Financial Intermediation, House Prices, and the Welfare Effects of the U.S. Great Recession*

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January 6, 2014

Abstract

This paper quantifies the welfare effects of the drop in aggregate house prices for leveraged and un-leveraged households in the Great Recession. We calibrate a dynamic general equilibrium model to the U.S. economy and simulate the 2007-2009 Great Recession as a contemporaneous shock to interest rate spreads and aggregate income. Our estimates show that borrowers lost significantly more in terms of welfare than savers. In counter-factual experiments we find this loss to be larger the higher the households' leverage. This last effect comes from non-linearity that is absent in a model with an always binding collateral constraint (i.e. constant leverage).

Keywords: Housing Wealth, Heterogeneous Agents, Welfare, Leverage

JEL Classification Numbers: D31, D58, D90, E21, E30, E44

*We thank Arpad Abraham, Alastair Ball, Almut Balleer, Christian Bayer, Alberto Bisin, Fabrice Collard, Russell Cooper, Antonia Diaz, Luigi Guiso, Piero Gottardi, Nikolay Hristov, Thomas Hintermaier, Guido Lorenzoni, Ramon Marimon, Nicola Pavoni, Fabrizio Perri, Vincenzo Quadrini and seminar participants at the EUI Macro Working Group, the XVII Workshop on Dynamic Macro in Vigo 2012, the Workshop on Institutions, Individual Behavior and Economic Outcomes in Argentiera 2012, the European Workshop in Macroeconomics 2013 at LSE, RWTH Aachen University 2012, Bocconi University 2012, University of Milan - Bicocca 2013, Federal Reserve Board of Governors 2013, the Ifo Institute Munich 2013, Bonn University 2013, the EEA Annual Congress in Gothenburg 2013, the Workshop on Macroeconomics, Financial Frictions and Asset Prices in Pavia 2013 for helpful comments on various drafts of this paper. All errors remain our own.

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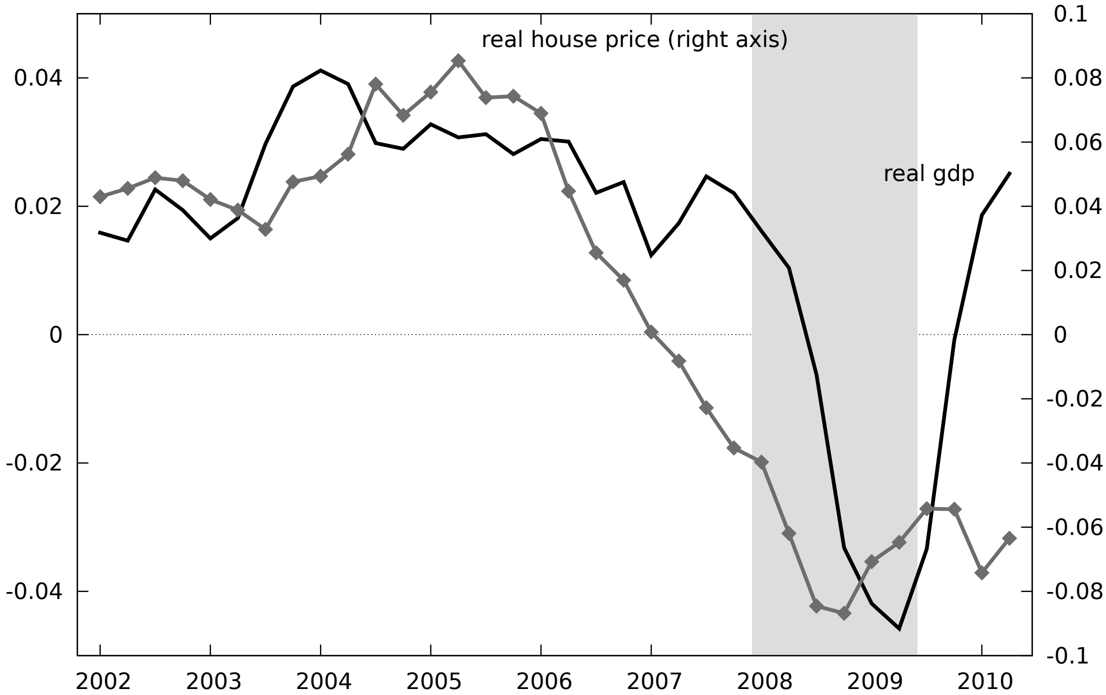
1 Introduction

The U.S. Great Recession was characterized by a large fall in GDP coupled with an unprecedented collapse in the housing market. This drop in aggregate house price between 2007:IV and 2009:II deeply affected a great number of U.S. households. [1] Figure 1 shows the de-trended quarterly series of US GDP and aggregate house prices. We observe a large drop of around 5.4% between the NBER recession dates, and a collapse in aggregate house prices of about 11%.

Figure 1: GDP growth vs. house price growth



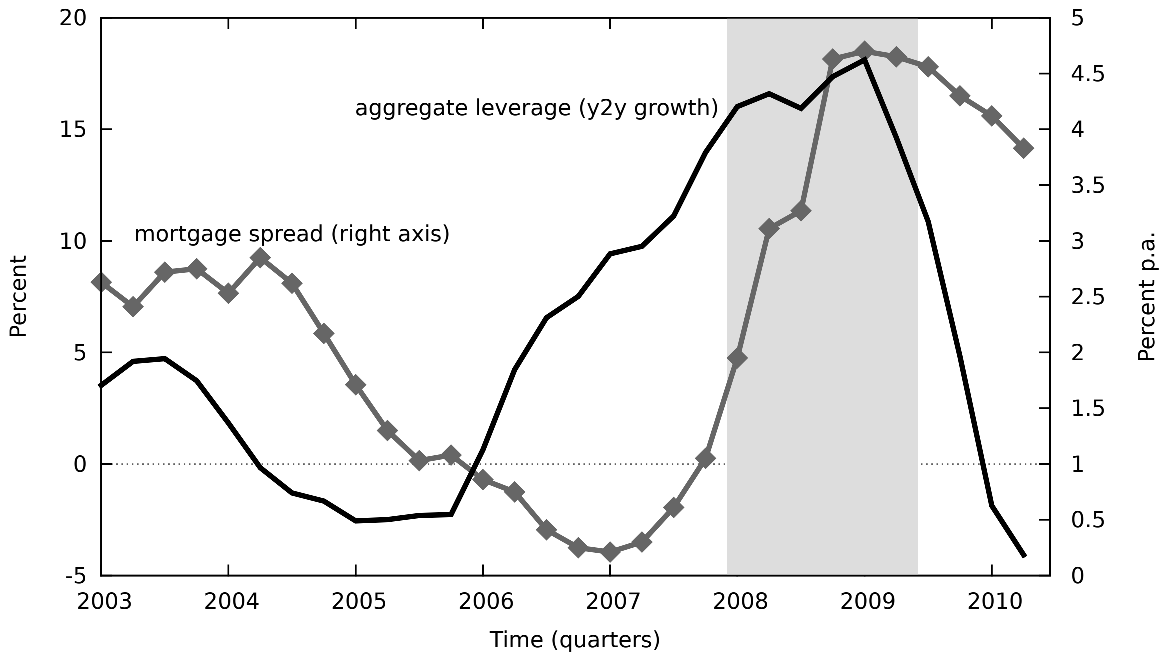
Notes: Shaded areas are NBER recession dates. The grey dotted-line is the Y2Y-growth rate of All-Transactions House Price Index for the United States deflated by CPI (less shelter); the black line is Y2Y growth of U.S. real GDP. For a detailed data description see appendix A.

The recession has also been linked turbulence in the financial markets and, in particular, the banking system. This fact has triggered a debate among economists and policy-makers about the welfare consequences of the financial innovation process that preceded the crisis and that possibly exacerbated the effects of the economic collapse. In fact, the last decade witnessed an increase in household indebtedness that coincided with a period of relaxing credit conditions. Both microeconomic and macroecronomic evidence show an increase in household leverage in the years preceding the recession. On the micro side, an analysis of Survey of Consumer Finance (SCF) data reveals that aggregate mortgage debt

[1](Iacoviello, 2011b) shows that housing wealth represents about half of total household net worth in 2008 and almost two third of median household total wealth

expanded by 59% between 2001 and 2007, despite a 19% increase in housing wealth. On the macro side, we observe around ten quarters of growth in leverage followed by sharp fall during the NBER recession dates, as seen in the mortgage to real estate ratio. Figure 2 plots the year-to-year growth rate of leverage and the spread between the mortgage interest rate and the federal funds rate. These two series show a negative correlation at the onset, and in the last quarters of the Great Recession. During the quarters preceding the crisis, spreads were particularly low and leverage was rising at an unprecedented rate. In mid-2008 however, interest rate spreads jumped to a level of about 4.5% while household leverage started to decline. Our interpretation is that, in the period of credit expansion (low spreads), the mortgage growth rate was faster than real estate inflation and leverage was increasing; the opposite happened in a period of credit contraction (high spreads).

Figure 2: Mortgage spread vs. Leverage



Notes: Shaded areas are NBER recession dates. The grey dotted-line shows the spread between the one-year amortizing adjustable mortgage rate (ARM) and the federal funds rate from 2002:I to 2010:II. Spreads of ARM over Fed Funds rate are shown in levels (percent p.a.). The black line is Y2Y growth of U.S. leverage defined as the ratio between mortgage and real estate series (taken from the balance sheet of U.S. households and nonprofit organizations). For a detailed data description see appendix A.

In the current paper we examine the effects of exogenous changes in interest rate spreads on endogenous aggregate house prices and, ultimately, on households' welfare. In this respect, we share the view that fluctuations in spreads largely reflect disturbances in the financial markets' assessments of credit risk (Bordo, 2008). Furthermore, we share the view of Adrian and Shin (2010) that variations in the price of default risk reflected variations in the effective risk-bearing capacity of the financial sector, which has been ultimately affected by aggregate portfolio losses.

The stylized facts highlighted in figures 1 and 2 motivate our interest in quantifying and isolating the impacts of financial and income shocks on aggregate house prices and, consequently, on households' welfare. In particular we address this question within a stochastic dynamic general equilibrium model with heterogeneous households and endogenous collateral constraints. In our model, households differ in their level of patience. This heterogeneity results into two types of agents: borrowers, who are potentially financial constrained; and savers, who are unconstrained.[2] Within this framework, we study the welfare effects of an endogenous drop in housing wealth for these two groups of households. The data in table 1 motivate the choice of this cross-sectional heterogeneity across households. Using panel data from the Survey of Consumer Finance (SCF) for the period from 2007 to 2009, the table shows that households with a positive net savings position (*savers*) show an average drop in housing wealth of 9.2% between 2007 and 2009. This is significantly lower than the equivalent number for households with a negative net savings position (*borrowers*, -16.6%).[3] Moreover we show that the drop in housing wealth for borrowers is increasing in the level of leverage in 2007:[4] while borrowers with initial levels of leverage greater than or equal to 67% show a drop of 23.5% in housing wealth, households that entered the recession with a lower level of leverage (less than 43%) show a much smaller drop in housing wealth.

Table 1: Summary Statistics from SCF panel 2007-2009

Household type$_{2007}$	*Savers*	*Borrowers*	*All households*
$\Delta_{07,09}$housing wealth	-9.2%	-16.6%	-12.9%
Leverage$_{2007}$	*< 43%*	*43 - 67%*	*>67%*
$\Delta_{07,09}$housing wealth	-12.9%	-16.5%	-23.5 %

[2]The structure of the economy is similar to Iacoviello (2011a) and Justiniano, Primiceri, and Tambalotti (2013) who present a quantitative analysis of the US Great Recession

[3]In table 1, saver and borrower status refers to households in 2007. Savers and borrowers are defined here - and throughout the paper- as households that show respectively a positive or a negative net asset position. A net asset position is defined as the sum of savings bonds, directly held bonds, the cash value of life insurances, certificates of deposits, quasi-liquid retirement accounts and all other types of transaction accounts minus the debt secured by primary residence, the debt secured by other residential property, credit card debt and other forms of debt. For a detailed description of data please refer to Appendix A

[4]Leverage is defined here - and thorough the remaining sections- as the ratio between net asset position and total housing wealth.

In the model economy, agents are fully rational and derive utility from both the consumption of perishable goods and of housing services coming from housing stock. Housing is the only physical asset in the economy and it is fixed in supply. This is motivated by the fact that previous and during the Great Recession, house prices were most volatile in geographical areas where the supply of houses was relatively fixed.[5] The financial friction arises because agents have to collateralize short positions of one-period financial asset by a fraction of the expected value of their available housing stock.

In this otherwise standard model, we introduce a competitive financial intermediation sector. All saving and borrowing is conducted though this sector, which faces exogenous shocks to its technology.[6] These shocks give rise to a spread between borrowing and lending such that the collateral constraint does not necessarily bind. In other words, it generates endogenous changes in the households' leverage. The second source of aggregate disturbance comes from standard aggregate income shocks that directly affect the households' endowment of the perishable good. This may be interpreted as a reduced form way to capture the cyclical behavior of productivity shocks.

We calibrate the model to the US economy and simulate the Great Recession as a contemporaneous negative income and financial shock that follows a period of moderate economic, credit expansion and increasing leverage. This characterization is due to the empirical observation that both income and financial intermediation were above (below) the long run trend before (after) the recession. To calibrate our key parameters we consider moments from both micro and macro data. In particular, we were able to match the leverage and the wealth share of borrowers relative to savers using from the Survey of Consumer Finances (SCF, waves 1998 - 2007). This calibration strategy, although different from the approach of most papers in the existing literature which target macro moments only, results in calibrated parameters that are compatible with recent contributions (Iacoviello and Guerrieri (2012)).

A very delicate issue for the calibration exercise is what time frame to use, and in particular, whether to incorporate a recession or not. We take the following stance. Our main goal is to maintain a close link between the model and the research question. We study the Great Recession as a state-contingent exogenous event that hit the US economy in late 2007, following a period characterized by banking innovation and increasing household leverage. Therefore, we consider the Great Recession as a low probabilistic event embedded in a business cycle framework. For this reason, we calibrate the model

[5]See figure IV in Mian and Sufi (2009).

[6]We consider a simple model for the financial intermediation in the spirit of Cooper and Ejarque (2000) and Cúrdia and Woodford (2010). Otherwise, the link to these studies is limited as the former looks at the business cycle properties of financial shocks within a representative agent framework, while the latter studies the implications of spread shocks for the optimal conduct of monetary policy.

to data including the quarters of the recession until 2009:II.[7] The structural nature of our exercise allows us to conduct counter-factual experiments in order to disentangle the quantitative effects of income and intermediation shocks on aggregate house prices and agents' welfare.

We have three major findings. First, we find that our benchmark model quantitatively explains the observed drop in house prices during the Great Recession. The majority of the effect is attributed to real income shocks. Financial intermediation shocks explain only a small percentage of the observed drop. This finding confirms that the observed behavior of aggregate house prices, before and after the Recession, could be partially related to changes in fully expected shocks. More importantly, we find that, in contrast to the widespread view, shocks in the financial sector have very limited quantitative effects on aggregate house prices.

Second, we find that borrowers significantly lost more than savers in the Great Recession. In particular we highlight a significant difference in the welfare effects of income and financial intermediation shocks. In the Great Recession, the negative income shock was the main driver behind the absolute drop in house prices and the absolute level of agents' welfare losses. The financial intermediation shock is instead the main determinant of changes in households' leverage before and after the house price drop.[8]. We show that increasing interest spreads had distributive effects, with savers gaining at the expense of borrowers. Accordingly we show that an increase in interest rate spreads forced borrowers to de-leverage and amplified their welfare losses of house price drop by 37.5% while causing a 66.7% welfare gain for savers. Moreover, counterfactual experiments show that the high leverage previous to the crisis made borrowers' welfare losses 25% bigger than if it would have occurred in a state of low leverage.

Third, we find that if we restrict the collateral constraint so that it always binds, the amplification effects given by leverage and de-leverage would have been underestimated; a model with always binding collateral constraint which reduces in fact the volatility of the aggregate leverage to zero. This is an important finding as previous studies (notably, Iacoviello (2005)) usually assume that the constraints are always binding. The intuition for this result is that when the growth rate of the borrowers' debt is forced to be proportional to changes in expected housing wealth, borrowers leverage up more slowly in expansions and de-leverage more slowly in contractions when compared to our bench-

[7]For the micro data, SCF is run every three years. We decided to include the 2009 wave and not to include the 2010 wave of the survey in the analysis in order to be consistent with the other calibrated parameters in the model. However, even when including the 2010 wave, the targeted values are very similar.

[8]This mechanism is in line with the microeconomic evidence of Mian and Sufi (2010), who found that an increase in credit supply, coupled with the effect of collateralized debt on increasing house prices, created an unprecedented increase in household leverage in the quarters preceding the crisis

mark model. This implies that when the crisis hits, borrowers have more outstanding debt in the benchmark model that they need to roll-over. In a recent paper, Iacoviello and Guerrieri (2012) explore the quantitative properties of occasionally binding collateral constraints and the relative non-linear effects coming from changes in the demand for housing.

The mechanism behind the three findings is the following. First, a negative realization of one or both of the exogenous shocks leads to credit contractions. In a credit contraction - given that it is more costly to roll over existing debt - borrowers choose optimally to reduce their indebtedness. If the reduction in debt is sufficiently large, borrowers need to reduce their housing stock. For a given supply of housing, house prices must therefore decrease. This causes borrowers to suffer in terms of both wealth and expected lifetime utility. On the other hand - because of the lower demand for debt - savers potentially face a lower interest rate on savings. This potentially hurts them by raising the price of future consumption. However, savers expecting house prices to rise again in the next period - can smooth their consumption by buying houses when their prices are depressed. Finally, savers gain in terms of wealth and do not suffer much in terms of expected lifetime utility. The size of this distributive effect depends crucially on how interest rates move. In this paper we quantitatively show what exactly distinguishes financial shocks from income shocks. Another important remark concerns the non-linearity generated by the collateral constraint. In states of the world where borrowers choose optimally to move away from the constraint, it becomes slack. That is, borrowers can choose the pace at which to reduce their debt, unlike the case in models with an always-binding constraint. This implies a change in the elasticity of the demand for debt and housing with respect to changes in house prices that could have non-negligible quantitative effects.

The present study is related to two important strands of literature. First, we relate to the recent literature that studies the financial sector as an autonomous source of macroeconomic fluctuations (Quadrini and Urban, 2012) and the literature that claims that financial frictions played a pre-eminent role in explaining the observed drop in US aggregate economic activity (Hall, 2011). Recently, Guerrieri and Lorenzoni (2011) find that a shock to the spread between the interest rate on borrowings and the interest rate on savings - in the presence of a collateral constraint that links debt to the level of durables - generates a decrease in the borrowers' demand for durables that grows stronger as agents get closer to the credit constraint. While their analysis abstracts from aggregate house prices and endogenous changes in wealth, we explicitly emphasize the channel that goes through the endogenous change in house prices.

Second, our analysis relates to recent studies on the distributive effects of the Great Recession. Compared to Glover, Heathcote, Krueger, and Ríos-Rull (2011) - a study on intergenerational redistribution during the Great Recession - we focus on a different dimension of agent heterogeneity and welfare, namely, redistribution between constrained agents (borrowers) and unconstrained agents (savers). Similar to Hur (2012), we find that the constrained agents always lose more than unconstrained agents.[9] Both of the aforementioned studies are silent about the inherent redistributive nature of financial shocks, the focus of this paper.[10]

The rest of the paper is structured as follows: In the following section we present the model. Section 3 presents the quantitative analysis. In section 4 we compare the predictions of the benchmark model to alternative specifications, including the case of an always binding constraint. Section 5 concludes.

2 Model

2.1 The physical economy

Uncertainty. Time is discrete and denoted by $t = 0, 1, \ldots$. In each period t, the world experiences one of Z possible exogenous events $z \in \mathcal{Z} = \{1, \ldots, Z\}$. The resolution of uncertainty is represented by an event tree Σ with root σ_0, which is given by a fixed event z_0 in which the economy starts at time 0. Each node is characterized by a history of events, denoted by $\sigma^t = (\sigma_0, \ldots, \sigma_t) \in \Sigma^t = \times_{k=0}^{t} \Sigma_k$. Each node has Z immediate successors $(\sigma_t z^+)$ and a unique predecessor (σ_t^-). The exogenous events follow a Markov process with transition matrix Π.

Agents and Endowments At each node σ_t there are two types of agents, borrowers (denoted by a subscript b) and savers (denoted by a subscript s). Borrowers and savers differ in their rates of time preference, in the sense that borrowers discount the future more than savers. Formally, we have $\beta_s > \beta_b$, where $\beta_i \in (0, 1)$ for $i = s, b$. Each group consists of infinitely many agents but the group size differs: denote by n_b and n_s the relative size of the borrower and saver groups. Note that we choose the normalization $n_b + n_s = 1$.

At each node σ_t, there is a perishable consumption good (non-durable consumption good). The total endowment of the perishable good is stochastic and depends on the

[9] Hur (2012) considers an overlapping generations model with collateral constraints; he finds that the constrained agents are mostly from the young cohort, and that those agents suffer the most during a recession.

[10] Another distinguishing element of our analysis to Hur (2012) and Guerrieri and Lorenzoni (2011), is that they consider the recession as an unanticipated event while, in our economy, agents take into account the probability of negative aggregate shocks when making decisions about the future.

realization of the shock alone, that is, $y(\sigma_t^-) = y(z)$, where $y : \mathcal{Z} \to R_{++}$ is a time-invariant function. Note that there is no idiosyncratic uncertainty, the endowment of the perishable good is the same for both types of households. In addition to the non-durable consumption good, agents trade houses. Houses are the only physical asset in the economy and are in fixed net supply. This is motivated by the fact that house prices were most volatile in counties where the supply of houses remained relatively fixed as shown by Mian and Sufi (2010). At period 0, agent $i = b, s$ owns a stock $h_i(\sigma_0^-) \geq 0$ of houses. We normalize $\sum_{i=b,s} h_i(\sigma_0^-) = 1$.

At node σ_t let $h_i(\sigma_t)$ denote agent $i's$ end-of-period stock of houses. We assume that houses are traded cum services. That is, buying a house allows the agent to enjoy the housing services in the same period: if agent i owns $h_i(\sigma_t)$ houses then he receives a service stream of $1 \cdot h_i(\sigma_t)$. Other than the service stream, houses do not yield any dividend payments.[11]

Markets. At each node, spot markets open and agents trade the perishable consumption good. We choose the perishable good as the numeraire and - without loss of generality - normalize its price to be equal to 1. Agents can trade housing in every period; that is, agents $i = s, b$ can buy a unit of housing at node σ_t at price $q(\sigma_t)$. As long as $h_i \geq 0$, there is no possibility of default since no promises are made when agents hold a positive amount of the physical asset. In addition to houses, there are two financial assets, debt and savings, both one-period securities. We denote agent i's end-of-period debt holdings by $d_i(\sigma_t)$ and end-of-period savings by $s_i(\sigma_t)$, respectively. Denote the prices of the respective securities by $p_j(\sigma_t)$ for $j = d, s$. We distinguish these two assets because their effective returns differ. Debt is assumed to be a security for which only negative (short) positions are allowed, that is, $d_i(\sigma_t) \leq 0$. For savings, agents can only take positive (long) positions, such that $s_i(\sigma_t) \geq 0$, for $i = b, s$ and all σ_t. Asset $j = d, s$ traded at σ_t promises a nominal pay-off $b_j(\sigma_t z)$ at any successor node $\sigma_t z$. We normalize $b_j(\sigma_t z) = 1$ for all $\sigma_t, \sigma_t z$. For the remainder of the paper, we will discuss pay offs in in terms of real interest rates: denote by $R_D(\sigma_t) = \frac{1}{p_d(\sigma_t)}$ the real interest rate on debt and $R(\sigma_t) = \frac{1}{p_s(\sigma_t)}$ the real interest rate on savings. We also restrict borrowers to hold zero savings and savers to hold zero debt. Formally, for all nodes σ_t, we have $d_b(\sigma_t) \leq 0$, $s_b(\sigma_t) = 0$, $d_s(\sigma_t) = 0$, and $s_s(\sigma_t) \geq 0$.[12]

[11] These assumptions are for simplicity. We could allow the service stream of houses to depend on the realization of the shock z or on the identity of the agent.

[12] This is only for the ease of exposition. When computing the equilibrium policy functions, we allow borrowers and savers to trade both assets, debt and savings. Borrowers will only want to take long positions in savings for high relative wealth shares. In the calibrated economy, this never occurs along the equilibrium path unless the initial wealth share of the borrowers is very high.

Collateral Requirements and Default. Similar to Kiyotaki and Moore (1997) we assume limits on debt obligations. Houses are distinguished from other assets by the fact that they are widely used as collateral for debt obligations (mortgages). As in Iacoviello and Neri (2010), the theoretical justification for collateral constraints is the ability of borrowers to default on their debt promises. If the borrowers default in some successor node $\sigma_t z^+$, lenders can seize the borrowers' assets, $q(\sigma_t z^+)h_b(\sigma_t)$ by paying a proportional transaction cost of $(1-m)E[q(\sigma_t z^+)|\sigma_t]h_b(\sigma_t)$ that is not redistributed. This transaction cost can be thought of as a loss associated with bankruptcy. Lenders will therefore never accept a debt contract where the borrowers' promises exceed the expected collateral value of housing. Formally, in each node σ_t, promises made by the borrower have to satisfy

$$R_D(\sigma_t)d(\sigma_t) + mE[q(\sigma_t z^+)|\sigma_t]h_b(\sigma_t) \geq 0. \tag{1}$$

Note that in some successor node $\tilde{z} \in \sigma_t z^+$ it might still be optimal for the borrowers to default ex-post. We assume throughout the analysis, however, that m is small enough that borrowers will never default in equilibrium:

Assumption 1

$$m \leq \frac{\min\left(q(\sigma_t z^+)\right)}{E[q(\sigma_t z^+)|\sigma_t]} \qquad \textit{for all } \sigma_t.$$

There is no default in equilibrium if and only if this condition is satisfied.[13] When solving the model equilibrium numerically, we assume that this condition holds and verify ex post that it is indeed satisfied for all prices along the equilibrium path. This allows us to treat debt as risk free.[14]

[13]Assuming default costs equal to zero, borrowers default in some successor node $\tilde{z} \in \sigma_t z^+$ iff

$$-mE[q(\sigma_t z^+)|\sigma_t]h_b(\sigma_t) + q(\tilde{z})h_b(\sigma_t) < 0,$$

That is, whenever the realized value of housing is smaller than the maximum amount promised. Since in any financial market equilibrium, house prices and - by the Inada conditions - h_b are strictly positive for a small enough m, this condition does not hold. As an alternative to a condition on m, we could just assume default costs are sufficiently high that it is never optimal for the borrowers to default.

[14]We evaluated the robustness of our results by replacing equation (1) by the following collateral requirement:

$$R_D(\sigma_t)d(\sigma_t) + m \cdot \min\left(q(\sigma_t z^+)\right) h_b(\sigma_t) \geq 0.$$

This is a tighter constraint and ensures that there is no default in equilibrium, independent of the value of m. While the qualitative implications remain unaffected, this specification implied slightly smaller quantitative effects on house prices and welfare. The intuition for the smaller quantitative effects is that leverage in states of high intermediation is lower compared to the benchmark model and the wealth distribution is therefore less sensitive to price changes. We stick to the collateral constraint as outlined in the main text because it has became standard in macroeconomic models with mortgage debt and thus increases the comparability of our results.

Utilities and budget constraints Agents $i = s, b$ maximize a time-separable utility function

$$U_i(c_i, h_i) = E_0 \sum_{t=0}^{\infty} \beta_i^t \, u(c_{s,t}, h_{s,t}) \tag{2}$$

where E_0 is the expectation operator at the the starting date $t = 0$. We consider period-by-period utility functions $u(c, h) : \mathbb{R}_{++} \times [0, 1] \rightarrow \mathbb{R}$ characterized by constant elasticity of substitution.

$$u(c, h) = \frac{\Psi(c, h)^{(1-\gamma)}}{1-\gamma}, \qquad \text{and} \qquad \Psi(c, h) = [\phi c^{\rho} + (1-\phi)h^{\rho}]^{\frac{1}{\rho}}$$

Note that this class of preferences is strictly monotone, continuously differentiable, strictly concave, and satisfies the Inada conditions for both c_i and h_i.

At each node, the savers' budget constraint is given by

$$c_s(\sigma_t) + q(\sigma_t)h_s(\sigma_t) + s_s(\sigma_t) \leq y(\sigma_t) + s_s(\sigma_t^-)R(\sigma_t^-) + q(\sigma_t)h_s(\sigma_t^-) + \Upsilon(s^t). \tag{3}$$

The right hand-side is the savers' available income. It consists of the endowment of the perishable good $y(\sigma_t)$, the gross return on savings, and the housing stock carried over from the previous period. Finally, $\Upsilon(s^t)$ are resources that are redistributed in a lump-sum fashion from the financial sector to the households, of which savers receive a share n_s, representing their share in the population. The reason why we need this re-distribution will be explained in detail below.

Analogously, the borrowers' budget constraint reads as

$$c_b(\sigma_t) + q(\sigma_t)h_b(\sigma_t) + d_b(\sigma_t) \leq y(\sigma_t) + d(\sigma_t^-)R_D(\sigma_t^-) + q(\sigma_t)h_b(\sigma_t^-) + \Upsilon(s^t). \tag{4}$$

The right hand-side is the borrowers' available income. It consists of the endowment of the perishable good $y(\sigma_t)$, the value of housing stock net of the debt burden from the previous period plus resources being redistributed from the financial sector to the households, of which borrowers receive the amount $\Upsilon(s^t)$.

Financial Intermediaries. Intermediaries demand aggregate deposits $S(\sigma_t)$ and supply aggregate debt $D(\sigma_t)$. The real pay-offs for each unit lent are given by the real interest rates, $R_D(\sigma_t)$ and $R(\sigma_t)$, respectively. The collateral constraints and assumption 1 make sure that debt is risk free. The key distortion in the intermediation sector is similar to that in Cooper and Ejarque (2000).[15] We assume that in each node σ_t only a fraction

[15] Another example for the inclusion of a supply-sided friction in the banking sector into an international macro model is Kalemli-Ozcan, Papaioannou, and Perri (2012).

of savings can be transformed into debt. This fraction is stochastic and depends on the realization of the current shock only. That is, $\theta(\sigma_t^- z) = \theta(z)$ and $\theta(z) : \mathcal{Z} \rightarrow (0, 1]$ is a time-invariant function.

This exogenous financial shock represents a reduced form way to model the risk-bearing capacity of the financial sector. In particular, changes in the intermediation technology θ potentially reflect changes in the value of equity associated with a risky asset portfolio or changes in monitoring by the bank managers as a consequence of changes in risk aversion. Consequently, while we remain agnostic about the exact foundation of the θ, we point out that the observed variations in the spread series in the period 2005-2009 mainly reflect changes in the households' price for risk rather than changes in the default risk.[16]

Financial intermediaries are otherwise risk neutral and maximize expected profits on their portfolio, that is,

$$\max_{D(\sigma_t), S(\sigma_t) \geq 0} R_D(\sigma_t) D(\sigma_t) - R(\sigma_t) S_i(\sigma_t) \tag{5}$$

subject to the constraint

$$D(\sigma_t) \leq \theta(\sigma_t) S(\sigma_t). \tag{6}$$

Because intermediaries operate in competitive markets with free entry, equilibrium interest rates are such that intermediaries make zero profits:

$$R_D(\sigma_t)\theta(\sigma_t) - R(\sigma_t) = 0. \tag{7}$$

This last relation implies that there is a spread between loan and deposit rates in this economy . In particular, the interest rate on debt is always at least as big as the interest rate on savings, or $R_D(\sigma_t) \geq R(\sigma_t)$.

Transfers from the Banking sector to the Household sector. Completing the description model, we specify the re-distribution function $\Upsilon(s^t)$. The intermediation process as outlined above implies an aggregate intermediation loss in terms of real resources that, in equilibrium, is given by $(1 - \theta(\sigma_t))S(\sigma_t)$. This can be easily verified by combining the households budget constraints, using market clearing conditions in the debt and savings markets, and the zero profit condition of financial intermediaries. The aggregate resource constraint, then, reads as:

$$n_b c_b(\sigma_t) + n_s c_s(\sigma_t) + (1 - \theta(\sigma_t))S(\sigma_t) = y(\sigma_t) + \Upsilon(s^t)$$

[16] The inclusion of a more detailed micro-founded banking sector is an interesting avenue that we leave for future research.

On the left hand side, we have the borrowers' and savers' consumption plus the resources 'eaten up' by the financial sector. On the right hand side we have aggregate income plus total transfers. In order to keep the intermediation process as a purely redistributive distortion, we choose $\Upsilon(s^t)$ such that all resources 'lost' in the intermediation sector are redistributed back to the agents, so that aggregate consumption is a function of aggregate income only. Therefore, aggregate transfers are defined as follows:

$$\Upsilon(s^t) \equiv (1-\theta(\sigma_t))S(\sigma_t) \tag{8}$$

We interpret this transfer as income generated by the intermediation sector that is redistributed back to the households because they are either the managers of the bank or the residual claimants on the portfolio revenues of the bank. The inclusion of the transfer function has two advantages. The first is that any effect of a θ shock on house prices and welfare comes through the effect on interest rates, and is not generated by an aggregate loss of resources. The second advantage is computational, as the re-distribution of resources makes sure that aggregate consumption is a function of aggregate endowment only, an essential requirement for the application of the concept of wealth recursive equilibria proposed by Kubler and Schmedders (2003) to our framework.

2.2 Financial Market Equilibrium with Intermediation and Houses as Collateral

The economy is a collection of period-by-period utility functions, impatience parameters, state-dependent endowments and state-dependent financial intermediation efficiency, aggregate transfers, transition probabilities, and the bankruptcy cost in case of default,

$$\mathcal{E} = \left(u, \left(\beta_i, y_i, h_i(\sigma_0^-)\right)_{i=b,s}, \theta, \Upsilon, \Pi, m\right).$$

Definition 1 *A financial markets equilibrium for an economy $\mathcal{E}$, initial housing stocks $(h_i(\sigma_0^-))_{i=b,s}$ and initial shock z_0 is a collection*

$$\Big((\bar{h}_b(\sigma_t), \bar{d}_b(\sigma_t), \bar{c}_b(\sigma_t)), (\bar{h}_s(\sigma_t), \bar{d}_s(\sigma_t), \bar{c}_s(\sigma_t)), (\bar{D}(\sigma_t), \bar{S}(\sigma_t)),$$
$$\bar{q}(\sigma_t), \bar{R}_D(\sigma_t), \bar{R}(\sigma_t), \bar{\Upsilon}(\sigma_t)\Big)_{\sigma_t \in \Sigma}$$

satisfying the following conditions:

(1) Markets clear for all $\sigma_t \in \Sigma$:

$$
\begin{aligned}
n_b \bar{h}_b(\sigma_t) + n_s \bar{h}_s(\sigma_t) &= 1 \\
\bar{D}(\sigma_t) + n_b \bar{d}_b(\sigma_t) &= 0 \\
\bar{S}(\sigma_t) - n_s \bar{s}_s(\sigma_t) &= 0
\end{aligned}
$$

(2) For borrowers,

$$(\bar{h}_b(\sigma_t), \bar{d}_b(\sigma_i), \bar{c}_b(\sigma_t)) \in \arg \max_{c_b \geq 0, h_b \geq 0, d_b \leq 0} U_b(c_b, h_b)$$

such that for all $\sigma_t \in \Sigma$

$$
\begin{gathered}
c_b(\sigma_t) + \bar{q}(\sigma_t) h_b(\sigma_t) + d_b(\sigma_t) \leq y(\sigma_t) + d_b(\sigma_t^-) \bar{R}_D(\sigma_t^-) + \bar{q}(\sigma_t) h_b(\sigma_t^-) + \bar{\Upsilon}(\sigma_t) \\
\bar{R}_D(\sigma_t) d_b(\sigma_t) + m \cdot E[\bar{q}(\sigma_t z) | \sigma_t] h_b(\sigma_t) \geq 0
\end{gathered}
$$

(3) For savers,

$$(\bar{h}_s(\sigma_t), \bar{s}_s(\sigma_i), \bar{c}_s(\sigma_t)) \in \arg \max_{c_s \geq 0, h_s \geq 0, s_s \geq 0} U_s(c_s, h_s)$$

such that for all $\sigma_t \in \Sigma$

$$c_s(\sigma_t) + \bar{q}(\sigma_t) h_s(\sigma_t) + s_s(\sigma_t) \leq y(\sigma_t) + s_s(\sigma_t^-) \bar{R}(\sigma_t^-) + \bar{q}(\sigma_t) h_s(\sigma_t^-) + \bar{\Upsilon}(\sigma_t)$$

(4) For financial intermediaries

$$(\bar{D}(\sigma_t), \bar{S}(\sigma_i)) \in \arg \max_{D \geq 0, S \geq 0} \bar{R}_D(\sigma_t) D(\sigma_t) - \bar{R}(\sigma_t) S(\sigma_t)$$

such that for all $\sigma_t \in \Sigma$

$$D(\sigma_t) \leq \theta(\sigma_t) S(\sigma_t)$$

(5) Free entry for financial intermediaries

$$\bar{R}_D(\sigma_t) \bar{D}(\sigma_t) - \bar{R}(\sigma_t) \bar{S}(\sigma_t) = 0$$

(6) Per-capita transfers are given by

$$\bar{\Upsilon}(\sigma_t) = (1 - \theta(\sigma_t)) \bar{S}(\sigma_t)$$

2.3 Wealth Recursive Equilibria

For the quantitative exercise, we define a wealth recursive formulation in the spirit of Kubler and Schmedders (2003). Since we have only two agents, the relative wealth of one agent, defined by a single value on the unit interval, uniquely define the complement of the other agent relative wealth; the borrowers' beginning-of-period wealth-share is :[17]

$$\omega_b(\sigma_t) = \frac{q(\sigma_t)h_b(\sigma_t^-) + R_D(\sigma_t^-)d(\sigma_t^-)}{q(\sigma_t)} \tag{9}$$

Note that the collateral constraints, the constraints on asset holdings, and the utility functions satisfying Inada-conditions, together with assumption 1, imply that the wealth share lies in the unit interval, $\omega_b \in [0, 1]$; by definition, $\omega_s = 1 - \omega_b$. The equilibrium policy function is then a function of the discrete exogenous state variable z and the financial wealth distribution is $\Omega = (\omega_b, 1 - \omega_b)$.

As we solve for an equilibrium numerically, we follow Kubler and Schmedders (2003) and compute ϵ-equilibria.[18] For the approximation of the equilibrium policy functions we adopt the time-iteration algorithm with linear interpolation proposed by Grill and Brumm (2010). That is, we approximate the equilibrium policy on a fine grid for the borrowers' wealth share. For points outside the grid we use linear piecewise interpolation. See appendix B for a detailed description of the algorithm.

3 Quantitative Analysis

This section studies the quantitative effects of the Great Recession on house prices and households' welfare. The Great Recession is modeled as contemporaneous negative shocks to both aggregate income and financial intermediation (mortgage rate spread). In this way, our simulation is driven by the empirical facts that motivated our research question. The next subsection outlines our calibration strategy. We then have a short section on the long-run stationary wealth distribution and we present our quantitative results on welfare effects.

3.1 Calibration

In the benchmark calibration, we assume an elasticity of substitution between houses and consumption equal to 1, so that $\rho = 0$. Risk aversion is set equal to $\gamma = 2$. These are

[17] Here, we used the market clearing conditions for the housing, debt, and savings markets and the fact that financial intermediaries make zero-profits in equilibrium, so that $h_b(\sigma_t^-) + h_s(\sigma_t^-) = 1$ and $R_D(\sigma_t^-)d_b(\sigma_t^-) + R(\sigma_t^-)s_s(\sigma_t^-) = 0$.

[18] For a definition and interpretation of ϵ-equilibria, we refer to the original text.

standard values used in the literature. In general, it is not straightforward to calibrate these parameters as macro and micro evidence span a relatively large sets of parameter estimates. As in (Glover, Heathcote, Krueger, and Ríos-Rull, 2011), the risk aversion γ is the crucial parameter for the elasticity of house prices with respect to aggregate shocks. The elasticity of substitution between consumption and savings plays an important role for the elasticity of welfare gains/losses to changes in the wealth distribution. Therefore, in section 4, we provide a sensitivity analysis for different values of the risk aversion parameter and allow for some substitutability between housing and non-durable consumption as recently found by Bajari, Chan, Krueger, and Miller (forthcoming). Notice that one period in the model corresponds to one quarter in the data.

The parameter ϕ is the expenditure share of non-durable consumption. We pick the value to match the average housing wealth over GDP in the data during the period 1998-2007. For aggregate housing wealth, we used the sum of the value of owner occupied real estate of private households plus the residential housing wealth of non-financial non-corporate private business. The savers' discount factor β_s is set so that the average interest rate on savings in the model matches the average return on savings, equal to 1.5% during 1998 - 2007 (at annualized level). The borrowers' discount factor β_b and m are jointly calibrated to match the average wealth share of the borrowers and the leverage ratio of the borrowers. Since there is not necessarily a one-to-one mapping between the parameters and their targets, we follow an iterative procedure to find values for β_s, β_b, m and ϕ. That is, we first guess values for the parameters and then compare the computed moments to their counterparts in the data. If they do not match, we change the values and repeat until they do. The procedure leads to a quite satisfactorily match between model and data moments.[19]

The relative population size of borrowers is set to 42%, corresponding to the fraction of borrowers in the SCF when using the weighted average share of households with a negative net asset position as defined in appendix A. This estimate is in line with the calibration in Iacoviello (2008).

The stochastic processes for the exogenous state variables y_t and θ_t are assumed to be independent. This is in line with the correlation in the data.[20] We assume that both aggregate income and the intermediation spread shock take two values each, that

[19]The variable definitions used to calculate the data moments are as close as possible to the definition of the model counterparts. For a detailed description of how we compute the relative wealth share and the leverage ratio in the data, see appendix A.

[20]We also conducted a VAR analysis for GDP growth and spreads for different lag-lengths and orderings and found no evidence for significant spillover terms and no contemporaneous correlations between GDP and mortgage spreads. Only in one specification (VAR of order two), the null of a Granger-causality of output growth on spreads is rejected, though the coefficients for individual lags of output were not significantly different from zero.

Table 2: calibration

Parameter	Value	Model	Data Target	Source
Preferences				
γ	2			Benchmark value from literature
ρ	0			Benchmark value from literature
ϕ	0.97	196%	196%	Average housing value over GDP (annualized) 1998 - 2009
β_s	0.996	1.5%	1.5%	Average return on savings (annualized)
β_b	0.988	11.7%	11.3%	Borrowers' financial wealth share (SCFaverage 1998-2009)
m	0.5	45%	44.4%	Borrowers' leverage ratio (SCF average between 1998-2009)
Relative population size				
n_b	0.42	42%	42%	Share of borrowers (SCF average 1998-2009)
Intermediation shock				
π^{θ}_{H}	0.565		56.5%	Probability of low spreads during 1998-2009:II
ρ_{θ}	0.868	0.868	0.868	Autocorrelation of spreads during 1998-2009:II
θ_L	0.9985	1.8 %	1.75 %	Average spread during 1998-2009:II (annualized)
θ_H	0.99207	1.27 %	1.27 %	Standard deviation of spread during 1998-2009:II (annualized)
Income shock				
π^{y}_{H}	0.85	15%	15%	Probability of recession 1980- 2009:II (NBER dates)
π^{y}_{LL}	0.8	5 quarters	5 quarters	Average duration of recession (NBER dates) 1980- 2009:II
y_L	0.9572	5%	5%	Average Peak to trough drop in GDP 1980-2009:II
y_H	1.0076			Normalization $E(y) = 1$

is $y_t = \{y_L, y_H\}$ and $\theta_t = \{\theta_L, \theta_H\}$. For the intermediation shock, we assume that the transition probabilities are given by:

$$\pi_{ij} = (1-\rho)\pi_j + \delta_{ij}\rho \qquad for\ i, j = H, L$$

where $\delta_{ij} = 1$ if $i = j$ and 0 otherwise; $\pi_j > 0$ is the unconditional probability of being in state j, and by definition we have $\sum_j \pi_j = 1$. The parameter ρ governs the persistence of the shock.[21] The unconditional probability of a high intermediation efficiency, $P(\theta = \theta_H)$, is set to 0.565, the fraction of quarters in which the U.S. experienced low spreads between 1998:I and 2009:II. We set $\theta_L = 0.99207$, $\theta_H = 0.9985$, and $\rho_\theta = 0.868$ so that we match the mean, standard deviation and the autocorrelation of the spreads in the data (for the data counterparts see table; for a description of the data see appendix A).

For the income shock, we choose y_H and y_L to match the mean, normalized to $E(y) = 1$, and an average peak-to-trough drop in GDP of 5% during a recession. The conditional probability of the low realization of y being in a recession today π^y_{LL} is chosen to match an average duration of a recession equal to five quarters. This is in line with the NBER recession dates between 1980:I and 2009:II. The transition probability of the high income realization conditional on high income today, $\pi^y_{HH} = 1 - (1 - \pi^y_{LL})\frac{1-\pi^y_H}{\pi^y_H}$, is obtained by setting the unconditional probability of a recession equal to 15% ($\pi_H = 0.85$). This is in line with NBER recession dates between 1980:I and 2009:II.

To summarize, the exogenous state space is then given by $\Sigma = \{(y_H, \theta_H), (y_L, \theta_H), (y_H, \theta_L), (y_L, \theta_L)\}$ and - given the assumption that income and intermediation processes are uncorrelated - the transition matrix for the exogenous process is just the Kronecker product of the individual transition probability matrices for the income shock and the intermediation shock. Table 2 summarizes the calibrated parameter values and the targets.

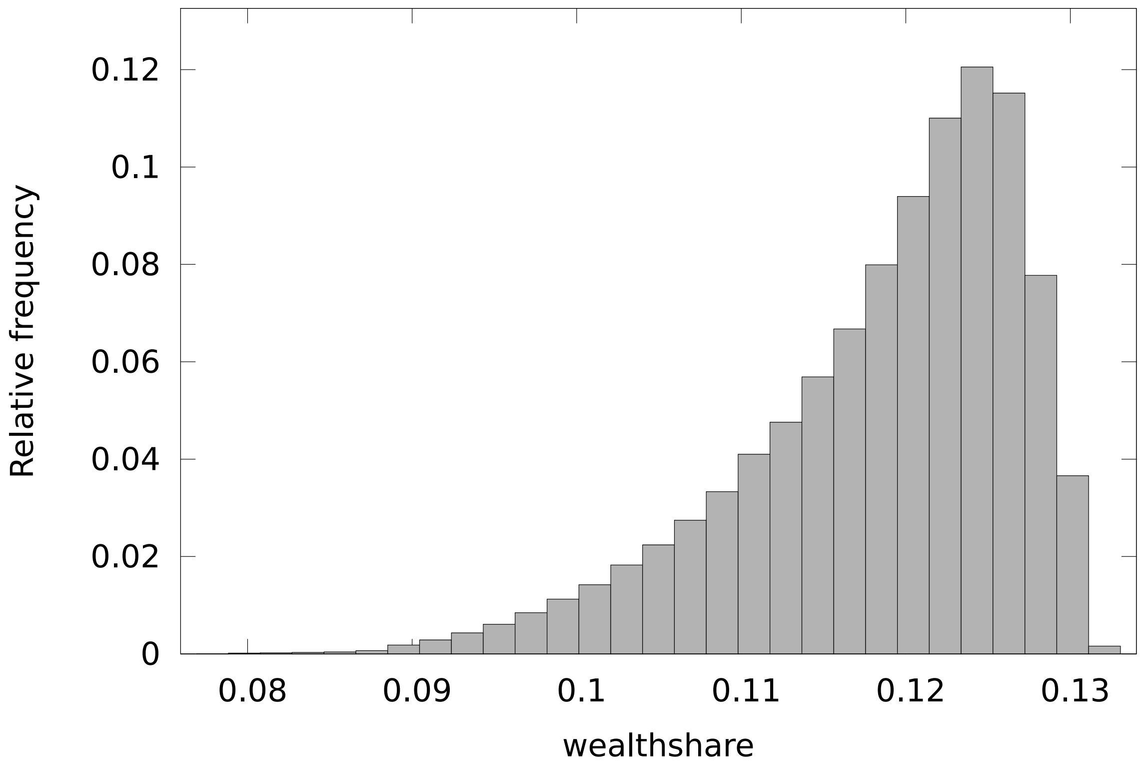
3.2 Stationary wealth distribution

Figure 3 shows the long-run stationary wealth distribution simulated over one million time periods.[22] Recall that the wealth distribution across agents is entirely summarized by the borrowers' fraction of wealth ω_b. On average, the borrowers hold 11.7% of the total wealth of the economy (which is equal to the value of housing q). The distribution of the borrowers' wealth share is concentrated around the mean and has a spike to the right at around 12.6%, which correspond to states of the world when there is a long period of credit and income expansion. In these states, the borrowers' collateral constraint is binding and the interest rate on borrowing is relatively low; demand for housing is high and expected house prices are therefore high. This marginally relaxes the constraint, so that aggregate debt and savings are high. Because house prices are rising and borrowers are accumulating housing, their wealth share increases. Conversely, negative realizations of aggregate shocks make the borrowers' wealth share drop. We will

[21] See Backus, Gregory, and Zin (1989) and Mendoza (1991)

[22] Because of the simple persistence rule used to discretize the exogenous processes, the high number of simulation periods makes sure that the exogenous processes have the same stochastic properties as their data counterparts.

Figure 3: Wealth distribution



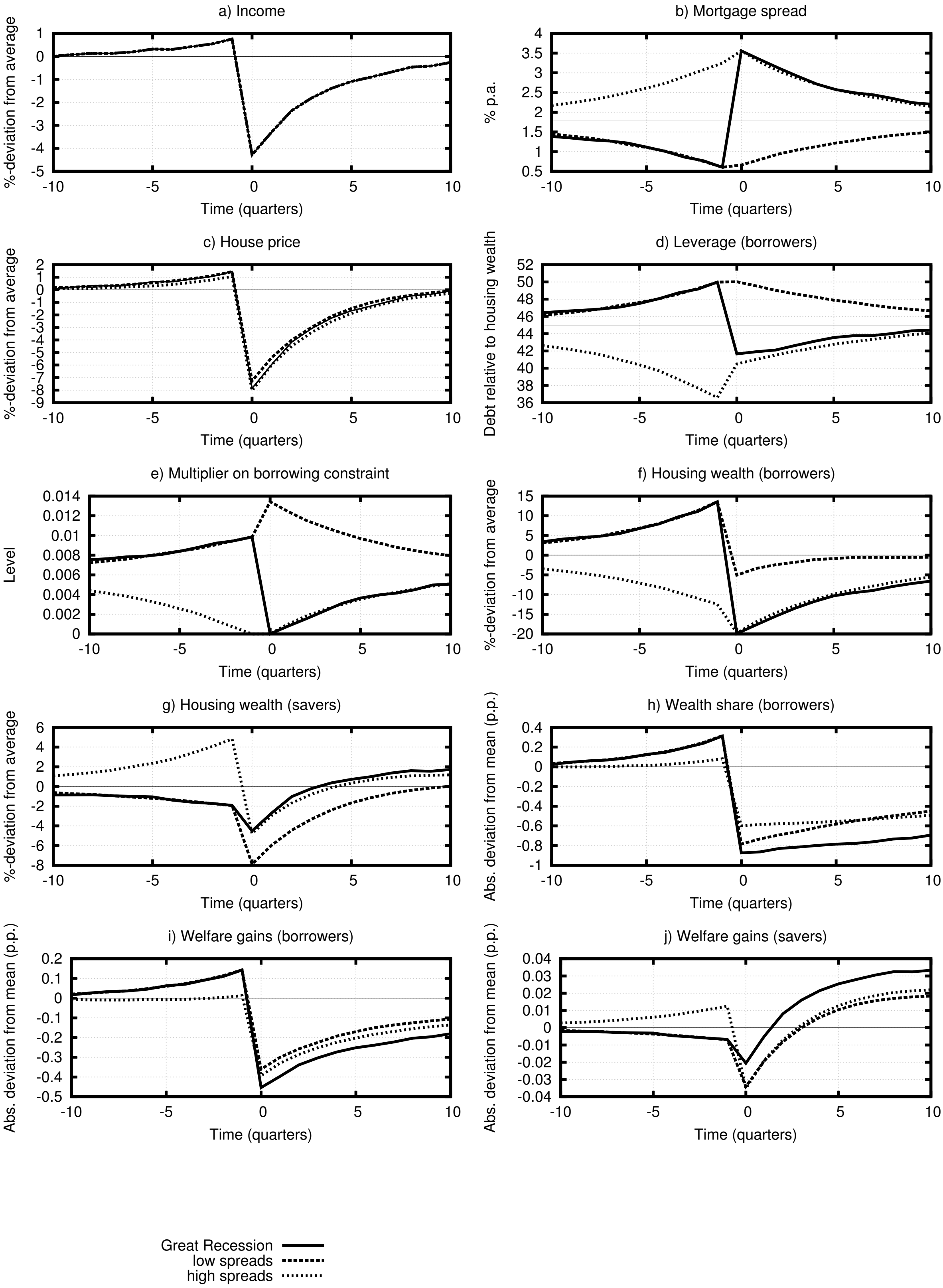
explain these mechanisms in detail in the following section(s).

3.3 Welfare effects in the Great Recession

We now turn to our main quantitative exercise, the estimation of welfare effects of the Great Recession. For this purpose we construct an event window around the Great Recession. We define the Great Recession as a state of the world with low income and high spreads that is preceded by a state of the world where income is high and spreads are low (i.e. intermediation is high). We then go along the equilibrium path of the simulated economy and select all sequences that match these criteria. In figure 4, we plot the average of selected realizations over all sequences including ten quarters preceding the crisis and ten quarters after the crisis. We compare the Great Recession to two counterfactual scenarios. First, we ask what would happen if spreads were low before and stayed low *during* the recession (this corresponds to the long dashed line in 4 which we label as *low-spreads* series). This experiment helps us to compare the welfare effects of a negative income shock when leverage is high or low before the shock realizes. Second, we look at a recession that occurs when spreads where already high before and during the crisis (short dashed line in figure 4 which we label as *high-spreads*). By comparing this scenario, with the Great Recession, we calculate the welfare effects of de-leveraging in the crisis.

Panel (a) and (b) show the evolution of income and mortgage spreads. In all scenarios, income first increases previous to the recession and then drops by 5 percent in period 0 when the recession hits. In the Great Recession, mortgage spread first decreases towards its lowest value in period -1 and then jump to 3.5 percent in period 0. In the *low-spreads* counterfactual scenario spreads decline and stay in their lowest realization in periods -1 and 0 and then return towards their long-run mean, around 1.75 percent per annum. Similarly in the *high-spreads* counterfactual scenario, spreads increase slowly previous to the recession, peaking at 3.5 percent p.a. in period 0 and then return slowly towards their long-run mean. From panel (c) it is evident that house-prices are clearly driven by aggregate income and not by mortgage spreads. Mortgage spreads, however, have an important impact on the borrowers' leverage ratio, defined as end-of-period leverage or $L_t^{EoP} = -\frac{d_t}{q_t h_{bt}}$; when spreads are low, borrowers leverage up by increasing their debt holdings faster than their housing wealth. This means they move towards the constraint. In our simulation, in the pre-crisis, leverage peaks at around 50 percent. When spreads increase in period 0, it becomes too costly for borrowers to roll-over their mortgages and de-leverage sharply so that the constraint gets slack. This is reflected by the multiplier associated with the collateral constraint that drops to zero. The time-path of leverage looks quiet different under the other two counterfactual scenarios. In the *low-spreads* case, borrowers stay leveraged also in period 0 and then de-leverage slowly following the path of spreads. In the *high-spreads* case, aggregate leverage is already low previous to the negative income shock and borrowers are pushed towards the collateral constraint in period 0 when house prices fall. This is because borrowers search to smooth the recession by borrowing up to the limit (which is tighter because the house price drops in the recession). This is also reflected by the increase in the multiplier on the collateral constraint shown in panel (e). Therefore, shocks to financial intermediation affects the borrowers' leverage ratio through the relative price of debt (the mortgage spread). Panels (f) and (g) show the paths for housing wealth for borrowers and savers, respectively. This figures illustrate the following. If mortgage spreads would have stayed low during the recession (*low-spreads* case), borrowers would have lost less in terms of housing wealth than in the benchmark scenario, whereas savers would have lost more housing wealth. The movements in leverage and housing wealth are reflected by the evolution of borrowers' wealth share, shown in panel (h). In this panel the solid line shows drop much more than the the long-dashed line. Importantly the wealth share recovers much slower after the Great Recession compared to the case when mortgage spreads would have stayed low during the crisis. This means that borrowers negative wealth shock is quite persistent in the Great Recession. Finally, panels (i) and (j) show the corresponding welfare gains

Figure 4: Great Recession (solid line) versus different intermediation regimes



for the two type of households (in consumption equivalents relative to long-run expected utility, for a formal definition see next paragraph). Borrowers lose the most in the Great Recession while savers lose the least when compared to the other counterfactual scenarios. Note that only after two or three quarters, savers' expected life-time utility becomes positive and stays persistently above zero. This indicates substantial redistributive forces that is connected to the discussion about the borrowers' relative wealth share.

These findings are quantitatively formalized in table 3. The table compares the model predictions with the data (we observe the on-impact change in house price, the change in housing wealth for borrowers and savers in the period 2007-2009) and - in addition - shows the average change in borrowers' wealth share and the welfare gains/losses in the recession for the two types of households, denoted by λ_b and λ_s, respectively. We define welfare gains in two ways. First, we define welfare gains of the recession as the compensation that is needed to make agents indifferent between the expected life-time utility in period -1 (i.e. the quarter that precedes the recession) and expected life-time utility in period 0 (i.e. the quarter when the recession hits). Negative numbers therefore reflect welfare losses of the recession. We refer to these numbers as '*on-impact welfare gains*'. Second, we report welfare gains of the expected life time-utility that agents have 7 periods[23] after the recession relative to the average expected life-time utility, that is $\sum_{\sigma=1}^{4} \pi_\sigma V_i(\omega(\sigma), \sigma)$ for $i = b, s$.[24] Also in this case we report the welfare gains in percent of total consumption compensation that is needed to make agents indifferent between the two alternatives. We refer to this second type as '*welfare gains after 7 periods*'.

Based on figure 4 and table 3 we can summarize the following two key findings:

1. High leverage makes the borrowers' wealth share more sensitive to house price changes.

2. A negative intermediation shock, when coupled with a negative income shock, results in higher (smaller) welfare losses for borrowers (savers).

Result 1 says that the higher the leverage ratio in the economy when entering a recession, the more the wealth gets distributed away from borrowers to savers. In other words, a given house price drop due to an aggregate income shock leads to more bigger wealth losses for borrowers to savers when there is more leverage prior to the shock. If the economy is experiencing high intermediation efficiency previous to a recession, the leverage ratio of borrowers will be high. The borrowers' wealth share will then be very sensitive to price changes.

[23]The recent recession lastet 7 quarters according to NBER recession dates.

[24]The probability π_σ is the unconditional (or stationary) probability that state $\sigma \in \Sigma$ occurs.

Table 3: Welfare effects of a recession (5 percent drop in income) for different spread regimes

	Δq	$\Delta(qh_b)$	$\Delta(qh_s)$	$\Delta\omega_b$	λ_b	λ_s
Data	-11	-16	-9	?	?	?
On impact, relative to pre-recession peak						
Great Recession	-9.18	-29.47	-2.65	-1.19	-0.60	-0.01
Low spreads	-8.59	-16.42	-6.07	-1.10	-0.50	-0.03
High spreads	-9.00	-8.34	-9.15	-0.68	-0.41	-0.05
After 7 periods, relative to long-run mean						
Great Recession	-1.29	-9.54	1.00	-0.78	-0.24	0.03
Low spreads	-1.01	-0.56	-1.13	-0.55	-0.15	0.01
High spreads	-1.37	-8.79	0.69	-0.54	-0.18	0.02

Notes: Column two shows the percentage change of the house price between date -1 and date 0, the period of the recession. Column three and four tabulate the percentage change in housing wealth between date -1 and 0 for borrowers and savers, respectively. Column four tabulates the absolute change of the borrowers' wealth share between date -1 and date 0 (in percentage points). Columns six and seven show the welfare gains of the recession in total consumption equivalents (relative to expected utility in period -1) for borrowers and savers, respectively. The Great recession is defined as a contemporaneous drop in income and financial intermediation (i.e. high spread) in period 0. The counterfactuals in row three (four) assume that financial intermediation is high (low) in both periods -1 and 0.

Result 2 deals with the second question raised in the introduction: whether a larger redistribution of wealth translates into more inequality in terms of welfare. We find that this crucially depends on whether the collateral constraint binds. That is, whether borrowers wish to stay up against the constraint, or move away from it. This result implies that the wealth loss from a recession only translates into a larger (smaller) welfare loss for borrowers (savers) when there is a simultaneous deterioration in the efficiency of financial intermediation. In particular, when spreads would have stayed low during the recession, shown in row three, the borrowers' welfare gain would have been 17 percent higher compared to the Great Recession. Savers would have lost three times more compared to the Great Recession. The intuition for both results is summarized in the following two paragraphs.

Intuition for Key Result 1 Let us now show the intuition behind these results graphically. To see the effects on the wealth distribution, we can rewrite the borrowers' wealth share in terms of the leverage ratio:

$$w_{b,t} = h_{b,t-1}(1 - L^{BoP}(q_t)) \tag{10}$$

where $L^{BoP}(q_t) = -\frac{R_{D,t-1}d_{b,t-1}}{q_t h_{b,t-1}}$ denotes the beginning of period leverage carried over from last period, evaluated at the house price of the current period.[25] Taking the total derivatives of the wealth share around $q_t = q_{t-1}$, one can see that the growth rate of the borrowers' wealth share is proportional to the growth rate of house prices and the proportionality factor is a function of leverage:

$$\frac{dw_{b,t}}{w_{b,t}} = \frac{L(q)}{1-L(q)}\frac{dq}{q}.$$

If financial intermediation efficiency is low and spreads are high, leverage is likely to be small and a given drop in house prices translates into a smaller drop in wealth. In other words, when borrowers' leverage is high, any aggregate price drop makes borrowers - on impact - relatively poorer in terms of wealth.

Of course, the price today is an equilibrium outcome; that is, the pricing function depends on the state of the economy. We have no closed form solution for this pricing function but we can plot the equilibrium house prices as a function of the wealth share using the simulated economy. This function is - for any realization of the exogenous shock $z \in \mathcal{Z}$ - decreasing in w_b, or

$$q = Q(w_b, z) \qquad \frac{\partial Q}{\partial w_b} < 0. \tag{11}$$

Given the promised value of previous-period debt, $R_{D,t-1}d_{b,t-1}$, and given the housing stock carried over from last period, $h_{b,t-1}$, the equilibrium wealth share in period t is implicitly defined by the solution to (10) and (11), or

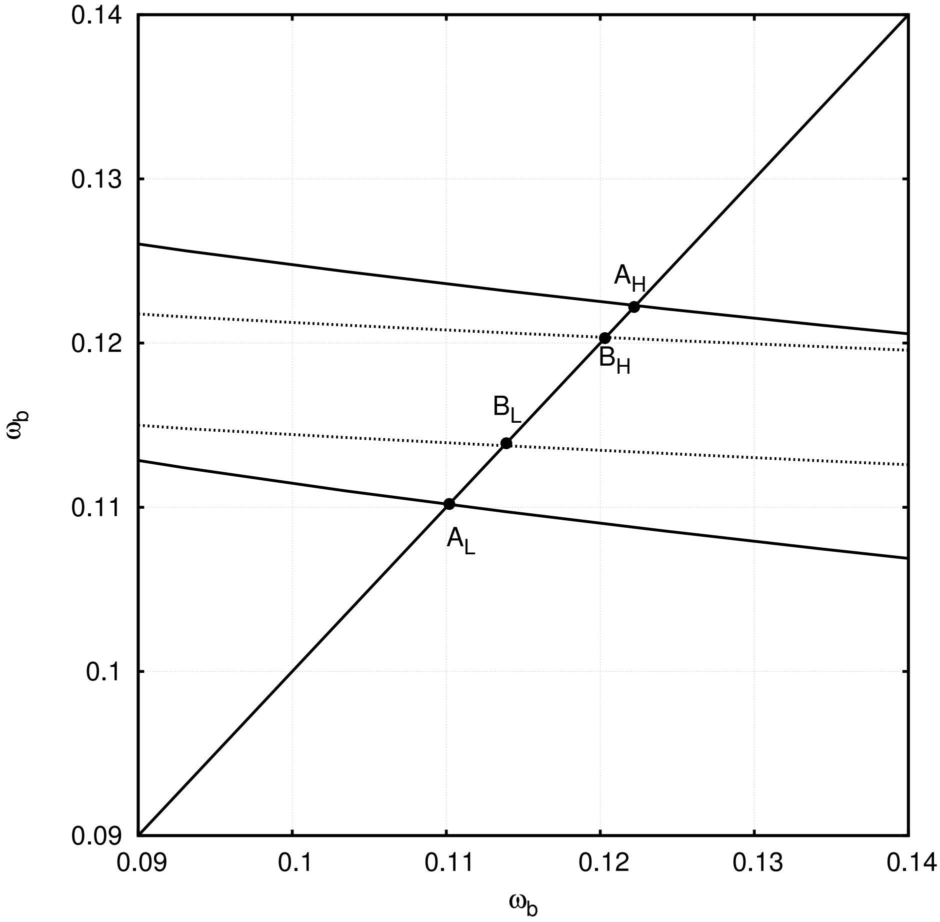
$$w_{b,t} = h_{b,t-1}\left(1 + \frac{R_{D,t-1}d_{b,t-1}}{Q(w_{b,t}, z_t)h_{b,t-1}}\right) \tag{12}$$

Figure 5 plots the left-hand side and right hand side of equation (12) as a function of the borrowers' wealth share w_b for different income realizations and for given assumptions on the level of debt and housing level. The solid line plots the right-hand side of equation 5 under the assumption that value of debt and housing stock in $t-1$ are relatively high (i.e. intermediation efficiency was high), while the dashed line assumes that debt and housing stock carried over from the previous period are low (i.e. financial efficiency was low).[26] When the previous period debt is high (solid line), the wealth share is more

[25] Note that by assumption 1, $L^{BoP}(q_t)$ is strictly smaller than one. This can be seen by the following. When leverage is high, most likely the collateral constraint is binding. Using the collateral constraint from last period and substituting it into the definition of beginning-of-period leverage, one obtains $m\frac{E_{t-1}(q_t)}{q_t}$. By assumption 1 and verified ex-post along the equilibrium path, this object is smaller than one.

[26] We set the respective values for housing stock and debt equal to the average value in period -1 of the event window above for the respective intermediation regime.

Figure 5: Response of equilibrium wealth share to a negative income shock, for previously high (solid lines) versus low intermediation (dashed lines)

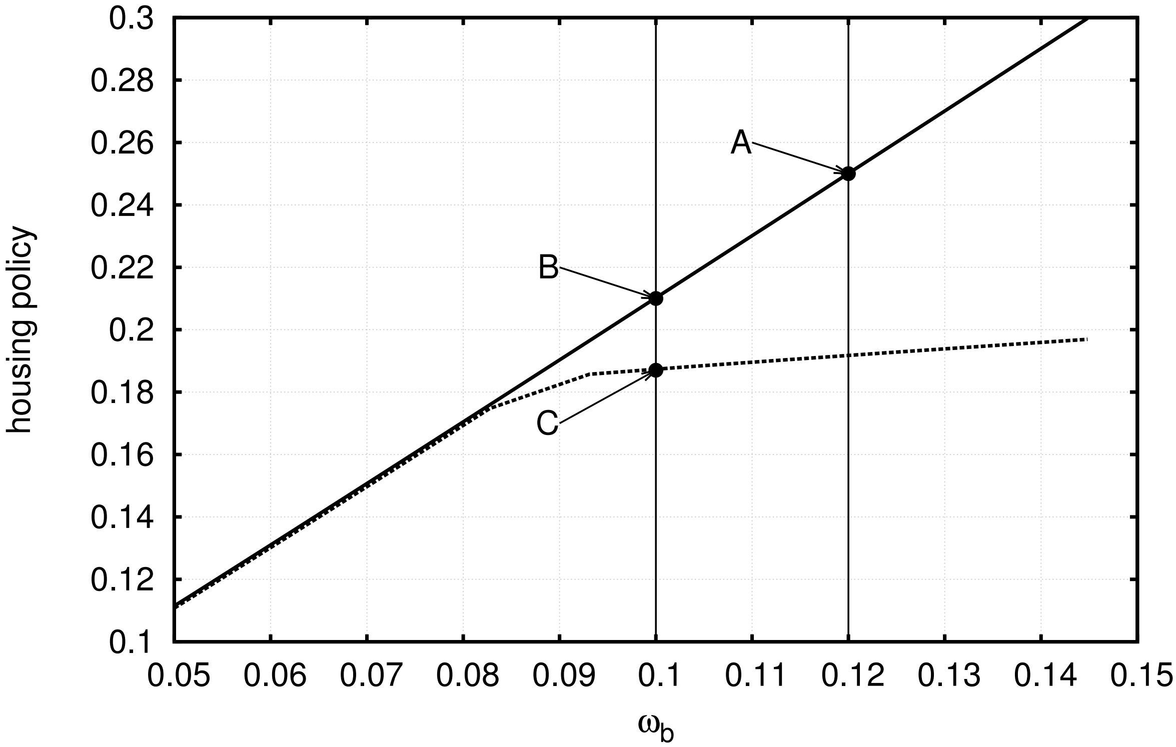


Notes: The figure plots the left-hand side (45 degree line) and the right hand side of equation (12) as a function of the borrowers' wealth share w_b and for different intermediation regimes. The solid lines show the right-hand side under the assumption that $h_{b,t-1}$ and $R_{D,t-1}d_{b,t-1}$ are relatively high (in absolute value) because of high financial intermediation. Given the assumption on debt and housing, point A_H materializes if income stays high whereas A_L is the wealth share when income drops to y_L. The dashed line shows the right-hand side under the assumption that $h_{b,t-1}$ and $R_{D,t-1}d_{b,t-1}$ are relatively low, that is for low intermediation. In this scenario, B_H is the wealth share that materializes when income stays high, whereas the wealth share drops to B_L when income falls to y_L

sensitive to exogenous shocks to income (drop from point A_H to A_L) compared to the case when debt carried over from last period is relatively low (drop from B_H to B_L). This illustrates the relationship between leverage and wealth dynamics during a recession: the effect comes from a different elasticity of wealth with respect to changes in prices which, in turn, depend on the aggregate state of financial intermediation.

Intuition for Key Result 2. Result (2) relates to combined income and negative intermediation shock. When house prices fall and there is a contemporaneous negative intermediation shock, borrowers face a higher interest rate on debt, which prevents them from rolling over the debt and moving away from the collateral constraint. This forces the borrowers to substantially decrease their stock of housing.

Figure 6: Equilibrium housing policy depends non-linear on wealth when financial intermediation efficiency changes from high to low



Notes: Solid line: housing policy as a function of the borrowers' wealth share, conditional on high financial intermediation efficiency. Dashed line: housing policy as a function of wealth, conditional on low financial efficiency. The vertical line intersecting at A is the borrowers' wealth share in a state with high income and high financial intermediation and the vertical line intersecting at B is borrowers' wealth share in the period when the Great Recession hits the economy.

Figure 6 plots the borrowers' housing stock policy function for high and low intermediation efficiency (respectively solid and dashed line). Following the Great Recession, the relative wealth of the borrower drops. As financial intermediation also drops during the recession from high to low efficiency, the housing stock drops from A to C. This is a substantially larger drop than would have occurred had the efficiency of intermediation stayed high. In this case, for the same drop in wealth, the decrease of the housing stock would have been less sharp (from A to B). In other words, the elasticity of demand for housing with respect to income shocks depends on the efficiency of the financial intermediation sector.

Summary of the welfare effects. First, both agents lose in response to an aggregate negative income shock, and borrowers always lose more than savers because they are financially constrained and unable to cushion themselves from negative shocks. Second, while borrowers experience a welfare loss in the case of a negative financial intermediation shock, savers are virtually unaffected. Third, in the simulated recession, we observe that the borrowers' welfare loss is larger than the algebraic sum of the welfare losses in response to negative income and intermediation shocks in isolation. The opposite is true for the savers. This comes from a non-linearity in the reaction of consumption that comes when borrowers are forced to de-leverage and move away from the collateral constraint. In such a scenario savers can even gain from the joint income and intermediation shock (relative to an income shock alone) because they become relatively wealthier.

This set of results leads to the conclusion that, following the Great Recession, while both types of agents experienced a welfare loss, savers could cushion themselves from the negative impact of the negative aggregate shocks by substituting their savings for depreciated houses. This conclusion, while qualitatively comparable with the recent findings of Hur (2012), highlights a different mechanism. In this model, savers are able to cushion themselves from the negative effects of the Great Recession because of the asymmetric effects of financial intermediation shocks and the high level of leverage prior to the shock.

An important remark relating to the magnitudes of the obtained welfare estimates concerns the error analysis of our numerical algorithm. That is, if the mistakes agents make using our algorithm are larger (in consumption equivalents) than the calculated welfare gains/losses, these numbers would have no quantitative validity. We find that the maximum relative Euler Error of our approximation is 3e-5 (or -4.5 in log(10)-scale). This implies that an agent, using our approximation of the equilibrium policy functions, would lose 30 Dollars for each million spent. For details see appendix B.4. We therefore conclude that our quantitative findings are valid and quantitatively meaningful.

3.4 Always binding collateral constraint

We solve the model employing a global solution method rather than the more widely used log-linearization method. This is necessary in order to take into account the fact that the collateral constraint is not always binding, but comes at the cost of a more complex numerical implementation. In this section we show how large is the cost of assuming always binding constraints in this framework.

To this end, we solve an alternative specification of the model by forcing the borrowers to have an always-binding constraint. In this case, the leverage ratio of the economy is always equal to $m\frac{E_{t-1}q_t}{q_t}$, which therefore needs to be re-calibrated for this specification in order to match the leverage ratio we find in data. The results are summarized in

Figure 7: Great Recession in benchmark model (solid line) versus always binding constraint (dashed line)

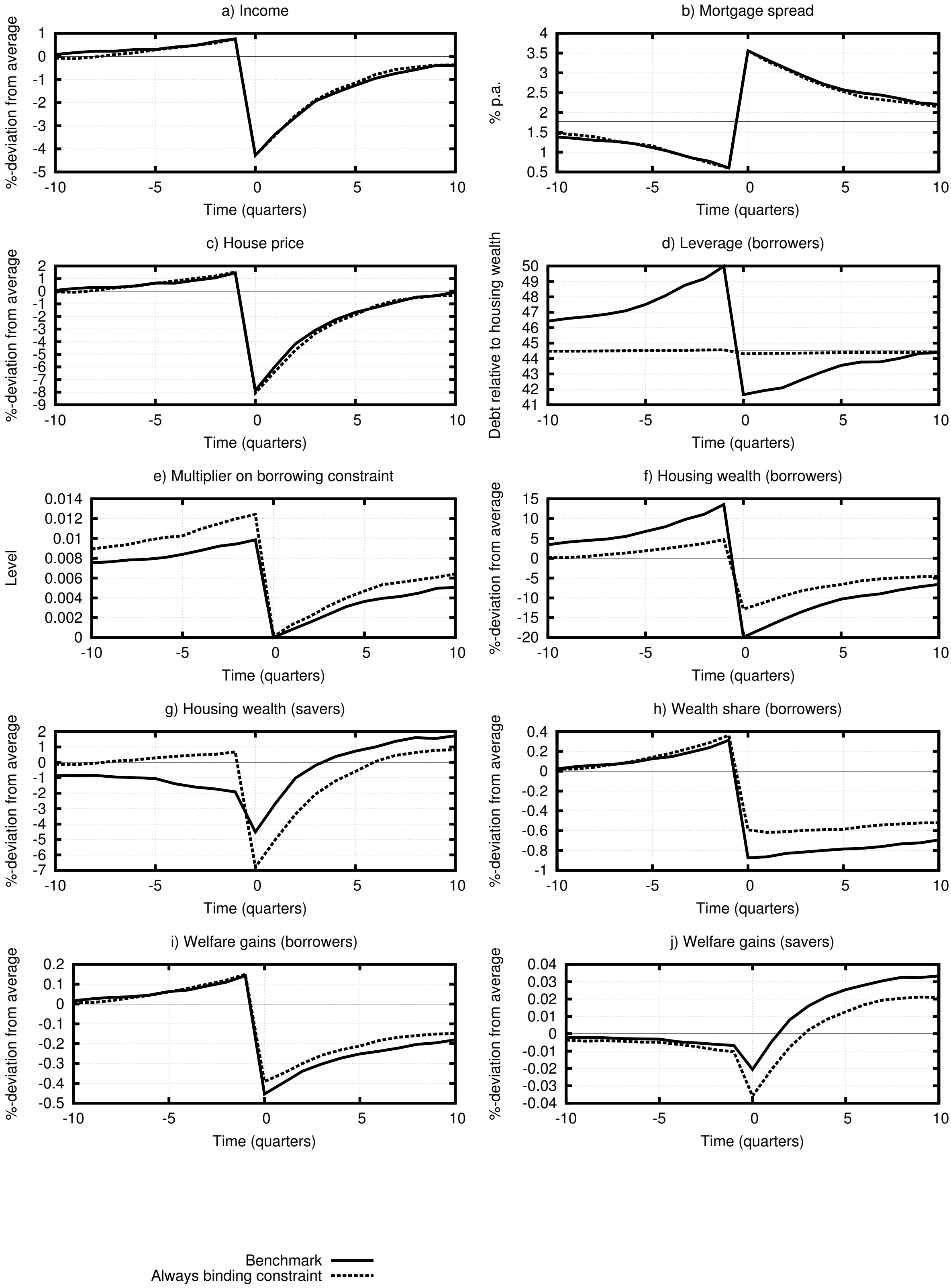


table 4. Compared to the benchmark model, we find that in a version of the model with always-binding collateral constraints: (i) the quantitative effects on house prices are larger relative to the benchmark model for a negative financial intermediation shock; ii) in the Great Recession, the welfare losses for borrowers (savers) are smaller (higher) in absolute terms. To summarize, the borrowers' welfare loss is lower by 0.07 percentage points (in absolute terms), while the savers' lose 0.04 percentage points more when compared to the benchmark model. Most importantly, the non-linearity of previous-period leverage completely vanishes, as the borrowers' wealth losses and the agents' welfare gains are just the algebraic sum of the effects when the economy is hit with each shock separately.

Table 4: Always binding collateral constraint

	Δq	$\Delta(qh_b)$	$\Delta(qh_s)$	$\Delta\omega_b$	λ_b	λ_s
Data	-11	-16	-9	?	?	?
On impact, relative to pre-recession peak						
Great Recession	-9.44	-16.72	-7.43	-0.95	-0.54	-0.03
Low spreads	-8.65	-14.83	-6.94	-0.86	-0.46	-0.04
High spreads	-8.58	-15.17	-6.86	-0.88	-0.47	-0.04
After 7 periods, relative to long-run mean						
Great Recession	-1.16	-5.73	0.06	-0.56	-0.18	0.02
Low spreads	-1.21	-2.91	-0.75	-0.23	-0.09	-0.00
High spreads	-1.05	-7.83	0.76	-0.83	-0.25	0.03

Notes: Column two is the change in house prices between period -1 (the period just before the shock occurs) and period 7 (following the start of the recession). Column three shows borrowers' start-of-period leverage ratio, defined as $L_{b,t}^{BoP}$ in the period of the shock $t = 0$; note that this leverage ratio is a function of the price today only (variables with subscript $t-1$ are given numbers). Column four shows the corresponding change in borrowers' financial wealth share between period -1 and period 7. Column five reports the borrowers' end-of-period leverage ratio, defined as L_t^{EoP} after the Great Recession - in period $t = 7$. Columns six and seven show the welfare gains/losses of borrowers and savers, respectively. All numbers are in percent.

The reason for these differences is that models with always binding constraint have the peculiarity of a constant elasticity of demand for debt with respect to changes in interest rate. In other words, following a spread shock, the borrowers' change in next period's debt has to be strictly proportional to the present discounted value of the drop in next period's housing wealth. When debt is costly, borrowers are prevented from moving away from the constraint. Aggregate debt moves less with respect to the benchmark case and this, in equilibrium, reduces the savers' ability to switch from savings to housing. This is the reason why house prices drop more in response to a negative intermediation shock. The elasticity of borrowers' wealth share to any given drop in house prices is always constant and given by $\frac{m^{AB}}{1-m^{AB}}$, where the superscript stands for 'always binding'.

Note that, in order to match the average leverage ratio in the data, $m^{AB} = 0.45$, which is lower than $m = 0.5$ in the benchmark calibration. The elasticity of the borrowers' wealth share is therefore constant, and is strictly less than one. This result suggests that the assumption of always-binding collateral constraints is not innocuous when making a welfare analysis.

4 Sensitivity Analysis

In this section we compare the quantitative implications of changing the elasticity of substitution between housing and non-durable consumption, and the coefficient of risk aversion. Note that, for all changes in these parameters, we re-calibrate the rest of the parameters that in order to match the targeted data moments. This allows us to compare the relative performance of each parameterization with the benchmark case.

4.1 Elasticity of substitution between housing and non-durable consumption

Here we conduct a sensitivity analysis for one of the two parameters that we fixed in the benchmark calibration to unity: the elasticity of substitution between housing and non-durable consumption. Table 5 summarizes the quantitative findings for a higher level of substitutability between housing and non-durable consumption, setting $\rho = 1.25$.[27]

Table 5 shows that, with increasing substitutability between housing and non-durable consumption, house prices (and therefore wealth) react more strongly to an intermediation shock when compared to the benchmark case. This, like in the case with the always-binding constraint, results in a decreased elasticity of demand for debt with respect to changes in the interest rate for borrowing. In addition, the Great Recession leads to smaller (bigger) welfare losses for borrowers' (savers') in this calibration. Borrowers are hurt less because they substitute housing for non-durable consumption, which is less painful when these goods are substitutes. This is also the reason why there is less redistribution in terms of welfare from borrowers to savers. Though, in absolute terms, savers lose more. Nevertheless, the key findings relating to the role of leverage in wealth dynamics and the role of the intermediation shock in a recession are unchanged.

[27] This parameter value is taken from Piazzesi, Schneider, and Tuzel (2007), who consider a representative agent framework with housing; As mentioned earlier in the paper, an elasticity of substitution larger than one between housing and non-durable consumption has also recently been found by Bajari, Chan, Krueger, and Miller (forthcoming).

Table 5: Welfare effects in model with higher elasticity of substitution between housing and non-durable consumption

	Δq	$\Delta(qh_b)$	$\Delta(qh_s)$	$\Delta\omega_b$	λ_b	λ_s
Data	-11	-16	-9	?	?	?
On impact, relative to pre-recession peak						
Great Recession	-9.10	-38.87	-0.34	-1.19	-0.57	-0.02
Low spreads	-8.54	-17.83	-5.78	-1.12	-0.49	-0.03
High spreads	-9.09	-6.71	-9.51	-0.47	-0.35	-0.06
After 7 periods, relative to long-run mean						
Great Recession	-0.96	-12.92	1.88	-0.82	-0.23	0.03
Low spreads	-1.05	0.92	-1.52	-0.64	-0.16	0.02
High spreads	-1.52	-11.74	0.90	-0.34	-0.13	0.00

4.2 Risk aversion

In this section we show quantitative analyses of Great Recession episodes for different values of the risk aversion parameter taken from the related literature. In particular, while the business cycle literature usually features a log-separable utility function with elasticity of substitution and risk aversion equal to unity, the macro-finance literature and recent contributions on the distributive effects of the Great Recession focus on a broader set of parameter values for risk aversion.[28] Table 6 summarizes the effects of the simulated Great Recession for the benchmark and other model specifications for different values of the risk aversion parameter.

As in Glover, Heathcote, Krueger, and Ríos-Rull (2011), the higher is the coefficient of risk aversion, the higher is the negative impact of a recession on equilibrium aggregate house prices. The the observed drop in the house price during the Great Recession is consistent for a risk aversion parameter between 2 and 3. The welfare analysis also confirms that bigger wealth shocks (due to the drop in house prices) translate into larger negative welfare effects for borrowers. This effect is again amplified by financial intermediation shocks, which make it more difficult to smooth negative income shocks. In contrast, savers are more able to cushion themselves from the negative effects of the Great Recession. The intuition is the same as in the benchmark model. Following the reduction in aggregate debt, savers are able to reallocate their portfolios from savings towards housing

[28] Glover, Heathcote, Krueger, and Ríos-Rull (2011) set the risk aversion equal to 3 in the benchmark case, and then conduct a sensitivity analysis. They find that the magnitude of equilibrium price responses increase non-monotonically as risk aversion increases. Piazzesi, Schneider, and Tuzel (2007), in a capital asset pricing model with housing, find that a model featuring a higher level of risk aversion better performs in matching the moments of housing returns.

Table 6: Welfare effects of the Great Recession, different risk aversion parameters

	Δq	$\Delta(qh_b)$	$\Delta(qh_s)$	$\Delta\omega_b$	λ_b	λ_s
Data	-11	-16	-9	?	?	?
On impact, relative to pre-recession peak						
$\gamma = 1$	-5.25	-22.52	-0.04	-0.60	-0.47	-0.04
$\gamma = 2$ (benchmark)	-9.18	-29.47	-2.65	-1.19	-0.60	-0.01
$\gamma = 3$	-12.80	-35.83	-4.84	-1.84	-0.73	0.02
$\gamma = 5$	-19.21	-37.31	-14.21	-2.33	-1.09	0.04
After 7 periods, relative to long-run mean						
$\gamma = 1$	-0.91	-6.95	0.72	-0.37	-0.15	0.01
$\gamma = 2$ (benchmark)	-1.29	-9.54	1.00	-0.78	-0.24	0.03
$\gamma = 3$	-1.38	-11.79	1.62	-1.21	-0.32	0.05
$\gamma = 5$	-1.40	-12.91	1.44	-1.52	-0.51	0.07

(when it is relatively cheap). Consequently, the higher is the coefficient of risk aversion, the smaller are the overall welfare losses for savers.

5 Conclusions

Using a dynamic general equilibrium model calibrated to the US economy, we evaluate the quantitative effects of (i) aggregate income shocks and (ii) shocks to financial intermediation on house prices and on the welfare of two types of agents: leveraged agents (borrowers) and non-leveraged agents (savers).

The quantification of welfare costs associated with the US Great Recession along this cross-section complements recent contributions (Glover, Heathcote, Krueger, and Ríos-Rull, 2011; Hur, 2012) and adds a new mechanism stemming from shocks to the capital market. Our set-up is well suited for the evaluation of the welfare consequences of credit supply shocks in a recession, and complements other recent studies by exploring the effects of financial intermediation shocks in a model with endogenous collateral constraints.

We find that, following a shock modeled on the Great Recession, all the agents in the economy experience a welfare loss, and borrowers always lose more than savers. This finding comes from the fact that savers, being unconstrained, change their portfolio allocations and smooth the negative shock by buying the deflated asset (housing). We find that a financial intermediation shock that occurs in a recession forces borrowers to de-leverage, and amplifies the re-distribution from savers to borrowers, which translate in higher welfare losses for the latter.

Finally, we find that, in a model where borrowers are always borrowing constrained, the non-linearity in the amplification mechanism coming from the financial intermediation shock vanishes, and the effects on wealth and welfare are smaller.

We provide a number of sensitivity checks. While the redistributive effects (both in terms of financial wealth and welfare) between borrowers and savers are decreasing in the substitutability between housing and non-durable consumption, the drop in house prices is bigger when risk aversion is stronger, leading to a proportional increase in redistribution.

Although the paper focuses on the distributive effects of the Great Recession on borrowers and savers, we do not explicitly consider the possibility that borrowers can default on their debt obligations. While this could potentially benefit borrowers at the expense of their creditors, empirical evidence suggests that this feature of the U.S. Great Recession was restricted to a subset of borrowers, the sub-primers, who are not explicitly modeled here. Adding this third form of heterogeneity to the analysis is, in our opinion, an interesting avenue for future research.

References

Adrian, T., and S. H. Shin (2010): "Financial Intermediaries and Monetary Economics," in *Handbook of Monetary Economics*, ed. by B. M. Friedman, and M. Woodford, vol. 3 of *Handbook of Monetary Economics*, chap. 12, pp. 601–650. Elsevier.

Backus, D. K., A. W. Gregory, and S. E. Zin (1989): "Risk premiums in the term structure: Evidence from artificial economies," *Journal of Monetary Economics*, 24(3), 371–399.

Bajari, P., P. Chan, D. Krueger, and D. Miller (forthcoming): "A Dynamic Model of Housing Demand: Estimation and Policy Implications," *International Economic Review*.

Bordo, M. D. (2008): "An Historical Perspective on the Crisis of 2007-2008," Working Paper 14569, National Bureau of Economic Research.

Cooper, R., and J. Ejarque (2000): "Financial Intermediation and Aggregate Fluctuations: A Quantitative Analysis," *Macroeconomic Dynamics*, 4(04), 423–447.

Cúrdia, V., and M. Woodford (2010): "Credit spreads and monetary policy," *Journal of Money, Credit and Banking*, 42(s1), 3–35.

Glover, A., J. Heathcote, D. Krueger, and J.-V. Ríos-Rull (2011): "Intergenerational Redistribution in the Great Recession," NBER Working Papers 16924, National Bureau of Economic Research, Inc.

Grill, M., and J. Brumm (2010): "Computing Equilibria in Dynamic Models with Occasionally Binding Constraints," 2010 Meeting Papers 695, Society for Economic Dynamics.

Guerrieri, V., and G. Lorenzoni (2011): "Credit Crises, Precautionary Savings, and the Liquidity Trap," NBER Working Papers 17583, National Bureau of Economic Research, Inc.

Hall, R. E. (2011): "The High Sensitivity of Economic Activity to Financial Frictions," *The Economic Journal*, 121, 351–378.

Hur, S. (2012): "The lost generation of the Great Recession," Mimeo.

Iacoviello, M. (2005): "House Prices, Borrowing Constraints, and Monetary Policy in the Business Cycle," *American Economic Review*, 95(3), 739–764.

——— (2008): "Household Debt and Income Inequality, 1963–2003," *Journal of Money, Credit and Banking*, 40(5), 929–965.

——— (2011a): "Financial Business Cycles," Mimeo, Federal Reserve Board.

——— (2011b): "Housing wealth and consumption," *International Encyclopedia of Housing and Home, Elsevier.*

Iacoviello, M., and L. Guerrieri (2012): "Collateral Constraints and Macroeconomic Asymmetries," Mimeo, Federal Reserve Board.

Iacoviello, M., and S. Neri (2010): "Housing Market Spillovers: Evidence from an Estimated DSGE Model," *American Economic Journal: Macroeconomics*, 2(2), 125–164.

Justiniano, A., G. E. Primiceri, and A. Tambalotti (2013): "Household Leveraging and Deleveraging," Working Paper 18941, National Bureau of Economic Research.

Kalemli-Ozcan, S., E. Papaioannou, and F. Perri (2012): "Global banks and crisis transmission," *Journal of International Economics*, 87(5), 495–510.

Kiyotaki, N., and J. Moore (1997): "Credit Cycles," *Journal of Political Economy*, 105(2), 211–248.

Kubler, F., and K. Schmedders (2003): "Stationary Equilibria in Asset-Pricing Models with Incomplete Markets and Collateral," *Econometrica*, 71(6), 1767–1793.

Mendoza, E. G. (1991): "Real Business Cycles in a Small Open Economy," *American Economic Review*, 81(4), 797–818.

Mian, A., and A. Sufi (2009): "The Consequences of Mortgage Credit Expansion: Evidence frome the U.S. Mortgage Default Crisis," *Quarterly Journal of Economics*, pp. 1499–1496.

——— (2010): "The Great Recession: Lessons from Microeconomic Data," *American Economic Review: Papers and Proceedings*, 100, 51–56.

Piazzesi, M., M. Schneider, and S. Tuzel (2007): "Housing, consumption and asset pricing," *Journal of Financial Economics*, 83(3), 531 – 569.

Quadrini, V., and J. J. Urban (2012): "Macroeconomic Effects of Financial Shocks," *American Economic Review*, 102(1), 238–71.

Zangwill, W., and C. Garcia (1981): *Pathways to solutions, fixed points, and equilibria*, Prentice-Hall series in computational mathematics. Prentice-Hall.

Appendix

A Data

The following series used in Figure 1 and Figure 2 are from Federal Reserve Economic Data: the federal funds rate, the one year mortgage interest rate (released by the Primary Mortgage Market Survey by Freddie Mac), the mortgage (defined as home mortgages from the balance sheet of U.S. households and nonprofit organizations) and real estate (defined as the market value of real estate from the balance sheet of U.S. households and nonprofit organizations). All series are at quarterly frequency. The series for house prices is the National Composite Home Price Index for the United States (the release is by S&P/Case-Shiller Home Price Indices). The spread has been calculated as the difference between the one year mortgage interest rate and the federal funds rate each quarter.

In the calibration section, we calculate housing wealth as percentage of US nominal GDP (yearly) by using historical data of the flows of funds tables from the Board of Governors. US nominal GDP is from the Bureau of Economic Analysis. Our definition of housing wealth includes the market value of real estate belonging to households, non-profit and non-financial non-corporate business.

The micro-data used for the calibration of the relative wealth distribution of borrowers and the leverage ratio are provided by the 1998 to 2009 waves of Survey of Consumer Finances (SCF). Unfortunately, the SCF does not provide information on the precise date at which households were interviewed. Consequently, we assume that the observed portfolios in 2009 reflect the distribution of household net worth at the end of 2007. Averaging for all the waves between 1998 and 2009 helps in targeting data moments that are not strongly influenced by the years preceding the Great Recession. Surveyed households have been partitioned into *borrowers* and *savers* depending on their net asset position. The net asset position is defined as the sum of savings bonds, directly held bonds, the cash value of life insurance, certificates of deposits, quasi-liquid retirement accounts and all other types of transaction accounts (we consider these aggregated values to be deposits in the model) minus the debt secured by primary residence (mortgages, home equity loans, etc.) and the debt secured by other residential property, credit card debt and other forms of debt (we refer to these aggregated values as debt in the model). If the net asset position is positive, we consider the household to be a saver in our model economy, otherwise we consider her to be a borrower. The reason to use a broad definition of aggregate deposits and debt in the data counterpart is that it is difficult to target borrowers and savers by strictly restricting attention to particular classes of debt. We moreover define net wealth per capita as the sum of the net asset position and the value of the primary residence and other residential properties, for both leveraged and net savers. Finally, we aggregate the net wealth of both groups (borrowers and savers) and we calculate the relative net wealth of borrowers as the ratio between their net wealth over the total net wealth in the economy. The leverage ratio of the borrowers is instead obtained as the weighted average mean (using SCF sample weights) of the net asset position over the value of primary and secondary residences. The reference values that are matched by the model are obtained by cutting the 5% tails of the distribution of net worth in each wave of the SCF This is done to cut the extreme observations that may bias the average values of net worth in the US economy. We want, in fact, to avoid the possibility of including in the range of borrower households that maintain large positions in the stock or housing markets and hold little savings.

B Numerical Details

The algorithm employed is an adoption of the time-iteration procedure with linear interpolation used in Grill and Brumm (2010). As we have only two agents, a fine grid for wealth is enough to deliver satisfactorily small Euler errors. For this reason, we do not adapt the grid around the points where the collateral constraint is binding, as proposed by Grill and Brumm (2010).

B.1 Equilibrium conditions

We want to describe the equilibrium in our economy in terms of policy functions that map the current state into current policies. Furthermore, we want to focus on recursive mappings - that is, time-invariant functions that satisfy the period-by-period first-order equilibrium conditions. In what follows, we characterize these equilibrium conditions in every detail. For each agent $i = b, s$, denote by $\nu_i(w, z)$ the Lagrange multiplier with respect to her budget constraint and by $\phi_i(w, z)$ the Kuhn- Tucker multiplier attached to her collateral constraint. In addition, we treat saving and debt as two separate assets: saving is an asset in which the agent can only take long positions, $s_i \geq 0$; debt is an asset with return R_D in which agents can only take short positions, $d_i \leq 0$. Denote the Kuhn-Tucker multipliers attached to these inequalities as χ_i and μ_i, respectively. Then, for each tuple consisting of wealth and exogenous state today $\sigma = (w, z)$, the (time-invariant) policy and pricing functions have to satisfy the following system of equations (we will show below how to solve for these time-invariant functions):

- Agent's first order conditions

$$\begin{aligned}
u_1(c_i(\sigma), h_i(\sigma)) - \nu_i(\sigma) &= 0 \\
u_2(c_i(\sigma), h_i(\sigma)) - q(\sigma)\nu_i(\sigma) &= 0 \\
-\nu_i(\sigma) + \beta^i E[\nu_i(\sigma^+)|\sigma]R(\sigma) + \chi_i(\sigma) &= 0, \qquad i = s, b \\
-\nu_i(\sigma) + \beta^i E[\nu_i(\sigma^+)|\sigma]R_D + \phi_i(\sigma)R_D(w, z) - \mu_i(\sigma) &= 0 \\
-\nu_i(\sigma)q(\sigma) + u_2(c_i(\sigma), h_i(\sigma)) + & \\
+\beta^i E[\nu_i(\sigma^+)q(\sigma^+)]|\sigma] + \phi_i(\sigma)mE[q(\sigma^+)|\sigma] &= 0
\end{aligned}$$

- Agent's budget constraints

$$\begin{aligned}
n_b y(s) + n_b \Upsilon(\sigma) + w \cdot q(\sigma) - d_b(\sigma) - s_b(\sigma) - q(\sigma)h_b(\sigma) - c_b(\sigma) &= 0 \\
n_s y(s) + n_s \Upsilon(\sigma) + (1 - w) \cdot q(\sigma) - d_s(\sigma) - s_s(\sigma) - q(\sigma)h_s(\sigma) - c_s(\sigma) &= 0
\end{aligned}$$

  NB: Here we have already used the definition for the borrower's wealth share and rewritten the budget constraints in these terms (see the law of motion for wealth below as a reminder of how we defined the wealth share).

- Zero profits in the financial sector

$$\theta(s) \cdot R_D(\sigma) - R(\sigma) = 0$$

- Market clearing in housing and financial sector

$$\begin{aligned} h_s(\sigma) + h_b(\sigma) - 1 &= 0 \\ d_b(\sigma) + d_s(\sigma) + \theta(s) \cdot (s_b(\sigma) + s_s(\sigma)) &= 0 \end{aligned}$$

- Transfers

$$\Upsilon(\sigma) - (1 - \theta(s))(s_b(\sigma) + s_s(\sigma)) = 0$$

- Complementary slackness conditions

$$\begin{aligned} \mu_i(\sigma) \geq 0, d_i(\sigma) \geq 0, &\quad \mu_i(\sigma) \perp d_i(\sigma) \\ \chi_i(\sigma) \geq 0, s_i(\sigma) \geq 0, &\quad \chi(\sigma) \perp s_i(\sigma), \qquad i = s, b \\ \phi_i(\sigma) \geq 0, CC_i(\sigma) \geq 0, &\quad \phi_i(\sigma) \perp CC_i(\sigma) \end{aligned}$$

  where $CC_i(\cdot)$ is the collateral constraint of agent i, that is,

$$CC_i(\sigma) \equiv R_D(\sigma) d_i(\sigma) + mE[q(\sigma^+)|\sigma] h_i(\sigma) \geq 0$$

- Implicit "Law of motion" for borrower's wealth share

$$w^+(\sigma, z^+) \equiv \frac{R_D(\sigma) d_b(\sigma) + R(\sigma) s_b(\sigma) + q(w^+(\sigma, z^+), z^+) h_b(\sigma)}{q(w^+(\sigma, z^+), z^+)}.$$

B.2 Algorithm

The structure of the above period-by-period equilibrium conditions can be summarized as follows: Given a guess for the policy and pricing functions in the next period - denoted by f^{prime} - we can compute the expectations in the agents' first order conditions. The functions that map current states to current policies - denoted by f - are then obtained by solving the static system of non-linear given in the previous subsection. More formally, the structure of the problem can be summarized as follows. For all tuples $\sigma = (w, z)$, we have

$$\psi(f^{prime})(\sigma, f(\sigma), \mu(\sigma)) = 0, \qquad \zeta(\sigma, f(\sigma)) \geq 0 \perp \mu(\sigma) \geq 0.$$

The system of equations $\psi[f^{prime}](\cdot)$ contains first order conditions of agents and the financial sector and market clearing conditions. The function $\zeta(\cdot)$ contains the sign restrictions and collateral constraints. $\mu(\cdot)$ denotes the respective Kuhn-Tucker multipliers. A recursive policy function f then solves $\psi[f](\sigma, f(\sigma)\mu(\sigma)) = 0$ such that the complementary slackness conditions are satisfied. The time iteration algorithm defined below finds the approximate recursive policy function iteratively.

In each iteration, taking as given a guess for f^{prime}, we obtain f by solving the above system of equations and then updating our guess by interpolating the obtained policy function on the implicitly defined next period wealth. The following box summarizes our algorithm in a form of Pseudo-code:

1. Select a grid $\mathcal{W}$, an initial guess f^{init} and an error tolerance ϵ. Set $f^{prime} = f^{init}$.
2. Make one time-iteration step:
   (a) For all $\sigma = (w, z)$, where $w \in \mathcal{W}$, find the function $f(\sigma)$ that solves
   $$\psi(f^{prime})(\sigma, f(\sigma), \mu(\sigma)) = 0, \qquad \zeta(\sigma, f(\sigma)) \geq 0 \perp \mu(\sigma) \geq 0.$$
   (b) Use the solution f and the guess f^{prime} to update wealth tomorrow and interpolate f on the obtained values for wealth tomorrow.
3. If $||f - f^{prime}|| < \epsilon$, go to step 4. Else set $f^{prime} = f$ and repeat step 2.
4. Set numerical solution $\tilde{f}$ equal to the solution of the infinite horizon problem, $\tilde{f} = f$.

B.3 Kuhn-Tucker equations (Garcia-Zangwill trick)

At each grid point - given the guesses of the policy functions for the next period - we have to solve a system of nonlinear equations, containing both inequalities and equalities. The period-by-period equilibrium conditions are basically standard Kuhn-Tucker (K-T) conditions. In order to employ standard non-linear equation solvers like fsolve in Matlab or Ziena's Knitro, it is computationally more stable to eliminate the inequalities and recast the problem as a system consisting of equations only. In this section we describe how to do this. In general, we can write the Kuhn-Tucker conditions of any convex NLP problem as:

$$\Delta f(x)' + \sum_{j=1}^{r} \lambda_j \Delta g_j(x)' + \sum_{j=1}^{s} \mu_j \Delta h_j(x)' = 0 \tag{13}$$
$$\lambda_j \geq 0, g_j(x) \geq 0, \qquad j = 1, \ldots, r$$
$$\lambda_j g_j(x) = 0, \qquad j = 1, \ldots, r$$
$$h_j(x) = 0, \qquad j = 1, \ldots, s$$

plus a constraint qualification restriction (CQ). The system in (13) are mixtures of equalities and inequalities. Since inequalities tend to be cumbersome and can potentially prevent numerical software from solving the NLP via path-following, we will rewrite the K-T conditions so that they are a system consisting solely of equations (Zangwill and Garcia, 1981). The reformulation is as follows. Let k be a positive integer, and given $\alpha \in \mathbb{R}^1$, define:

$$\alpha^+ = [\max\{0, \alpha\}]^k$$
$$\alpha^- = [\max\{0, -\alpha\}]^k.$$

Hence, we always have $\alpha^+ \geq 0$, $\alpha^- \geq 0$, and $\alpha^+\alpha^- = 0$. Note also that both variables, $\alpha^+$and α^-, are $(k-1)$-continuously differentiable. Using this transformation, we can recast the K-T conditions and create the Kuhn-Tucker *equations* (Zangwill and Garcia, 1981):

$$\Delta f(x)' + \sum_{j=1}^{r} \alpha_j^+ \Delta g_j(x)' + \sum_{j=1}^{s} \mu_j \Delta h_j(x)' = 0 \tag{14}$$
$$\alpha_j^- - g_j(x) = 0, \qquad j = 1, \ldots, r$$
$$h_j(x) = 0, \qquad j = 1, \ldots, s$$

where $\alpha = (\alpha_1, \ldots, \alpha_r) \in R^r$ and (α^+, α^-) are defined as above. Note that the (K-T) equations defined here are precisely equivalent to the K-T conditions in (13). In particular, if (x^*, α^*, μ^*) satisfies the K-T equations, then (x^*, λ^*, μ^*) satisfies the the K-T conditions with $\lambda_j^* \equiv (\alpha_j^*)^+$, $j = 1, \ldots, r$. Conversely, if (x^*, λ^*, μ^*) satisfies the K-T conditions in (13), then (x^*, α^*, μ^*) satisfies the K-T equations in (14) with

$$\alpha_j^* \equiv \begin{cases} (\lambda_j^*)^{1/k} & \text{if } g_j(x^*) = 0 \\ -(g(x^*)_j)^{1/k} & \text{if } g_j(x^*) > 0 \end{cases} \qquad j = 1, \ldots, r.$$

B.4 Numerical Accuracy

In order to measure the accuracy of our approximation procedure, we calculate two statistics: first, we compute the relative Euler errors along the equilibrium path for very long time series. Second, for each exogenous shock, we randomly draw 3000 points from the wealth grid and compute the relative Euler Errors. To summarize the findings: for all simulated models, the maximum relative Euler Error is 3e-5 (or -4.5 in log(10)-scale). This implies that an agent, using our approximation of the equilibrium policy functions, would lose 30 Dollars for each million spent. It is important to compare this number to the welfare gains we obtain in the benchmark model. The borrowers' welfare loss on impact of an financial intermediation shock is 0.07 percentage points, that is, in log(10) scale, equal to -3.15. This number is one order of magnitude bigger, so even when netting these numbers by the mistakes that agents make, we conclude that our quantitative findings are still valid.