would have profitable deviations). Moreover, as expected by the fact that broad-suitability workers find jobs faster than their limited-suitability counterparts, expected worker suitability declines over the spell of unemployment. The hiring protocol of Gonzalez and Shi (2010) captures the declining quality of unemployment pool in a straightforward and intuitive way.

Lemma 3. *In any equilibrium in which the labor market is segmented by unemployment duration, $q_\tau > 0$ for all τ. Hence, the complementary slackness condition (13) holds with equality.*

Lemma 4. *Beliefs about worker's expected suitability for a given job, μ_τ, are decreasing in τ.*

Finally, since the employment prospects of workers deteriorate over time, the value of unemployment is strictly decreasing in unemployment duration. This result would also hold in a model with skill depreciation only. However, as mentioned above, the deterioration of employment prospects would not be fast enough to rationalize convex job-finding rates in this case.

Proposition 3. *In any equilibrium in which the labor market is segmented by unemployment duration, the value of unemployment, U^*_τ, is decreasing in τ.*

2.3 Quantitative Extension

Endogenous Search Effort. The framework presented above can be easily extended to incorporate an extra force of duration dependence: declining search effort. I model search effort as a participation decision: a measure of unemployed workers will not be searching for jobs. This modeling choice is motivated by the empirical evidence on workers' search effort, presented in Section 3. Using data from Krueger and Mueller (2011), I show that the intensive margin of search effort (time to devoted to search) is insignificant for generating job-offers. On the other hand, the extensive margin of participation in job-search is found to be significant for generating job-offers. Therefore, the appropriate measure of search effort in Krueger and Mueller (2011) data is the participation margin and this explains my modeling choice.

In each period an unemployed worker is hit by an IID search cost shock $\tilde{c}$. The support of $\tilde{c}$ is a bounded interval in the real line, $\text{supp}(\tilde{c}) = [-K_1, K_2]$, and its CDF is a continuous strictly increasing function $F(\tilde{c})$. The Bellman equation for an unemployed worker of duration τ can be written as:

$$U_\tau(w,q) = b + \int_{-K_1}^{K_2} \max\left\{ -\tilde{c} + \beta(1-\nu)\mu_\tau x(q)\big(E_\tau(w) - U^*_{\tau+1}\big), 0 \right\} dF(\tilde{c}) + \beta(1-\nu)U^*_{\tau+1} \quad (16)$$

The idea here is that if the search cost drawn at a period is low enough, then the worker participates in the labor market facing the job-finding prospects analyzed above. If the search cost is high though, the worker does not participate in the labor market and she enters next period as unemployed.

One can apply the standard quantile transformation to write equation (16) in a more concrete form. Define the function $c'(z) \equiv F^{-1}(z)$, where z is a uniform random variable with $[0,1]$ support: $z \sim \mathbb{U}_{[0,1]}$.[9] After the change of variables $\tilde{c} \equiv c'(z)$, the value function of unemployment can be written as:

$$U_\tau(w,q) = b + \int_0^1 \max\left\{ -c'(z) + \beta(1-\nu)\mu_\tau x(q)\big(E_\tau(w) - U^*_{\tau+1}\big), 0 \right\} dz + \beta(1-\nu)U^*_{\tau+1} \quad (17)$$

Since $F(\tilde{c})$ is strictly increasing, its inverse is strictly increasing as well. Hence, the value function can be written as:

$$U_\tau(w,q) = b + \max_{s\in[0,1]} \int_0^s -c'(z) + \beta(1-\nu)\mu_\tau x(q)\big(E_\tau(w) - U^*_{\tau+1}\big) dz + \beta(1-\nu)U^*_{\tau+1} \quad (18)$$

Assuming that $c'(z)$ is integrable, it has a well-defined antiderivative function $c(z)$. If one assumes that $c(0) = 0$, one can write the value function in the familiar form:

$$U_\tau(w,q) = \max_{s\in[0,1]} \left\{ b - c(s) + \beta(1-\nu)s\mu_\tau x(q)\big(E_\tau(w) - U^*_{\tau+1}\big) + \beta(1-\nu)U^*_{\tau+1} \right\} \quad (19)$$

[9] It is trivial to show that $c'(z)$ has the same CDF as $\tilde{c}$

The interpretation of s is different, though: instead of denoting the intensity of job search activity (intensive margin), here s denotes the probability to participate in the labor market (extensive margin). This interpretation rests on the microfoundation presented above, in which the basic assumption is that search cost shocks are IID over time. Alternatively, one could think of this microfoundation as follows: only a measure s of unemployed workers of duration τ participates in the labor market when applying to a job offering wage w with a queue length q, while a measure $1-s$ does not search for jobs. To summarize, equations (16) and (19) are equivalent and produce the same answer concerning worker job-search effort, supported by two different interpretations.

The FOCs for this problem are straightforward to interpret: the probability to participate in the labor market equalizes the marginal cost of participation with its marginal return. Evaluating the FOCs in equilibrium yields:

$$c'(s_\tau^*) = \beta(1-\nu)\mu_\tau x(q_\tau)\left[\frac{w_\tau}{1-\beta(1-\nu)} - U_{\tau+1}^*\right] \tag{20}$$

Assuming the standard power search cost function, $c(s) = \phi\frac{s^\eta}{\eta}$, and a Cobb-Douglas matching function, equation (20) becomes:[10]

$$s_\tau^* = \left\{\beta(1-\nu)\phi^{-1}\mu_\tau q_\tau^{\alpha-1}\left[\frac{w_\tau}{1-\beta(1-\nu)} - U_{\tau+1}^*\right]\right\}^{\frac{1}{\eta-1}} \tag{21}$$

Substituting back into (19) and a bit of algebra leads to:

$$q_\tau^\alpha \frac{w_\tau}{1-\beta(1-\nu)} = q_\tau^\alpha U_{\tau+1}^* + \frac{q_\tau}{\beta(1-\nu)\mu_\tau}\left\{\frac{U_\tau^* - b - \beta(1-\nu)U_{\tau+1}^*}{\phi^{\frac{1}{1-\eta}}\frac{\eta-1}{\eta}}\right\}^{\frac{\eta-1}{\eta}} \tag{22}$$

This is the enriched version of constraint (13) for the case with endogenous participation choice. One can substitute this constraint into firms' profit and take FOCs with respect to q_τ. This gives an expression for the job-finding rate, $q_\tau^{\alpha-1}$, as a function of $U_{\tau+1}^*$, μ_τ and parameters only.:

$$q_\tau^{\alpha-1} = \frac{1}{\beta(1-\nu)\mu_\tau}\left\{\frac{U_\tau^* - b - \beta(1-\nu)U_{\tau+1}^*}{\phi^{\frac{1}{1-\eta}}\frac{\eta-1}{\eta}}\right\}^{\frac{\eta-1}{\eta}} \frac{1}{\alpha\left(\frac{y_\tau}{(1-\beta(1-\nu))} - U_{\tau+1}^*\right)} \tag{23}$$

[10]See Pissarides (2000), Mukoyama et al. (2014), and Gomme and Lkhagvasuren (2015).

Finally, one could substitute back into (22) to obtain an expression for equilibrium wage:

$$\frac{w_\tau}{1-\beta(1-\nu)} = \alpha\Big(\frac{y_\tau}{1-\beta(1-\nu)} - U^*_{\tau+1}\Big) + U^*_{\tau+1} = \alpha\frac{y_\tau}{1-\beta(1-\nu)} + (1-\alpha)U^*_{\tau+1} \tag{24}$$

Based on (24) it is trivial to calculate the value of a filled vacancy for the firm:

$$J(w_\tau) = \frac{y_\tau - w_\tau}{1-\beta(1-\nu)} = (1-\alpha)\left[\frac{y_\tau}{1-\beta(1-\nu)} - U^*_{\tau+1}\right] \tag{25}$$

Proposition (2) ensures that $J(w_\tau)$ is decreasing in τ; thus, the surplus of the match, $\frac{y_\tau}{1-\beta(1-\nu)} - U^*_{\tau+1}$, must also be decreasing in τ. Moreover, simple substitution into the Free Entry condition and the participation FOCs yields:

$$q_\tau = \kappa^{\frac{1}{\alpha}}(1-\alpha)^{-\frac{1}{\alpha}}\left[\frac{y_\tau}{1-\beta(1-\nu)} - U^*_{\tau+1}\right]^{-\frac{1}{\alpha}} \tag{26}$$

$$s^*_\tau = \left\{\beta(1-\nu)\phi^{-1}\mu_\tau\kappa^{\frac{\alpha-1}{\alpha}}(1-\alpha)^{\frac{1-\alpha}{\alpha}}\alpha\left[\frac{y_\tau}{1-\beta(1-\nu)} - U^*_{\tau+1}\right]^{\frac{1}{\alpha}}\right\}^{\frac{1}{\eta-1}} \tag{27}$$

which proves that under this specific parameterization the optimally chosen search effort is decreasing over unemployment duration.

Lemma 5. *Under a power search cost function and a Cobb-Douglas matching function, workers' participation probability is decreasing in τ.*

Exogenous Separations. I also introduce exogenous separation shocks for the quantitative analysis: an employed worker loses her job with probability δ each period. Since workers now move from employment to unemployment, I need to take a stance on how their unobserved feature evolves when they enter unemployment. I assume that every time a worker reenters unemployment her suitability type is redrawn. Moreover, the probability to be a broad-suitability worker is decreasing in duration, such that all workers in the same submarket have the same probability to be suitable for a given job. In other words, given π, a^H, a^L, and the equilibrium queue lengths, one can construct the sequence $\{\mu_\tau\}_{\tau\leq T}$, following the Bayes rule in (5), for some workers who will be unemployed for at least T periods after they entered the market. I assume that every employed worker in submarket τ, when entering unemployment, has a probability π^*_τ to be of high ability, with π^*_τ be defined as the solution to the equation: $\mu_\tau = \pi^*_\tau a^H + (1-\pi^*_\tau)a^L$ or just $\pi^*_\tau = \frac{\mu_\tau - a^L}{a^H - a^L}$. This assumption implies that there is a measure of π broad-suitability workers in the unemployment pool at every instant. It also imposes that the fraction of broad-suitability workers at each submarket is decreasing in duration and, most importantly, perfectly known to firms.

What this assumption rules out is the possibility of firms' expectation regarding the measure of broad-suitability workers in a submarket be different than the actual one. In other words, this assumption combines employer discrimination in callbacks and dynamic selection in hiring into one mechanism. This is a natural assumption in the present paper for, at least, two reasons: (i) the model does not feature a separate interview stage, thus it cannot have distinct implications for callbacks and hires. (ii) The difference between employer discrimination in callbacks and dynamic selection in hiring is quantitatively meaningful for the job-finding rate only if the workers who are discriminated in the interview stage would end up hired if interviewed by firms. Jarosch and Pilossoph (2015), in the context of an equilibrium search model, report that this happens very rarely. If firms' discrimination at the hiring stage does not result in extra jobs being lost, then it is not crucial to consider its effects for duration dependence separetely. In other words, this paper assumes, based on the results of Jarosch and Pilossoph (2015), that employer discrimination is just a *means* through which dynamic selection takes place. Because of that, however, the results of the quantitative section should be interpreted as providing an upper bound for the magnitude of unobserved heterogeneity and a lower bound for the magnitude of true duration dependence.

3 Empirical Evidence

This section presents the empirical evidence that model the will speak to in Section 5 of the paper.

Job-Finding Rates. The main focus of this paper is to decompose duration dependence in unemployment into its key channels. The empirical evidence for duration dependence comes from the empirical relationship between the observed job-finding probability and unemployment duration in the Current Population Survey (CPS). Specifically, I follow Kroft et al. (2016) and Jarosch and Pilossoph (2015) and estimate that relationship in two steps.

First, I pool CPS data from 1994 to 2014, following the matching process outlined in Nekarda (2009), for workers between 25 and 54 years old. I regress the dummy for finding a job on unemployment duration and a standard set of demographic controls via weighted

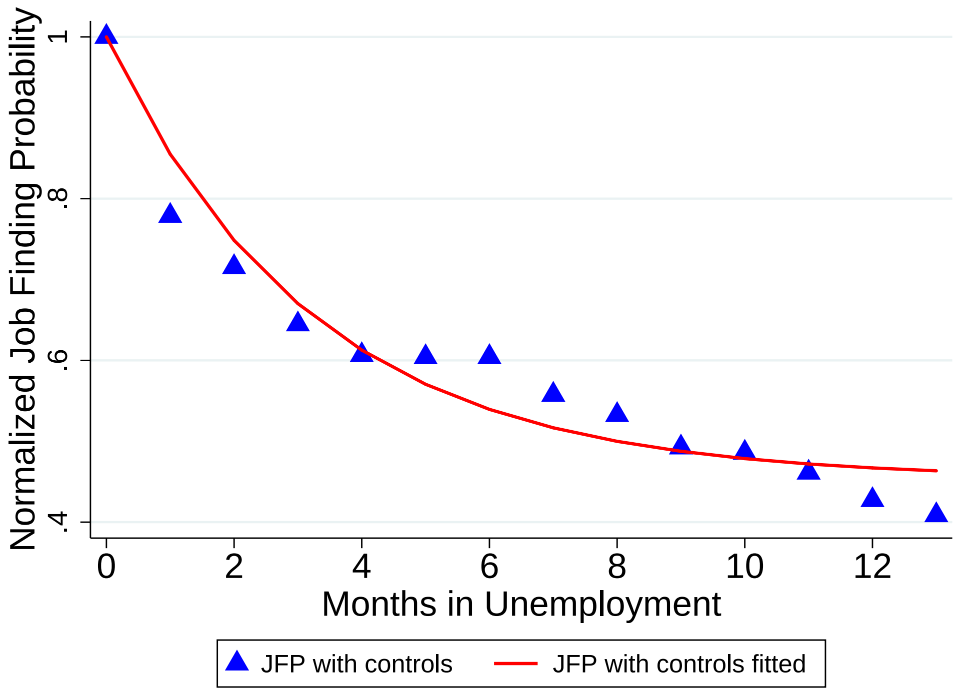

Figure 1: Normalized Job-Finding Probabilities by Unemployment Duration. Data from CPS, 1994-2014, workers between 25 and 54 years old

nonlinear least squares.[11] Second, I estimate an exponential function for the average job-finding probability at duration τ relative to the average job-finding probability of workers who have been unemployed one month or less:

$$\frac{f(\tau)}{f(1)} = b_0 + (1 - b_0) \times \exp(-b_1 \times \tau) \tag{28}$$

The empirical estimates are $\hat{b}_0 = 0.480$ and $\hat{b}_1 = 0.329$, very close to the estimates of Jarosch and Pilossoph (2015) and Kroft et al. (2016). Figure 1 plots the normalized job-finding probabilities (i.e. relative to the level in the first month) along with the fitted curve implied by specification (27). This fitted curve will serve as the main evaluation test of the model: the predicted job-finding probabilities will be compared to the estimates of specification (27), as a test for the success of the model to replicate the job-finding profile.

Reemployment Wages. In my model only workers suitable for a vacancy are hired.

[11]The controls include a gender dummy, a fifth degree polynomial in age, three race dummies (white/black/other), four education category dummies, and gender interactions with all these covariates.

Unobserved heterogeneity implies that some workers are suitable for more jobs than other workers but this heterogeneity is not directly reflected in reemployment wages. Hence, the appropriate measure of wages for this paper should strip out the effects of unobserved heterogeneity in wages. Unfortunately, this cannot be done with CPS data. CPS follows workers for only eight months, hence there are very few workers that move from unemployment to employment twice. Thus, fixed effects cannot be used to strip out the effect of unobserved heterogeneity.

Fortunately, there are good measurements of this effect available in the literature. Schmieder et al. (2016) and Autor et al. (2015) provide causal estimates of non-employment duration on wages. They find that for each additional month in non-employment, wages decline by a bit less than one and two and a half percent, respectively. The sample in Autor et al. (2015), though, consists of SSDI applicants with low labor force attachment, hence this number may be too large for this paper. Moreover, Ortego-Marti (2017), controlling for unoberved heterogeneity with fixed effects in the PSID, finds that an extra month in non-employment lowers wage by one point two percent. Thus, I will use a monthly wage loss of one percent to discipline the decline of human capital in my model. Finally, note that both Schmieder et al. (2016) and Ortego-Marti (2017) report that this drop in wages is linear; that is, it is almost equal for each month in non-employment. I will exploit this feature of the data in my identification strategy, presented in Section 4.2.

Search Effort. It is notoriously difficult to obtain reliable measurements of job-search effort (see Hornstein and Kudlyak (2016)). Mukoyama et al. (2014), DeLoach and Kurt (2013), and Gomme and Lkhagvasuren (2015) use minutes devoted to job-search activities as their measure of search effort. Merging the American Time Use Survey (ATUS) with CPS allows them to obtain evidence of how this measure changes over the business cycle. These papers find mixed evidence regarding the cyclicality of time devoted to job-search.

A recent source of reliable evidence is the New Jersey survey of Krueger and Mueller (2011)— KM from now on. I choose to use that data for the following reasons: first, it is a panel survey. They followed the same unemployed workers over time; on the other hand, the ATUS is a cross-sectional survey. As a result, fixed effects cannot be incorporated directly. One needs to project time devoted to search by using the methods of job-search from CPS, as in Mukoyama et al. (2014). This method yields a monthly measure, yet it is plagued by the well-known reporting problems of CPS. Second, the KM survey was conducted on a weekly basis. Hence, the self-reported evidence on job-search are probably more accurate than those coming from CPS, which is a monthly survey. Finally, Krueger and Mueller (2011) oversampled long-term unemployed workers, which guarantees reliable reports of search effort for workers with high duration of unemployment.

I run two simple fixed effects regressions using the KM data. First, to determine the proper measure of search effort, I use the following specification:

$$Offer_{it} = a_i + \beta \times SE_{it-1} + \gamma_t + \epsilon_{it} \tag{29}$$

where $Offer_{it}$ is a dummy of whether the individual was offered a job in week t, SE_{t-1} is a measure of search intensity in week $t-1$, and γ_t a week fixed effect. When SE_{it-1} is the number of hours devoted to job-search (intensive margin), the estimate $\hat{\beta}$ is not significant, with a t-statistic equal to -0.5: one extra hour of job search has an insignificant effect on generating job-offers. This result was also obtained in a more sophisticated way by Krueger and Mueller (2011) and challenges the use of time devoted to search as the appropriate measure of search effort. On the other hand, when SE_{it-1} is a dummy variable of whether the individual did anything to find a job in week $t-1$ (extensive margin), the estimate of β appears to be significant, with a t-statistic of 5.23. Hence, I choose to work with the extensive margin of participation as the proper measure of search effort in the KM data.

Second, I regress the dummy of search effort on unemployment duration and an individual fixed effect:

$$SE_{it} = a_i + \beta \times \tau_{it} + \gamma \times \tau_{it}^2 + \delta_t + u_{it} \tag{30}$$

where τ is the unemployment duration of the individual in week t of the survey. The coefficient $\hat{\beta}$ is estimated to be equal to -0.006, with a t-statistic of -2.90, and $\hat{\gamma} = 9 \times 10^{-6}$, with a t-statistic of 0.98. I choose to discipline the decline in search effort in my model assuming a monthly linear drop in participation of around two percent. However, the KM survey was conducted from October 2009 to April 2010, a period of mass unemployment in New Jersey. Hence, the measured discouragement effects are likely higher than the effects in normal times, hence this measurement should be interpreted as the upper bound for the elasticity of workers' search effort for an extra week in unemployment.

It is worth mentioning that this finding is consistent with the evidence reported in Faberman and Kudlyak (2014). Using data from a job website, they find that the weekly number of submitted applications declines as job-search continues, controlling for individual fixed effects. In their data, the drop seems to have convex and not linear shape, though.

Callback Rates. To inform the distribution of unobserved heterogeneity in my model I use data from the audit study of Kroft et al. (2013), as reported in Kroft et al. (2016). This paper uses an audit study approach: they submitted carefully constructed fictitious job applications to posted job openings to investigate whether the duration of non-employment affects the likelihood to receive a callback when applying for a job. Kroft et al. (2013) report a steep decline of callbacks along duration, which can be seen in Figure 2.

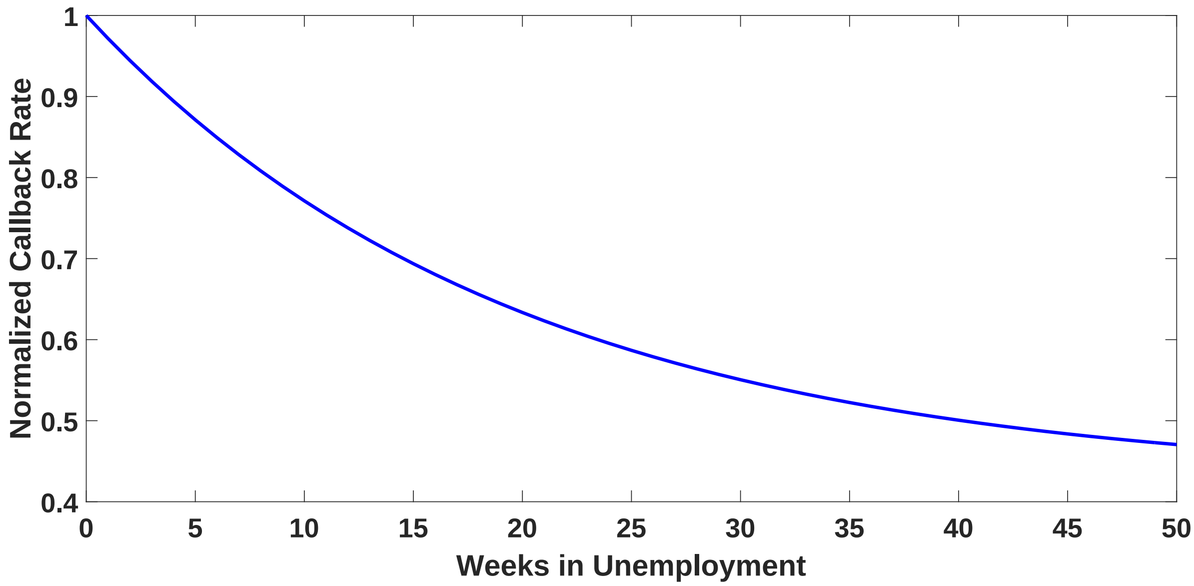

Figure 2: Normalized Callback Probabilities by Unemployment Duration as approximated by equation (27) and reported in Kroft et al. (2016)

Unfortunately, the external validity of this evidence is far from established. Jarosch and Pilossoph (2015) offer a thoughtful summary of the literature on audit studies and I summarize their main points here. Ghayad (2013) finds similar results as Kroft et al. (2013); Oberholzer-Gee (2008) finds declining callbacks only for very long unemployment spells; Eriksson and Rooth (2014) find large drops in callbacks for medium and low skilled jobs but not for high skilled jobs; most importantly, Farber et al. (2017) find no evidence of duration dependence in callbacks.

Farber et al. (2017) attribute most of the difference with Kroft et al. (2013) on the age composition of their samples: the former focus on older job applicants (mid-thirties to mid-fifties) while the latter on younger job-applicants (mid-twenties). There is an intuitive mechanism behind that difference: older applicants have longer employment histories that may outweigh any recent employment experience when resumes are evaluated by potential employers. Younger job-seekers, however, have short employment histories, hence recent unemployment experience may get higher weight in the evaluation of their applications. The fact that the applicants in Eriksson and Rooth (2014) and Ghayad (2013) are all in their twenties, with no more than five or six years of experience, supports that conclusion. Employers seem to employ unemployment duration as a signal of workers' quality in cases where the information on workers' CVs is not rich enough to allow for an informed decision (young workers, low-skilled applicants or applicants in slack labor markets; see Kroft et al. (2013)).

Jarosch and Pilossoph (2015), in the context of an equilibrium search model, find that interviews lost to statistical discrimination that would otherwise have led to jobs are very rare. Firms discriminate against long-term unemployed because they correctly anticipate being unable to form a viable match with them. Based on this result, my interpretation of the audit studies results is that discrimination in callbacks is an informed response to workers' unobserved characteristics. In other words, employers' beliefs, as captured by duration dependence in callbacks, are informative about unobservable worker quality among the population of job-seekers. I choose to discipline unobserved heterogeneity with the callback results from Kroft et al. (2013) because, given that their applicants were relatively young, the use of unemployment duration as a signal in this study has the best chance of being informative regarding the underlying worker characteristics, among the available audit studies.[12]

4 Quantitative Analysis

4.1 Identification

The first step is to show that the structure of the model is sufficient to separately identify the effect of each mechanism contributing to duration dependence. To put it differently, that there are some features of the data that the model would fail to capture if it did not incorporate all channels of duration dependence. In the model unemployed workers who participate in the labor market face duration dependence *caused* by skill depreciation and the declining quality of the unemployment pool. Hence, it should be shown that the

[12] Another piece of evidence supporting this interpretation is the other main finding of Kroft et al. (2013), namely that employers discriminate more in tighter labor markets. In tighter labor markets workers are evaluated more often by firms, hence unemployment duration is a more informative signal of worker's unobserved quality.

effects of these two forces are not observationally equivalent through the lens of the model. Workers' participation choice only *amplifies* these two channels but it does not constitute a separate mechanism of duration dependence that needs to be identified. The magnitude of this amplification will be disciplined directly by the Krueger and Mueller (2011) data on participation presented above.

Consider a version of the model in which the only force creating duration dependence is the declining expected suitability of the unemployment pool. Workers' on-the-job productivity stays constant over the spell of unemployment. In search models, workers are paid a share of the match surplus on top of their unemployment value. In this model, this fact is captured by the following equilibrium relationships:

$$w_\tau = \alpha y_\tau + (1-\alpha)\big(1-\beta(1-\nu)\big)U^*_{\tau+1} \tag{31}$$

$$w_{\tau+1} = \alpha y_{\tau+1} + (1-\alpha)\big(1-\beta(1-\nu)\big)U^*_{\tau+2} \tag{32}$$

Without skill loss, the productivity term is constant across τ. Hence, the wage drop is a fraction of the drop in the value of unemployment over the unemployment spell:

$$w_\tau - w_{\tau+1} = (1-\alpha)\big(1-\beta(1-\nu)\big)\Big(U^*_{\tau+1} - U^*_{\tau+2}\Big) \tag{33}$$

In a model without skill loss, the decline in the value of unemployment is very small for two reasons. First, worker's productivity on-the-job is constant, which mechanically shuts down an important component of the decline. Second, firms need to be compensated for offering higher wages to workers of short durations, hence the queue lengths are higher early in unemployment and decreasing over the spell. That is, the waiting time to find a job is less for suitable workers at higher durations. This force tends to increase the value of unemployment over the unemployment spell and makes the decline in equilibrium wages even smaller.

In quantitative terms, wage loss in this model is around 10% of the wage decline found in the data, as can be seen in Figure 7. In other words, it is impossible in this model to cook up a distribution of unobserved heterogeneity to replicate the wage drop we see in the data. The larger is the drop in wages, the larger needs to be the decrease in queue lengths to make firms ex ante indifferent between workers. Therefore, this will always create a countervailing decrease in waiting times over the spell, making the total drop in the value of unemployment and wages quantitatively insignificant.

Turning to the version of the model without suitability considerations, assume that workers are homogeneous and productive for all jobs. On-the-job productivity, though, declines with unemployment duration, capturing skill loss. Following the arguments of Proposition 2 one can show that this model predicts decreasing wages and job-finding rates. Thus, in principle, a version of the model with only human capital could rationalize both data series. This is not true, though: the model cannot rationalize the small linear drop in wages and the large convex drop in job-finding rates at the same time.

To show that, let me derive the equilibrium expressions for job-finding rates and wages in a model with only human capital depreciation:

$$w_\tau = \alpha\big(y_\tau - \Delta_\tau\big) + \Delta_\tau \tag{34}$$

$$q_\tau^{\alpha-1} \equiv f_\tau = \frac{1-\alpha}{\kappa}^{\frac{1-\alpha}{\alpha}} \big(y_\tau - \Delta_\tau\big)^{\frac{1-\alpha}{\alpha}} \tag{35}$$

where $\Delta_\tau \equiv \big(1 - \beta(1-\nu)(1-\delta)\big)U^*_{\tau+1}$. The results in Schmieder et al. (2016) and Ortego-Marti (2017) show that the decline of reemployment wages over the unemployment spell is roughly linear. Hence, the path of human capital in the model should be calibrated such that the term $y_\tau - \Delta_\tau$ falls linearly. However, if this is the case, then the term $\big(y_\tau - \Delta_\tau\big)^{\frac{1-\alpha}{\alpha}}$ is a concave function. According to the Petrongolo and Pissarides (2001) survey, “A plausible range for the empirical elasticity on unemployment is 0.5 to 0.7...”, thus the range of the term $\frac{1-\alpha}{\alpha}$ is from around 0.43 to 1, making it a concave function over τ. However, it is clear from the data that the drop in job-finding rates has a convex shape: it is large for short durations and small for high durations.

Intuitively, the productivity drop for workers of short durations is not enough to rationalize sharp declines in job-offer rates. Workers who are unemployed for few periods have almost the same productivity as workers unemployed for one period. Firms find it optimal to respond with slightly decreasing job-offer probabilities. Workers who have accumulated a lot of periods in unemployment have lost a large part of their productivity. As a result, they face sharp declines in job-finding rates, which also contradicts the data. The model needs the composition effect to produce job-finding rates that decline fast in low durations and slow in high durations. The intuition for this is, again, that when meeting rates are high and the worker fails to find a job, the probability to be suitable for a given job drops very fast. This force is needed on top of productivity drop to produce convex-shaped job-finding rates.

The argument analyzed above also suggests which data series informs each parameter of the model. The unobserved heterogeneity parameters are identified by the shape of callback rates coming from Kroft et al. (2013). The evidence on wages will determine the drop of productivity, y_τ; it will also pin down the vacancy creation cost κ through the Free Entry condition. Finally, the search cost parameters will be chosen so that participation in the model matches the weekly evidence on participation coming from the survey data of Krueger and Mueller (2011).

4.2 Calibration

I set a period in the model to be a week. I fix the maximum number of weeks in unemployment at $T = 50$. I normalize the productivity of newly unemployed workers to $y_1 = 1$ and impose a linearly depreciating log productivity, motivated by the findings of Schmieder et al. (2016) and Ortego-Marti (2016, 2017). I impose a standard Cobb-Douglas matching function, as in equations (1) – (3).

Several parameters are set outside the model. The discount factor β is set to 0.999, consistent with a 5% annual interest rate. I target an average 40-year career for workers, implying $\nu = 5 \times 10^{-4}$. The separation rate δ is set to 0.009 to match the monthly separation rate of 3.4% from Shimer (2012). I set the value of leisure to $b = 0.69$, as in Fernández-Blanco and Preugschat (2016), which lies in the middle of the range of estimates provided by Chodorow-Reich and Karabarbounis (2015). Finally, I follow Shimer (2005b) and set the elasticity of the matching function with respect to unemployment to 0.72, which lies towards the upper end of the range of estimates reported in Petrongolo and Pissarides (2001).

Table 1: Exogenously Set Parameters

Parameter	**Description**	**Value**	**Source**
β	Discount Factor	0.999	Annual Interest Rate of 5 %
ν	Death Probability	5×10^{-4}	40 year working life
δ	Separation Probability	0.009	Shimer (2012)
b	Value of Leisure	0.69	Fernández-Blanco and Preugschat (2016)
α	Matching Function Elasticity	0.72	Shimer (2005b)

There are seven parameters that are calibrated so the model matches the data reported in Section 3: π, a^H, a^L, d, κ, η and ϕ. Following the identification strategy outlined in the previous section, I choose the unobserved heterogeneity parameters (π, a^H and a^L) to mimic the evidence in callback rates; d and κ to capture the empirical decline in wages; and the search cost parameters (η and ϕ) to replicate data on weekly participation. More specifically, following Fernández-Blanco and Preugschat (2016), I target the average, standard deviation and skewness of the callback rates from Kroft et al. (2013) to pin down π, a^H and a^L. The targets for d and κ are the slope of reemployment wages and the average reemployment wage, based on Schmieder et al. (2017) and Ortego-Marti (2016, 2017). Finally, η and ϕ are pinned down by the average and standard deviation of the participation profile over the unemployment spell from the Krueger and Mueller (2011) survey.

Table 2: Jointly Calibrated Parameters

Parameter	**Description**	**Value**	**Target**
π	Share of broad-suitability workers	0.59	Average Callback Rate
a^H	Jobs suitable for broad-suitability workers	0.16	St. dev. of Callback Rates
a^L	Jobs suitable for limited-suitability workers	0.07	Skeweness of Callback Rates
d	Step of Productivity Decline	-1.17%	Slope of Reemployment Wages
κ	Vacancy Cost	3.72	Average Reemployment Wage
η	Search Cost Elasticity	5.1	St. dev. of Participation Profile
ϕ	Disutility of Search	0.84	Average Participation Probability

It is important to notice that the evidence on job-finding rates over the unemployment spell is not included in the calibration targets. That is, none of the parameters is chosen such that the model replicates the job-finding data of Figure 1. On the contrary, the ability of the model to produce a duration dependence profile close to the observed one will be used as the main evaluation test for its validity.

Table 3: Targeted Moments

Parameter	**Value**	**Target**	**Data**	**Model**
π	0.59	Average Callback Rate	0.64	0.66
a^H	0.16	St. dev. of Callback Rates	0.15	0.14
a^L	0.07	Skeweness of Callback Rates	0.86	0.92
d	-1.17%	Slope of Reemployment Wages	-0.009	-0.01
κ	3.72	Average Reemployment Wage	0.94	0.93
η	5.1	St. dev. of Participation Profile	0.07	0.06
ϕ	0.84	Average Participation Probability	0.88	0.86

As can be seen in Figure 3, the model does a good job matching the targeted features of the data. More specifically, it matches the participation and wage profile very accurately and slightly understimates the drop in callback probabiltiy from Kroft et al. (2013). The value of vacancy cost is at the upper end of estimates reported in the literature but, reassuringly, is roughly equal to 2.5 months production in the average match. This estimate is close to the values reported in other papers using directed search models, like Menzio and Shi (2011) or Flemming (2016). Finally, the model predicts a relatively low but realistic fraction of long-term unemployed equal to 12 %, though this was not included in calibration targets.

The unobservable heterogeneity parameters are broadly in line with the values reported in Fernández-Blanco and Preugschat (2016). The step of human capital decline is very close to the empirical estimates in the literature, including Ortego-Marti (2016, 2017), Schmieder et al. (2016) and Autor et al. (2015). Finally, there are very few estimates for the search cost parameters available in the literature. The most common approach is to normalize $\phi = 1$ and set $\eta = 2$. My estimates imply a higher elasticity, reflecting the large drop of participation in the KM survey. As mentioned earlier, though, this is likely an overestimate of the response of participation to the returns to job-search. This observation implies that the calibrated values used here are likely an upper bound for the actual population values of ϕ and η .

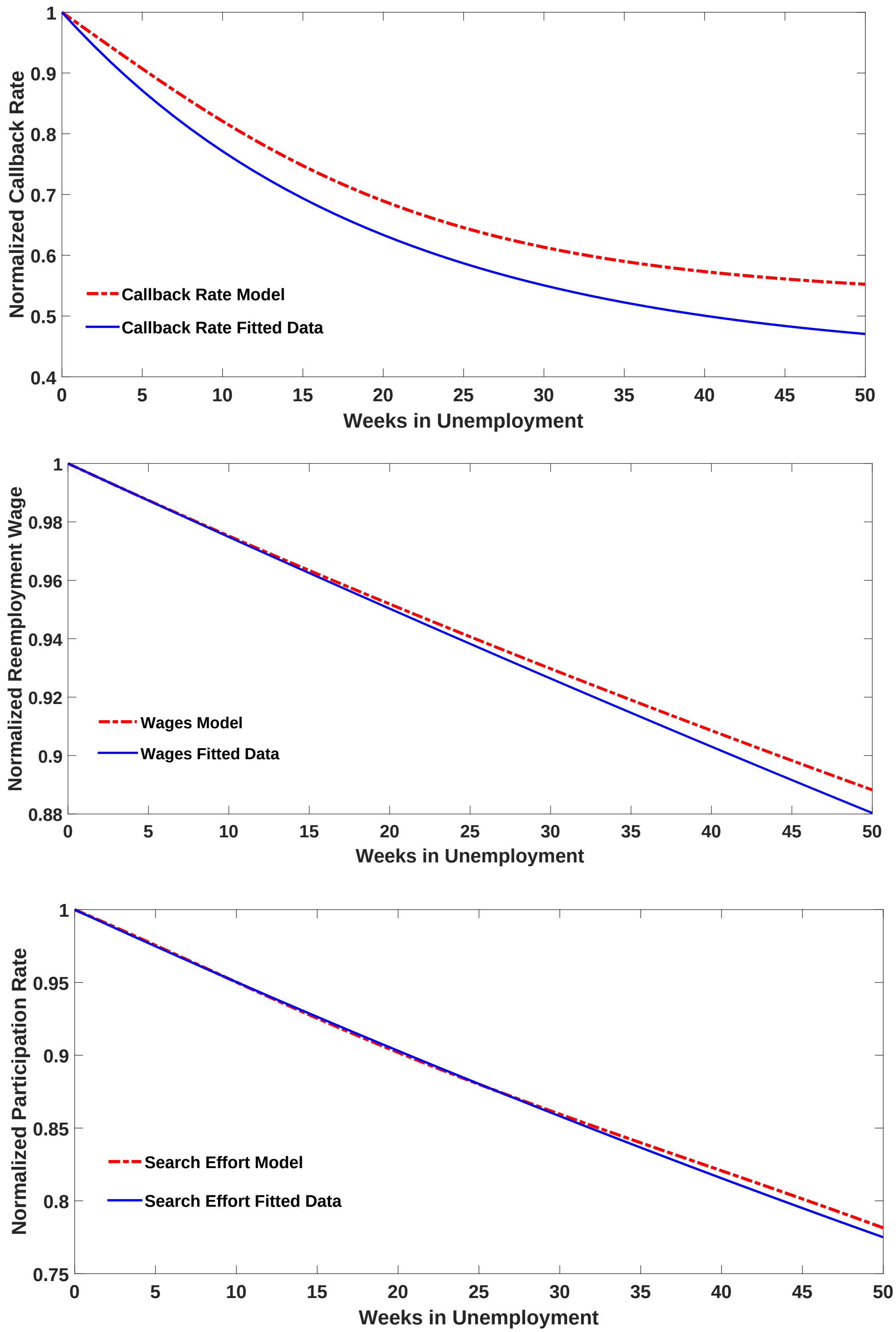

Figure 3: Model and Targets: Normalized Callback Probabilities, Wages and Participation Probabilities by Unemployment Duration

5 Results

Duration Dependence and Decomposition. As shown in the previous section, the parameter values in the model were chosen such that the model matches the available data on the channels creating duration dependence in unemployment. An important evaluation test for the model is whether it is able to predict a realistic duration dependence profile; that is, is the model-implied job-finding rate close to the observed one?

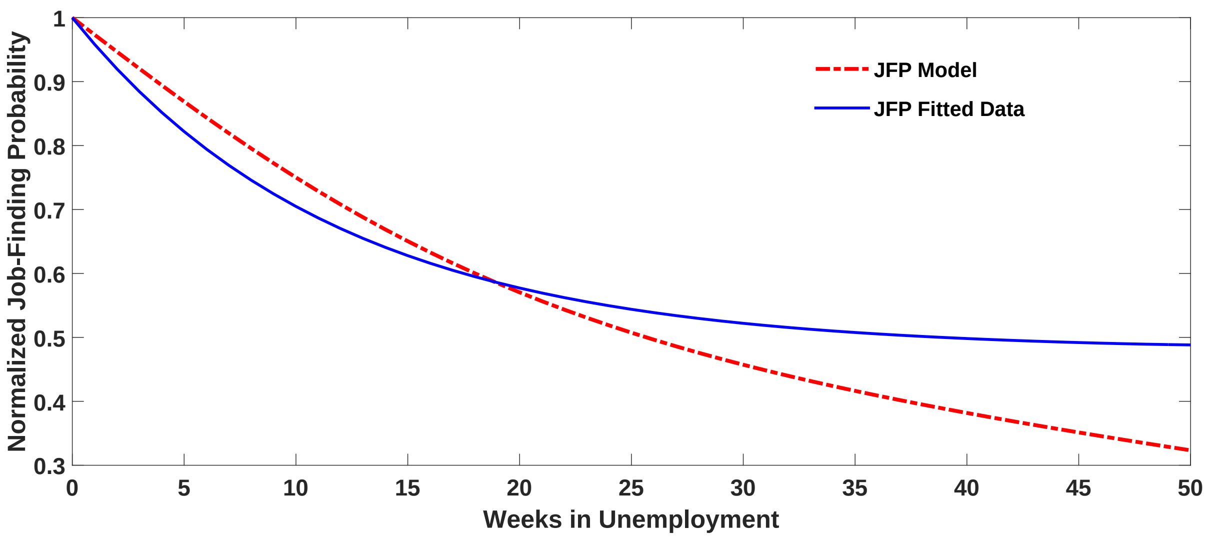

Figure 4: Normalized Job-finding Probabilities by Unemployment Duration

As can be seen in Figure 4, the model predicts a duration dependence profile very close to the one observed in CPS, even though job-finding rates were not included in the calibration targets. More specifically, the model provides an excellent match for unemployment spells up to six months and mildly overpredicts the steep decline in job-finding rates at higher durations. It is reassuring that even though parameter values were chosen to make the model consistent with micro data on the *sources* of duration dependence, the model matches unemployment exit probabilities accurately. This fact demonstrates that the model is an appropriate framework to be used for evaluating the quantitative significance of the three mechanisms contributing to duration dependence in unemployment.

Let the forces behind duration dependence considered in this paper form the set:

$$I = \{\text{Unobserved Heterogeneity, Skill Depreciation, Search Effort}\}$$

To accurately evaluate the contribution of each channel $i \in I$ to duration dependence one should be able to answer the counterfactual question: "How large of a decline in job-finding probability would we observe had channel i been absent?". Moreover, the complete answer to this question should take into account the absence of the effect of channel i on the other channels $j \in I$. In other words, for the appropriate decomposition exercise, one should be able to strip out not only the direct effect of i on job-finding rates but also the interactions of i with the rest of the channels contributing to duration dependence. The model built in this paper will be used to perform this counterfactual exercise.

To be more specific, let the parameters capturing unobserved heterogeneity among workers be summarized by a vector $\xi = [\pi \; a^H \; a^L]$. The full model predicts the following equilibrium job-finding rate for each duration τ:

$$f_\tau = s_\tau(\mu_\tau, y_\tau) \times \mu_\tau(\xi, x_\tau) \times x_\tau(\mu_\tau, y_\tau) \tag{36}$$

where y_τ is the productivity level of workers with duration τ. To evaluate the effects of skill loss, a version of the model without it will be used to compute the alternative job-finding profile f^1:

$$f^1_\tau = s_\tau(\mu_\tau) \times \mu_\tau(\xi, x_\tau) \times x_\tau(\mu_\tau) \tag{37}$$

Finally, to evaluate the effects of search effort, a version of the model without it will be used to compute the following job-finding profile:

$$f^2_\tau = \mu_\tau(\xi, x_\tau) \times x_\tau(\mu_\tau) \tag{38}$$

Notice that the profile f^2 includes only dynamic selection as a source of duration dependence. Hence, it captures the model's prediction regarding the magnitude of unobserved heterogeneity.[13]

The results of this exercise can be seen in Figure 5. Generally, and consistent with the findings of Alvarez et al. (2016) and Ahn and Hamilton (2016), unobserved worker heterogeneity accounts for the largest part of total duration dependence in unemployment. The role of skill loss and search effort, though, is quantitatively significant, especially at longer durations. The effect of skill loss plays a major role for spells greater than six months, while declining search effort affects the job-finding rate of all unemployed of duration greater than nine months in a uniform way. Interestingly, search effort slightly mitigates duration

[13] Actually, given the fact that the model cannot distinguish dynamic selection from employer discrimination, this estimate should be interpreted as an upper bound for the importance of unobserved worker heterogeneity through the lens of the model. See, also, the end of Section 2.3.

dependence for medium-term unemployed workers.

The model-implied significance of the forces causing within-worker duration dependence can rationalize the findings of Abraham et al. (2016) and Bentolila et al. (2017). These authors find strong within-person duration dependence in the data. Importantly, my model can also shed light to the quantitative effect of each mechanism contributing to this result. The effect of skill loss is sizable during the whole unemployment spell but it becomes especially pronounced at durations longer than six months. On the other hand, the impact of declining search effort is small and uniform for spells longer than nine months. Finally, the initial steep decline in job-finding probabilities can be attributed mostly to dynamic selection, that is, to the fact that "good" workers find job faster than "bad" workers.

The model can reconcile the empirical estimates in the following way. In data sets in which there is substantial mass of unemployed workers with unemployment duration longer than six months, the aggregate contribution of within-person duration dependence is expected to be significant. In data sets in which most unemployed workers have relatively short unemployment durations, as in US for instance, unobserved heterogeneity can account for the largest part of job-finding differences over the unemployment spell.

To put the importance of each channel in perspective, it would be instructive to go deeper in the decomposition. As expected by the non-linear nature of the model, the order of the decomposition matters for the magnitude of each channel.[14] That is, instead of the process above, one could evaluate the effects of search effort by using the following counterfactual job-finding profile:

$$f_\tau^3 = \mu_\tau(\xi, x_\tau) \times x_\tau(\mu_\tau, y_\tau) \tag{39}$$

In this case, the effect of human capital would be measured by evaluating the difference between f^3 and f^2.

[14]Reassuringly, though, the order does not change the key messages of the exercise.

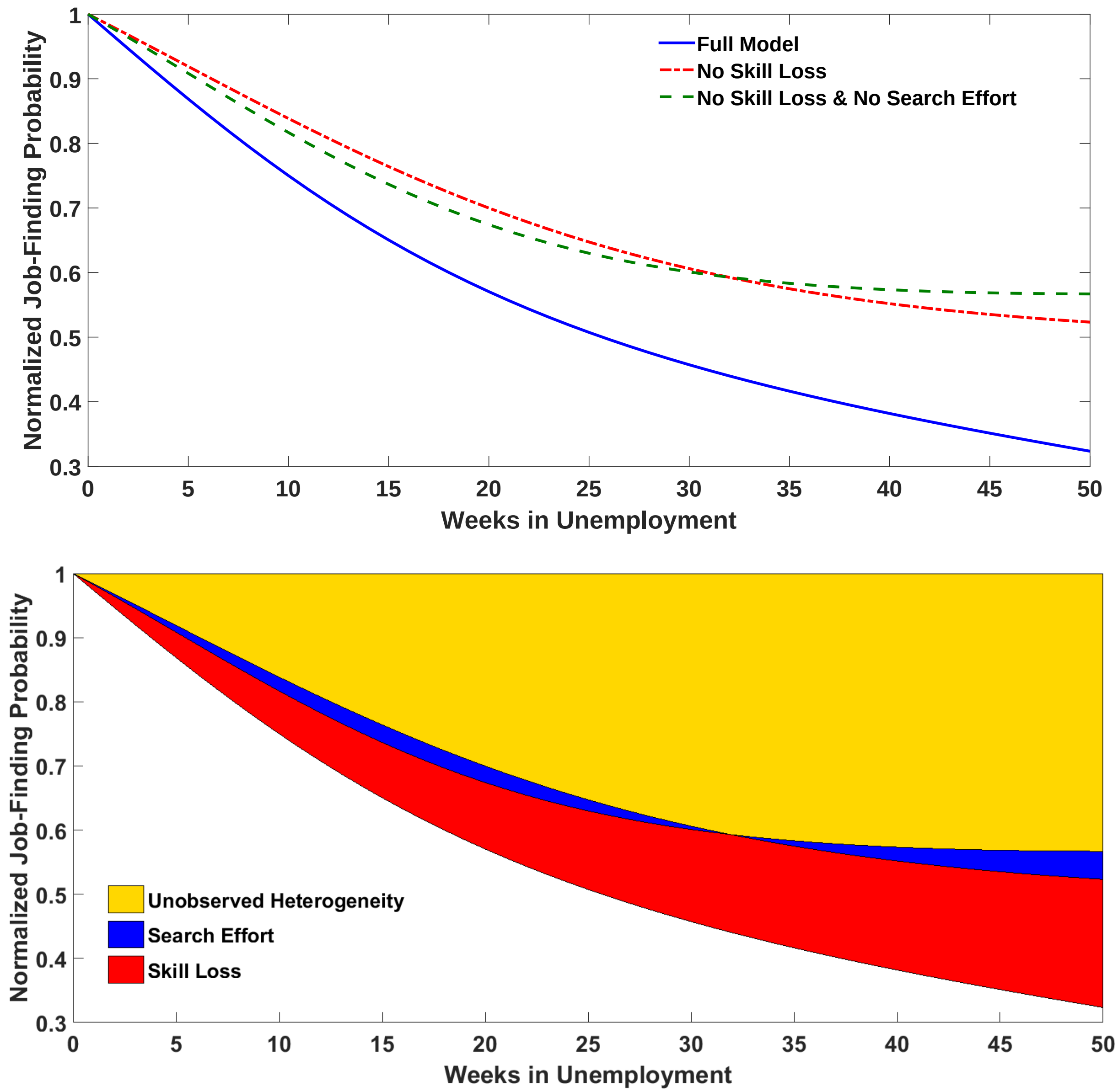

Figure 5: Decomposition of Normalized Job-finding Probabilities by Unemployment Duration

To evaluate the effects of each mechanism, one can define the following ratios, based on the weekly losses in job-finding:

$$R_\tau = \frac{f_\tau - f_\tau^2}{1 - f_\tau} \tag{40}$$

$$R_\tau^{HC} = \frac{0.5(f_\tau^2 - f_\tau^3) + 0.5(f_\tau - f_\tau^1)}{1 - f_\tau} \tag{41}$$

$$R_\tau^{SE} = \frac{0.5(f_\tau^2 - f_\tau^1) + 0.5(f_\tau - f_\tau^3)}{1 - f_\tau} \tag{42}$$

The first ratio, R_τ, measures the contribution of within-worker channels on the total amount of duration dependence predicted by the model, for different stages of unemployment duration. The next two ratios perform a similar measurement but focus on skill loss and search effort decline as sources of duration dependence. R_τ^{HC} captures the contribution of human capital depreciation to overall duration dependence. It is the average of two decomposition scenarios: (i) begin from the full model and shut down skill loss; (ii) add skill loss to a model that contains only unobserved heterogeneity. Each scenario of those would quantify a part of duration dependence due to skill loss. The answers, though, will differ because of the non-linear interactions of the model's mechanisms. To make the exercise robust, I take the average of all possible decomposition exercises for each force.

Correspondingly, R_τ^{SE} measures the impact of declining search effort on the model-implied overall duration dependence. It is the average of the corresponding scenarios analyzed above for human capital depreciation. Notice that all ratio measures are indexed by the length of the unemployment spell, τ. This is done to highlight the fact that at different lengths of an unemployment spell, the quantitative importance of each mechanism for the observed duration dependence will be different.

However, one could shut down the mechanisms following many different sequential orders. Specifically, the decomposition analyzed above does not consider a version of the model with only within-worker duration dependence. For robustness, in Figure 6 below, I present the averages for all different permutations of decompositions for each mechanism, including versions without unobserved heterogeneity. The details of each calculation can be found in Appendix II. They key messages are the same, though, regardless of the sequential order of the decomposition.

Plotting the ratio measures in Figure 6 is illuminating. First, consider the classic question, "How large is the part of duration dependence that is attributed to unobserved worker differences?". The model-implied response is, "It depends on the length of the unemployment spells of the workers at hand". Overall, when comparing unemployed at most durations, unobserved heterogeneity accounts for the largest share of the differences in job-finding. For long-term unemployed workers, though, the accumulated effects of skill loss and search effort are quantitatively significant and account for almost half of the observed differences in job-finding between workers.

Digging into the structure of within-person duration dependence is, also, revealing. As expected from Figure 5, the effect of declining search effort is quantitatively small and symmetric for workers unemployed longer than nine months. On the contrary, the impact of skill loss needs some time to accumulate and become quantitatively significant. For spells longer than six months, skill loss becomes an important cause of duration dependence.

To make these findings clearer, consider the following two comparisons. First, in the US, a newly unemployed worker has a 30% greater chance of finding a job, compared to an observationally similar worker who is jobless for three months. According to my model, 85% of this disparity can be attributed to unobserved differences between the average newly unemployed and the average worker who is unemployed for three months, while skill loss and search effort account for a modest 15%. Second, when comparing a worker unemployed for six months with a worker unemployed for a year or more, the former has a 12% greater chance of finding a job. The model attributes one half of that disparity to unobserved worker differences and the other half to a combination of skill decay and lower search effort exhibited by workers who are unemployed for a year or more. Importantly, the model implies that skill loss accounts for a vastly larger part of the disparity than the decline in search effort.

The quantitative results of the model highlight the importance of policies tight to the length of spell of different unemployed workers to improve their job-finding prospects. According to the model, there is little space for policymakers to improve the job-finding rates of the short-term unemployed. The prospects of these workers decline very fast due to the declining average quality of the unemployment pool. On the other hand, the model points to the appropriate short-term policy responses for improving the job-finding prospects of medium- and long-run unemployed workers. Specifically, the model implies that policymakers should invest in both job-training and job-search assistance programs to fight long-term unemployment. The impact of job-training programs, though, is expected to be greater.

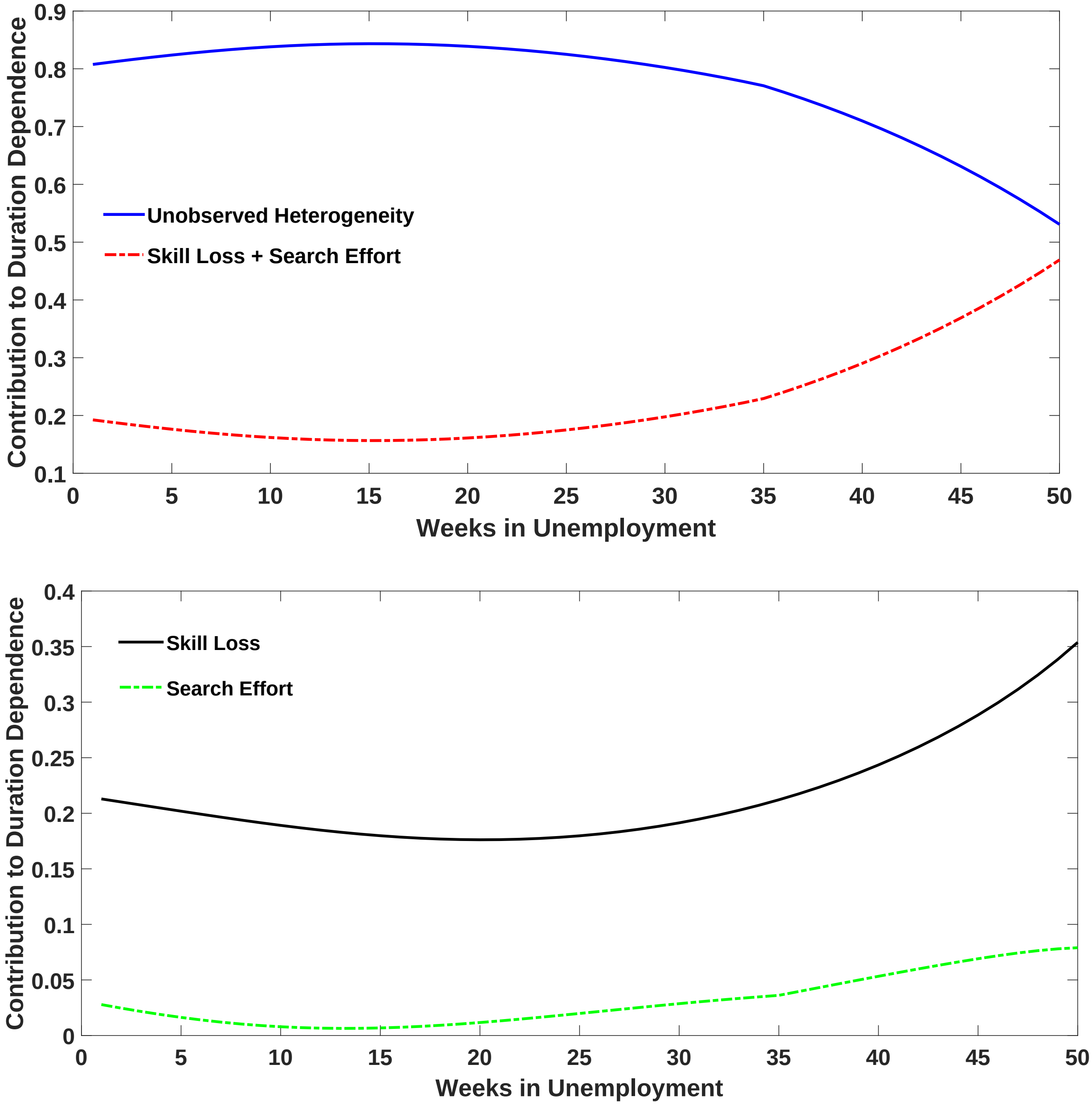

Figure 6: Contribution of Various Channels to Duration Dependence by Unemployment Duration

It is worth emphasizing that the model-implied policy implications are consistent with the findings of Card et al. (2016) regarding the impact of active labor market policies on fighting long-term unemployment. In a meta-study of various active labor market programs around the world, Card et al. (2016) find that more than 70% of job-training and job-search assistance programs in their sample had a significant positive impact for the long-term unemployed. Importantly, though, they also find that the effects of active labor market programs are more positive for the long-term unemployed that short- or medium-term unemployed. Moreover, they find that job-training programs are significantly more likely to bring about positive impacts for the long-term unemployed than for workers at shorter durations. The model developed in this paper illuminates how the interaction of unobserved heterogeneity with skill loss and declining search effort can generate these policy conclusions.

Showing the predicted paths for reemployment wages in Figure 7 is useful to understand the mechanics of the model. As mentioned earlier, in versions of the model without skill loss, reemployment wages drop by less than 0.1% per month, a counterfactual prediction. This highlights the importance of skill depreciation in order the model to match the data, as well as the fact that wages in the model are pinned down by the speed of skill depreciation. The fact that wages are higher in the version of the model with endogenous search effort is a result of directed search. Wages price waiting times in competitive search; waiting times (job-finding rates) are higher (lower) in the model with search effort, hence wages needs to be higher to compensate workers who do not differ in productivity over their unemployment spell.

Finally, to highlight the importance of unobserved worker heterogeneity it would be useful to consider the predicted job-finding profile of a model in which this mechanism is absent. Figure 8 plots the normalized job-finding profiles for the full model and for a model that contains only skill loss and search effort. The concave shape of the profile makes clear that a model without unobserved worker differences and learning cannot generate the large drop in job-finding observed in the first months of the spell. Skill losses need time to accumulate to make firms not willing to hire the long-term unemployed. Hence, unobserved heterogeneity is crucial for the model to capture the empirical pattern of duration dependence in unemployment.

Intuition: How the Model works. At this point it may be instructive to explain how the different channels of the model interact to make its predictions consistent with the data. First, consider the submarkets populated by job-seekers with short unemployment

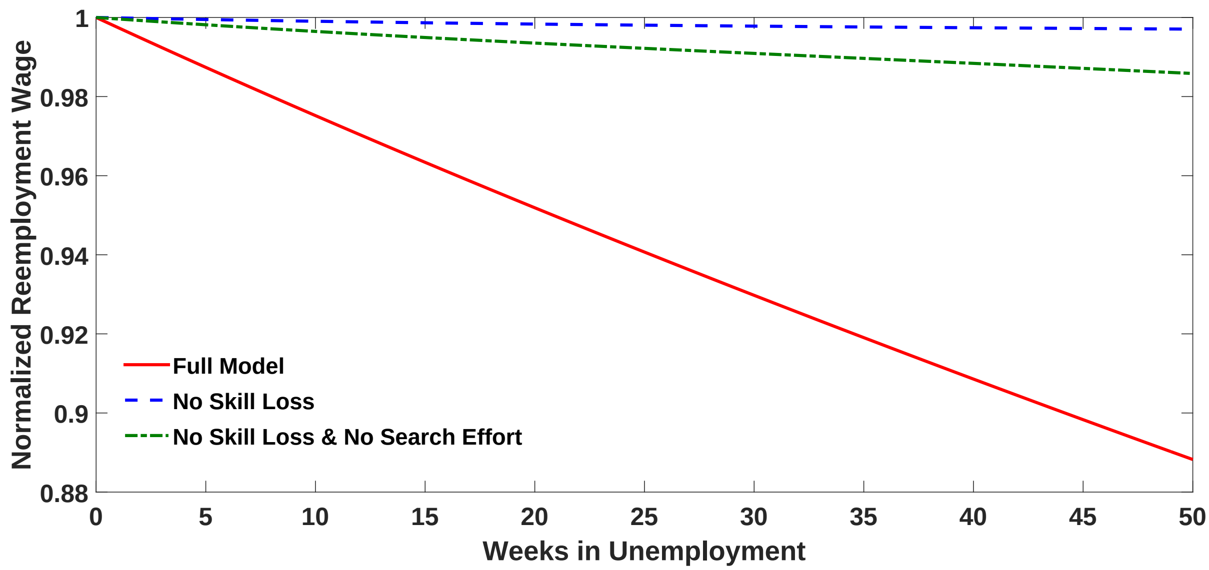

Figure 7: Normalized Reemployment Wages by Unemployment Duration

spells. Workers who are found suitable in these submarkets have relatively high productivity, leading to high match surplus. As a result, firms post a lot of vacancies directed to newly unemployed workers, since they are very productive and have good chances of being suitable for a job's tasks. Given the high probability of job applications be reviewed by firms, if a worker fails to find a job early on her spell, this says a lot about her quality: unemployment duration is a very informative signal for short durations, because of the large number of worker-firm meetings. Thus, μ_τ drops very fast in the first few periods of unemployment. Moreover, since the returns to job-search are high for the newly unemployed, most workers engage in job search in the beginning of their unemployment spell.

As their unemployment spell evolves, workers become less productive, due to skill loss. Hence, the match surplus declines and firms offer less job opportunities to workers with high unemployment durations: x_τ declines, following the path of skill depreciation. However, because worker-firm meetings are scarce, the probability a worker to be tested is low. Thus, failing to find a job is not very informative about workers' quality: μ_τ drops slowly for high durations of unemployment. Finally, the returns to job search decrease, hence workers of high τ exhibit lower effort to find jobs: s_τ drops due to discouragement.

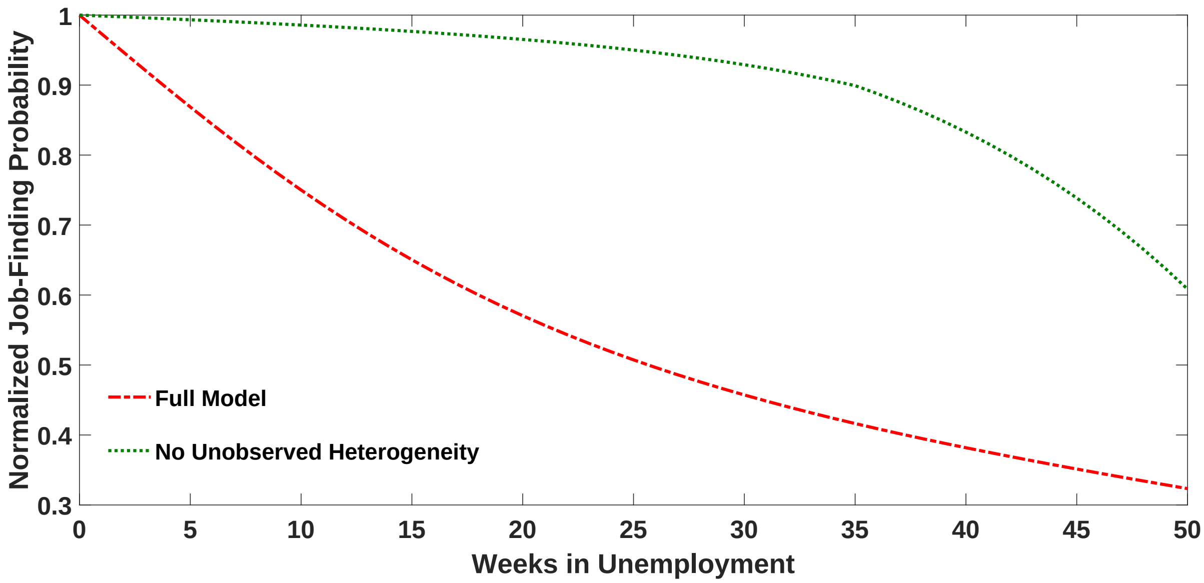

Figure 8: Normalized Job-Finding Probabilities by Unemployment Duration

6 Discussion and Related Literature

This paper is related to several strands of literature in Macroeconomics and Labor Economics. This section describes how the paper fits into these strands and how its contributions advance the relevant lines of work.

Competitive search. The model in this paper uses the machinery of competitive search, developed by Moen (1997), Acemoglu and Shimer (1999a,b), Burdett et al. (2001), Mortensen and Wright (2002), Shi (2002, 2006), Shimer (2005a), and Inderst (2005). It generalizes the competitive search framework to an environment in which interacting non-stationary forces cause duration dependence in unemployment. It establishes the equivalence between competitive search equilibrium and the solution of an auxiliary optimization problem, in the tradition of Acemoglu and Shimer (1999a,b), and characterizes the equilibrium analytically. It develops an algorithm to compute the equilibrium of directed search models that fully exploits the fixed-point structure of the auxiliary optimization problem. However, this paper remains silent regarding the efficiency properties of equilibrium, a theme analyzed very often in the competitive search literature.

Models of Duration Dependence. This is the strand of the literature this paper is most related to. Early contributions include the random search models of Lockwood (1991), Pissarides (1992), Blanchard and Diamond (1994) and Acemoglu (1995). Gonzalez and Shi (2010) provide the first directed search framework that could speak to the question of duration dependence. They construct a model in which workers learn about their type while searching for a job to explain the stylized fact that reemployment wages are decreasing in unemployment duration. This paper uses a variation of the matching process developed in Gonzalez and Shi (2010). Their paper is an exclusively theoretical contribution (provides no quantitative results) and, most importantly, has an important counterfactual implication: duration dependence in unemployment is positive. The hazard rates within individual workers out of unemployment are increasing over the unemployment spell. This paper fixes that problem by introducing skill loss in the model. As I show above, my model is capable of successfully rationalizing both job-finding rates and reemployment wages data, as well as providing answers to relevant quantitative questions.

The most recent and closely related papers are Fernández-Blanco and Preugschat (2016), Jarosch and Pilossoph (2015) and Doppelt (2014). Fernández-Blanco and Preugschat (2016) build a directed search model to rationalize the evidence presented in the audit study of Kroft et al. (2013). Jarosch and Pilossoph (2015) evaluate the quantitative significance of firm discrimination against long-term unemployed on job-finding rates. Their results imply that the contribution of stigma to job-finding rates is weak, a finding this paper takes seriously and builds upon. Doppelt (2014) is a skillful quantitative extension of Gonzalez and Shi (2010) that features learning about a worker's quality over her whole career.

A shared limitation of these studies is that the only source of duration dependence is unemployment stigma— meaning, employer discrimination against long-term unemployed in hiring. As a result, they remain silent regarding the importance of skill depreciation and search effort decline for observed duration dependence.[15] Moreover, their elegance and significance notwithstanding, these studies have some troubling implications. Reemployment wages in Fernández-Blanco and Preugschat (2016) may increase with unemployment duration; the firm-worker meeting rates in Jarosch and Pilossoph (2015) are exogenous, so they cannot perform counterfactuals useful for policy analysis; finally, Doppelt (2014) shares the counterfactual result of Gonzalez and Shi (2010), with a minority of workers facing increasing job-finding rates over the unemployment spell. This paper predicts unambiguously decreasing job-finding rates and reemployment wages for all workers in the labor market, while the

[15] To be fair, Doppelt (2014) provides a version of his model with skill depreciation. However, skill decay in Doppelt's model attenuates the drop in job-finding rates! In other words, skill depreciation improves the prospects of workers in Doppelt's model, which is at odds with the empirical findings of Card et al. (2016).

meeting rates are endogenous objects, determined in equilibrium.

It should be mentioned, however, that through the lens of the model analyzed here, dynamic selection in job-finding and employer discrimination in interviews cannot be distinguished. The model does not incorporate a separate interview stage in the hiring process; hence, it is not capable of providing an estimate of the effect of employer discrimination at the interview stage. Since dynamic selection is conflated with employer discrimination, the results of my model should be interpreted as an upper bound for the magnitude of unobserved worker heterogeneity and a lower bound for the magnitude of within-worker duration dependence. Jarosch and Pilossoph (2015), however, show that employer discrimination at the interview stage has a small effect on duration dependence.

Other relevant contributions include Flemming (2016) and Potter (2017). Flemming (2016) rationalizes duration dependence in unemployment with learning-by-doing and home production. Her model predicts unambiguously decreasing job-finding rates and reemployment wages but the decline of the model-implied job-finding rates does not have the convex shape found in the data. This result highlights the importance of a composition/learning mechanism to account for the convex shape of job-finding rates. Potter (2017) builds a partial equilibrium model to emphasize the effect of learning on workers' search intensity, one of the mechanisms at work in this paper too. He uses the data from Krueger and Mueller (2011) survey but he works with the intensive margin of search effort, which is shown not to have a significant effect on job-finding probability. Finally, the consequences of human capital depreciation in random search models are studied by Ortego-Marti (2016, 2017) and Laureys (2014), in a line of work initiated by Pissarides (1992) and Ljungqvist and Sargent (1998).

Empirical Work. Turning to the empirical front, the first paper that analyzed duration dependence in unemployment was Kaitz (1970). After him, a large econometric literature tried to measure duration dependence by estimating duration models with observational data. This literature made progress by imposing strong parametric identifying assumptions to identify within-worker duration from unobserved heterogeneity. It is nicely summarized by Van den Berg (2001) and Machin and Manning (1999). More related to this paper is a series of recent contributions that estimate within-worker duration dependence either by using sophisticated econometric techniques or more reduced form methods. The former include Alvarez et al. (2016), Ahn and Hamilton (2016) and Bentolila et al. (2017), while the main paper in the latter is Abraham et al. (2016). Abraham et al. (2016) and Bentolila et al. (2017) find a strong role for within-worker duration dependence. The results in Ahn and Hamilton (2016) and Alvarez et al. (2016) emphasize the importance of unobserved heterogeneity but they still find a small positive role for within-worker duration dependence.

This paper analyzes duration dependence through the lens of an equilibrium search model. Hence, it can measure the magnitude of the effects of specific mechanisms on observed duration dependence. The empirical approaches are primarily concerned with distinguishing within-worker duration dependence from dynamic selection, thus they are not equipped to measure the magnitude of different mechanisms contributing to within-worker duration dependence. Other relevant empirical contributions include the papers measuring the effect of unemployment duration on reemployment wages (Schmieder et al. (2016), Autor et al. (2015), Nekoei and Weber (2017)), on search effort (Krueger and Mueller (2011), Faberman and Kudlyak (2014)), as well as a series of influential audit studies (Kroft et al. (2013), Eriksson and Rooth (2014), Ghayad (2013), Oberholzer-Gee (2008), Farber et al. (2017)). Finally, Card et al. (2016) is a meta-study of active labor market policies, the results of which are fully consistent with the results of this paper.

Methodology. This paper uses an equilibrium search model to assign magnitudes to forces causing duration dependence. To measure the effect of each channel, it computes counterfactual job-finding profiles over the unemployment spell. For each counterfactual, a specific mechanism of duration dependence is shut down and the difference of the predicted job-finding profile with the profile of the full model is attributed to the missing channel. This methodology is employed by many recent studies in Macroeconomics: Burdett et al. (2016), Jarosch (2014), and Wolcott (2017) use rich search models to perform similar quantitative decompositions. Moreover, Fernández-Blanco and Preugschat (2016), Jarosch and Pilossoph (2015) and Doppelt (2014) employ this methodology to evaluate the effects of employer discrimination against long-term unemployed in hiring. I am not aware of any paper, though, that uses a directed search model to evaluate the effects of dynamic selection, skill loss and search effort decline.

7 Conclusion

In short, this paper makes two contributions: (i) it introduces a directed search model of the labor market, featuring unobserved worker differences, skill loss in unemployment, and endogenous job-search effort; (ii) it combines the structure of the model with data on reemployment wages and search effort to evaluate the significance of each mechanism for the observed duration dependence in unemployment. The results of interest include: (i) the model successfully replicates the job-finding profile in US data, even though the latter was not used to pin down any model parameters; (ii) in agreement with recent empirical literature, overall, the most important factor behind the total observed duration dependence is unobserved worker heterogeneity; (iii) the bulk of the effect of unobserved worker heterogeneity is concentrated in the first few months of the unemployment spell; more than 40% of the differences among workers at longer spells should be attributed to skill loss and declining search effort; (iv) skill loss is quantitatively more important for drops in job-finding at spells greater than six months. These results have sharp implications about how active labor market programs should be tailored to help short- and long-term unemployed workers find jobs.

To conclude, let me summarize some future research directions based on this paper. First, an interesting direction would be to make the model stochastic to incorporate aggregate shocks. This would be a useful framework to study the effects of extensive unemployment benefits on duration dependence in recessions. Second, one could study the efficiency properties of the framework developed here and compare the results with Fernández-Blanco and Preugschat (2016). Third, the modeling of human capital could be extended to reflect skills as measured directly in the data, following Macaluso (2017). Finally, and more broadly, it would be of interest for one to study the forces evaluated in this paper in a model of stock-flow matching (Ebrahimy and Shimer (2010)).

References

Abraham, K. G., Haltiwanger, J. C., Sandusky, K., and Spletzer, J. (2016). The consequences of long term unemployment: Evidence from matched employer-employee data. Technical report, National Bureau of Economic Research.

Acemoglu, D. (1995). Public policy in a model of long-term unemployment. *Economica*, pages 161–178.

Acemoglu, D. and Shimer, R. (1999a). Efficient unemployment insurance. *Journal of political Economy*, 107(5):893–928.

Acemoglu, D. and Shimer, R. (1999b). Holdups and efficiency with search frictions. *International Economic Review*, 40(4):827–849.

Ahn, H. J. and Hamilton, J. D. (2016). Heterogeneity and unemployment dynamics. Technical report, National Bureau of Economic Research.

Alvarez, F. E., Borovičková, K., and Shimer, R. (2016). Decomposing duration dependence in a stopping time model. Technical report, National Bureau of Economic Research.

Autor, D. H., Maestas, N., Mullen, K. J., and Strand, A. (2015). Does delay cause decay? the effect of administrative decision time on the labor force participation and earnings of disability applicants. Technical report, National Bureau of Economic Research.

Bentolila, S., García-Pérez, J. I., and Jansen, M. (2017). Are the spanish long-term unemployed unemployable? *SERIEs*, 8(1):1–41.

Blanchard, O. J. and Diamond, P. (1994). Ranking, unemployment duration, and wages. *The Review of Economic Studies*, 61(3):417–434.

Burdett, K., Carrillo-Tudela, C., and Coles, M. (2016). Wage inequality: A structural decomposition. *Review of Economic Dynamics*, 19:20–37.

Burdett, K., Shi, S., and Wright, R. (2001). Pricing and matching with frictions. *Journal of Political Economy*, 109(5):1060–1085.

Card, D., Kluve, J., and Weber, A. (2016). Active labour market policies and long-term unemployment. *Long-Term Unemployment After the Great Recession: Causes and remedies*, page 11.

DeLoach, S. B. and Kurt, M. (2013). Discouraging workers: Estimating the impacts of macroeconomic shocks on the search intensity of the unemployed. *Journal of Labor Research*, 34(4):433–454.

Doppelt, R. (2014). The hazards of unemployment. *New York university working paper*.

Ebrahimy, E. and Shimer, R. (2010). Stock–flow matching. *Journal of Economic Theory*, 145(4):1325–1353.

Elsby, M. W. and Hobijn, B. (2010). The labor market in the great recession. *Brookings Papers on Economic Activity: Spring 2010*, page 1.

Eriksson, S. and Rooth, D.-O. (2014). Do employers use unemployment as a sorting criterion when hiring? evidence from a field experiment. *The American Economic Review*, 104(3):1014–1039.

Faberman, R. J. and Kudlyak, M. (2014). The intensity of job search and search duration.

Farber, H. S., Silverman, D., and Von Wachter, T. M. (2017). Factors determining callbacks to job applications by the unemployed: An audit study. *RSF*.

Fernández-Blanco, J. and Preugschat, E. (2016). On the effects of ranking by unemployment duration.

Flemming, J. (2016). Skill accumulation in the market and at home.

Galenianos, M. and Kircher, P. (2012). On the game-theoretic foundations of competitive search equilibrium. *International economic review*, 53(1):1–21.

Ghayad, R. (2013). The jobless trap. *Northeastern University*.

Gomme, P. and Lkhagvasuren, D. (2015). Worker search effort as an amplification mechanism. *Journal of Monetary Economics*, 75:106–122.

Gonzalez, F. M. and Shi, S. (2010). An equilibrium theory of learning, search, and wages. *Econometrica*, 78(2):509–537.

Guerrieri, V., Shimer, R., and Wright, R. (2010). Adverse selection in competitive search equilibrium. *Econometrica*, 78(6):1823–1862.

Hornstein, A. and Kudlyak, M. (2016). Estimating matching efficiency with variable search effort.

Inderst, R. (2005). Competitive search markets with heterogeneous workers. *European Economic Review*, 49(6):1525–1542.

Jarosch, G. (2014). Searching for job security and the consequences of job loss. *V University of Chicago, Job Market Paper*.

Jarosch, G. and Pilossoph, L. (2015). Statistical discrimination and duration dependence in the job finding rate.

Kaitz, H. B. (1970). Analyzing the length of spells of unemployment. *Monthly Lab. Rev.*, 93:11.

Kroft, K., Lange, F., and Notowidigdo, M. J. (2013). Duration dependence and labor market conditions: Evidence from a field experiment. *The Quarterly Journal of Economics*, 128(3):1123–1167.

Kroft, K., Lange, F., Notowidigdo, M. J., and Katz, L. F. (2016). Long-term unemployment and the great recession: the role of composition, duration dependence, and nonparticipation. *Journal of Labor Economics*, 34(S1):S7–S54.

Krueger, A. B., Cramer, J., and Cho, D. (2014). Are the long-term unemployed on the margins of the labor market? *Brookings papers on economic activity*, 2014(1):229–299.

Krueger, A. B. and Mueller, A. (2011). Job search, emotional well-being, and job finding in a period of mass unemployment: Evidence from high-frequency longitudinal data. *Brookings Papers on Economic Activity*, 2011(1):1–57.

Laureys, L. (2014). The cost of human capital depreciation during unemployment.

Ljungqvist, L. and Sargent, T. J. (1998). The european unemployment dilemma. *Journal of political Economy*, 106(3):514–550.

Lockwood, B. (1991). Information externalities in the labour market and the duration of unemployment. *The Review of Economic Studies*, 58(4):733–753.

Macaluso, C. (2017). Skill remoteness and post-layoff labor market outcomes.

Machin, S. and Manning, A. (1999). The causes and consequences of longterm unemployment in europe. *Handbook of labor economics*, 3:3085–3139.

Menzio, G. and Shi, S. (2010). Directed search on the job, heterogeneity, and aggregate fluctuations. *The American Economic Review*, 100(2):327–332.

Menzio, G. and Shi, S. (2011). Efficient search on the job and the business cycle. *Journal of Political Economy*, 119(3):468–510.

Moen, E. R. (1997). Competitive search equilibrium. *Journal of political Economy*, 105(2):385–411.

Mortensen, D. T. and Wright, R. (2002). Competitive pricing and efficiency in search equilibrium. *International economic review*, 43(1):1–20.

Mukoyama, T., Patterson, C., and Sahin, A. (2014). Job search behavior over the business cycle.

Nekarda, C. J. (2009). A longitudinal analysis of the current population survey: Assessing the cyclical bias of geographic mobility.

Nekoei, A. and Weber, A. (2017). Does extending unemployment benefits improve job quality? *The American Economic Review*, 107(2):527–561.

Oberholzer-Gee, F. (2008). Nonemployment stigma as rational herding: A field experiment. *Journal of Economic Behavior & Organization*, 65(1):30–40.

Ortego-Marti, V. (2016). Unemployment history and frictional wage dispersion. *Journal of Monetary Economics*, 78:5–22.

Ortego-Marti, V. (2017). Loss of skill during unemployment and tfp differences across countries. *European Economic Review*.

Petrongolo, B. and Pissarides, C. A. (2001). Looking into the black box: A survey of the matching function. *Journal of Economic literature*, 39(2):390–431.

Pissarides, C. A. (1992). Loss of skill during unemployment and the persistence of employment shocks. *The Quarterly Journal of Economics*, 107(4):1371–1391.

Pissarides, C. A. (2000). *Equilibrium unemployment theory.* MIT press.

Potter, T. (2017). Learning and job search dynamics during the great recession. Technical report, LeBow College of Business, Drexel University.

Schmieder, J. F., von Wachter, T., and Bender, S. (2016). The effect of unemployment benefits and nonemployment durations on wages. *The American Economic Review*, 106(3):739–777.

Shi, S. (2002). A directed search model of inequality with heterogeneous skills and skill-biased technology. *The Review of Economic Studies*, 69(2):467–491.

Shi, S. (2006). Wage differentials, discrimination and efficiency. *European Economic Review*, 50(4):849–875.

Shimer, R. (2005a). The assignment of workers to jobs in an economy with coordination frictions. *Journal of Political Economy*, 113(5):996–1025.

Shimer, R. (2005b). The cyclical behavior of equilibrium unemployment and vacancies. *American economic review*, pages 25–49.

Shimer, R. (2012). Reassessing the ins and outs of unemployment. *Review of Economic Dynamics*, 15(2):127–148.

Van den Berg, G. J. (2001). Duration models: specification, identification and multiple durations. *Handbook of econometrics*, 5:3381–3460.

Wolcott, E. L. (2017). Employment inequality: Why do the low-skilled work less now?

A Appendix I: Proofs

Lemma A.1 (Equilibrium $\mapsto$ Auxiliary Problem). *Let $w_\tau^* \in \mathbb{W}_{\tau,\mu}^*$ and $q_\tau^* = Q_{\tau,\mu}^*(w_\tau^*)$, where $\{\mathbb{W}_{\tau,\mu}^*, \{Q_{\tau,\mu}^*\}_{\tau \leq T}, \mathbf{U}^*\}$ be an equilibrium allocation; then $\{w_\tau^*, q_\tau^*\}_{\tau \leq T}$ solve problem (12) under constraints (13), (14) and (15), with $U_{\tau,\mu}(w_\tau^*, q_\tau^*) = U_{\tau,\mu}^*$ if $q_\tau^* > 0$.*

Proof. First, notice that the Beliefs Updating condition ensures that the constraint (15) is satisfied. Also, note that Optimal Application ensures that constraint (13) is satisfied.

Now, suppose that some w_τ^* and q_τ^* do not maximize (12). That is, there are $q_\tau' > 0$ and a w_τ' that achieve a strictly positive value for the firm, while satisfying constraints (13), (14) and (15). Formally:

$$-\kappa + \lambda(q_\tau') \frac{y_\tau - w_\tau'}{1 - \beta(1-\nu)} > 0$$

while $U_{\tau,\mu}(w_\tau', q_\tau') = U_{\tau,\mu}^*$. By the definition of competitive search equilibrium, it has to be the case that $U_{\tau,\mu}(w_\tau', Q_{\tau,\mu}^*(w_\tau')) \leq U_{\tau,\mu}^*$, due to Rational Expectations. Hence, $U_{\tau,\mu}(w_\tau', q_\tau') \geq U_{\tau,\mu}(w_\tau', Q_{\tau,\mu}^*(w_\tau'))$. By definition:

$$U_{\tau,\mu}(w_\tau', q_\tau') = b + \beta(1-\nu)\left[\mu x(q_\tau')\left(\frac{w_\tau'}{1-\beta(1-\nu)} - U_{\tau+1,\mu'(q_\tau')}^*\right) + U_{\tau+1,\mu'(q_\tau')}^*\right]$$
$$U_{\tau,\mu}(w_\tau', Q_{\tau,\mu}^*(w_\tau^{'})) = b + \beta(1-\nu)\left[\mu x(Q_{\tau,\mu}^*(w_\tau^{'}))\left(\frac{w_\tau'}{1-\beta(1-\nu)} - U_{\tau+1,\mu'(Q_{\tau,\mu}^*)}^*\right) + U_{\tau+1,\mu'(Q_{\tau,\mu}^*)}^*\right]$$

where $\mu'(q_\tau') = H(x(q_\tau'), \mu)$ and $\mu'(Q_{\tau,\mu}^*)) = H(x(Q_{\tau,\mu}^*)), \mu)$.

There are two scenarios making the inequality $U_{\tau,\mu}(w_\tau', q_\tau') \geq U_{\tau,\mu}(w_\tau', Q_{\tau,\mu}^*(w_\tau'))$ hold:

- <u>Case 1</u>: $x(Q_{\tau,\mu}^*(w_\tau^{'})) \leq x(q_\tau')$, $\mu'(q_\tau') \leq \mu'(Q_{\tau,\mu}^*)$, and $U_{\tau+1,\mu'(q_\tau')}^* \leq U_{\tau+1,\mu'(Q_{\tau,\mu}^*)}^*$.[16] Then, the following chain is true: $x(Q_{\tau,\mu}^*(w_\tau^{'})) \leq x(q_\tau') \Leftrightarrow Q_{\tau,\mu}^*(w_\tau^{'}) \geq q_\tau' \Leftrightarrow \lambda(Q_{\tau,\mu}^*(w_\tau^{'})) \geq \lambda(q_\tau')$.

[16] The value of unemployment is strictly increasing in expected suitability; for a formal proof see Gonzalez and Shi (2010), Theorem 3.1.

As a result:

$$-\kappa + \lambda(Q^*_{\tau,\mu}(w'_\tau))\frac{y_\tau - w'_\tau}{1-\beta(1-\nu)} > 0$$

which contradicts the Profit Maximization and Free Entry conditions of equilibrium.

- Case 2: $x(Q^*_{\tau,\mu}(w'_\tau)) \geq x(q'_\tau)$, $\mu'(q'_\tau) \geq \mu'(Q^*_{\tau,\mu})$, and $U^*_{\tau+1,\mu'(q'_\tau)} \geq U^*_{\tau+1,\mu'(Q^*_{\tau,\mu})}$. Then, one can find a slightly greater queue length, q''_τ, such that $q''_\tau > q'_\tau$, but also $U^*_{\tau+1,\mu'(q''_\tau)} \geq U^*_{\tau+1,\mu'(q'_\tau)}$, which yields the same value of unemployment for the worker at duration τ.

  As a result:

$$-\kappa + \lambda(q''_\tau)\frac{y_\tau - w'_\tau}{1-\beta(1-\nu)} > 0$$

  which contradicts the Profit Maximization and Free Entry conditions of equilibrium.

■

Lemma A.2 (Auxiliary Problem ↦ Equilibrium). *If some $\{w^*_\tau, q^*_\tau\}_{\tau\leq T}$ solve problem (12) under constraints (13), (14) and (15), then there exists an equilibrium $\{\mathbb{W}^*_{\tau,\mu}, \{Q^*_{\tau,\mu}\}_{\tau\leq T}, \mathbf{U}^*\}$ such that $w^*_\tau \in \mathbb{W}^*_{\tau,\mu}$ and $q^*_\tau = Q^*_{\tau,\mu}(w^*_\tau)$, $\forall\tau \leq T$.*

Proof. Let me start with the constructive part of the claim. It is straightforward to construct $\{\mu_\tau\}_{\tau\leq T}$ as a function of $\{q^*_\tau\}_{\tau\leq T}$. Define $\mathbb{W}^*_{\tau,\mu} = \{w^*_\tau\}_{\tau\leq T}$ and $Q^*_{\tau,\mu}(w^*_\tau) = q^*_\tau$, $\forall\tau \leq T$, given the constructed series of expected suitability. Set the following recursively:

$$U^*_{\tau,\mu_\tau} = b + \beta(1-\nu)\left[\mu_\tau x(q^*_\tau)\left(\frac{w^*_\tau}{1-\beta(1-\nu)} - U(w^*_{\tau+1}, q^*_{\tau+1})\right) + U(w^*_{\tau+1}, q^*_{\tau+1})\right]$$

and

$$U^*_{T,\mu_T} = b + \beta(1-\nu)\left[\mu_T x(q^*_T)\left(\frac{w^*_T}{1-\beta(1-\nu)} - U(w^*_T, q^*_T)\right) + U(w^*_T, q^*_T)\right]$$

Now, define $Q^*_{\tau,\mu}(w)$ to satisfy:

$$U^*_{\tau,\mu_\tau} = b + \beta(1-\nu)\left[\mu_\tau x(Q^*_{\tau,\mu}(w)\left(\frac{w}{1-\beta(1-\nu)} - U^*_{\tau+1,\mu_{\tau+1}}\right) + U^*_{\tau+1,\mu_{\tau+1}}\right]$$

and

$$U^*_{T,\mu_T} = b + \beta(1-\nu)\left[\mu_T x(Q^*_{\tau,\mu}(w)\left(\frac{w}{1-\beta(1-\nu)} - U^*_{T,\mu_T}\right) + U^*_{T,\mu_T}\right]$$

or $Q^*_{\tau,\mu}(w) = 0$ if there is no solution to any of these equations.

By construction, $\{\mathbb{W}^*_{\tau,\mu}, \{Q^*_{\tau,\mu}\}_{\tau\leq T}, \mathbf{U}^*\}$ satisfy the Profit Maximization, Free Entry and Beliefs Updating conditions. It remains to be shown that it satisfies Optimal Application.[17] Suppose to the contrary that there are equilibrium w'_τ and $Q^*_{\tau,\mu}(w'_\tau) > 0$ that yield greater utility to the worker than $U^*_{\tau,\mu}$:

$$U^*_{\tau,\mu} < b + \beta(1-\nu)\left[\mu x(Q^*_{\tau,\mu}(w'_\tau)\left(\frac{w'_\tau}{1-\beta(1-\nu)} - U^*_{\tau+1,\mu'(Q^*_{\tau,\mu}(w'_\tau))}\right) + U^*_{\tau+1,\mu'(Q^*_{\tau,\mu}(w'_\tau))}\right]$$

But then there is a $q'_\tau > Q^*_{\tau,\mu}(w'_\tau) > 0$ such that:[18]

$$U^*_{\tau,\mu} = b + \beta(1-\nu)\left[\mu x(q'_\tau)\left(\frac{w'_\tau}{1-\beta(1-\nu)} - U^*_{\tau+1,\mu'(q'_\tau)}\right) + U^*_{\tau+1,\mu'(q'_\tau)}\right]$$

Then it is true that $\lambda(q'_\tau) > \lambda(Q^*_{\tau,\mu}(w'_\tau))$; that is, (w'_τ, q'_τ) yield strictly greater profit to the firm. That is, I have shown that (w'_τ, q'_τ) yield strictly greater profit than (w^*_τ, q^*_τ) while satisfying constraints (13), (14) and (15), a contradiction.

■

Proposition A.1. *There exists an equilibrium in which the labor market is segmented by unemployment duration.*

Proof. First, consider the simple firms' maximization problem (12) under constraint (13) only. The objective function is a continuous function. Also, every q_τ is bounded below by zero and constraint (13) puts an upper bound on it for every duration τ. Therefore, Weierstrass

[17] If Optimal Application is satisfied, then the Rational Expectations condition holds by the construction of $Q^*_{\tau,\mu}(\cdot)$ and $\{U^*_{\tau,\mu}\}_{\tau\leq T}$.

[18] Again, Gonzalez and Shi (2010) prove that the value of unemployment is strictly increasing and, as a result, continuous.

Theorem ensures the existence of a solution to this simple maximization problem.

To proceed, let $f : K \to K$, where $K \equiv [a^L, a^H]^T \times [\frac{b}{1-\beta(1-\nu)}, \frac{y_1}{1-\beta(1-\nu)}]^T$ is a compact set. I define f to be the composite correspondence $f \equiv \psi \circ g$, where ψ and g are defined as follows. First, let $z \equiv \big(\{\mu_\tau\}_{\tau \leq T}, \{U_\tau\}_{\tau \leq T}\big)$ and $g(z)$ be defined as the set of elements $\{q_\tau, w_\tau, U^*_\tau\}_{\tau \leq T}$ that satisfy the zero-profit condition (14) and solve the firms' profit maximization problem (12) under constraint (13). U^*_τ is obtained by using the complementary slackness condition (13). Second, let ψ be defined as $\psi\big(\{w_\tau\}_{\tau \leq T}, \{q_\tau\}_{\tau \leq T}, \{U^*_\tau\}_{\tau \leq T}\big) \equiv \big(\{\mu'_\tau\}_{\tau \leq T}, \{U'_\tau\}_{\tau \leq T}\big)$, where $\{\mu'_\tau\}_{\tau \leq T}$ is uniquely determined by the Bayesian updating equation (15) with $\mu_1 = \pi a^H + (1-\pi)a^L$ and $U'_\tau = U^*_\tau$ for all τ. Notice that the equilibrium can be identified as a fixed point of f.

I need to show that f is a continuous function. First, ψ is obviously continuous. It remains to be shown that $g(z)$ is singleton and continuous for every $z \in K$. After substituting constraint (13) in (12) firms' problem becomes:

$$V^*_\tau = \max_{q_\tau} -\kappa + \lambda(q_\tau)\left(\frac{y_\tau}{1-\beta(1-\nu)} - U^*_{\tau+1}\right) + q_\tau\left(\frac{U^*_{\tau+1}}{\mu_\tau} - \frac{U^*_\tau - b}{\beta(1-\nu)\mu_\tau}\right), \quad \forall \tau \leq T$$

Having assumed that $\lambda(\cdot)$ is strictly concave ensures that this function is strictly concave in q_τ, thus there is a unique optimum. Hence, g is a function. Finally, the Maximum Theorem guarantees that g is continuous at $z \in K$. Therefore, the composite function f is also continuous. Hence, Brouwer's Fixed Point Theorem ensures that f has a fixed point in K, so there is an equilibrium with segmented labor markets.

■

Proposition A.2. *In any equilibrium in which the labor market is segmented by unemployment duration, q_τ is increasing and w_τ is decreasing in τ; also, the difference $y_\tau - w_\tau$ is decreasing in τ. Hence, the value of a filled vacancy, $J(w_\tau)$, is decreasing in τ.*

Proof. First, notice that by constraint (14) (Free Entry) the sequences $y_\tau - w_\tau$ and q_τ must move in opposite directions. That is, since $\lambda(q_\tau)\frac{y_\tau - w_\tau}{1-\beta(1-\nu)} = \kappa$, for all τ, and $\lambda(\cdot)$ is strictly increasing, in order the Free Entry condition to hold, $y_\tau - w_\tau$ and q_τ have to be moving in opposite directions over different submarkets.

I will prove this statement by induction. To begin with, I will show that it is true for $\tau = T-1$ and $\tau = T$. Following the algebra developed in section 4.1, one can compute equilibrium wages and the value of a filled job for submarkets $T-1$ and T as follows:

$$
\begin{aligned}
w_{T-1} &= \alpha y_{T-1} + (1-\alpha)\big(1-\beta(1-\nu)\big)U_T^* \\
w_T &= \alpha y_T + (1-\alpha)\big(1-\beta(1-\nu)\big)U_T^* \\
y_{T-1} - w_{T-1} &= (1-\alpha)\big(y_{T-1} - (1-\beta(1-\nu)U_T^*\big) \\
y_T - w_T &= (1-\alpha)\big(y_T - (1-\beta(1-\nu)U_T^*\big)
\end{aligned}
$$

Subtracting the last two equalities yields:

$$(y_{T-1} - w_{T-1}) - (y_T - w_T) = (1-\alpha)(y_{T-1} - y_T)$$

or just

$$w_{T-1} - w_T = \alpha(y_{T-1} - y_T) < y_{T-1} - y_T$$

The difference $w_{T-1} - w_T$ is positive and smaller than $y_{T-1} - y_T$. Also, $(y_{T-1} - w_{T-1}) > (y_T - w_T)$, hence q_T has to be greater than q_{T-1}.

To proceed with the induction, assume that $w_\tau > w_{\tau+1}$, $(y_\tau - w_\tau) > (y_{\tau+1} - w_{\tau+1})$ and $q_\tau < q_{\tau+1}$; to complete the proof it needs to be shown that $w_{\tau-1} > w_\tau$, $(y_{\tau-1} - w_{\tau-1}) > (y_\tau - w_\tau)$ and $q_{\tau-1} < q_\tau$. Subtracting wages yields:

$$w_{\tau-1} - w_\tau = \alpha(y_{\tau-1} - y_\tau) + (1-\alpha)(1-\beta(1-\nu))\big(U_\tau^* - U_{\tau+1}^*\big)$$

The idea here is to use the information on w_τ and $w_{\tau+1}$ to gain information on the difference $U^*_{\tau+1} - U^*_{\tau+2}$ which, in turn, will be useful for bounding the difference $U^*_\tau - U^*_{\tau+1}$ and the difference in wages.

Using the standard expression for equilibrium wages yields:

$$U^*_{\tau+1} = \frac{w_\tau - \alpha y_\tau}{(1-\alpha)(1-\beta(1-\nu))}$$
$$U^*_{\tau+2} = \frac{w_{\tau+1} - \alpha y_{\tau+1}}{(1-\alpha)(1-\beta(1-\nu))}$$

or just:

$$U^*_{\tau+1} - U^*_{\tau+2} = \frac{w_\tau - w_{\tau+1} - \alpha(y_\tau - y_{\tau+1})}{(1-\alpha)(1-\beta(1-\nu))}$$

Now, consider the difference $U^*_\tau - U^*_{\tau+1}$:

$$\begin{aligned}
U^*_\tau - U^*_{\tau+1} = b + \beta(1-\nu)\left[\mu_\tau x(q_\tau)\left(\frac{w_\tau}{1-\beta(1-\nu)} - U^*_{\tau+1}\right) + U^*_{\tau+1}\right] - \\
- b - \beta(1-\nu)\left[\mu_{\tau+1} x(q_{\tau+1})\left(\frac{w_{\tau+1}}{1-\beta(1-\nu)} - U^*_{\tau+2}\right) + U^*_{\tau+2}\right] \geq \\
\geq \beta(1-\nu)\left[\mu_{\tau+1} x(q_{\tau+1})\left(\frac{w_\tau}{1-\beta(1-\nu)} - U^*_{\tau+1} - \frac{w_{\tau+1}}{1-\beta(1-\nu)} + U^*_{\tau+2}\right) + U^*_{\tau+1} - U^*_{\tau+2}\right] = \\
= \beta(1-\nu)\left[\mu_{\tau+1} x(q_{\tau+1})\left(\frac{w_\tau}{1-\beta(1-\nu)} - \frac{w_{\tau+1}}{1-\beta(1-\nu)}\right) + \left(1-\mu_{\tau+1} x(q_{\tau+1})\right)\left(U^*_{\tau+1} - U^*_{\tau+2}\right)\right] = \\
= \beta(1-\nu)\left[\mu_{\tau+1} x(q_{\tau+1})\left(\frac{w_\tau}{1-\beta(1-\nu)} - \frac{w_{\tau+1}}{1-\beta(1-\nu)}\right) + \left(1-\mu_{\tau+1} x(q_{\tau+1})\right)\frac{w_\tau - w_{\tau+1} - \alpha(y_\tau - y_{\tau+1})}{(1-\alpha)(1-\beta(1-\nu))}\right] = \\
= \beta(1-\nu)\frac{(1-\alpha\mu_{\tau+1} x(q_{\tau+1}))(w_\tau - w_{\tau+1}) - \alpha(1-\mu_{\tau+1} x(q_{\tau+1}))(y_\tau - y_{\tau+1})}{(1-\alpha)(1-\beta(1-\nu))}
\end{aligned}$$

Hence, the difference $w_{\tau-1} - w_\tau$ can be bounded as:

$$w_{\tau-1} - w_\tau = \alpha(y_{\tau-1} - y_\tau) + (1-\alpha)(1-\beta(1-\nu))\big(U^*_\tau - U^*_{\tau+1}\big) \geq$$

$$\geq \alpha(y_{\tau-1}-y_\tau)+\beta(1-\nu)(1-\alpha\mu_{\tau+1}x(q_{\tau+1}))(w_\tau-w_{\tau+1})-\beta(1-\nu)\alpha(1-\mu_{\tau+1}x(q_{\tau+1}))(y_\tau-y_{\tau+1})$$

Assuming a linear drop D in workers' productivity, as in the quantitative analysis of the paper, yields:

$$w_{\tau-1}-w_\tau = \alpha D y_{\tau-1}-\beta(1-\nu)\alpha(1-\mu_{\tau+1}x(q_{\tau+1}))Dy_\tau+\beta(1-\nu)(1-\alpha\mu_{\tau+1}x(q_{\tau+1}))(w_\tau-w_{\tau+1}) \geq$$

$$\geq \alpha D y_\tau\big(1-\beta(1-\nu)(1-\mu_{\tau+1}x(q_{\tau+1}))\big) + \beta(1-\nu)(1-\alpha\mu_{\tau+1}x(q_{\tau+1}))(w_\tau - w_{\tau+1}) \geq 0$$

Now, consider the following differences:

$$(y_{\tau-1} - w_{\tau-1}) - (y_\tau - w_\tau) = (1-\alpha)(y_{\tau-1} - y_\tau) - (1-\alpha)(1-\beta(1-\nu))\big(U^*_\tau - U^*_{\tau+1}\big) \geq$$

$$\geq (1-\alpha)(y_{\tau-1}-y_\tau)-\beta(1-\nu)(1-\alpha\mu_{\tau+1}x(q_{\tau+1}))(w_\tau-w_{\tau+1})+\beta(1-\nu)\alpha(1-\mu_{\tau+1}x(q_{\tau+1}))(y_\tau-y_{\tau+1}) \geq$$

$$\geq (1-\alpha)(y_{\tau-1}-y_\tau)-\beta(1-\nu)(1-\mu_{\tau+1}x(q_{\tau+1}))(w_\tau-w_{\tau+1})+\beta(1-\nu)(1-\mu_{\tau+1}x(q_{\tau+1}))(y_\tau-y_{\tau+1}) =$$

$$= (1-\alpha)(y_{\tau-1} - y_\tau) + \beta(1-\nu)(1-\mu_{\tau+1}x(q_{\tau+1}))\big((y_\tau - w_\tau) - (y_{\tau+1} - w_{\tau+1})\big) \geq 0$$

Finally, since q_τ and $y_\tau - w_\tau$ move in opposite directions in equilibrium, the last step proves that $q_{\tau-1} \leq q_\tau$.

■

Lemma A.3. *In any equilibrium in which the labor market is segmented by unemployment duration, $q_\tau > 0$ for all τ. Hence, the complementary slackness condition (13) holds with equality.*

Proof. Suppose that there exists at least one duration group of workers such that its associated queue is 0. Let us denote by τ_0 the first duration for which the queue length is 0.

All queues associated with longer durations must also be 0, since $y_\tau < y_{\tau_0}$ for all $\tau > \tau_0$ and Proposition A.2 proved that the value of a filled vacancy is decreasing in τ. Then, the unemployment value of workers with unemployment duration greater than or equal to τ_0 must be $\frac{b}{1-\beta(1-\nu)}$, as they will remain unemployed forever.

Let w_{τ_0} be the profit maximizing wage for workers of duration τ_0. Given that $y_{\tau_0} > b$, there exists an arbitrarily small, but positive ϵ such that $b + \epsilon < y_{\tau_0}$. Consider now the alternative wage $w'_{\tau_0} = b + \epsilon$. This wage offer will attract a positive queue of workers and delivers strictly higher profits than w_{τ_0}, so w_{τ_0} and $q_{\tau_0} = 0$ cannot be profit-maximizing. Therefore, $q_\tau > 0$ for all τ in any equilbrium. ∎

Lemma A.4. *Beliefs about worker's expected suitability for a given job, μ_τ, are decreasing in τ.*

Proof. By construction $a^L \leq \mu_\tau \leq a^H$ for all τ. It is straightforward to notice that:

$$\begin{aligned}
\mu_{\tau+1} &\leq \mu_\tau \Leftrightarrow \\
a^H - \frac{(a^H - \mu_\tau)(1 - x_\tau a^L)}{1 - x_\tau \mu_\tau} &\leq \mu_\tau \Leftrightarrow \\
(1 - x_\tau \mu_\tau) a^H - (a^H - \mu_\tau)(1 - x_\tau a^L) &\leq (1 - x_\tau \mu_\tau)\mu_\tau \Leftrightarrow \\
(1 - x_\tau \mu_\tau)(a^H - \mu_\tau) &\leq (1 - x_\tau a^L)(a^H - \mu_\tau) \Leftrightarrow \\
a^L &\leq \mu_\tau
\end{aligned}$$

which is always true, since in equilibrium $q_\tau > 0$. ∎

Proposition A.3. *In any equilibrium in which the labor market is segmented by unemployment duration, the value of unemployment, U^*_τ, is decreasing in τ.*

Proof. I prove this by induction. First, it is straightforward to notice that $U^*_{T-1} \geq U^*_T$ since

$q_{T-1} < q_T$, $\mu_{T-1} \geq \mu_T$, $w_{T-1} \geq w_T$ and $x(\cdot)$ is strictly decreasing:

$$U^*_{T-1} = b + \beta(1-\nu)\left[\mu_{T-1}x(q_{T-1})\left(\frac{w_{T-1}}{1-\beta(1-\nu)} - U^*_T\right) + U^*_T\right]$$
$$U^*_T = b + \beta(1-\nu)\left[\mu_T x(q_T)\left(\frac{w_T}{1-\beta(1-\nu)} - U^*_T\right) + U^*_T\right]$$

Now assume that $U^*_{\tau+1} \leq U^*_\tau$ to show that $U^*_\tau \leq U^*_{\tau-1}$:

$$U^*_{\tau-1} - U^*_\tau = b + \beta(1-\nu)\left[\mu_{\tau-1}x(q_{\tau-1})\left(\frac{w_{\tau-1}}{1-\beta(1-\nu)} - U^*_\tau\right) + U^*_\tau\right] -$$
$$- b - \beta(1-\nu)\left[\mu_\tau x(q_\tau)\left(\frac{w_\tau}{1-\beta(1-\nu)} - U^*_{\tau+1}\right) + U^*_{\tau+1}\right] \geq$$
$$\geq \beta(1-\nu)\left[\mu_{\tau-1}x(q_{\tau-1})\left(\frac{w_{\tau-1}}{1-\beta(1-\nu)} - U^*_\tau - \frac{w_\tau}{1-\beta(1-\nu)} + U^*_{\tau+1}\right) + U^*_\tau - U^*_{\tau+1}\right] =$$
$$= \beta(1-\nu)\left[\mu_{\tau-1}x(q_{\tau-1})\left(\frac{w_{\tau-1}}{1-\beta(1-\nu)} - \frac{w_\tau}{1-\beta(1-\nu)}\right) + \left(1-\mu_{\tau-1}x(q_{\tau-1})\right)\left(U^*_\tau - U^*_{\tau+1}\right)\right] \geq 0$$

■

B Appendix II: Quantitative Model

B.1 The Fixed-Point Problem

The proof of equilibrium existence applies Brouwer's fixed point theorem on the auxiliary optimization problem of maximizing (12) under the constraints (13), (14) and (15). Recall that the objective is the firm's value of posting a vacancy, given that is supplies the worker with her market value, the Free Entry condition holds and beliefs about worker quality follow Bayes rule. The structure of this problem implies a straightforward algorithm for the computation of equilibrium.

The algorithm rests on the structure of this auxiliary problem and I conjecture that it could be used for all block recursive directed search models. It is similar in spirit to the famous Menzio and Shi (2011) method but it differs from their work in that it exploits the tractability of firm's FOCs in the auxiliary problem, instead of the worker's optimal submarket choice.

The equilibrium in my model is a fixed point in the space of workers' market values, $\{U^*_\tau\}_{\tau \leq T}$. One can see that by carefully inspecting the auxiliary optimization problem as analyzed in section 2.3. First, note that the market value constraint (22) defines a relationship between w_τ, q_τ, μ_τ and U^*_τ, $U^*_{\tau+1}$:

$$q_\tau^\alpha \frac{w_\tau}{1-\beta(1-\nu)} = q_\tau^\alpha U^*_{\tau+1} + \frac{q_\tau}{\beta(1-\nu)\mu_\tau}\left\{\frac{U^*_\tau - b - \beta(1-\nu)U^*_{\tau+1}}{\phi^{\frac{1}{1-\eta}}\frac{\eta-1}{\eta}}\right\}^{\frac{\eta-1}{\eta}}$$

This relationship is substituted into the objective function of the auxiliary problem to strip out wage from the value of a vacancy. Taking FOCs with respect to q_τ yields an expression for queue lengths as a function of μ_τ, U^*_τ and $U^*_{\tau+1}$:

$$q_\tau^{\alpha-1} = \frac{1}{\beta(1-\nu)\mu_\tau}\left\{\frac{U^*_\tau - b - \beta(1-\nu)U^*_{\tau+1}}{\phi^{\frac{1}{1-\eta}}\frac{\eta-1}{\eta}}\right\}^{\frac{\eta-1}{\eta}} \frac{1}{\alpha\left(\frac{y_\tau}{(1-\beta(1-\nu))} - U^*_{\tau+1}\right)}$$

The next step is to substitute this expression back into the market value constraint (22). This will lead to an expression of equilibrium wages as a function of workers' market values, as in equation (24):

$$\frac{w_\tau}{1-\beta(1-\nu)} = \alpha\left(\frac{y_\tau}{1-\beta(1-\nu)} - U^*_{\tau+1}\right) + U^*_{\tau+1} = \alpha\frac{y_\tau}{1-\beta(1-\nu)} + (1-\alpha)U^*_{\tau+1}$$

Notice that the beliefs do not appear in that equation. This is a result of the assumption made in Gonzalez-Shi hiring protocol that unsuitable workers are never hired, as well as of the assumption that workers redraw their types when enter unemployment. As a result, wages do not directly reflect the probability for a successful match.

Next, one can substitute the expression for wages in the Free Entry condition and express equilibrium queue lengths as a function of market values only, as in equation (26):

$$q_\tau = \kappa^{\frac{1}{\alpha}}(1-\alpha)^{-\frac{1}{\alpha}}\left[\frac{y_\tau}{1-\beta(1-\nu)} - U^*_{\tau+1}\right]^{-\frac{1}{\alpha}}$$

Hence, a guess of $\{U^*_\tau\}_{\tau\leq T}$ pins down the sequences $\{w_\tau\}_{\tau\leq T}$ and $\{q_\tau\}_{\tau\leq T}$, via equations (24) and (26). Notice that the optimal choice of search effort is given by equation (27):

$$s^*_\tau = \left\{\beta(1-\nu)\phi^{-1}\mu_\tau\kappa^{\frac{\alpha-1}{\alpha}}(1-\alpha)^{\frac{1-\alpha}{\alpha}}\alpha\left[\frac{y_\tau}{1-\beta(1-\nu)} - U^*_{\tau+1}\right]^{\frac{1}{\alpha}}\right\}^{\frac{1}{\eta-1}}$$

Finally, given that μ_1 is pinned down by the unobserved heterogeneity parameters, one can use (27) to compute s_1. Then, μ_2 is a function of q_1, s_1 and μ_1. Using (27) again helps pin down s_2 and, by iteration, all subsequent $\{s_\tau\}_{\tau\leq T}$ and $\{\mu_\tau\}_{\tau\leq T}$. One then can use the definition of the value of unemployment to compute a new sequence $\{U'^*_\tau\}_{\tau\leq T}$, based on the values of $\{w_\tau\}_{\tau\leq T}$, $\{q_\tau\}_{\tau\leq T}$, $\{s_\tau\}_{\tau\leq T}$ and $\{\mu_\tau\}_{\tau\leq T}$. If $\{U'^*_\tau\}_{\tau\leq T}$ is close to the initial guess $\{U^*_\tau\}_{\tau\leq T}$, the fixed point is computed; if not, the algorithm should repeat the process.

In short, the algorithm works as follows:

1. Guess a sequence $\{U^*_\tau\}_{\tau\leq T}$.

2. Use the structure of the auxiliary problem to compute $\{w_\tau\}_{\tau\leq T}$ and $\{q_\tau\}_{\tau\leq T}$, via equations (24) and (26).

3. Use workers' FOCs for optimal search effort and Bayes rule to compute $\{s_\tau\}_{\tau\leq T}$ and $\{\mu_\tau\}_{\tau\leq T}$.

4. Use the definition of the value of unemployment to compute the updated $\{U'^*_\tau\}_{\tau\leq T}$.

5. If $\{U'^{*}_{\tau}\}_{\tau \leq T}$ is close to $\{U^{*}_{\tau}\}_{\tau \leq T}$, stop; otherwise, go to step 1 and repeat.

B.2 Average Contribution of Each Mechanism

Recall that the full model predicts the following equilibrium job-finding rate for each duration τ:

$$f_\tau = s_\tau(\mu_\tau, y_\tau) \times \mu_\tau(\xi, x_\tau) \times x_\tau(\mu_\tau, y_\tau)$$

Now, one can construct all the possible scenarios by shutting down one or two mechanisms in each version:[19]

$$f^1_\tau = s_\tau(\mu_\tau) \times \mu_\tau(\xi, x_\tau) \times x_\tau(\mu_\tau)$$

$$f^2_\tau = \mu_\tau(\xi, x_\tau) \times x_\tau(\mu_\tau)$$

$$f^3_\tau = \mu_\tau(\xi, x_\tau) \times x_\tau(\mu_\tau, y_\tau)$$

$$f^4_\tau = s_\tau(y_\tau) \times x_\tau(y_\tau)$$

$$f^5_\tau = x_\tau(y_\tau)$$

$$f^6_\tau = \mu_\tau(\xi, x_\tau)$$

The top panel of Figure 6 reports the results for the following ratios:

$$R^{UH}_\tau = \frac{0.5(1 - f^6_\tau) + 0.5(f^4_\tau - f_\tau)}{1 - f_\tau}$$
$$R^{WW}_\tau = \frac{0.5(1 - f^4_\tau) + 0.5(f^6_\tau - f_\tau)}{1 - f_\tau}$$

where R^{WW}_τ aggregates the effects of skill loss and search effort (within-worker duration dependence).

[19] The only uninteresting case is a model that contains only endogenous search effort. This simply is the standard directed search environment that does not predict negative duration dependence.

The bottom panel of Figure 6 reports the results for the following ratios:

$$R_\tau^{HC} = \frac{\frac{1}{3}(f_\tau^6 - f_\tau^3) + \frac{1}{3}(f_\tau^1 - f_\tau) + \frac{1}{3}(1 - f_\tau^5)}{1 - f_\tau}$$
$$R_\tau^{SE} = \frac{\frac{1}{3}(f_\tau^6 - f_\tau^1) + \frac{1}{3}(f_\tau^3 - f_\tau) + \frac{1}{3}(f_\tau^5 - f_\tau^6)}{1 - f_\tau}$$

In other words, each ratio is the average of the effect of each mechanism on job-finding rate over different model scenarios; in each scenario either the mechanism is shut down at various sequential orders or is the only force creating negative duration dependence (this only applies to unobserved heterogeneity and skill loss; see footnote 19).