Advertising Arbitrage

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Abstract

Speculators often advertise arbitrage opportunities in which they invest. We show that such behavior is the arbitrageur's optimal response to the limits to arbitrage due to limited attention by other investors: insofar as advertising persuades other investors, the arbitrageur accelerates the correction of mispricing. Advertising induces under-diversification: a risk-averse arbitrageur who observes mispricing of several assets will optimally advertise a single asset, and overweigh this asset in his portfolio; a risk-neutral arbitrageur will invests his whole wealth in this asset. In picking their investments, arbitrageurs will consider not only their initial mispricing, but also their "advertisability" and the quality of public information disclosed in the future. When there are multiple arbitrageurs, externalities in advertising can induce them to pick and advertise the same asset, even when this is inefficient.

Introduction

Professional investors often "talk up their book", namely, they openly advertise their positions to other investors. Examples range from well-known investors like Charles Icahn or Warren Buffet respectively buying large stakes in Apple and IBM, while claiming their shares to be greatly undervalued, to small activist investors like Carson Block and Glaucus Research Group shorting Chinese companies allegedly involved in fraudulent accounting while publicly recommending to sell their shares. At first, these practices may seem odd: why would an informed investor publicly disclose his private knowledge about a company he invests in? The answer put forward in this paper is that an investor who identifies arbitrage opportunities (hereafter, "arbitrageurs") has the incentive to advertise his positions to accelerate the correction of its mispricing, in a situation where they are unable to correct the mispricing with their trades and investors pay limited attention to each investment opportunity. Absent advertising, in this situation prices may diverge even further from fundamentals after the arbitrageur has invested due to noise trading. Conversely, if successful, advertising nudges the market price closer to fundamentals, thus allowing the arbitrageur to close his position earlier and reinvest his funds elsewhere.

Put it otherwise, advertising is a way to relax the limits to arbitrage that may arise from investors' inattention and noise trader risk and/or low quality of public information. This is exactly what Ljungqvist and Qian (2014) document in a study of reports by 17 arbitrageurs that shorted 113 US listed companies between 2006 and 2011. Their evidence shows that these arbitrageurs overcome limits to arbitrage (which in their case also include severe short-sales constraints) precisely by advertising: once they have taken a short position, "they reveal their information to the market". The explanation provided by Ljungqvist and Qian fits perfectly with our setting, where arbitrageurs are assumed to be price-takers: the arbitrageurs that they analyze are so small and constrained that they cannot hope to correct the mispricing simply by shorting the targeted stocks aggressively. "The apparent aim is to engage the one group of investors who are not constrained: the target company's current shareholders ... If [these investors] can be persuaded to sell, this will not only correct the mispricing but also reduce noise trader risk by accelerating price discovery" (p. 3).

Of course, other investors must guard against the danger that arbitrageurs advertise deals just to manipulate market prices and take advantage of them. However, this does appear to be the case in the data analyzed by Ljungqvist and Qian, where other investors tend to listen to arbitrageurs. Also in other instances the market tends to heed the recommendations of well-known professional investors: the price of Apple rose by 5% on 13

August 2013 following Icahn's recommendation to buy it, while the shares of Minzhong Food Corp. dropped by 50% following announcements by Glaucus Research Group. If investors are rational, these price reaction cannot be just the outcome of successful manipulation: these advertisements must be typically informative. This accords with Benabou and Laroque (1992), who show that market gurus can affect prices only by being truthful on average, even when they only have soft information.[1] For simplicity, in our model arbitrageurs are assumed to advertise hard information, so that they cannot lie in order to manipulate the market: they share their private information not to mislead the market but to insure against a possible liquidity shock.

We have four main results. First, we show that, even when he knows that several assets are mispriced, an arbitrageur will want to concentrate his advertising effort on a single one: he prefers to focus the attention of other investors on a single asset so as to eliminate mispricing in that asset as far as possible, rather than disperse his effort across different assets, and eliminate little mispricing in each of them. This is because this maximizes the chances that the price of this asset will converge to its true value quickly, thus allowing the arbitrageur to redeploy his limited wealth on other mispriced assets.

Second, and relatedly, the concentration of advertising activity on a single asset leads to under-diversification in portfolio choice: even a risk-averse arbitrageur will want to overweigh the asset that he advertises, while a risk-neutral arbitrageur will hold only that asset.

Thirdly, arbitrageurs will tend to pick the assets that they advertise and hold not only based on the magnitude of their mispricing but also on how "advertisable" they are, namely, how suitable they are for being effectively advertised: other things equal, advertising is likely to be more effective for simple assets than for complex ones, and for asset classes with which investors are already familiar than novel, unfamiliar ones. We also show that richer arbitrageurs will prefer more "advertisable" assets, since they will have more at stake once they concentrate their portfolio on the asset that they advertise.

Fourthly, due to complementarity in advertising, multiple arbitrageurs will tend to advertise the same asset, and may end up being collectively "trapped" in an inefficient portfolio choice, where they all advertise an asset that has not the greatest return among those for which they have information. For instance, they could collectively go for an asset that is

[1] In their model the guru's information cannot be justified with hard evidence. Instead, the guru is believed to be honest with certain probability and to be profit maximizing (opportunistic) with complementary probability. If the guru is opportunistic and receives positive private information about the asset, he sends a negative message which depresses the asset price. The guru buys the asset cheaply and obtains a high return on his investment. Benabou and Laroque conclude that gurus can manipulate markets if they have some reputational capital.

highly "advertisable" even though it is not the most mispriced one: this may explain why sometime the market appears to focus on the minor mispricing of some assets, while failing to correct huge mispricing in others, especially complex ones, such as RMBS and CDOs before the subprime financial crisis.

Our model naturally bridges two strands of research: that on limited attention in asset markets, which studies portfolio choice and asset pricing if investors cannot process all available price-relevant information (Barber and Odean (2008), DellaVigna and Pollet (2009), Huberman and Regev (2001), Peng and Xiong (2006), Van Nieuwerburgh and Veldkamp (2009, 2010)), and the research based on limits to arbitrage, which shows that arbitrageurs may be unable to eliminate quickly all mispricing when capital moves slowly (see Shleifer and Vishny (1997), and Gromb and Vayanos (2010), among others). In our setting, the assumption that investors have limited attention is the reason why advertising can play a role: advertising may succeed precisely by catching the attention of investors, namely by inducing them to devote their scarce processing ability to the arbitrage opportunity chosen by the arbitrageur.

However, our setting produces interesting results that are absent from both of these strands of research. Since our arbitrageurs have unlimited information-processing capacity (indeed are perfectly informed about several potential arbitrage opportunities), in principle they could choose well-diversified portfolios. Yet, just as investors with fixed information-processing capacity, they choose under-diversified portfolios, because they need to be as efficient as possible in advertising: the limited attention of the investors to whom they direct their advertising exerts a "contagion" on the arbitrageurs' own portfolio choices. Advertising also adds a missing dimension in limits-to-arbitrage models: by advertising, arbitrageurs can effectively relax limits to arbitrage, and speed up endogenously the movement of capital towards arbitrage opportunities.

The paper is organized as follows. In section 1 we introduce the model. First, we characterize the arbitrageur's advertising in section 2. In section 3 we study the arbitrageur's portfolio choice assuming risk averse arbitrageur, then we introduce risk neutrality and obtain additional results in section 4. Section 5 is devoted to the analysis of strategic interaction among many arbitrageurs. In the end of the last section we discuss the results.

1 Environment

We start the analysis with a baseline model with a single arbitrageur and numerous risk-neutral investors. There are three periods: $t = 0, 1, 2$, and there is a continuum of assets ($i \in N$) traded at dates $t = 0, 1$, which deliver a return $\theta_i \in \{0, 1\}$ at $t = 2$. There is no discounting between periods. At $t = 0$ investors' prior beliefs about asset i's return is given by $\Pr(\theta_i = 1) = \pi_i \in [\underline{\pi}, \overline{\pi}]$, $i \in N$, $0 < \underline{\pi} < \overline{\pi} < 1$.

At $t = 1$ a noisy public signal $s_i \in \{0, 1\}$ about θ_i, $i \in N$ becomes available. The signal is correct ($s_i = \theta_i$) with probability $\gamma_i \in [0, 1)$ and is an uninformative random variable ϵ_i with probability $1 - \gamma_i$. Its distribution is the same as that of θ_i: $\Pr(\epsilon_i = 1) = \pi_i$, $i \in N$, but it is independent of θ_i.

At $t = 0$, the arbitrageur privately learns θ_i for a finite subset of assets $i \in M$ and decides in which assets to take positions. Investors do not know the set M and believe that any asset $i \in N$ can be in M with the same probability. The arbitrageur can credibly communicate θ_i about any $i \in M$ to investors by exerting "advertising effort" $e_i \geq 0$. Investors have limited attention, in the sense that the can only learn θ_i if the arbitrageur advertises assets i. But even so, advertising need not be successful:

Assumption 1. *Investors learn the true realization of θ_i at $t = 1$ only if the advertising is successful, which happens with probability $q_i = \min[a_i e_i, 1]$, $a_i \in (0, 1]$ for any $i \in M$.*

With complementary probability, the advertising fails and investors learn true θ_i only at $t = 2$. Parameter a_i captures the extent to which information about asset i is "advertisable", and therefore stands for various reasons that facilitate communication about the asset. For instance, a_i may be high when the investing public is very receptive about information related to asset i, either because they already hold it in their portfolio, or because the asset belongs to a relatively well-known asset class. Alternatively, a_i may be large if the evidence found by the arbitrageur is very convincing, that is, the arbitrageur has explicit and credible information about θ_i. Investors' attention may also be affected by the asset's previous performance, for instance, by how often the asset was in the news previously.

For simplicity, we assume that at $t = 0$ the arbitrageur can take a position x_i in any asset $i \in N$ and at $t = 1$ he has to liquidate all positions. In an extension we show that basic results hold even if the arbitrageur is not forced to liquidate at $t = 1$ with probability one.

Assets that do not belong to the set M are of no interest for the arbitrageur because he has no private information about them; hence, without loss of generality we consider assets in M. The timing is as follows (also summarized by the picture below).

At $t = 0$ asset i's price is $p_0^i = \pi_i$. The arbitrageur takes positions x_i and decides on advertising efforts e_i, $i \in M$. At $t = 1$ for each $i \in M$ signal s_i is realized. With probability $q_i = \min[a_i e_i, 1]$ the arbitrageur's advertising succeeds and investors learn θ_i; with complementary probability, investors rely on s_i. The asset i's price p_1^i is realized. The arbitrageur liquidates all his positions x_i, $i \in M$. Finally, at $t = 2$ all θ_i, $i \in M$ realize.

Figure 1: Timing for each asset $i \in M$

0	1	2
Asset price: p_0^i	Public signal s_i	θ_i is realized
Arbitrageur's position: x_i	Investors learn θ_i with probability q_i	
Arbitrageur's effort: e_i	Asset price: p_1^i	
	Arbitrageur liquidates	

The arbitrageur's utility $V(c, e)$ at $t = 1$ is a function of his monetary payoff $c = \sum_i x_i p_1^i$ and his total advertising effort $e = \sum_i e_i \geq 0$. We assume the utility function to be increasing in the monetary payoff, decreasing in advertising effort, and not convex: $V_c > 0$, $V_{cc} \leq 0$, $V_e < 0$ for $e > 0$, $V_e(.,0) = 0$, $V_{ee} \leq 0$. We also assume that the cost of advertising is not increasing with monetary payoff $V_{ec} \geq 0$: the marginal cost of advertising is not increasing with the arbitrageur's monetary payoff.

Assumption 2. *The arbitrageur has limited resources $w > 0$ at $t = 0$.*

At $t = 0$ the arbitrageur can allocate resources w among investments x_i. Denoting by $y_i = |x_i p_i^0|$ the absolute market value of the arbitrageur's position in asset i at $t = 0$, his budget constraint is

$$\sum_{i \in M} y_i \leq w. \tag{1}$$

Notice that (1) also imposes a constraint on the arbitrageur's short positions: this is because in reality both long and short positions require some collateral. For brevity and without loss of generality we will focus on the case of undervalued assets:

Assumption 3. *All assets in M are undervalued $\theta_i = 1$, $i \in M$.*

Clearly, the arbitrageur may only want to take long positions in these assets $x_i \geq 0$, $i \in M$. All results hold if we allow for $\theta_i = 0$ in M and study short positions.

We assume arbitrageur's trades to be small compared to the market volume of any asset.

Assumption 4. *Arbitrageur's trades do not affect prices.*

In other words, the arbitrageur can affect asset prices only by advertising his private information.

Assumption 5. *Perfect advertising is prohibitively costly: $V(\frac{w}{\pi}, 1) - V(0, 1) < |V_e(\frac{w}{\pi}, 1)|$.*

This assumption obtains from the following condition $\frac{\partial}{\partial e_i}[q_i V(\frac{w}{\underline{\pi}}, e_i) + (1-q_i)V(0, e_i)] < 0$ for $e_i = 1$. This condition ensures that even if the arbitrageur were to invest all his wealth w in the most underpriced asset ($p_i^0 = \underline{\pi}$), and this asset happened to be the easiest to advertise ($a_i = 1$), the arbitrageur would not choose an advertising level $e_i = 1$ for that asset such that $q_i = 1$ and investors learn θ_i for sure. In other words, the marginal cost of advertising effort $e_i = 1$ is sufficiently high. This assumption is natural and also simplifies the analysis. We can consider $q_i < 1$ for any $i \in M$ without loss of generality.

2 Concentrated advertising

Having described the environment, we solve for the arbitrageur's advertising effort and portfolio choice. At $t = 0$ risk neutral investors have prior beliefs π_i about any asset $i \in M$, so that the price of each asset i is $p_0^i = \pi_i$. At $t = 1$ investors learn θ_i with probability q_i, in which case the price becomes $p_1^i = \theta_i$. With complementary probability $1 - q_i$, investors do not learn θ_i and rely only on the public signal s_i, in which case the price is $p_1^i = E[\theta_i|s_i] = (1-\gamma_i)\pi_i + \gamma_i s_i$. The signal s_i is correct with probability γ_i, the prior about θ_i is π_i, therefore Bayesian investors' expectation is $E[\theta_i|s_i] = (1-\gamma_i)E[\theta_i|\epsilon_i = s_i] + \gamma_i E[\theta_i|\theta_i = s_i] = (1-\gamma_i)\pi_i + \gamma_i s_i$.

The per dollar return from investing in the asset at $t = 0$ is $\tilde{r}_i = \frac{p_1^i}{p_0^i}$. The return can take three values: $r_i^H = \frac{1}{\pi_i}$ if the advertising succeeds; $r_i^M = 1 - \gamma_i + \frac{\gamma_i}{\pi_i}$ if the advertising fails and $s_i = 1$; $r_i^L = 1 - \gamma_i$ if the advertising fails and $s_i = 0$.

At $t = 0$ the arbitrageur knows $\theta_i = 1$, for $i \in M$. From the arbitrageur's perspective $\Pr(s_i = 1|\theta_i = 1) = \gamma_i \Pr[\theta_i = 1|\theta_i = 1] + (1-\gamma_i)\Pr[\epsilon_i = 1|\theta_i = 1] = \gamma_i + (1-\gamma_i)\pi_i$. For brevity denote $t_i = \Pr(s_i = 1|\theta_i = 1)$ and $1 - t_i = \Pr(s_i = 0|\theta_i = 1)$, $i \in M$. The distribution of asset i's per dollar return from the arbitrageur's point of view is the following

$$\tilde{r}_i = \begin{cases} r_i^H & \text{with probability } q_i \\ r_i^M & \text{with probability } (1-q_i)t_i \\ r_i^L & \text{with probability } (1-q_i)(1-t_i) \end{cases}, i \in N. \tag{2}$$

The arbitrageur's investments are characterized by his portfolio $\mathbf{y} = (y_1, ..., y_M)$, and his advertising efforts by $\mathbf{e} = (e_1, ..., e_M)$. At $t = 1$ the arbitrageur's final wealth is $c = \sum_{i=1}^{M} \tilde{r}_i y_i$. For instance, if the arbitrageur were to advertise all assets and investors were to learn all θ_i, $i \in M$ at $t = 1$ the arbitrageur's monetary payoff would be $c = \sum_{i=1}^{M} r_i^H y_i$, which happens with probability $\prod_{i \in M} q_i$.

At $t = 0$ the arbitrageur maximizes his expected utility taking (1) and (2) into account. The return on each asset $i \in M$ has three possible realizations, so that for two two assets we have nine possible realizations of monetary payoff, for M assets we have 3^M possible realizations. In general the expression for expected utility is very cumbersome. To write the expected utility in a relatively concise way we pick any two assets i and $j \neq i$ from M, and consider four states: advertising of both assets i and j is successful, only advertising of asset i is successful, only advertising of asset j is successful, and advertising of neither i nor j is successful. If the advertising of asset i is not successful, its return can be described by a binary random variable $\rho_i \in \{r_M, r_L\}$, with $\Pr(\rho = r_M) = t_i$. Analogously for j. The returns of all assets $\tilde{r}_i$, $i \in M$ are independent. For brevity denote by $\tilde{r}_{-ij} = \sum\limits_{k \neq i,j} \tilde{r}_k y_k$ the return on other assets in M except i and j. We can write down the arbitrageur's expected utility at $t = 0$ in the following way

$$\begin{aligned} E[V|\mathbf{y},\mathbf{e}] = q_i q_j E[V(y_i r_i^H + y_j r_j^H + \tilde{r}_{-ij}, e)] + q_i(1-q_j)E[V(y_i r_i^H + y_j \rho_j + \tilde{r}_{-ij}, e)] + \\ (1-q_i)q_j E[V(y_i \rho_i + y_j r_j^H + \tilde{r}_{-ij}, e)] + (1-q_i)(1-q_j)E[V(y_i \rho_i + y_j \rho_j + \tilde{r}_{-ij}, e)]. \end{aligned} \tag{3}$$

The arbitrageur's portfolio choice and advertising decisions solve:

$$\max_{\{\mathbf{y}\geq 0,\mathbf{e}\geq 0\}} E[V|\mathbf{y},\mathbf{e}],\ s.t.\ \sum_i y_i \leq w,\ q_i = \min[a_i e_i, 1],\ \forall i \in M. \tag{4}$$

We start solving the arbitrageur's problem by characterizing his advertising decisions.

Lemma 1. *In any solution to the arbitrageur's problem advertising never succeeds for sure:* $q_i < 1$ *for all* i.

All proofs are in the appendix. Because advertising is relatively costly (assumption 5), the arbitrageur never advertises an asset so much that $q_i = 1$.

Proposition 1. *The arbitrageur advertises only one asset:* $e_i > 0$ *for some* $i \in M$ *and* $e_j = 0$ *for any* $j \neq i$.

The proof is straightforward if the arbitrageur is risk neutral. Intuitively, a risk-neutral arbitrageur invests in an asset with the highest expected return. Naturally, the arbitrageur advertises an asset only if he invests in it. Therefore, a risk-neutral arbitrageur does not advertise two assets. If the arbitrageur is risk-averse the result is not so obvious. One may think that a risk -averse arbitrageur would choose to invest in and advertise several assets in order to diversify risks. This is not true. The detailed proof is in the appendix. We illustrate the intuition with a simple example with two symmetric identical assets, and uninformative public signal $s_i = \epsilon_i$.

Example with two assets. M contains two identical assets $i = 1, 2$ such that $\gamma_1 = \gamma_2 = 0$, $r_1^L = r_2^L = 1$, $r_1^H = r_1^M = r_2^H = r_2^M = r$, $a_1 = a_2 = 1$. For the sake of illustration assume the arbitrageur's cost of effort is zero but the arbitrageur has one unit capacity of advertising effort. To illustrate, suppose he can either allocate his effort equally to both assets $e_1 = e_2 = 1/2$ or he can put all his effort in one of the assets $e_i = 1$, $e_{-i} = 0$, $i = 1, 2$. Also we assume that $w = 2$ and the arbitrageur invests $y_i = y_2 = 1$ in each of the assets. We can show that advertising both assets delivers a lower expected payoff than advertising one of the assets.

Suppose the arbitrageur advertises both assets $e_1 = e_2 = 1/2$. With probability $(1 - e_1)(1 - e_2) = 1/4$ his advertising is not successful for both assets, and the arbitrageur gets monetary payoff $y_1 r_1^L + y_2 r_2^L = 2$ delivers $y_2 r_2^L = 1$. With probability 1/4 his advertising is successful for both assets and he gets $y_1 r_1^H + y_2 r_2^H = 2r$, with probability 1/2 his advertising is successful for one of the assets and he gets $1 + r$. His expected payoff is $E[V|e_1 = \frac{1}{2}, e_2 = \frac{1}{2}] = \frac{1}{4}V(2) + \frac{1}{4}V(2r) + \frac{1}{2}V(1 + r)$.

Suppose the arbitrageur advertises only one asset, setting for instance $e_1 = 1$, $e_2 = 0$. With probability $e_1 = 1$ his advertising about asset 1 is successful. His advertising about asset 2 is never successful. Hence, he gets return $1 + r$ for sure and his expected payoff is $E[V|e_1 = 1, e_2 = 0] = V(1 + r)$.

The difference in payoffs is $E[V|e_1 = 1, e_2 = 0] - E[V|e_1 = \frac{1}{2}, e_2 = \frac{1}{2}] = \frac{1}{2}V(1 + r) - \frac{1}{4}V(2) - \frac{1}{4}V(2r)$. Given that the arbitrageur is risk averse we have $v_1 - v_2 > 0$, that is the arbitrageur prefers to advertise only one asset. This somewhat counter-intuitive result is actually very natural. Advertising both assets results in a riskier lottery than the lottery corresponding to advertising only one of the assets: in fact the former lottery is a mean preserving spread of the latter. The risk averse arbitrageur prefers the latter lottery and advertises only one asset.

General intuition behind the result goes as follows. The risk-averse arbitrageur actively tries to insure against a bad outcome of no information at $t = 1$ by advertising and increasing the probability of information arriving at $t = 1$. For a given portfolio choice, the arbitrageur prefers to allocate all his advertising effort to one asset. This happens precisely because the arbitrageur is risk-averse and prefers a sure outcome over a lottery. By concentrating his advertising effort on one asset, the arbitrageur obtains a more secure bet than by spreading it over several assets. In the latter case, many assets may pay off with some probability and the final payoff is very uncertain. In the former case, instead, the arbitrageur gets a safer lottery: when he advertises a single asset, it is most likely that at time $t = 1$ this asset will deliver a high return; instead, the other assets that he does not advertise are most likely not to deliver high return. For both, the payoff involves little risk. This parallels the choice of the "job market paper" in the academic job market: typically, candidates come to the market with a single strong paper. Betting a future career on a single paper looks like a very risky strategy. In contrast, our analysis suggests that this strategy is the safest, as the candidate will have to advertise his project and fight for the market's attention.

3 Overweighing of the advertised asset by the arbitrageur

Proposition 1 greatly simplifies the analysis. Because only one asset $i \in M$ is advertised, $q_j = 0$ for $j \neq i$ and the expression for the arbitrageur's utility (3) can be written as

$$E[V|\mathbf{y}, e_{-i} = 0] = q_i E[V(y_i r_i^H + \sum_{j \neq i} \rho_j y_j, e_i)] + (1 - q_i) E[V(\sum_{j \in M} \rho_j y_j, e_i)]. \tag{5}$$

The arbitrageur's optimization problem (4) can be solved in the following manner. For each $i \in M$ one can find $\mathbf{e}(i)$ and $\mathbf{y}(i)$ that maximize (5) subject to $\sum_i y_i \leq w$ and $q_i = a_i e_i$. According to lemma 1 $q_i < 1$ and we can consider $e_i \in [0, 1/a_i]$ without loss of generality. For any given $e_i \in [0, 1/a_i]$, $q_i = a_i e_i$ is fixed and one can find a portfolio $\mathbf{y}(e_i)$ that maximizes (5) subject to $\sum_i y_i \leq w$. For each e_i denote by $E[V|e_i]$ the corresponding maximal value. Function $E[V|e_i]$ is bounded for $e_i \in [0, 1/a_i]$ therefore it achieves a maximum for some e_i^*, denote the maximal value $E[V]_i$, note there maybe be multiple levels of e_i that deliver $E[V]_i$. By advertising asset $i \in M$ the arbitrageur can get at most $E[V]_i$. In optimum the arbitrageur advertises an asset $i^* \in \arg\max_j E[V]_j$, note that there maybe be multiple assets that deliver the same maximal payoff. The level of advertising is given by $e_{i^*}^* \in \arg\max_{e_i} E[V|e_i]$ and portfolio choice is $\mathbf{y}(e_{i^*}^*)$.

As the above argument illustrates: once the arbitrageur has chosen the asset to advertise i^* and has chosen his advertising effort $e_{i^*}^*$ his portfolio choice problem is a standard diversification problem. The only difference is that the likelihood that he would get a high return on investment in asset i^* is enhanced by advertising. In general one can expect the arbitrageur to take a large position in asset i^* and take small positions in remaining assets to reduce the overall riskiness of his portfolio. To make this point most clearly, we concentrate on a symmetric case where, in the absence of advertising, the arbitrageur would choose an equal-weighted portfolio. We show that in the presence of advertising he will overweight the advertised asset in his portfolio.

Assumption 6. *Assets in M differ only in terms of advertisability: $\gamma_i = \gamma$ and $\pi_i = \pi$ for all $i \in M$ and $a_i \neq a_j$ for any $i \neq j$.*

As a benchmark case we solve for optimal portfolio allocation when advertising is not possible, that is $e = 0$. In this case all M assets are equivalent from the arbitrageur's point of view: $t_i = t$, $r_i^M = r^M$, $r_i^L = r^L$ for all $i \in M$.

Lemma 2. *If advertising is not possible, the arbitrageur is risk-averse and assumption 6 holds, the arbitrageur takes equal positions in all assets in M.*

The lemma is intuitive. Given that assets are identical but independent, a risk averse arbitrageur fully diversifies his portfolio: he takes equal positions in all assets in M. This is not the case when advertising is possible.

Proposition 2. *If advertising is possible, the arbitrageur is risk-averse and assumption 6 holds, the arbitrageur advertises the most advertisable asset and invests more in this asset than in any other asset: for $i = \arg\max_{j \in M} a_j$ we have $y_i > y_j$ for any $j \neq i$. Investments in other assets are the same $y_j = y$ for $j \neq i$.*

This result is intuitive. Recall that, by Proposition 1, only one asset is advertised. Clearly, the most advertisable asset is advertised, as it has a higher expected return for a given level of advertising effort than any other asset from the viewpoint of the arbitrageur. Proposition 2 states that, for this reason, the arbitrageur overweighs this asset in his portfolio compared to others.

Propositions 1 and 2 establish that the arbitrageur's advertising and investment will be concentrated under a general utility function V. We would like to go one step further and characterize explicitly which asset is chosen for advertising depending on parameters such as asset's potential return, quality of public signal and advertisability. In order to do so we concentrate on the case of a risk neutral arbitrageur, and later on use this specification as a workhorse for extensions.

4 Risk-neutral arbitrageur

From now on the arbitrageur is risk-neutral with respect to his monetary payoff c and has quadratic effort cost.

Assumption 7. $V(c, e) = c - e^2/2$.

We drop assumption 6 about symmetry of assets and consider M assets with different expected return ($1/\pi_i \neq 1/\pi_j$), different informativeness of the public signal ($\gamma_i \neq \gamma_j$), and different advertisability ($a_i \neq a_j$ fro any $i \neq j$). According to Proposition 1 the risk-neutral arbitrageur advertises only one asset (for convenience asset i, that is $e_i > 0$). It turns out that he also invests all his wealth w in this asset. To understand why, first observe that the risk neutral arbitrageur cares only about the expected return but not about the risk. Second, suppose he were to invest in a second asset $j \neq i$ that he does not advertise $e_j = 0$.

This would be consistent with optimality if the expected returns of the two assets were the same (otherwise the arbitrageur would strictly prefer one of the assets). But, if the asset j which is not advertised gives the same return as the advertised asset i, it must yield an even higher expected return if it were advertised. Hence, the arbitrageur will benefit by advertising asset j instead of asset i: by choosing $e'_j = e_i > 0$, $e'_i = 0$ and $y'_j = w$ he will increase the expected return of asset j. This contradicts the initial assumption that it is optimal to invest in both assets. Hence, the arbitrageur advertises and invests all his wealth in the same asset.

Suppose the arbitrageur invests all his wealth in asset k ($y_k = w$). Given that his advertising succeeds with probability $q_k = a_k e_k$ for $e_k \leq 1/a_k$, his advertising effort should maximize his expected payoff:

$$\max_{e_k \in [0,1/a_k]} \{a_k e_k r_k^H w + (1 - a_k e_k)[t_k r_k^M + (1 - t_k) r_k^L]w - e_k^2/2\}. \tag{6}$$

The solution is $e_k^* = a_k(1 - \gamma_k^2)\frac{1-\pi_k}{\pi_k}w$, it is interior because assumption 5 implies $w/\underline{\pi} < 1$ and $e_k^* < 1/a_k$. Substituting for e_k^* in (6) one can express the highest expected payoff the arbitrageur can get by investing in asset k and advertising it:

$$E[V|\pi_k, \gamma_k, a_k] = w[1 + \gamma_k^2(\frac{1}{\pi_k} - 1)] + \frac{w^2 a_k^2}{2}(1 - \gamma_k^2)^2(\frac{1}{\pi_k} - 1)^2). \tag{7}$$

Lemma 3. *The arbitrageur invests $y_i^* = w$ in asset $i = \arg\max_{k \in M} E[V|\pi_k, \gamma_k, a_k]$ and advertises it, his advertising effort is $e_i^* = a_i(1 - \gamma_i^2)\frac{1-\pi_i}{\pi_i}w$.*

This result explicitly shows how the arbitrageur weighs different parameters: potential return $\frac{1}{\pi_k}$, advertisability a_k and quality of public information γ_k. Note that all three asset's characteristics are desirable from the arbitrageur's viewpoint:

$$\begin{aligned}
\frac{\partial E[V|\pi, \gamma, a]}{\partial(1/\pi)} &= w\gamma^2 + a^2 w^2 (1 - \gamma^2)^2 (\frac{1}{\pi} - 1) > 0, \\
\frac{\partial E[V|\pi, \gamma, a]}{\partial a} &= w^2 a (1 - \gamma^2)^2 (\frac{1}{\pi} - 1)^2 > 0, \\
\frac{\partial E[V|\pi, \gamma, a]}{\partial \gamma} &= 2\gamma w(\frac{1}{\pi} - 1)[1 - wa^2(1 - \gamma^2)(\frac{1}{\pi} - 1)] > 0.
\end{aligned} \tag{8}$$

The first two inequalities are obvious, while the last one follows from assumption 5 that guarantees $w/\pi < 1$. Therefore, the arbitrageur faces trade-offs when he chooses in which asset to invest. For instance, he may need to compare an asset with high advertisability a_i and low potential return $1/\pi_i$ with an asset with low advertisability a_k and high potential return $1/\pi_k$.

In order to better understand the arbitrageur's preferences, we plot his indifference curves for different parameter values. Figure 2 illustrates which combinations of advertisability a and potential return $1/\pi$ deliver the same expected utility to the arbitrageur depending on the informativeness of the public signal γ. The four indifference curves in each graph correspond to $w = 0.1$ and net expected return $(E[V|\pi, \gamma, a] - w)/w$ equal to $10\%, 20\%, 30\%$ and 40%.

Figure 2: Indifference curves between potential return and advertisability for different quality of public information.

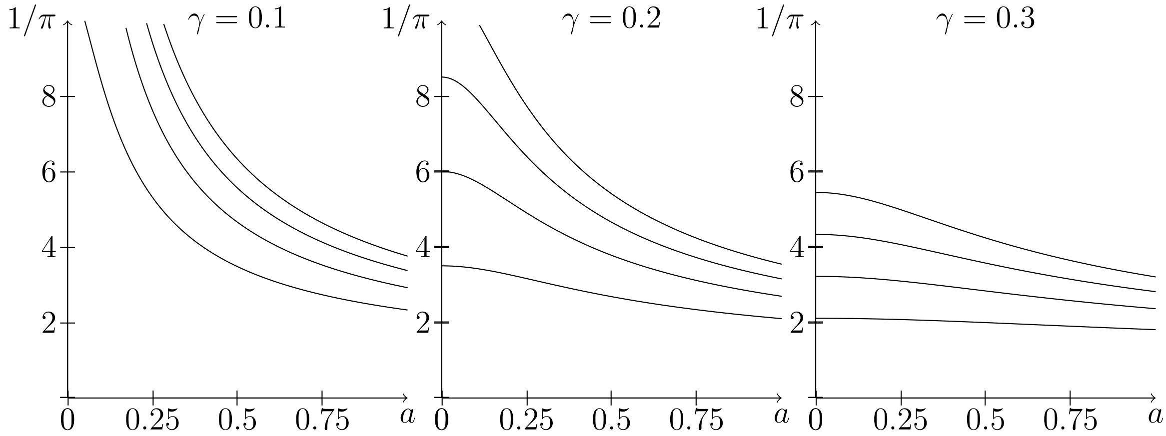
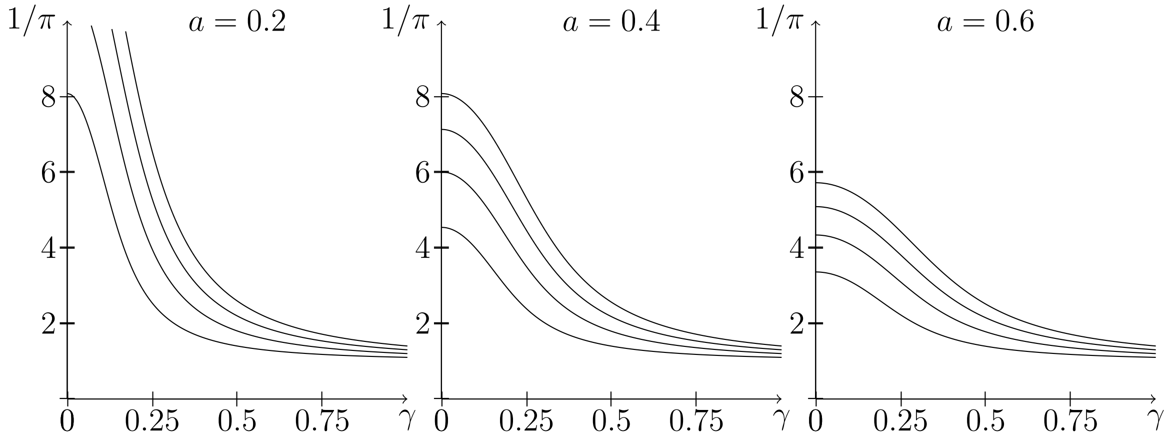


Figure 3: Indifference curves between potential return and quality of public information for different advertisability.



As can be seen from the figures, when the public signal is imprecise (low γ), advertisability is an attractive property for an asset, in the sense that the arbitrageur is ready to forgo an asset that can be expected to yield a large return in exchange for a more advertisable one. When instead $\gamma = 0.3$, he is no longer willing to do so, because information about the asset's value can be expected to be freely impounded in the market price without advertising: in this case, advertisability is not that valuable for an arbitrageur. Analogous reasoning applies for Figure 3: for low values of advertisability a, the arbitrageur is willing to pick an asset with significantly lower potential return if if features a more informative signal. Instead, he is no longer willing to do so if the initial asset is easily advertisable (high a).

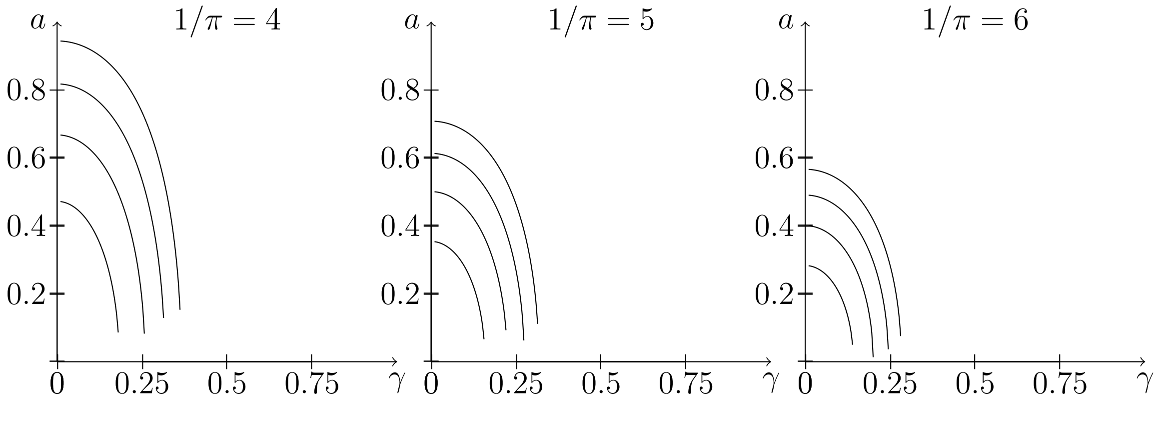
Formally, these results can be established in terms marginal rates of substitution (MRS) between different asset's characteristics from the point of view of the arbitrageur.

$$\begin{aligned}
MRS_a^{\gamma}(\gamma, a, \frac{1}{\pi}, w) &= \frac{\frac{\partial E[V|\pi,\gamma,a]}{\partial \gamma}}{\frac{\partial E[V|\pi,\gamma,a]}{\partial a}} = \frac{2\gamma}{1-\gamma^2}[\frac{1}{wa(1-\gamma^2)(\frac{1}{\pi}-1)} - a],\\
MRS_{1/\pi}^{\gamma}(\gamma, a, \frac{1}{\pi}, w) &= \frac{\frac{\partial E[V|\pi,\gamma,a]}{\partial \gamma}}{\frac{\partial E[V|\pi,\gamma,a]}{\partial (1/\pi)}} = \frac{2\gamma(\frac{1}{\pi}-1)}{\frac{1}{1-wa^2(1-\gamma^2)(\frac{1}{\pi}-1)} - (1-\gamma^2)},\\
MRS_a^{1/\pi}(\gamma, a, \frac{1}{\pi}, w) &= \frac{\frac{\partial E[V|\pi,\gamma,a]}{\partial (1/\pi)}}{\frac{\partial E[V|\pi,\gamma,a]}{\partial a}} = \frac{a}{\frac{1}{\pi}-1} + \frac{\gamma^2}{wa(1-\gamma^2)^2(\frac{1}{\pi}-1)^2}.
\end{aligned} \tag{9}$$

Proposition 3. *1) The higher the level of advertisability, the more the arbitrageur values advertisability relative to informativeness of the public signal; the higher the informativeness of the public signal, the more the arbitrageur values its informativeness relative to advertisability:* $MRS_a^{\gamma}(\gamma, a, \frac{1}{\pi}, w)$ *decreases in* a *and increases in* γ.

2) The larger the arbitrageur's initial wealth (w)*, the more he values advertisability relative to the informativeness of the public signal and to the asset's potential return, and the less he values it relative to the asset's potential return:* $MRS_a^{\gamma}(\gamma, a, \frac{1}{\pi}, w)$, $MRS_a^{1/\pi}(\gamma, a, \frac{1}{\pi}, w)$, *and* $MRS_{1/\pi}^{\gamma}(\gamma, a, \frac{1}{\pi}, w)$ *decrease with* w.

Figure 4: Indifference curves between advertisability and quality of public information for different potential return.



The proposition follows from (9). Figure 4 illustrates the first result of Proposition 3. Even though there is a natural trade-off between the advertisability and the informativeness of the public signal, this trade-off becomes less pronounced once the arbitrageur considers extremes: the most advertisable assets or the assets with the most informative public signals. In other words, one may expect a natural specialization of arbitrageurs: those who pursue arbitrage strategies involving assets with informative public signals and poor advertisability and those who prefer to invest in advertisable assets with relatively uninformative public signals. The remaining part of the proposition shows that relatively wealthy arbitrageurs are likely to invest in advertisable assets. An arbitrageur with deep pockets has a lot to loose if the advertising does not succeed: so his incentives to talk up his book and advertise the asset are high. Conversely, wealth-constrained arbitrageurs would prefer to invest in assets for which relatively informative public signals are available.

5 Multiple arbitrageurs

In many circumstances, one would expect several arbitrageurs to have the same information about certain assets. If several arbitrageurs independently acquire private information about different assets and do not share this information, one can consider each of them in isolation. In this case each of them behaves as described in previous sections. The analysis becomes instead non-trivial if several arbitrageurs have private information about the same set of assets. This is the case analyzed in this section.

5.1 Homogeneous arbitrageurs

Consider $L \geq 2$ identical arbitrageurs that at $t = 0$ share information about a set of assets $M \in N$. To begin we assume independent arbitrageurs: at $t = 0$ each arbitrageur $l \in M$ chooses his investments $\mathbf{y}_l$ and advertising efforts $\mathbf{e}_l$ taking the behavior of other arbitrageurs as given. The advertising efforts of arbitrageurs are complementary in the following sense: for any asset $i \in M$ advertised by several arbitragers $e_i^l \geq 0$, $l \in M$, the probability that investors learn true θ_i at $t = 1$ is $q_i = a_i \sum_l e_i^l$. As before, we want to avoid perfect advertising $q_i = 1$, to do so we modify assumption 5 to the case of multiple arbitrageurs and assume $Lw < \underline{\pi}$.

Possible realizations of asset i's returns are characterized by equation (2) as before. When arbitrageurs choose their investments and advertising efforts they have common information about the set of assets M, hence the game among arbitrageurs is one of complete information. We look for a Nash equilibrium in pure strategies $\mathbf{y}_l^*$, $\mathbf{e}_l^*$, $l = 1, ..., L$. By the same argument as in the beginning of section 4 each arbitrageur invests in a single asset and advertises it.

Lemma 4. *In equilibrium all L arbitrageurs invest in the same asset.*

Suppose otherwise, some arbitrageurs invest in asst j and some others in asset $k \neq j$, then the expected return of both assets must be the same. If an arbitrageur who invests in asset j deviates, invests in assets k and advertises it the expected return of asset k would increase, and the arbitrageur would benefit. It follows that all arbitrageurs must invest in the same asset in equilibrium. An equilibrium in which all arbitrageurs invest in asset j and advertise it exists if and only if no arbitrageur wants to deviate, invest in a different asset and advertise it. If the arbitrageur deviates, he chooses an asset in M different from j that maximizes his expected payoff in autarky $h_j = \underset{k \in M \setminus j}{\arg\max}\, E[V|\pi_k, \gamma_k, a_k]$, the corresponding expected payoff is

$$V^a_{-j} = w[1 + \gamma^2_{h_j}(\frac{1}{\pi_{h_j}} - 1)] + \frac{w^2 a^2_{h_j}}{2}(1 - \gamma^2_{h_j})^2(\frac{1}{\pi_{h_j}} - 1)^2. \tag{10}$$

If all arbitrageurs invest in asset j, each arbitrageur $l \in L$ chooses his advertising effort in order to maximize his expected payoff

$$\max_{e^l_j \in [0,1/a_j]} \{a_j(e^l_j + \sum_{m \neq l} e^m_j) r^H_j w + (1 - a_j(e^l_j + \sum_{m \neq l} e^m_j))[t_j r^M_j + (1 - t_j) r^L_j]w - (e^l_j)^2/2\}.$$

Each arbitrageur chooses $e_j = a_j(1 - \gamma^2_j)\frac{1-\pi_j}{\pi_j}w$, so that $q_j = La^2_j(1 - \gamma^2_j)\frac{1-\pi_j}{\pi_j}w$. Note that assumption $Lw < \underline{\pi}$ guarantees $q_j < 1$. Substituting for advertising efforts we obtain each arbitrageur's expected payoff if all arbitrageurs invests in asset j and advertise it:

$$V_j(L) = w[1 + \gamma^2_j(\frac{1}{\pi_j} - 1)] + (L - \frac{1}{2})w^2 a^2_j(1 - \gamma^2_j)^2(\frac{1}{\pi_j} - 1)^2. \tag{11}$$

Proposition 4. *There exists an equilibrium in which all arbitrageurs invest in asset j and advertise it if and only if $V_j(L) \geq V^a_{-j}$.*

Condition $V_j(L) \geq V^a_{-j}$ insures that each arbitrager prefers to invest in the asset that is already advertised by other $L - 1$ arbitrageurs. Note that V_j increases with L, that is an equilibrium with all arbitrageurs advertising any asset $j \in M$ is more likely to exist when M is high. This is because of complementarity of advertising efforts of different arbitrageurs. If some arbitrageurs advertise asset j this asset becomes attractive for other arbitrageurs because it's expected return is high. Thus in equilibrium, if many arbitrageurs already advertise an asset, an arbitrageur also prefers to advertise the same asset. Because of this complementarity multiple equilibria are possible with different assets being advertised. Clearly, some equilibria may be inefficient, that is arbitrageurs would prefer a different equilibrium. The following example illustrates this.

Example with two assets. Consider assets $i = 1, 2$ and assume $\pi_2 = \pi_1 = \pi$, $a_2 > 0$, $a_1 = 0$, $\gamma_2 = 0$ and $\gamma_1 > 0$. According to Proposition 4 an equilibrium with all arbitrageurs investing in asset 1 exists if and only if

$$V_1(2) = w[1 + \gamma_1^2(\frac{1}{\pi} - 1)] \geq w + \frac{w^2 a_2^2}{2}(\frac{1}{\pi} - 1)^2 = V_{-1}^a.$$

Similarly, an equilibrium with all arbitrageurs investing in asset 2 exists if and only if

$$V_2(2) = w + \frac{3w^2 a_2^2}{2}(\frac{1}{\pi} - 1)^2 \geq w[1 + \gamma_1^2(\frac{1}{\pi} - 1)] = V_{-2}^a.$$

Assume $\frac{3}{2}wa_2^2(\frac{1}{\pi} - 1) \geq \gamma_1^2 \geq \frac{1}{2}wa_2^2(\frac{1}{\pi} - 1)$ so that both equilibria exist. It is easy to see that the former equilibrium results in a lower expected payoff to both arbitrageurs that the latter one. Indeed, in the former equilibrium the arbitrageurs choose not to invest in the second asset and advertise it because individually it is relatively expensive to do so. Yet, if both arbitrageurs would advertise, they would enjoy positive externalities: if an arbitrageur invests in an asset and advertises it, other arbitrageurs benefit because the expected return of the asset increases. Therefore, arbitrageurs are jointly better off from investing in asset 2 and advertising it.

5.2 Heterogeneous arbitrageurs

In reality the arbitrageurs may differ in their characteristics. We capture this possibility by considering two types of arbitragers, A and B. There are $L_\tau \geq 1$ arbitrageurs of type $\tau = \{A, B\}$, each of them possesses resources w_τ. The game among arbitrageurs proceeds as in the homogeneous case. We look for a Nash equilibrium in pure strategies, $\mathbf{y}_l$, $\mathbf{e}_l$, $l = 1, ..., L_A + L_B$. Note, that for all arbitrageurs of the same type τ Lemma 4 holds: in equilibrium they invest in the same asset and advertise it.

It follows that two kinds of equilibria are possible: either both types of arbitrageurs invest the same asset, or each type invests in a different asset. For brevity we assume that there is one arbitrageur of each type $L_A = L_B = 1$. The results easily generalize to $L_A > 1$ and $L_B > 1$.

Similarly to the analysis in the previous section, for each arbitrageur τ define the maximum expected payoff he can get by investing in any asset in M except j

$$V_{-j}^a(\tau) = w_\tau[1 + \gamma_{h_{j\tau}}^2(\frac{1}{\pi_{h_{j\tau}}} - 1)] + \frac{w_\tau^2 a_{h_{j\tau}}^2}{2}(1 - \gamma_{h_{j\tau}}^2)^2(\frac{1}{\pi_{h_{j\tau}}} - 1)^2.$$

Suppose both arbitrageurs invest in asset j, each of them would choose advertising effort $e_j^\tau = a_j(1 - \gamma_j^2)\frac{1-\pi_j}{\pi_j}w_\tau$, $\tau \in \{A, B\}$. The expected payoff of each arbitrageur $\tau \in \{A, B\}$ in this case is:

$$V_j(\tau) = w_\tau[1 + \gamma_j^2(\frac{1}{\pi_j} - 1)] + \frac{1}{2}w_\tau^2 a_j^2(1 - \gamma_j^2)^2(\frac{1}{\pi_j} - 1)^2 + w_\tau w_{-\tau} a_j^2(1 - \gamma_j^2)^2(\frac{1}{\pi_j} - 1)^2.$$

Proposition 5. *1) A pooling equilibrium in which both arbitrageurs invest in the same asset j and advertise it exists if and only if $V_j(\tau) \geq V^a_{-j}(\tau)$ for $\tau = A, B$.*

2) The separating equilibrium in which arbitrageur of type A invests in asset $j(A)$, while arbitrageur of type B invests in asset $j(B) \neq j(A)$ exists if and only if for $\tau = A, B$

$$
\begin{aligned}
j(\tau) &= \arg\max_{k \in M} \left[w_\tau [1 + \gamma_k^2 (\frac{1}{\pi_k} - 1)] + \frac{w_\tau^2 a_k^2}{2} (1 - \gamma_k^2)^2 (\frac{1}{\pi_k} - 1)^2 \right], \\
V_{j(-\tau)}(\tau) &\leq \max_{k \in M} \left[w_\tau [1 + \gamma_k^2 (\frac{1}{\pi_k} - 1)] + \frac{w_\tau^2 a_k^2}{2} (1 - \gamma_k^2)^2 (\frac{1}{\pi_k} - 1)^2 \right].
\end{aligned}
\tag{12}
$$

Condition $V_j(\tau) \geq V^a_{-j}(\tau)$ grantees that both types prefer to invest in the same asset together rather than deviate and invest on their own. Condition (12) consist of two parts. The first part states that each type of the arbitrageur invests in the asset that delivers the highest expect return to the arbitrageur of the corresponding type. The second part states that no type of the arbitrageur wants to deviate and invest in an asset the other type invests in.

Corollary 1. *If a pooling equilibrium and the separating equilibrium exist, both types are better off in the poling equilibrium.*

In a separating equilibrium each type gets his autarky payoff. In a pooling equilibrium condition $V_j(\tau) \geq V^a_{-j}(\tau)$ guarantees that each type is weakly better off than in autarky. In principle, multiple pooling equilibria are possible, because the arbitrageurs may coordinate on different assets.

5.3 Discussion

If arbitrageurs are identical, their interests are congruent. They will try to coordinate in order to advertise and invest in the most profitable asset. This implies, that if arbitrageurs make their decisions sequentially, the first arbitrageur to move should advertise and invest in the most profitable asset. The others would optimally follow. The analysis is not so trivial if arbitrageurs are heterogeneous. It can happen that some of equilibria are preferred by some arbitrageurs, and other equilibria are preferred by other arbitrageurs. For instance, an arbitrageur with little wealth $w_A < w_B$ may prefer an equilibrium where both arbitrageurs invest in the same asset with high advertisability a_j in order to free ride on the advertising effort of the arbitrageur with $w_B > w_A$. At the same time, the wealthy arbitrageur may prefer an equilibrium where both arbitrageurs invest in the same asset with low advertisability a_i in order to economize on the advertising effort. This implies that if advertising decisions of the arbitrageurs were sequential, the first arbitrageur to advertise could start advertising the asset he likes the most and get the other arbitrageur to follow.

Several important aspects of the model deserve a closer look. We have assumed advertisability of an asset to be exogenous, while in reality it is very likely to be affected by the allocation of attention by investing public. Investors may choose to pay attention to some assets anticipating the advertising by arbitrageurs. In other words there can be a complementarity between decisions of investors and arbitrageurs that may be interesting to analyze.

Appendix

Proof of lemma 1.

If $e_j^* = 0$, then $q_j = \min[a_j e_j, 1] = 0$. Consider $e_i > 0$ for some $i \in M$ $q_i < 1$. Fix $\mathbf{y}^*$ and e_k^*, $k \neq i$. To see that $q_i < 1$ suppose instead that $q_i = 1$ and $e_i = \frac{1}{a_i}$: then, the first order condition with respect to e_i would require

$$a_i e_i E[V_e(\sum_k \tilde{r}_k y_k, \frac{1}{a_i} + \sum_{k \neq i} e_k))] \geq -a_i E[V(\frac{y_i}{\pi_i} + \sum_{k \neq i} \tilde{r}_k y_k, \frac{1}{a_i} + \sum_{k \neq i} e_k)].$$

First, $\tilde{r}_i \leq \frac{1}{\pi_i}$, $\pi_i \geq \underline{\pi}$ and $\sum_k y_k \leq w$ implies $\sum_k \tilde{r}_k y_k \leq \frac{w}{\underline{\pi}}$. Second, $a_i \leq 1$ implies $\frac{1}{a_i} + \sum_{k \neq i} e_k \geq 1$. Together with $V_{ce} \geq 0$ and $V_{ee} \leq 0$ this implies the left hand side is smaller than $V_e(\frac{w}{\underline{\pi}}, 1)$. Together with $V_c > 0$ and $V_e < 0$ this implies the right hand side is greater than $-V(\frac{w}{\underline{\pi}}, 1)$, which contradicts assumption 5. Thus $q_i < 1$, $i \in M$. QED.

Proof of proposition 1. Consider a solution $\mathbf{y}^*$, $\mathbf{e}^*$ to (4). Since $V_e(., 0) = 0$, $\gamma_i < 1$ and $a_i > 0$ for any $i \in M$ we must have $e_i^* > 0$ for some $i \in M$. First, notice that if the arbitrageur advertises asset i, he must have invested in it. Indeed if $y_j^* = 0$ then optimally $e_j^* = 0$, $j = 1, ..., M$, therefore $e_i^* > 0$ implies $y_i^* > 0$. Suppose there exists $j \neq i$ such that $e_j^* > 0$. This implies $y_j^* > 0$. Let $\hat{e} = e_i^* + e_j^*$, consider e_i and e_j such that $e_j = \hat{e} - e_i$.

Lemma 1 implies that $q_i < 1$, $q_j < 0$. A necessary condition for the maximum of the arbitrageur's expected payoff is that e_i and e_j maximize $E[V|\mathbf{y}, \mathbf{e}]$ subject to $e_j = \hat{e} - e_i$. Substitute for e_j in (3). Suppose $e_j^* > 0$. The first order condition for an interior solution requires $\frac{\partial E(V|\bar{y}, \bar{e})}{\partial e_i}|_{e_j = \hat{e} - e_i} = 0$.

We now show that this is not a maximum, and that an interior solution with $e_i > 0$ and $e_j > 0$ is not possible. To do so compute

$$\frac{\partial^2 E(V|\bar{y}, \bar{e})}{\partial^2 e_i}|_{e_j = \hat{e} - e_i} = -a_i a_j E[V(y_i r_i^H + y_j r_j^H + \sum_{k \neq i,j} \tilde{r}_k y_k)] + a_i a_j E[V(y_i r_i^H + y_j \rho_j + \sum_{k \neq i,j} \tilde{r}_k y_k)] + a_i a_j E[V(y_i \rho_i + y_j r_j^H + \sum_{k \neq i,j} \tilde{r}_k y_k)] - a_i a_j E[V(y_i \rho_i + y_j \rho_j + \sum_{k \neq i,j} \tilde{r}_k y_k)].$$

We will show that if $V(v, e)$ is concave in c then $\frac{\partial^2 E(V|\bar{y}, \bar{e})}{\partial^2 e_i}|_{e_j = \hat{e} - e_i} > 0$, that is in optimum either e_i^*, or e_j^* should be zero. First, note that $\frac{\partial^2 E(V|\bar{y}, \bar{e})}{\partial^2 e_i}|_{e_j = \hat{e} - e_i} \geq 0$ is equivalent to

$$\frac{1}{2} E[V(y_i r_i^H + y_j \rho_j + \sum_{k \neq i,j} \tilde{r}_k y_k)] + \frac{1}{2} E[V(y_i \rho_i + y_j r_j^H + \sum_{k \neq i,j} \tilde{r}_k y_k)] \geq \frac{1}{2} E[V(y_i r_i^H + y_j r_j^H + \sum_{k \neq i,j} \tilde{r}_k y_k)] + \frac{1}{2} E[V(y_i \rho_i + y_j \rho_j + \sum_{k \neq i,j} \tilde{r}_k y_k)]. \tag{13}$$

Recall that ρ_i is a binary random variable: $\Pr\{\rho_i = r_i^M\} = t_i$ and $\Pr\{\rho_i = r_i^L\} = 1 - t_i$ for all $i \in M$. Note that the right hand side of (13) corresponds to a payoff from a compound lottery $\tilde{x}_{RHS}$. The left hand side of (13) corresponds to a payoff from a compound lottery $\tilde{x}_{LHS}$. Below we will show that $\tilde{x}_{RHS}$ is a mean preserving spread of $\tilde{x}_{LHS}$.

First, note that returns on assets $k \neq i, j$ do not matter for the comparison. Next, consider assets i and j. The table below lists possible monetary returns from assets i and j with corresponding probabilities in lotteries $\tilde{x}_{LHS}$ and $\tilde{x}_{RHS}$.

return on assets i, j	probability in $\tilde{x}_{LHS}$	probability in $\tilde{x}_{RHS}$
$r_i^H y_i + r_j^H y_j$	0	$\frac{1}{2}$
$r_i^H y_i + r_j^M y_j$	$\frac{1}{2}t_j$	0
$r_i^H y_i + r_j^L y_j$	$\frac{1}{2}(1-t_j)$	0
$r_i^M y_i + r_j^H y_j$	$\frac{1}{2}t_i$	0
$r_i^L y_i + r_j^H y_j$	$\frac{1}{2}(1-t_i)$	0
$r_i^M y_i + r_j^M y_j$	0	$\frac{1}{2}t_i t_j$
$r_i^M y_i + r_j^L y_j$	0	$\frac{1}{2}t_i(1-t_j)$
$r_i^L y_i + r_j^M y_j$	0	$\frac{1}{2}(1-t_i)t_j$
$r_i^L y_i + r_j^L y_j$	0	$\frac{1}{2}(1-t_i)(1-t_j)$

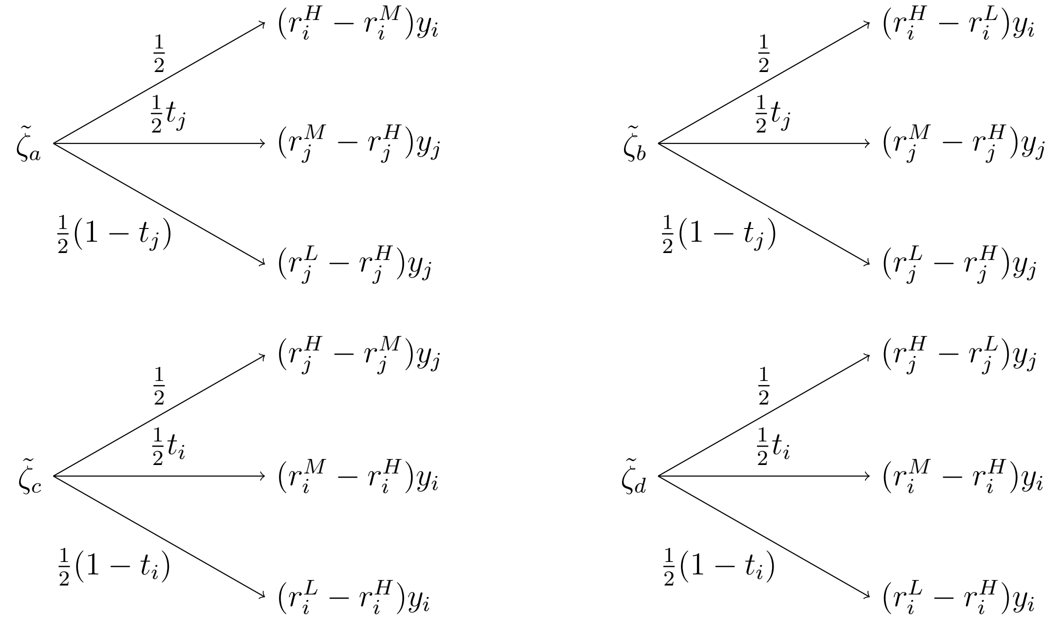
It is easy to check that both lotteries have the same expected monetary return. One can find a random variable $\tilde{\zeta}$ with zero mean such that $\tilde{x}_R = \tilde{x}_L + \tilde{\zeta}$, that is RHS lottery is a mean preserving spread of the LHS lottery. To see this, construct $\tilde{\zeta}$ as a compound lottery of four lotteries in the following manner:

$$\tilde{\zeta} = \begin{cases} \tilde{\zeta}_a & \text{with probability } \frac{1}{2}t_i \\ \tilde{\zeta}_b & \text{with probability } \frac{1}{2}(1-t_i) \\ \tilde{\zeta}_c & \text{with probability } \frac{1}{2}t_j \\ \tilde{\zeta}_d & \text{with probability } \frac{1}{2}(1-t_j) \end{cases}$$

so that each lottery $\tilde{\zeta}_a$, $\tilde{\zeta}_b$, $\tilde{\zeta}_c$, $\tilde{\zeta}_d$ is played in the node with the same probability in the LHS lottery described in the table before. Each of these lotteries should map outcomes of the LHS lottery into outcomes of the RHS lottery, this can be done with the following lotteries.

One can substitute and verify that $\tilde{x}_R = \tilde{x}_L + \tilde{\zeta}$. Since $r_k^H = \frac{1}{\pi_k} > 1 - \gamma_k = r_K^L$, $y_k > 0$, $k = i, j$ lottery $\tilde{\zeta}$ is not degenerate. Its mean is zero $E[\tilde{\zeta}] = \frac{1}{4}t_i[(r_i^H - r_i^M)y_i + t_j(r_j^M - r_j^H)y_j + (1-t_j)(r_j^L - r_j^H)y_j] + \frac{1}{4}(1-t_i)[(r_i^H - r_i^L)y_i + t_j(r_j^M - r_j^H)y_j + (1-t_j)(r_j^L - r_j^H)y_j] + \frac{1}{4}t_j[(r_j^H - r_j^M)y_j + t_i(r_i^M - r_i^H)y_i + (1-t_i)(r_i^L - r_i^H)y_i] + \frac{1}{4}(1-t_j)[(r_j^H - r_j^L)y_j + t_i(r_i^M - r_i^H)y_i + (1-t_i)(r_i^L - r_i^H)y_i] = \frac{1}{4}[t_i(r_i^H - r_i^M)y_i + (1-t_i)(r_i^H - r_i^L)y_i + t_j(r_j^M - r_j^H)y_j + (1-t_j)(r_j^L - r_j^H)y_j] + \frac{1}{4}[t_j(r_j^H -$

Figure 5: Description of lotteries.



$r_j^M)y_j+(1-t_j)(r_j^H-r_j^L)y_j+t_i(r_i^M-r_i^H)y_i+(1-t_i)(r_i^L-r_i^H)y_i] = \frac{1}{4}[r_i^H y_i - t_i r_i^M y_i - (1-t_i)r_i^L y_i - r_j^H y_j + t_j r_j^M y_j + (1-t_j)r_j^L y_j] + \frac{1}{4}[r_j^H y_j - t_j r_j^M y_j - (1-t_j)r_j^L y_j - r_i^H y_i + t_i r_i^M y_i + (1-t_i)r_i^L y_i] = 0.$

Now consider separately two cases of a risk-averse arbitrageur and a risk neutral arbitrageur.

1. If the arbitrageur is risk-averse, that is $V(c,e)$ is concave in c, then $\frac{\partial^2 E(V|\bar{y},\bar{e})}{\partial^2 e_i}|_{e_j=\hat{e}-e_i} > 0$. In this case the arbitrageur will never choose $e_i > 0$ and $e_j = \hat{e} - e_i > 0$, because setting $e_i = 0$ or $e_j = 0$ would increase payoff. This implies that $e_i^* > 0$ and $e_j^* > 0$ can't be optimal. In other words, $e_i^* = \hat{e} > 0$ for some $i \in M$ implies $e_j^* = 0$ for any $j \neq i$: only one asset is advertised by a risk-averse arbitrageur.

2. If the arbitrageur is risk-neutral, i.e. $V(c,e)$ is linear in c, and the arbitrageur advertises both assets $e_i > 0$, $e_j > 0$, it must be the case that he invests in both assets $y_i > 0$, $y_j > 0$. It follows that both assets have the same expected return. Given that $q_i < 1$, $q_j < 1$ from lemma 1, there is a profitable deviation for an arbitrageur. He can choose $e_i' = e_i + e_j$, $e_j' = 0$, $y_i' = y_i + y_j$, $y_j' = 0$ and benefit, because the return on asset i would increase due to extra advertising and, hence, overall return on his investment would increase. Thus a risk neutral arbitrageur also advertises only one asset. QED.

Proof of lemma 2. When advertising is not possible, the arbitrageur's portfolio choice

$\mathbf{y}$ must satisfy his resource constraint $\sum_i y_i = w$ and maximize

$$E[V|\mathbf{y}] = t^2 E[V(y_k r^M + y_i r^M + \sum_{j\neq i,k} y_j\rho_j)] + t(1-t)E[V(y_k r^M + y_i r^L + \sum_{j\neq i,k} y_j\rho_j)] + \\ (1-t)tE[V(y_k r^L + y_i r^M + \sum_{j\neq i,k} y_j\rho_j)] + (1-t)^2 E[V(y_k r^L + y_i r^L + \sum_{j\neq i,k} y_j\rho_j)] \tag{14}$$

The arbitrageur's is risk-averse, hence his objective is strictly concave in $\mathbf{y}$. The set of possible values is compact $\sum_i y_i = w$, $y_i \geq 0$. So an optimal portfolio exists and is unique. Take asset k with $y_k^* \geq 0$ and fix $\bar{y} = y_i^* + y_k^*$, and y_j^* for $j \neq k, i$. Maximize (15) subject to $y_i = \bar{y} - y_k$ and y_j^* for $j \neq i, k$. The solution to this problem should deliver $y_k = y_k^*$ and $y_i^* = \bar{y} - y_k^*$. The first order condition is

$$\begin{aligned} &t^2 E[V'(y_k r^M + (\bar{y} - y_k)r^M + \sum_{j\neq i,k} y_j^*\rho_j)](r^M - r^M) + \\ &t(1-t)E[V'(y_k r^M + (\bar{y} - y_k)r^L + \sum_{j\neq i,k} y_j^*\rho_j)](r^M - r^L) + \\ &(1-t)tE[V'(y_k r^L + (\bar{y} - y_k)r^M + \sum_{j\neq i,k} y_j^*\rho_j)](r^L - r^M) + \\ &(1-t)^2 E[V'(y_k r^L + (\bar{y} - y_k)r^L + \sum_{j\neq i,k} y_j^*\rho_j)](r^L - r^L) = 0. \end{aligned} \tag{15}$$

As the first and the last term of the left-hand side of (15) are zero, equation (15) becomes: $E[V'(y_k r^M + (\bar{y} - y_k)r^L + \sum_{j\neq i,k} y_j^*\rho_j)] = E[V'(y_k r^L + (\bar{y} - y_k)r^M + \sum_{j\neq i,k} y_j^*\rho_j)]$, which implies $y_k^* = y_i^* = \bar{y}/2$. One can check that corner solutions $y_k = 0$, $y_k = \bar{y}$ do not satisfy the necessary condition because V is concave. A similar argument for any couple of other assets i and $j \neq i$ would imply $y_j^* = y_i^*$. As the number of assets in M is M, we get $y_i^* = w/M$. QED.

Proof of proposition 2. Recall that, by assumption, when advertising effort is zero, its marginal cost is zero: $V_e(.,0) = 0$. First, the arbitrageur must advertise the asset with the highest advertisability $i = \arg\max_{k\in M} a_k$. Suppose otherwise $e_i = 0$ and $e_j = e > 0$ for some $j \neq i$. Denote corresponding investments y_i and $y_j = y > 0$. This is not optimal because the arbitrageur can get a higher utility by switching around both advertising effort and investment levels between the two assets. Namely, by setting $y_i' = y_j = y$, $y_j' = y_i$, $e_i' = e_j = e$ and $e_j' = e_i$. Indeed, investments y_j' and y_i deliver identical returns. Yet, investment y_i' dominates investment y_j in terms of first order stochastic dominance, as the table below illustrates:

return on investment	probability for $y_i' = y$, $e_i' = e$	probability for $y_j = y$, $e_j = e$.
$r^H y$	$a_i e$	$a_j e$
$r^M y$	$(1-a_i e)t$	$(1-a_j e)t$
$r^L y$	$(1-a_i e)(1-t)$	$(1-a_j e)(1-t)$

Since $i = \arg\max_{k\in M} a_k$, it must be that $e_i > 0$.

Second, it is straightforward to show that the arbitrageur invests equal amounts in the assets that he does not advertise. The argument is the same as in the proof of lemma 2.

To prove that $y_i > y_j$, $j \neq i$, let's rewrite the arbitrageur's expected utility as follows:

$$\begin{aligned}
E[V|\mathbf{y},e] = {} & ea_i t E[V(y_i r^H + y_j r^M + \sum_{k\neq i,j} y_k \rho_k, e)] + \\
& ea_i(1-t)E[V(y_i r^H + y_j r^L + \sum_{k\neq i,j} y_k \rho_k), e] + \\
& (1-ea_i)t^2 E[V(y_i r^M + y_j r^M + \sum_{k\neq i,j} y_k \rho_k, e)] + \\
& (1-ea_i)t(1-t)E[V(y_i r^M + y_j r^L + \sum_{k\neq i,j} y_k \rho_k, e)] + \\
& (1-ea_i)(1-t)tE[V(y_i r^L + y_j r^M + \sum_{k\neq i,j} y_k \rho_k, e)] + \\
& (1-ea_i)(1-t)^2 E[V(y_i r^L + y_j r^L + \sum_{k\neq i,j} y_k \rho_k, e)].
\end{aligned} \tag{16}$$

As before, let us fix all optimal y_k^*, $k \neq j, i$ and set $\bar{y} = y_i^* + y_j^* > 0$. Consider then optimization of (16) over y_i given the constraint $y_j = \bar{y} - y_i$. The first order necessary condition with respect to y_i is:

$$\begin{aligned}
& ea_i t E[V'(y_i r^H + y_j r^M + \sum_{k\neq i,j} y_k \rho_k, e)](r^H - r^M) + \\
& ea_i(1-t)E[V'(y_i r^H + y_j r^L + \sum_{k\neq i,j} y_k \rho_k), e](r^H - r^L) + \\
& (1-ea_i)t^2 E[V'(y_i r^M + y_j r^M + \sum_{k\neq i,j} y_k \rho_k, e)](r^M - r^M) + \\
& (1-ea_i)t(1-t)E[V'(y_i r^M + y_j r^L + \sum_{k\neq i,j} y_k \rho_k, e)](r^M - r^L) + \\
& (1-ea_i)(1-t)tE[V'(y_i r^L + y_j r^M + \sum_{k\neq i,j} y_k \rho_k, e)](r^L - r^M) + \\
& (1-ea_i)(1-t)^2 E[V'(y_i r^L + y_j r^L + \sum_{k\neq i,j} y_k \rho_k, e)](r^L - r^L) = 0.
\end{aligned} \tag{17}$$

This boils down to

$$ea_itE[V'(y_ir^H + y_jr^M + \sum_{k\neq i,j} y_k\rho_k, e)](r^H - r^M)+$$

$$ea_i(1-t)E[V'(y_ir^H + y_jr^L + \sum_{k\neq i,j} y_k\rho_k), e](r^H - r^L) =$$

$$(r^M - r^L)(1-ea_i)(1-t)tE[V'(y_ir^L + y_jr^M + \sum_{k\neq i,j} y_k\rho_k, e) - V'(y_ir^M + y_jr^L + \sum_{k\neq i,j} y_k\rho_k, e)].$$

The LHS is positive for any $e > 0$. Given that V is concave the RHS is positive if and only if $y_ir^L + y_jr^M < y_ir^M + y_jr^L$, this implies $y_i > y_j$. QED.

Proof of proposition 3. Differentiate $E[V|\pi,\gamma,a] = w[1+\gamma^2(\frac{1}{\pi}-1)] + \frac{w^2a^2}{2}(1-\gamma^2)^2(\frac{1}{\pi}-1)^2)$ and obtain $\frac{\partial E[V|\pi,\gamma,a]}{\partial a} = w^2a(1-\gamma^2)^2(\frac{1}{\pi}-1)^2$, $\frac{\partial E[V|\pi,\gamma,a]}{\partial w} = 1+\gamma^2(\frac{1}{\pi}-1) + wa^2(1-\gamma^2)^2(\frac{1}{\pi}-1)^2$. First, $\frac{\partial^2 E[V|\pi,\gamma,a]}{\partial a\partial\gamma} = -4\gamma w^2a(1-\gamma^2)(\frac{1}{\pi}-1)^2 < 0$ proves 1). Second, $\frac{\partial^2 E[V|\pi,\gamma,a]}{\partial w\partial a} = 2wa(1-\gamma^2)^2(\frac{1}{\pi}-1)^2 > 0$ proves 2). Finally, $\frac{\partial^2 E[V|\pi,\gamma,a]}{\partial w\partial\gamma} = 2\gamma(\frac{1}{\pi}-1)[1-2wa^2(1-\gamma^2)(\frac{1}{\pi}-1)] < 0$ whenever $2wa^2(1-\gamma^2)(\frac{1}{\pi}-1) > 1$, that is if and only if $\pi < 2wa^2(1-\pi)$ and $\gamma < \gamma^* = \sqrt{1-\frac{\pi}{2wa^2(1-\pi)}}$, which proves 3). QED.

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