Wages and Informality in Developing Countries*

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Abstract

We develop an equilibrium wage-posting model with heterogeneous firms that decide to locate in the formal or the informal sector and workers who search randomly on and off the job. We estimate the model on Brazilian labor force survey data. In equilibrium firms of equal productivity locate in different sectors, a fact observed in the data. Wages are characterized by compensating differentials. We show that tightening enforcement does not increase unemployment and increases wages, total output and welfare by enabling better allocation of workers to higher productivity jobs and improving competition in the formal labor market.

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1 Introduction

The informal economy is a large component of many developing and even some OECD countries. In Brazil, where we draw our data from, over 40% of the entire workforce is employed in the informal sector. Firms operating in the informal sector do not comply with labor market statutes, including minimum wage laws and firing regulations and do not pay social security contributions of any sort. It is often argued that as a result they are the engine of growth because they lead to effective labor market deregulation improving flexibility and reducing labor costs. On the other hand, informality implies that workers are excluded from a number of benefits, including health coverage and unemployment compensation and the government does not collect income and corporate taxes, reducing the size of the tax base. Moreover, since smaller firms can evade regulation more easily informality can be viewed as a subsidy for smaller and often less productive firms. Understanding how these tradeoffs balance out and what they imply about the effects of informality is a key policy question for developing countries.

Magnac (1991) presents two competing views of informality. The most traditional view associates informality with a subsistence sector in a segmented labor market, with access to the formal sector restricted by minimum wage, tax laws and other labor regulations. In this view, segmentation between the sectors is maintained by restricting access of formal jobs for lower productivity individuals through legal barriers including minimum wages and other labor market regulations. A similar, but slightly more nuanced view sees the allocation of workers to the sectors by comparative advantage as in a Roy (1951) economy. In equilibrium wages will reflect amenity differences and given their skills and comparative advantage individuals will be indifferent between the two sectors. In both cases the informal sector expands the range of jobs available and leads to increases in employment and welfare. Magnac (1991) concludes that this latter competitive view is a better description of reality than the former dualist view. However, none of these two explanations are completely satisfactory when we consider data on the dynamics of informality. In particular we observe low skill workers in both sectors and more importantly we observe transitions between the two either directly or via unemployment, within relatively short periods of time. This is not easy to reconcile either with the rationing view of formal jobs or with comparative advantage, at least if we think of the latter as being a more or less permanent characteristic.[1]

[1] For example, Maloney (1999) shows no evidence of segmented markets for Mexico, where transitions between formal and informal sector seem to be equally probable in both directions. Barros, Sedlacek and Varandas (1990), Neri (2002) and Curi and Menezes-Filho (2006) analyze Brazil and also point to the significant mobility between sectors. Furthermore, Maloney et al (2007) shows for the Dominican Republic that informal workers are more satisfied than formal workers in terms of job flexibility. For Argentina, Pratap and Quintin's (2006) findings suggest that informal workers can be as well off as similar

In this paper we offer a new perspective on informality that can explain key facts, namely that low skill workers are to be found in both sectors and indeed transit between the two, while at the same time the informal sector pays substantially less than the formal one. The key element of our model are search frictions that generate profit opportunities for firms to post jobs in both sectors. Specifically, identical low skill workers search randomly (on and off the job) and they may receive offers either from formal or informal firms, which are heterogeneous in their productivity. Search frictions imply that firms at a given productivity level may be able to make positive profits in both sectors, accounting for compliance costs in terms of fines for those caught. This is because in a world where all firms are formal, one could deviate and be informal avoiding all the costs of regulation without having to fully compensate the workers for the loss in amenities (pension contributions, severance, UI etc.) since search frictions prevent workers from immediately locating a better alternative. This can continue up until there are enough informal firms so that wages are competed up enough to equalize profits between the formal and the informal sector. The extent of frictions in the formal and the informal sector determine the extent to which wages in the two sectors can diverge and the prevalence of informality at each productivity level.

In equilibrium the lowest part of the productivity distribution is populated only by informal firms, since regulatory costs (such as minimum wages) are too high to make formal employment profitable. However over a large segment of the support of the productivity distribution formal and informal firms coexist and profits are equalized across sectors (given productivity). The increased probability of detection for larger informal firms together with the regulatory costs means that informal firms are much more prevalent in lower levels of productivity and are much smaller. Formal firms are more productive, pay better and become larger. The end result is that search frictions increases the prevalence of low productivity firms, which also reduces the probability of workers matching with higher productivity firms, thus reducing output. This is consistent with the picture of informality offered by La Porta and Shleifer (2008) across a number of developing countries. Moreover, we use their data to show that the strong overlap of the productivity distributions predicted by our model is indeed a feature of the data.

The presence of informality has implications for the wage distribution both within and between sectors. Importantly, equilibrium wages are on average higher in the formal sector, just as in the data, because, as mentioned above, formal firms are more likely to be found in the higher parts of the productivity and thus pay higher wages (similarly to Burdett and Mortensen, 1998). At the same time informal

formal workers.

firms mostly pay more than formal ones of *equal* productivity, which reflects compensating differentials. However, as we suggested above, the differential is lower than the loss of benefits from being informal.

The increased allocation of workers to low productivity jobs caused by informality reduces output and thus welfare, despite the fact it reduces the coverage of costly regulation. Our results show that reducing informality by increasing enforcement does not increase unemployment and increases welfare by enabling the reallocation of workers to higher productivity jobs. As a consequence overall wages go up and inequality is decreased. Of course these results keep labor market regulation fixed and it may still be the case that greater welfare gains may be obtainable from deregulation. However, it seems that an intermediate world where the informal sector is tolerated at the current levels of enforcement is not a welfare enhancing policy.

The model in this paper extends that of Burdett and Mortensen (1998) and more generally the literature on equilibrium search models with heterogeneous firms and on-the-job search[2] by allowing endogenous choice of sector by firms, with no ex-ante entry restriction and estimating the model on microeconomic data.[3] Alternative approaches have taken a macroeconomic perspective and have been based on the Mortensen-Pissarides (1994) matching model, such as the paper of Albrecht, Navarro and Vroman (2009) who model the informal sector as unregulated self-employment with fixed productivity, while allowing for match heterogeneity subject to shocks in the formal sector. They calibrate their model to match key facts in some main Latin American economies and then they simulate the impact on informality of payroll taxes and severance pay. Bosch and Esteban-Pretel (2012) also specify a matching model with ex ante homogeneous workers and firms and match specific effects that are subject to shocks (which differ depending on whether the job is formal or not). Jobs that have a low match specific effect become informal. In that paper the two markets are subject to the same frictions and direct job flows only take place from the informal to the formal sector although jobs may change status between formal and informal following a match specific shock. They calibrate the model to aggregate Brazilian data and use it to interpret the aggregate fluctuations and analyze the effects of labor market policy on job market flows. Prado (2011) uses a Ghironi and Melitz (2005) model with formal and informal sectors calibrated for 28 OECD countries and Brazil to estimate the welfare effects of informality and labor market regulation.

[2]See e.g. Burdett and Mortensen (1998), Van den Berg and Ridder (1998), Van den Berg (2003) and Bontemps, Robin and Van den Berg (2000).

[3]Bradley, Postel-Vinay and Turon (2013) estimate a multiple sector model with search frictions. There are a number of differences from our model. First firms cannot choose their sector. Second the wage distribution and employment rates in the public sector are exogenous policy parameters. Theirs is a model of how the private sector operates given the existence of a public sector.

Ulyssea (2013) takes a microeconomic perspective and uses firm level data from Brazil to model whether firms choose to register as formal or not and whether, while registered as formal they hire informal workers. He thus allows for both an intensive and an extensive margin for informality, which is an important phenomenon.[4] Our model, which is based on individual worker data, is consistent with such behavior (but does not explain it) because there is no distinction between one firm posting some vacancies in the formal sector and some in the informal one as opposed to two different firms with the same productivity posting vacancies in each of the two sectors respectively; this is because of constant returns to scale in production. Finally, Lopez-Garcia (2013) estimates a partial equilibrium model of career choice using Chilean microdata, where workers can choose to be in the formal or informal sector based on comparative advantage. The distinguishing feature of his model is that he endogenizes education choice.

Our model was conceived with the economies of Latin America in mind and more generally for economies where a substantial informal and formal sectors flourish side by side and with substantial mobility between them (see Maloney, 1999 for example). It allows us to discuss the relative merits of increasing enforcement in this context. We use data from Brazil where informality of labor is about 40 percent of the labor force.[5] Our main data source is the Brazilian labor Force Survey, *Pesquisa Mensal de Emprego*, which provides a rotating panel of individuals sampled from the six main metropolitan regions of Brazil.

In the next section, we present the data and some facts about informality in Brazil that help motivate the model. In Section 3, we describe the theoretical model followed by a description of our specification choices and the estimation method in Section 4. In Section 5, we present and comment on the main results. In Section 6, we examine the effects of changes in the levels of enforcement. Conclusions are in Section 7.

2 The Data and motivating facts

Our main source of data consists of a panel of individuals of working age, sampled by the labor force survey of Brazil, *Pesquisa Mensal de Emprego* (PME). PME was designed and conducted by the National

[4]Other related papers are for example Gabriel Ulyssea (2010), El-Badaoui, Strobl and Walsh (2010), Boeri and Garibaldi (2005), and Fugazza and Jacques (2003). They use a more simplified structure for dual economies than that of Albrecht et al. (2009) and Bosch and Esteban-Pretel(2012).

[5]We consider informal the unregistered employees and the self-employed. Narita (2012) treats the self-employed as a separate third sector and part of a life-cycle choice of individuals. However, her model does not model firm choice of sector.

Statistics Bureau to follow individuals of the six main metropolitan regions of Brazil. Each individual is interviewed during four consecutive months, then for another four consecutive months one year after their entry into the sample. The sample period starts in January 2002 and goes until December 2007. However, we opted to use only PME from year 2002. The first reason is that our model assumes steady-state, which is an assumption hard to defend over a long period of time. The second reason is that PME from year 2002 contains retrospective information about duration of the actual employment, which we need to identify job-to-job transitions.

For the purpose of this paper, we select workers from age 23, where the chance of returning to full time education is very low, up to age 65 who are found to be either unemployed,[6] working as an employee (registered or unregistered) or self employed. We define a worker as formal if she/he is a registered employee.[7] The remaining workers – unregistered employees and the self employed – are thus considered informal in this paper. Over the period we consider about 40% of workers are informal, 50% are formal and the rest unemployed. Our study focuses on the low skill labor market and we select workers with 8 or less years of education.

In Brazil, there is a federal minimum wage, which should be the minimum paid to all formal employees. The average legal minimum wage over the sample period is of 300 reais per month.[8] Workers under a formal contract found to earn less than the minimum wage were removed from the sample (8% of formal workers). We believe this is due to reporting error and we similarly discard the 5% lowest wages out of the informal workers sample, thus excluding mostly the zero-wage earners and some part-time jobs. We also trim the very top wages (0.01% highest of the sample).

We follow individuals for up to four months or until their first move (if that is sooner). This can be job-to-job, unemployment-to-job or job-to-unemployment, where the job can be in the formal or in the informal sector. We use the first move to compute transitions to minimize the effects of attrition (see also Jolivet et al., 2006). The estimation method is adapted to account for this by generating a simulated moment equivalent to the one we construct in the data.[9] At the date of the first interview, we observe

[6] We take out unemployed whose last job was not as an employee or self employed. By doing so, we exclude mostly unemployed who once was inactive, e.g. individuals whose behavior deviate from the predictions of our model.

[7] The job is registered if the worker reports having a worker's card signed by the firm, which means that the workers is protected by the Employment laws.

[8] All wages are in reais of June of 2008.

[9] We do not use the entire sixteen-months window of PME due to attrition problems. In the four month window attrition ranges from 20%-26% depending on region and gender. Ribas and Soares (2008) state that there are three main reasons for attrition that are equally important: (i) people leave the house and unlike other similar surveys such as the PSID or BHPS there is no effort to recontact this person (ii) vacation months - seasonality (iii) the nature of the survey - monthly, large

TABLE 1
Description of Data, by region and sex

	Sao Paulo		Salvador	
	Males	Females	Males	Females
Number of Individuals	42,321	19,261	21,342	9,638
Unemployed	3,472	3,127	2,265	2,070
Formal	19,369	7,324	8,033	2,366
Informal	19,480	8,810	11,044	5,202
Informality Rate	0.46	0.46	0.52	0.54
Transitions (% of workers by initial status)				
Unemployed-Formal	7.82	4.14	4.46	1.63
Unemployed-Informal	38.00	16.62	20.45	7.14
Formal-Formal	1.58	1.23	2.49	2.02
Formal-Unemployed	1.99	2.02	1.93	1.27
Formal-Informal	0.45	0.39	0.41	0.11
Informal-Informal	6.01	4.78	5.10	3.10
Informal-Unemployed	5.49	4.00	3.09	2.04
Informal-Formal	0.47	0.23	0.29	0.12
Log Wages				
Formal Sector				
Mean	6.74	6.44	6.31	6.08
Std. Dev.	0.45	0.38	0.40	0.32
Obs.	18,514	6,654	5,937	1,240
Informal Sector				
Mean	6.43	6.07	5.88	5.61
Std. Dev.	0.60	0.60	0.57	0.56
Obs.	17,817	7,562	9,233	3,600

Note: Brazilian Labor Force Survey 2002-2007, low education individuals aged 23-65. Transitions are the first move from the initial state observed at the first interview within the first four months of the observation window. The complement of each set of transitions (say from formal) are those who remained in the original state, which for workers is the original firm. So, for example, formal-formal means moving from one formal firm to another formal firm.

the worker's employment status and the wage earned. From the subsequent three months, we observe the job status, the time in the current job, and construct the transition indicators. We identify job-to-job transitions using the survey question on job duration.[10] For example, we classify a worker as a non-mover in the third month of the interview if she/he does not change status (e.g. remains formal) and declares that the current spell has lasted more than three months, i.e. more than the period that passed since the last interview.

In estimation we use workers with less than 9 years of education, who we assume are homogeneous - an assumption we discuss when introducing the model. Firms on the other hand are taken to be heterogeneous. Thus, Table 1 displays the composition of workers, the informality rate and turnover information for our low education sample at the date of the first interview by region and sex, pooled over the years we use. Both the formal and the informal sector are substantial and workers are split more or less half and half in Sao Paulo. In Salvador the informal sector is larger than the formal one. But in both cases these very low skilled workers are found in substantial numbers in both sectors. Informality is higher among females, regardless of the region.

Over the observation period large proportions of the unemployed take up informal jobs and a smaller fraction obtain formal jobs. Exits from unemployment are faster in Sao Paulo than Salvador and men find jobs quicker. However the key point here is that the unskilled unemployed obtain jobs in both sectors in substantial numbers. There is substantial mobility between informal jobs, and much less among formal ones. Finally, some workers move across sectors without an intervening unemployment spell. These facts led us to a model that allows transitions in all directions and where the same worker may be found in either formal or informal employment.

In Table 1 we also show summary statistics of wages by region and sex and formal versus informal sector. Formal workers are paid on average more than informal ones, but wages in the informal sector are more dispersed, features that our model was designed to be able to replicate. In both markets men are paid more than women, while workers in Sao Paulo are paid more than those in Salvador. Finally, wage inequality, measured by the standard deviation, is higher in the informal sector across all groups.

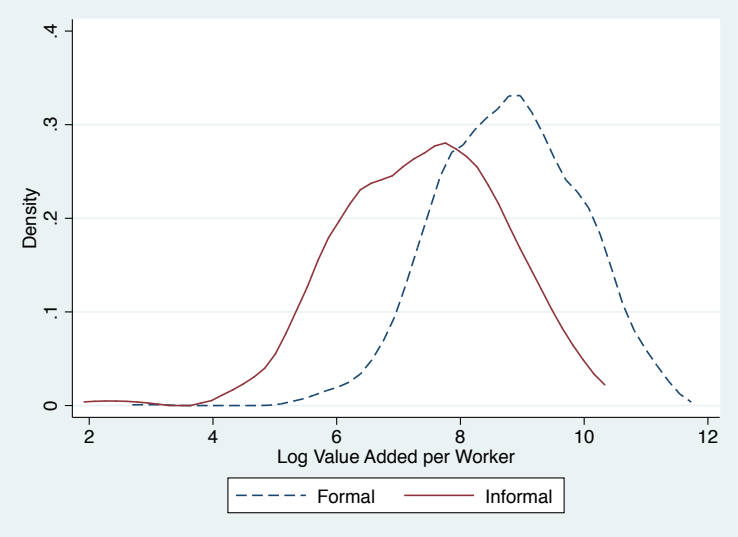
La Porta and Shleifer (2008) report that informal firms are much less productive than formal ones. However, another feature that is revealed when comparing formal and informal firms is that the distributions of productivity overlap over a very large range of the support of productivities: in Figure 1 we have

questionnaire - people refuse to be interviewed.

[10]This question is only available in PME after year 2002.

FIGURE 1

The Distribution of Log Productivity for Formal and Informal Firms in Brazil



used data provided by La Porta and Shleifer (2008) and drawn from several World Bank Surveys[11] to look at the distribution productivity of formal and informal firms.[12] Two important features are revealed by this comparison: first, for a good range of low productivities only informal firms operate; this is because regulatory costs such as minimum wages, taxes etc. make it unprofitable to be formal. The implication from this is that informal firms create employment opportunities at the low end, where current regulations make it prohibitive for formal firms to operate. Second, for a large part of the support of productivity formal and informal firms coexist. In other words there is no clear segmentation - even approximately so. This substantial overlap is all the more interesting given that the sample includes firms of all sizes and does not condition on small (formal and informal) firms. The implication is that in equilibrium, and given current regulatory costs a proportion of informal forms could become formal and still be profitable; on the other hand the existence of informal firms probably increases the density of firms with low productivity. These key characteristics of the data will be replicated and explained by our model and will play a central role in understanding the implications of the counterfactual policy simulations.

As documented by Ulyssea (2010) informality in Brazil increased steadily from 1989 to 2004. The tail end of this trend can be seen in Table 2, where overall informality rates are reported over all education groups and both genders. The 1988 constitutional change raising the costs of formal employment (mainly through increases in severance payments),[13] trade liberalization and various macroeconomic factors have

[11] The Enterprise Survey, the Micro Survey and the Informal Survey

[12] We refer the reader to La Porta and Shleifer (2008) for details on the measurement of productivity.

[13] See Gonzaga (2003) and Paes de Barros and Corseuil (2004).

TABLE 2
Working Status, by year

	2002	2003	2004	2005	2006	2007	Total
Unemployed	10.4	11.1	10.5	9.4	9.6	8.7	10.0
Formal	49.4	48.1	48.5	50.2	50.8	52.2	49.8
Informal	40.3	40.9	41.1	40.4	39.6	39.1	40.2

Note: Brazilian Labor Force Survey 2002-2007, individuals aged 23-65. The values are percentages of individuals according to their working status at the first interview.

all been associated with this increase in informality. Bosch, Goni-Pacchioni and Maloney (2012) find that the increase in severance pay explains a 3-4 percentage point increase in informality in the period 1988-2002. While Goldberg and Pavnick (2003) found no relationship between the share of informality and the reduction in trade protection in Brazil, Menezes-Filho and Muendler (2011) conclude that trade liberalization increased the proportion of informal workers (including the self-employed). Bosch, Goni-Pacchioni and Maloney (2012) study the period 1983-2002, covering major shocks to the Brazilian economy caused by the Asian and Russian crises and leading to stabilization policies to control hyperinflation. They find that the informal sector tends to absorb more labor during downturns because the formal salaried sector stops creating new jobs and the overall informal sector does not.

More recently, informality has decreased (Ulyssea, 2010) and a possible determinant of this reverse trend is the increasing cost of informality due to stricter enforcement after the mid-90s. Almeida and Carneiro (2012) find that labor inspections raise unemployment but reduce informality, linked to a reduction in self employment and an increase in formal employment.

Our model is that of a stationary steady state and is not designed to explain trends or macroeconomic fluctuations. However we can quantify the impact of policies such as increased enforcement on informality, which can inform the interpretation of the trends, although this is complicated by the numerous changes taking place at the same time.

3 The Equilibrium Search Model

We now develop a stationary equilibrium model that is capable of replicating the key features of the data we described and provides a way of carrying out counterfactual analysis. In our model, we make the simplifying assumption that workers are homogeneous within a submarket but firms heterogeneous.

Thus we consider a pool of low skilled workers assumed homogeneous (conditional on gender and state of residence), who will typically engage in jobs requiring low training input. The model explains the cross sectional variance of wages by the dispersion of productivity among firms. In general it is not possible to separately identify the contribution of worker and firm unobserved heterogeneity without matched employer-employee data. This is not available for informal firms, as far as we know. Nevertheless, there is suggestive evidence that skill heterogeneity is much less important for low educated workers. One potential source of heterogeneity is skill accumulation over age. However, wage growth for this low education group is no more than 1.2% a year, which is consistent with low training content of the jobs they are engaged in and some improvement of wages due to job search.[14] Another possible source of heterogeneity is ability: in the US, where we have data, the gradient of wages with respect to ability is much lower for the statutory schooling group than it is for those with higher levels of education (Abbott et al., 2013). Thus, while allowing for heterogeneity in skills may be interesting, it is both complex and probably not of first order importance for this low skill labor market.[15]

The model will be estimated separately for males and females and by region. Separate estimation by sex implicitly assumes that the labor markets are segmented for these groups and they do not compete directly; this assumption is clearly a limitation and we employ it for reasons of tractability - it is a way of controlling for these observable differences and better in our view than mixing the groups and ignoring differences among them. We also estimate the model separately for two regions with clearly distinct labor markets, namely Sao Paulo and Salvador. The former is a well developed low unemployment economy, while the latter is characterized by very high levels of unemployment.[16] Separating these regions is important, because both the job destruction rates and the arrival rates are likely to be very different. Our empirical work treats these as independent local labor markets.

There are two sectors in our economy, the formal and the informal one. Firms in the formal sector are subject to corporation tax on profits, have to pay social security contributions for their workers and severance pay upon laying them off and have to pay workers a minimum wage. Workers in formal firms

[14] In our sample average annual wage growth is as follows. Formal sector Sao Paulo: Males 1.20%, Females 0.42%. Formal sector Salvador: Males 1.00%, Females 0.44%. Informal sector Sao Paulo: Males 0.35%, Females -0.18%. Informal sector Salvador: Males 0.87%, Females 0.40%.

[15] Similar to our approach, Ridder and Van den Berg (2003) also assume homogeneous workers within each of their segmented markets. Bontemps, Robin and Van den Berg (1999) do include worker heterogeneity within market but only through differences in the value of leisure - not skill. Similarly, Shephard (2009) only allows for differences in the value of leisure when he studies the incidence of tax credits in a wage posting model.

[16] Over the period of analysis (2002-2007), the GDP per capita in 2008 prices for Salvador was 3.9 thousand dollars, whereas for Sao Paulo this figure was almost three times as much, 11.2 thousand dollars.

are subject to income tax and are eligible for unemployment insurance, which is funded by taxes. All these costs and associated benefits do not affect informal firms and their workers; however, firms are monitored and if caught not complying they pay a fine.

In this environment firms, which differ in their productivity, maximize profits by choosing the wage rate they will offer workers and the sector in which they will post their vacancy in. Workers who meet with firms randomly, either accept or reject an offer, as in Burdett and Mortensen (1998). Given existing evidence the low skilled workers are unlikely to have much bargaining power and thus wage posting reflects well the reality in low wage labor markets.[17] As we show, imperfect monitoring of compliance with the legal framework creates profitable opportunities for firms (and particularly those with lower productivity) to ignore the regulations and operate in the informal sector.

Workers seek to maximize their expected lifetime income. The flow utility of workers depends linearly on the wage they receive plus the value of the social security contributions made by the firm on their behalf, which we include in the wage measure and are net of any taxes due. Workers also value severance pay and unemployment insurance as will be evident in the value function. The economy is subject to search frictions and workers search both when unemployed and when they are employed. They also receive competing offers from both sectors. Subscripts with value 0 denote the unemployed, with value 1 denote the formal sector and with value 2 the informal one.

3.1 Workers

Workers are homogeneous and maximize the expected lifetime income discounted at a rate of r. At any instant, unemployed workers receive an income stream b, taken to be constant across individuals, regardless of their history. Let $W_1(w)$ and $W_2(w)$ denote the values of a wage contract paying w in the formal (sector 1) and the informal sectors (sector 2). Their support is denoted as $[\underline{W_i}, \overline{W_i}]$ for i= 1 and 2. Finally, U denotes the value of unemployment.

Individuals receive job offers according to a Poisson process with arrival rate λ_{ij}, where $i = 0,1,2$ denotes the current state (unemployed, or working in the formal/informal sectors) and j denotes the source of the offer. An offer is an employment contract promising a *fixed* wage and, implicitly, specific outside options. In particular, a worker can receive offers from either sector – indeed we also allow formal workers to receive offers from the informal sector and some of these offers may be worth accepting –

[17] For example, Cahuc, Postel-Vinay and Robin (2006) show that the unskilled have zero bargaining power.

and can be laid off at sector-specific rates $\lambda_{i0}, i = 1,2$. Later we show how these arrival rates can be made endogenous. Finally, in our model there is only exogenous job destruction, which is not restrictive given we only have worker level data. Distinguishing between endogenous job destruction - due say to productivity shocks or price shocks, or alternatively exogenous separation would require the use of matched employer-employee data, including for informal firms. In any case, introducing shocks in a model with search frictions can be particularly complicated.

Lastly, $F_i,\ i = 1,2$, denote the (equilibrium) distribution of (present values of) contracts from which workers sample their offers. These distributions define the pay strategy of firms in each sector and are therefore endogenous; we will show in some detail how they are determined.

The definition of the wage in the formal sector represents the entire monetary compensation for the worker: thus it is after tax but *before* social security deductions, which are effectively part of their compensation as it entitles them to a pension and to health benefits. Pay also includes contributions to pensions made by the employer on behalf of the worker. In the informal sector no taxes or contributions are made so the wage is just the gross wage.

The value functions for each state, namely employed in the informal sector, employed in the formal sector and unemployed describe the optimal behavior of workers. As usual these values combine the immediate gains of being in the sector (i.e the wage, w) together with the resulting option values, such as the possibility of moving to better jobs within or between sector or the impact of exogenous shocks, such as the possibility of job destruction leading to unemployment. Thus the value of working in the informal sector, is

$$rW_2(w) = w + \lambda_{20}\left[U - W_2(w)\right] + \lambda_{21}\,\mathbb{E}_{F_1}\max\{W - W_2(w), 0\} + \lambda_{22}\,\mathbb{E}_{F_2}\max\{W - W_2(w), 0\},$$

where $\mathbb{E}_{F_j}, j = 1,2$, takes expectations over a generic contract value W distributed as F_j in sector j. Later in solving for equilibrium it is useful to rewrite this expression after integrating by parts,[18]

$$rW_2(w) = w + \lambda_{20}\left[U - W_2(w)\right] + \lambda_{21}\int_{W_2(w)}^{\overline{W}_1} \overline{F}_1(x)\,dx + \lambda_{22}\int_{W_2(w)}^{\overline{W}_2} \overline{F}_2(x)\,dx, \tag{1}$$

[18] We make use of the following property. For any random variable X with distribution (CDF) F on $[\underline{x},\overline{x}]$, and for all $u \in \mathbb{R}$,

$$\mathbb{E}_F \max\{X - u, 0\} = \int_{\underline{x}}^{\overline{x}} \max\{x - u, 0\}\,dF(x) = \int_u^{\overline{x}} \overline{F}(x)\,dx.$$

where overlines on distribution functions denote survival functions: $\overline{F} = 1 - F$ and where U is the value of being unemployed, defined below. Thus the flow utility in the informal sector is the wage rate (w) plus the loss that the individual may incur if laid off, which happens at rate λ_{20}, as well as the "capital gain" of obtaining a better offer either from the formal or the informal sector with rates λ_{21} and λ_{22} respectively.

A similar expression can be derived for the value of working in the formal sector. The key difference here will be in the definition of the wage, which we discussed before and in the expression for the loss incurred when moving to unemployment. We write the value of employment in the formal sector (using the second expression derived above) as

$$rW_1(w) = w + \lambda_{10}\left[U + UI + s \cdot w - W_1(w)\right] + \lambda_{11}\int_{W_1(w)}^{\overline{W}_1} \overline{F}_1(x)\,dx + \lambda_{12}\int_{W_1(w)}^{\overline{W}_2} \overline{F}_2(x)\,dx. \tag{2}$$

The cost of becoming unemployed is mitigated by two factors. The first is unemployment insurance (UI), which for simplicity we assume is paid upfront.[19] The second term is severance pay $s \cdot w$, s being the compensation rate in the case of termination of employment. Severance may matter in equilibrium if minimum wages bite because minimum wages prevent a complete adjustment of wages to counteract this expected payment (see Lazear, 1990). As we show below, we determine the level of UI endogenously based on the tax rate used to fund it and on the overall number of unemployed. For a given wage, both UI and severance pay increase the value of employment in the formal sector – and in the informal sector since a transition between the two is possible – and both affect the equilibrium distribution of wages. The only difference of UI from severance pay is that the firm directly pays the latter, whereas UI is funded by general taxation. This distinction will be of importance when we define the firm's problem.[20] Finally, since there are no shocks to productivity, jobs are only closed down because of exogenous job destruction, the rates of which may differ depending on the sector, (λ_{10} and λ_{20} respectively).

To write the value of unemployment note that in equilibrium, firms will only offer acceptable wages so that the value at the minimum offered wage, $\underline{W}_1$ and $\underline{W}_2$, are greater than U, otherwise no production would take place. So the equilibrium value of unemployment is such that

$$rU = b + \lambda_{01}(\mu_1 - U) + \lambda_{02}(\mu_2 - U), \tag{3}$$

[19] Specifically it avoids making the duration of unemployment a state variable if UI is time limited for example.

[20] In practice UI acts as public insurance, while severance payment is just deferred pay with no real effects unless the minimum wage bites. This distinction is lost in our model since workers are risk neutral.

where $\mu_1 = \int_{\underline{W}_1}^{\overline{W}_1} x dF_1(x), \mu_2 = \int_{\underline{W}_2}^{\overline{W}_2} x dF_2(x)$ denote the mean contract values offered in the formal and the informal sector respectively, and b is the flow-value of leisure.

Contract values reflect the benefits, opportunities and costs of working in each sector. They are increasing functions of wages, yet the wage rate alone is not sufficient for ranking jobs across sectors, because each sector comes with different future opportunities. It is thus possible that a move across sectors is accompanied by a pay-cut. However in this model mobility within the sector can only take place when accompanied by a wage rise, which has to imply a move to a higher productivity firm. This is because there is no other source of heterogeneity (such as productivity shocks) and because firms do not respond to outside offers. While it is possible to modify the model to achieve this, for example by allowing some layoffs to lead to immediate job transitions (say because of notice), or by allowing for productivity shocks, we do not believe this is a first order issue for the problem at hand and we opted for the relative simplicity of the current framework.

3.2 Steady-State Worker Flows

The value functions discussed above define the optimal choices of workers and are conditional on the wage offer distributions in the formal and informal sectors. The latter are equilibrium objects. To derive them we need to define the steady state flows of workers between the three states (unemployment, formal and informal employment) as well as the behavior of the firms. In steady state, the stocks of workers and firms in each sector and in each part of the contract value distribution remain stable. We now define these flows and use them to solve for the steady state stocks and for the relationship between the equilibrium contract offer distribution and accepted offers.

Define the fraction of the labor force in each sector to be $m_i,\ i = 1,2$, and the unemployment rate to be $u = 1 - m_1 - m_2$. Let $G_1(W)$ and $G_2(W)$ be the distributions of accepted contract values in the formal and informal sectors, respectively: they denote the proportion of the stock of individuals with a contract value lower than or equal to W, respectively. The following equation states that in equilibrium the flow of workers out of each part of the formal sector distribution has to equal the inflow. Thus, for

any $W \in [\underline{W}_1, \overline{W}_1]$,

$$\left[\lambda_{10} + \lambda_{11}\overline{F}_1(W)\right] m_1 G_1(W) + \lambda_{12} m_1 \int_{\underline{W}_1}^{W} \overline{F}_2(x)\, dG_1(x) = \lambda_{01} u F_1(W) + \lambda_{21} m_2 \int_{\underline{W}_2}^{W} [F_1(W) - F_1(x)]\, dG_2(x). \quad (4)$$

The mass of workers in the formal sector at or below contract value $G_1(W)$ is $m_1 G_1(W)$. Some of these are destroyed because of exogenous layoffs (λ_{10}), receipt of offers valued more than W from other formal firms, and receipt of acceptable offers from the informal sector.[21] On the right hand side is the balancing job creation. Jobs are created with contract values below W in the formal sector, when the unemployed accept offers less than W or workers in the informal sector receive and accept offers whose value is lower than W.[22]

Similarly we can also define the equilibrium flow equation for the informal sector. For $W \in [\underline{W}_2, \overline{W}_2]$,

$$\left[\lambda_{20} + \lambda_{22}\overline{F}_2(W)\right] m_2 G_2(W) + \lambda_{21} m_2 \int_{\underline{W}_2}^{W} \overline{F}_1(x)\, dG_2(x) = \lambda_{02} u F_2(W) + \lambda_{12} m_1 \int_{\underline{W}_1}^{W} [F_2(W) - F_2(x)]\, dG_1(x). \quad (5)$$

In Appendix A we show how to (uniquely) solve equations (4) and (5) for the distributions of accepted contracts G_1 and G_2 given the distribution of offered contracts F_1 and F_2. There exists an equilibrium relationship between the distribution of accepted (G) and offered (F) contract values:

$$m_i G_i(W) = \frac{\lambda_{0i} F_i(W) - \Phi(W)}{d_i(W)} u, \quad i = 1, 2, \quad (6)$$

where $\Phi(W) \equiv \Phi[F_1, F_2](W)$ is an operator on F_1 and F_2 that is derived in Appendix A, and that is nil for

[21] This is reflected in the integral since departures from all parts of $G_1(W)$ need to be recorded and not only the ones leading to higher contract values than W.

[22] The flow equation considers outflows from the distribution of values below W in the formal sector. For someone having a job with value less than W this can happen either if one receives an offer from within the formal sector from a job offering more than W an event that occurs with probability $\lambda_{11}\overline{F}_1(W)$ or by receiving any improved offer with value greater than W from the informal sector. This explains the integral on the left hand side of the equation and a similar symmetric argument explains the integral on the right hand side

all $W \leq \max\{\underline{W}_1, \underline{W}_2\}$, and where, in the denominator,

$$d_i(W) = \lambda_{i0} + \lambda_{i1}\overline{F}_1(W) + \lambda_{i2}\overline{F}_2(W), \quad i = 1,2, \tag{7}$$

are the total job destruction rates in sectors 1 and 2 at contract values below W. This expression is the key to estimating the contract offer distributions (F_1 and F_2) based on the accepted offers that we observe.

Straightforwardly, we can also derive expressions for the proportion of workers in each sector and in unemployment, by setting W in equation (6) equal to its largest value for the respective sector and making use of the fact that $m_1 + m_2 + u = 1$:

$$\frac{m_1}{u} = \frac{\lambda_{01} - \Phi(\overline{W}_1)}{\lambda_{10} + \lambda_{12}\overline{F}_2(\overline{W}_1)}, \quad \frac{m_2}{u} = \frac{\lambda_{02} + \Phi(\overline{W}_2)}{\lambda_{20} + \lambda_{21}\overline{F}_1(\overline{W}_2)}, \quad \frac{1}{u} = 1 + \frac{m_1}{u} + \frac{m_2}{u}. \tag{8}$$

Hence, knowledge of the distribution of wage offers by the formal sector, F_1, and the informal sector F_2, allows us to infer the steady state stocks of employment (m_1 and m_2) and unemployment (u) as well as the equilibrium distribution of accepted wages G_1 and G_2 that are observable. This is not a full characterization of equilibrium; we now need to show how the offer distributions F_1 and F_2 and the decision to post offers in one or the other sector are determined. This depends on firm behavior to which we now turn.

3.3 Firms

Firms maximize profits by choosing in which sector to operate[23] and the wage they will post, which determines the size of their labor force, given their specific productivity p. This productivity captures both technology differences across firms as well as price differences, both of which are assumed to be constant over time, a restrictive assumption made to keep the model tractable. Introducing productivity shocks via p would be interesting but identification would require one to observe matched employer employee data. Even if such data can be obtained for formal firms they are not available for informal

[23] We can either think of firms as posting vacancies in one specific sector only or posting some vacancies in the formal sector and others in the informal. Because of the absence of complementarities between workers and constant returns to scale there is no analytical difference. We can think for example as a firm that has both formal and informal workers as two firms, one in each sector.

ones (to our knowledge). Finally, we assume constant returns to scale.[24]

In the formal sector there are a number of costs associated with hiring a worker at a wage rate w. These include payroll taxes (τ), corporate taxes on profits (t) and severance payments ($s \cdot w$) to workers who are laid off. Finally, these firms may be subject to minimum wage laws $w_{\min}$, which imply that firms cannot necessarily adjust pay to offset the effects of severance pay (Lazear, 1990).

Informal labor markets are monitored randomly by the government authorities whose role is to enforce tax and labor laws. When caught evading regulations a firm has to pay a fine depending on its size. We denote as $C(\ell_2)$ the expected cost of informality, assumed convex in firm size ℓ_2. This function will have to be estimated from the data, based on firm behavior.

The strategy of the firm is to choose a contract value (or wage) to offer any worker it contacts. The strategy will determine the attractiveness and hence the size of the firm: in equilibrium there is a tradeoff between low labor costs and size because a worker a low paying firm is more likely to receive an improved contract and leave. There are no adjustment costs and, conditional on the wage they pay workers, no dynamics in the decision problem of the firms. They just choose a wage and thus implicitly a contract value W to maximize profit flows.

Specifically, formal and informal firms will offer optimal contracts $K_1(p)$ and $K_2(p)$ respectively that solve the following profit maximization problems given productivity p,

$$\pi_1(p) = \max_{W \geq \max\{U, W_1(w_{\min})\}} (1-t)\left[p - (1+\tau+\lambda_{10}s)w_1(W)\right]\ell_1(W), \tag{9}$$

$$\pi_2(p) = \max_{W \geq U} \left[p - w_2(W)\right]\ell_2(W) - C(\ell_2(W)), \tag{10}$$

where $\ell_1(W)$ and $\ell_2(W)$ are the size of informal and formal firms respectively, offering a wage contract worth W, and $w_j(W)$ denotes the wage to be paid to a worker in sector j corresponding to a contract value W.

The wage functions $w_1(W)$ and $w_2(W)$ that the firm has to take into account in this optimization problem are defined by the value functions of the workers $W_1(w) = W$ and $W_2(w) = W$, from equations

[24] Dealing with decreasing returns to scale in a model with random search is very complicated because it leads to interactions in the pay of workers. It is certainly an interesting area of research but beyond the scope of this paper.

(2) and (1) respectively. Inverting them for the wage we get

$$(1+\lambda_{10}s)w_1(W) = (r+\lambda_{10})W - \lambda_{10}(U+UI) - \lambda_{11}\int_W^{\overline{W}_1} \overline{F}_1(x)\,dx - \lambda_{12}\int_W^{\overline{W}_2} \overline{F}_2(x)\,dx, \tag{11}$$

and

$$w_2(W) = (r+\lambda_{20})W - \lambda_{20}U - \lambda_{21}\int_W^{\overline{W}_1} \overline{F}_1(x)\,dx - \lambda_{22}\int_W^{\overline{W}_2} \overline{F}_2(x)\,dx. \tag{12}$$

In steady-state, the flow of workers leaving the workforce of any firm ($d_1(W)\ell_1(W)$ and $d_2(W)\ell_2(W)$ for the two sectors respectively) should be equal to the inflow of new hires. Hence,[25]

$$\ell_i(W) = \frac{M}{N_i}\frac{h_i(W)}{d_i(W)}, \quad i = 1,2, \tag{13}$$

where M is the total number of workers, N_1 and N_2 are the (endogenous) measures of firms in the formal and informal sector respectively, and $h_1(W)$ and $h_2(W)$ denote the share of contacts between firms and workers willing to accept a job paid W, i.e.

$$h_i(W) = \lambda_{0i}u + \lambda_{1i}m_1G_1(W) + \lambda_{2i}m_2G_2(W), \quad i = 1,2, \tag{14}$$

and $d_1(W)$ and $d_2(W)$ are the total job destruction rates (equation (7)).

3.4 Equilibrium Productivity Distributions

A key element of our model is that firms decide whether to post vacancies in the formal or the informal sector as well as what wage to post. In equilibrium, given productivity, all strategies will yield equal profits, a property we now use to determine how firms locate. Because of the various costs of employing workers in the formal sector, we can expect firms with lower productivity to locate in the informal sector, at least in the presence of minimum wages, if expected fines for informality are not too high. However, there may be a range of productivities over which, in equilibrium, firms are indifferent between the two sectors; indeed this turns out to be the case, which as we saw is also a feature of the data. This is a particularly important property of the model with key implications for our understanding of informality and its effects. Of course, the fact that firms of both types coexist over a productivity range does not mean

[25] These are expressions equivalent to those of Burdett and Mortensen (1998)

they will have the same size or pay the same wages; quite the contrary and we will discuss this later.

We assume that there exist a number of potential entrants, N, with a proper distribution of productivity $\Gamma(p)$ on $[\underline{p},\overline{p}]$ (i.e. $\Gamma(\underline{p})=0$ and $\Gamma(\overline{p})=1$). In equilibrium we will obtain a measure of productivities in each sector. We denote the equilibrium measure of productivity in each sector by $\Gamma_i(p)$ ($i=1,2$). At the equilibrium, each firm maximizes profit flows given the equilibrium contract distributions. Hence,

$$\Gamma_i(p) = n_i F_i\left[K_i(p)\right], \quad n_i = \frac{N_i}{N}, \quad i=1,2, \tag{15}$$

with $n_0 = 1-n_1-n_2$ denoting the fraction of inactive firms at the equilibrium.

Denote the support of the measure for informal firms to be $[\underline{p}_2,\overline{p}_2]$ and for formal firms $[\underline{p}_1,\overline{p}]$, where it is possible to have overlap in the supports, i.e. $\overline{p}_2 > \underline{p}_1$. As discussed above, we expect that the equilibrium is such that the initial interval of productivity will be occupied by informal firms only, i.e. $\underline{p}_2 < \underline{p}_1$ because labor market regulation may make it too costly for low productivity firms to post in the formal sector. Informal firms can bypass these regulations and post wages below the minimum. For $\underline{p}_1 \leq p \leq \overline{p}_2$, firms operate in both sectors. We also allow for the possibility that there is a range of productivities ($p > \overline{p}_2$) where firms operate only in the formal sector. Thus equilibrium will b characterized by the following regimes.

1. **Inactivity**: For $\underline{p} \leq p < \underline{p}_2$, $\pi_1(p) < 0$, $\pi_2(p) < 0$, and $\Gamma_1(p) = \Gamma_2(p) = 0$. This is important to consider when discussing counterfactual policy experiments which may lead to firm exit.

2. **Informal sector only**: For $\underline{p}_2 \leq p \leq \underline{p}_1$, $\pi_1(p) < \pi_2(p)$, $\Gamma_1(p) = 0$, and $\Gamma_2(p) = \Gamma(p) - \Gamma(\underline{p}_2)$. It is possible that this interval is just zero, meaning that the first relevant interval is the next one. The existence of this interval depends on the relative importance of formal labor market regulation and the costs of informality.

3. **Overlapping region**: In this region formal and informal firms of identical productivity coexist and make the same profits: For $\underline{p}_1 \leq p \leq \overline{p}_2$, $\pi_1(p) = \pi_2(p)$, and

$$\Gamma_1(p) + \Gamma_2(p) = \Gamma(p) - \Gamma(\underline{p}_2).$$

4. **Formal sector only**: For all $p \geq \overline{p}_2$, $\pi_1(p) > \pi_2(p)$, $\Gamma_2(p) = \Gamma_2(\overline{p}_2)$, and

$$\Gamma_1(p) = \Gamma(p) - \Gamma_2(\overline{p}_2) - \Gamma(\underline{p}_2).$$

If there is a range of productivities where only formal firms operate, this will be in the higher range. Implicit in this assertion is that the profits of informal firms are increasing slower with respect to productivity than those of formal ones, possibly because of rapidly increasing enforcement costs. For example, if the probability of detection as well as the fines increase fast enough with firm size, this will lead to convex costs of informality, making participation in that sector unprofitable as firm size increases. However, we cannot exclude the possibility that informal firms operate at all levels of productivity.

To see the intuition of why in equilibrium firms of equal productivity may differ as to whether they are formal or informal consider a world where all firms are formal. Then one firm can deviate by not complying with the regulations. Assuming enforcement costs are not too high (low probability of detection and/or low fines) it will be able to reduce its overall costs by avoiding taxes and other regulatory costs. Part of the savings will go to increased wages to compensate workers for loss of benefits such as UI and severance pay - a compensating wage differential.[26] However, the firm may not even need to compensate workers fully because search frictions prevent them from locating alternative employment immediately. The process can continue until profits are equalized across sectors. The new equilibrium wages and firm size in each sector will depend on the speed with which workers can locate alternative jobs in the formal or the informal sector (either directly or via unemployment). The nature of this equilibrium can explain two seemingly contradictory assertions: first as already mentioned, there can be a compensating differential for working in the informal sector as we would expect. At the same time average formal workers will be paid more than informal ones (as in the data) due to a composition effect: there are more formal jobs at higher levels of productivity than at lower ones. The computation of the equilibrium is described in Appendix B.

[26] Note that we are treating social security contributions in the formal sector as part of compensation.

4 Specification and Estimation

We develop a new estimation strategy suitable for this model, which is more complicated than the one for the standard Burdett-Mortensen model[27] because of the endogenous choice of sector activity. The market equilibrium defines two distributions of labor contracts *and* two distributions for the productivities of firms operating in the formal and informal sectors. We first estimate the distribution of contract values consistent with equilibrium in both sectors. Then, the distributions of firm productivities can be identified based on the restrictions from profit-maximization.

4.1 Offer Distributions and Transition Rates

The first step in our procedure is the estimation of the accepted offer distributions (F, F_2) based on the distribution of wages observed in the data. In Section 3.2, we have derived the way the distributions of *offered contracts*, $F_i(W), i = 1,2$, are related to the *accepted contract* distributions $G_i(W), i = 1,2$ (see equation (6)). However, in practice we can only estimate directly from the data the accepted *wage* distributions, $G_1^*(w)$ and $G_2^*(w)$. These are related to the accepted contract distributions by the equality $G_i^*(w) = G_i[W_i(w)], i = 1,2$, where $W_i(w)$ is the value of wage contract w in the sector i, which in turn depends on (F_1, F_2) (see equations (1) and (2)). The estimation method uses equation (6) and the equations for the value functions to find the pair (F_1, F_2) that minimizes the distance between the theoretical distributions of accepted wages $(G_1 \circ W_1, G_2 \circ W_2)$ to their estimated counterparts $(\hat{G}_1^*, \hat{G}_2^*)$, as described below.

The offered contract distributions can be estimated nonparametrically. However, we simplify the estimation problem by using a parametric distribution as an approximation, namely a non standard beta distribution:

$$F_i(x) = \text{betacdf}\left(\frac{x - \underline{W}_i}{\overline{W}_i - \underline{W}_i}; \alpha_i, \beta_i\right) \; i = 1,2; \; \underline{W}_i \leq x \leq \overline{W}_i,$$

where $\text{betacdf}(\cdot; \alpha, \beta)$ is the CDF of a beta distribution with parameters α and β. The advantage of the beta distribution is that it has bounded support and it is very flexible in the shapes it can take, without sacrificing parsimony as it depends on only two parameters. An important practical advantage of a parametric specification for the F_i is that it guarantees the smoothness of the offer densities $f_1 = F_1'$ and

[27] see Bontemps *et al.* (2000)

$f_2 = F_2'$, used in the calculation of the function Φ (of equation (6)) and of the transition rates.[28]

To estimate the parameters we use the method of moments. We match the distribution of wages for each sector and the transition rates implied by the model to those observed in the data. Given the above specification, we need to estimate the six arrival rates and the two job destruction rates all denoted by $\boldsymbol{\lambda} = (\lambda_{ij})_{i,j=0,1,2}$ and six further parameters $\boldsymbol{\theta} = (\underline{W}_1, \overline{W}_1, \underline{W}_2, \overline{W}_2, \alpha_1, \beta_1, \alpha_2, \beta_2)$ characterizing the offer distribution. Our algorithm estimates $\boldsymbol{\theta}$ given the $\boldsymbol{\lambda}$. We then update the latter, and iterate. This turned out to be a very quick procedure in practice.

Step 1: Offer distributions given transition rates. Given the $\boldsymbol{\lambda}$ we estimate $\boldsymbol{\theta}$ by minimizing the quadratic distance

$$Q_1(\boldsymbol{\theta}|\boldsymbol{\lambda}) = \sum_{i=1}^{2} \sum_{k=0}^{M} \left(\widehat{G}_i^*(w_{ik}) - G_i(W_{ik}) \right)^2, \tag{16}$$

where $W_{ik}, k = 0, ..., M$, defines a grid on the space of contract values, separately for both sectors $i = 1, 2$; $\widehat{G}_i^*(w_{ik})$ is the estimated wage distribution for sector i estimated from the data and evaluated at the implied grid for wages: $w_{ik} \equiv w_i(W_{ik})$, using equations (11) and (12); and $G_i(W_{ik})$ is the distribution of contracts in the population of employed workers implied by the model and which depends on all parameters ($\boldsymbol{\theta}$ and $\boldsymbol{\lambda}$).[29] This distribution can be computed on the basis of the equations in our theoretical section for a given set of parameters, as discussed above.

Step 2: Transition rates given offer distributions. In a similar way as we estimate $\boldsymbol{\theta}$ given $\boldsymbol{\lambda}$, we can estimate $\boldsymbol{\lambda}$ given $\boldsymbol{\theta}$ by matching the appropriate moments. In the data we observe the proportion of workers in state $i = 0, 1, 2$ at the beginning of the survey moving to state $j = 0, 1, 2$ before the end of the survey, lasting T periods ($\widehat{D}_{ij}$). We can use the model to compute the theoretical counterparts to these proportions (D_{ij}) as we show in Appendix C. For example the proportion who were in a formal sector job and move to an alternative job within the same sector is given by

$$D_{11} = \int_{\underline{W}_1}^{\overline{W}_1} \frac{\lambda_{11} \overline{F}_1(x)}{d_1(x)} (1 - e^{-d_1(x)T}) \, dG_1(x).$$

[28] More flexible distributions, such as mixtures of beta distributions, can be used for increased precision.

[29] We use Chebyshev nodes for the grid of contract values and we replace the integrals in contract spaces by Clenshaw-Curtis (CC) quadrature approximations. Computational details are provided in Appendix C.

Now, in equilibrium,

$$\frac{h_1(x)}{d_1(x)} = m_1 \frac{dG_1(x)}{dF_1(x)},$$

allowing to replace dG_1 by dF_1 inside the integral. Then CC-quadrature can be used to approximate the integral.

We thus estimate $\boldsymbol{\lambda}$ given $\boldsymbol{\theta}$ by minimizing the criterion

$$Q_2(\boldsymbol{\lambda}|\boldsymbol{\theta}) = \sum_{i,j=0,1,2} \left(\widehat{D}_{ij} - D_{ij}\right)^2,$$

where $\widehat{D}_{ij}$ is the empirical counterpart of D_{ij}.

4.2 Value of Leisure

As mentioned above we allow unemployment insurance to be determined endogenously: in Brazil about 8.5% of receipts from labor taxes fund UI. Hence we compute the implied amount using the government budget constraint

$$0.085\tau \int_{\underline{w}_1}^{\overline{w}_1} x \, d\widehat{G}_1^*(x) = UI \cdot D_{10}.$$

where D_{10} is the average transition probability from a formal sector job to unemployment and where $\widehat{G}_1^*$ is the estimated wage distribution and where τ has the same value as in the model (0.285). Remember that UI is paid to workers at the moment of transition into unemployment; hence this calculation is useful for constructing an amount that is consistent with the expected expenditure by Brazil and with the way we model UI.

Having estimated the contract values in both sectors and having set U to be equal to $\underline{W}_2$ (the legal minimum wage is not enforced in the informal sector and hence the minimum observed wage is the reservation wage) we can use the value function for the unemployed (3) to estimate the value of leisure, b.[30]

[30] At present we have not allowed for wages to be measured with error. If we did, this may affect the estimation of the distributions G and the value of leisure b.. We have mitigated the effects of measurement error by trimming the wage data to exclude formal jobs paying less than the minimum wage and by removing the lowest 5% of extremely low informal wages.

4.3 Productivity Distribution

Estimation up to this point has not made use of the firm profit functions, the costs of informality, or indeed the distribution of productivities: the arrival rates, the job destruction rates and the wage offer distribution can all be identified purely from the distribution of wages and the transition rates between employment states. The offer distribution, however, implicitly depends on the costs of informality. Thus, counterfactual policy simulations require an explicit specification and estimation of the costs of informality, which will allow us to compute the new equilibrium.

The costs of informality reflect the expected fines from government audit activities as well as other costs, including restricted access to finance. We specify the cost function as $C(\ell_2) = c\ell_2^{\gamma}$, with $c > 0$ and $\gamma \geq 1$ being the parameters to be estimated and $\ell_2(W)$ the size of informal firms offering contract W, inferred by the model. This specification captures the idea that small firms are unlikely to be caught and face lower costs of being informal, while large firms face disproportionately higher enforcement costs.

The next step involves estimating the productivity distributions in each sector, which result from the choice of sector by firms, as well as the costs of informality (c and γ) and the number of firms in the formal and the informal sector n_1 and n_2.

Computation of the estimates is based on an iterative algorithm. We start by guessing c, γ, n_1 and n_2. Given these we can compute the equilibrium labor force size by sector and productivity level, which will then be used to derive the relationship between offered contracts and productivities. The labor force size in each sector i ($i = 1,2$) is given by $\ell_i(W) = \frac{M}{N}\frac{1}{n_i}\frac{h_i(W)}{d_i(W)}$ (equation (13)). We use these to derive the support of the productivity distributions using the first order conditions for the firm's optimization problem (see (9), (10))

$$K_1^{-1}(W) = (1+\tau+\lambda_{10}s)\left[w_1(W) + w_1'(W)\frac{\ell_1(W)}{\ell_1'(W)}\right], \tag{17}$$

$$K_2^{-1}(W) = w_2(W) + w_2'(W)\frac{\ell_2(W)}{\ell_2'(W)} + c\gamma\ell_2(W)^{\gamma-1}, \tag{18}$$

where the expressions for $w_i'(W)$ and for $\ell_i'(W)$, as well as further details, are given in Appendix C. Based on the support of the contract offer distributions estimated earlier (F_1 and F_2) we use the two equations above to compute the support of the productivity distributions: for each point of the contract grids, W_{ik}, we can calculate a corresponding point on the productivity grid $p_{ik} = K_i^{-1}(W_{ik})$. The productivity

distributions are then computed as

$$\Gamma_i(p_{ik}) = n_i \cdot F_i(W_{ik}),\ i = 1,2,\ k = 0,...,N.$$

The productivity bounds defining the range of operation for the informal and the formal sector respectively are estimated as the endpoints of the support, i.e. $\underline{p}_2 = p_{20}$, $\underline{p}_1 = p_{10}$, $\overline{p}_2 = p_{2N}$, $\overline{p}_1 = p_{1N}$.

To update the estimates of c and γ, and n_1 and n_2, we use the equilibrium conditions requiring that at the lowest productivity point informal profits are zero, $\pi_2(\underline{p}_2) = 0$ (free entry), and on the overlapping part of the support profits across sectors are equalized, $\pi_1(p) = \pi_2(p) > 0$ for $p \in [\underline{p}_1, \overline{p}_2]$. We first estimate c and γ, and $\frac{n_1}{n_1+n_2}$ and $\frac{n_2}{n_1+n_2}$, the relative proportion of formal and informal firm respectively, so as to minimize

$$\pi_2(p_{20})^2 + \sum_{k,k'=0}^{M} \mathscr{K}(p_{1k} - p_{2k'})[\pi_1(p_{1k}) - \pi_2(p_{2k'})]^2,$$

where $\mathscr{K}$ is a kernel density.

The process above estimates at each step the distribution of productivity *conditional* on firms being active (i.e. $\Gamma(p)/(n_1+n_2)$), which is nonparametrically identified and whose lowest support point is $\underline{p}_2$, as well as the costs of informality and the relative size of the formal and the informal sector. However, the *total* proportion of active firms (n_1+n_2), needed for counterfactual simulations that allow for entry and exit of firms, is not identified without further structure on $\Gamma(p)$ allowing us to extrapolate out of sample to the left of $\underline{p}_2$. We thus assume the unconditional distribution $\Gamma(p)$ (including that is the inactive firms with productivity below $\underline{p}_2$) is lognormal with parameters μ and σ^2. These are estimated alongside n_1+n_2 by minimizing the distance (on a grid) between the parametric form and the non parametrically estimated distribution imposing the structure

$$\Gamma(p) = \begin{cases} \Gamma(p_{20}) + (n_1+n_2)\frac{\Gamma_2(p)}{n_1+n_2}, & \forall p \in [\underline{p}_2, \underline{p}_1], \\ \Gamma(p_{20}) + (n_1+n_2)\left[\frac{\Gamma_1(p)}{n_1+n_2} + \frac{\Gamma_2(p)}{n_1+n_2}\right], & \forall p \in [\underline{p}_1, \overline{p}_2], \\ \Gamma(p_{20}) + (n_1+n_2)\left[\frac{n_2}{n_1+n_2} + \frac{\Gamma_1(p)}{n_1+n_2}\right], & \forall p \in [\overline{p}_2, \overline{p}_1]. \end{cases}$$

where $\Gamma(p_{20})$ is the proportion of inactive firms.

The number of active workers M is directly observable. However, this is not the case for the total

number of active firms N that needs to be estimated. Usually the level does not matter but here it does because it determines firm size, which in turn is needed to determine the costs of informality (because of they are nonlinear in size). We thus estimate N/M targeting the average size of formal firms, that is observed in an alternative Brazilian firm level data set. The entire process is then iterated until convergence.

5 Estimation Results

In what follows the policy parameters are set to the values applicable to Brazil at the time of our data: The corporation tax rate is $t = 23\%$, the payroll tax rate is $\tau = 28.5\%$ and the severance pay rate is $s = 12\%$. We use a discount rate of $r = 0.5\%$ a month. As we already discussed we focus on low educated individuals and present estimates separately for males and females and for two contrasting regions of Brazil: wealthy and dynamic Sao Paulo and the poorer region of Salvador. Following estimation we discuss the impact of policies towards informality.

5.1 The Model Fit

Table 3 compares moments from our low education sample and the simulations. The model fits transitions remarkably well. The distribution of wages is also very well replicated for the median, the 25th and the 75th percentiles, although the fit is not as good in the tails. The stocks of formal employment, informal employment and unemployment fit best for females in Salvador. In other cases we tend to underestimate a bit formal employment and unemployment with more people being allocated to the informal sector relative to the data. Nevertheless, the model seems to be able to replicate the key patterns in the data with relatively few parameters.

The model generates predictions for the size of formal and informal firms that can be compared to those from external sources. We obtained the average firm size in the formal sector using RAIS 2003, containing a census of registered employees.[31] We also constructed this information for the informal sector, including both self employed and unregistered employees, using ECINF 2003, a survey of small firms (less than 5 workers). To compare the model predictions with the latter we censored the model predictions at 5 workers as well. We used observations of low education males and females living in the metropolitan regions of Sao Paulo and Salvador, to match our sample in this paper. Table 3 shows how

[31] This was kindly provided by Carlos Corseuil.

TABLE 3
Model Fit

	Sao Paulo				Salvador			
	Males		Females		Males		Females	
	Actual	Model	Actual	Model	Actual	Model	Actual	Model
Formal Employment (m_1)	0.458	0.384	0.380	0.300	0.376	0.273	0.246	0.256
Informal Employment (m_2)	0.460	0.551	0.457	0.571	0.518	0.636	0.540	0.576
Unemployment (u)	0.082	0.065	0.162	0.129	0.106	0.091	0.215	0.168
Transitions								
D_{01}	0.078	0.078	0.041	0.041	0.045	0.045	0.016	0.016
D_{02}	0.380	0.380	0.166	0.166	0.205	0.205	0.071	0.071
D_{10}	0.020	0.020	0.020	0.020	0.019	0.019	0.013	0.013
D_{11}	0.016	0.016	0.012	0.012	0.025	0.025	0.020	0.020
D_{12}	0.005	0.005	0.004	0.004	0.004	0.004	0.001	0.001
D_{20}	0.055	0.055	0.040	0.040	0.031	0.031	0.020	0.020
D_{22}	0.060	0.060	0.048	0.048	0.051	0.051	0.031	0.031
D_{21}	0.005	0.005	0.002	0.002	0.003	0.003	0.001	0.001
Formal Wages (log)								
P10	6.20	6.15	6.01	5.86	5.87	5.37	5.75	5.54
P25	6.41	6.48	6.15	6.17	6.01	6.06	5.84	5.89
Median	6.67	6.72	6.35	6.40	6.24	6.38	5.99	6.01
P75	6.99	6.98	6.59	6.58	6.50	6.53	6.22	6.19
P90	7.31	7.13	6.90	6.74	6.85	6.66	6.48	6.27
Informal Wages (log)								
P10	5.70	5.16	5.30	5.20	5.16	4.42	4.88	4.79
P25	6.03	5.94	5.67	5.72	5.52	5.55	5.20	5.25
Median	6.36	6.35	6.00	6.04	5.80	5.85	5.58	5.53
P75	6.78	6.73	6.37	6.36	6.17	6.06	5.86	5.74
P90	7.21	6.93	6.80	6.45	6.58	6.23	6.28	5.91
Mean Firm Size[1]								
formal	28.8	28.8	37.4	37.4	34.9	34.9	57.2	57.2
informal	-	7.8	-	9.7	-	10.0	-	6.7
informal (censored$\leq$5 workers)[2]	1.4	1.6	1.3	1.2	1.3	1.5	1.2	1.2
Proportion of Formal Firms $[n_1/(n_1+n_2)]$								
Mean	-	0.16	-	0.12	-	0.11	-	0.05
Std. Dev.	-	(0.02)	-	(0.02)	-	(0.02)	-	(0.02)

Notes:
[1]The formal sector data on firm average size were kindly provided by Carlos Corseuil based on *RAIS 2003* (Census of registered employees). The informal sector data uses own calculations from *ECINF 2003.*
[2]The *ECINF* data only contains firms with no more than five workers, which is the reason we also present the model predictions for informal firms censored at that point.

the model fits the observed firm size. Once we take censoring into account the fit is excellent. Note that informal firms are much smaller than formal ones, which is correctly predicted by the model.

5.2 Search Frictions and the Level of Informality

Table 4 shows the estimates for the job destruction and the job arrival rates (on a monthly frequency) with associated standard errors obtained using 500 bootstrap replications.[32] In general job destruction is quite low from the informal sector. For example for males an informal job spell is expected to last (before a transition to unemployment) for nearly six years in Sao Paulo and about 10 years in Salvador. Formal jobs last 2-3 times longer than that – for all practical purposes they can be taken as lasting forever, except of course that they may be interrupted by a transition to an alternative employer.

Focusing on males in Sao Paulo, it takes on average three years to obtain a formal job from unemployment but only a few months to obtain an informal one. So clearly the informal sector offers a relatively quick return to employment. However, from an informal job it is virtually impossible to obtain a formal job offer, which is an important factor preventing workers in the informal sector from moving up to higher productivity jobs. Individuals in the formal sector mostly obtained their jobs from unemployment. Alternative job offers for someone in the formal sector arrive on average every 4.8 years from the formal sector and every 18 months from an informal job. In the formal sector alternative jobs take on average just over one year to arrive.

In Salvador, arrival rates when unemployed and job destruction rates are both lower, implying substantially longer unemployment *and* employment spells. However, mobility while in work is much higher. On average a formal male worker waits for two years for an alternative formal offer and about 7.5 months for an informal job offer. Turnover between informal jobs is also higher. These differences across regions reflect themselves in the implied unemployment rates: that of Salvador is about 1.3 times that of Sao Paulo (Table 3), which mirrors the data.

The overwhelming feature of the female labor market is one of much lower destruction rates, lower arrival rates when unemployed and lower rates of job offers when employed. This is particularly so in the poorer region of Salvador. These low rates are a direct reflection of the higher level of unemployment and lower turnover of women as we see in the data.

[32] The unit of time is a month. Subscript 0 refers to unemployment, 1 refers to the formal sector and 2 to the informal. The arrival rates λ_{ij} denote an offer arriving from sector j to someone currently in sector i.

The implied proportion of formal firms is significantly smaller in Salvador than in Sao Paulo (see Table 3). The informal firms here include those of the self employed, who are typically owner-only businesses.[33]

5.3 The Cost of Informality and the Value of Leisure

Table 5 presents the implied cost to the firm of remaining informal. This cost arises from random monitoring and imposition of fines. It may also reflect other costs such as limited access to finance. We report the parameters of the cost function, $C(\ell) = c\ell^{\gamma}$, and the mean cost per unit of profit. The costs are linear in firm size for females in Sao Paulo and strictly convex in all other submarkets. In both cases the gradient c is very high for both males and females. These features imply that informality will be concentrated among smaller firms as implied by both model and data (see Table 3). This implication is achieved without ever targeting informal firm size.

In the last column of Table 5 we present the estimated flow value of leisure. These are lower in Sao Paulo for each gender respectively. Moreover, women value leisure much more than men, possibly reflecting the demands of families and home production, which are estimated to be higher in Salvador than in Sao Paulo.

5.4 Formal and Informal Sector Productivity and Wages

A key feature of the equilibrium we describe is that at a given level of productivity, both formal and informal firms can coexist: frictions ensure that firms can adjust their wage policy (given costs of informality and the costs involved in being formal) so that profits are equalized across the two sectors for a given level of productivity. Thus, policies that reduce informality will not necessarily shut down all jobs in this part of the productivity distribution; on the other hand this should not be taken to imply that such an exercise will be costless, because lower levels of productivity may be able to sustain only smaller and fewer formal firms, given the amount of competition for workers and the overall regulatory costs. We consider these issues by first describing the equilibrium that results from our estimates and subsequently by counterfactual simulations.

Based on the estimates we can back out the implied allocation of workers to the formal and the

[33] 82% of the self employed run business on their own. Source: author's calculations using PME 2002-2007.

TABLE 4
Transition Parameters

	λ_{10}	λ_{20}	λ_{01}	λ_{02}	λ_{11}	λ_{22}	λ_{12}	λ_{21}
Sao Paulo								
Males	0.0051	0.0146	0.0261	0.1271	0.0175	0.0725	0.0533	0.0018
	(0.0003)	(0.0005)	(0.0018)	(0.0042)	(0.0020)	(0.0050)	(0.0121)	(0.0002)
Females	0.0051	0.0105	0.0116	0.0466	0.0112	0.0610	0.0478	0.0008
	(0.0005)	(0.0006)	(0.0013)	(0.0027)	(0.0020)	(0.0074)	(0.0247)	(0.0002)
Salvador								
Males	0.0049	0.0081	0.0128	0.0588	0.0405	0.0902	0.1340	0.0012
	(0.0004)	(0.0005)	(0.0015)	(0.0032)	(0.0083)	(0.0129)	(0.0424)	(0.0003)
Females	0.0032	0.0052	0.0043	0.0187	0.0338	0.0462	0.0489	0.0004
	(0.0006)	(0.0005)	(0.0009)	(0.0017)	(0.0213)	(0.0117)	(0.1112)	(0.0002)

Notes: The unit of time is a month Bootstrap standard errors in parentheses, based on 500 replications.

TABLE 5
Cost of Informality and Value of Leisure

	c	γ	mean cost-profit ratio	b
Sao Paulo				
Males	95.5	1.5	0.179	-1,308.0
	(3.3)	(0.2)	(0.036)	(158.4)
Females	83.5	1.0	0.453	-214.9
	(1.9)	(0.2)	(0.073)	(63.9)
Salvador				
Males	85.5	1.8	0.232	-295.5
	(2.7)	(0.1)	(0.025)	(55.9)
Females	34.5	1.9	0.162	28.4
	(2.4)	(0.2)	(0.029)	(27.2)

Note: Bootstrap standard errors in parentheses, based on 500 replications.

informal sector for different levels of productivity, as well as the pay structure. The results are presented in Table 6 for low education males, and in Table 7 for low education women, in Sao Paulo and Salvador, respectively.

Figure 2 displays the predicted distributions for log-productivity for formal and informal firms separately. In both Sao Paulo and Salvador the distributions overlap and informal firms are concentrated in the lower part of the productivity distribution with only three percent or less of formal firms operating below the 25th percentile, a fact that reflects the information presented in our data section. At higher levels of productivity formal firms become rapidly larger and overtake informal ones in size substantially. The distributions are more to the left that those of Figure 1, but that is expected as we focus on unskilled workers, while the data from La Porta and Shleifer include all workers. Nevertheless, the model reproduces this key feature of the data.

Finally, we also find that most productivity percentiles are higher for males than females, and for Sao Paulo than for Salvador, reflecting the overall lower productivity of jobs in Salvador and the fact that women are employed in low productivity occupations.

One of the most interesting features of the model is the implied wage structure. First, comparing wages and productivities the implied rents accruing to firms (i.e. profit divided by output - see profit rate column in Tables 6 and 7) in both sectors are substantial because of the search frictions. This is important for understanding why policies that increase compliance costs do not increase unemployment as we show later. Interestingly, the profit rates are not monotonic in productivity: rents are higher for both low and high productivity firms. This is an unusual feature in a wage posting model, where high productivity firms have increasingly more monopsony power. Here, the competition between sectors tends to reduce firms' monopsony power in the lower part of overlapping region.

Second, the results justify two seemingly contradictory statements. Wages are on average higher in the formal sector than in the informal one, because the formal firms become increasingly large as productivity increases and wages increase with productivity (as indeed in the standard Burdett and Mortensen model): this is a composition effect. However, given productivity, for the most part formal firms pay less than informal ones: this is a compensating differential for UI, pension entitlements and severance pay, although frictions and different job arrival rates will imply that the relationship is not one-to-one with these benefits. This differential declines and even gets reversed at the highest levels of productivity in Salvador.

TABLE 6

Wages and profits, Males - Estimates by productivity

Productivity		Cumulative	Fraction of	Fraction of	Wage (log)		Profit rate		Value (log)		Firm size	
Percentiles	(log)	worker share	formal firms	formal workers	Formal	Informal	Formal	Informal	Formal	Informal	Formal	Informal
					Sao Paulo							
10th	5.916	0.009	0.000	0.000	-	-	-	-	-	11.343	-	1.5
25th	6.205	0.039	0.031	0.048	-	4.297	-	0.77	11.380	11.367	2.5	2.3
50th	6.505	0.108	0.070	0.141	5.063	5.627	0.54	0.50	11.466	11.404	5.6	4.3
75th	6.904	0.310	0.146	0.239	6.151	6.352	0.30	0.32	11.585	11.482	15.1	13.3
90th	7.183	0.542	0.190	0.382	6.607	6.624	0.23	0.32	11.725	11.547	38.6	25.0
99th	7.869	0.901	0.418	0.620	7.059	7.019	0.31	0.50	11.978	11.695	107.4	47.3
$\overline{p}$	8.889	0.989	0.910	0.957	7.334	7.488	0.59	0.72	12.181	11.942	134.5	61.9
					Salvador							
10th	4.782	0.005	0.000	0.000	-	-	-	-	-	10.767	-	1.0
25th	5.274	0.023	0.021	0.054	-	-	-	-	10.868	10.791	1.9	1.5
50th	5.684	0.061	0.042	0.077	-	4.415	-	0.63	10.932	10.830	3.3	3.0
75th	6.220	0.195	0.097	0.110	4.851	5.546	0.52	0.34	11.048	10.911	9.0	12.0
90th	6.611	0.399	0.112	0.218	6.056	5.852	0.22	0.31	11.222	10.978	41.5	30.8
99th	7.309	0.861	0.170	0.330	6.601	6.226	0.24	0.39	11.498	11.132	241.9	100.2
$\overline{p}$	8.176	0.993	0.910	0.949	6.837	6.770	0.54	0.57	11.676	11.469	322.3	176.6

Note: Cumulative worker share = fraction of all workers employed at firms with productivity less than p; Fraction of formal firms = probability of drawing a formal job conditional on drawing a job of productivity p; Fraction of formal workers = share of formal workers among employees at jobs of productivity p; Wage is wage offer $K_i(p)$ of firms of productivity p; Profit rate = profit flow $\pi_1(p)$ divided by output $p\ell_i(p)$; Value is $W_i(K_i(p))$; Size is $\ell_i(p)$.$\bar{p}$ corresponds to the 0.999 quantile of the productivity distribution - effectively the max.

TABLE 7

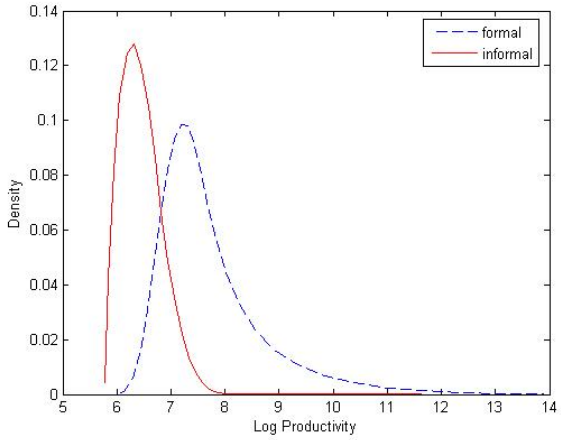
Wages and profits, Females - Estimates by productivity

Productivity		Cumulative	Fraction of	Fraction of	Wage (log)		Profit rate		Value (log)		Firm size	
Percentiles	(log)	worker share	formal firms	formal workers	Formal	Informal	Formal	Informal	Formal	Informal	Formal	Informal
					Sao Paulo							
10th	5.808	0.009	0.000	0.000	-	2.566	-	0.71	-	11.039	-	1.7
25th	5.999	0.041	0.000	0.000	-	4.780	-	0.50	-	11.063	-	2.6
50th	6.201	0.107	0.029	0.037	4.194	5.498	0.64	0.33	11.105	11.103	5.5	5.3
75th	6.479	0.311	0.137	0.127	5.653	6.039	0.35	0.23	11.211	11.185	12.8	18.5
90th	6.693	0.508	0.278	0.253	6.033	6.264	0.26	0.25	11.299	11.253	23.7	36.6
99th	7.400	0.908	0.558	0.644	6.665	6.611	0.30	0.49	11.606	11.408	99.3	69.0
$\overline{p}$	8.700	0.993	0.954	0.968	6.961	7.049	0.61	0.79	11.820	11.663	126.0	86.2
					Salvador							
10th	4.998	0.008	0.000	0.000	-	2.966	-	0.85	-	10.514	-	0.9
25th	5.121	0.019	0.000	0.000	-	3.897	-	0.68	-	10.528	-	1.1
50th	5.407	0.063	0.000	0.000	-	4.792	-	0.42	-	10.574	-	2.2
75th	5.848	0.222	0.016	0.005	-	5.406	-	0.27	10.713	10.678	3.3	9.8
90th	6.199	0.436	0.115	0.158	5.537	5.640	0.25	0.28	10.924	10.764	25.0	25.2
99th	7.236	0.808	0.234	0.709	6.338	6.141	0.32	0.53	11.332	11.066	588.9	73.9
$\overline{p}$	8.107	0.997	0.867	0.979	6.476	6.495	0.60	0.73	11.446	11.329	650.7	92.4

Note: Cumulative worker share = fraction of all workers employed at firms with productivity less than p; Fraction of formal firms = probability of drawing a formal job conditional on drawing a job of productivity p; Fraction of formal workers = share of formal workers among employees at jobs of productivity p; Wage is wage offer $K_i(p)$ of firms of productivity p; Profit rate = profit flow $\pi_1(p)$ divided by output $p\ell_i(p)$; Value is $W_i(K_i(p))$; Size is $\ell_i(p)$.

FIGURE 2

The Predicted Distribution of Log Productivity for Formal and Informal Firms



1

TABLE 8
Comparing male and female wages, by productivity

	Sao Paulo				Salvador			
	Formal		Informal		Formal		Informal	
Productivity	Males	Females	Males	Females	Males	Females	Males	Females
6.000	-	-	-	4.780	3.494	3.143	5.315	5.533
6.250	-	4.194	4.297	5.722	4.851	5.745	5.546	5.640
6.500	5.063	5.653	5.627	6.039	5.677	6.106	5.717	5.824
6.750	5.917	6.173	6.171	6.264	6.185	6.266	5.963	5.907
7.000	6.333	6.399	6.499	6.447	6.462	6.338	6.059	5.987
7.250	6.607	6.583	6.624	6.530	6.601	6.338	6.226	6.141
Mean	6.760	6.429	6.401	6.065	6.301	6.009	5.823	5.549

The overall picture is similar for women with some small differences: first formal firms in Salvador start operating at a higher point of the distribution of productivity. Second the wage structure and the distribution of productivities differ: male wages in the formal sector are more dispersed than those of females in both regions.

Table 8 presents male and female wages for the two regions by sector and overall, at the same productivity level. In all cases, but the formal sector of Salvador, women are paid more conditional on productivity, at lower productivity levels. This is reversed at the higher productivity levels. Thus women in most cases seem to work in more competitive labor markets with lower monopsony power for firms. However, on average women are paid less than men because most of them work in lower productivity (and hence lower paid) jobs. In other words the model interprets discrimination as being due to the type of jobs in the female labor market.

6 Policy Analysis: Increasing the Cost of Informality

We now explore the implications of this model for policies relating to increasing the cost of informality. The equilibrium setting is important because the policy will affect the optimal location of firms and wage posting in both sectors. Hence our model emphasizes the inextricable link between firm and worker outcomes as the policy environment changes. However, the model, as it is, is still incomplete as it does not specify how arrival rates adjust to changing labor market conditions. For example if a policy increases the number of firms competing for workers in say the formal sector this may affect the rate at

which workers meet firms. Thus, before running the simulations we first explain how we endogenize job offer arrival rates.

6.1 Endogenous Arrival Rates

To capture the relationship between arrival rates and search activity we specify a matching function $f(\xi)$, describing the flow of matches as a function of market tightness (ξ - the effective number of firms divided by the effective number of job seekers).[34] This is combined with an assumption about the way these contacts are allocated between the formal and the informal sector. We then identify the parameters of this model by imposing the restriction implied by the matching functions on the job arrival rates separately for each submarket using minimum distance.

Define market tightness as

$$\xi = \frac{n_1 + \alpha n_2}{u + s_1 m_1 + s_2 m_2}. \tag{19}$$

where s_1 and s_2 are the search effort of those employed in the formal and the informal sector respectively relative the search effort of the unemployed s_0, which we normalize to one. Define the flow of contacts by the matching function $f(\xi) = \phi \xi^{\eta}$. We assume that the probability of an offer from the formal sector is $n_1/(n_1 + \alpha n_2)$ and the informal one $\alpha n_2/(n_1 + \alpha n_2)$, where α denotes the relative *visibility* of informal vacancies in the market. Given these specifications, the job offer arrival rates to workers in state $i = 0, 1, 2$ from the formal and the informal sector, can be written respectively as

$$\lambda_{i1} = \frac{n_1}{(n_1 + \alpha n_2)} s_i f(\xi); \quad \lambda_{i2} = \frac{\alpha n_2}{(n_1 + \alpha n_2)} s_i f(\xi). \tag{20}$$

We fit the above for a fixed level of the elasticity of the matching function η, which requires variation in the matching process to be identified. Combining the arrival rates and market tightness across our submarkets implies an $\eta = 0.34$, which is within the range of estimates in the literature.[35] We use this value but also show below the sensitivity of the matching function parameters to a higher value of $\eta = 0.5$.

The estimates of the matching function that relates these transition rates to market tightness are shown in Table 9. According to these estimates informal jobs are approximately 2-4 times easier to locate than

[34] Since firms will hire anyone they meet offering them their posted wage the effective number of firms relative to the effective number of job seekers is equivalent to the number of vacancies per worker searching.

[35] The elasticity of the matching function with respect to vacancies η is usually estimated in the range 0.3-0.5 (see, Petrongolo and Pissarides, 2001).

TABLE 9
Matching Function Estimates

	s_1	s_2	α	ϕ		ξ
				$\eta = 0.3$	$\eta = 0.5$	
Sao Paulo						
Males	0.545	0.320	3.031	0.170	0.128	4.116
	(0.084)	(0.023)	(0.366)	(0.005)	(0.004)	(0.786)
Females	0.997	0.690	3.799	0.108	0.088	2.804
	(0.303)	(0.090)	(1.021)	(0.009)	(0.006)	(1.175)
Salvador						
Males	2.718	0.813	3.521	0.142	0.121	2.253
	(0.661)	(0.120)	(0.886)	(0.011)	(0.011)	(0.668)
Females	5.285	1.283	2.066	0.101	0.114	0.551
	(5.370)	(0.347)	(1.757)	(0.025)	(0.042)	(0.667)

Note: The matching function is $f(\xi) = \phi\xi^{\eta}$ with tightness $\xi = \frac{n_1+\alpha n_2}{u+s_1 m_1+s_2 m_2}$. Bootstrap standard errors in parentheses.

formal ones. In Sao Paulo search intensity is higher among the unemployed than among the employed. This result is reversed in Salvador except for informal male workers.

6.2 Counterfactual Analysis

In the subsequent simulations we examine the effects of increasing enforcement costs. To economize in space, we present simulations for males in Sao Paulo only, which is sufficient to understand the implications of the model. We increase the parameter γ successively from its estimated value of 1.5 all the way up to 2.0, making the costs of informality higher and more convex. This is equivalent to a policy where increasing attention is paid to larger informal firms. The effects of increasing γ on costs are highly nonlinear. For example, before any behavioral effects take place, increasing γ from 1.5 to 1.8 increases average costs of informality by 70%, while increasing γ to 2 increases average costs by a factor of 2.5; this is unevenly distributed with the cost increasing by about 20% at the lower decile of productivity and being multiplied by over 4 times at the top decile. The last simulation where γ is increased to 2 is quite extreme and is presented so as to show what happens when we come close to eliminating informality, which requires extrapolation far out of sample.

Below we will be presenting results for two simulations: when we allow the arrival rates to adapt to changes in market tightness (based on the matching function we estimated above) and when we keep them fixed. When we use the endogenous arrival rates the model fit worsens slightly because we are

using a more restrictive model with fewer parameters. We thus compare each simulation to its respective baseline - although these are very similar.

We start by presenting the bottom line, i.e. the welfare effects. Our welfare measure includes the total value going to workers and firms in each sector, to the unemployed including future expected flows due to layoffs or improved wages, as well as net government revenue from taxes and enforcement. These are shown in the last row of Table 10. As we increase enforcement we achieve progressively higher overall welfare. However, this does not go on forever: when the informal sector has shrank to a small group of firms there do not seem to be any further gains because profits of firms start declining faster than any further gains from reallocating the few remaining informal workers. Finally, it is interesting to note that government revenue (counted in total welfare) does not change much throughout these simulations: the increased informality costs for those remaining informal and the fact that many of the more productive firms become formal acts to balance the change revenues.

Breaking the changes in welfare down we see that the value of workers increases (in all states) while firm profits per worker decline within each sector. The fundamental reason welfare goes up is because the increased costs of informality reallocates workers from the informal sector to the formal one where productivity and pay are higher. The same direction of effects occurs with fixed arrival rates, but the gains are much lower, because the rate at which this reallocation takes place is lower relative to the case where the arrival rates respond endogenously. Specifically, workers can find formal firms and move up the ladder of productivity faster both because more firms locate in the formal sector and because the arrival rates towards formal firms improve. We now look at the mechanisms underlying this welfare improvement.

Table 11 shows that increasing the cost of informality increases the number of firms operating in the formal sector and decreases informal ones. As we may expect it also leads to some firms exiting (n_0 increases) because of the increased cost of being informal. The increased proportion of firms in the formal sector in itself increases the number of workers who obtain formal jobs (from the equilibrium flow conditions) as shown in the last column where we keep arrival rates fixed. When arrival rates adjust the flow of workers matching with jobs in the formal sector increases substantially and an even larger proportion of workers become formal, which is the key mechanism for increasing welfare. Importantly unemployment does not increase (indeed it tends to decrease slightly). Thus, informal firms are not extending employment opportunities where they would not have existed but create an over-representation

TABLE 10
Welfare Effects of Increasing the Costs of Informality

	Endogenous λ					Exogenous λ	
	Baseline	Change from Baseline				Baseline	Change
	Level	$\gamma = 1.7$	$\gamma = 1.8$	$\gamma = 1.9$	$\gamma = 2.0$	Level	$\gamma = 2.0$
Formal worker $[rE(W_1)]$	717.8	6.0	225.5	268.3	283.6	683.4	357.3
Informal worker $[rE(W_2)]$	557.9	18.7	146.3	229.4	262.9	500.1	273.3
Unemployed $[rU]$	492.2	36.0	154.8	233.3	262.9	418.8	297.2
Formal firm $[E(\pi_1) * N_1/M_1]$	740.3	-55.6	-196.1	-38.1	-233.1	1,053.0	-603.5
Informal firm $[E(\pi_2) * N_2/M_2]$	271.3	-46.0	-22.7	-128.4	-129.1	517.9	-266.4
Total welfare per worker	1,459.5	55.2	254.3	514.5	338.1	1,633.3	89.3

Notes: The two baseline columns correspond to the levels of these variables as predicted by the model. The remaining columns are changes from the respective baseline. Formal and informal profits are total profits in the sector per worker. N_1 (N_2) is the number of firms in the formal (informal) sector. M_1 (M_2) is the number of workers in the formal (informal) sector.

TABLE 11
Changes in Worker and Firm Allocation across Sectors as Costs of Informality Increase

	Endogenous λ					Exogenous λ	
	Baseline	Change from Baseline				Baseline	Change
	Level	$\gamma = 1.7$	$\gamma = 1.8$	$\gamma = 1.9$	$\gamma = 2.0$	Level	$\gamma = 2.0$
m_1	0.415	0.167	0.174	0.324	0.342	0.384	0.117
m_2	0.546	-0.166	-0.172	-0.320	-0.337	0.551	-0.087
u	0.038	-0.002	-0.003	-0.004	-0.005	0.065	-0.030
n_1	0.098	0.089	0.024	0.111	0.123	0.110	0.006
n_2	0.570	-0.170	-0.138	-0.322	-0.337	0.576	-0.270
n_0	0.333	0.082	0.114	0.211	0.215	0.314	0.264
Formal firm size	33.8	-8.8	3.5	-6.2	-10.2	28.8	2.9
Informal firm size	7.7	0.7	-0.6	-0.4	2.1	7.8	4.1

Note: The two baseline columns correspond to the levels of these variables as predicted by the model. The remaining columns are changes from the respective baseline.

of low productivity firms and make locating higher productivity firms harder. This leads to lower output (because of workers misallocating more to low productivity firms) and thus lowers welfare. The increase in formal sector workers as a result of increased enforcement is consistent with the quasi-experimental evidence from Brazil by Almeida and Carneiro (2012).

The increase in the number of formal workers and firms are associated with opposing competitive forces: as firms move out of the informal sector it becomes easier for the remaining informal firms to hire and retain workers, pushing wages down and mitigating the cost increases from increased enforcement costs. However, as workers migrate to the formal sector matching with workers becomes harder in the informal one and easier in the formal. Finally, the rate at which firms switch from the informal to the formal sector or to inactivity (exit) differs by productivity, endogenously changing its distribution in each sector. These effects are reinforced by the change in the arrival rates. For example an increase in the number of formal workers decreases the rate at which the unemployed locate formal jobs, while an increase in formal firms increases the matching rate into the formal sector and towards higher productivity formal firms. All these changes will lead to changes in the wage posting strategy of firms in each sector, but the net effect is unclear and needs to be determined empirically.

Table 12 shows the change in the distribution of productivity as enforcement costs are increased. All changes are due to composition changes as firms enter or exit production or move sectors. At first the productivity in both sectors declines: as the relatively more productive (and hence larger) informal firms face increasing costs they move to the formal sector; although they are relatively more productive in the informal sector they still tend to be lower productivity relative the formal one, bringing the overall productivity in both sectors down. Eventually though the combination of exit of the lower productivity informal firms and the migration of the even more productive ones to the formal sector improves the distribution of productivity in both sectors. The minimum levels of productivity for both informal ($\underline{p}_2$) and formal sectors increase ($\underline{p}_1$). The former as a direct effect of the increasing costs of being informal and the latter as an equilibrium reaction to the changing competition at the lower part of the distribution: as fewer informal firms remain at the bottom of the distribution they can match with workers more easily (as also reflected in the decline of informal wages at the bottom) making it harder for the lowest productivity formal firms to attract workers. This pushes up the point at which it becomes profitable to enter as a formal firm. Thus increasing enforcement does cut lower productivity jobs as we expected. Importantly, this is not at the expense of overall employment.

TABLE 12

Effects of Changes in the Cost of Informality on Productivity

	Endogenous λ					Exogenous λ	
	Baseline	Change from Baseline				Baseline	Change
	Level	$\gamma = 1.7$	$\gamma = 1.8$	$\gamma = 1.9$	$\gamma = 2.0$	Level	$\gamma = 2.0$
log Formal Productivity							
P10	6.444	-0.434	0.456	0.368	0.759	6.302	1.089
P25	6.691	-0.319	0.493	0.355	0.589	6.656	0.837
Median	7.060	-0.239	0.367	0.166	0.306	6.898	0.709
P75	7.420	-0.253	0.203	0.005	0.106	7.208	0.581
P90	7.830	-0.443	-0.026	-0.188	-0.074	7.528	0.452
Mean	7.469	-0.419	0.124	-0.095	0.053	7.163	0.711
log Informal Productivity							
P10	5.936	0.069	0.212	0.389	0.405	5.916	0.482
P25	6.189	-0.088	0.186	0.352	0.398	6.206	0.426
Median	6.451	-0.057	0.200	0.370	0.469	6.499	0.457
P75	6.701	0.003	0.251	0.414	0.607	6.772	0.595
P90	6.948	0.208	0.314	0.458	0.748	7.039	0.779
Mean	6.522	0.073	0.237	0.395	0.579	6.606	0.622
Minimum thresholds							
$\underline{p}_2$	5.839	0.092	0.230	0.416	0.425	5.799	0.524
$\underline{p}_1$	5.977	0.009	0.374	0.537	1.186	6.046	1.031

Note: The two baseline columns correspond to the (log) productivities as predicted by the model. The remaining columns are changes from the respective baseline.

TABLE 13
Effects of Changes in the Cost of Informality on Wages

	Endogenous λ					Exogenous λ	
	Baseline	Change from Baseline				Baseline	Change
	Level	$\gamma = 1.7$	$\gamma = 1.8$	$\gamma = 1.9$	$\gamma = 2.0$	Level	$\gamma = 2.0$
log Formal Wages							
P10	6.403	-0.173	0.105	0.103	0.237	6.151	0.609
P25	6.605	-0.090	0.212	0.231	0.247	6.481	0.483
Median	6.776	-0.056	0.280	0.295	0.309	6.715	0.434
P75	6.928	0.037	0.324	0.270	0.281	6.982	0.334
P90	7.136	-0.029	0.234	0.231	0.185	7.132	0.288
Mean	6.807	-0.052	0.254	0.248	0.252	6.760	0.413
log Informal Wages							
P10	5.346	-1.186	-0.031	0.113	0.195	5.160	0.660
P25	5.941	-0.451	-0.073	-0.014	0.043	5.941	0.266
Median	6.292	-0.076	0.072	-0.024	0.022	6.352	0.122
P75	6.636	-0.139	-0.047	-0.001	0.037	6.735	0.025
P90	6.813	-0.005	-0.036	0.014	0.129	6.930	-0.013
Mean	6.335	-0.156	0.007	0.025	0.090	6.401	0.099
Overall Wage Inequality							
P75/P25	1.67	0.12	0.42	0.32	0.09	1.94	-0.26
P90/P10	2.83	0.61	0.25	-0.25	-0.38	4.15	-1.53

Note: The two baseline columns correspond to the (log) wages as predicted by the model. The remaining columns are changes from the respective baseline.

The effects on wages, shown in Table 13, are driven by opposing forces on competition for workers, namely the change in the number of firms competing for workers in each sector and the changing number of workers being matched with formal and informal jobs. In addition the change in the distribution of productivity we documented above will also drive wages higher in each sector: productivity is a key driver of wages in our model. Thus, following an initial decline, overall wages increase particularly in the formal sector. This explains why welfare increases as much with increased enforcement: workers are reallocated to more productive firms *and* competition for these workers pushes wages up within that sector. The scarcity of workers in the informal sector has a similar but smaller effect, as there are fewer and fewer firms competing for the workers. The change in inequality is ambiguous with the 90/10 declining but the 75/25 inequality increasing.

7 Conclusions

Informality is extremely common in developing countries. However its effects are not well understood. On the one hand informal firms are often portrayed as regulation busters that offer a much needed competitive fringe in countries with over-regulated labor markets. Hence they are considered job creators and tolerating them with relatively low enforcement levels is often considered an indirect way by which employment protection legislation can be relaxed without the political cost of explicit labor market deregulation. However, in an economy with search frictions the story may not be that simple because low quality informal firms may end up making it harder for workers to find the higher quality formal jobs by congesting the matching process.

The relative importance of job creation and de facto deregulation through informality vis a vis the effects of search frictions is an empirical matter. We thus set up a model in which workers search randomly for jobs. Job offers take the form of a "take it or leave it" wage offer as in Burdett and Mortensen (1998), which we extend to allow for two sectors: firms, which are heterogeneous in productivity, can choose what wage to post *and* in which sector to operate. Our model is motivated by the empirical observation that low skill workers are observed working both in formal and informal jobs and given productivity levels some firms choose to be informal while others become formal. This overlap in the distributions of productivity is an important feature of the data and of our model model and has key implications on how we understand informality. In other words, given a particular enforcement strategy by the government

and given regulatory costs (taxes, minimum wages, severance etc.) it is possible in equilibrium for a firm to make equal profits with alternative wage strategies in the formal and the informal sector respectively. Thus segmentation is endogenously determined by an interplay between search frictions, the institutional requirements for formal firms and the penalties of informality.

We estimate the model using data from two regions of Brazil - Sao Paulo and the poorer region of Salvador so as to understand how different labor market conditions affect the results. Mostly the implications of the model do not differ qualitatively across regions, with some important exceptions. However, it is always the case that the value of working in the formal sector is higher than the value in the informal one, but the search costs prevent workers waiting for a formal job. Wages themselves display compensating differentials in the sense that informal firms pay more than formal ones of equal productivity (for most of the support except right at the top), but this is not enough to equalize values. However, as in the data, wages in the formal sector are on average higher than those in the informal one because formal firms are on average more productive and pay more.

Given the estimates we simulate increases in enforcement that affect disproportionately larger informal firms. We find that this leaves unemployment virtually unchanged and reallocates workers to the formal sector increasing overall welfare. Thus the intuitive result that allowing a competitive fringe by tolerating informality improves the labor market, reduces unemployment and increases welfare no longer holds up in the presence of labor market frictions. This is no theoretical curiosity but a clear conclusion of empirical analysis in a labor market with formal regulations and a substantial informal sector.

The key conclusion from our results is that in a market with search frictions the informal sector has important detrimental effects: by endogenously segmenting the labor market, it reduces the competition for workers and makes it harder for workers to locate to higher productivity firms. Increasing the costs of informality improves the allocation of workers to better firms, increases wages and increases overall welfare. Interestingly policies reducing informality do not increase unemployment. One reason for this is that firms are making very high rents and can absorb the increasing cost of regulation. With exogenous arrival rates welfare still goes up, but by less; the full effect is only felt as we allow matching rates to respond to changes in market tightness.

Finally, our model focuses on one key aspect of informality: the way firms decide to post vacancies in the formal or the informal sector and how this affects the allocation of workers to more or less productive jobs. There are other aspects that need further research. In particular understanding the investment

strategies of firms and how the presence of informality affects investment is important for completing the picture of how informality may affect growth. The fact that informal firms have such low productivity and employ a high proportion of the workforce is suggestive that informality may reduce overall investment.

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APPENDIX - Not for publication

A Equilibrium Offer and Accepted Contract Distributions

In this section, we derive G_1 and G_2 from F_1 and F_2.

By equation (4), for any $W \in [\underline{W}_1, \overline{W}_1]$,

$$\left[\lambda_{10} + \lambda_{11}\overline{F}_1(W)\right] m_1 G_1(W) + \lambda_{12} m_1 \int_{\underline{W}_1}^{W} \overline{F}_2(x)\, dG_1(x) \\ = \lambda_{01} u F_1(W) + \lambda_{21} m_2 \int_{\underline{W}_2}^{W} [F_1(W) - F_1(x)]\, dG_2(x).$$

Making use of the identities (integration by parts):

$$\int_{\underline{W}_1}^{W} \overline{F}_2(x)\, dG_1(x) = \overline{F}_2(W) G_1(W) + \int_{\underline{W}_1}^{W} G_1(x)\, dF_2(x),$$
$$\int_{\underline{W}_2}^{W} [F_1(W) - F_1(x)]\, dG_2(x) = \int_{\underline{W}_2}^{W} G_2(x) dF_1(x),$$

we can rewrite this equation as

$$d_1(W)\frac{m_1}{u} G_1(W) = \lambda_{01} F_1(W) - \Phi(W), \tag{A.1}$$

where $d_1(W) = \lambda_{10} + \lambda_{11}\overline{F}_1(W) + \lambda_{12}\overline{F}_2(W)$, and

$$\Phi(W) = \lambda_{12} \int_{\underline{W}_1}^{W} \frac{m_1}{u} G_1(x)\, dF_2(x) - \lambda_{21} \int_{\underline{W}_2}^{W} \frac{m_2}{u} G_2(x)\, dF_1(x). \tag{A.2}$$

Turning to the informal sector, equation (5) indicates that for $W \in [\underline{W}_2, \overline{W}_2]$,

$$\left[\lambda_{20} + \lambda_{22}\overline{F}_2(W)\right] m_2 G_2(W) + \lambda_{21} m_2 \int_{\underline{W}_2}^{W} \overline{F}_1(x)\, dG_2(x) \\ = \lambda_{02} u F_2(W) + \lambda_{12} m_1 \int_{\underline{W}_1}^{W} [F_2(W) - F_2(x)]\, dG_1(x).$$

Using the same integrations by part, we obtain that

$$d_2(W)\frac{m_2}{u} G_2(W) = \lambda_{02} F_2(W) + \Phi(W), \tag{A.3}$$

where $d_2(W) = \lambda_{20} + \lambda_{21}\overline{F}_1(W) + \lambda_{22}\overline{F}_2(W)$.

Next, multiplying equation (A.1) by $\frac{\lambda_{12} f_2(W)}{d_1(W)}$ (with $f_2 = F_2'$) and equation (A.3) by $-\frac{\lambda_{21} f_1(W)}{d_2(W)}$, and adding the two resulting equations, we obtain the first-order differential equation

$$\Phi' = A - B\Phi, \tag{A.4}$$

where

$$A = \lambda_{01} F_1 \frac{\lambda_{12} f_2}{d_1} - \lambda_{02} F_2 \frac{\lambda_{21} f_1}{d_2},$$
$$B = \frac{\lambda_{12} f_2}{d_1} + \frac{\lambda_{21} f_1}{d_2},$$

with boundary condition $\Phi(U) = 0$ (in fact $\Phi(W) = 0, \forall W \leq \max\{\underline{W}_1, \underline{W}_2\}$).

The solution of differential equation (A.4) is given by

$$\Phi(W) = \frac{\int_U^W e^{\int_U^x B(x')dx'} A(x)\,dx}{e^{\int_U^W B(x)\,dx}}. \tag{A.5}$$

Substituting this solution back into equations (A.1) and (A.3) we obtain the equilibrium relationship between the distribution of offered (F) and accepted (G).

B Computing the Equilibrium

In this section we describe the computation of the equilibrium. We assume known the number of workers M, the potential number of firms N and the potential distribution of productivity Γ.

B.1 Exogenous Transition Rates

Assume that rates λ_{ij} are fixed.

1. Define contract value offer distribution F_1 and F_2, with supports bounds $\underline{W}_2 = U < \underline{W}_1 < \overline{W}_2 < \overline{W}_1$. Note that. from equation (3),
$$\underline{W}_2 = U = \frac{b + \lambda_{01}\mu_1 + \lambda_{02}\mu_2}{r + \lambda_{01} + \lambda_{02}}.$$
Define the shares of firms in each sector $n_i, i = 1,2$, with $n_1 + n_2 \leq 1$.

2. Use steady-state flow conditions in Appendix (A) to derive m_1, m_2, u and G_1, G_2 from F_1, F_2.

3. Profit maximization then implies that optimal decision rules satisfy
$$p = K_1^{-1}(W) = (1 + \tau + \lambda_{10}s)[w_1(W) + w_1'(W)\frac{\ell_1(W)}{\ell_1'(W)}],$$
$$p = K_2^{-1}(W) = w_2(W) + w_2'(W)\frac{\ell_2(W)}{\ell_2'(W)} + c\gamma\ell_2(W)^{\gamma-1},$$
with
$$\begin{aligned}
(1+\lambda_{10}s)w_1(W) &= (r+\lambda_{10})W - \lambda_{10}(U+UI) - \lambda_{11}\int_W^{\overline{W}_1}\overline{F}_1(x)\,dx - \lambda_{12}\int_W^{\overline{W}_2}\overline{F}_2(x)\,dx,\\
(1+\lambda_{10}s)w_1'(W) &= r + \lambda_{10} + \lambda_{11}\overline{F}_1(W_1(w)) + \lambda_{12}\overline{F}_2(W_1(w)),\\
\ell_i(W) &= \frac{M}{N}\frac{1}{n_i}\frac{h_i(W)}{d_i(W)},\ i = 1,2,\\
d_i(W) &= \lambda_{i0} + \lambda_{i1}\overline{F}_1(W) + \lambda_{i2}\overline{F}_2(W),\\
h_i(W) &= \lambda_{0i}u + \lambda_{1i}m_1G_1(W) + \lambda_{2i}m_2G_2(W),\\
G_i'(W) &= \frac{1}{m_i}\frac{h_i(W)}{d_i(W)}F_i'(W).
\end{aligned}$$

4. Then calculate productivity distributions
$$\Gamma_i(K_i^{-1}(W)) = n_iF_i(W),\ i = 1,2.$$

5. Consistency with the predetermined distribution of productivity Γ requires that

$$\Gamma(p) = \begin{cases} \Gamma(\underline{p}_2) + \Gamma_2(p), & \forall p \in [\underline{p}_2, \underline{p}_1], \\ \Gamma(\underline{p}_2) + \Gamma_1(p) + \Gamma_2(p), & \forall p \in [\underline{p}_1, \overline{p}_2], \\ \Gamma(\underline{p}_2) + n_2 + \Gamma_1(p), & \forall p \in [\overline{p}_2, \overline{p}_1], \end{cases}$$

   with $\Gamma(\underline{p}_2) + n_1 + n_2 = 1$.

6. If this consistency restriction is not satisfied, reiterate that sequence with another guess of F_1, F_2 and n_1, n_2.

In practice we discretize functions and approximate integrals as described in the estimation section, and we search for discrete approximations of F_1 and F_2, as well as shares n_1, n_1 so as to minimize a distance between Γ and its prediction. The dimensionality of the optimization problem can be reduced by using simple parametric approximations for F_1, F_2 such as the beta distribution used in the estimation section.

B.2 Endogenous Transition Rates

If the transition rates are endogenous then

$$\lambda_{i1} = \frac{n_1}{(n_1 + \alpha n_2)} s_i f(\xi); \quad \lambda_{i2} = \frac{\alpha n_2}{(n_1 + \alpha n_2)} s_i f(\xi); \quad \xi = \frac{n_1 + \alpha n_2}{u + s_1 m_1 + s_2 m_2}.$$

So step 2 cannot be calculated because tightness ξ is a function of m_1, m_2. Let $m_i(\xi)$ and $u(\xi)$ denote the shares of workers in formal/informal sectors and unemployed that can be calculated in step 2 given ξ. Then ξ is the fixed point

$$\xi = \frac{n_1 + \alpha n_2}{u(\xi) + s_1 m_1(\xi) + s_2 m_2(xi)}.$$

Solving for ξ must be embedded in step 2 if rates are endogenous.

C Estimation

Let F_1 and F_2 be two candidate offer distributions, defined on the spaces of contract present values. Although we could implement this procedure nonparametrically, we use non standard beta distributions as approximations:

$$F_i(x) = \text{betacdf}\left(\frac{x - \underline{W}_i}{\overline{W}_i - \underline{W}_i}; \alpha_i, \beta_i \right) \; i = 1, 2; \; \underline{W}_i \leq x \leq \overline{W}_i,$$

where $\text{betacdf}(\cdot; \alpha, \beta)$ is the CDF of a beta distribution with parameters α and β. An important practical reason why a (flexible) parametric specification is useful is that, in order to calculate the function Φ and other transition rates (see below) we need to calculate offer densities $f_1 = F_1'$ and $f_2 = F_2'$. Assuming a parametric specification guarantees the smoothness of both the distribution function and its derivative.

To estimate the parameters we use the method of moments. We match the distribution of wages for each sector and the transition rates implied by the model to those observed in the data. Given the above specification, we need to estimate the six arrival rates and the two job destruction rates all denoted by $\boldsymbol{\lambda} = (\lambda_{ij})_{i,j=0,1,2}$ and six further parameters $\boldsymbol{\theta} = (\underline{W}_1, \overline{W}_1, \underline{W}_2, \overline{W}_2, \alpha_1, \beta_1, \alpha_2, \beta_2)$ characterizing the offer distribution. Our algorithm estimates $\boldsymbol{\theta}$ given the $\boldsymbol{\lambda}$, then $\boldsymbol{\lambda}$ given $\boldsymbol{\theta}$, and iterates until convergence. Although we could estimate all parameters at the same time, this turned out to be a very quick procedure in practice.

C.1 Contract Offer Distributions

We start by taking the arrival rates $\boldsymbol{\lambda}$ as given to estimate $\boldsymbol{\theta}$ as follows. Let $z_k = \cos(k\pi/K), k = 0, ..., K$, be $K+1$ Chebychev nodes on $[-1,1]$. These nodes allow to define grids on $[\underline{W}_1, \overline{W}_1]$ and $[\underline{W}_2, \overline{W}_2]$ as

$$W_{ik} = \frac{\underline{W}_i + \overline{W}_i}{2} + \frac{\underline{W}_i - \overline{W}_i}{2} z_k, \; i = 1,2, \; k = 0, ..., K.$$

For each point on the grids, one can calculate a corresponding wage w_{ik} using equations (11) and (12), and replacing integrals by quadrature approximations. The appropriate quadrature for Chebychev nodes is the Clenshaw-Curtis (CC) quadrature, whose weights ω_k can be easily calculated using Fast Fourier Transform (FFT) (see Waldvogel, 2006). For example, we have

$$\begin{aligned}(1+\lambda_{10}s)w_{1k} = (r+\lambda_{10})W_{1k} - \lambda_{10}(\underline{W}_2 + UI) \\ - \lambda_{11}\frac{\underline{W}_1 - \overline{W}_1}{2}\sum_{\ell=0}^{K}\omega_\ell \mathbf{1}_{(W_{1\ell}>W_{1\ell})}\overline{F}_1(W_{1\ell}) \\ - \lambda_{12}\frac{\underline{W}_2 - \overline{W}_2}{2}\sum_{\ell=0}^{K}\omega_\ell \mathbf{1}_{(W_{2\ell}>W_{1\ell})}\overline{F}_2(W_{2\ell}),\end{aligned}$$

where $\mathbf{1}_{(\cdot)}$ is the indicator function. A similar expression can be obtained to determine wage nodes for the informal sector, w_{2k}.

Then we search for $\boldsymbol{\theta}$ minimizing

$$Q_1(\boldsymbol{\theta}|\boldsymbol{\lambda}) = \sum_{i=1,2}\sum_{k=0}^{K}\left(\widehat{G}_i^*(w_{ik}) - G_i(W_{ik})\right)^2,$$

where $G_i(W_{ik})$ is calculated using equation (6), and replacing integrals by CC-quadrature approximations, and $\widehat{G}_i^*$ is an estimate of wage distribution functions.

Note that, assuming that $U = \underline{W}_2 \leq \underline{W}_1$ and $\overline{W}_2 \leq \overline{W}_1$, we have

$$(1+\lambda_{10}s)\underline{w}_1 = (r+\lambda_{10})\underline{W}_1 - \lambda_{10}(\underline{W}_2 + UI) - \lambda_{11}(\mu_1 - \underline{W}_1) - \lambda_{12}\int_{\underline{W}_1}^{\overline{W}_2}\overline{F}_2(x)\,dx, \tag{C.1}$$

$$(1+\lambda_{10}s)\overline{w}_1 = (r+\lambda_{10})\overline{W}_1 - \lambda_{10}(\underline{W}_2 + UI), \tag{C.2}$$

$$\underline{w}_2 = r\underline{W}_2 - \lambda_{21}(\mu_1 - \underline{W}_2) - \lambda_{22}(\mu_2 - \underline{W}_2), \tag{C.3}$$

$$\overline{w}_2 = (r+\lambda_{20})\overline{W}_2 - \lambda_{20}\underline{W}_2 - \lambda_{21}\int_{\overline{W}_2}^{\overline{W}_1}\overline{F}_1(x)\,dx, \tag{C.4}$$

where $[\underline{w}_1, \overline{w}_1]$ and $[\underline{w}_2, \overline{w}_2]$ are the observed wage supports in the formal and informal sectors, respectively, and with

$$\mu_i = \underline{W}_i + (\overline{W}_i - \underline{W}_i)\frac{\alpha_i}{\alpha_i + \beta_i}, \; i = 1,2.$$

Hence, we can simplify the estimation problem slightly by using equations (C.2) and (C.3) to substitute observed wage bounds $\underline{w}_2$ and $\overline{w}_1$ for $\underline{W}_2 = U$ and $\overline{W}_1$ (given the α, β and $\underline{W}_1, \overline{W}_2$).

C.2 Transition Rates

In a similar way as we estimate $\boldsymbol{\theta}$ given $\boldsymbol{\lambda}$, we can estimate $\boldsymbol{\lambda}$ given $\boldsymbol{\theta}$. Natural counterparts to the theoretical transition rates can be calculated from observed flows between states (0: unemployment; 1: working in the formal sector; and 2: working in the informal sector).

From the labor force survey, we calculate the intensity of transitions from unemployment to job ($\widehat{D}_{0j}$; $j = 1,2$), from a formal sector job to unemployment, to another job in the same sector or to the informal sector ($\widehat{D}_{1j}$; $j = 0,1,2$) and similar ones for a workers initially in the informal sector ($\widehat{D}_{2j}; j = 0,1,2$). We then estimate our transition parameters using the method of moments. We choose the parameters so as to match the observed transition rates between sectors.

Consider first the workers who are unemployed at the date of the first interview, that we follow over T periods. Workers are not heterogeneous in this model and hence the remaining unemployment duration is exponentially distributed. Thus the implied proportion of those who move out of unemployment and into a job in sector j over the time period of observation T is

$$D_{0j} = \frac{\lambda_{0j}}{\lambda_{01}+\lambda_{02}}(1-e^{-(\lambda_{01}+\lambda_{02})T}),\ j = 1,2 \tag{C.5}$$

Now consider workers in the formal sector. Over T periods the proportion making a transition to an alternative job in the same sector, to a job in the informal sector or to unemployment is, respectively, for $i = 1,2$,

$$\begin{aligned} D_{ij} &= \int_{\underline{W}_i}^{\overline{W}_i} \frac{\lambda_{ij}\overline{F}_i(x)}{d_i(x)}(1-e^{-d_i(x)T})dG_i(x),\ j = 1,2, \\ D_{i0} &= \int_{\underline{W}_i}^{\overline{W}_i} \frac{\lambda_{i0}}{d_i(x)}(1-e^{-d_i(x)T})dG_i(x). \end{aligned} \tag{C.6}$$

where $d_i(W) = \lambda_{i0} + \lambda_{i1}\overline{F}_1(W) + \lambda_{i2}\overline{F}_2(W)$. Now, in equilibrium,

$$\frac{dG_i(x)}{dF_i(x)} = \frac{1}{m_i}\frac{h_i(x)}{d_i(x)}. \tag{C.7}$$

with $h_i(W) = \lambda_{0i}u + \lambda_{1i}m_1G_1(W) + \lambda_{2i}m_2G_2(W)$. This allows to replace the derivative of G_i by that of F_i inside the integral. This is useful as we have parameterized F_1, F_2 using a continuous distribution. Then CC-quadrature can be used to approximate the integral.

These are the model counterparts for these empirical moments as functions of the arrival rates, the job destruction rates, the offers distributions F_i and as a function of the equilibrium contract values distributions G_i ($i = 1,2$). Contract offers and equilibrium distributions are related by a complex function as explained in Appendix A.

We can thus estimate $\boldsymbol{\lambda}$ given $\boldsymbol{\theta}$ by minimizing the criterion

$$Q_2(\boldsymbol{\lambda}|\boldsymbol{\theta}) = \sum_{i,j=0,1,2}\left(\widehat{D}_{ij} - D_{ij}\right)^2,$$

where $\widehat{D}_{ij}$ is the empirical counterpart of D_{ij}.

We could minimize the two criteria Q_1 and Q_2 jointly but it is numerically faster to use a nested algorithm.

C.3 Productivity Distribution

Up to this point, there has been no need to use the firm profit functions, or indeed the distribution of productivities. To complete estimation we need to estimate the cost function of informality. This will allow us to characterize the choice of firms to locate in either sector and ultimately to carry out counterfactual simulations.

We specify the cost function as $C = c\ell_2(W)^\gamma$, with c and γ being the parameters to be estimated. Given values for c and γ, and $n_2 \leq 1$, we solve for the labor force size in the formal sector ($\ell_1(W) = \frac{M}{N}\frac{1}{n_1}\frac{h_1(W)}{d_1(W)}$) and in the informal sector ($\ell_2(W) = \frac{M}{N}\frac{1}{n_2}\frac{h_2(W)}{d_2(W)}$). From the firm's maximization problem in each sector, we next derive the support of the distribution of formal and informal productivities, i.e. $p_1 = K_1^{-1}(W)$ and $p_2 = K_2^{-1}(W)$ respectively. The first order conditions for the firm's optimization problem (see (9), (10)) gives

$$p_1 = K_1^{-1}(W) = (1+\tau+\lambda_{10}s)\left[w_1(W) + w_1'(W)\frac{\ell_1(W)}{\ell_1'(W)}\right], \tag{C.8}$$

$$p_2 = K_2^{-1}(W) = w_2(W) + w_2'(W)\frac{\ell_2(W)}{\ell_2'(W)} + c\gamma\ell_2(W)^{\gamma-1}, \tag{C.9}$$

where the expressions for $w_1'(W)$, $i = 1,2$, are given by

$$\begin{aligned} w_1'(W) &= \frac{r+\lambda_{10}+\lambda_{11}\overline{F}_1(W_1(w))+\lambda_{12}\overline{F}_2(W_1(w))}{1+\lambda_{10}s}, \\ w_2'(W) &= r+\lambda_{20}+\lambda_{21}\overline{F}_1(W_2(w))+\lambda_{22}\overline{F}_2(W_2(w)), \end{aligned}$$

and where firm sizes can be differentiated using

$$\frac{\ell_i}{\ell_i'} = \frac{h_i/d_i}{(h_i/d_i)'} = \frac{h_i d_i}{h_i' d_i - h_i d_i'}, \quad h_i' = \lambda_{0i}u + \lambda_{1i}\frac{h_1}{d_1}F_1' + \lambda_{2i}\frac{h_2}{d_2}F_2', \quad d_i' = \lambda_{i1}\overline{F}_1' + \lambda_{i2}\overline{F}_2'.$$

For each point of the contract grids, W_{ik}, one can thus calculate a point p_{ik} on a productivity grid, with $\underline{p}_2 = p_{20}$, $\underline{p}_1 = p_{10}$, $\overline{p}_2 = p_{2N}$ and $\overline{p}_1 = p_{1N}$:

$$p_{1k} = (1+\tau+\lambda_{10}s)\left[w_1(W_{1k}) + w_1'(W_{1k})\left.\frac{h_1^2}{h_1' d_1 - h_1 d_1'}\right|_{W=W_{1k}}\right],\ k = 0,...,K,$$

and

$$p_{2\ell} = w_2(W_{2\ell}) + w_2'(W_{2\ell})\left.\frac{h_2 d_2}{h_2' d_2 - h_2 d_2'}\right|_{W=W_{2\ell}} + \widetilde{c}\times\gamma\left(\frac{1}{\widetilde{n}_2}\frac{h_2(W_{2\ell})}{d_2(W_{2\ell})}\right)^{\gamma-1},\ \ell = 0,...,K,$$

for $\widetilde{n}_2 = \frac{n_2}{n_1+n_2}$ and $\widetilde{c} = c\left(\frac{M}{(n_1+n_2)N}\right)^{\gamma-1} = c\left(\frac{M}{N_1+N_2}\right)^{\gamma-1}$ (where N_1, N_2 are the total numbers of formal and informal firms operating in the economy at the equilibrium).

Moreover, we have

$$\begin{aligned} \pi_1(p_{1k}) &= (1-t)(1+\tau+\lambda_{10}s)w_1'(W_{1k})\frac{\ell_1(W_{1k})^2}{\ell_1'(W_{1k})} \\ &= \frac{M}{N}\frac{1}{n_1+n_2}\left[\frac{1}{\widetilde{n}_1}\times(1-t)(1+\tau+\lambda_{10}s)w_1'(W_{1k})\left.\frac{h_1^2}{h_1' d_1 - h_1 d_1'}\right|_{W=W_{1k}}\right], \end{aligned}$$

for $\widetilde{n}_1 = \frac{n_1}{n_1+n_2}$, and

$$\begin{aligned} \pi_2(p_{2\ell}) &= w_2'(W_{2\ell})\frac{\ell_2(W_{2\ell})^2}{\ell_2'(W_{2\ell})} + c(\gamma-1)\ell_2(W_{2\ell})^\gamma \\ &= \frac{M}{N_1+N_2}\left[\frac{1}{\widetilde{n}_2}\times w_2'(W_{2\ell})\left.\frac{h_2^2}{h_2' d_2 - h_2 d_2'}\right|_{W=W_{2\ell}} + \widetilde{c}\times(\gamma-1)\left(\frac{1}{\widetilde{n}_2}\frac{h_2(W_{2\ell})}{d_2(W_{2\ell})}\right)^\gamma\right]. \end{aligned}$$

Note that both profit functions are proportional to $\frac{M}{N_1+N_2}$, which is not necessarily known at this stage if the number of informal firms in the economy is not directly observed.

Equilibrium conditions require that $\pi_2(\underline{p}_2) = 0$, and $\pi_1(p) = \pi_2(p) > 0$ for $p \in [\underline{p}_1, \overline{p}_2]$. We thus estimate $\widetilde{c}$ and γ, and $\widetilde{n}_1 \geq 0$ and $\widetilde{n}_2 \geq 0$ such that $\widetilde{n}_1 + \widetilde{n}_2 = 1$, so as to minimize

$$\pi_2(p_{20})^2 + \sum_{k,\ell=0}^{K} \mathscr{K}(p_{1k} - p_{2\ell})[\pi_1(p_{1k}) - \pi_2(p_{2\ell})]^2,$$

where $\mathscr{K}$ is a kernel density. Notice that a direct estimate of $\widetilde{n}_1, \widetilde{n}_2$ immediately follows if estimates of the numbers of formal and informal firms (N_1 and N_2) are available:

$$\widetilde{n}_i = \frac{N_i}{N_1 + N_2}.$$

Then define

$$\widetilde{\Gamma}_i(p_{ik}) = \widetilde{n}_i F_i(W_{ik}),\ k = 0, ..., K,$$

and interpolate for all $p \in [p_{i0}, p_{iK}]$, and set $\widetilde{\Gamma}(p) = 0$ for $p < p_{i0}$ and $\widetilde{\Gamma}(p) = \widetilde{n}_i$ for $p > p_{iK}$. The unconditional productivity distribution Γ satisfies

$$\Gamma(p) = \begin{cases} \Gamma(p_{20}) + (n_1 + n_2)\widetilde{\Gamma}_2(p), & \forall p \in [\underline{p}_2, \underline{p}_1], \\ \Gamma(p_{20}) + (n_1 + n_2)\left[\widetilde{\Gamma}_1(p) + \widetilde{\Gamma}_2(p)\right], & \forall p \in [\underline{p}_1, \overline{p}_2], \\ \Gamma(p_{20}) + (n_1 + n_2)\left[\widetilde{n}_2 + \widetilde{\Gamma}_1(p)\right], & \forall p \in [\overline{p}_2, \overline{p}_1]. \end{cases}$$

By postulating a parametric form for Γ, say a log-normal with parameters μ and σ^2, one can then estimate $n_1 + n_2, \mu, \sigma^2$ by maximizing the fit on a grid for $[p_{20}, p_{1K}]$.

Finally, $\frac{M}{N_1+N_2}$ can be estimated by fitting the average firm size in the informal sector. The cost parameter c is then $c = \widetilde{c}\left(\frac{M}{N_1+N_2}\right)^{1-\gamma}$.