Income and Top Income Mobility

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Motivation
Income mobility

Long been claimed that the US economy generates

- Much income inequality in a given year
- In exchange for greater mobility and thus less permanent inequality

Two pieces of evidence suggest otherwise

1. Americans enjoy no more income mobility than their European peers

(see e.g. Burkhauser and Puopure, 1997; Aaberge et al., 2002; Chen, 2009)

2. Rise in cross-sectional inequality in the US is associated with a decline in income mobility (Koupczuk et al., 2007)

Top income mobility

- Those with the highest income might also have variable

  incomes (capital income, tax adjustments, etc)
- Could it be that some of the high top income shares are

  simply due to high gains being realized in any one year?
- The concentration of “power” argument depends on whether

  the same people are top income holders year after year
- A measure of top income mobility is needed to address these issues

Previous evidence on income mobility

	$G(Z)$	$G(Z_R)$	$\frac{G(Z_R) - G(Z)}{G(Z_R)}$
Denmark, 80-90	0.220	0.239	0.080
Norway, 80-90	0.256	0.275	0.069
Sweden, 80-90	0.234	0.252	0.073
US, 80-90	0.378	0.404	0.065
Germany, 83-88	0.224	0.240	0.065
US, 83-88	0.323	0.340	0.048
US, 1960	0.272	0.291	0.065
US, 1980	0.301	0.329	0.058
US, 2001	0.426	0.440	0.032

Intepreting the evidence

Following Shorrocks (1978), these studies define

- Mobility as the <u>share</u> of cross-sectional inequality that is transitory

Implications:

1. Mobility is not necessarily higher in a society where changes in relative incomes occur more frequently or are of greater magnitude

2. High mobility may not necessarily equalize permanent income and raise social welfare more than low mobility

Contrasts to the conventional view of income mobility:

> *"If income mobility were very high, the degree of inequality in any given year would be unimportant, because the distribution of lifetime income would be very even" (Krugman, 1992)*

This talk is based on results from

- Aaberge, R. And M. Mogstad (2015): Income Mobility as an Equalizer of Permanent Income, Mimeo.
- Aaberge, R., T. Atkinson and J. Modalsli (2015): “The Ins and Outs of Top Income Mobility”, Mimeo.

These papers

Proposes and applies a general framework for comparison of income distributions according to income and top income mobility

1. Introduces mobility curves representing the notions of income and top income mobility as equalizers of permanent income

    - Similar role as the Lorenz curve in analysis of income inequality

2. Introduces dominance criteria for partial rankings
    - 1 order dominance
    - 2nd order upward and downward dominance

    - Transfer principles provide normative justification

These papers (cont)

3. Derives a general family of rank-dependent income and top income mobility measures, summarizing the income mobility and top income mobility curves
   - Intuitive social welfare interpretation

4. Characterizes relationship between two subfamilies and 2nd order upward and downward dominance
   - Differ in sensitivity to mobility in lower vs. upper part of distribution

5. Applies the framework to re-examine recent conclusions about income mobility over time and across countries, and to describe the development of top income mobility in Norway from 1967 to 2011

Outline

1. MOBILITY CURVE
2. PARTIAL RANKING
3. COMPLETE RANKING
4. RELATION TO TRADITIONAL MOBILITY MEASURES
5. EMPIRICAL ANALYSIS

Mobility curve

Let L_Z and L_{Z_R} denote the Lorenz curves for the distribution F_Z of the observed permanent income *Z* and the distribution F_{Z_R} of the hypothetical reference permanent income when there is no mobility

$$M(u) = L_Z(u) - L_{Z_R}(u)$$

- Equality in permanent income may be due to:
  - Equality in the cross-sectional income distributions
  - Changes in relative incomes over time, i.e. income mobility

Mobility curve: Annuity

The annuity income is defined by

$$Z = \frac{Y_T + \sum_{t=1}^{T-1} Y_t \prod_{j=1+t}^{T} (1+r_j)}{1 + \sum_{t=1}^{T-1} \prod_{j=1+t}^{T} (1+r_j)},$$

which yields

$$M(u) = L_Z(u) - \sum_{t=1}^{T} \frac{\mu_t}{\mu_Z} b_t L_t(u)$$

where

$$b_t = \frac{\prod_{j=t+1}^{T} (1+r_j)}{1 + \sum_{s=1}^{T-1} \prod_{j=s+1}^{T} (1+r_j)}, \quad t = 1, 2, \ldots, T-1,$$

and $\mu_t = EY_t$ and $\mu_Z = \sum_{t=1}^{T} b_t \mu_t$

Figure 1. Lorenz curves in the distributions of observed and reference annuity income

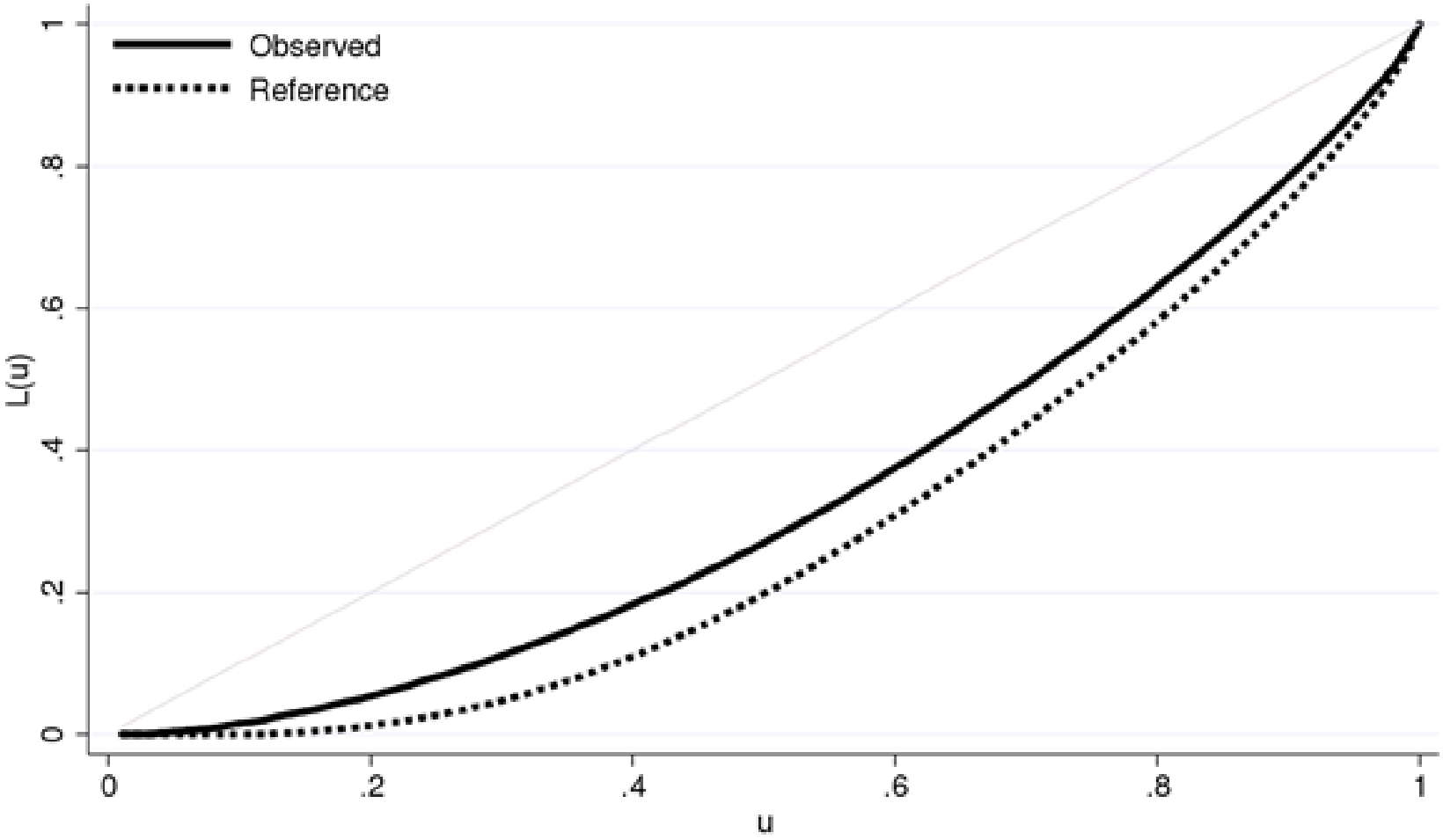
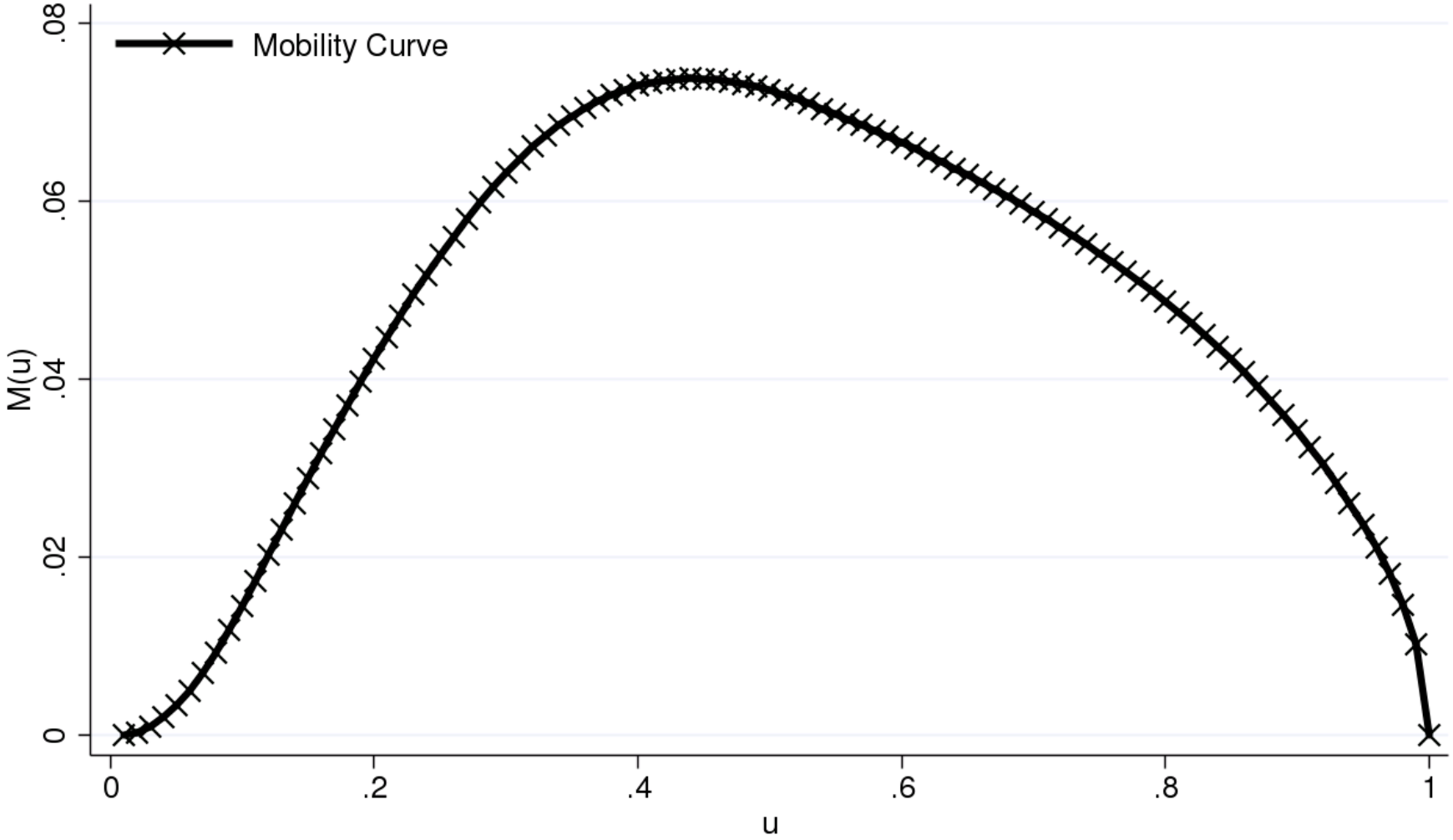


Figure 2. Mobility curve from the distributions of observed and reference annuity income



The derivative of the Mobility curve

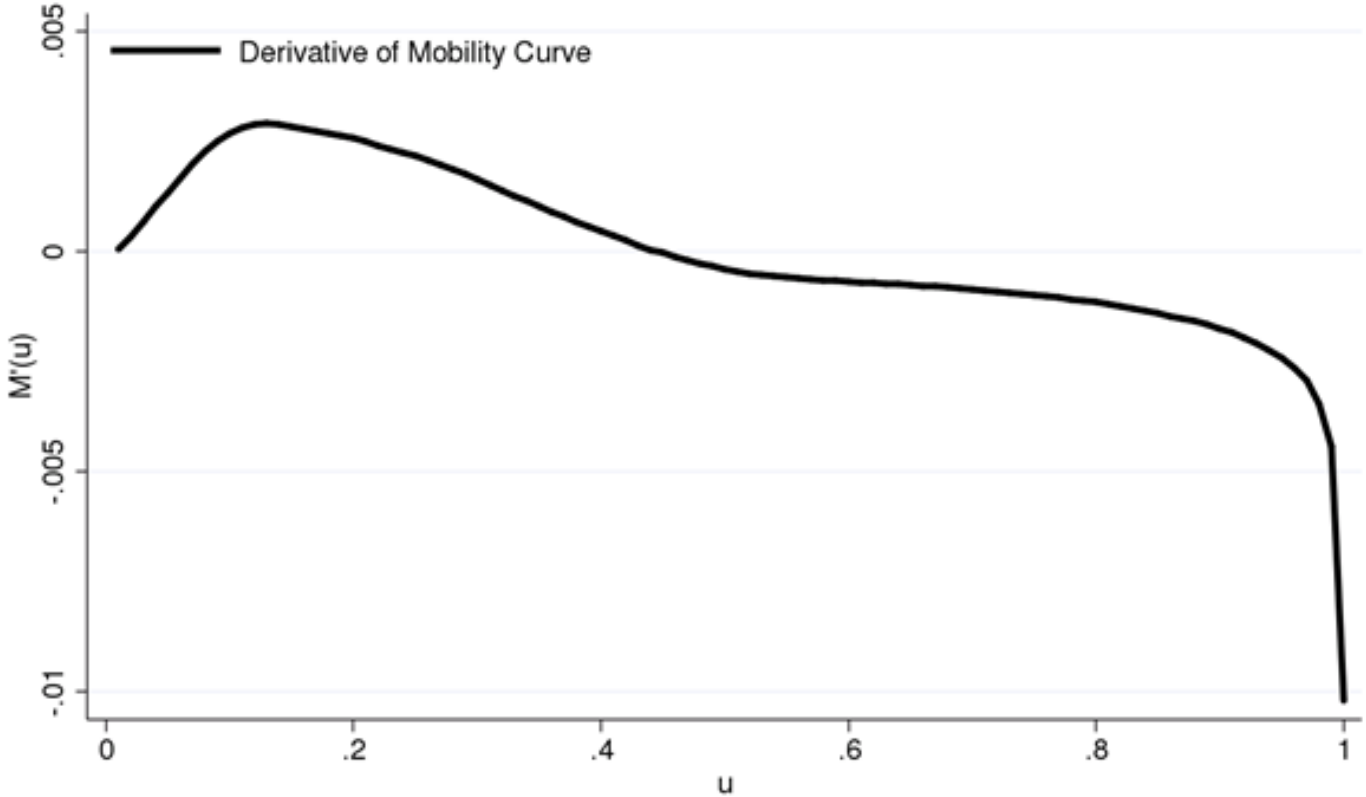
The derivative of M is given by

$$M'(u) = \frac{F_Z^{-1}(u)}{\mu_Z} - \frac{F_{Z_R}^{-1}(u)}{\mu_{Z_R}}, \; u \in [0,1].$$

Individuals for which $M'(u)$ is positive (negative) become better (worse) off because of income mobility:

Their shares of total income are higher (lower) than what they would have been in the absence of changes in relative incomes over time.

Figure 3. Derivative of the mobility curve



The poorest 44 percent of the population benefits from income mobility, at the cost of the richest 56 percent.

The gain peak at the 13th percentile where mobility increases the share of total income by 0.29 percentage points (from .07 percent with Z_R to 0.36 percent with Z).

1st degree dominance and transfer principle

Definition 2.1. *A mobility curve M_1 is said to* ***first-degree dominate*** *a mobility curve M_2 if*

$$M_1(u) \geq M_2(u) \quad \text{for all} \quad u \in [0,1]$$

and the inequality holds strictly for some $u \in \langle 0,1 \rangle$.

Definition 2.2. A Pigou-Dalton permanent income transfer *is a transfer in the permanent income distribution F from a person of rank t with income* $F^{-1}(t)$ *to a person of rank s with income* $F^{-1}(s)$, *where* $0 < s < t \leq 1$, *such that the period-specific income distributions are kept unchanged.*

Equivalence result

Theorem 2.1. *Let* M_1 and M_2 *be members of* ***M****. Then the following statements are equivalent,*

(i) $$M_1(u) \geq M_2(u) \quad \textit{for all} \quad u \in [0,1]$$

(ii) M_1 *can be attained from* M_2 *by Pigou-Dalton permanent income transfers*

Figure 4. Mobility curves for individuals born in urban and rural municipalities

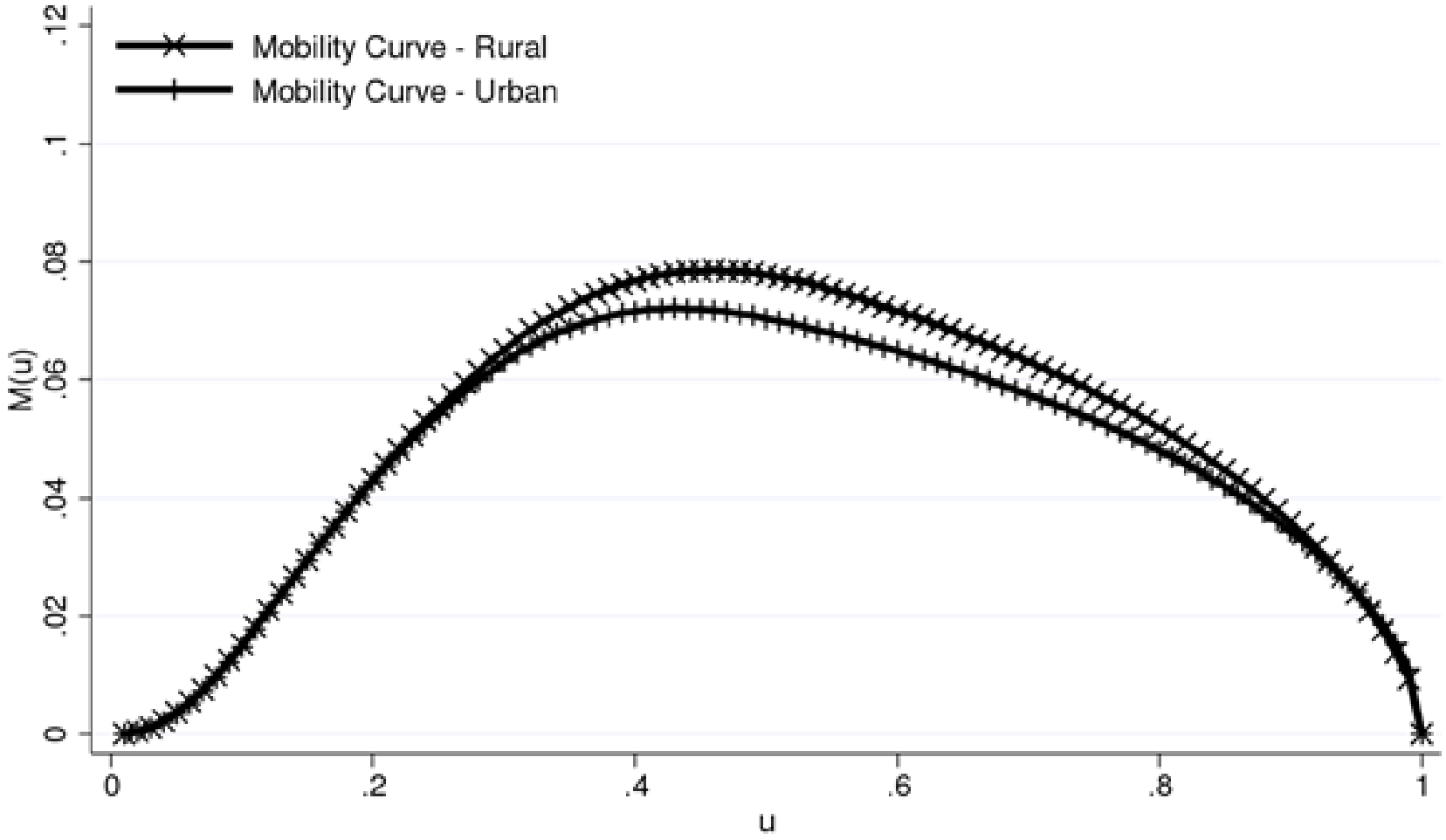
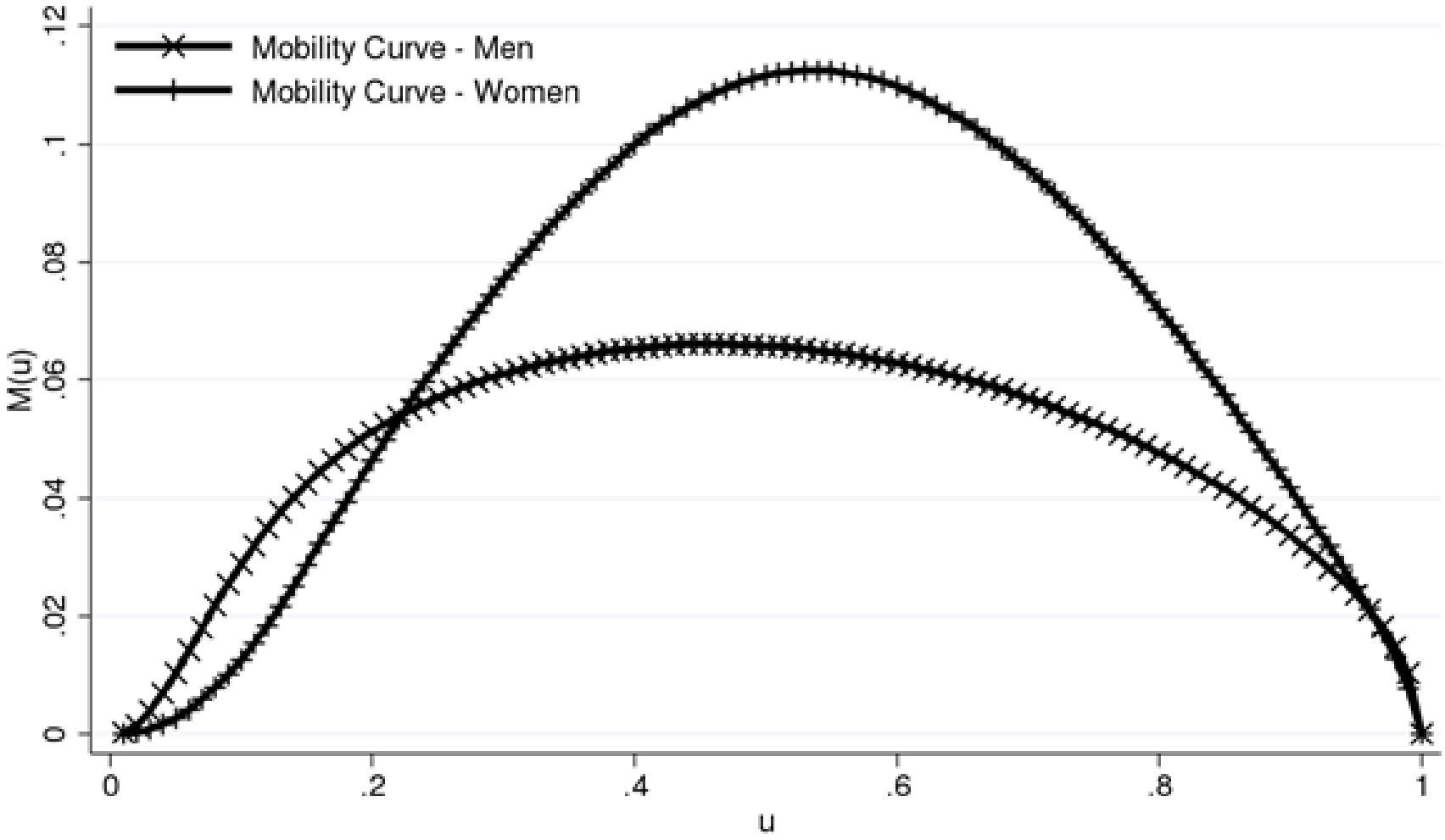


Figure 5. Mobility curves for men and women



Second degree dominance

Definition 2.3A. *A mobility curve M_1 is said to* ***second-degree upward dominate*** *a mobility curve M_2 if*

$$\int_0^u M_1(\mathrm{t})\,dt \geq \int_0^u M_2(\mathrm{t})\,dt \quad \textit{for all} \;\; u \in [0,1]$$

and the inequality holds strictly for some $u \in \langle 0,1 \rangle$.

Definition 2.3B. *A mobility curve M_1 is said to* ***second-degree downward dominate*** *a mobility curve M_2 if*

$$\int_u^1 M_1(\mathrm{t})\,dt \geq \int_u^1 M_2(\mathrm{t})\,dt \quad \textit{for all} \;\; u \in [0,1]$$

and the inequality holds strictly for some $u \in \langle 0,1 \rangle$.

Figure 5. Mobility curves for men and women

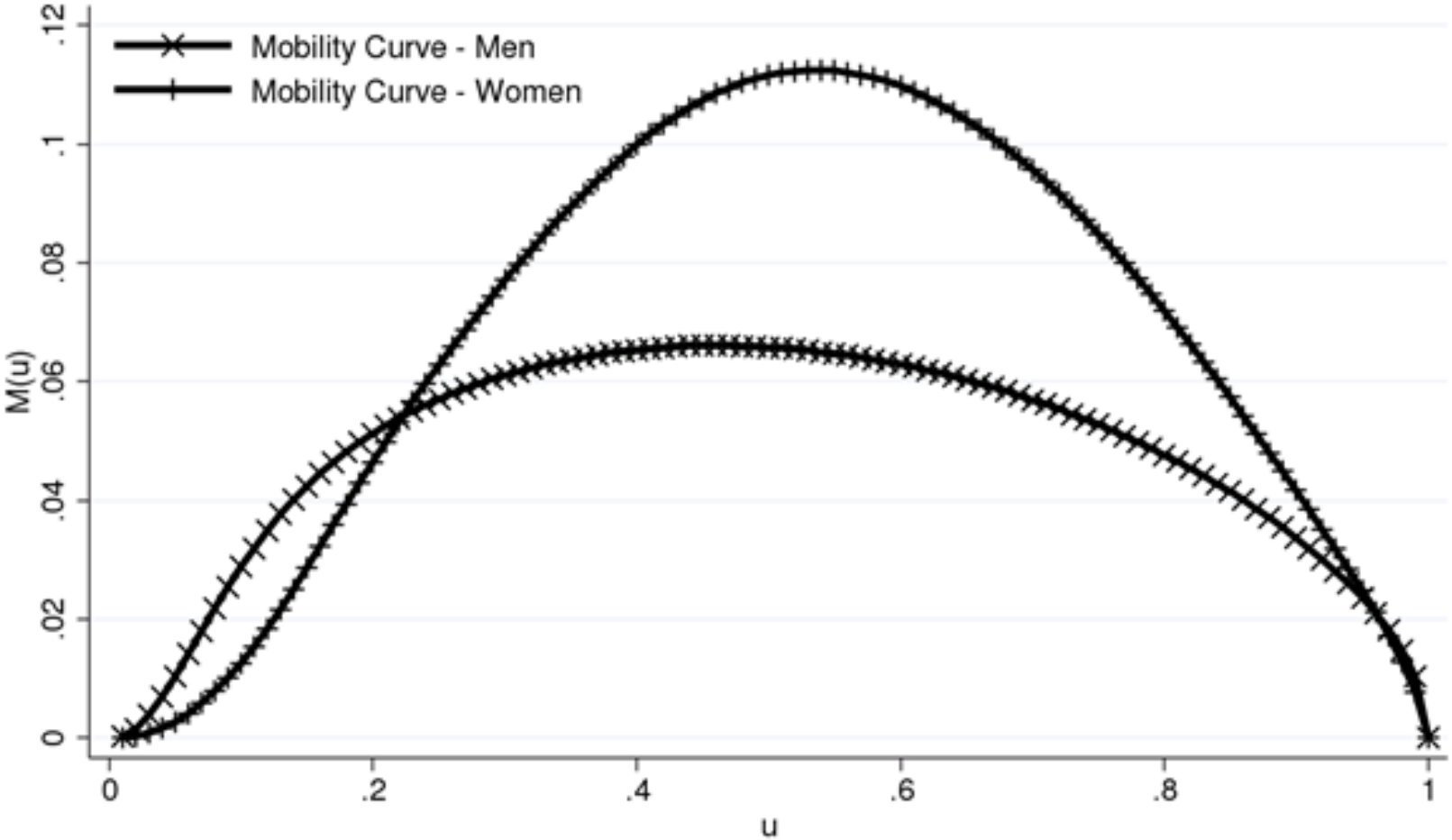
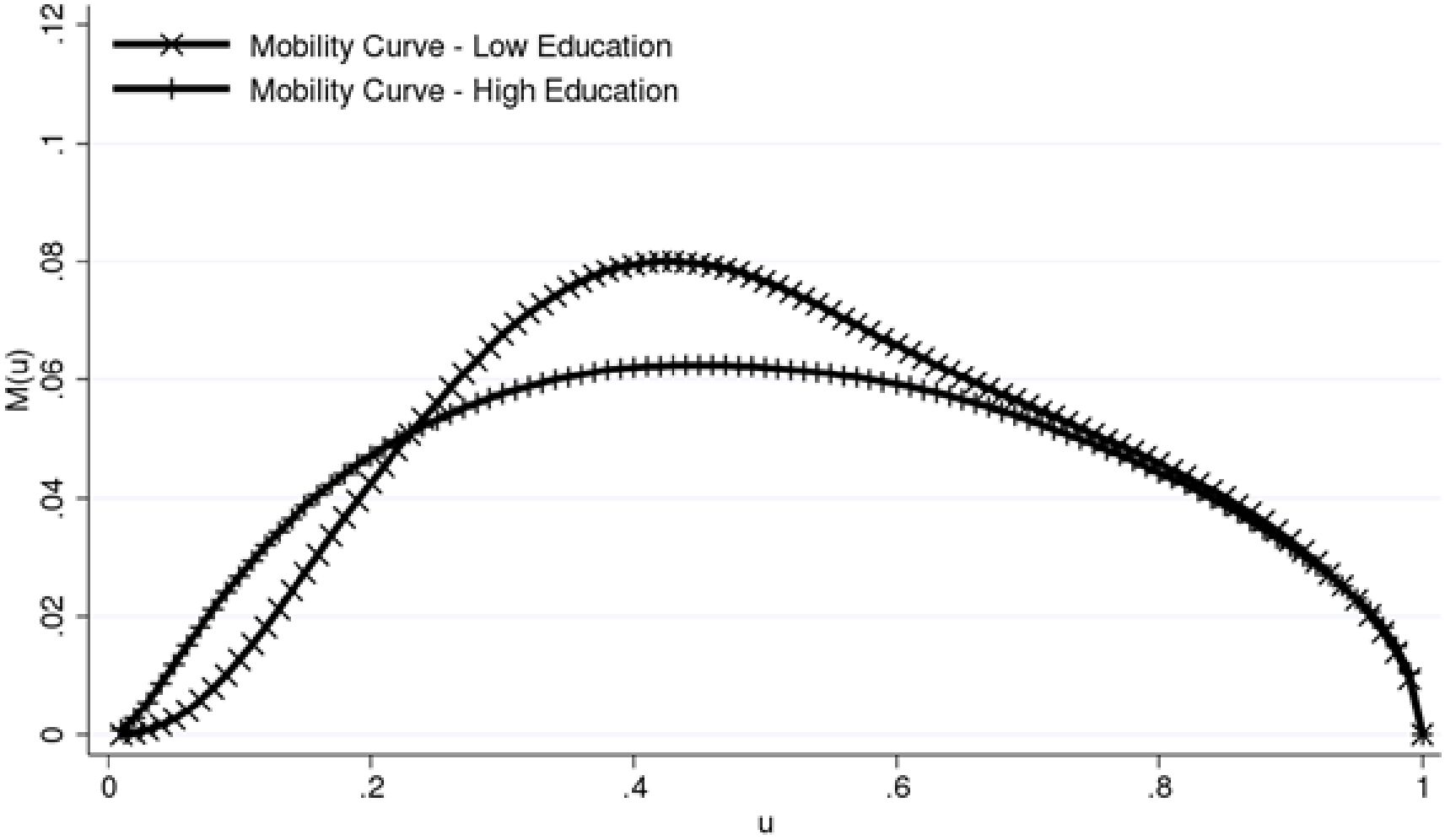


Figure 6. Mobility curves for individuals with low and high education



Summary measures of mobility

Let $\succeq$ be an ordering defined on the family ***M*** of mobility curves.

Assume that $M_1 \succeq M_2$ if $M_1(u) \geq M_2(u)$ *for all* $u \in [0,1]$

By further assuming that $\succeq$ is continuous, transitive, complete, and satisfies the following axiom

Independence condition: *Let M_1, M_2 and M_3 be members of **M** and let* $\alpha \in [0,1]$. *Then* $M_1 \succeq M_2$ *implies* $\alpha M_1 + (1-\alpha)M_3 \succeq \alpha M_2 + (1-\alpha)M_3$

then $\succeq$ can be represented by the following family of mobility measures

$$\Lambda_p(M) = \int_0^1 p(u)dM(u),$$

where the weighting function *p* is a positive and non-increasing.

Summary measures of mobility (cont)

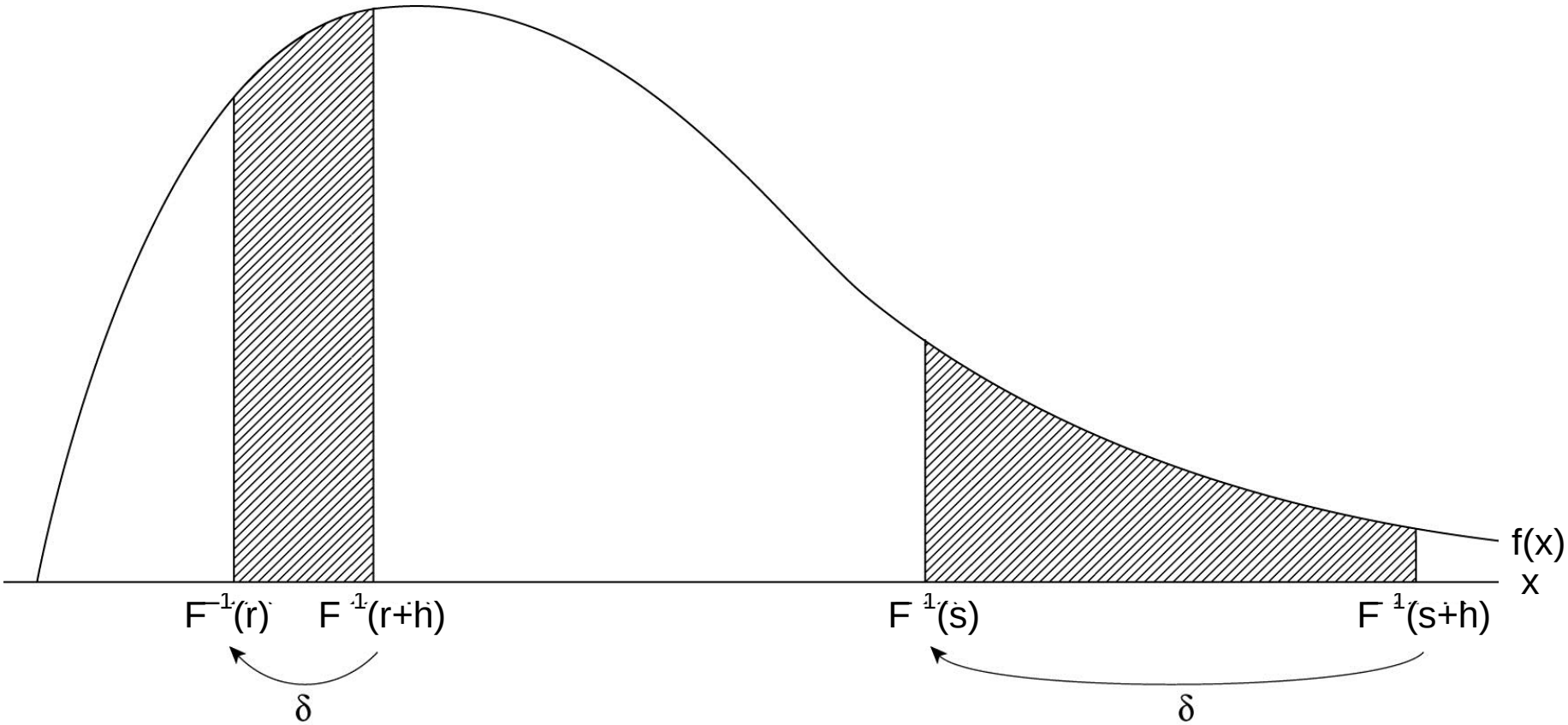
Inserting for $M(u) = L_Z(u) - L_{Z_R}(u)$ in Λ_p yields

$$\Lambda_p(M) = \int_0^1 p(u)dL_Z(u) - \int_0^1 p(u)dL_{Z_R}(u) = J_p(L_{Z_R}) - J_p(L_Z),$$

where the inequality measure $J_p(L)$ for the Lorenz curve L of distribution F with mean μ is defined by

$$J_p(L) = 1 - \int_0^1 p(u)d\,L(u) = 1 - \frac{1}{\mu}\int_0^1 p(u)F^{-1}(u)du\,.$$

Transfer principles



Equivalence results

THEOREM 2.2A. *Let M_1 and M_2 be members of* ***M****. Then the following statements are equivalent,*

(i) M_1 *second-degree upward dominates* M_2

(ii) $\Lambda_p(M_1) > \Lambda_p(M_2)$ for all non-increasing convex p *such that* $p'(1) = 0$

(iii) $\Lambda_p(M_1) > \Lambda_p(M_2)$ *for all p being such that* Λ_p *obeys the principle of DPTS*

THEOREM 2.2B. *Let M_1 and M_2 be members of* ***M****. Then the following statements are equivalent,*

(i) M_1 *second-degree downward dominates* M_2

(ii) $\Lambda_p(M_1) > \Lambda_p(M_2)$ *for all non-increasing concave p such that* $p'(0) = 0$

(iii) $\Lambda_p(M_1) > \Lambda_p(M_2)$ *for all p being such that* Λ_p *obeys the principle of UPTS*

Parametric subfamilies

Mobility measures associated with upward dominance

$$\Lambda_p(M) = \Lambda_{1,k}(M) \equiv (k+1)\int_0^1 (1-u)^k \, d\left(L_Z(u) - L_{Z_R}(u)\right) = G_k(L_{Z_R}) - G_k(L_Z), k \geq 1$$

where $G_k(L) = 1-(k+1)\int_0^1 (1-u)^k \, dL(u)$ is equal to the extended Gini family of inequality measures introduced by Donaldon and Weymark (1980)

Mobility measures associated with downward dominance

$$\Lambda_p(M) = \Lambda_{2,k}(M) \equiv (k+1)\int_0^1 \left(1-u^k\right) d\left(L_Z(u) - L_{Z_R}(u)\right) = D_k(L_{Z_R}) - D_k(L_Z), k \geq 1$$

where $D_k(L) = 1-(k+1)\int_0^1 \left(1-u^k\right) dL(u), k \geq 1$ is equal to the Lorenz family of inequality measures introduced by Aaberge (2000).

Limits

For *k=1* we find that both weighting functions form the following mobility measure,

$$\Lambda_p(M) = \Lambda_G(M) \equiv \int_0^1 (1-u)\, d\left(L_Z(u) - L_{Z_R}(u)\right) = G(L_{Z_R}) - G(L_Z),$$

where G is the Gini coefficients.

When $k \to \infty$ we get that

$$\Lambda_{1,k}(M) \to \frac{F_Z^{-1}(0+)}{\mu_Z} - \frac{F_{Z_R}^{-1}(0+)}{\mu_{Z_R}}$$

and

$$\Lambda_{1,k}(M) \to 0$$

Social welfare interpretation

Yaari (1988) introduced the so-called dual family of social welfare functions:

$$W_p(F) = \int_0^1 p(u) F^{-1}(u) du$$

$W_p(F)$ admits the following decomposition: $W_p(F) = \mu\left(1 - J_p(L)\right),$

where *F* is an income distribution with mean μ and Lorenz curve *L*.

The welfare produced by the permanent income distribution:

$$W_p\left(F_Z\right) = W_p\left(F_{Z_R}\right) + \mu_Z \Lambda_p(M).$$

Inequality, mobility and social welfare

	$G(Z_R)$	$G(Z_R) - G(Z)$	$W(Z) - W(Z_R)$	% Effect
Males	0.312	0.096	27 056	12.2
Females	0.457	0.135	18 331	19.8
Rural	0.412	0.101	19 733	14.7
Urban	0.417	0.095	20 779	14.0
Low Educ	0.431	0.097	18 055	14.6
High Educ	0.334	0.091	25 212	12.0
All	0.417	0.096	20 118	14.2

Traditional mobility measures

Following Shorrocks (1978), income mobility measures can be derived from a factor decomposition of inequality measures and be written as

$$\tilde{\Lambda}_p(L_Z) = \frac{J_p(L_{Z_R}) - J_p(L_Z)}{J_p(L_{Z_R})} = \frac{W_p(F_Z) - W_p(F_{Z_R})}{\mu_Z - W_p(F_{Z_R})},$$

when the rank-dependent family of inequality measures form the basis for the measurement of inequality.

$$\begin{aligned} W_p(F_Z) &= W_p(F_{Z_R}) + \tilde{\Lambda}_p(L_Z)(\mu_Z - W_p(F_{Z_R})) \\ &= W_p(F_{Z_R}) + \mu_Z \tilde{\Lambda}_p(L_Z) J_p(F_{Z_R}) \end{aligned}$$

- Do NOT separate changes in the cross-sectional distributions from changes in relative incomes over time

Comparison of mobility estimates

	$G(Z_R)$	$G(Z_R) - G(Z)$	$\frac{G(Z_R) - G(Z)}{G(Z_R)}$
Males	0.312	0.096	0.308
Females	0.457	0.135	0.294
Rural	0.412	0.101	0.246
Urban	0.417	0.095	0.227
Low Ed	0.431	0.097	0.225
High Ed	0.334	0.091	0.270
All	0.417	0.096	0.230

	$G(Z_R)$	$G(Z_R) - G(Z)$	$\frac{G(Z_R) - G(Z)}{G(Z_R)}$
Denmark, 80-90	0.239	0.019	0.080
Norway, 80-90	0.275	0.019	0.069
Sweden, 80-90	0.252	0.018	0.073
US, 80-90	0.404	0.026	0.065
Germany, 83-88	0.240	0.015	0.065
US, 83-88	0.340	0.016	0.048
US, 60-64	0.291	0.019	0.065
US, 80-84	0.329	0.019	0.058
US, 01-05	0.440	0.014	0.032

Top income mobility

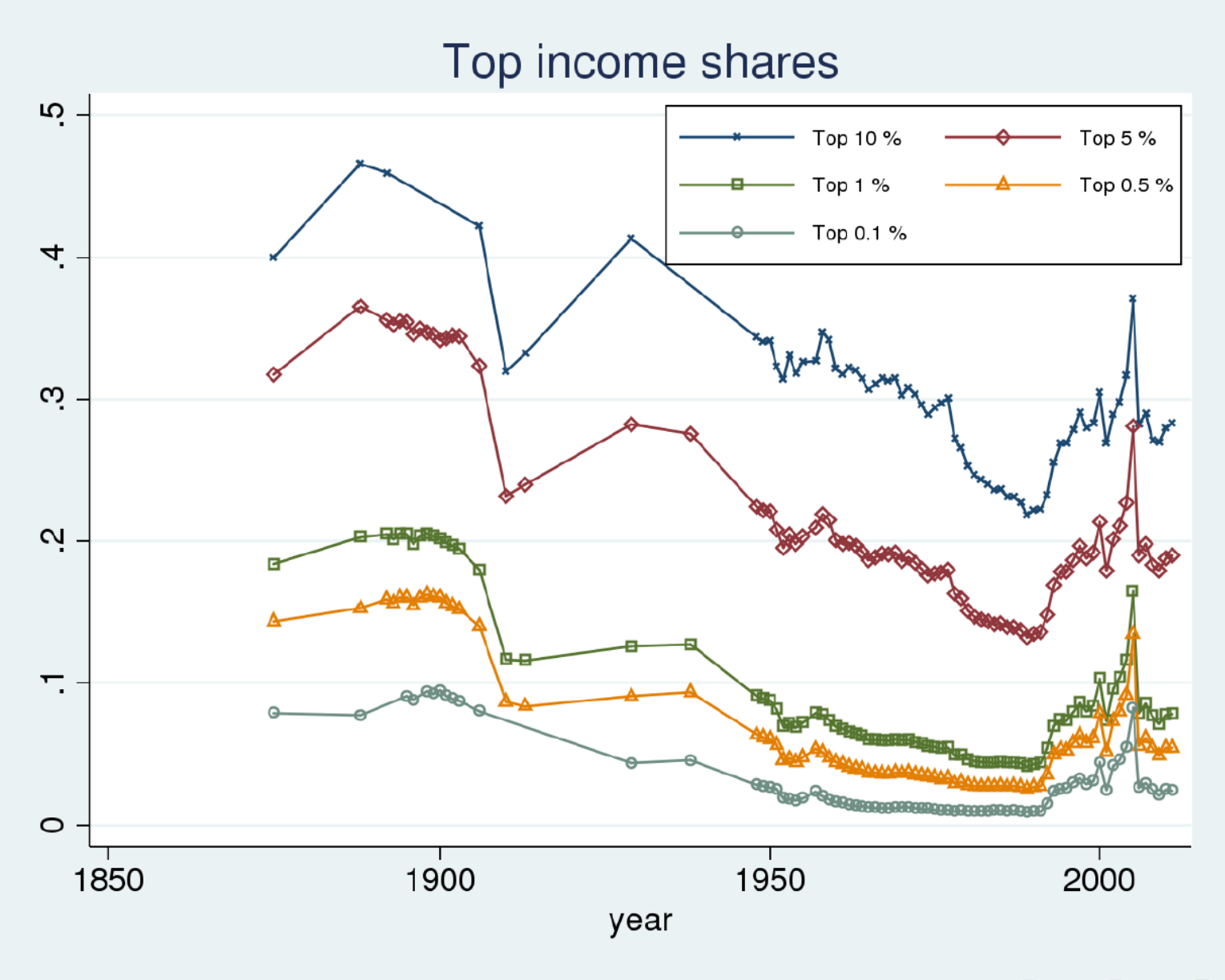
Outline

1. TOP INCOME MOBILITY CURVE
2. SUMMARY MEASURES OF TOP INCOME MOBILITY
3. EMPIRICAL RESULTS

Literature on top income mobility

- Studies on top income mobility typically use the "matrix approach":
  - Auten et al (2013) on top 1 per cent in USA: moderately persistent; little change in mobility 1991-2009
  - Björklund et al (2012): Intergenerational mobility in Sweden: Find strong persistence, but do not examine changes over time
- Matrices do not reveal the magnitude of income changes underlying mobility
- Our approach accounts for the income changes. Previous examples:
  - Shorrocks (1978): Mobility is when inequality falls if the accounting period is extended
  - Aaberge et al (2002): "No mobility" is constant *rankings*, not constant *relative income*
  - Aaberge and Mogstad (2013): Formalized as an (absolute) reduction in inequality
- Some studies use multi-year averages to study top incomes, but no benchmarks are provided

Top income shares in Norway, 1875-2011



Conceptual framework

- Key concept: comparing the cross-section income distribution to a distribution of permanent income
- Denote income in period t as X_t, with mean μ_t and Lorenz curve L_t
- For r years, define permanent income X as

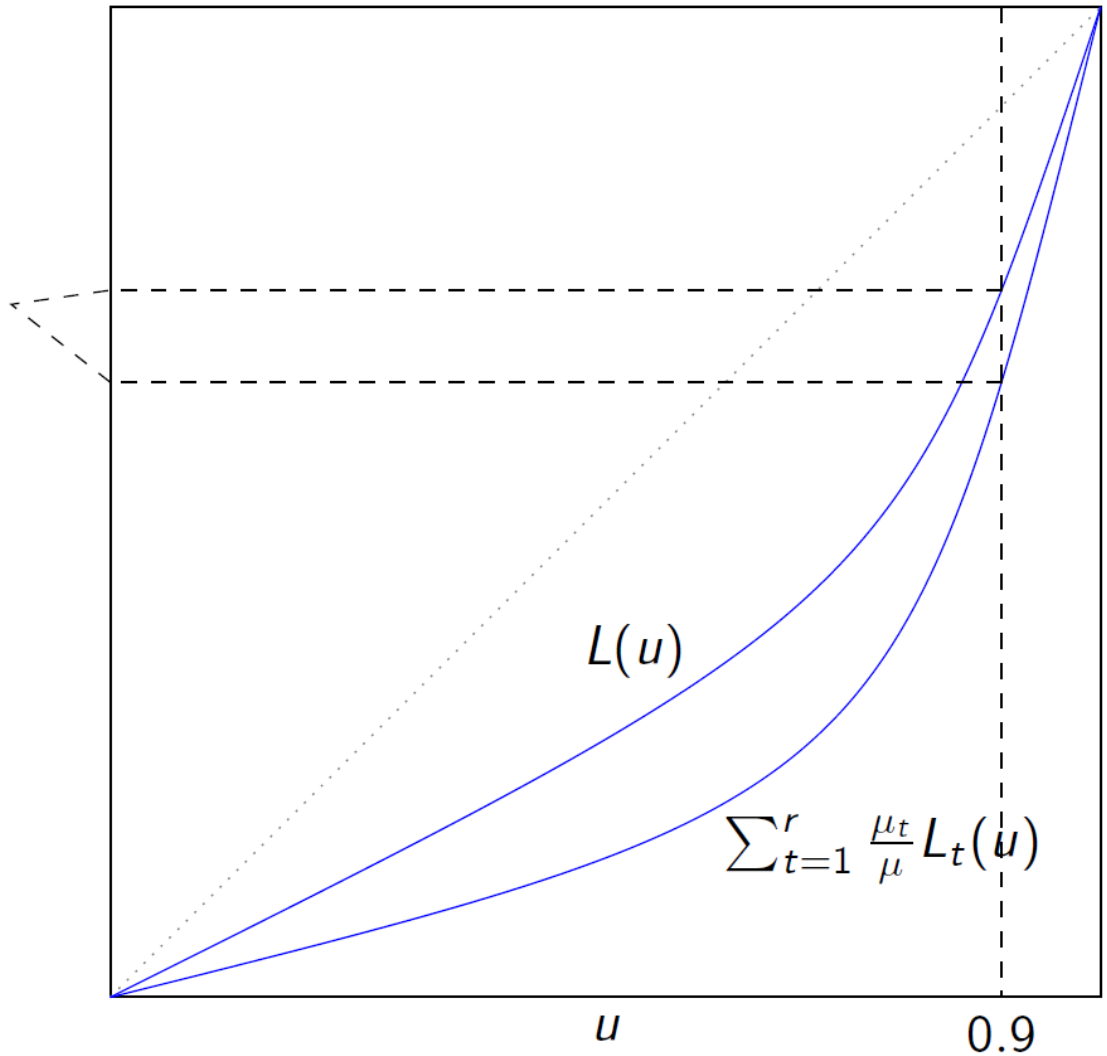
$$X = \sum_{t=1}^{r} X_t$$

- with corresponding mean μ and Lorenz curve L
- For comparison, use the *sum of the cross-section income distributions* for the same r years

Explanation: Top income and mobility

Top income mobility
at 0.9

$T(0.9)$



Definition of top income mobility

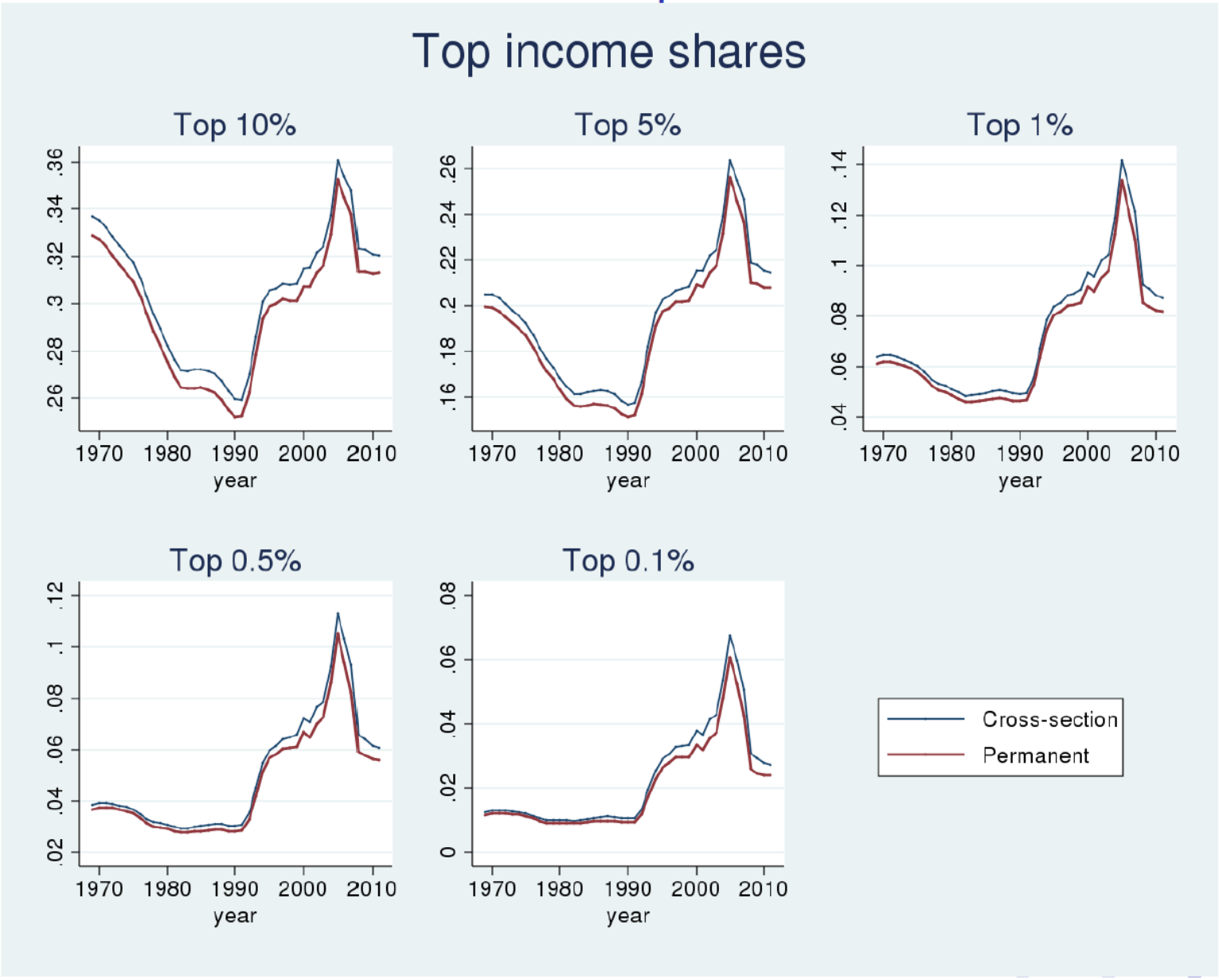
$$T(u) = \sum_{t=1}^{r} \frac{\mu_t}{\mu} (1 - L_t(u)) - (1 - L(u))$$
$$= \sum_{t=1}^{r} \frac{\mu_t}{\mu} (L(u) - L_t(u))$$

- Like top incomes, can be examined at specific points, for example $u = 0.9$ for top 10%
- We will follow the convention of examining the distribution at the top 10%, 5%, 1%, 0.5% and 0.1%

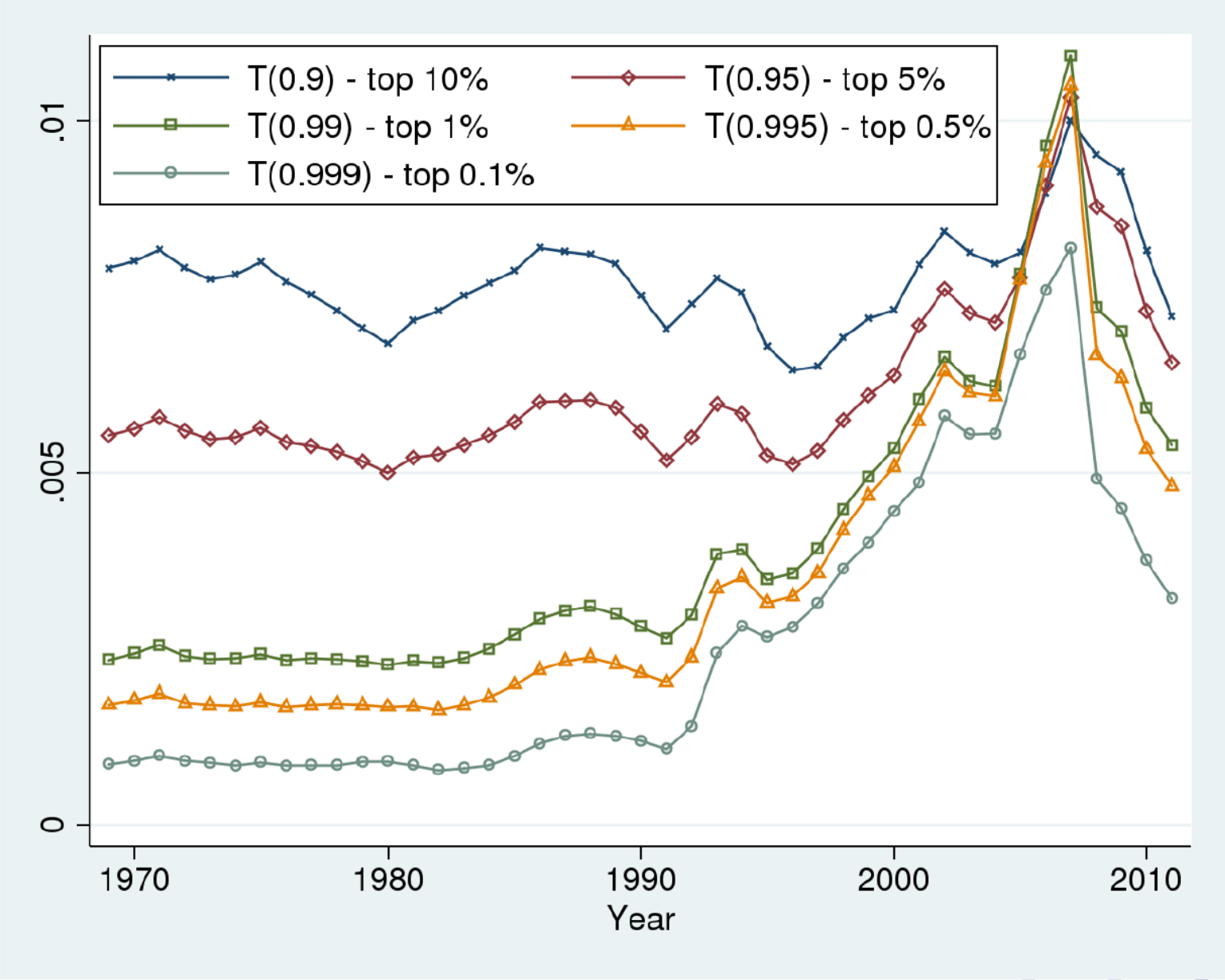
Application: Norway 1967-2011

- Income data for the entire Norwegian working population
- Administrative data from tax authorities, linked to population register
- Income definition: Ordinary income ("Alminnelig inntekt"): income less basic deduction
- Permanent income duration: 3 years ($r = 3$)
- Population used here: All adults resident in Norway for rolling 3-year windows
- Notation: "1969" on axis means incomes for 1967, 1968 and 1969
- Sample size 1969: 2.6 million individuals; 2011: 3.7 million

Permanent and cross-section top incomes over time



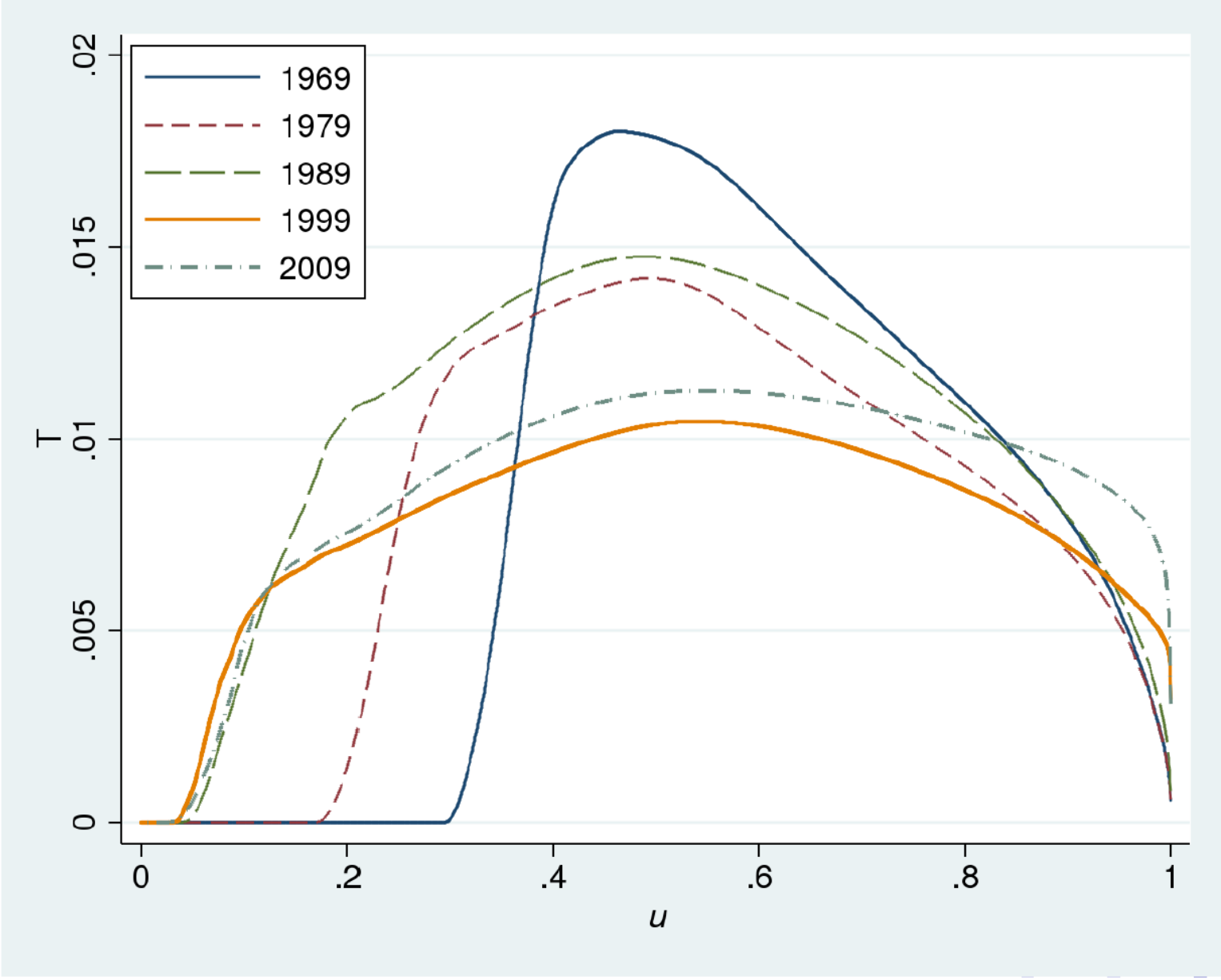
$T(u)$: Top income mobility



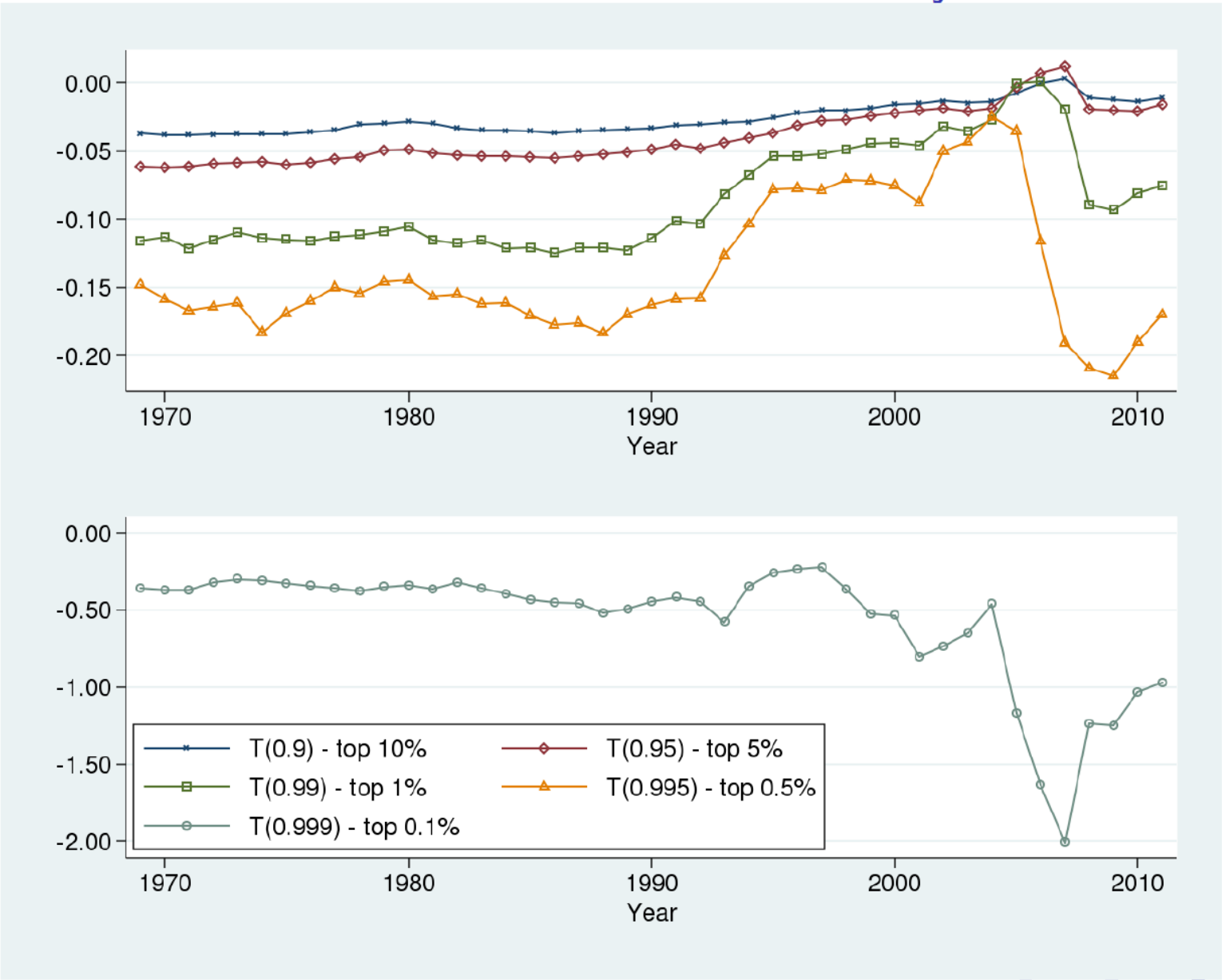
Top income mobility: Results

- Top income mobility was stable and low throughout the 1970s and 1980s
- Sharp increase for the upper quantiles starting around 1992
  - Capital market liberalization from mid-1980s; effect probably postponed by recession
  - Reduced marginal tax on capital income
- Peak I: Dividend tax introduced 2001, removed 2002 (peak in 2000 in anticipation of reform)
- Peak II: Dividend tax reintroduced 2006
  - Very high dividend income in 2005 (anticipation)
  - Lower dividend income in 2006 and onwards
- Despite the large changes, overall top income mobility is low
  - Rarely above one percentage point

The entire mobility curve



The derivative of T: "Losses" from mobility



Summary measures of top income mobility

- To compare mobility across the entire upper half of the distribution, we integrate downwards:

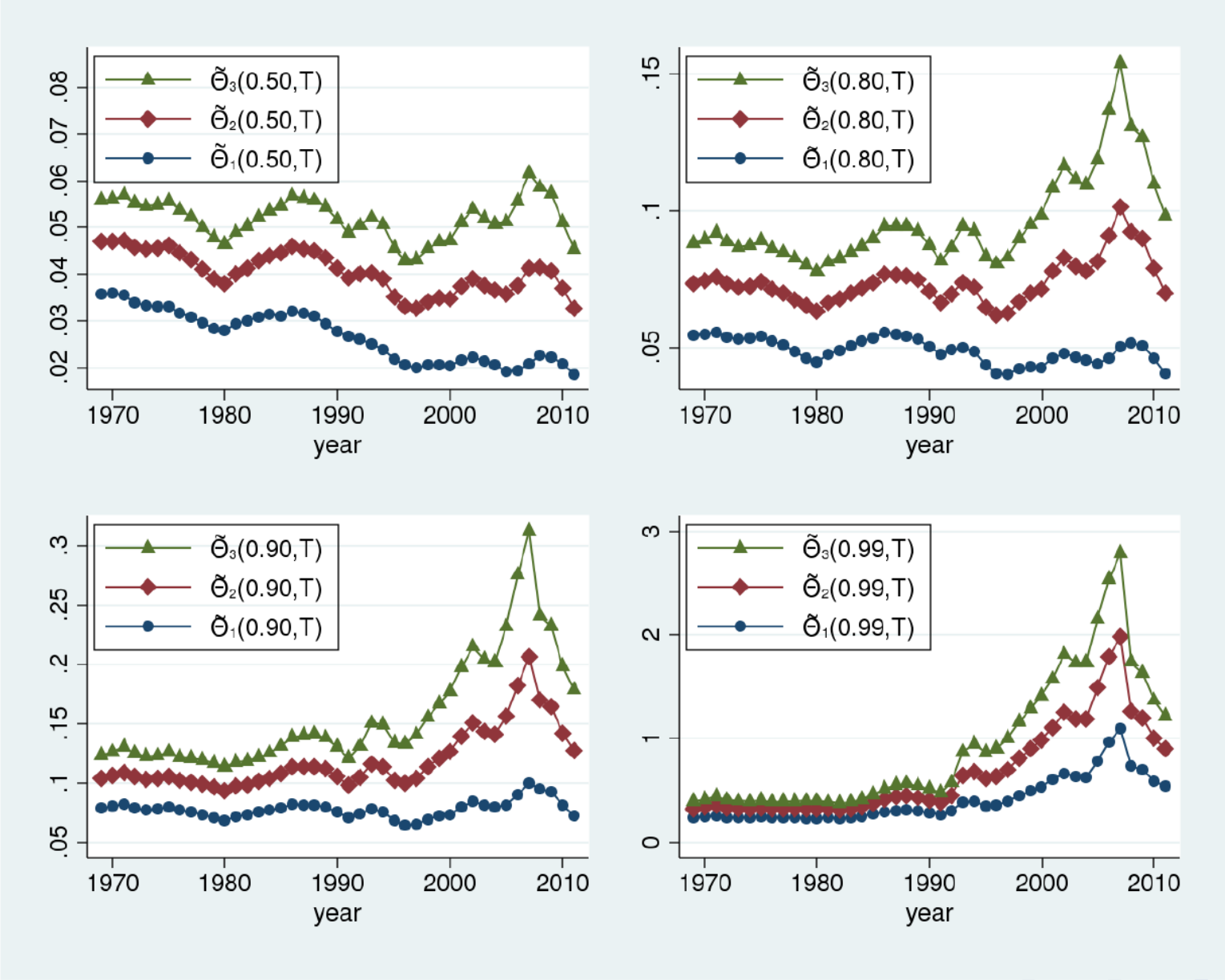
$$\tilde{\Theta}_k(a;T) = \frac{k}{(1-a)^k \mu} \int_a^1 (s-a)^{k-1} \left(\sum_{t=1}^{r} F_t^{-1}(s) - F^{-1}(s) \right) \mathrm{d}s \ , k = 1,$$

- For $k = 1$, this is the differences in permanent and average cross-section expected income, conditional on being in the upper half of the distribution:

$$\tilde{\Theta}_1(a;T) = \frac{1}{\mu} \left[\sum_{t=1}^{r} E\left(X_t \middle| X_t \geq F_t^{-1}(a) \right) - E\left(X \middle| X \geq F^{-1}(a) \right) \right]$$

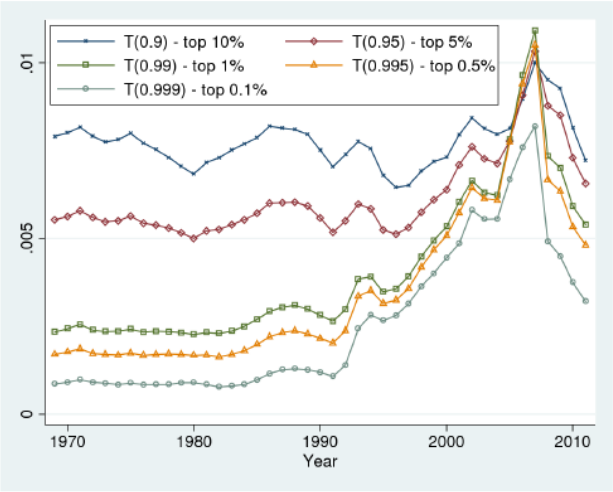
- $\tilde{\Theta}$ with higher k incorporate terms for the spread as well, giving higher weight to the upper end of the distribution.

Summary measures of top income mobility



The effects of tax reforms

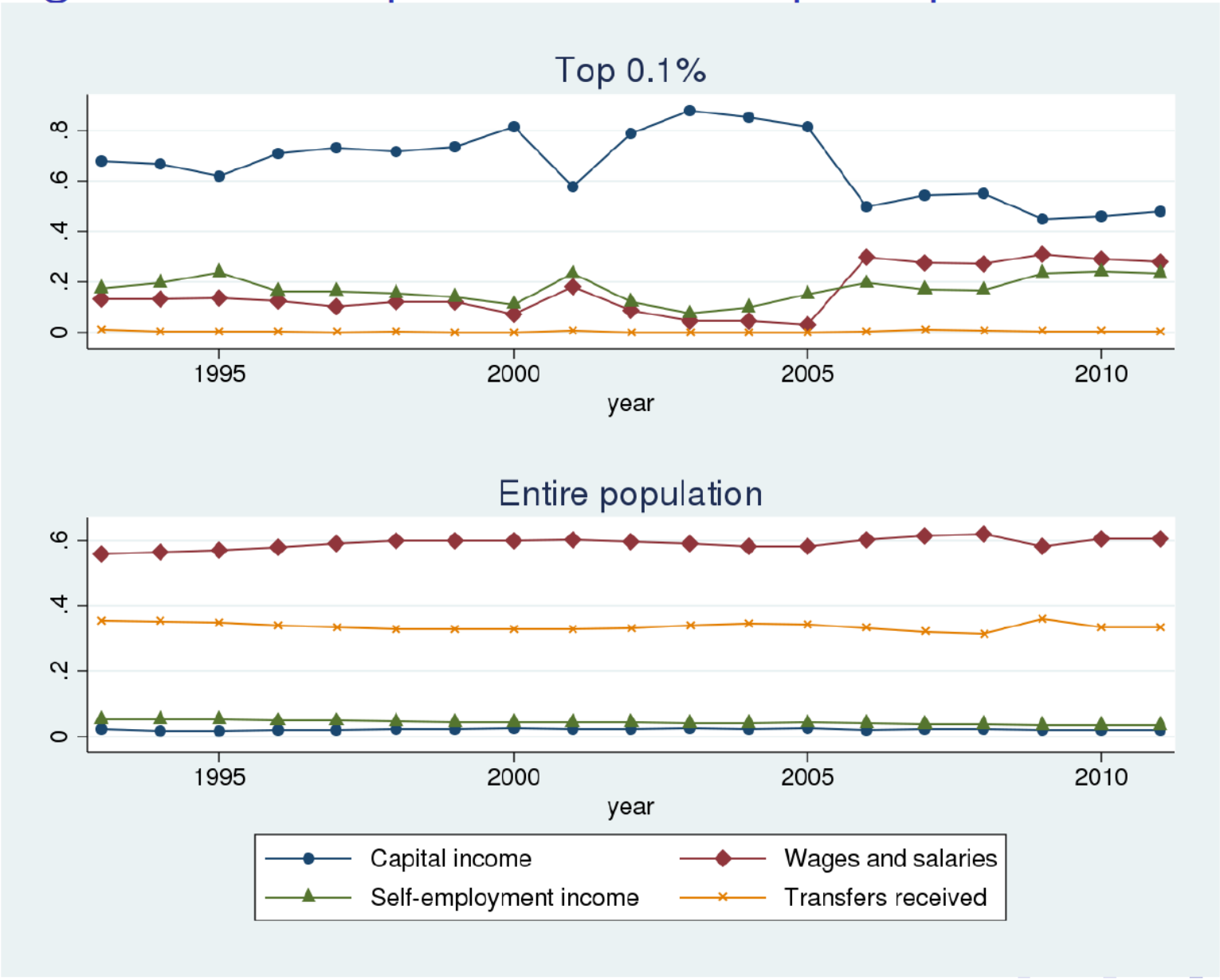
- Large effects of tax reforms in the 2000s
  - Dividend tax introduced 2001, removed 2002
  - Dividend tax introduced 2006
- Effect 1: large dividends realised year before reform $\rightarrow$ high top income shares in that year
- Effect 2: less dividends realised after reform, and/or dividend gains spread over more years than before



The composition of incomes among individuals

- Composition of the top income holders: Group individuals by their **predominant source of income**
  - Capital income
  - Wages and salaries
  - Self-employment income
  - Transfers received

Changes in the composition of the top 0.1 per cent

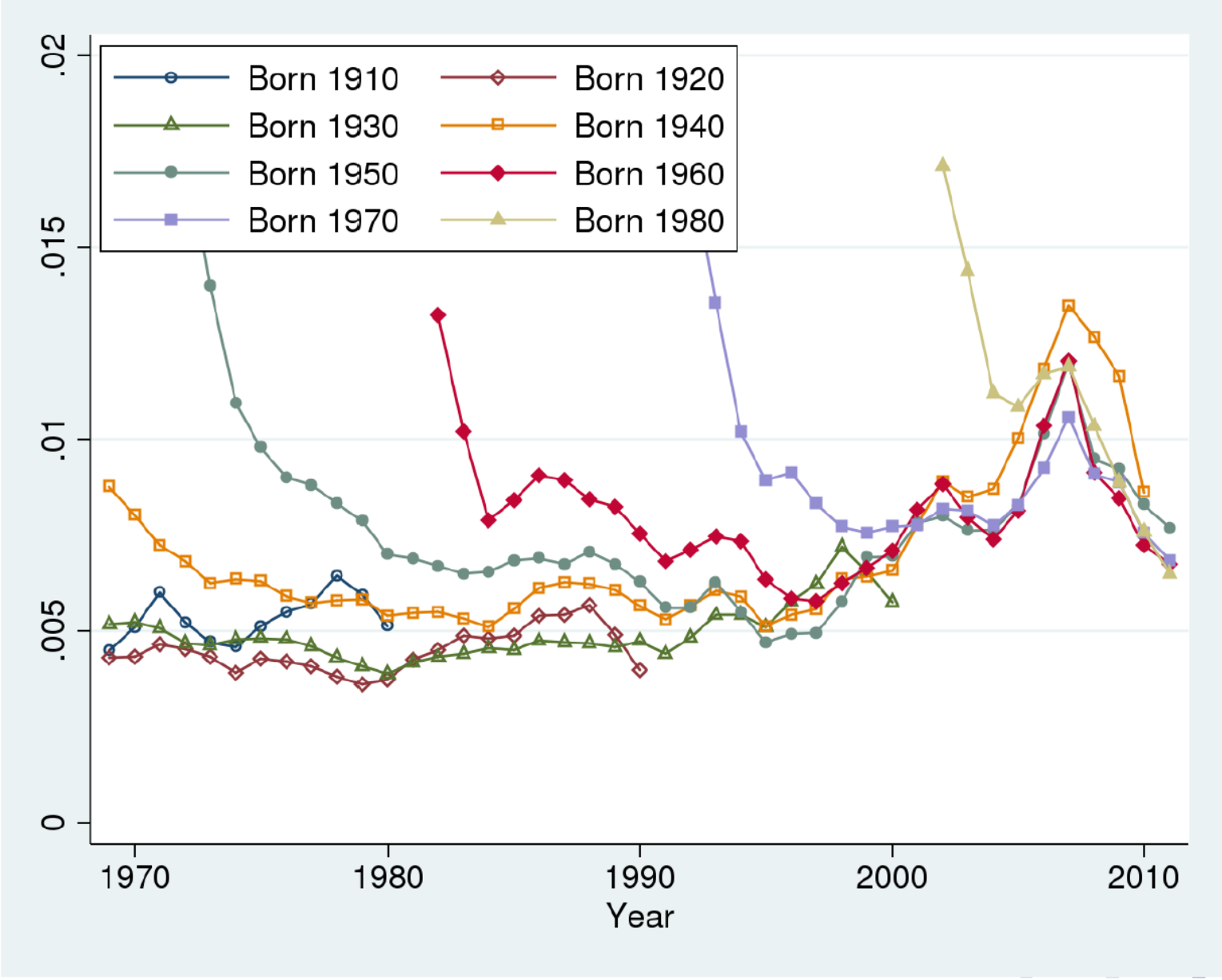


Conclusions

- Top income mobility can be defined using the "equalization of permanent income" paradigm
- Large changes in top income mobility over time, driven by structural effects as well as short-term reforms
- However, low mobility
  - → The people at the top have very high incomes even when we increase the time horizon

- Top income mobility can be defined using the "equalization of permanent income" paradigm
- Large changes in top income mobility over time, driven by structural effects as well as short-term reforms
- However, low mobility
  - → The people at the top have very high incomes even when we increase the time horizon
- A strong life cycle profile in mobility; however, demographic forces do not drive the aggregate trend in top income mobility

Extra: Top income mobility within cohorts: Top 5%



Extra: Top income mobility within cohorts: Top 0.1%