Product Mix and Firm Productivity Responses to Trade Competition

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1 Introduction

In this paper, we document how demand shocks in export markets lead French multi-product exporters to re-allocate the product mix sold in those destinations. We develop a theoretical model of multi-product firms that highlight the specific demand and cost conditions needed to generate all those empirical predictions. We then show how the increased competition from the demand shocks in export markets – and the associated product mix responses – lead to substantial productivity improvements for multi-product exporters.

Recent studies using detailed micro-level datasets on firms, plants, and the products they produce have documented vast differences in all measurable performance metrics across those different units. Those studies have also documented that these performance differences are systematically related to participation in international markets (see, e.g., Mayer and Ottaviano (2008) for Europe, and Bernard, Jensen, Redding and Schott (2012) for the U.S.): Exporting firms and plants are bigger, more productive, more profitable, and less likely to exit than non-exporters. And better performing firms and plants export a larger number of products to a larger number of destinations. Exporters are larger in terms of employment, output, revenue and profit. Similar patterns also emerge across the set of products sold by multi-product firms. There is a stable performance ranking for firms based on the products' performance in any given market, or in worldwide sales. Thus, better performing products in one market are most likely to be the better performing products in any other market (including the global export market). This also applies to the products' selection into a destination, so better performing products are also sold in a larger set of destinations.

Given this heterogeneity, trade shocks induce many different reallocations across firms and products. Some of these reallocations are driven by 'selection effects' that determine which products are sold where (across domestic and export markets), along with firm entry/exit decisions (into/out of any given export market, or overall entry/exit of the firm). Other reallocations are driven by 'competition effects' whereby – conditional on selection (a given set of products sold in a given market) – trade affects the relative market shares of those products. Both types of reallocations generate (endogenous) productivity changes that are independent of 'technology' (the production function at the product-level). This creates an additional channel for the aggregate gains from trade.

Unfortunately, measuring the direct impact of trade on those reallocations across firms is a very hard task. On one hand, shocks that affect trade are also likely to affect the distribution of market shares across firms. On the other hand, changes in market shares across firms likely reflect many technological factors (not related to reallocations). Looking at reallocations across products within firms obviates some of these problems. Recent theoretical models of multi-product firms highlight how trade induces a similar pattern of reallocations within firms as it does across firms. And measuring reallocations within multi-product firms has several advantages: Trade shocks that are exogenous to individual firms can be identified much more easily than at a higher level of aggregation; Controls for any technology changes at the firm-level are also possible; and reallocations can be measured for the same set of narrowly defined products sold by same firm across destinations or over time. Moreover, multi-product firms dominate world production and trade flows. Hence, reallocations within multi-product firms have the potential to generate large changes in aggregate productivity. Empirically, we find very strong evidence for the effects of trade shocks on those reallocations, and ultimately on the productivity of multi-product firms. The overall impact on aggregate French manufacturing productivity is substantial.

2 Previous Evidence on Trade-Induced Reallocations

In a previous paper, Mayer, Melitz and Ottaviano (2014), we investigated, both theoretically and empirically, the mechanics of these reallocations within multi-product firms. We used a comprehensive firm-level data on annual shipments by all French exporters to all countries in the world (not including the French domestic market) for a set of more than 10,000 goods. Firm-level exports are collected by French customs and include export sales for each 8-digit (combined nomenclature or NC8) product by destination country. Our focus then was on the cross-section of firm-product exports across destinations (for a single year, 2003). We presented evidence that French multi-product firms indeed exhibit a stable ranking of products in terms of their shares of export sales across export destinations with 'core' products being sold in a larger number of destinations (and commanding larger market shares across destinations). We used the term 'skewness' to refer to the dispersion of these export market shares in any destination and showed that this skewness consistently varied with destination characteristics such as GDP and geography: French firms sold relatively more of their best performing products in bigger, more centrally-located destinations (where competition from other exporters and domestic producers is tougher).

Other research has also documented similar patterns of product reallocations (within multi-product firms) over time following trade liberalization. For the case of CUSFTA/NAFYA, Baldwin and Gu (2009), Bernard, Redding and Schott (2011), and Iacovone and Javorcik (2008) all report that Canadian, U.S., and Mexican multi-product firms reduced the number of products they produce during these trade-liberalization episodes. Baldwin and Gu (2009) and Bernard, Redding and Schott (2011) further report that CUSFTA induced a significant increase in the skewness of production across products. Iacovone and Javorcik (2008) separately measure the skewness of Mexican firms' export sales to the US. They report an increase in this skewness following NAFTA: They show that Mexican firms expanded their exports of their better performing products (higher market shares) significantly more than those for their worse performing exported products during the period of trade expansion from $1994-2003$.

As prices are rarely observed, there is little direct evidence on how markups, prices and costs are related across products supplied with different productivity, and how they respond to trade liberalization.[1] A notable exception is the recent paper by DeLoecker, Goldberg, Pavcnik and Khandelwal (2012) who exploit unique information on the prices and quantities of Indian firms' products over India's trade liberalization period from $1989-2003$. They also document that better performing firms (higher sales and productivity) produce more products. They then focus on markups. Across firms, they document that those better performing firms set higher markups. They also document a similar patter across the products produced by a given multi-product firm, which sets relatively higher markups on their better performing products (lower marginal cost and higher market shares). In addition, they show strong evidence for endogenous markup adjustments via imperfect pass-through from products' marginal costs to their prices: Only a portion of marginal cost decreases are passed on to consumers in the form of lower prices, while the remaining portion goes to higher markups. This is consistent with recent firm-level evidence on exchange rate pass-through. Berman, Martin and Mayer (2012) analyze the heterogeneous reaction of exporters to real exchange rate changes using a rich French firm-level data set with destination specific export values and volumes on the period $1995-2005$. They find that on average firms react to depreciation by increasing their markup. They also find that high-performance firms increases their markup significantly more – implying that the pass-through rate is significantly lower for better performing firms.

3 Reallocations Over Time

We now document how changes *within* a destination market over time induce a similar pattern of reallocations as the ones we previously described. More specifically, we show that demand

[1] Prices are typically backed-out as unit values based on reported quantity information, which is extremely noisy.

shocks in any given destination market induce firms to skew their product level export sales to that destination towards their best performing products. In terms of first moments, we show that these demand shocks also lead to strong positive responses in both the intensive and extensive margins of export sales to that destination.

3.1 Data

We use the same data as Mayer, Melitz and Ottaviano (2014), the only difference being multiple years $1995-2005$ instead of a single year 2003. Besides what we already discussed, the reporting criteria for all firms operating in the French metropolitan territory are as follows. For within EU exports, the firm's annual trade value exceeds 100,000 Euros;[2] and for exports outside the EU, the exported value to a destination exceeds 1,000 Euros or a weight of a ton. Despite these limitations, the database is nearly comprehensive. For instance, in 2003, 100,033 firms report exports across 229 destination countries (or territories) for 10,072 products. This represents data on over 2 million shipments.

We restrict our analysis to export data in manufacturing industries, mostly eliminating firms in the service and wholesale/distribution sector to ensure that firms take part in the production of the goods they export.[3] This leaves us with data on over a million shipments by firms in the whole range of manufacturing sectors.[4]

3.2 Measuring Trade Shocks

Consider a firm i who exports a number of products s in industry I to destination d in year t. We measure industries (I) at the 3-digit ISIC level (35 different classifications across French manufacturing). We consider several measures of demand shocks that affect this export flow. At the most aggregate level we use the variation in GDP in d, $\log GDP_{d,t}$. At the industry level I, we use total imports into d *excluding* French exports, $\log M^I_{d,t}$. We can also use our detailed product-level shipment data to construct a firm i-specific demand shock:

$$\text{shock}^I_{i,d,t} \equiv \overline{\log M^s_{d,t}} \quad \text{for all products } s \in I \text{ exported by firm } i \text{ to } d \text{ in year } t_0, \tag{1}$$

[2] If that threshold is not met, firms can choose to report under a simplified scheme without supplying export destinations. However, in practice, many firms under that threshold report the detailed export destination information.

[3] Some large distributors such as Carrefour account for a disproportionate number of annual shipments.

[4] In a robustness check, we also drop observations for firms that the French national statistical institute reports as having an affiliate abroad. This avoids the issue that multinational firms may substitute exports of some of their best performing products with affiliate production in the destination country, thus reducing noise in the product export skewness. Results are quantitatively very similar in all regressions.

where $M^s_{d,t}$ represents total imports into d (again, excluding French exports) for product s. For world trade, the finest level of product level of aggregation is the HS-6 level (from UN-COMTRADE and CEPII-BACI), which is slightly more aggregated than our NC8 classification for French exports (roughly 5,300 HS products per year versus 10,000 NC8 products per year). The construction of this last trade shock is very similar to the one for the industry level imports $\log M^I_{d,t}$, except that we only use imports into d for the precise product categories that firm i exports to d.[5] In order to ensure that this demand shock is exogenous to the firm, we use the set of products exported by the firm in its first export year in our sample (1995, or later if the firm starts exporting later on in our sample), and then exclude this year from our subsequent analysis. Note that we use an un-weighted average so that the shocks for all exported products s (within an industry I) are represented proportionately.

For all of these demand shocks $X_t = GDP_{d,t}, M^I_{d,t}, M^s_{d,t}$, we compute the first difference as the Davis-Haltiwanger growth rate: $\tilde{\Delta} X_t \equiv (X_t - X_{t-1}) / (.5X_t + .5X_{t-1})$. This measure of the first difference preserves observations when the shock switches from 0 to a positive number, and has a maximum growth rate of -2 or 2. This is mostly relevant for our measure of the firm-specific trade shock, where the product-level imports into d, $M^s_{d,t}$ can often switch between 0 and positive values. Whenever $X_{t-1}, X_t > 0$, $\tilde{\Delta} X_t$ is monotonic in $\Delta \log X_t$ and approximately linear for typical growth rates ($|\Delta \log X_t| < 2$).[6] We thus obtain our three measures of trade shocks in first differences: $\tilde{\Delta} GDP_{d,t}, \tilde{\Delta} M^I_{d,t}, \overline{\tilde{\Delta} M^s_{d,t}}$. For the firm i-specific shock $\overline{\tilde{\Delta} M^s_{d,t}}$, we take the un-weighted average of the growth rates for all products exported by the firm in $t-1$.

3.3 The Impact of Demand Shocks on Trade Margins and Skewness

Before focusing on the effects of the demand shocks on the skewness of export sales, we first show how the demand shocks affect firm export sales at the intensive and extensive margins (the first moments of the distribution of product export sales). Table 1 reports how our three demand shocks (in first differences) affect changes in firm exports to destination d in ISIC I (so each observation represents a firm-destination-ISIC combination). We decompose the firm's export response to each shock into an intensive margin (average exports per product) and an extensive margin (number of exported products). We clearly see how all three demand shocks induce very strong (and highly

[5] There is a one-to-many matching between the NC8 and HS6 product classifications, so every NC8 product is assigned a unique HS6 classification. We use the same $M^s_{d,t}$ data for any NC8 product s within the same HS6 classification.

[6] Switching to first difference growth rates measured as $\Delta \log X_t$ (and dropping products with zero trade in the trade shock average) does not materially affect any of our results.

significant) positive responses for both margins. This confirms that our demand shocks capture important changes in the local demand faced by French exporters.[7]

Table 1: Demand shocks and local exports

Dependent Variable	Δ log Exports per Product			Δ log # Products Exported		
$\tilde{\Delta}$ GDP Shock	0.486^a			0.147^a		
	(0.046)			(0.016)		
$\tilde{\Delta}$ Trade Shock		0.273^a			0.075^a	
		(0.009)			(0.004)	
$\tilde{\Delta}$ Trade Shock - ISIC			0.038^a			0.014^a
			(0.005)			(0.002)
Observations	396740	402522	402522	396740	402522	402522

Standard errors in parentheses: $^c < 0.1$, $^b < 0.05$, $^a < 0.01$. All regressions include year dummies, and standard errors clustered at the level relevant for the variable of interest: destination country for columns (1) and (4), firm-destination for columns (2) and (5) and ISIC-destination for columns (3) and (6).

We now investigate the consequences of those demand shocks for the skewness of export sales (independent of the level of product sales). In Mayer, Melitz and Ottaviano (2014), we focused on those effects in the cross-section across destinations. Here, we examine the response of skewness within a destination over time using our new demand shocks. We rely on the Theil index as our measure of skewness due to its aggregation properties: We will later aggregate the export responses at the destination-ISIC level up to the firm-level – in order to generate predictions for firm-level productivity. Thus, our measure of skewness for the distribution of firm i's exports to destination d in industry I, $x^s_{i,d,t}$, is the Theil index:

$$T^I_{i,d,t} \equiv \sum_{s \in I} \frac{x^s_{i,d,t}}{x^I_{i,d,t}} \log\left(\frac{x^I_{i,d,t}}{x^s_{i,d,t}}\right), \quad x^I_{idt} \equiv \sum_{s \in I} x^s_{i,d,t}. \tag{2}$$

Table 2 reports regressions of this skewness measure on all three demand shocks (jointly) – at the firm-destination-ISIC level. In the first column, we use a specification in (log) levels (FE), and use firm-destination-ISIC fixed effects to isolate the variation over time. In the second column, we return to our specification in first differences (FD). In the third column we add the firm-destination-ISIC fixed effects to this specification in first differences (FD-FE). This controls for

[7] Specifications using the log levels of the shocks and firm-destination-ISIC fixed-effects yield similar results. Other specifications including the three covariates show that those demand shocks are different enough to be estimated jointly while each keeping its positive sign and statistical significance.

any trend growth rate in our demand shocks over time. Across all three specifications, we see that positive (negative) demand shocks induce a highly significant increase in the skewness of firm export sales to a destination.[8] Next, we construct a measure of the change in skewness restricted to the subset of products s exported in both periods (for the first difference specifications). This alternate skewness measure $\Delta T_{i,d,t}^{I,\text{const}}$ isolates changes that are driven only by the intensive margin of exports. The two first-difference specifications using this alternate skewness measure are reported in the last two columns. The effects of our firm-level trade shock is still highly significant (well beyond the 1% level), whereas the effects based on the ISIC-level trade shock are teetering at the 5% significance level. However, those columns show that the GDP shocks tend to proportionately change the export sales of 'incumbent' products. The effect of the GDP shock on skewness thus comes entirely from the extensive margin (new products exported in response to higher GDP levels in a destination).[9]

Table 2: Demand shocks and local skewness

Dependent Variable	$T_{i,d,t}^{I}$	$\Delta T_{i,d,t}^{I}$		$\Delta T_{i,d,t}^{I,\text{const}}$	
Specification	FE	FD	FD-FE	FD	FD-FE
GDP Shock	0.076^a				
	(0.016)				
Trade Shock	0.047^a				
	(0.005)				
Trade Shock - ISIC	0.002^a				
	(0.000)				
$\tilde{\Delta}$ GDP Shock		0.067^a	0.068^a	-0.005	-0.004
		(0.012)	(0.016)	(0.008)	(0.009)
$\tilde{\Delta}$ Trade Shock		0.036^a	0.032^a	0.012^a	0.012^a
		(0.005)	(0.006)	(0.003)	(0.003)
$\tilde{\Delta}$ Trade Shock - ISIC		0.006^a	0.004	0.002	0.004^b
		(0.002)	(0.003)	(0.001)	(0.002)
Observations	474506	396740	396740	437626	437626

Standard errors in parentheses: $^c < 0.1$, $^b < 0.05$, $^a < 0.01$. All regressions include year dummies, and standard errors clustered at the level of the destination country.

[8] The effect of all three shocks are weakened a little bit due to some collinearity. Even in the FD-FE specification, the ISIC-level shock remains significant at the 1% level when it is entered on its own.

[9] These newly exported products have substantially smaller market shares than the incumbent products and therefore contribute to an increase in the skewness of export sales.

4 Theory

In the previous section we documented the pattern of product reallocations in response to demand shocks in export markets. We now develop a theoretical model of multi-product firms that highlights the specific demand conditions needed to generate this pattern. Our theory shows that these demand conditions imply that the demand shocks lead to increased competition for exporters in those markets. In addition, we show how this increased competition and the associated product mix responses lead to higher firm productivity. Finally, we also show that the demand conditions needed to predict the observed pattern of reallocations to demand shocks are the same that are needed to predict the relation among firm/product performance measures discussed in Section 2.

In particular, we first present a flexible model with both single- and multi-product firms characterizing the properties of the demand system that are needed to predict the observed relation between firm/product performance measures. We then show that these properties *necessarily* imply the documented the pattern of product reallocations in response to demand shocks in export markets. This is our main point of departure relative to the recent literature focusing on more flexible demand systems in monopolistic competition models of trade.[10]

4.1 Closed Economy

To better highlight the role played by the properties of the demand system, we initially start with a closed economy. We will then move to the open economy to discuss the trade shocks and their effects.

Multi-Product Production with Additive Separable Utility

Consider an economy populated by L^w identical households, each consisting of η workers. Thus, each worker supplies $1/\eta$ efficiency units of labor inelastically so that labor supply equals L^w while the total number of consumers equals $L^c = \eta L^w$. Labor is the only productive factor and efficiency units of labor per worker are chosen as numeraire so that each consumer earns unit wage (and household income is η). We think of a demand shock as a change in η for given L^w, as this changes the number of consumers L^c for given number of workers.[11]

[10] See, for instance, Arkolakis, Costinot, Donaldson and Rodriguez-Clare (2012), Zhelobodko, Kokovin, Parenti and Thisse (2012), Fabinger and Weyl (2012 and 2014), Mrazova and Neary (2014), Parenti, Ushchev and Thisse (2014).

[11] In the same vein, Parenti, Ushchev and Thisse (2014) consider a number of consumers $L_c = L$, each inelastically supplying y efficiency units so that labor supply equals $L^w = yL = yL^c$. Clearly, imposing $\eta = 1/y$ delivers our parametrization. However, they then study the comparative statics with respect to changes in L and y. Both changes

Utility is assumed to be additive separable over a continuum of imperfectly substitutable products indexed $i \in [0, M]$ where M is the measure of products available. The typical consumer in any household solves the following utility maximization problem:

$$\max_{x_i \geq 0} \int_0^M u(x_i) di \text{ s.t. } \int_0^M p_i x_i di = 1,$$

where $u(x_i)$ is the sub-utility associated with the consumption of x_i units of product i and expenditure equals unit wage. We assume that this sub-utlity exhibits the following properties:

(A1) $u(x_i) \geq 0$ with equality for $x_i = 0$; $u'(x_i) > 0$ and $u''(x_i) < 0$ for $x_i \geq 0$.

The first order conditions for the consumer's problem determine the *inverse* demand function:

$$p_i = \frac{u'(x_i)}{\lambda}, \text{ with } \lambda = \int_0^M u'(x_i) x_i di, \tag{3}$$

where $\lambda > 0$ is the marginal utility of income. Larger λ shifts inverse demand downwards, reducing the price the consumer is willing to pay for any level of consumption. Concavity of $u(x_i)$ ensures that x_i satisfying (3) also meets the second order condition for the consumer's problem. Note that λ is an increasing function of M and x_i.

Products are supplied by firms that may be single- or multi-product. Market structure is monopolistically competitive as in Mayer, Melitz and Ottaviano (2014) in that each product is supplied by only one firm and each firm supplies a countable number of the continuum of products. Technology exhibits increasing returns to scale associated with a fixed production cost, along with a constant marginal cost. The fixed cost f is the same for all products while the marginal cost v differs across them. For a given firm, products are indexed in increasing order m of marginal cost from a 'core product' indexed by $m = 0$. Moreover, entry incurs a sunk cost f^e. Only after this cost is incurred, entrants randomly draw their marginal cost levels for their core products from a common continuous differentiable distribution with density function $\gamma(c)$ and cumulative density function $\Gamma(c)$ defined over the support $[0, \infty)$. We use $M(c)$ to denote the number of products supplied by a firm with core marginal cost c and $v(m, c)$ to denote the marginal cost of its product m. We assume $v(m, c) = cz(m)$ with $z(0) = 1$ and $z'(m) > 0$.[12]

thus affect labor supply, which we want instead to hold constant.

[12] The assumption $z'(m) > 0$ will generate the within-firm ranking of products discussed in Section 2. In the limit case when $z'(m)$ is infinite, all firms are single-product.

An entrant supplying product i with marginal cost v solves the profit maximization problem:

$$\max_{q_i \geq 0} \pi(q_i) = p_i q_i - v q_i - f,$$

subject to the market clearing condition for its output $q_i = x_i L^c$ and inverse demand given by (3).[13] The optimal level of output $q_v = x_v L^c$ satisfies the first order condition:

$$u'(x_v) + u''(x_v)x_v = \lambda v, \tag{4}$$

where $r(x_v) = \phi(x_v)/\lambda$ is the *marginal* revenue associated with a given variety. Markup pricing is revealed by rewriting (4) as

$$p(x_v) = \frac{v}{1 - \varepsilon_p(x_v)}, \tag{5}$$

where

$$\varepsilon_p(x_v) \equiv -\frac{u''(x_v)x_v}{u'(x_v)} \tag{6}$$

is the elasticity of inverse demand, such that $\varepsilon_p(x_v) \in (0, 1)$.[14] This is a measure of the concavity of $u(x_v)$.[15]

The optimal level of output $x_v$must also satisfy the second order condition for profit maximization:

$$\phi'(x_v) \equiv 2u''(x_v) + u'''(x_v)x_v < 0, \tag{7}$$

which can be restated is terms of the elasticity of marginal revenue $\varepsilon_r(x_v)$ as:

$$\varepsilon_r(x_v) \equiv -\frac{\phi'(x_v)x_v}{\phi(x_v)} > 0. \tag{8}$$

This requires the inverse demand to be not too convex and implies that, for any given λ, a unique solution $x_v(\lambda v)$ exists for (4).[16] This is our second assumption:

(A2) $\varepsilon_r(x_v) > 0$.

[13] This problem is faced by any entrant, no matter whether single- or multi-product, as our assumptions rule out cannibalization within and between entrants' product ranges.

[14] As in Zhelobodko, Kokovin, Parenti and Thisse (2012), we use $\varepsilon_p(x_v)$ to denote the elasticity of $p(x_v)$ with respect to x_v. This is the inverse of the price elasticity of demand that would be denoted $\varepsilon_x(p_v)$.

[15] This elasticity should not be confused with the elasticity of utility $u'(x_v)x_v/u(x_v)$, which is an inverse measure of "love of variety". As discussed by Neary and Mrazova (2014), this elasticity is important for welfare analysis. In Zhelobodko et al (2012) $\varepsilon_p(x_v)$ is called "relative love of variety".

[16] In Mrazova and Neary (2014), (8) is equivalently stated as $\rho(x_v) < 2$ where $\rho(x_v) \equiv -[u'''(x_v)x_v]/u''(x_v) = 2 - \varepsilon_r(x_v)[1 - \varepsilon_p(x_v)]/\varepsilon_p(x_v)$ measures the convexity of inverse demand.

Hence, (A1) and (A2) are necessary and sufficient conditions for the consumers' and firms' optimization problems. When satisfied, they imply a unique output and price level for all varieties $x_v > 0$ and $p(x_v) > 0$, and for any given $\lambda > 0$. As all admissible additive separable preferences must satisfy (A1) and (A2), for all subsequent results we will assume that (A1) and (A2) always hold without explicitly mentioning those assumptions each time.

Several implications of cost heterogeneity for product performance matching some key findings discussed in Section 2 can be derived from those optimization conditions. In particular, as long as (A1) and (A2) hold, lower cost firms/products are associated with lower price, larger output, larger revenue and larger profit.[17]

Free Entry Equilibrium

In equilibrium consumers maximize utility, firms maximize profits, and their optimal choices in the product and labor markets are mutually compatible. To characterize this equilibrium outcome, it is useful to make the dependence of maximized operating profit π_v and profit-maximizing output x_v on the endogenous marginal utility of income explicit. In particular, we define $\pi_v = \pi^*(v, \lambda)L^c$ and $x_v = x^*(\lambda v)$ with

$$\pi^*(v, \lambda) = \max_x \left[\frac{u'(x)}{\lambda} - v\right] x,$$
$$x^*(\lambda v) = \arg\max_x \left[\frac{u'(x)}{\lambda} - v\right] x,$$

so that, given (A1) and (A2), we obtain:

$$\frac{\partial x^*(\lambda v)}{\partial v} = \frac{\lambda}{u''(x_v)\left[2 - \rho(x_v)\right]} < 0, \qquad (9)$$
$$\frac{\partial x^*(\lambda v)}{\partial \lambda} = \frac{v}{u''(x_v)\left[2 - \rho(x_v)\right]} < 0.$$

By the envelope theorem, maximized profit is decreasing in both its arguments:

$$\frac{\partial \pi^*(v, \lambda)}{\partial v} = -x^*(\lambda v) < 0, \qquad (10)$$
$$\frac{\partial \pi^*(v, \lambda)}{\partial \lambda} = -\frac{u'(x^*(\lambda v))x^*(\lambda v)}{\lambda^2} < 0.$$

The fact that maximized profit is decreasing in marginal cost implies that only products with

[17] See the appendix for a proof.

marginal cost v below some cost cutoff $\widehat{v}$ can be profitably produced. At the same time, entrants that do not find it profitable to sell even their core products will decide not to produce at all. Thus, the product cutoff level $\widehat{v}$ is also the firm cutoff level $\widehat{c}$ for core competency: entrants drawing a core marginal cost $c > \widehat{c}$ exit immediately without producing.

Given $L_c = \eta L^w$ and $v(0,c) = c$, the indifference condition for the marginal producer is:

$$\pi^*(\widehat{c}, \lambda)\eta L^w = f. \tag{11}$$

Since $\pi^*(c, \lambda)$ is decreasing in both c and λ, this cutoff condition has a unique solution $\widehat{c}(\lambda)$. For a given measure of entrants N_e, $\widehat{c}(\lambda)$ determines the fraction of those entrants that eventually produce: $\Gamma(\widehat{c}(\lambda))$.

All prospective entrants are identical ex-ante. Free entry then requires that expected profit equal the sunk entry cost. Post-entry, an entrant with a core cost draw $c \leq \widehat{c}$ earns profit:

$$\Pi^*(c, \lambda) \equiv \sum_{m=0}^{M(c)-1} \left[\pi^*\left(cz(m), \lambda\right) L^c - f\right],$$

where $M(c)$ is the number of products the entrant supplies with marginal cost $cz(m) \leq \widehat{c}$. Hence, upon entry, the expected profit of an entrant is

$$\begin{aligned} \int_0^{\widehat{c}} \Pi^*(c, \lambda)\gamma(c)dc &= \int_0^{\widehat{c}} \left\{ \sum_{\{m|cz(m)\leq\widehat{c}\}} \left[\pi^*\left(cz(m), \lambda\right) L^c - f\right] \right\} \gamma(c)dc \\ &= \sum_{m=0}^{\infty} \left[\int_0^{\widehat{c}/z(m)} \left[\pi^*\left(cz(m), \lambda\right) L^c - f\right] \gamma(c)dc \right]. \end{aligned}$$

The free entry condition can then be re-stated as

$$\int_0^{\widehat{c}} \Pi^*(c, \lambda)\gamma(c)dc = \sum_{m=0}^{\infty} \left[\int_0^{\widehat{c}/z(m)} \left[\pi^*\left(cz(m), \lambda\right) \eta L^w - f\right] \gamma(c)dc \right] = f^e \tag{12}$$

Equations (11) and (12) jointly determine the equilibrium cost cutoff $\widehat{c}^*$ and the marginal utility of income λ^*. As both $\Pi^*(c, \lambda)$ and $\widehat{c}(\lambda)$ decrease in λ, this solution $(\widehat{c}^*, \lambda^*)$ exists and is unique. The labor market clearing condition

$$N_e \left\{ f^e + \sum_{m=0}^{\infty} \left[\int_0^{\widehat{c}/z(m)} \left[cz(m)x^*\left(cz(m), \lambda\right) \eta L^w + f\right] \gamma(c)dc \right] \right\} = L^w, \tag{13}$$

evaluated for $\widehat{c} = \widehat{c}^*$, then determines the equilibrium number of entrants N_e^* and producers $N^* = \Gamma(\widehat{c}^*)\, N_e^*$.

Reconciling Empirical Facts with Preferences

We have already argued that, as long as (A1) and (A2) hold, lower cost firms/products are associated with lower price, larger output, larger revenue and larger profit. These implications match some of the empirical relationships among firm/product performance measures highlighted in Section 2, and are common across all preferences compatible with utility and profit maximization. However, additional restrictions are needed in order to capture the additional empirical regularities we described.

First, De Loecker, Goldberg, Pavcnik and Khandelwal (2012) show that lower costs are associated with larger markups so that cost advantages are not fully passed through to prices. As the pass-through from cost to price can be expressed as

$$\theta(x_v) = \frac{d \ln p(x_v)}{d \ln v} = \frac{\varepsilon_p(x_v)}{\varepsilon_r(x_v)}, \tag{14}$$

a necessary and sufficient condition for $\theta(x_v) < 1$ is that the elasticity of *marginal* revenue is larger than the elasticity of inverse demand: $\varepsilon_r(x_v) > \varepsilon_p(x_v)$.[18] This condition can be equivalently stated as requiring that the elasticity of inverse demand increases in consumption. This is our third assumption:[19]

$$\textbf{(B1)}\ \varepsilon_p'(x_v) > 0.$$

Second, Berman, Martin and Mayer (2012) find that high-performance firms react to a real exchange depreciation by increasing significantly more their markup. This happens when the pass-through decreases with firm performance: $\theta'(x_v) < 0$. Given (B1), a necessary condition for decreasing pass-through to be predicted by our model is that marginal revenue is increasing in x_v

[18] See also proof in the appendix.

[19] In the terminology of Neary and Mrazova (2014) $\varepsilon_p'(x_v) > 0$ defines the "subconvex" case, with "subconvexity" of an inverse demand function $p(x)$ at an arbitrary point x_c being equivalent to the function being less convex at that point than a CES demand function with the same elasticity. In the terminology of Zhelobodko, Kokovin, Parenti and Thisse (2012), in this case preferences are said to display an increasing "relative love of variety" (RLV) as consumers care less about variety when their consumption level is lower. The RLV is, thus, increasing if and only if the demand for a variety becomes more elastic when the price of this variety rises. Also the "Adjustable pass-through" (Apt) class of demand functions proposed by Fabinger and Weyl (2012) satisfies (B1). This assumption is, instead, weaker than the assumption by Arkolakis, Costinot, Donaldson and Rodriguez-Clare (2012) that the demand function of any product is log-concave in log-prices. Log-concavity implies (B1) but not vice versa and, thus, log-concavity is not necessary to associate lower cost with larger markups (see the appendix for proofs).

(hence decreasing in v).[20] This is our fourth assumption:

$$\textbf{(B2)}\ \varepsilon_r'(x_v) > 0.$$

Third and last, empirically lower cost firms/products are associated with larger employment. This is the case if and only if the elasticity of marginal revenue is smaller than one, otherwise products associated with negligible marginal cost would be produced with negligible employment.[21] This is our fifth and final assumption:

$$\textbf{(B3)}\ \varepsilon_r(x_v) < 1$$

To summarize, for the model to be consistent with the existing evidence on heterogeneous firm performance, the following properties of demand have to hold: $0 < \varepsilon_p(x_v) < 1$, $0 < \varepsilon_r(x_v) < 1$, $\varepsilon_p^{'}(x_v) > 0$ and $\varepsilon_p^{'}(x_v) > 0$, or equivalently $0 < \varepsilon_p(x_v) < \varepsilon_r(x_v) < 1$ and $\varepsilon_r'(x_v) > 0$.[22] Figure 1 provides a graphical representation of demand and marginal revenue curve satisfying these properties. These additional properties of demand must be satisfied in order to generate predictions that are not counter-factual with respect to the empirical findings summarized in Section 2. Importantly, those empirical findings rule out the case of CES and log-convex demand.

Demand Shock

We now discuss the implications of our model under these empirically relevant assumptions (B1)-(B3). We focus on the effects of a demand shock (a change in η) on the product range, the product mix and the productivity of multi-product firms. In so doing, we distinguish between 'long run' effects when entry is free and 'short run' effects when entry is, instead, restricted.

Long Run The following proposition holds:[23]

[20] See proof in appendix.

[21] See appendix for proof.

[22] Although our results have been derived for an additive separable utility function, they eventually depend on the properties of the associated inverse demand. They thus may hold also for utility or expenditure functions that are not additive separable but still share those properties. In this respect, our results suggest that the taxonomy of demand systems proposed by Mrazova and Neary (2014) in terms of $1/\varepsilon_p(x_v)$ and $\rho(x_v)$ – or equivalently in terms of $\varepsilon_p(x_v)$ and $\varepsilon_r(x_v)$ – could be fruitfully enriched also in terms of $\rho'(x_v)$ – or equivalently in terms of $\varepsilon_r'(x_v)$ – to cover additional comparative statics implications that are crucial when products are associated with heterogeneous costs.

[23] In the case of non-separable preferences, Parenti, Thisse an Ushchev (2014) characterize general conditions on profits such that larger market size leads to lower cutoff, but point out that general conditions on demand are unavailable due to dependence on the cost distribution.

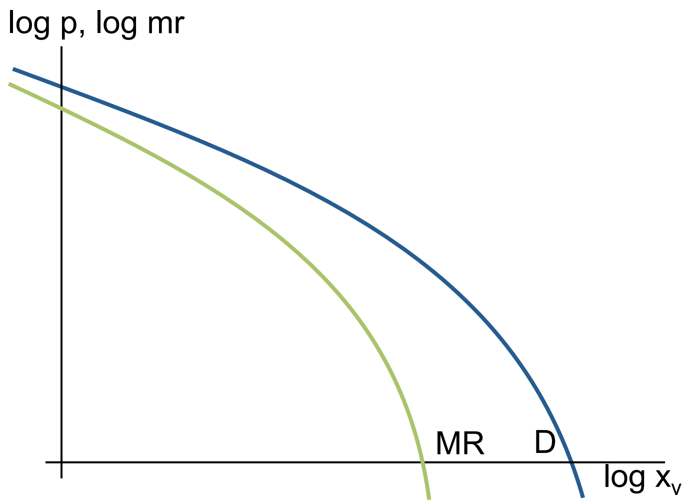

Figure 1: Graphical Representation of Demand Assumptions

Proposition 1 ***(Closed Economy)*** *Consider an additive separable utility function satisfying (A1)-(A2) and (B1)-(B3). Then, a positive demand shock (i) increases the marginal utility of income, (ii) reduces the firm cost cutoff, (iii) reallocates output, revenue and employment from higher to lower cost products, and (iv) increases (decreases) profit for low (high) cost products.*

Proof. See appendix. ■

Hence, we generate a clear cut result for the effects of a demand shock along the intensive firm/product margin. Note, however, that, since these effects are accompanied by a changing number of entrants, the impact of the demand shock on the number of producers and products supplied is ambiguous without further assumptions on the distribution of marginal cost $\Gamma(c)$.

Proposition 1 implies that a positive demand shock induces multi-product incumbents to (weakly) shed some more costly non-core products (and some single-product incumbents to stop producing altogether). It also induces multi-product incumbents to shift output, revenue and employment towards better-performing products (with lower marginal cost). Since those products already had larger output, revenue and employment before the shock, this leads to an increase in the 'skewness' of output, revenue and employment. Given our demand assumptions, higher skewness, in turn, leads to higher firm-productivity (the average productivity across the constant range of products produced both before and after the demand shock). Specifically, let M be the number of these products and index them by $m = 0, ..., M - 1$ in increasing order of marginal cost so that

$v_0 < v_1 < ... < v_{M-1}$. The average productivity computed for this set of products is then

$$\Phi = \frac{\sum_{m=0}^{M-1} x_m}{\sum_{m=0}^{M-1} v_m x_m} = \frac{\sum_{m=0}^{M-1} \frac{1}{v_m} v_m x_m}{\sum_{m=0}^{M-1} v_m x_m} = \frac{\sum_{m=0}^{M-1} \frac{1}{v_m} \ell_m}{\sum_{m=0}^{M-1} \ell_m} = \sum_{m=0}^{M-1} s_m \frac{1}{v_m},$$

where $\ell_m = v_m x_m$ is employment in product m, x_m is its output, and $s_m = \ell_m / \sum_{m=0}^{M-1} \ell_m$ is the product's employment share. Hence, Φ is the employment weighted average of products' productivity levels $1/v_m$'s. When the marginal utility of income increases due to the positive demand shock, firm productivity Φ increases.[24]

Short Run So far we have consider a long-run scenario in which firms can freely enter the market. We now consider an alternative short-run scenario in which the number of incumbents is fixed at $\overline{N}$. In this scenario (12) no longer holds, and we assume that the sunk entry costs have already been incurred (thus no employment associated with the entry cost f^e). The short-run equilibrium is then characterized by *two* conditions. The first is the zero cutoff profit condition (11):

$$\pi^*(\widehat{c}, \lambda)\eta L_w = f.$$

The second is the labor market clearing condition obtained from (13) after imposing $N_e = \overline{N}$ and $f^e = 0$:

$$\overline{N} \sum_{m=0}^{\infty} \left[\int_0^{\widehat{c}/z(m)} \left[cz(m) x^* \left(cz(m), \lambda \right) \eta L^w + f \right] \gamma(c) dc \right] = L^w. \tag{15}$$

These conditions pin down $\widehat{c}$ and λ for fixed $\overline{N}$. They imply that the all results in the previous proposition also hold in the short run, except (ii). This is because a positive demand shock has an ambiguous impact on the firm cost cutoff in the short-run (this also depends on distributional assumptions for the core cost draw).

5 Open Economy

With our empirical application in mind, we consider a simplified three-country economy consisting of a Home country (H: France) and a Foreign country (F: RoW) both exporting to a Destination country (D). For $l, h \in \{H, F, D\}$ trade from country l to country h is subject to both a variable iceberg cost $\tau_{lh} > 1$ and a fixed export cost f_h^x.

[24] See appendix for proofs.

Country D is the focus of the analysis and is assumed to be 'small' from the point of view of both H and F so that changes in destination D-specific variables do not affect equilibrium variables for either H or F (apart from the export cutoff to D and thus the margins of trade to D). Since wages in H and F are fixed, we choose labor as the numeraire. The short-run equilibrium is then characterized by D's zero cutoff profit condition with a fixed numbers of incumbents for D, H and F, and fixed domestic cutoffs for H and F. The long-run equilibrium is characterized by D's zero cutoff profit and free conditions with fixed domestic cutoffs and fixed numbers of incumbents for H and F.

To emphasize competition in the destination market D, we focus on a situation in which D does not export (and trade is thus unbalanced). The same qualitative results hold when one allows also for exports from D.[25] Under the assumption that firms located in D do not export, the long run equilibrium is characterized by the following *four* conditions: the zero cutoff profit for domestic sales in D

$$\pi^*_{DD}(\widehat{c}_{DD}, \lambda_D)\eta L^w_D = f; \tag{16}$$

the zero cutoff profit for export sales from $l \in \{H, F\}$ to D

$$\pi^*_{lD}(\tau_{lD}\widehat{c}_{lD}, \lambda_D)\eta L^w_D = f^x_D; \tag{17}$$

the free entry in D

$$\sum_{m=0}^{\infty}\left[\int_0^{\widehat{c}_{DD}/z(m)} \left[\pi^*_{DD}\left(cz(m), \lambda\right)\eta L^w_D - f\right]\gamma(c)dc\right] = f^e; \tag{18}$$

and the labor market clearing in D

$$N^e_D\left\{f^e + \sum_{m=0}^{\infty}\left[\int_0^{\widehat{c}_{DD}/z(m)} \left[cz(m)x^*_{DD}\left(cz(m), \lambda\right)\eta L^w_D + f\right]\gamma(c)dc\right]\right\} = L^w_D. \tag{19}$$

Given

$$\pi^*_{DD}(c, \lambda_D) = \max_x \left[\frac{u'(x)}{\lambda_D} - c\right]x, \tag{20}$$

conditions (16) and (18) pin down $\widehat{c}_{DD}$ and λ_D. Then, given

$$x^*_{DD}(c, \lambda_D) = \arg\max_x \left[\frac{u'(x)}{\lambda_D} - c\right]x, \tag{21}$$

[25] See our appendix for this extension.

condition (19) pins down the number of entrants N_D^e and producers $N_D^p = \Gamma(\widehat{c}_{DD}) N_D^e$.

The measure of products sold in D includes also those exported from $l \in \{H, F\}$. This is a fraction $\Gamma(\widehat{c}_{lD})$ of the fixed measure of incumbent products $\overline{M}_l^i$ in $l \in \{H, F\}$:

$$N_{lD}^x = \Gamma(\widehat{c}_{lD}) \overline{M}_l^i.$$

To determine $\widehat{c}_{lD}$, we write

$$\pi_{lD}^*(\tau_{lD} c, \lambda_D) = \max_x \left[\frac{u'(x)}{\lambda_D} - \tau_{lD} c \right] x \tag{22}$$

and

$$x_{lD}^*(\tau_{lD} \lambda_D c) = \arg\max_x \left[\frac{u'(x)}{\lambda_D} - \tau_{lD} c \right] x, \tag{23}$$

with $\tau_{lD} > 1$. Then, given λ_D, condition (17) pins down $\widehat{c}_{lD}$. With $f_D^X > f$ (17) also implies

$$\tau_{lD} \widehat{c}_{HD} < \widehat{c}_{DD}.$$

Consider now the effects of raising η. By comparing the corresponding expressions in the closed and the open economies, we see that *all closed economy results fully apply to D variables* ($\widehat{c}_{DD}$, λ_D, N_D^e, N_D^p) *in the open economy*. In particular, by (16) and (18), larger L_D^c increases λ_D and decreases $\widehat{c}_{DD}$ when the elasticity of inverse demand is increasing in consumption. Hence, accounting also for the response of the export cutoff $\widehat{c}_{lD}$, the following result holds:

Proposition 2 ***(Open Economy)*** *Consider an additive separable utility function satisfying (A1)-(A2) and (B1-B3). Then a positive demand shock in an export market (i) increases the local marginal utility of income, (ii) reduces the local firm cost cutoff, (iii) reallocates output, revenue and employment from higher to lower cost exported products, and (iv) increases (decreases) profit for low (high) exported cost products. Moreover, as long as the export fixed cost is large enough, the positive demand shock also decreases the export cost cutoff and increases the number of exported products, and aggregate exports.*

Proof. See appendix. ■

When the fixed export cost is large enough, the demand shock in D has opposite effects on the domestic cutoff (which increases) and the export cutoff (which decreases). This is due to the fact that the demand shock, in equilibrium, induces a rotation of the residual demand curve faced by each product. Thus, this demand shifts in for worse-performing products while it shifts out for the better performing ones. When the fixed export cost is high enough, the cutoff exported product has a relatively larger market share and thus benefits from the outward shift of a portion of the demand curve. Thus, even though competition and selection is tougher in D following the demand shock, an exporting firm will then respond by increasing the set of products exported to D.

In the short run the number of incumbent firms in D is fixed at $\overline{N}_D^i$, (18) does not hold, and there is no employment associated with the entry cost f_D^e. Then the short-run equilibrium is characterized by the following *three* conditions: the zero cutoff profit condition (16) for domestic sales in D; the zero cutoff profit (17) for export sales to D; and the labor market clearing condition

$$\overline{N}_D^i \left\{ \sum_{m=0}^{\infty} \left[\int_0^{\widehat{c}_{DD}/z(m)} \left[cz(m) x_{DD}^* \left(cz(m), \lambda \right) \eta L_D^w + f \right] \gamma(c) dc \right] \right\} = L_D^w. \tag{24}$$

Conditions (16) and (24) pin down $\widehat{c}_{DD}$ and λ_D for fixed $\overline{N}_D^i$. These are the *same as in the short-run closed economy*, so all corresponding short-run comparative statics results for larger η apply here too. Given λ_D, export cutoffs $\widehat{c}_{HD}$ and $\widehat{c}_{FD}$ are the determined by (17).

To conclude, we have shown that, under our demand assumptions, a positive demand shock in a destination market D will generate an increase in competition that induces exporters to reallocate their export sales towards their better performing products. This reallocation, in turn, will generate an increase in firm productivity. We now empirically test this prediction linking trade shocks and firm productivity.

6 Trade Competition and Productivity

Our theoretical model highlights how our measured demand shocks induce increases in competition for exporters to those destinations; and how the increased competition generates increases in productivity by shifting market shares and employment towards better performing products. We now directly test this link between increased trade competition (induced by the demand shocks in export markets) and productivity. We focus on the portion of the product reallocations and induced productivity increases *within* the firm. We can then use our constructed demand shocks (exogenous to the firm) and trace out their impact on firm productivity and product reallocations (at the firm level).

We obtain our measure of firm productivity by merging our firm-level trade data with firm-level production data. This latter dataset contains various measures of firm outputs and inputs. As we are interested in picking up productivity fluctuations at a yearly frequency, we focus on labor productivity measured as deflated value added per worker (using sector-specific price deflators). We then separately control for the impact of changes in factor intensities and returns to scale (or variable utilization of labor) on labor productivity. Note that this firm-level productivity measure aggregates (using labor shares) to the overall deflated value-added per worker for manufacturing. So long as our sector-specific price indices are accurately measured, this aggregate productivity measure accurately tracks a welfare-relevant quantity index – even though we do not have access to firm-level prices. In other words, the effect of pure markup changes at the sector level are netted-out of our productivity measure. We can thus report a welfare-relevant aggregate productivity change by aggregating our firm-level productivity changes using the observed changes in labor shares.[26]

6.1 Firm-Level Trade Shock

We cannot separately measure productivity at the destination or product level – only at the firm level. We thus need to aggregate our destination-industry measures of demand shocks to the firm-level. We use the firm's market shares in those destinations and industries to perform this aggregation.[27] In a first step, we perform this aggregation over all export destinations and obtain our firm-level demand shock in (log) levels and first difference:

$$\text{shock}_{i,t} \equiv \sum_{d,I} \frac{x^I_{i,d,t_0}}{x_{i,t_0}} \times \text{shock}^I_{i,d,t}, \qquad \tilde{\Delta}\text{shock}_{i,t} = \sum_{d,I} \frac{x^I_{i,d,t-1}}{x_{i,t-1}} \times \tilde{\Delta}\text{shock}^I_{i,d,t},$$

where $x_{i,t} \equiv \sum_{d,I} x^I_{i,d,t}$ represents firm i's total exports in year t. As was the case for the construction of our firm-level destination shock (see 1), we only use the firm-level information on exported products and market shares in prior years (the year of first export sales t_0 for the demand shock in levels and lagged year $t-1$ for the first difference between t and $t-1$). This ensures the exogeneity of our constructed firm-level demand shocks (exogenous to firm-level actions in year $t > t_0$ for levels, and exogenous to firm-level changes Δ_t for first differences). In particular, changes in the

[26] At the firm-level, an increase in markups across all products will be picked-up in our firm productivity measure – even though this does not reflect a welfare-relevant increase in output. But if this is the case, then this firm's labor share will decrease, and its productivity will carry a smaller weight in the aggregate index. In addition, increases in firm productivity are very strongly correlated with increases in labor shares – so this cannot be a pervasive component of our measured productivity changes.

[27] Later, when we focus on the impact of trade competition on skewness, we will show that those market shares are the relevant weights to aggregate the destination-specific skewness measures.

set of exported products or exported market shares are not reflected in the demand shock.[28]

Ideally, this aggregation would also incorporate a firm's exposure to demand shocks in its domestic (French) market. This is not possible for two reasons: most importantly, we do not observe the product-level breakdown of the firms' sales in the French market (we only observe total domestic sales across products); in addition, world exports into France would not be exogenous to firm-level technology changes in France. Therefore, we need to adjust our export-specific demand shock using the firm's export intensity to obtain an overall firm-level demand shock:

$$\text{shock\_intens}_{i,t} = \frac{x_{i,t_0}}{x_{i,t_0} + x_{i,F,t_0}} \text{shock}_{i,t}, \qquad \tilde{\Delta}\text{shock\_intens}_{i,t} = \frac{x_{i,t-1}}{x_{i,t-1} + x_{i,F,t-1}} \tilde{\Delta}\text{shock}_{i,t},$$

where $x_{i,F,t}$ denotes firm i's total (across products) sales to the French domestic market in year t (and the ratio thus measures firm i's export intensity). Once again, we only use prior year's information on firm-level sales to construct this overall demand shock. Note that this adjustment using export intensity is equivalent to assuming a demand shock of zero in the French market and including that market in our aggregation by market share relative to total firm sales $x_{i,t} + x_{i,F,t}$.

6.2 Impact of the Trade Shock on Firm Productivity

In this section, we investigate the direct link between this firm-level demand shock and firm productivity. Here, we focus exclusively on our firm-specific demand shock from (1), as this is our only shock that exhibits firm-level variation within destinations. Our measure of productivity is the log of deflated value-added per worker. In order to control for changes in capital intensity, we use the log of capital per worker (K_{it}/L_{it}). We also control for unobserved changes in labor utilization and returns to scale by using the log of raw materials (including energy use), R_{it}. Then, increases in worker effort or higher returns to scale will be reflected in the impact of raw materials use on labor productivity. As there is no issue with zeros for all these firm-level variables, we directly measure the growth rate of those variable using simple first differences of the log levels.

We begin with a graphical representation of the strong positive relationship between firm-level productivity and our constructed demand shock. Figure 2 illustrates the correlation between those variables in first differences for the largest French exporters (representing 50% of French exports in 1996). Panel (a) is the unconditional scatter plot for those variables, while panel (b) shows the added-variable plot for the first-difference regression of productivity on the trade shock,

[28] Lileeva and Trefler (2010) and Hummels et al (2014) use a similar strategy to construct firm-level trade-related trade shocks.

with additional controls for capital intensity, raw materials (both in log first-differences) and time dummies. Those figures clearly highlight the very strong positive response of the large exporters' productivity to changes in trade competition in export markets (captured by the demand shock).

Figure 2: Exporters Representing 50% of French Trade in 1996: First Difference 1996-2005

(a) Unconditional

(b) Conditional

Table 3 shows how this result generalizes to our full sample of firms and our three different specifications (FE, FD, FD-FE). Our theoretical model emphasizes how a multi-product firm's productivity responds to the demand shock via its effect on competition and product reallocations in the firm's export markets. Thus, we assumed that the firm's technology at the product level (the marginal cost $v(m, c)$ for each product m) was exogenous (in particular, in respect to demand fluctuations in export markets). However, there is a substantial literature examining how this technology responds to export market conditions via various forms of innovation or investment choices made by the firm. We feel that the timing dimension of our first difference specifications – especially our FD-FE specifications which nets out any firm-level growth trends – eliminates this technology response channel: It is highly unlikely that a firm's innovations or investment responses to the trade shock in a given year (especially the innovation in the trade shock relative to trend) would be reflected contemporaneously in the firm's productivity. However, we will also show some additional robustness checks that address this potential technology response.

The first three columns of Table 3 show that, across our three timing specifications, there is a stable and very strong response of firm productivity to the trade shock. Since our measure of productivity as value added per worker incorporates neither the impact of changes in input intensities nor the effects of non-constant returns to scale, we directly control for these effects in the next set of regressions. In the last 3 columns of Table 3, we add controls for capital per worker and raw material use (including energy). Both of these controls are highly significant: not surprisingly, increases in capital intensity are reflected in labor productivity; and we find that increases in raw materials use are also associated with higher labor productivity. This would be the case if there are increasing returns to scale in the value-added production function, or if labor utilization/effort increases with scale (in the short-run). However, the very strong effect of the trade shock on firm productivity remains when these controls are added – and they remain highly significant, well beyond the 1% significance level. (From here on out, we will keep those controls in all of our firm productivity regressions.)

Table 3: Baseline Resultsl: Impact of Trade Shock on Firm Productivity

Dependent Variable	log prod.	Δ log prod.		log prod.	Δ log prod.	
Specification	FE	FD	FD-FE	FE	FD	FD-FE
log (trade shock × export intens.)	0.094^a			0.073^a		
	(0.019)			(0.018)		
$\tilde{\Delta}$ (trade shock × export intens.)		0.134^a	0.116^a		0.108^a	0.096^a
		(0.024)	(0.028)		(0.024)	(0.028)
log capital stock per worker				0.228^a		
				(0.007)		
log raw materials				0.091^a		
				(0.004)		
Δ log capital stock per worker					0.327^a	0.358^a
					(0.008)	(0.009)
Δ log raw materials					0.100^a	0.093^a
					(0.004)	(0.004)
Observations	213877	188328	188328	201627	174931	174931

Standard errors (clustered at the firm level) in parentheses: $^c < 0.1$, $^b < 0.05$, $^a < 0.01$

We now describe several robustness checks that further single-out our theoretical mechanism operating through the demand-side product reallocations for multi-product firms. In the next table we regress our capital intensity measure on our trade shock; the results in Table 4 show that there is no response of investment to the trade shock. This represents another way to show that the short-run timing for the demand shocks precludes a contemporaneous technology response: if this were the case, we would expect to see some of this response reflected in higher investment (along with other responses along the technology dimension).

Table 4: K/L Does Not respond to Trade Shocks

Dependent Variable	ln K/L	Δ ln K/L	Δ ln K/L
Specification	FE	FD	FD-FE
log (trade shock $\times$ export intens.)	-0.018		
	(0.018)		
$\tilde{\Delta}$ (trade shock $\times$ export intens.)		-0.003	-0.005
		(0.017)	(0.020)
Observations	212745	186171	186171

Standard errors in parentheses: $^{c} < 0.1$, $^{b} < 0.05$, $^{a} < 0.01$

Next we use a different strategy to control for the effects of non-constant returns to scale or variable labor utilization: in Table 5, we split our sample between year intervals where firms increase/decrease employment. If the effects of the trade shock on productivity were driven by scale effects or higher labor utilization/effort, then we would expect to see the productivity responses concentrated in the split of the sample where firms are expanding employment (and also expanding more generally). Yet, Table 5 shows that this is not the case: the effect of the trade shock on productivity is just as strong (even a bit stronger) in the sub-sample of years where firms are decreasing employment; and in both cases, the coefficients have a similar magnitude to our baseline results in Table 3.[29]

In order to further single-out our theoretical mechanism operating through the demand-side product reallocations for multi-product firms, we now report two different types of falsification tests. Our first test highlights that the link between productivity and the trade shocks is only operative for multi-product firms. Table 6 reports the same regression (with controls) as our baseline results from Table 3, but only for single-product exporters. This new table clearly shows that this there is no evidence of this link among this subset of firms. Next, we show that this productivity-trade link is only operative for firms with a substantial exposure to export markets (measured by export intensity). Similarly to single-product firms, we would not expect to find a

[29] Since we are splitting our sample across firms, we no longer rely on the two specifications with firm fixed-effects and only show results for the FD specification.

Table 5: Robustness to Scale Effects

Sample	Employment Increase	Employment Decrease
Dependent Variable	Δ log productivity	Δ log productivity
Specification	FD	FD
$\tilde{\Delta}$ (trade shock $\times$ export intens.)	0.135^a	0.156^a
	(0.035)	(0.045)
Δ log capital stock per worker	0.288^a	0.332^a
	(0.012)	(0.013)
Δ log raw materials	0.091^a	0.097^a
	(0.005)	(0.005)
Observations	69642	65268

Standard errors in parentheses: $^c < 0.1$, $^b < 0.05$, $^a < 0.01$

significant productivity-trade link among firms with very low export intensity. This is indeed the case. In Table 7, we re-run our baseline specification using the trade shock before it is interacted with export intensity. The first three columns report the results for the quartile of firms with the lowest export intensity, and highlight that there is no evidence of the productivity-trade link for those firms. On the other hand, we clearly see from the last three columns that this effect is very strong and powerful for the quartile of firms with the highest export intensity.[30]

[30] Since the trade shock as not been interacted with export intensity, the coefficients for this top quartile represent significantly higher magnitudes than the average coefficients across the whole sample reported in Table 3 (since export intensity is always below 1). This is also confirmed by a specification with the interacted trade shock restricted to this same top quartile of firms.

Table 6: Reduced Form: Single Product Firms

Dependent Variable	log prod.	Δ log prod.	
Specification	FE	FD	FD-FE
log (trade shock $\times$ export intens.)	0.005		
	(0.050)		
log capital stock per worker	0.269^a		
	(0.016)		
log raw materials	0.101^a		
	(0.010)		
$\tilde{\Delta}$ (trade shock $\times$ export intens.)		-0.021	-0.138^c
		(0.062)	(0.079)
Δ log capital stock per worker		0.368^a	0.415^a
		(0.020)	(0.028)
Δ log raw materials		0.114^a	0.090^a
		(0.010)	(0.013)
Observations	32870	25330	25330

Standard errors in parentheses: $^c < 0.1$, $^b < 0.05$, $^a < 0.01$

Table 7: Reduced Form: Low/High export intensity

Sample	exp. intens. quartile # 1			exp. intens. quartile # 4		
Dependent Variable	log prod.	Δ log prod.		log prod.	Δ log prod.	
Specification	FE	FD	FD-FE	FE	FD	FD-FE
log trade shock	0.009			0.068^a		
	(0.006)			(0.014)		
log capital stock per worker	0.278^a			0.217^a		
	(0.022)			(0.015)		
log raw materials	0.070^a			0.128^a		
	(0.006)			(0.010)		
$\tilde{\Delta}$ trade shock		0.000	-0.002		0.096^a	0.100^a
		(0.007)	(0.009)		(0.017)	(0.021)
Δ log capital stock per worker		0.323^a	0.367^a		0.325^a	0.368^a
		(0.016)	(0.020)		(0.014)	(0.016)
Δ log raw materials		0.070^a	0.057^a		0.129^a	0.123^a
		(0.006)	(0.006)		(0.008)	(0.010)
Observations	49227	38894	38894	53125	46347	46347

Standard errors in parentheses: $^c < 0.1$, $^b < 0.05$, $^a < 0.01$

7 Trade Competition and Product Reallocations at the Firm-Level

In order to further highlight the demand-side product reallocations channel for the productivity-trade link, we now show that our trade shock aggregated to the firm level strongly predicts product reallocations towards better performing products (higher market shares) at the firm level; that is the firm's global product mix (the distribution of product sales across all destinations). Our theoretical model highlighted how (under our demand assumptions) those product reallocations would then lead to higher firm productivity.

Our previous results highlighted how demand shocks lead to reallocations towards better performing products at the destination-industry level. Intuitively, since there is a stable ranking of products at the firm level (better performing products in one market are most likely to be the better performing products in other markets – as we previously discussed), then reallocations towards better performing products within destinations should also be reflected in the reallocations of global sales/production towards better performing products; and the strength of this link between the skewness of sales at the destination and global levels should depend on the importance of the destination in the firm's global sales. Our chosen measure of skewness, the Theil index, makes this intuition precise. It is the only measure of skewness that exhibits a stable decomposition from the skewness of global sales into the skewness of destination-level sales (see Jost 2007).[31] Specifically, let $T_{i,t}$ be firm i's Theil index for the skewness of its global exports by product $x^s_{i,t} \equiv \sum_d x^s_{i,d,t}$. (the sum of exports for that product across all destinations).[32] Then this global Theil can be decomposed into a market-share weighted average of the within-destination Theils $T_{i,d,t}$ and a "between-destination" Theil index that measures differences in the distribution of product-level market shares across destinations:[33]

$$T_{i,t} = \sum_d \frac{x_{i,d,t}}{x_{i,t}} T_{i,d,t} - \sum_d \frac{x_{i,d,t}}{x_{i,t}} T^B_{i,d,t}, \tag{25}$$

[31] This decomposition property is similar – but not identical – to the within/between decomposition of Theil indices across populations. In the latter, the sample is split into subsamples. In our case, the same observation (in this case, product sales) is split into "destinations" and the global measure reflects the sum across "destinations".

[32] The Theil index is defined in the same way as the destination level Theil in (2).

[33] For simplicity, we omit the industry referencing I for the destination Theils. The decomposition across industries follows a similar pattern.

where the between-destination Theil $T^B_{i,d,t}$ is defined as

$$T^B_{i,d,t} = \sum_s \frac{x^s_{i,d,t}}{x_{i,d,t}} \log\left(\frac{x^s_{i,d,t}/x_{i,d,t}}{x^s_{i,t}/x_{i,t}}\right).$$

Note that the weights used in this decomposition for both the within- and between-destination Theils are the firm's export shares $x_{i,d,t}/x_{i,t}$ across destinations d. The between-destination Theil $T^B_{i,d,t}$ measures the deviation in a product's market share in a destination d, $x^s_{i,d,t}/x_{i,d,t}$, from that product's global market share $x^s_{i,t}/x_{i,t}$ and then averages these deviations across destinations. It is positive and converges to zero as the distributions of product market shares in different destination become increasingly similar.

To better understand the logic behind (25), note that it implies that the average of the within-destination Theil indices can be decomposed into the sum of two positive elements: the global Theil index, and the between-destination Theil index. This decomposition can be interpreted as a decomposition of variance/dispersion. The dispersion observed in the destination level product exports must be explained either by dispersion in global product exports (global Theil index), or by the fact that the distribution of product sales varies across destinations (between-destination Theil index).

A simple example helps to clarify this point. Take a firm with 2 products and 2 destinations. In each destination, exports of one product are x, and exports of the other product are $2x$. This leads to the same value for the within-destination Theil indices of $(1/3)\ln(1/3) + (2/3)\ln(2/3)$, and hence the same value for the average within-destination Theil index. Hence, if the same product is the better performing product in each market (with $2x$ exports), then the distributions will be synchronized across destinations and the between-destination Theil will be zero: all of the dispersion is explained by the global Theil index, whose value is equal to the common value of the two within-destination Theil indices. On the other hand, if the opposite products perform better in each market, global sales are $3x$ for each product. There is thus no variation in global product sales, and the global Theil index is zero. Accordingly, all of the variation in the within-destination Theil indices is explained by the between-destination Theil.

The theoretical model of Bernard, Redding and Schott (2011) with CES demand predicts that the between-destination Theil index would be exactly zero when measured on a common set of exported products across destinations. With linear demand, Mayer, Melitz and Ottaviano (2014) show (theoretically and empirically) that this between-destination index would deviate from zero because skewness varies across destinations. We have shown earlier that this result holds for a larger class of demand systems such that the elasticity and the convexity of inverse demand increase with consumption. Yet, even in these cases, the between-destination Theil is predicted to be small because the ranking of the product sales is very stable across destinations. This leads to a prediction that the market-share weighted average of the destination Theils should be strongly correlated with the firm's global Theil. Empirically, this prediction is strongly confirmed as shown in Figure 3.

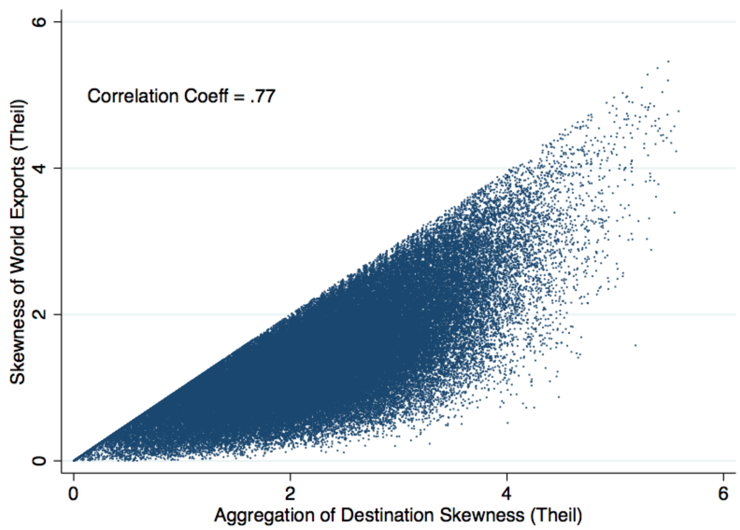

Figure 3: Correlation Between Global Skewness and Average Local Skewness

This high correlation between destination and global skewness of product sales enables us to move from our previous predictions for the effects of the trade shocks on skewness at the destination-level to a new prediction at the firm-level. By aggregating the trade shocks across destinations using the firm's market share in each destination, we have constructed a firm-level trade shock that predicts changes in the weighted average of destination skewness $T_{i,d,t}$ – and hence will predict changes in the firm's global skewness $T_{i,t}$ (given the high correlation between the two indices). This result is confirmed by our regression of the firms' global Theil on our trade shock measures, reported in the first three columns of Table 8. Our firm-level trade shock has a strong and highly significant (again, well beyond the 1% significance level) impact on the skewness of global exports. In this regression, we have also added back our two more aggregated measures of demand shocks: the industry (ISIC-3) level trade shock and the GDP shock (aggregated to the firm-level using the same market share weights across destinations). The industry level trade shock – which was already substantially weaker than the firm-level trade shock in the destination-level regressions

– is no longer significant at the firm level. The GDP shock is not significant in the (log) levels regressions, but is very strong and significant in the two first-difference specifications.

Table 8: The Impact of Demand Shocks on the Global Product Mix (Firm Level)

Dependent Variable	$T_{i,t}$	$\Delta T_{i,t}$		Exp. Intens$_{i,t}$	Δ Exp. Intens$_{i,t}$	
Specification	FE	FD	FD-FE	FE	FD	FD-FE
log GDP shock	-0.001			0.003^{a}		
	(0.004)			(0.001)		
log trade shock	0.045^{a}			0.014^{a}		
	(0.009)			(0.003)		
log trade shock - ISIC	-0.001			0.000		
	(0.001)			(0.000)		
$\tilde{\Delta}$ GDP shock		0.118^{a}	0.107^{a}		0.032^{a}	0.035^{a}
		(0.031)	(0.038)		(0.010)	(0.012)
$\tilde{\Delta}$ trade shock		0.057^{a}	0.050^{a}		0.019^{a}	0.016^{a}
		(0.011)	(0.013)		(0.003)	(0.004)
$\tilde{\Delta}$ trade shock - ISIC		-0.003	-0.010		0.002	0.000
		(0.005)	(0.007)		(0.002)	(0.002)
	(0.110)	(0.004)	(0.004)	(0.030)	(0.001)	(0.001)
Observations	117851	117851	117851	110565	107283	107283

Standard errors in parentheses: $^{c} < 0.1$, $^{b} < 0.05$, $^{a} < 0.01$

Our global Theil measure $T_{i,t}$ measures the skewness of export sales across all destinations, but it does not entirely reflect the skewness of production levels across the firm's product range. That is because we cannot measure the breakdown of product-level sales on the French domestic market. Ultimately, it is the distribution of labor allocation across products (and the induced distribution of production levels) that determines a firm's labor productivity – conditional on its technology (the production functions for each individual product). As highlighted by our theoretical model, the export market demand shocks generate two different types of reallocations that both contribute to an increased skewness of production levels for the firm: reallocations within the set of exported products, which generate the increased skewness of global exports that we just discussed; but also reallocations from non-exported products towards the better performing exported products (including the extensive margin of newly exported products that we documented at the destination-level). Although we cannot measure the domestic product-level sales, we can measure a single statistic that reflects this reallocation from non-exported to exported goods: the firm's export intensity. We can thus test whether the firm-level trade shocks also induce an increase in the firm's export intensity. Those regressions are reported in the last three columns of Table 8, and confirm that our firm-level trade shock has a very strong and highly significant positive impact on a firm's export intensity.[34] The impact of the GDP coefficient is also strong and significant, whereas the industry-level trade shock remains insignificant. Thus, our firm-level trade shock and GDP shock both predict the two types of reallocations towards better performing products that we highlighted in our theoretical model (as a response to increased competition in export markets). Holding the firm's product-level technology fixed, these reallocations both generate the subtantial increases in firm-level productivity that we previously documented.

8 Can the Measured Product Reallocations Explain the Entire Impact of Trade Competition on Productivity?

To be completed...

9 Conclusion

To be completed...

10 References

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[34] Since the export intensity is a ratio, we do not apply a log-transformation to that variable. However, specifications using the log of export intensity yield very similar results.

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Appendix

A Higher cost is associated with higher price, lower output, lower revenue and lower profit.

First, implicit differentiation of (4) yields

$$\frac{dx_v}{dv} = \frac{\lambda}{\phi'(x_v)} < 0 \tag{A.1}$$

where the sign is dictated by (A2). This shows that higher marginal cost leads to smaller output $q_v = x_v L_v$. Second, by (3), under (A1) smaller output implies higher price as

$$\frac{dp_v}{dx_v} = \frac{u''(x_v)}{\lambda} < 0$$

so higher marginal cost leads to higher price

$$\frac{dp_v}{dv} = \frac{dp_v}{dx_v}\frac{dx_v}{dv} = \frac{u''(x_v)}{\phi'(x_v)} > 0.$$

Third, given $\varepsilon_p(x_v) \in (0,1)$, higher marginal cost is associated with lower revenue $r_v L_v = p_v x_v L_v$ as

$$\frac{dr_v}{dv} = \frac{dp_v x_v}{dv} = [1 - \varepsilon_p(x_v)]\frac{dx_v}{dv} < 0$$

which shows that the negative effect of cost on output dominates its positive effect on price. Fourth, by the envelope theorem, also profit is a decreasing function of marginal cost as

$$\frac{d\pi_v}{dv} = -x_v < 0 \tag{A.2}$$

where $\pi_v = (p_v - v)x_v L^c$ is operating profit. Hence, (A1) and (A2) are necessary and sufficient for higher cost products to associated with higher price, lower output, lower revenue and lower profit.

B Lower cost is associated with higher markup

From (5), the (multiplicative) markup equals

$$m_v = \frac{p_v}{v} = \frac{1}{1 - \varepsilon_p(x_v)} \tag{B.1}$$

Given that in (a) we have already shown that x_v is a decreasing function of v, markup m_v will be decreasing or increasing in v depending on whether the elasticity of inverse demand $\varepsilon_p^{'}(x_v)$ is positive or negative. Moreover, that $\varepsilon_p^{'}(x_v) \gtreqless 0$ is equivalent to $\varepsilon_r(x_v) \gtreqless \varepsilon_p(x_v)$ can be seen by noting that we can write

$$\frac{\varepsilon_p'(x_v)x_v}{\varepsilon_p(x_v)} = \frac{1-\varepsilon_p(x_v)}{\varepsilon_p(x_v)}\left[\varepsilon_r(x_v) - \varepsilon_p(x_v)\right]$$

so that, given $\varepsilon_p(x_v) \in (0,1)$, $\varepsilon_p^{'}(x_v) \gtreqless 0$ if and only if $\varepsilon_r(x_v) \gtreqless \varepsilon_p(x_v)$.

Arkolakis, Costinot, Donaldson and Rodriguez-Clare (2012) assume that demand functions are log-concave in log-prices for all differentiated goods (this is their assumption "A3"). To compare (B1) with this assumption, invert our inverse demand

$$p_i = \frac{u'(x_i)}{\lambda}, \text{ with } \lambda = \int_0^M u'(x_i)x_i di$$

to obtain (direct) demand

$$x_i = u'^{-1}\left(e^{\ln p_i + \ln \lambda}\right). \tag{B.2}$$

Then, define the function

$$d(y) = u'^{-1}\left(e^y\right)$$

and rewrite (B.2) as

$$x_i = d(\ln p_i + \ln \lambda)$$

This is the way Arkolakis, Costinot, Donaldson and Rodriguez-Clare (2012) state their demand system in the separable case. Clearly, under (A1) $d(y)$ is strictly decreasing so that we have $d'(y) < 0$. Arkolakis, Costinot, Donaldson and Rodriguez-Clare (2012) define "log-concavity" as

$$d''(y) < 0$$

i.e. $d''(y)$ is concave. As the inverse of a strictly decreasing strictly concave real function is strictly concave, strict concavity of $d(y)$ implies strict concavity of its inverse

$$y_i = \ln u'(x_i).$$

The concavity of this function depends on the sign of its second derivative

$$\frac{d^2 \ln u'(x_i)}{dx_i^2} = \frac{u'''(x_i)u'(x_i) - u''(x_i)u''(x_i)}{[u'(x_i)]^2},$$

which, under (A1), is negative if and only if

$$\frac{u'''(x_i)x_i}{u''(x_i)} - \frac{u''(x_i)x_i}{u'(x_i)} > 0$$

or equivalently

$$\frac{\varepsilon_r(x_v)}{\varepsilon_p(x_v)} > \frac{2 - \varepsilon_p(x_v)}{1 - \varepsilon_p(x_v)}$$

As the term on the right hand side is larger than one, the assumption of "log-concavity" in the sense of Arkolakis, Costinot, Donaldson and Rodriguez-Clare (2012) is more restrictive than our assumption (B1).

C Pass-through is incomplete and lower cost is associated smaller pass-through

Define the pass-through as

$$\theta(x_v) \equiv \frac{d \ln p(x_v)}{d \ln v} = \frac{d \ln p(x_v)}{d \ln x_v} \frac{d \ln x_v}{d \ln v}$$

By definition we have

$$\frac{d \ln p(x_v)}{d \ln x_v} = -\varepsilon_p(x_v)$$

while implicit differentiation of (3) gives

$$\frac{dx_v}{dv} = \frac{\lambda}{\phi'(x_v)}$$

so that

$$\frac{d \ln x_v}{d \ln v} = \frac{\phi(x_v)}{\phi'(x_v)x_v} = -\frac{1}{\varepsilon_r(x_v)}$$

where the first equality is granted by (3) and the second equality is granted by definition. Accordingly, the pass-through can be expressed as

$$\theta(x_v) = \frac{\varepsilon_p(x_v)}{\varepsilon_r(x_v)} \tag{C.1}$$

with $\theta(x_v) < 1$ when (B1) holds. Differentiating (C.1) show that

$$\frac{\varepsilon_p'(x_v)x_v}{\varepsilon_p(x_v)} < \frac{\varepsilon_r'(x_v)x_v}{\varepsilon_r(x_v)}$$

is necessary and sufficient for $\theta'(x_v) < 0$. Then, given (B1) $\varepsilon_p'(x_v) > 0$, $\varepsilon_r'(x_v) > 0$ is necessary for $\theta'(x_v) < 0$.

In Berman, Martin and Mayer (2102) an exporter to destination market D maximizes

$$\max_{q_i \geq 0} \pi_D(q_i) = \frac{p_i}{\epsilon_D} q_i - v q_i - f$$

subject to

$$p_i = \frac{u'(x_i)}{\lambda}, \text{ with } \lambda = \int_0^M u'(x_i) x_i di$$

where p_i is the price denominated in the destination's currency, v is the marginal cost denominated in the currency of the firm's country, and ϵ_D is the exchange rate. The FOC for profit maximization is

$$u'(x_i) + u''(x_i)x_i = \epsilon \lambda c$$

which implies that the pass-through from marginal cost to price is homomorphic to the pass-through from exchange rate to price.

D Lower cost is associated with larger employment

Employment in the supply of a product with marginal cost v equals $v x_v L^c$. Differentiation yields

$$\frac{dvx_v}{dv} = \left(1 + \frac{\partial x_v}{\partial v} \frac{v}{x_v}\right) x_v = \left(1 - \frac{1}{\varepsilon_r(x_v)}\right) x_v$$

where the second equality is granted by (3) together with the definitions of $\varepsilon_p(x_v)$ and $\varepsilon_r(x_v)$. Hence, employment decreases with marginal cost if and only if $\varepsilon_r(x_v) < 1$.

E Long-Run Response to a Demand Shock in Closed Economy (Proposition 1)

The effects of a demand shock on $\widehat{c}^*$ and λ can be characterized extending the analysis by Zhelobodko et al (2012) to the case of multiproduct firms. The reason is that, as these effects are fully determined by (11) and (12), changing η is paramount to changing L^c, which is the comparative statics experiment studied by Zhelobodko et al (2012).

Too see this, rewrite expressions (10) in terms of elasticities as

$$\epsilon_{\pi^*/\lambda}(\lambda v) = \frac{\partial \pi^*(v,\lambda)}{\partial \lambda}\frac{\lambda}{\pi^*(v,\lambda)} = -\frac{1}{1-\frac{c}{\frac{u'(x^*(\lambda v))}{\lambda}}} = -\frac{1}{\varepsilon_p(x^*(\lambda v))} \tag{E.1}$$

$$\epsilon_{\pi^*/v}(\lambda v) = -\frac{\partial \pi^*(v,\lambda)}{\partial v}\frac{v}{\pi^*(v,\lambda)} = -\frac{v}{p(x^*(\lambda v))-v} = -\frac{1-\varepsilon_p(x^*(\lambda v))}{\varepsilon_p(x^*(\lambda v))} \tag{E.2}$$

To obtain $d\lambda^*/dL^c > 0$ divide (12) by L^c to yield

$$\sum_{m=0}^{\infty}\int_0^{\widehat{c}/z(m)} \pi^*\left(cz(m),\lambda\right)\gamma(c)dc = \frac{f_e}{L^c} + \frac{f}{L^c}\sum_{m=0}^{\infty}\Gamma\left(\frac{\widehat{c}}{z(m)}\right) \tag{E.3}$$

where regularity conditions are assumed on $z(m)$ to make the series on the right hand side converge. Then differentiate with respect to L^c to get

$$\begin{aligned}
&\frac{\partial\lambda}{\partial L^c}\sum_{m=0}^{\infty}\int_0^{\widehat{c}/z(m)} \frac{\partial\pi^*\left(cz(m),\lambda\right)}{\partial\lambda}\gamma(c)dc + \frac{\partial\widehat{c}}{\partial L^c}\sum_{m=0}^{\infty}\pi^*\left(\widehat{c},\lambda\right)\frac{\gamma(\widehat{c}/z(m))}{z(m)} \\
&= -\frac{f_e}{(L^c)^2} - \frac{f}{(L^c)^2}\sum_{m=0}^{\infty}\Gamma\left(\frac{\widehat{c}}{z(m)}\right) + \frac{\partial\widehat{c}}{\partial L^c}\frac{f}{L^c}\sum_{m=0}^{\infty}\frac{\gamma\left(\widehat{c}/z(m)\right)}{z(m)}
\end{aligned}$$

$$\begin{aligned}
&\frac{\partial\lambda}{\partial L^c}\sum_{m=0}^{\infty}\int_0^{\widehat{c}/z(m)} \frac{\partial\pi^*\left(cz(m),\lambda\right)}{\partial\lambda}\gamma(c)dc + \frac{\partial\widehat{c}}{\partial L^c}\sum_{m=0}^{\infty}\left[\pi^*\left(\widehat{c},\lambda\right) - \frac{f}{L^c}\right]\frac{\gamma(\widehat{c}/z(m))}{z(m)} \\
&= -\frac{f_e}{(L^c)^2} - \frac{f}{(L^c)^2}\sum_{m=0}^{\infty}\Gamma\left(\frac{\widehat{c}}{z(m)}\right)
\end{aligned} \tag{E.4}$$

By (11), the term between square brackets equals zero while again (12) implies

$$-\frac{f_e}{(L^c)^2} - \frac{f}{(L^c)^2}\sum_{m=0}^{\infty}\Gamma\left(\frac{\widehat{c}}{z(m)}\right) = -\frac{\sum_{m=0}^{\infty}\int_0^{\widehat{c}/z(m)}\pi^*\left(cz(m),\lambda\right)\gamma(c)dc}{L^c}$$

Exploiting these results, (E.4) can be restated in terms of elasticities as

$$\frac{\partial\lambda}{\partial L^c}\sum_{m=0}^{\infty}\int_0^{\widehat{c}/z(m)} \frac{\partial\pi^*\left(cz(m),\lambda\right)}{\partial\lambda}\gamma(c)dc = -\frac{\sum_{m=0}^{\infty}\int_0^{\widehat{c}/z(m)}\pi^*\left(cz(m),\lambda\right)\gamma(c)dc}{L^c}$$

$$\frac{\partial\lambda}{\partial L^c}\frac{L^c}{\lambda}\sum_{m=0}^{\infty}\int_0^{\widehat{c}/z(m)}\frac{\partial\pi^*(cz(m),\lambda)}{\partial\lambda}\frac{\lambda}{\pi^*(cz(m),\lambda)}\pi^*(cz(m),\lambda)\gamma(c)dc$$
$$= -\sum_{m=0}^{\infty}\int_0^{\widehat{c}/z(m)}\pi^*(cz(m),\lambda)\gamma(c)dc$$

$$\epsilon_{\lambda/L^c}(\lambda\widehat{c})\sum_{m=0}^{\infty}\int_0^{\widehat{c}/z(m)}\epsilon_{\pi^*/\lambda}\pi^*(cz(m),\lambda)\gamma(c)dc = -\sum_{m=0}^{\infty}\int_0^{\widehat{c}/z(m)}\pi^*(cz(m),\lambda)\gamma(c)dc$$

$$\epsilon_{\lambda/L^c}(\lambda\widehat{c}) = -\frac{\sum_{m=0}^{\infty}\int_0^{\widehat{c}/z(m)}\pi^*(cz(m),\lambda)\gamma(c)dc}{\sum_{m=0}^{\infty}\int_0^{\widehat{c}/z(m)}\pi^*(cz(m),\lambda)\epsilon_{\pi^*/\lambda}\gamma(c)dc} = \frac{\sum_{m=0}^{\infty}\int_0^{\widehat{c}/z(m)}\pi^*(cz(m),\lambda)\gamma(c)dc}{\sum_{m=0}^{\infty}\int_0^{\widehat{c}/z(m)}\frac{\pi^*(cz(m),\lambda)}{\varepsilon_p(x^*(\lambda cz(m)))}\gamma(c)dc} \quad \text{(E.5)}$$

where the last equality is granted by (E.1). The restriction $\varepsilon_p(x^*(\lambda v)) \in (0,1)$ implies $\epsilon_{\lambda/L^c} \in (0,1)$. This proves point (i) in the proposition.

To determine the sign of $\epsilon_{\widehat{c}/L^c}(\lambda\widehat{c})$, differentiate (11) with respect to L^c and get

$$\frac{\partial\pi^*(\widehat{c},\lambda)}{\partial\widehat{c}}\frac{\partial\widehat{c}}{\partial L^c} + \frac{\partial\pi^*(\widehat{c},\lambda)}{\partial\lambda}\frac{\partial\lambda}{\partial L^c} = -\frac{f}{(L^c)^2} = -\frac{\pi^*(\widehat{c},\lambda)}{L^c}$$

where the last equality is also granted by (11), or in terms of elasticities

$$\epsilon_{\pi^*/\widehat{c}}\epsilon_{\widehat{c}/L^c} + \epsilon_{\pi^*/\lambda}\epsilon_{\lambda/L^c} = -1$$

After substituting for (E.1), (E.2) and (E.5), this equation can be solved for $\epsilon_{\widehat{c}/L^c}$ to yield

$$\epsilon_{\widehat{c}/L^c}(\lambda\widehat{c}) = \frac{\varepsilon_p(x^*(\lambda\widehat{c})) - \epsilon_{\lambda/L^c}}{1-\varepsilon_p(x^*(\lambda\widehat{c}))} = \frac{\varepsilon_p(x^*(\lambda\widehat{c}))}{1-\varepsilon_p(x^*(\lambda\widehat{c}))}\left[1 - \frac{\sum_{m=0}^{\infty}\int_0^{\widehat{c}/z(m)}\frac{\pi^*(cz(m),\lambda)}{\varepsilon_p(x^*(\lambda\widehat{c}))}\gamma(c)dc}{\sum_{m=0}^{\infty}\int_0^{\widehat{c}/z(m)}\frac{\pi^*(cz(m),\lambda)}{\varepsilon_p(x^*(\lambda cz(m)))}\gamma(c)dc}\right]$$

The sign of this expression is dictated by the sign of the term in square brackets given $\varepsilon_p(x^*(\lambda v)) \in (0,1)$. As $x^*(\lambda v) > x^*(\lambda\widehat{c})$ for all $v < \widehat{c}$, then $\varepsilon_p(x^*(\lambda v)) > \varepsilon_p(x^*(\lambda\widehat{c}))$ for all $v < \widehat{c}$ when $\varepsilon_p'(x^*(\lambda\widehat{c})) > 0$. In this case, $\epsilon_{\widehat{c}/L^c}(\lambda\widehat{c}) < 0$ follows. Vice versa, $\epsilon_{\widehat{c}/L^c}(\lambda\widehat{c}) > 0$ follows from $\varepsilon_p'(x^*(\lambda\widehat{c})) < 0$. This proves that larger $L^c = \eta L^w$ reduces (increases) the cost cutoff when the elasticity of inverse demand is increasing in consumption, i.e. when (B1) holds. This proves point (ii) in the proposition.

The effects of a demand shock on intensive margin reallocations can be assessed as follows. Start with point (iii) in the proposition. By result (e), larger $L_c = \eta L_w$ increases the marginal utility of income λ. Hence, the impact of larger ηL_w on output, revenue and employment ratios has the same sign as the impact of λ on them.

Consider first the impact on the output ratio $x_{\underline{v}}/x_{\overline{v}}$:

$$\frac{d\left(\frac{x_{\underline{v}}}{x_{\overline{v}}}\right)}{d\lambda} = \frac{\frac{dx_{\underline{v}}}{d\lambda}x_{\overline{v}} - x_{\underline{v}}\frac{dx_{\overline{v}}}{d\lambda}}{(x_{\overline{v}})^2}$$

The sign of the right hand side is dictated by the numerator. Implicit differentiation of (4) can be used to substitute for $dx_{\underline{c}}/d\lambda$ and $dx_{\overline{v}}/d\lambda$, rewriting the numerator first as

$$\frac{\frac{\underline{c}}{\phi(x_{\underline{v}})}}{\phi'(x_{\underline{v}})\frac{x_{\underline{c}}}{\phi(x_{\underline{v}})}} - \frac{\frac{\overline{v}}{\phi(x_{\underline{v}})}}{\frac{x_{\overline{v}}}{\phi(x_{\overline{v}})}\phi'(x_{\overline{v}})}$$

and then, using (4) and the definition of the elasticity of marginal revenue in (8), as

$$\varepsilon_r(x_{\underline{v}}) - \varepsilon_r(x_{\overline{v}}).$$

Repeating the same argument for any pair of values $x_{\underline{v}}$ and $x_{\overline{v}}$ such that $x_{\underline{v}} > x_{\overline{v}}$ implies that $d\left(x_{\underline{v}}/x_{\overline{v}}\right)/d\lambda > 0$ as long as $\varepsilon_r'(x_v) > 0$, which is the case under (B2).

Consider now impact on the revenue ratio

$$\frac{d\left(r_{\underline{v}}/r_{\overline{v}}\right)}{d\lambda}\frac{\lambda}{r_{\underline{v}}/r_{\overline{v}}} = \frac{\partial r_{\underline{v}}}{\partial x_{\underline{v}}}\frac{\partial x_{\underline{v}}}{\partial \lambda}\frac{\lambda}{r_{\underline{v}}} - \frac{\partial r_{\overline{v}}}{\partial x_{\overline{v}}}\frac{\partial x_{\overline{v}}}{\partial \lambda}\frac{\lambda}{r_{\overline{v}}} = \varepsilon_r(x_{\underline{v}})\frac{\lambda}{r_{\underline{v}}} - \varepsilon_r(x_{\overline{v}})\frac{\lambda}{r_{\overline{v}}}$$

By result (a), we have $x_{\underline{v}} > x_{\overline{v}}$ and $r_{\underline{v}} > r_{\overline{v}}$. Then, $\varepsilon_r'(x_v) > 0$ implies $d\left(r_{\underline{v}}/r_{\overline{v}}\right)/d\lambda > 0$.

Last, consider the impact on the employment ratio. Larger λ implies a reallocation from the higher to lower cost products when $\underline{v}x_{\underline{v}}/\overline{v}x_{\overline{v}} > 1$ and $d\left(\underline{v}x_{\underline{v}}/\overline{v}x_{\overline{v}}\right)/d\lambda > 0$. By result (d), the employment ratio is larger than one under (B3) $\varepsilon_r(x_v < 1$. Moreover, as $\underline{v}$ and $\overline{v}$ are given, the employment ratio increases with λ when the output ratio increases in λ, that is, when $\varepsilon_r'(x_v) > 0$, which is again the case under (B2).

Finally, consider point (iv) in the proposition. To see how changes in η affect $\pi^*(v,\lambda)L^c$, differentiate to obtain

$$\begin{aligned}\frac{\partial \pi^*(v,\lambda)L^c}{\partial L_c} &= \pi^*(v,\lambda) + \frac{\partial \pi^*(v,\lambda)}{\partial \lambda}\frac{\partial \lambda}{\partial L^c}L^c \\ &= \pi^*(v,\lambda)\left(1 + \frac{\partial \pi^*(v,\lambda)}{\partial \lambda}\frac{\lambda}{\pi^*(v,\lambda)}\frac{\partial \lambda}{\partial L^c}\frac{L^c}{\lambda}\right) \\ &= \pi^*(v,\lambda)\left(1 + \epsilon_{\pi^*/\lambda}(\lambda v)\epsilon_{\lambda/L^c}(\lambda\widehat{c})\right)\end{aligned}$$

which can be rewritten as

$$\frac{\partial \pi^*(v,\lambda)L^c}{\partial L^c}\frac{L^c}{\pi^*(v,\lambda)L^c} = 1 + \epsilon_{\pi^*/\lambda}(\lambda v)\epsilon_{\lambda/L^c}(\lambda\widehat{c})$$

and thus as

$$\epsilon_{\pi^* L^c/L^c}(v,\lambda,\widehat{c}) = 1 + \epsilon_{\pi^*/\lambda}(\lambda v)\epsilon_{\lambda/L^c}(\lambda\widehat{c}) \tag{E.6}$$

where $\epsilon_{\lambda/L^c}(\lambda\widehat{c})$ is common across products while $\epsilon_{\pi^*/\lambda}(\lambda c)$ is product-specific and is given by (E.1) both in the short and in the long run. By (E.1), expression (E.6) can be restated as

$$\epsilon_{\pi^* L^c/L^c}(v,\lambda,\widehat{c}) = 1 - \frac{\epsilon_{\lambda/L^c}(\lambda\widehat{c})}{\varepsilon_p(x^*(\lambda v))}$$

with $\varepsilon_p(x^*(\lambda v)) \in (0,1)$. In the long run, by (E.5), we also have $\epsilon_{\lambda/L^c}(\lambda\widehat{c}) \in (0,1)$ so that $\pi^*(v,\lambda)L^c$ increases (decreases) with L^c for low v and decreases (increases) with λ for high v as long as $\varepsilon_p^{'}(x_v) > (<)0$. In the short run, the same results hold when the change in L^c is driven by η (as $\partial\lambda/\partial\eta > 0$ implies $\epsilon_{\lambda/L^c}(\lambda\widehat{c}) > 0$).

F A positive demand shock increases the productivity of multi-product firms

By result (d), larger λ increases the employment ratio $\ell_m/\ell_{m'}$ of a lower cost product m to a higher cost one m'. As the employment ratio equals the employment share ratio

$$\frac{\ell_m}{\ell_{m'}} = \frac{\frac{\ell_m}{\sum_{m=0}^{M-1}\ell_m}}{\frac{\ell_{m'}}{\sum_{m=0}^{M-1}\ell_m}} = \frac{s_m}{s_{m'}},$$

by increasing $\ell_m/\ell_{m'}$, larger λ also increases s_m relative to $s_{m'}$ for any $m < m'$. Given that productivity levels $1/v_m$'s are fixed, a higher share ratio in turn implies that larger λ increases average productivity as this is an average in which the weighting is being shifted from lower productivity (higher cost) products like m' towards higher productivity (lower cost) products like m.[35]

[35] Everything would hold weakly if we relaxed the ranking condition $v_0 < v_1 < ... < v_{M-1}$ to $v_0 \leq v_1 \leq ... \leq v_{M-1}$.

G Short-Run Response to a Demand Shock in Closed Economy

To see what happens when η changes in the short run, differentiate (11) to yield

$$\frac{\partial \pi^*(\widehat{c},\lambda)}{\partial \lambda}\frac{\partial \lambda}{\partial \eta}+\frac{\partial \pi^*(\widehat{c},\lambda)}{\partial \widehat{c}}\frac{\partial \widehat{c}}{\partial \eta}=-\frac{f}{\eta^2 L_w}$$

Then rewrite (15) as

$$\sum_{m=0}^{\infty}\int_0^{\widehat{c}/z(m)} cz(m)x^*\left(cz(m),\lambda\right)\gamma(c)dc=\frac{1}{\eta\overline{N}}-\frac{f}{\eta L^w}\sum_{m=0}^{\infty}\Gamma(\widehat{c}/z(m))$$

and differentiate to obtain

$$\left[\sum_{m=0}^{\infty}\int_0^{\widehat{c}/z(m)} cz(m)\frac{\partial x^*\left(cz(m),\lambda\right)}{\partial\lambda}\gamma(c)dc\right]\frac{\partial\lambda}{\partial\eta}+\sum_{m=0}^{\infty}\left[\left(\widehat{c}x^*\left(\widehat{c},\lambda\right)+\frac{f}{\eta L^w}\right)\frac{\gamma(\widehat{c}/z(m))}{z(m)}\right]\frac{\partial\widehat{c}}{\partial\eta}$$
$$=-\frac{L^w-f\overline{N}\sum_{m=0}^{\infty}\Gamma(\widehat{c}/z(m))}{\eta^2 L^w\overline{N}}<0$$

where the negative sign is due to the fact that $L^w-f\overline{N}\sum_{m=0}^{\infty}\Gamma(\widehat{c}/z(m))>0$ is labor absorbed by variable production costs. In matrix notation we can rewrite

$$\begin{bmatrix}\frac{\partial\pi^*(\widehat{c},\lambda)}{\partial\lambda} & \frac{\partial\pi^*(\widehat{c},\lambda)}{\partial\widehat{c}}\\ \int_o^{\widehat{c}} c\frac{\partial x^*(\lambda c)}{\partial\lambda}\gamma(c)dc & \sum_{m=0}^{\infty}\left[\left(\widehat{c}x^*\left(\widehat{c},\lambda\right)+\frac{f}{\eta L^w}\right)\frac{\gamma(\widehat{c}/z(m))}{z(m)}\right]\end{bmatrix}\begin{bmatrix}\frac{\partial\lambda}{\partial\eta}\\ \frac{\partial\widehat{c}}{\partial\eta}\end{bmatrix}=\begin{bmatrix}-\frac{f}{\eta(L_w)^2}\\ -\frac{L^w-f\overline{N}\sum_{m=0}^{\infty}\Gamma(\widehat{c}/z(m))}{\eta^2L^w\overline{N}}\end{bmatrix}$$

which can be solved by Cramer rule to obtain

$$\frac{\partial\lambda}{\partial\eta}=\frac{-\frac{f}{\eta(L_w)^2}\left[\sum_{m=0}^{\infty}\left(\widehat{c}x^*\left(\widehat{c},\lambda\right)+\frac{f}{\eta L^w}\right)\frac{\gamma(\widehat{c}/z(m))}{z(m)}\right]\gamma(\widehat{c})+\frac{L^w-f\overline{N}\sum_{m=0}^{\infty}\Gamma(\widehat{c}/z(m))}{\eta^2L_w\overline{N}}\frac{\partial\pi^*(\widehat{c},g)}{\partial\widehat{c}}}{\frac{\partial\pi^*(\widehat{c},\lambda)}{\partial\lambda}\left[\sum_{m=0}^{\infty}\left(\widehat{c}x^*\left(\widehat{c},\lambda\right)+\frac{f}{\eta L^w}\right)\frac{\gamma(\widehat{c}/z(m))}{z(m)}\right]-\frac{\partial\pi^*(\widehat{c},\lambda)}{\partial\widehat{c}}\int_o^{\widehat{c}}c\frac{\partial x^*(\lambda c)}{\partial\lambda}\gamma(c)dc}>0$$

and

$$\frac{\partial\widehat{c}}{\partial\eta}=\frac{-\frac{L^w-f\overline{N}\sum_{m=0}^{\infty}\Gamma(\widehat{c}/z(m))}{\eta^2L^w\overline{N}}\frac{\partial\pi^*(\widehat{c},\lambda)}{\partial\lambda}+\frac{f}{\eta(L_w)^2}\left[\int_o^{\widehat{c}}c\frac{\partial x^*(\lambda c)}{\partial\lambda}\gamma(c)dc\right]}{\frac{\partial\pi^*(\widehat{c},\lambda)}{\partial\lambda}\left[\sum_{m=0}^{\infty}\left(\widehat{c}x^*\left(\widehat{c},\lambda\right)+\frac{f}{\eta L^w}\right)\frac{\gamma(\widehat{c}/z(m))}{z(m)}\right]-\frac{\partial\pi^*(\widehat{c},\lambda)}{\partial\widehat{c}}\int_o^{\widehat{c}}c\frac{\partial x^*(\lambda c)}{\partial\lambda}\gamma(c)dc}\gtreqless 0$$

where the sign of $\partial\lambda/\partial\eta$ is dictated by (9) and (10), while the sign of $\partial\widehat{c}/\partial\eta$ depends on distributional assumptions. The fact that the sign of $\partial\lambda/\partial\eta$ is positive while the sign of $\partial\widehat{c}/\partial\eta$ is ambiguous implies that result (f) as well as all results in Proposition 1 except (ii) also hold in the short run.

H Response to a Demand Shock in Open Economy

To see what happens to the export cutoff when L_D^c changes due to changing η, rewrite (17) as

$$\pi_{lD}^*(\tau_{lD}\widehat{c}_{lD}, \lambda_D) = \frac{f_D^x}{\eta L_D^c}$$

with $l \in \{H, F\}$. Differentiation with respect to L_D^c gives

$$\frac{\partial \pi_{lD}^*(\tau_{lD}\widehat{c}_{lD}, \lambda_D)}{\partial \widehat{c}_{lD}} \frac{\partial \widehat{c}_{lD}}{\partial L_D^c} + \frac{\partial \pi_{lD}^*(\tau_{lD}\widehat{c}_{lD}, \lambda_D)}{\partial \lambda_D} \frac{\partial \lambda_D}{\partial L_D^c} = -\frac{f_D^x}{\left(L_D^c\right)^2}$$

which can be restated as

$$\frac{\partial \widehat{c}_{lD}}{\partial L_D^c} = \frac{-\frac{\partial \pi_{lD}^*(\tau_{lD}\widehat{c}_{lD}, \lambda_D)}{\partial \lambda_D} \frac{\partial \lambda_D}{\partial L_D^c} - \frac{f_D^x}{\left(L_D^c\right)^2}}{\frac{\partial \pi_{lD}^*(\tau_{lD}\widehat{c}_{lD}, \lambda_D)}{\partial \widehat{c}_{lD}}}$$

Given $\partial \pi_{lD}^*/\partial \lambda_D < 0$, $\partial \pi_{lD}^*/\partial \widehat{c}_{lD} < 0$ (as $\pi^*(v, \lambda)$ is a decreasing function of v) and $\partial \lambda_D/\partial L_D^c > 0$ when (A1)-(A2) hold, the export cutoff increases with L_D^c as long as f_D^x is large enough. Hence, as η rises, smaller $\widehat{c}_{DD}$ for $\varepsilon_p^{'}(x_c) > 0$ is accompanied by larger $\widehat{c}_{lD}$ for large enough f_D^x. This leads to a larger number of exported products $M_{lD}^x = \Gamma\left(\widehat{c}_{lD}\right) \overline{M}_l^i$.

Aggregate exports from country $l \in \{H, F\}$ are

$$EXP_{lD} = \overline{N}_l^i \sum_{m=0}^{\infty} \left[\int_0^{\widehat{c}_{lD}/z(m)} \left[x_{lD}^*\left(cz(m), \lambda\right) L_D^c + f\right] \gamma(c) dc \right]$$

where $\overline{N}_l^i$ is the fixed measure of incumbent firms in l. Dividing by $\overline{N}_l^i$ and diffentiating gives

$$\begin{aligned}
\frac{1}{\overline{N}_l^i} \frac{\partial EXP_{lD}}{\partial L_D^c} = & \left\{ \sum_{m=0}^{\infty} \left[\int_0^{\widehat{c}_{lD}/z(m)} \frac{\partial x_{lD}^*\left(cz(m), \lambda\right)}{\partial \lambda_D} L_D^c \gamma(c) dc \right] \right\} \frac{\partial \lambda_D}{\partial L_D^c} \\
& + \sum_{m=0}^{\infty} \left[\int_0^{\widehat{c}_{lD}/z(m)} x_{lD}^*\left(cz(m), \lambda\right) \gamma(c) dc \right] \\
& + \left\{ \sum_{m=0}^{\infty} \left[x_{lD}^*\left(\widehat{c}_{lD}, \lambda\right) L_D^c + f\right] \frac{\gamma(\widehat{c}_{lD}/z(m))}{z(m)} \right\} \frac{\partial \widehat{c}_{lD}}{\partial L_D^c}
\end{aligned}$$

Given $\partial \pi_{lD}^*/\partial \lambda_D < 0$ and $\partial \lambda_D/\partial L_D^c > 0$ when (A1)-(A2) hold, the first term on the right hand side is negative. Hence, we get $\partial EXP_{lD}/\partial L_D^c > 0$ as long as f_D^x is large enough to make $\partial \widehat{c}_{lD}/\partial L_D^c$ positive and large enough.

I Allowing for the Destination Country to Export

In the main text we have analyzed a situation in which D does not export. This assumption is not crucial, either in the long or the short run. Consider the *long run*. With exports from D the free entry condition would become

$$f^e = \sum_{m=0}^{\infty} \left[\int_0^{\widehat{c}_{DD}/z(m)} \left[\pi_{DD}^* \left(cz(m), \lambda\right) \eta L_D^w - f\right] \gamma(c) dc \right] + \sum_{m=0}^{\infty} \left[\int_0^{\widehat{c}_{DH}/z(m)} \left[\pi_{DH}^* \left(cz(m), \lambda\right) \eta L_H^w - f_H^x\right] \gamma(c) dc \right] + \sum_{m=0}^{\infty} \left[\int_0^{\widehat{c}_{DF}/z(m)} \left[\pi_{DF}^* \left(cz(m), \lambda\right) \eta L_F^w - f_F^x\right] \gamma(c) dc \right]$$

where the second and third terms on the right hand side are constant for our comparative statics analysis with respect to η. In particular, comparison with (18) reveals that allowing for exports from D is homomorphic to reducing the entry cost f_D^e to

$$f_D^e - \sum_{m=0}^{\infty} \left\{ \int_0^{\widehat{c}_{DH}/z(m)} \left[\pi_{DH}^* \left(cz(m), \lambda\right) \eta L_H^w - f_H^x\right] \gamma(c) dc + \int_0^{\widehat{c}_{DF}/z(m)} \left[\pi_{DF}^* \left(cz(m), \lambda\right) \eta L_F^w - f_F^x\right] \gamma(c) dc \right\}.$$

The assumption of D not exporting is also not crucial in the *short run*. With exports from D the labor market clearing condition would become

$$L_D^w = \overline{N}_D^i \left\{ f^e + \sum_{m=0}^{\infty} \left[\int_0^{\widehat{c}_{DD}/z(m)} \left[cz(m) x_{DD}^* \left(cz(m), \lambda\right) \eta L_D^w + f\right] \gamma(c) dc \right] + \sum_{m=0}^{\infty} \left[\int_0^{\widehat{c}_{DH}/z(m)} \left[cz(m) x_{DH}^* \left(cz(m), \lambda\right) \eta L_H^w + f_H^x\right] \gamma(c) dc \right] + \sum_{m=0}^{\infty} \left[\int_0^{\widehat{c}_{DF}/z(m)} \left[cz(m) x_{DF}^* \left(cz(m), \lambda\right) \eta L_F^w + f_F^x\right] \gamma(c) dc \right] \right\}$$

where the second and third terms on the right hand side are again constant for our comparative statics analysis with respect to η. In particular, as comparison with (19) and (24) reveals, by introducing another fixed source of employment for D's workers, allowing for exports from D is homomorphic to introducing some sort of fictitious 'entry cost'

$$f^e_{D?} \equiv \sum_{m=0}^{\infty} \left[\int_0^{\widehat{c}_{DH}/z(m)} \left[cz(m) x^*_{DH} \left(cz(m), \lambda \right) \eta L^w_H + f^x_H \right] \gamma(c) dc \right.$$
$$\left. + \int_0^{\widehat{c}_{DF}/z(m)} \left[cz(m) x^*_{DF} \left(cz(m), \lambda \right) \eta L^w_F + f^x_F \right] \gamma(c) dc \right]$$

also in the short run.