The Simple Economics of Capital Structure, Corporate Structure, and Information Policy*

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Abstract

Should different risky projects be financed separately through different companies or should they be bundled into a unified corporate structure? How does the optimal capital structure mix of debt and equity depend on the corporate structure? What is the optimal amount of information and variability in the project returns that are observed by the company? This paper addresses these questions by recasting the tradeoff theory of capital structure in terms of the theory of monopoly pricing. Drawing a close parallel between project bundling and product bundling, we show that it is optimal to bundle projects that have relatively high expected returns and low variability; the corresponding capital structure involves relatively low debt and high equity. Projects with low expectation and high variability are optimally financed separately and with a high debt to equity ratio. The firm has a preference for extremes when choosing the variability of returns.

JEL Classification Codes: D40, G32, G34, L11

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1 Introduction

Should a firm organize and finance its affiliates as separately incorporated subsidiaries or as branches whose liabilities represent claims on the parent institution? At least since Lewellen (1971), joint financing of risky projects is perceived to generate purely financial synergies and favor integrated corporate structures. Joint financing should reduce the probability of bankruptcy by allowing the firm to use the proceeds of a successful project to save an unsuccessful one, which otherwise would have failed. Diversification and tax gains can then be obtained by economizing on bankruptcy costs and increasing borrowing capacity.

This paper amends this claim by uncovering and exploiting a close formal connection between the *capital and corporate structure* problems studied in the corporate finance literature and the *monopoly product bundling* problem studied in the industrial organization literature (see Adams and Yellen (1976), Johnson and Myatt (2006), and Fang and Norman (2006)). Exploiting this connection, we derive three set results on capital structure, corporate structure, and information policy.

- First, the connection with monopoly pricing allows us to express the choice of optimal capital structure as a *modified* version of the classic Lerner formula as an optimal debt level markup depending on the elasticity of underlying return distribution.

- Second, the connection between product and project bundling allows us to provide sufficient conditions on the optimality of joint financing for low levels of debt, which arise when the variance is low with respect to the mean, and on the optimality of separate financing for high levels of debt, which arise when the variance is low with respect to the mean.

- Third, using the same principles we derive results on the optimal information policy. We predict that low levels of debt are connected to little information provision but high levels of debt are connected to extensive information provision.

The literature on pure product bundling asks whether it is profit maximizing for a monopolist to sell products separately or jointly as a package.[1] The key insight obtained in

[1] Our problem is related to the problem of pure rather than mixed bundling. Mixed bundling consists in offering the products separately as well as bundled—see McAfee, McMillan, and Whinston (1989).

that literature is that product bundling reduces the effective dispersion of the distribution of buyer's valuations and increases the elasticity at the mean valuation. In the language of Johnson and Myatt (2006), bundling induces an anti-clockwise rotation of the demand curve: the demand for two (identical) bundled products is lower than the original demand for prices above the mean valuation, and higher for prices below the mean valuation. This observation allows Fang and Norman (2006) to show that monopoly bundling is profitable (unprofitable) if *both* bundling and separation prices are below (above) the mean.

To understand the connection between the industrial organization and the corporate finance problems, consider a firm with two ex-ante identical projects with normal returns.[2] The firm maximizes its value by choosing the optimal corporate structure, i.e. deciding to finance the two projects jointly or separately, and the optimal capital structure, i.e. choosing the optimal mix of standard debt and tax-disadvantaged equity. As shown in the left-panel in Figure 1, if the projects are financed separately with nominal debt equal to the vertical line, the probability of default for each of them is equal the horizontal line. The debt value is equal to the area to the left of the debt obligation and above the horizontal line (in yellow) and the part of the area below the horizontal line and above the distribution of returns that is not lost to bankruptcy (in pink). The equity value is given by the fraction of the area to the right of the vertical line and above the distribution that is not lost to tax authorities (in blue).

By rotating the same figure, we can draw a connection between the optimal choice of capital structure and the standard monopoly pricing problem. As shown by the middle panel, the distribution of project returns can be re-interpreted as the distribution of consumer valuations. The probability of staying afloat (when the realized return turns out to be above the nominal debt level) is equivalent to the probability of selling (when the consumer's valuation is above the price). Expected creditor profits conditional on staying afloat, which are equal to the debt multiplied by the probability of staying afloat, in the case of product bundling correspond to monopoly profits, equal to the price multiplied by the probability of selling. Expected equity profits are equivalent to the consumer welfare in the product bundling problem. Therefore the capital structure that maximizes firm value when equity is fully taxed and all the returns are lost in case of bankruptcy is the

[2]Proposition 2 shows that this logic applies more generally when returns have a symmetric log-concave distribution.

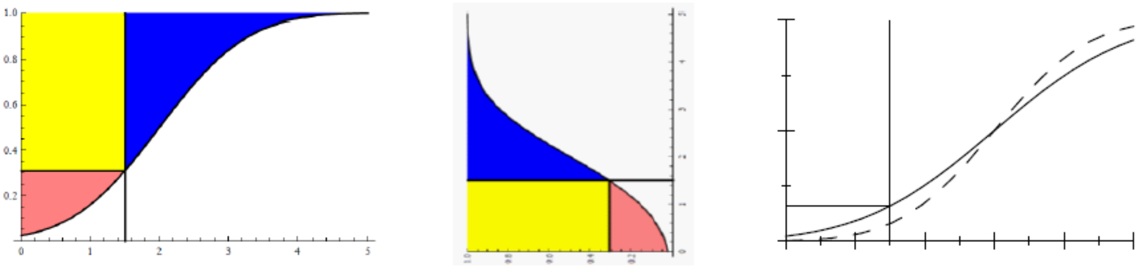


Figure 1: **Connection between capital and corporate structure and product bundling.** The left panel depicts the distribution of returns (continuous line), the nominal debt level (vertical line), the probability of bankruptcy (horizontal line), the value of debt (yellow plus a fraction of the pink areas) and the value of the equity (a fraction of the blue area). The middle panel depicts the distribution of valuations (continuous line), the probability of selling (vertical line), the price (horizontal line), the monopoly profits (yellow area), the consumer welfare (in blue) and the deadweight loss (pink area). In the right panel, the continuous curve represents the probability distribution of returns when the projects are financed separately, while the dashed curve represents the distribution associated to the average return when the projects are financed jointly.

same as the price that maximizes monopoly profits.

We can use the same reasoning to draw a connection between the choice of corporate structure and the bundling problem. As shown in the right-hand side panel, when the projects are financed jointly, the relevant distribution of returns is the dashed curve, representing the distribution of the average returns of the two projects. The density of the average of two identical normals is more peaked around the mean and therefore the distribution of the average is below the individual distribution for returns below the mean and above it for returns above the mean. The reduction in the dispersion of returns induced by joint financing is akin to the reduction in the dispersion of buyers' valuations induced by product bundling. Therefore in the particular case of full taxation and full bankruptcy costs, financing jointly versus separately is the same question as selling products as a bundle versus selling them separately.

This connection allows us to generate three sets of results. First, we revisit the trade-off theory of corporate finance which notes that, by increasing leverage, the company increases the bankruptcy costs but reduces the tax bill. This problem is similar to the classical monopoly problem which highlights that, by increasing the price, the monopoly reduces sales at the margin but increases revenues inframarginally. In the trade-off theory, however, the effects are weighted by the fraction of losses in case of bankruptcy and by the fraction of returns of equity lost to tax authorities. An increase in the leverage induces losses at the margin that are proportional to the fraction of bankruptcy costs, and generates inframarginal gains that are proportional to the fraction of tax rate. This connection allows us to express the optimal capital structure as a modified version of the classic Lerner formula used to express the monopoly price as an optimal markup depending on the elasticity of underlying return distribution. We also derive simple comparative statics predictions on the optimal capital structure.

Second, we generate general sufficient conditions for the optimal corporate structure. For a given level of nominal debt, the probability of bankruptcy is reduced with joint financing if and only if the nominal debt is below the average return, as shown in the figure. The value of the debt conditional on staying afloat is therefore higher. If, in addition, the default costs are lower than the tax rate, the additional default losses are compensated by the reduction in the tax bill. As a result, the total value of the firm is higher. Therefore if bankruptcy costs are lower than the tax rates, projects should be financed jointly if both debt levels are below the mean. In the case of a normal distribution this happens if the variance is low with respect to the mean. Instead, if the bankruptcy costs are higher than the tax rate and the debt levels are above the mean return, not only the bankruptcy probability is reduced with separate financing but also the value of the firm is higher. In the case of a normal distribution this happens if the variance is high with respect to the mean.

Third, we draw general implications for the optimal choice of information policy, which generate different return distributions. As shown by the figure, the choice of corporate structure boils down to a choice of dispersion. By financing the projects jointly, the firm commits to providing less information because the resulting distribution is less dispersed. In the last part of the paper, we allow the firm to choose the optimal degree of dispersion. Using the same principles, we show that for low levels of debt, it is optimal for the firm to commit to a return distribution that contains little information (resulting in a concentrated distribution of returns), but for high levels of debt, the firm it is optimal for the firm to give a lot of information (resulting in a distribution of returns with high dispersion).

So far the literatures on project and product bundling have developed in parallel, but departing from opposite premises. For the problem of product bundling, the naive intuition prevailing before Adams and Yellen's (1976) seminal contribution favored product separation. Hence, Adams and Yellen (1976) stressed the somewhat counterintuitive advantages of bundling products. For the problem of project bundling, instead, Lewellen's (1971) initial view found the advantages of project bundling more intuitive. Hence, Leland (2007) and Banal-Estañol, Ottaviani, and Winton (2013) and Banal-Estañol and Ottaviani (2013) stress the more subtle advantages of separation. In reality, the decision of whether to bundle or separate is driven by similar economic forces in the two problems of product pricing and project financing.

We proceed in Section 2 by presenting the baseline problem of the optimal capital structure. It proves useful to offer a graphical representation of the breakdown of the value of the firm into the value of debt and the value of equity. In Section 3 we uncover a tight connection between capital structure theory and monopoly pricing theory. We then reinterpret the optimal capital structure in terms of the classic Lerner formula familiar from monopoly pricing. In Section 4 we further develop this connection to analyze the problem of project bundling in parallel to the problem of pure product bundling from the industrial organization literature. We exploit this parallel to compare the profitability of joint and separate financing of a number of ex-ante identical, independent, symmetric projects. We then extend the results to analyze the effect of asymmetry, heterogeneity, and correlation in the distribution of returns. In Section 5 we turn to the problem of the selection of the stochastic properties of a project. Section 6 concludes. The Appendix collects the proofs omitted from the text.

2 Model

This section formulates our simple model of debt financing by a company endowed with a number of projects. The company could itself be a financial institution such as an insurance, a securities firm or a bank, which debt-holders do not have deposit insurance. The model is designed to address the question of how this firm should design its corporate liability structure. In the rest of the paper we derive results for special cases of this model.

A risk-neutral firm (bank, or entrepreneur) has n projects. Each project i yields at $t = 2$ a random return r_i with density $f_i(r_i)$ and distribution $F_i(r_i)$ over the non-negative support $(\underline{r}_i, \overline{r}_i)$.[3] Returns are possibly correlated across projects.

[3]Note that if the realized returns could be negative, we would need to specify whether the creditor is liable when the company defaults.

The firm can raise external finance from creditors at $t = 0$. The firm can assign its projects to different limited liability companies set up at no cost, and then seek independent financing for the projects assigned to each company. All the projects bundled in the same company are financed jointly, but independently from projects financed in the other companies set up by the same firm. The firm's problem is to decide how to group (or "bundle") projects into companies and how much debt each of these company issues.

Creditors lend money by way of standard debt contracts at $t = 1$. The debt contract posited here is the optimal contractual arrangement if we assume that returns are private information and can be verified only at a cost (equal to the bankruptcy cost), as in the costly state verification model of Townsend (1978) and Gale and Hellwig (1985).

Creditors operate in a perfectly competitive market and therefore expect to make zero expected profits from lending. This is equivalent to assuming that each company j makes a take-it-or-leave-it repayment offer to a single creditor. Corporation j, consisting of n_j projects, promises to repay $n_j r'_j$ at $t = 2$ in exchange of $n_j D_j$ at $t = 1$. Thus, the promised per-project repayment r'_j depends on the per-project level of debt issued, D_j. According to our accounting convention, the outstanding debt obligation that the company promises to repay comprises interest and principal.

Each company replays the creditor when its return is sufficient to cover the promised repayment, r^*_j. If instead the company is unable to meet its outstanding obligation, the company goes bankrupt and the ownership of the company's returns is transferred to the creditor. Due to bankruptcy costs, the creditor loses a fraction β ($0 \leq \beta \leq 1$) of the realized returns and receives a fixed per-project payoff, α, which can be positive, zero or negative.[4] This formulation includes as particular cases (i) our baseline specification with proportional bankruptcy costs ($\alpha = 0$), (ii) a specification with per-project fixed recovery ($\beta = 1$ and $\alpha > 0$) and (iii) a specification with per-project fixed bankruptcy cost ($\beta = 0$ and $\alpha < 0$).[5]

We could distinguish between the preferences of a regulator versus those of a company by including social costs, beyond the losses of the firm, in the fixed payoff. In formal terms, the regulator would have a more negative α, which accounts for social costs. If the firm is

[4] In oder to avoid that the company recovers more than r, we need to make additional assumptions on α. For example, we could assume that $\alpha < \underline{r}$.

[5] For estimates of bankruptcy costs and other costs of financial distress across industries see, for example, Warner (1977), Weiss (1990), and Korteweg (2006).

a bank, this cost could include the impact of the bank's failure upon the payment system, and the costs of destroying informational assets which have a value in the relationship with the bank's clients (Freixas et al. 2007).

We denote the equity value of the corporation, if it consists of n_j projects, as $n_j E_j$. We assume that, while debt payments are tax deductible and therefore exempt from taxes, (inside) equity is subject to a proportional corporate tax τ ($0 \leq \tau \leq 1$), which captures the tax disadvantage (or other net costs) of equity relative to debt. For notational simplicity, we assume that β or τ or both are strictly greater than 0, to avoid the uninteresting case in which debt is at the maximum level (if $\alpha > 0$), at the minimum (if $\alpha < 0$) or it is irrelevant (if $\alpha = 0$).

2.1 Breakdown of Value

In the analysis that follows we will make repeated reference to a useful graphical representation of the per-project value of the debt and equity of a company. For any distribution function F_i, the expected return of the project is, integrating by parts,

$$\int_{\underline{r}_i}^{\overline{r}_i} x dF_i(x) = \int_0^{\overline{r}_i} [1 - F_i(x)]\, dx,$$

whereas for any repayment rate r the expected gross return obtained by the creditor and the value of the debt is equal to

$$D_i = (r - \alpha)(1 - F_i(r)) + (1 - \beta) \int_0^r (F_i(r) - F_i(x)) dx + \alpha, \tag{1}$$

and the expected value of the equity is

$$E_i = (1 - \tau) \left[\int_r^{\overline{r}_i} x dF_i(x) - r(1 - F_i(r)) \right] = (1 - \tau) \int_r^{\overline{r}_i} [1 - F_i(x)]\, dx, \tag{2}$$

and therefore the value of the company is given by

$$EV_i = D_i + E_i = (r - \alpha)(1 - F_i(r)) + (1 - \beta) \int_0^r (F_i(r) - F_i(x)) dx + (1 - \tau) \int_r^{\overline{r}_i} [1 - F_i(x)]\, dx + \alpha, \tag{3}$$

Figure 2 depicts the division of the (ex-post) per-project present value and the difference between the ex-post and the ex-ante net present value. The ex-ante net present value corresponds to all the colored area, above the distribution function (and below 1). Within this area, the expected net return for the borrower associated to any interest rate r is

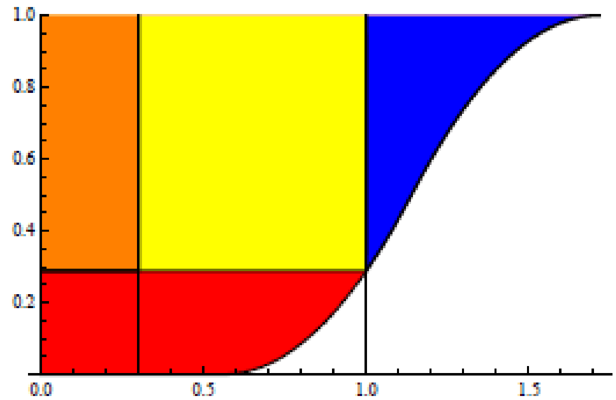


Figure 2: **Split of Expected Returns.** The area above the distribution function represents the project's expected return. For a given interest rate r, the expected return for the borrower corresponds to the area above the distribution function and to the right of r (in blue). The expected return of creditor is the sum of (1) the area above $F(r)$ and to the left of r and (2) the area below $F(r)$ and to the left of b.

equal to a fraction $(1-\tau)$ of the area to the right of r (in blue). The remaining of this area is lost through the expected tax bill. Beyond a fixed payment α (independent of r), the expected return for the creditor is the area to the left of r: and to the right of α and above of the distribution function (in yellow), a fraction $(1-\beta)$ of the area below $F(r)$ and the distribution of returns (in red). The remaining part of this area corresponds to the expected bankruptcy costs. The difference between the ex-ante and the ex-post net present value is a fraction β of the right red area and a fraction τ of the blue area.

In term of objectives, the maximizes the sum of the debt (yellow plus a discounted value of the red areas) plus a discounted equity value (blue area), where the discount for part of the debt represent the bankruptcy costs and the discount for the equity represents the disadvantage of using equity with respect to debt financing (e.g. due to tax considerations). In the limit, if the discount on equity is extremely large, the firm maximizes the debt value, whereas if it is zero, the firm do not use debt at all. If the discount on bankruptcy cost is zero, the firm uses only debt.

3 Optimal Capital Structure: Tradeoff Theory Anew

In this section, we draw a connection with the product bundling problem studied in the industrial organizational literature. This connection allows us to express the optimal capital structure with a modified version of the classical Lerner formula used to express the optimal monopoly price.

3.1 Connection to Monopoly Pricing

Consider the standard monopoly problem. A profit maximizing monopolist sells n indivisible products indexed by $i = 1, 2$. Consumers' valuation (also known as reservation price) for product i, r_i, is distributed as a continuous random variable with symmetric density $f_i(r_i)$ and distribution $F_i(r_i)$. Suppose that the products' marginal costs are given by α and the fixed per projects' costs by C. Suppose that products are independent in consumption, so that consumers' valuation for the bundle consisting of m products is equal to the sum of the valuations for the component products, $r_1 + ... + r_m$.

The monopoly profits from selling a single product at a price r is given by

$$(r - \alpha)\left[1 - F_i(r)\right] - C, \tag{4}$$

given that the demand at price r is equal to $1 - F_i(r)$. Maximizing (4) is equivalent to maximizing the value of the company in the project bundling problem (3) for the case in which equity and bankruptcy costs are extremely costly ($t = 1$ and $\beta = 1$). Substituting $C = -\alpha$, the net profits of the monopoly in the project financing problem (4) are exactly the same as the value of the company in the project bundling case (3). The profits of the monopoly, gross of the fixed costs C, are also represented by the yellow area in Figure 2. The blue area corresponds to the consumer welfare (willingness to pay in excess of the price) and the red area to the deadweight loss.

3.2 Adjusted Lerner Formula

For a single product, the value of the company is maximized for a level of leverage r^* that satisfies

$$-\left(\beta r^* - \alpha\right) f(r^*) + \tau\left[1 - F(r^*)\right] = 0. \tag{5}$$

As in the trade-off theory, by increasing leverage, the company increases the bankruptcy costs (first term) but reduces the tax bill (second term). In the classical profit-maximization problem (equivalent to $\beta = \tau = 1$ in our problem), an increase in price reduces sales at the margin (first term) but increases revenues inframarginally (second term). In the current problem, the first effect is weighted by the bankruptcy costs β and the second by the tax discount τ. An increase in r induces losses at the margin proportional to the bankruptcy costs and generates inframarginal gains proportional to the tax discount. Instead, in the monopoly case, losses and gains are in full. Graphically in Figure 2, an increase in r increases the red area (which we obtain partially, with a weight β) and reduces the height of the yellow area (which we obtain in full) but, at the same time, it reduces the blue area (which we obtain partially, with a weight τ) and increases the base of the yellow area. Increasing r has smaller losses if α is larger, which is equivalent to higher marginal costs in the monopoly problem. Graphically, this corresponds to a larger orange area.

Rearranging (5), we obtain the following result

Proposition 1 *The optimal level of leverage satisfies*

$$\frac{(r^* - \alpha/\beta)}{r^*} = \frac{\tau}{\beta\varepsilon(r^*)} \text{ if } \beta > 0 \text{ and } r^* = -\frac{\alpha}{\tau}\varepsilon(r^*) \text{ if } \beta = 0$$

where $\varepsilon(r^*)$ represents the elasticity of the returns $\varepsilon(r^*) = rf(r)/\left[1 - F(r)\right]$.

The first equation is an "adjusted" Lerner formula with the addition of relative weights τ/β (in the monopoly case $\tau = \beta = 1$). If the bankruptcy costs are proportional ($\alpha = 0$) the optimal level of leverage satisfies

$$\varepsilon(r^*) = \frac{\tau}{\beta}$$

which is an adjusted version of the monopoly revenue maximization, in which the profit-maximizing price arises if the elasticity is equal to 1.

As an example, for a uniform distribution over the support $[L, U]$, with density $f(x) = 1/(U - L)$ and distribution $F(x) = (x - L)/(U - L)$ between L and U, we have that, if the solution is interior,

$$r^* = \frac{\tau U + \alpha}{\tau + \beta}.$$

As expected, r^* is decreasing in β and increasing in τ in the proportional case ($\alpha = 0$) and in the case of fixed bankruptcy costs ($\beta = 0$ and $\alpha < 0$) (the derivative with respect to τ has the same sign as $U\beta - \alpha$). We can also see that as the fixed recovery increases, r^* increases.

4 Optimal Corporate Structure: Project Bundling

In this section, we present our main result, consisting in a sufficient condition on the choice of joint versus separate financing. We make use again of the connection with the pure bundling problem.

4.1 Connection to Product Bundling

Next, consider joining m projects within the same company. Denote by r the *per project* repayment, so that mr is the total repayment promised to investors. Following the same reasoning as before, for any per-project repayment rate r the gross expected net returns of the creditors are

$$m(r-\alpha)(1-G(mr))+(1-\beta)\int_0^{mr}(G(mr)-G(mx))dx+m\alpha, \tag{6}$$

where G is the distribution function of the sum of m random variables with distribution F. Noting that the distribution of the sum computed at mr is

$$G\left(mr\right)=\Pr\left(r_1+...+r_m\leq mr\right)=\Pr\left(\frac{r_1+...+r_m}{m}\leq r\right)=:H(r),$$

where H is the distribution function of the average, the returns per project are equal to

$$(r-\alpha)(1-H(r))+(1-\beta)\int_0^{r}(H(r)-H(x))dx+\alpha \tag{7}$$

In the case the projects are ex-ante identical, the per-project expected net return of the creditor ($F_i\equiv F$) is exactly the same as in (1), replacing F by H. Similarly, the per-project inside equity and the per-project value of the company in the ex-ante identical case ($F_i\equiv F$) are exactly as in (2) and (3), replacing F by H.

We can again see the connection with the monopoly problem and, in particular, with the pure product bundling problem. The per-product monopoly profits for a given per-product price r is given by

$$(r-\alpha)\left[1-H\left(r\right)\right]-C. \tag{8}$$

Maximizing (8) is equivalent to maximizing the value of the company in the project bundling problem. Again, with the same substitution $C=-a$, the net profits of the monopoly in the project financing problem (8) are exactly the same as the value of the company in the project bundling case (7).

4.2 Joint versus Separate Financing

We are now ready to state our main result. Suppose that we have two ex-ante identical projects with log-concave and symmetric-around-the-mean probability density.[6]

Proposition 2 *Assume that the density f is symmetric around the mean μ and log-concave; realizations are independent. The firm prefers to finance n projects separately if*

$$\tau \leq \beta \text{ and } (\beta\mu - \alpha)\, h(\mu) < \tau/2,$$

but prefers to finance all of them jointly if

$$\tau \geq \beta \text{ and } (\beta\mu - \alpha)\, f(\mu) > \tau/2,$$

where h is the density of the average of m random variables with density f.

For example, when the returns of two project are normally distributed, $N(\mu, \sigma^2)$, and $\alpha = 0$, the conditions for separate financing are $\tau \leq \beta$ and $\beta\mu < \tau\sqrt{\pi/4}\sigma$ and those for joint financing are $\tau \geq \beta$ and $\beta\mu < \tau\sqrt{\pi/2}\sigma$. When returns are uniformly distributed in the interval $[L, U]$, the conditions are $\tau \leq \beta$ and $[\beta(U + L) - 2\alpha]/(U - L) < \tau/2$ for separate financing and $\tau \geq \beta$ and $[\beta(U + L) - 2\alpha]/(U - L) > \tau$ for joint financing.

Figure 3 represents the mean-variance region of parameters for which separate and joint financing of two projects is optimal in the case in which $\beta = \tau$. Joint financing is optimal in the strong and light red areas whereas separate financing is optimal in the strong and light blue areas. The strong red and blue areas are those covered by the joint and separate financing conditions of the proposition. Separation holds for a larger region of parameters if the mean returns are low and if the variance is high.

Corollary 1 *The greater the externality (α), the more possible that sufficient condition on separate financing is satisfied and more difficult the one for joint.*

[6] A random variable is log-concave if the logarithm of the probability density function is concave. Symmetry and log-concavity are satisfied by many common parametric densities, such as the uniform, normal, logistic, Laplace, beta (when $a = b$), truncated Student's t and any truncations or linear combinations of these distributions. See Bagnoli and Bergstrom (2005) for other examples and applications.

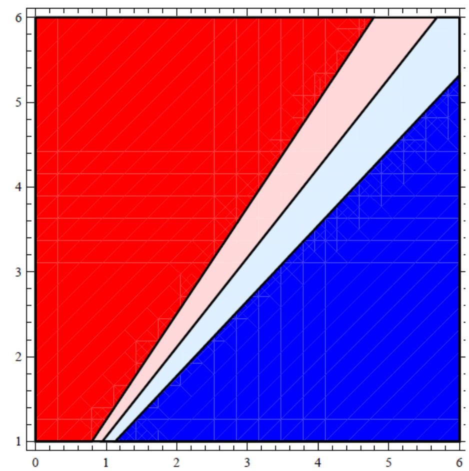


Figure 3: **Separate and joint financing for the normal distribution.** This figure represents the mean (vertical axis) - standard deviation (horizontal axis) region of parameters for which separate and joint financing of two projects is optimal in the case in which $\beta = \tau$. Joint financing is optimal in the strong and light red areas whereas separate financing is optimal in the strong and light blue areas. The strong red and blue areas are those covered by the joint and separate financing conditions of the proposition. Separation holds for a larger region of parameters if the mean returns are low and if the variance is high.

Corollary 2 *In the case of fixed per-project bankruptcy costs, ($\beta = 0$, $\alpha < 0$), then if $\tau = 0$, firms always prefer to finance jointly.*

Corollary 3 *A monopoly seller ($\tau = \beta = 1$) should sell the products separately at a higher price if*

$$(\mu - \alpha)h(\mu) < 1/2,$$

but should sell them jointly at a higher price if'

$$(\mu - \alpha)f(\mu) > 1/2.$$

4.3 Asymmetric Binary Distribution

In order to see the effects of a non-symmetric distribution we now consider the binary case. That is, $F(r) = 0$ if $r < r_L$, $F(r) = 1 - p$ if $r_L < r < r_H$, and $F(r) = 1$ if $r_H < r$. Again, the analysis is going to be based on the relative weights β/τ, which we denote for notational simplicity $w \equiv \beta/\tau$. We therefore assume in this section that $\tau > 0$.

Among all the rates with the same probability of bankruptcy, it is optimal to choose the largest to minimize tax payments (unless $\tau = 1$, in which case the firm is indifferent).

Therefore, in the case of separate financing, the firm choose either r_L or r_H. If $r = r_L$ is chosen, there is no probability of bankruptcy,

$$EV_i(r_L) \equiv r_L + (1-\tau)p(r_H - r_L) \tag{9}$$

whereas in the case of $r = r_H$ there are no taxes

$$EV_i(r_H) \equiv pr_H + (1-\beta)(1-p)r_L \tag{10}$$

and therefore $EV_i(r_H) > EV_i(r_L)$ as long as the inefficiency from choosing r_H if, bankruptcy costs are low with respect to the taxes, or in other words, if

$$w \equiv \frac{\beta}{\tau} < w_S^* \equiv \frac{p\left(r_H - r_L\right)}{(1-p)r_L} \tag{11}$$

Following the same procedure, in the case of joint financing, we have three possible rates, $r = r_L$, $r = (r_L + r_H)/2$, or $r = r_H$. The following lemma characterizes when it is optimal to use each of them.

Lemma 1 *If $w > w_J^*$ then $r^* = r_L$, if $w_J^* > w > w_J^{**}$ then $r^* = (r_L + r_H)/2$ and if $w < w_J^{**}$ then $r^* = r_H$, where*

$$w_J^* \equiv \frac{p\left(r_H - r_L\right)(1-p/2)}{(1-p)^2 r_L} \text{ and } w_J^{**} \equiv \frac{p^2\left(r_H - r_L\right)/2}{2p(1-p)\left(r_H + r_L\right)/2} \tag{12}$$

It is easy to show that $w_J^* > w_S^* > w_J^{**}$ and therefore we have the following proposition comparing joint and separate financing:

Proposition 3 *Joint financing is better than separate financing if and only*

$$\frac{p\left(r_H - r_L\right)/2}{(1-p)r_L} \equiv w^{***} < w$$

4.4 Heterogeneous Projects

Assume here that projects are not ex-ante identical.

Proposition 4 *If the average distribution, defined as* $J(p) \equiv [F_1(p) + F_2(p)]/2$*, cross the distribution of the average,* H*, from above and only once, i.e.*

$$J(p) \gtreqless H(p) \Leftrightarrow p \lesseqgtr \widehat{p} \text{ for some } \widehat{p} \qquad \text{(Condition 1)}$$

then projects should be financed together if $f_1(\widehat{p})(\widehat{p} - c) > 1 - H(\widehat{p})$ *and* $f_2(\widehat{p})(\widehat{p} - c) > 1 - H(\widehat{p})$ *and separately if* $h(\widehat{p})(\widehat{p} - c) < 1 - H(\widehat{p})$.

Notice that the average distribution is a well defined distribution function with density $(f_1(x) + f_2(x))/2$ and expectation $(\mu_1 + \mu_2)/2$. However, it may not be symmetric around its mean (for example when both are normal and $\mu_1 \neq \mu_2$ and $\sigma_1 \neq \sigma_2$). If the average density is symmetric around the average of the means (as in the first two examples below but unlike in the third), then this is equivalent to ensure that the average distribution is more peaked (at the average of the means) than the average of the distributions. In this case, the crossing point is $\widehat{p} = \frac{\mu_1 + \mu_2}{2}$.

Example 1: Normal with Non-Identical Means (FOSD) If $\theta_i \sim N(\mu_i, \sigma^2)$ and independent for $i = 1, 2$ then $\frac{\theta_1 + \theta_2}{2} \sim N\left(\frac{\mu_1 + \mu_2}{2}, \frac{\sigma^2}{2}\right)$. In this case the average density is symmetric around the average expectation and therefore $\widehat{p} = \frac{\mu_1 + \mu_2}{2}$ (and hence $1 - H(\widehat{p}) = 1/2$). Moreover, the condition is always satisfied.

Corollary 4 *Condition 1 is always satisfied if the two distributions have the same variance and therefore finance the projects together if* $(\mu_1 + \mu_2) e^{-\frac{(-\mu_1 + \mu_2)^2}{8\sigma^2}} > \sqrt{2\pi}\sigma$ *and separately if* $\mu_1 + \mu_2 < \sqrt{\pi}\sigma$

Example 2: Normal with Non-Identical Variances (SOSD) If $\theta_i \sim N(\mu, \sigma_i^2)$ and independent for $i = 1, 2$ then $\frac{\theta_1 + \theta_2}{2} \sim N\left(\mu, \frac{\sigma_1^2 + \sigma_2^2}{4}\right)$. Again, the average density is symmetric around the mean and therefore $\widehat{p} = \mu$ (and $1 - H(\widehat{p}) = 1/2$). Here the condition is satisfied if the variances are not very different.

Corollary 5 *Provided that they have the same mean, Condition 1 is satisfied if the two variances are not very different; i.e. denoting* $\sigma_2 = k\sigma_1$ *condition 1 is satisfied iff* $0.36 \leq k \leq 2.76$. *In this case bundle if* $\mu > \sqrt{\pi/2}\sigma_1$ *and* $\mu > \sqrt{\pi/2}\sigma_2$ *separate if* $\mu < \sqrt{\pi/8\left(\sigma_1^2 + \sigma_2^2\right)}$

Example 3: Normal with Non-Identical Means and Variances If $\theta_i \sim N(\mu_i, \sigma_i^2)$ and independent for $i = 1, 2$ then $\frac{\theta_1+\theta_2}{2} \sim N\left(\frac{\mu_1+\mu_2}{2}, \frac{\sigma_1^2+\sigma_2^2}{4}\right)$. In this case, it is possible that the average distribution is not symmetric around its mean. As a result, $\widehat{p} \neq \frac{\mu_1+\mu_2}{2}$ in general (and therefore $1 - H(\widehat{p}) \neq 1/2$).

Corollary 6 *Condition 1 is satisfied iff* $g(k, s) = 4k/\sqrt{1+k^2} - ke^{-s} + e^{-\frac{s}{k^2}} > 0$ *where* $\sigma_2 = k\sigma_1$ *and* $s = (-\mu_1 + \mu_2)^2 / \sigma_1^2$.

Notice that this condition is more likely to be satisfied when s is increased, i.e., when the difference between the means is larger given that

$$\frac{\partial g}{\partial s} = -(-s)ke^{-s} - e^{-\frac{s}{k^2}}(-\frac{1}{k^2}) > 0.$$

In addition, we have verified numerically that this condition is more likely to be satisfied when k is reduced, as we saw in the case with equal means. Algebraically, we have that

$$\frac{\partial g}{\partial k} = \frac{4(1+k^2)}{\sqrt{(1+k^2)}} - e^{-s} - e^{-\frac{s}{k^2}}(s\frac{2}{k^3}).$$

4.5 Correlated Normal Projects

If $\theta_1, \theta_2 \sim N(\mu, \sigma^2)$ and their correlation coefficient is equal to ρ, then $\frac{\theta_1+\theta_2}{2} \sim N\left(\mu, (1+\rho)\frac{\sigma^2}{2}\right)$. In this case the average distribution is (weakly) more peaked than the original distributions (strictly for $\rho < 1$) As a consequence the distributions cross at a point $\widehat{p} = \mu$, and we have

$$F(p) \gtreqless H(p) \Leftrightarrow p \lesseqgtr \widehat{p}.$$

As a consequence

$$2p(1 - F(p)) \lesseqgtr 2p(1 - H(p)) \Leftrightarrow p \lesseqgtr \widehat{p}.$$

For $\rho < 1$, following the same procedure as above, we get the following sufficient conditions

$$\text{bundle if } f(\widehat{p})(\widehat{p} - c) = \frac{(\mu - c)}{\sqrt{2\pi\sigma^2}} > \frac{1}{2} \Leftrightarrow \mu - c > \sqrt{\pi/2}\sigma, \tag{13}$$

$$\text{separate if } h(\widehat{p})(\widehat{p} - c) = \frac{(\mu - c)}{\sqrt{\pi(1+\rho)\sigma^2}} < \frac{1}{2} \Leftrightarrow \mu - c < \sqrt{\pi(1+\rho)/4}\sigma. \tag{14}$$

This is true as long as the two distributions are not perfectly correlated. If $\rho = 1$ then $F(p) \equiv H(p)$ and there is no diversification and the optimal prices and profits are equal.

As correlation decreases, diversification means that the bundle's demand is more elastic at the mean and bundle prices tend to the mean (although not necessarily in a monotonic way, see below). In the extreme, when the correlation is close to -1, the demand for the bundled products is almost completely elastic at the mean. As a result, keeping products separate is never *unambiguously* optimal because the optimal bundle price is always marginally below the mean (i.e. (14) is not satisfied if $\mu > c$ and $\rho = -1$).

Note that if independent prices are below the mean (i.e. bundling is profitable and condition (13) is satisfied), then the less correlation the more profitable the bundle. Indeed, optimal prices and profits will continue to increase until the mean as correlation decreases. Instead, if bundle prices are above the mean for a given correlation (i.e. diversification is bad and condition (14) is satisfied), then decreasing the correlation does not always result in a reduction in profits for the bundle. Bundle prices will end up below the mean and therefore bundle profits might be higher than with separate products (condition (14) is not satisfied in the limit). A technical curiosity is that for ρ very close to 1, the conditions are almost necessary and sufficient.

5 Project Selection

We now turn to the firm's incentives for choosing projects with different return distributions. This choice can be the result of the firm's orientation regarding the intensity of the resolution of uncertainty about the profitability of the project. Our two-period model simply captures the resolution of uncertainty in the most basic form.

To illustrate this point, add an underlying quality of the project, H or $L < H$, with a prior $\hat{r} = \Pr(H)$. A highly variable return distribution (resulting in a clockwise rotation) corresponds to the case in which the returns realized in $t = 2$ are a highly informative signal about underlying quality of the project. Consider a family of distribution functions $F(r; s)$, where the real valued parameter $s \in [\underline{s}, \overline{s}]$ captures how informative returns are about the underlying quality of the project. As in Johnson and Myatt (2006), a higher value of s induces a clockwise rotation of the distribution of r around the prior $\hat{r}$ through a mean-preserving spread with

$$\frac{dF(r;s)}{ds} \gtreqless 0 \text{ for } r \lesseqgtr \hat{r} \tag{15}$$

over $r \in (0, 1)$, with $F(\hat{r}; s) = \hat{r}$. A signal structure that results in such a rotation in the posterior distribution is more informative in the sense of Blackwell; see Ganuza and Penalva's (2010) Theorem 2. As an immediate implication of the mean-preserving spread, the density at the mean, $f(\hat{r}; s)$, is strictly decreasing in s. The most intense resolution of uncertainty the firm could choose, $s = \overline{s}$, corresponds to a project whose return realization is equal to the quality of the project—in which case the distribution $F(r; \overline{s}) = \hat{r}$ for $r < 1$ and $F(1; \overline{s}) = 1$. For $s = \underline{s}$, at the other extreme, the return realization is constant—in which case the distribution $F(r; \underline{s}) = 0$ for $r < \hat{r}$ and $F(r; \underline{s}) = 1$ for $r \geq \hat{r}$.

Extending the analysis from the previous section, we find that the firm has a preference for extreme variabilities of returns. Consider the choice of dispersion from a given interval $[\underline{s}, \overline{s}]$, for given level of average return. When debt levels are relatively low, an increase in dispersion makes a given debt level riskier. Instead, when debt levels are relatively high, an increase in dispersion makes a given debt level safer. If $\beta = \tau$, the firm value is maximized at one of two extreme dispersion levels: either (i) lowest dispersion with corresponding relatively low leverage or (ii) highest dispersion with relatively high leverage. Building on the previous results, we have that:

- If $\beta < \tau$ and $(\beta\mu - \alpha) f_s(\mu) > \tau/2$ for $s = \overline{s}$, the firm optimally chooses minimum dispersion, relatively low levels of debt for all dispersion levels, and a decrease in dispersion makes debt safer.

- If $\beta > \tau$ and $(\beta\mu - \alpha) f_s(\mu) \leq \tau/2$ for $s = \underline{s}$, the firm optimally chooses maximum dispersion, relatively high levels of debt for all dispersion levels, and a increase in dispersion makes debt safer.

We can generalize Proposition 2 as follows:

Proposition 5 *(i) If $\beta > \tau$ and debt leverage is above the mean for the distribution with the lowest possible dispersion (or for a range of dispersions), then the firm chooses the maximum dispersion $s = \overline{s}$, resulting in high debt and returns with a flat distribution corresponding to a completely inelastic demand. (i) If $\beta < \tau$ and debt leverage is below the mean for the distribution with the highest dispersion, then the firm chooses the minimum dispersion possible $s = \underline{s}$, resulting in very little debt and inelastic returns corresponding to a horizontal demand. If $\beta = \tau$ one of the two extremes is optimal.*

6 Conclusion

This paper studies the problem of how projects should be financed (within the same company or in separate special purpose vehicles?) in a world of taxes and bankruptcy costs. Focusing on the purely financial effects, we derive simple but general conditions for when projects should be financed jointly or separately. Our characterization relies on the close formal connection between the problems of project and product bundling we uncover.

As we show, joint financing is unprofitable if debt levels are high and bankruptcy is likely. By financing jointly the two projects, the probability of bankruptcy is increased. Given that there is a high probability of having a low return for the second project, a project with a high return runs a serious risk of being dragged down. On the other hand, if the debt levels are low and bankruptcy is unlikely, the probability of bankruptcy is reduced by joint financing. Firms with a low realization for one project but an average return for the second project, end up staying afloat if the two projects are financed jointly.

So far the literature on project and product bundling have developed in parallel, but departing from opposite positions. For the problem of product bundling, the naive intuition prevailing before Adams and Yellen's (1976) contribution was favoring product separation—hence Adams and Yellen stressed the somewhat counter-intuitive advantages of bundling products. For the problem of project bundling, Lewellen (1971) stressed the intuitive advantages of bundling. As we show in this paper, similar forces drive the decision of whether to bundle or separate in the two problems.

Appendix

Proof of Proposition 2

The proof follows in three steps. First, we need to show that for $r > \mu$,

$$EV_S(r) \equiv (r-\eta)(1-F(r)) + (1-\beta)\int_0^r [F(r)-F(x)]\,dx + (1-\tau)\int_r^{\overline{r}} [1-F(x)]\,dx + \eta >$$

$$(r-\eta)(1-H(r)) + (1-\beta)\int_0^r [H(r)-H(x)]\,dx + (1-\tau)\int_r^{\overline{r}} [1-H(x)]\,dx + \eta \equiv EV_J(r)$$

Second, $EV_S(r)$ and $EV_J(r)$ need to be single-peaked. Third, the derivative of $EV_J(r)$ at μ is positive if and only if

$$[-(\beta r-\eta)\,h(r) + \tau(1-H(r))]_{r=\mu} = -(\beta\mu-\eta)h(\mu) + \tau/2 > 0$$

or $(\beta\mu-\eta)\,h(\mu) < \tau/2$.The second-order condition is always satisfied given that the second-order derivative is given by

$$-(1-\gamma+1-\nu)h(r)-(1-\gamma)rh'(r)$$

From Proschan (1965), we know that if two independent random variables have a log-concave and symmetric density then the average is more peaked than the individual random variable. As a consequence, the distributions cross at $r=\mu$, and we have

$$F(r) \gtreqless H\left(r\right) \Leftrightarrow r \lesseqgtr \mu \tag{16}$$

and as a result

$$r\left[1-F(r)\right] \lesseqgtr r\left[1-H\left(r\right)\right] \Leftrightarrow r \lesseqgtr \mu. \tag{17}$$

If r_m^*, the lowest r such that $r\left[1-H\left(r\right)\right]=1$, is such that $r_m^*<\mu$, then $r_m^*<r_i^*$. Indeed, although r_i^* exists by assumption, it is not possible that $r_i^*<r_m^*$ because, by (17) and concavity of the profit function, we have that for $r<r_m^*$, $r\left[1-F(r)\right]<r\left[1-H\left(r\right)\right]<r_m^*\left[1-H\left(r_m^*\right)\right]=1$. As a result, from (16) and monotonicity of F, we conclude that the probability of bankruptcy is lower with joint financing, $H\left(r_m^*\right)<F(r_m^*)<F(r_i^*)$.

On the other hand if r_m^* is such that $r_m^*>\mu$, then $r_m^*>r_i^*$. Indeed, given that the creditor's proceeds at $r=0$ are equal to 0 and they are higher than 1 at $r=r_m^*$, $r_m^*\left[1-F(r_m^*)\right]>r_m^*\left[1-H\left(r_m^*\right)\right]=1$, by the intermediate value theorem there exists some $r_i^*<r_m^*$ at which $r_i^*\left[1-F\left(r_i^*\right)\right]=1$. As a result, from (16) and monotonicity of H, we have that the probability of bankruptcy is lower with separate financing, $F(r_i^*)<H\left(r_i^*\right)<H(r_m^*)$.

By single-peakedness, r_m^* is such that $r_m^*>\mu$ if and only if the following two conditions hold

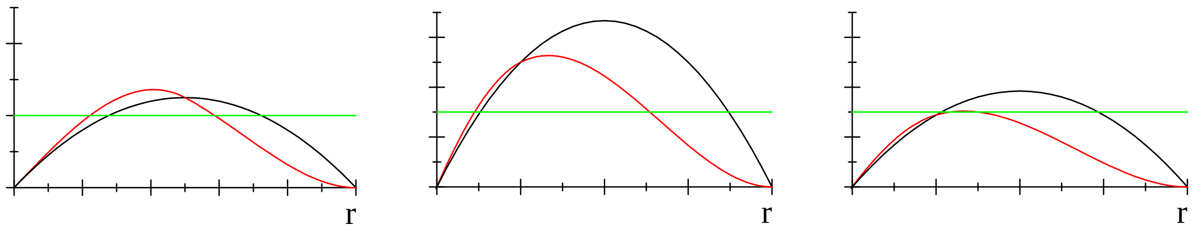
$$\left.\frac{\partial r\left[1-H\left(r\right)\right]}{\partial r}\right|_{r=\mu}>0 \text{ and } \left.r\left[1-H\left(r\right)\right]\right|_{r=\mu}<I,$$

which are equivalent to

$$h(\mu)\mu<\frac{1}{2} \text{ and } \mu<2I,$$

as claimed.

The figure shown below depicts the creditor's expected gross return under separate financing (equal to $r\left[1-F(r)\right]$, displayed in black) and joint financing (equal to $r\left[1-H\left(r\right)\right]$, in red) as a function of the gross interest rate, r, for different distribution functions in each panel. Note that $r\left[1-F(r)\right]$ and $r\left[1-H\left(r\right)\right]$ cross at $r=\mu$, where $F(\mu)=H(\mu)=1/2$. In addition, joint financing yields a higher return than separate financing, $r\left[1-H\left(r\right)\right]>r\left[1-F(r)\right]$, whenever $r<\mu$, and vice-versa for $r>\mu$. The green line corresponds to the initial outlay (equal to I) that creditors must recover in order to break even.



In the left and middle panels, the (lowest) return level r_m^* at which the creditors break even is such that $r_m^*<\mu$. Hence, joint financing results in a lower interest rate, as well as in a lower probability of bankruptcy. The right panel shows that for separate financing to result in a lower interest rate two conditions must be satisfied: (i) the creditor's returns under separate financing is increasing at $r=\mu$ (i.e., $\left[1-H\left(\mu\right)\right]-\mu h\left(\mu\right)=1/2-\mu h\left(\mu\right)>0$), so that the maximum expected returns under separate financing is higher than under joint financing, and (ii) the rate at which the creditor breaks even is to the right of the point at which the expected returns under separate and joint financing cross (i.e., $\mu\left[1-H\left(\mu\right)\right]=\mu\left[1-F\left(\mu\right)\right]=\mu/2<I$).

Proof of Lemma 1

The inefficiencies for $r=r_L$ are

$$\tau p\left(r_H-r_L\right) \tag{18}$$

for $r=(r_L+r_H)/2$

$$\beta(1-p)^2 r_L+\tau p^2\left(r_H-r_L\right)/2 \tag{19}$$

and for $r=r_H$

$$\beta(1-p)^2 r_L+\beta 2p(1-p)\left(r_H+r_L\right)/2 \tag{20}$$

and therefore $EV_m(r_H)>EV_m((r_L+r_H)/2)$ as long as

$$\beta 2p(1-p)\left(r_H+r_L\right)/2<\tau p^2\left(r_H-r_L\right)/2 \tag{21}$$

We can then check that $EV_J(r_H) > EV_J(r_L)$ (see appendix below). If this condition is not satisfied, then the intermediate is preferred to r_L as long as

$$\beta(1-p)^2 r_L < \tau p \left(r_H - r_L\right)\left(1 - p/2\right) \tag{22}$$

If this condition is not satisfied then then r_L is preferred (again, it should be the case that if this condition is not satisfied, then r_L is preferred to r_H. Therefore if $w > w^*$ then $r^* = r_L$, if $w_J^* > w > w_J^{**}$ and if $w < w_J^{**}$ then $r^* = r_H$, where

$$w_J^* \equiv \frac{p\left(r_H - r_L\right)\left(1 - p/2\right)}{(1-p)^2 r_L} \text{ and } w_J^{**} \equiv \frac{p^2\left(r_H - r_L\right)/2}{2p(1-p)\left(r_H + r_L\right)/2} \tag{23}$$

Clearly $w_J^* > w_J^{**}$ as this reduces to $(2-p)\left(r_H + r_L\right) > (1-p) r_L$, which is clearly satisfied.

Proof of Proposition 3

It is easy to show that $w_J^* > w_S^* > w_J^{**}$ and therefore we have the following cases (i) if $\frac{\beta}{\tau} > w_J^*$ in we go for r_L in both joint or separate, (ii) if $w_J^* > \frac{\beta}{\tau} > w_S^*$ then in separate we go for r_L and in joint for $(r_L + r_L)/2$, (iii) if $w_S^* > \frac{\beta}{\tau} > w_J^{**}$ then in separate we go for r_H and in joint for $(r_L + r_L)/2$ and (iv) if $w_J^{**} > \frac{\beta}{\tau}$ we go for r_H in both cases. In (i), we have the same inefficiencies, in (ii) we have that joint is better because the inefficiencies with $(r_L + r_L)/2$ are lower than those with r_L which are the same as in separate. In (iii), we have that the inefficiencies in joint are lower if and only if

$$\frac{p\left(r_H - r_L\right)/2}{(1-p) r_L} \equiv w^{***} < \frac{\beta}{\tau}$$

but remember that in this case we have that $w_S^* > \frac{\beta}{\tau} > w_J^{**}$. It is easy to check that $w_S^* > w^{***} > w_J^{**}$. Finally in (iv), clearly, separate is better as the inefficiency as the losses in joint are greater than in separation because it is easy to check that

$$\beta(1-p)^2 r_L + \beta 2p(1-p)\left(r_H + r_L\right)/2 > \beta(1-p) r_L. \tag{24}$$

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