

# Advertising Arbitrage

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Comments are welcome!

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# Motivation

Textbook: arbitrageur takes a position in a mispriced asset and waits until the price converges to the fundamental.

Real world: arbitrageurs often disclose and actively advertise their positions.

Why? This paper:

- ▶ Arbitrageur has private information about some assets.
- ▶ He has to liquidate positions before the information is reflected in prices.
- ▶ He may advertise his positions and **reveal private information**, accelerating the convergence of prices to fundamentals.

## Examples

### 1. **David Einhorn (Greenlight Capital):**

- ▶ 2008, Lehman Brothers (CAP \$60 billion): bankruptcy
- ▶ 2012, Martin Marietta Materials (CAP \$4.6 billion): 14% ↓

### 2. **Carson Block (Muddy Waters Research):**

- ▶ 2011, false accounting in Sino-Forest (CAP \$5billion): 82% ↓

### 3. **Glaucus Research Group:**

- ▶ 2013, China Minzhong Food Corp (CAP \$500 million): 50% ↓

### 4. **Carl Icahn:**

- ▶ 2013, tweeted about Apple (CAP \$ 450 billion): 5% ↑

### 5. Ljungqvist and Qian (2014) study 17 advertising arbitrageurs between 2006 and 2011 (113 cases), average price drop:

- ▶ 7.4% on the report release date,
- ▶ 26.4% in the three subsequent months.

# Questions

Questions we want to answer:

- ▶ How do arbitrageurs allocate their advertising effort?
- ▶ How does advertising affect portfolio choices of arbitrageurs?
- ▶ How does reputation building affect advertising?
- ▶ Does advertising relax limits to arbitrage? Is it efficient?

## Main results:

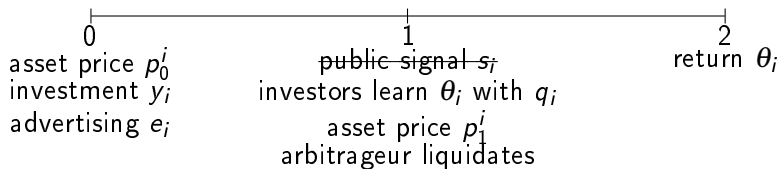
1. **Concentrated advertising:** even if arbitrageur identifies several mispriced assets, he advertises a single one.
2. **Portfolio under-diversification:** high exposure to advertised asset.
3. **Advertising intensity** is high if
  - ▶ asset is severely mispriced,
  - ▶ public information about asset is noisy,
  - ▶ asset is “advertisable” (familiar, simple asset is advertised more than new, complex one).
4. **Reputation is important:**
  - ▶ if information is soft, price reaction to advertising is proportional to arbitrageur’s reputation,
  - ▶ high-skill arbitrageur has higher incentive to build reputation than low-skill one.
5. **Potentially inefficient coordination among arbitrageurs:**
  - ▶ they advertise the same asset.

# Environment

- ▶  $t=0,1,2$ .
- ▶ many risk neutral investors
- ▶ continuum of assets  $i \in N$ , return  $\theta_i \in \{0,1\}$  at  $t = 2$
- ▶ investors' prior belief:  $\Pr\{\theta_i = 1\} = \pi_i \in [\underline{\pi}, \bar{\pi}]$ .

# Information and Advertising

- ▶ at  $t = 0$  arbitrageur learns  $\theta_i$  for assets  $i \in M \subset N$ ,  $M$  - finite.
- ▶ at  $t = 0$  arbitrageur invests  $y_i$  and exerts advertising effort  $e_i$  for  $i \in M$ ,
- ▶ investors can learn  $\theta_i$  at  $t = 1$  with probability  $q_i(e_i)$



Simplified: **no independent public signal** at  $t = 1$ , (in the paper the signal of precision  $\gamma \in [0, 1)$ ).

## Arbitrageur's preferences

$V(c, e)$  is not convex in  $c$  and concave in  $e$ ,  $V_{ce} = 0$ .

- ▶  $e = \sum_{i=1}^M e_i$  - total advertising effort.
- ▶  $\tilde{r}_i = \frac{p_1^i}{p_0^i}$  - return on asset  $i \in N$  at  $t = 1$
- ▶  $c = \sum_{i=1}^N y_i \tilde{r}_i$  monetary payoff at  $t = 1$ .
- ▶ budget constraint  $\sum_{i=1}^N y_i \leq w$ ,



## Assumptions

**Assumption 1:** The probability that investors learn  $\theta_i$  at  $t = 1$  is proportional to advertising effort:

$$q_i = \min[a_i e_i, 1]. \quad (1)$$

here  $a_i \in (0, 1]$  is the "advertisability" of asset  $i$ .

**Assumption 2:** Limited resources:  $w < \infty$ .

**Assumption 3:** Arbitrageur's trades do not affect prices.

**Assumption 4:** All assets in  $M$  are undervalued:  $\theta_i = 1$ ,  $i \in M$ .

**Assumption 5:** Perfect advertising is prohibitively costly:

$$V\left(\frac{w}{\underline{\pi}}, 1\right) - V(0, 1) < |V_e\left(\frac{w}{\underline{\pi}}, 1\right)|. \quad (2)$$

## Arbitrageur's view on date $t = 1$ returns

$t = 0$ : prior  $\pi_i$  of risk neutral investors determines the price

$$p_0^i = E_0[\theta_i] = \pi_i$$

$t = 1$ : advertising either succeeds  $S$  or fails  $F$

- ▶ with probability  $q_i$  advertising succeeds, investors learn  $\theta_i = 1$

$$p_1^i(S) = E_1[\theta_i | \theta_i = 1] = 1$$

- ▶ with probability  $1 - q_i$  advertising fails, the price is

$$p_1^i(F) = E_0[\theta_i] = \pi_i$$

Arbitrageur knows  $\theta_i = 1$  at  $t = 0$ , from his viewpoint

$$\tilde{r}_i = \frac{p_1^i}{p_0^i} = \begin{cases} \frac{1}{\pi_i} & \text{with probability } q_i \\ 1 & \text{with probability } 1 - q_i \end{cases} \quad (3)$$

## Arbitrageur's problem ( $M$ contains two assets)

Denote  $r_i^H = \frac{1}{\pi_i}$ ,  $r_i^L = 1$ ,  $i = 1, 2$ .

Arbitrageur's expected utility at  $t = 0$  is:

$$\begin{aligned} E[V|\mathbf{y}, \mathbf{e}] = & q_1 q_2 V(y_1 r_1^H + y_2 r_2^H, \mathbf{e}) + \\ & q_1 (1 - q_2) V(y_1 r_1^H + y_2 r_2^L, \mathbf{e}) + \\ & (1 - q_1) q_2 V(y_1 r_1^L + y_2 r_2^H, \mathbf{e}) + \\ & (1 - q_1) (1 - q_2) V(y_1 r_1^L + y_2 r_2^L, \mathbf{e}). \end{aligned} \quad (4)$$

Investments  $\mathbf{y} = (y_1, y_2)$  and advertising  $\mathbf{e} = (e_1, e_2)$  solve:

$$\max_{\{\mathbf{y} \geq 0, \mathbf{e} \geq 0\}} E[V|\mathbf{y}, \mathbf{e}], \text{ s.t. } \sum_i y_i \leq w, q_i = \min[a_i e_i, 1], \forall i \in M. \quad (5)$$

# Advertising

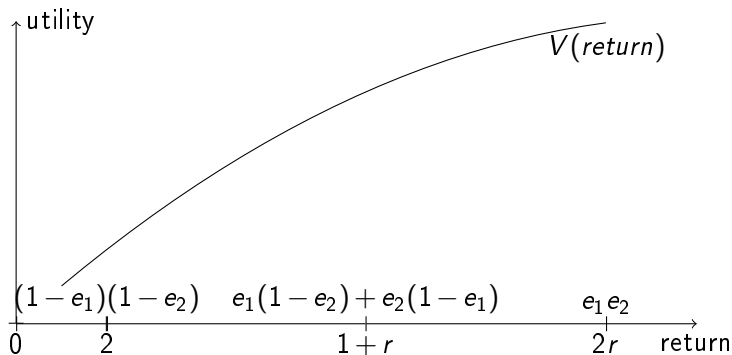
**Proposition 1:** *The arbitrageur advertises only one asset:  $e_i > 0$  for some  $i \in M$  and  $e_j = 0$  for  $j \neq i$ .*

Idea: Advertising determines the **probability distribution** of monetary payoffs at  $t = 1$ , but **not the monetary payoffs**.

Arbitrageur is risk averse: he does not like uncertainty  $\rightarrow$  he prefers lotteries with "concentrated" probability mass on one outcome  $\rightarrow$  he advertises only one asset.

Example: identical independent assets  $\tilde{r}_i \in \{1, r\}$ ,  $i = 1, 2$

$y_1 = y_2 = 1$ ,  $r > 1$ , and  $q_1 = e_1 \in \{0, \frac{1}{2}, 1\}$ ,  $q_2 = e_2 = 1 - e_1$ .



$$E[V|e_1 = \frac{1}{2}, e_2 = \frac{1}{2}] \sim \frac{1}{4} V(2) + \frac{1}{4} V(2r) + \frac{1}{2} V(1+r),$$

$$E[V|e_1 = 1, e_2 = 0] \sim V(1+r) \geq E[V|e_1 = \frac{1}{2}, e_2 = \frac{1}{2}].$$

$\frac{1}{2} V(1+r) \geq \frac{1}{4} V(2) + \frac{1}{4} V(2r)$  because  $V(\cdot)$  is not convex.

# Green Light Capital homepage (David Einhorn)



[Information on David's book "Fooling Some of the People All of The Time"](#)

[Sohn Investment Conference Presentation, May 5, 2014](#)

# Muddy Waters webpage

## Research

Report Title	Company	Date
<b>NEW</b> NQ Admits Widespread Data Tampering, but Claims it is not a Fraud	NQ Mobile Inc.	2014-06-04
NQ: You Can't Fool All of the People All of the Time	NQ Mobile Inc.	2014-04-13
MW Offer to NQ's Independent Committee	NQ Mobile Inc.	2013-12-19
NQ's US Veneer: Withholding Facts, Conned Men, and a Convicted Racketeer	NQ Mobile Inc.	2013-11-12
If You Believe in Yidatong, You'll Believe in Santa Claus	NQ Mobile Inc.	2013-11-06
Chinese Media Views on NQ	NQ Mobile Inc.	2013-11-01
NQ's Top Ten Lies Since Friday	NQ Mobile Inc.	2013-10-29
Initiating Coverage on NQ Mobile Inc. (NYSE: NQ) – Strong Sell	NQ Mobile Inc.	2013-10-24



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### Recent Reports

**NQ Admits Widespread Data Tampering, but Claims it is not a Fraud**

NQ admitted widespread tampering with data reviewed during the investigation.

# Glaucus Research webpage

## **MOST RECENT PUBLICATIONS:**

**Asia Plastic Recycling Holding Limited (TWSE: 1337)**

*May 5, 2014*

**Glaucus Research issues an Open Letter to the Market on Asia Plastic Recycling (TWSE: 1337) and maintains Strong Sell Rating**

*May 1, 2014*

**Glaucus Research Issues a Supplemental Report on Asia Plastic Recycling (TWSE: 1337) and maintains Strong Sell Rating**

*April 28, 2014*

**Glaucus Research issues a rebuttal to Asia Plastic Recycling (TWSE: 1337) and maintains Strong Sell Rating**

*April 24, 2014*

**Glaucus Research initiates coverage on Asia Plastic Recycling (TWSE: 1337) with a Strong Sell Rating**

翻譯成中文之執行摘要副本可在此處找到



## Overweighting of the advertised asset.

**Assumption 6:** Assets in  $M$  differ only in terms of advertisability.

**Lemma 2:** *When advertising is not possible, the arbitrageur is risk-averse, and Assumption 6 holds, the arbitrageur takes equal positions in all assets in  $M$ .*

**Proposition 2:** *When advertising is possible, the arbitrageur is risk-averse, and Assumption 6 holds, the arbitrageur advertises the most advertisable asset and invests more in it than in any other asset. Investments in other assets are the same.*

## Advertising by risk-neutral arbitrageur

$$V(c, e) = c - e^2/2$$

Note: risk-neutral arbitrageur invests in one asset.

If he invests  $w$  in asset  $k \in M$ , then his optimal advertising effort

$$e_k^* = a_k(1 - \gamma_k^2)\left(\frac{1}{\pi_k} - 1\right)w. \quad (6)$$

**Remark 1:** *The optimal advertising effort increases with the asset's "advertisability" ( $a_k$ ) and mispricing ( $1/\pi_k$ ), and decreases with the precision of the public signal ( $\gamma_k$ ).*

## Investment by risk-neutral arbitrageur

Risk-neutral arbitrageur chooses asset  $k \in M$  that maximizes

$$E[V|\pi_k, \gamma_k, a_k] = w \left[ 1 + \gamma_k^2 \left( \frac{1}{\pi_k} - 1 \right) \right] + \frac{w^2 a_k^2}{2} (1 - \gamma_k^2)^2 \left( \frac{1}{\pi_k} - 1 \right)^2 .$$

**Remark 2:** *Arbitrageur prefers an asset that is more advertisable (high  $a_k$ ), more significantly mispriced (high  $1/\pi_k$ ), and with more precise public information (high  $\gamma_k$ ).*

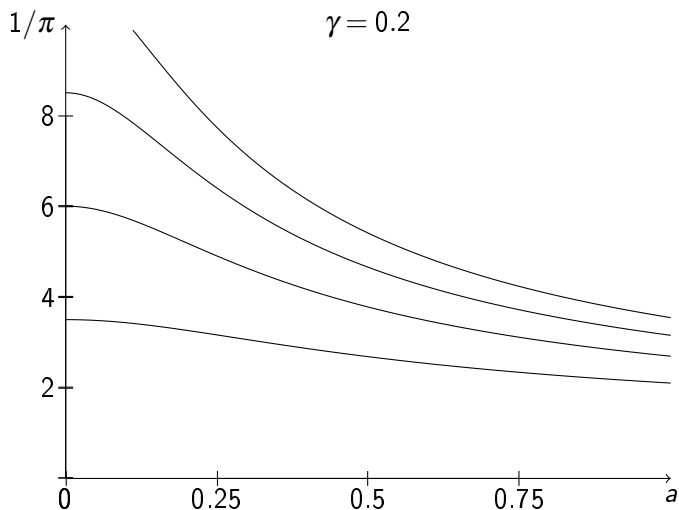
## Preferences over assets

**Proposition 3:** 1) *The greater an asset's advertisability, the more the arbitrageur values advertisability relative to informativeness of the public signal; the greater the informativeness of the public signal, the more he values its informativeness relative to advertisability.*

2) *The larger the arbitrageur's initial wealth ( $w$ ), the more he values advertisability relative to the asset's potential return and to the informativeness of the public signal, and the less he values the informativeness of the public signal relative to the asset's potential return.*

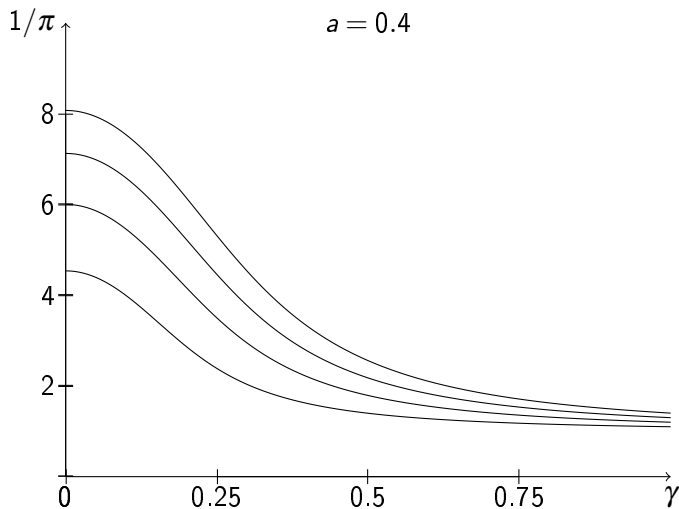
# Illustration 1

Figure: Indifference curves between potential return and advertisability.



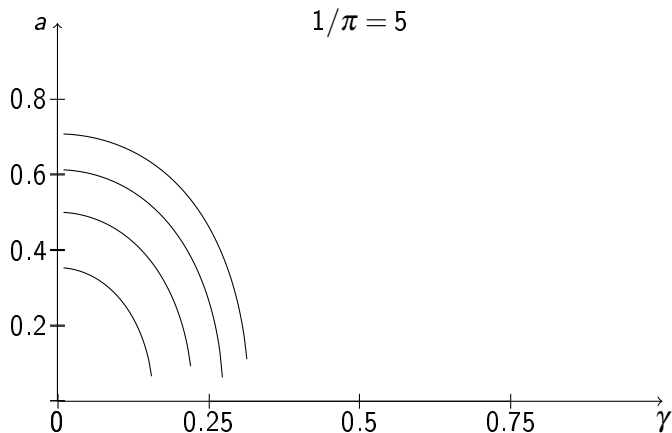
## Illustration 2

Figure: Indifference curves between potential return and quality of public information.



## Illustration 3

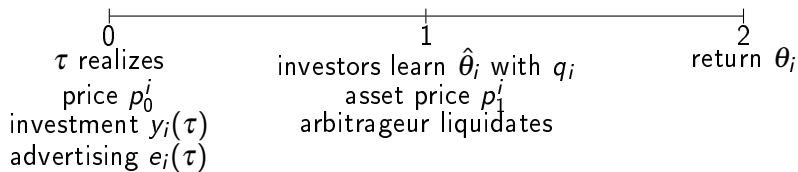
**Figure:** Indifference curves between advertisability and quality of public information.



# Static model with reputation

Modified environment:

- ▶ arbitrageur has private information about only one asset  $i \in N$
- ▶ arbitrageur's information  $\hat{\theta}_i$  is soft (little hard data)
- ▶ arbitrageur's type  $\tau \in \{L, H\}$  is private information
  - with  $\mu$  he is high-skill ( $\tau = H$ ) and info is perfect  $\hat{\theta}_i = \theta_i$ ,
  - with  $1 - \mu$  he is low-skill ( $\tau = L$ ) and info is pure noise.





## Solution to the static model

if advertising fails at  $t = 1$ , the price is  $p_1^i(F) = E[\theta_i] = \pi_i$ .

if it succeeds, investors posterior is  $\mu_1(S)$  and the price is

$$p_1^i(S) = E[\theta_i | \hat{\theta}_i = 1] = \mu_1(S) + (1 - \mu_1(S))\pi_i \quad (7)$$

Price reaction:

$$\Delta \equiv p_1^i(S) - p_1^i(F) = \mu_1(S)(1 - \pi_i). \quad (8)$$

Bayesian updating:

$$\mu_1(S) = \frac{\mu a_i e_H^*}{\mu a_i e_H^* + (1 - \mu) a_i e_L^*}. \quad (9)$$

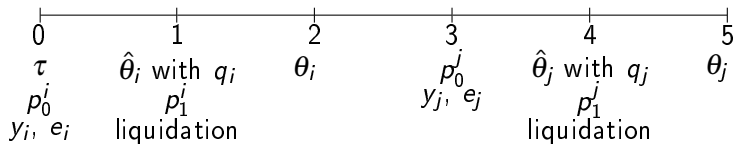
## Price reaction to advertising

Arbitrageur's problem:

$$\max_{e_\tau \in [0, 1/a_i]} a_i e_\tau \frac{\Delta}{\pi_i} w + w - e_\tau^2/2, \tau \in \{L, H\}. \quad (10)$$

**Proposition 4:** *The two types of arbitrageur exert the same level of effort, therefore  $\mu_1(S) = \mu$  and  $\Delta = \mu(1 - \pi_i)$ .*

## Reputation building (repeated interaction)



Key difference: at  $t = 2$  investors can compare  $\hat{\theta}_i$  and  $\theta_j$

**Proposition 5:** *In equilibrium with advertising the high-skill arbitrageur advertises more at  $t = 0$  than the low-skill one:  $e_H^* > e_L^*$ . Success in advertising signals the arbitrageur's skill:  $\mu_1(S) > \mu$ .*

**Proposition 6:** *With reputation-building, the equilibrium price reaction at  $t = 1$  is greater than in the static model:  $\Delta_1^* > \Delta^*$ .*

## Multiple arbitrageurs

- ▶  $L \geq 2$  identical arbitrageurs
- ▶ at  $t = 0$  they share **perfect** information about assets in  $M$
- ▶ each arbitrageur takes behavior of others as given,
- ▶ each arbitrageur chooses  $\mathbf{y}_l$  and  $\mathbf{e}_l$ ,  $l = 1, \dots, L$ ,
- ▶ investors learn return  $\theta_i$  of asset  $i \in M$  at  $t = 1$  with probability  $q_i = a_i \sum_l e_i^l$ .

Nash equilibrium in pure strategies

## Advertising by multiple arbitrageurs

**Lemma 4** All  $L$  arbitrageurs invest in the same asset.

Idea: if others advertise  $j$  it becomes attractive for an arbitrageur.

Multiple equilibria are possible, some are inefficient.

**Corollary 1:** Any asset  $j \in M$  for which arbitrageurs have information can be advertised in equilibrium by all of them, if the number of arbitrageurs  $L$  exceeds a critical threshold  $\underline{L}(j) < \infty$  and their individual resources are limited  $w < \underline{\pi}/L$ .

It can happen that in equilibrium arbitrageurs invest in  $i$  while they are jointly better off from investing in and advertising  $j$ .

## Predictions that have already been tested

*(i) Advertising accelerates price discovery, and on average it increases arbitrageurs' profits.*

Ljungqvist and Qian (2014): stocks drop by 7.4% on the report release date, and by 26.4% in the three subsequent months.

*(ii) Advertising of hard information and advertising by reputable arbitrageurs has greater price impact.*

Ljungqvist and Qian (2014): reports based on actual data have a strong price impact, those that contain only opinions have no significant effect; prices react more strongly to reports by arbitrageurs with good track record.

*(iii) Different arbitrageurs will tend to advertise the same opportunities and to exploit them simultaneously.*

Zuckerman (2012): upon being publicly identified as overvalued by managers of large US equity hedge funds, stocks were shorted by several funds at once.

## Predictions that are not yet tested

*(i) Arbitrageurs concentrate advertising on one asset at a time.*

*(ii) Arbitrageurs should overweight advertised assets in their portfolios.*

*(iii) Arbitrageurs are more likely to advertise an asset – and to do so more intensively – if it is more severely mispriced and/or more advertisable than others. They will also advertise an asset more heavily when public information on it is less accurate (for instance, stocks that are not covered by analysts).*