NEW YORK UNIVERSITY SCHOOL OF LAW

NYU Center for Law, Economics and Organization

Exchange Efficiency with Weak Property Rights

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October 2012

LAW & ECONOMICS RESEARCH PAPER SERIES
WORKING PAPER NO. 12-35

Exchange Efficiency with Weak Property Rights*

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Abstract

We show that the first welfare theorem obtains independently of the strength of property rights protection. In an exchange setting, a large class of legal rules (what we call generalized liability rules) are exchange-efficient. Included in this class are property rules (generalized liability rules with very large damages Ds), standard liability rules (generalized liability rules with Ds that track the owner's valuation), and even rules which afford possessory interests only very weak protection (generalized liability rules with very small Ds). This result corrects a previous misconception in the literature, and yields the provocative conclusion that strong property rights are not required for exchange efficiency.

1 Introduction

The first welfare theorem is the main intellectual justification for the free market system. This theorem is proved under the assumption that the owner of a good cannot be forced to trade it. Similarly, virtually all the theoretical results demonstrating the efficiency of decentralized exchange, assume that agents can hold on to their endowment, if they so choose.[1] Thus allocative efficiency obtains if *exchange is voluntary* or, in other words, if

*We thank Ronen Avraham, Omri Ben-Shahar, Oliver Board, Ryan Bubb, Peter DiCola, Richard Epstein, Lee Fennell, Clay Gillette, David Gilo, Lewis Kornhauser, Saul Levmore, Anup Malani, Yoram Margalioth, Dotan Oliar, Michele Piccione, Ariel Porat, Eric Posner, Ariel Rubinstein, Steve Shavell, Henry Smith, Asher Wolinsky and conference and workshop participants at New York University, Northwestern University, Tel Aviv Univeristy and the 2011 Annual Meeting of the American Law and Economics Association for helpful comments.

[1] The few exceptions are reviewed in Section 2.

property rights are strongly protected.[2]

This paper shows that under relatively mild assumptions strong property rights are not necessary for exchange efficiency. We study an exchange economy in which an agent may temporarily possess a good, and during this time enjoy the benefits of consumption. But the good may be taken away without the agent's consent. If the good is taken by another agent the first, dispossessed agent receives a court-ordered monetary compensation D (damages), which may be much lower than the value of the object to the dispossessed agent. This is a relatively weak form of protection. This type of entitlement protection is called "liability rule" protection in legal parlance. The strong form of property rights assumed in most economic models obtains by setting $D = \infty$.

In this setup, the law and economics literature holds that decentralized trading might fail to secure exchange-efficiency. The argument is the following. Suppose D is low, and so property rights are weakly protected. Under this liability rule, an agent with value $u_j > D$ will find it profitable to pay damages and take the object from its current possessor, even if the latter values the object at $u_i > u_j$. This inefficiency, however, is resolved if the possessor can "bribe" the taker into giving up the right to take (paying the taker $u_j - D + \varepsilon$ will suffice). That is, in a two-person economy efficiency prevails. The alleged difficulty arises under multi-party decentralized trading, where many agents can take, or threaten to take, the asset. Quoting from the seminal paper in this literature:

> "Consider the situation of an owner and a particular taker who values the car less highly than does the owner (but above the level of damages). The owner would like to bargain with the taker and pay him not to take the car. However, it would be irrational for the owner to pay this taker not to take the car, and then another and another. Therefore, the potential taker will tend to take the car even though the owner values it more highly. The general point, in other words, is that when courts err and set damages too low, bargaining by owners will be

[2] The capacity to *own* something is also seen as a value *per se*. To many social thinkers, voluntary exchange represents the absence of economic coercion; it is what puts the "free" in free markets. For example, Ludwig von Mises writes:

> "The essential teaching of liberalism is that social cooperation and the division of labor can be achieved only in a system of private ownership of the means of production, i.e., within a market society, or capitalism." (von Mises, 1948, p. 48)

Similarly, Ayn Rand writes:

> "Capitalism is a social system based on the recognition of individual rights, including property rights, in which all property is privately owned." (Rand, 1967, p. 19)

> effectively infeasible, and socially undesirable takings will occur." (Kaplow and Shavell 1996, pp. 765-66)

By this account, liability rules are inefficient in a decentralized economy, despite the efficiency-enhancing properties of bilateral contracting, because not all relevant parties can enter into a global contract. Apparently, then, decentralized contracting does not yield efficiency in this setting. This argument is seen as an efficiency rationale for strong property rights as the best rule for protecting entitlements in a large economy.

In this paper we show that this reasoning is incomplete, and that efficiency does in fact prevail even under decentralized contracting. The conventional argument against liability rules fails to account for the recursive nature of the problem: the first taker is also herself subject to taking by the second taker. The second taker, in turn, is subject to taking by the third taker, and so on.[3] Once this effect is properly accounted for, we show that the incentives to take are reduced to the point where there is an efficient equilibrium in this exchange economy. In other words, we show that the first welfare theorem holds regardless of the degree of property rights protection.

This result is provocative, to be sure, because it forces us to take seriously very unfamiliar systems of protecting entitlements. But it should not be interpreted *too* provocatively, for example as arguing for the irrelevance of the state. The state plays a crucial role in the efficiency result, by enforcing the (possibly small) damages awards and the bilateral contracts which need to be executed in equilibrium.

If, as we show, a liability rule is just as exchange-efficient as a property rule, then it is natural to ask whether there is some other rationale that favors property rules over liability rules? For example, a property rule is generally believed to provide better incentives to invest in an asset, compared to any liability rule. Could this feature explain the prevalence of property rules? This seems plausible. And yet, we argue that there are environments in which a liability rule provides better incentives to invest, compared to a property rule.

The next obvious question, then, is whether liability rules are ever observed in reality. As it turns out, liability rules are very common in the law. Therefore we believe that our model, despite being very stylized, is more than an abstract exercise. We refer to Section 9.5 for a more in-depth discussion.

The remainder of the paper proceeds as follows. Section 2 summarizes the relevant literature. Section 3 presents a motivating example. Section 4 introduces our general framework

[3] Kaplow and Shavell, in a footnote, acknowledge that the first taker's incentive to take should be weaker, if the first taker expects to be himself subject to a taking by the second taker. See Kaplow and Shavell (1996), p. 766, note 167.

of analysis. Section 5 presents our solution concept. Section 6 proves that an efficient equilibrium exists under a large class of legal rules, including both property rules and liability rules. Section 7 interprets the monotonic selection condition, a condition which is necessary to obtain efficiency. Section 8 shows that an efficient equilibrium can be implemented through a standard non-cooperative bargaining game. Section 9 discusses ex ante investment efficiency and other matters. Section 10 concludes.

2 Related Literature

The identification of property rules ($D = \infty$) and liability rules (smaller D) as the two common legal approaches to the protection of entitlements harkens back to the seminal article by Calabresi and Melamed (1972). Subsequent contributions set-out to delineate the efficiency properties of property rules and liability rules. See, e.g., Ayres and Talley (1995), Kaplow and Shavell (1995, 1996), Bebchuk (2001) and Bar-Gill and Bebchuk (2010).

Our main finding, the exchange efficiency result, belongs to the literature on competitive equilibrium and its variants, rather than the literature on contracts and the Coase theorem. The reason the Coase theorem does not apply in our framework, as we mentioned, is that all relevant parties cannot get together and enter into a contract, or a grand bargain. That being said, our discussion of investment-efficiency of different entitlement rules (Section 9.1) draws heavily on the contract theory literature, which studies how the allocation of property rights affects incentives to make non-contractible investments (see, e.g., Hart 1995).

The existence of only weak property rights has been studied in the innovation context, where the asset that is only weakly protetcted, if at all, is information. The focus of this literature has been to identify strategies for extracting value in the absence of property rights. See, e.g., Anton and Yao (1994, 2002). This literature has also considered the choices of an employee who discovers an innovation for which no property rights exist and must choose between keeping the innovation private or disclosing the innovation to the employer. See, e.g., Anton and Yao (1995); Baccara and Razin (2006).

A related set of papers is Piccione and Rubinstein (2004, 2007) who study economies in which the stronger may take from the weaker. We interpret Piccione and Rubinstein's research agenda as inquiring into resource allocations in a weak state, a state in which entitlements are defended by force and agents differ in the force they have. Our setting, by contrast, features rules that are effectively enforced by the state and apply equally to every agent. Moreover, Piccione and Rubinstein's notion of allocational efficiency is very different from ours. They consider it Pareto-optimal for one agent (the strongest) to get all the resources in the economy, even if his valuations are lower than those of some other agent. According to our efficiency criterion, such an allocation would not be optimal, because we require that the goods are, at any point in time, in the hands of those who value them most. In this sense, our efficiency result is stronger than Piccione and Rubinstein's, but it requires the machinery of a well-functioning state (enforcement of damages awards and side contracts among agents).

Also tangentially related is the literature on the appropriability of investment in intellectual property and the optimal patent length. This literature sometimes argues that short patent lengths give sufficient incentives to invest in innovation (see, e.g., Boldrin and Levine 2002).

3 Motivating Example

In this section we present a simple example illustrating that exchange efficiency can obtain with weak property rights. The setup in this example is inspired by the Kaplow and Shavell quote on page 2.

The initial owner, party 0, enjoys a per-period value $\overline{u}$ from using an asset. In each period $t = 1, 2, ..$ a taker t appears. For all takers, the per-period value of the asset is $\underline{u} < \overline{u}$. Time is discounted at rate δ. Each taker can unilaterally acquire ownership by paying the current owner damages in the amount of $D = d/(1-\delta)$, where $d < \underline{u}$. (In applying liability rules, courts often try to set damages that are fully compensatory, i.e., $D = \overline{u}/(1-\delta)$ for the initial owner and $D = \underline{u}/(1-\delta)$ for other parties. In practice, damages are often undercompensatory, and that is believed to give rise to inefficiency)

Let's assume that in each period $t = 1, 2, ...$ the beginning-of-period owner makes a take-it-or-leave-it offer to party t. The owner offers m to party t and, if accepted, party t forgoes the right to take the asset. If m is rejected, then party t chooses whether to take the asset and pay the owner damages of $d/(1-\delta)$, or walk away.

Proposition 1 *There exists an efficient equilibrium in which the original owner successfully bargains with each successive taker and pays each of them m^* to go away. The unique m^* compatible with this equilibrium is $m^* = \underline{u} - d$.*

Proof. We will construct an equilibrium in which any party t who were to become an owner would bargain with all future takers and pay them m^* to go away; takers $t+1, t+2, ...$ will each accept m^* from any current owner and in exchange refrain from exercising their right to take the asset. Off the equilibrium path, if a taker were to reject the owner's offer, he would then exercise his right to take.

Subgame in which the taker has rejected the owner's offer

In this subgame the taker (party t, where necessarily $t \geq 1$) has a further choice to make, i.e., whether to take the asset or walk away. If he takes he pays D and then, following the conjectured equilibrium, pays m^* in each future period to retain possession of the asset forever. The taker's lifetime payoff associated with the taking option is

$$\underline{u} - \frac{d}{1-\delta} + \sum_{t=1}^{\infty} \delta^t \cdot [\underline{u} - m^*] \, . \tag{1}$$

Whenever this payoff exceeds 0, that is when

$$m^* \leq \frac{\underline{u} - d}{\delta}, \tag{2}$$

the payoff from taking exceeds that from walking away. If this condition is satisfied, then any taker who were to reject an offer would then exercise his right to take and obtain a lifetime payoff of (1) from following the equilibrium strategy in the future.

Subgame in which the owner chooses which m (if any) to offer the taker

The previous analysis shows that the subgame perfect equilibrium strategy for a taker is to accept any offer m greater or equal to (1). Now let us consider the beginning-of-period owner's behavior. If the owner makes a "serious" offer he will offer the smallest m compatible with the offer being accepted. This we denote by $\underline{m}$ equal to (1). Assuming that the owner is of type $t \geq 1$, his lifetime payoff from offering this minimum acceptable $\underline{m}$ is

$$\begin{aligned} & \underline{u} - \underline{m} + \sum_{t=1}^{\infty} \delta^t \cdot [\underline{u} - m^*] \\ & = \frac{d}{1-\delta}. \end{aligned} \tag{3}$$

where the equality follows from substituting (1) for $\underline{m}$. Now, what is the payoff to the owner of type $t \geq 1$ who makes an offer which is rejected? Then the asset will be taken, which *also* delivers a payoff of $\frac{d}{1-\delta}$. This argument shows that any owner of type $t = 1, 2, ...$ is *indifferent* between offering the minimum acceptable $\underline{m}$ and keeping the asset, or giving up the asset in exchange for damages. In our equilibrium the owner does offer $\underline{m}$ and the offer is accepted.[4] What about an owner of type 0? That owner strictly prefers to offer $\underline{m}$, because then his lifetime payoff is

$$\overline{u} - \underline{m} + \sum_{t=1}^{\infty} \delta^t \cdot [\overline{u} - m^*] \, ,$$

[4] If we choose the opposite tie-breaker for this indifference, we get the same results.

which strictly exceeds (3) and hence is strictly greater than $\frac{d}{1-\delta}$, the payoff from giving up the object and getting damages.

Computation of the bribe m^*

It remains to compute the equilibrium bribe m^*. This equilibrium bribe coincides with $\underline{m}$, the offer made by each owner. Since, by definition, $\underline{m}$ equals (1), we have

$$m^* = \underline{u} - \frac{d}{1-\delta} + \sum_{t=1}^{\infty} \delta^t \cdot [\underline{u} - m^*] .$$

Solving for m^* yields $m^* = \underline{u} - d$. Notice that $m^* = \underline{u} - d$ satisfies (2), which means that it is rational for the taker, if he were to reject the offer, to then exercise his right to take. The original owner enjoys a surplus of $\sum_{t=0}^{\infty} \delta^t \cdot [\overline{u} - m^*] = \frac{\overline{u}-(\underline{u}-d)}{1-\delta}$. This is greater than $\frac{d}{1-\delta}$, which shows that the original owner strictly prefers to bargain rather than to be expropriated. ■

The equilibrium outcome is efficient because the asset remains forever with the original owner, party 0. This suggests that the conventional argument, as presented in the Kaplow and Shavell quote on page 2, must be revisited. However, one wants to be careful and not infer too much from an example. Would efficiency still obtain if we changed the bargaining protocol? Or if we relaxed the assumption that all takers have the same low value $\underline{u}$ for the asset? To answer these questions, we turn to the general model.

4 The Model

4.1 The Economy

Time runs discrete $t = 1, 2, ...T$. All parties discount the future at rate $\delta < 1$. There is a single asset (a durable good) which is owned by party 0 at the beginning of period 1. In each period t a different potential taker shows up and a bargaining game takes place between the beginning-of-period owner and the potential taker which determines who owns the asset in that period. The party who is not the owner at the end of the period exits the game forever. Parties are indexed by the period in which they show up to take. Figure 1 represents the timing.

If party i owns the asset at the end of a period she enjoys a per-period return equal to u_i from owning the asset during that period. Party i's per-period return, u_i, is constant across periods. So, if party 1 owns the asset for three periods then her discounted value from owning

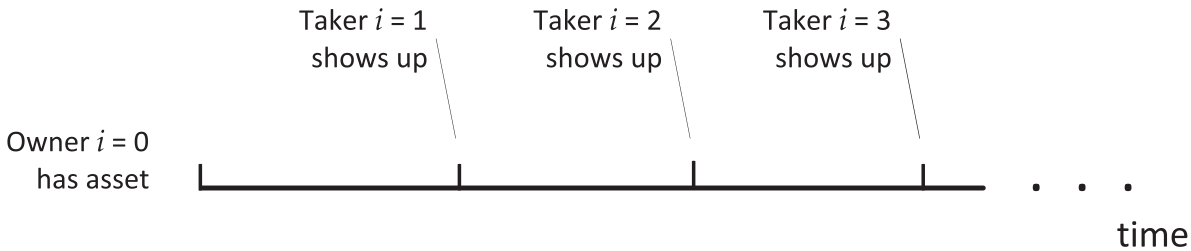

Figure 1: Timeline: in each period, a new taker shows up and bargains with the current owner.

the asset will be $u_1 (1 + \delta + \delta^2)$. The sequence $\{u_t\}_{t=0}^{T}$ of per-period returns, or use values, which encodes the order in which takers with different valuations show up, is a part of the model.

In the last period, T, there are two scenarios. Scenario 1: The world ends after period T or, due to depreciation, the asset looses all value after period T; or Scenario 2: Takers appear for only T periods, which means that whoever owns the asset in period T gets to keep it forever after and enjoy the associated stream of benefits, discounted at the appropriate rate. Our analysis will cover both scenarios.

We denote the beginning-of-period owner by i and the period j taker by j. We adopt the notational convention that $i < j$. Therefore, when i and j meet, the first will necessarily be the owner, and the second will be the period j taker. One of the two will, of course, be the owner in period $j + 1$.

Next, we define how entitlements are protected in our model.

4.2 The Law

Entitlements are protected by a "generalized liability rule," which we define as a rule that allows the taker to take the asset as long as he pays the previous owner damages in the amount of $D_{j,i}$ (the taker enjoys a "right to take," if you will).[5] The measure of damages is very general because it is allowed to depend on the identity of the owner (i) and of the taker (j). When damages are very large the "generalized liability rule" coincides with a property rule - a rule that prevents any transfer of the asset without the current owner's consent.

[5] We will use the word "owner" instead of the possibly more precise "possessor."

When damages are very small, property rights are only weakly protected.[6,7]

5 The Solution Concept: Bilateral Bargaining Solution

We want to avoid being tied down to a specific bargaining protocol (say, one where owners make a take it or leave it offer, or one where takers make such offers). So, in what follows we introduce a "broader" solution concept which only encodes some minimal restrictions which "many" bargaining protocols satisfy. This solution concept, which we term *bilateral bargaining solution*, is defined in this section. In Section 5.4 below, and again later in Section 8, we connect this solution concept with the Nash equilibria of a class of full-information bargaining games.[8]

5.1 Bilateral Bargaining Solution: An Example

To build some familiarity with the bilateral bargaining solution, let us analyze a simple example. Consider a one-shot bargaining game between just two players, an owner i and a taker j. This game could entail a take-it-or-leave-it offer made by one of the two players to the other, or some more complicated bargaining protocol. Suppose property rights are strongly protected (as is usually assumed in economics). Given complete information, we expect the outcome to be efficient in many (but not necessarily all) bargaining protocols. Efficiency requires that the asset is traded if and only if $u_i < u_j$. When trade takes place, we

[6] Several special cases, resonating with legal practice, are worth noting: The law generally views damages as compensation for deprived use. If owner i loses an asset to a taker in period j, then this owner loses a stream of discounted per-period use values. In Scenario 1, where the world ends after period T, this stream of use values, and the corresonding damages, equal $D_{j,i} = \sum_{t=j}^{T} \delta^{t-j} u_i$; in Scenario 2, where takers appear for only T periods, the stream of use values, and the corresonding damages, equal $D_i = u_i/(1-\delta)$. Often courts charged with assessing damages cannot observe individual use values; rather they use a common estimate u for all owners' per-period use values. Damages are then $D_j = \sum_{t=j}^{T} \delta^{t-j} u$ in Scenario 1 and $D = u/(1-\delta)$ in Scenario 2. Note that, in this last case, $D_{j,i} \equiv D$ is independent of i and j. In these special cases, damages reflect the hypothetical multi-period use values that were deprived by the taking. In theory, compensatory damages should reflect also lost proceeds from potential future sales of the asset and saved costs of bribes that the owner would need to pay future takers. In reality, however, courts cannot be expected to calculate such ideal compensatory damages. In any event, our main results apply to any general measure of damages $D_{j,i}$, which includes ideal compensatory damages.

[7] One might equivalently recast this legal framework as positing the existence of two entitlements: the owner's entitlement to keep the object, and the taker's entitlement to take it; with both entitlements being protected by a property rule. In this equivalent framework, the level of damages defines the relative strength of these two entitlements.

[8] In Section 9.2 we discuss briefly how our analysis would extend to an environment with private information.

expect a transfer $p_{j,i}$ to be paid to the owner which could be as low as u_i and as high as u_j. Its precise value will depend on the specific bargaining protocol (who makes the first offer, etc.).

The equilibrium outcome of any game within this family of bargaining protocols could be, equivalently, expressed by the following set of conditions.

$$V_{j,i} = \max\{u_j - p_{j,i}, 0\}$$
$$p_{j,i} \geq u_i$$

In this formulation $p_{j,i}$ represents an *exogenously specified* price. Any such price can be interpreted as corresponding to the non-cooperative equilibrium outcome under a specific bargaining protocol. $V_{j,i}$ is endogenous, and can be interpreted as the value to agent j of playing the non-cooperative game. The inequality requires the price to be high enough that agent i wants to sell (recall that we are now assuming strong property rights). If $p_{j,i}$ is set too high then $V_{j,i}$ is zero, i.e., agent j does not buy the object. Thus no-trade outcomes can be captured in this formulation. If we seek to capture an efficient outcome, then when $u_i < u_j$ we can choose a price $p_{j,i}$ in the interval (u_i, u_j), which ensures that $V_{j,i} > 0$, meaning that agent j ends up with the object. If instead $u_i \geq u_j$, then the prices $p_{j,i}$ that solve the above conditions have the property that the taker's value must be zero (no trade). In either case, prices exist that support the efficient outcome. More broadly, this example illustrates how the outcomes of many possible bargaining games can be captured using these conditions.

These two conditions are a version of a bilateral bargaining solution. Let us now define this solution concept within our more complex bargaining environment.

5.2 Bilateral Bargaining Solution: Definition

When taker j meets owner i three outcomes are possible:

(a) Taker j takes and pays the legally stipulated damages $D_{j,i}$;

(b) Taker j purchases at a price of $p_{j,i} \geq 0$, which only makes sense if damages are very high, that is, if there is a price $p_{j,i} < D_{j,i}$ which the owner will accept (e.g., because the taker would rather walk away than pay the high damages);

(c) Taker j walks away in exchange for $m_{j,i} \geq 0$.(We call this payment a bribe but it is, in fact, the legally enforceable price of the right to take.)

We formulate these three outcomes as mutually exclusive, and there is no loss of generality in this stipulation.[9]

The reader might wonder about the presence of option (b). When might a taker wish to purchase the object instead of taking it? This will happen when damages are very high, which means we are in a world of strong property rights.

Based on these three outcomes let us define a bilateral bargaining solution.

Fix $p_{j,i}$ and $m_{j,i}$. Denote by $V_{j,i}$ taker j's value at the beginning of period j, given that the asset is held by party i. The taker's value is

$$V_{j,i} = \max\left\{u_j + \delta V_{j,j+1} - \min\left[D_{j,i}, p_{j,i}\right], m_{j,i}\right\}. \tag{4}$$

The value of the taking party at the beginning of period j is the larger of (1) the benefit from consuming the asset in period j (u_j), minus the cost of either taking ($D_{j,i}$) or purchasing ($p_{j,i}$) the object, whichever is cheaper, plus the value of being an owner at the beginning of period $j+1$, and facing taker $j+1$ ($V_{j,j+1}$)[10]; and (2) the value of the bribe $m_{j,i}$ received from party i. The *max* operator expresses the notion that, if the bribe is too small then the taker does not have to accept it and can choose to take instead.

The owner's value is given by

$$V_{i,j} = \begin{cases} u_i + \delta V_{i,j+1} - m_{j,i} & \text{if } V_{j,i} = m_{j,i} \\ \min\left[D_{j,i}, p_{ji}\right] & \text{otherwise} \end{cases} \tag{5}$$

This recursive formulation says that the value of party i owning the asset at the beginning of period j is either the value of consuming in period j and continuing as an owner for one more period, minus the bribe paid to party j; or else the damages if expropriated or the price if the asset is traded. The fact that the i's value depends on j's simply expresses the feasibility constraint in the economy: if i keeps the asset then j does not acquire it, and vice versa.

We now spell out conditions on $p_{j,i}$ and $m_{j,i}$ which we expect should be met in many bargaining games. First,

$$p_{j,i} \geq u_i + \delta V_{i,j+1}. \tag{6}$$

This constraint says that if the object is sold (as opposed to taken), the owner must be willing to sell.

[9] Indeed, (a) and (c) cannot be jointly effected since they conflict in the allocation of the object. The same goes for (b) and (c). And one of (a) or (b) is redundant for the taker.

[10] According to our convention $j > i$ and so there is no ambiguity in denoting by $V_{j,i}$ the taker's value and $V_{i,j}$ the owner's value at the beginning of period j. For example, $V_{2,4}$ denotes the value of owner 2 in period 4, and $V_{4,2}$ the value of taker 4 in the same period, when facing owner 2.

The second condition we impose on $p_{j,i}$ and $m_{j,i}$ is this:

$$u_i + \delta V_{i,j+1} - m_{j,i} \geq \min\left[D_{j,i}, p_{j,i}\right].$$

This condition must hold because $m_{j,i}$ represents the bribe which owner i is willing to pay taker j to go away, and so owner i must prefer this option to the alternative which is $\min\left[D_{j,i}, p_{j,i}\right]$. This condition can be rewritten as

$$m_{j,i} \leq u_i + \delta V_{i,j+1} - \min\left[D_{j,i}, p_{j,i}\right]$$

and more precisely stated to avoid the possibility of "reverse bribes" as

$$0 \leq m_{j,i} \leq \max\left\{u_i + \delta V_{i,j+1} - \min\left[D_{j,i}, p_{j,i}\right], 0\right\}. \tag{7}$$

Finally, the terminal condition says that whoever is the owner in period $T+1$ gets:

$$V_{i,T+1} = f\left(u_i\right), \tag{8}$$

where $f(\cdot)$ is any nondecreasing function. This formulation is sufficiently flexible to capture the two scenarios listed in Section 4.1: $f(\cdot) \equiv 0$ captures Scenario 1 in which the world ends in period $T+1$; $f(u) = u/(1-\delta)$ captures Scenario 2 in which the owner in period T gets to keep the asset forever after and enjoy the associated stream of benefits, discounted at the appropriate rate.

Definition 1 *Fix* $\{D_{j,i}\}_{\substack{j=1,\ldots,T \\ i<j}}, \{u_j\}_{j=0}^{T}, \delta$. *A* ***bilateral bargaining solution*** *is a bi-vector of prices and bribes* $\{p_{j,i}, m_{j,i}\}_{\substack{j=1,\ldots,T \\ i<j}}$ *which satisfy conditions (4) through (8). The associated asset allocation is that the asset changes hands at the beginning of period* j *if* $V_{j,i} = u_j + \delta V_{j,j+1} - \min\left[D_{j,i}, p_{j,i}\right]$, *and it does not change hands if* $V_{j,i} = m_{j,i}$.

5.3 Discussion of the Bilateral Bargaining Solution Concept

The word "bilateral" is meant to emphasize the fact that exchange is decentralized. No grand Coasian bargain among all players is possible here.

There is a "price-taking flavor" to conditions (6) and (7), in the sense that we do not write down an explicit bargaining game through which $p_{j,i}$ and $m_{j,i}$ are formed. Along the same lines, note that in a bilateral bargaining solution both "prices," namely $p_{j,i}$ and $m_{j,i}$, are identified in each period. These prices are in addition to the third "price" $D_{j,i}$ which is legally stipulated. Of course, only one out of these three "prices" is actually observed in the equilibrium of any bargaining game. The rest are "out of equilibrium."[11]

Formulation (4) appears to give the taker a lot of bargaing power, by endowing the taker with the operators max and min. But this is not the case; formulation (4) does not pre-determine the allocation of bargaining power. This goes back to the price-taking nature of the solution concept. We, the modeler, retain the freedom of choosing the p's and the m's. Choosing large p's and small m's (compatible with constraints 6 and 7) corresponds to giving owners more bargaining power relative to takers. For example, say damages are set very high and the owner is a tough bargainer who uses the magnitude of damages as a "bargaining chip" in the determination of the price p at which he is willing to trade away the object. We would capture this scenario by setting $p_{j,i}$ close to $D_{j,i}$. In sum, our formulation (4) does not pre-determine the allocation of bargaining power.

5.4 Connection Between Bilateral Bargaining Solution and Subgame Perfect Equilibria of a Family of Dynamic Bargaining Games

Consider a dynamic game in which, in each period t, a bargaining game takes place between the owner and the taker.

Definition 2 *A* ***dynamic bargaining game*** *is a sequence of stage games indexed by the identity of its players, an owner and a taker. Each period's stage game can have one of three distinct outcomes: (a) Taker j takes the asset and pays legally stipulated damages $D_{j,i}$; (b) Taker j purchases the asset at a price of $p_{j,i} \geq 0$; (c) Taker j goes away in exchange for $m_{j,i} \geq 0$. In the first two cases the taker becomes the new owner in the next period's stage game. Otherwise the identity of the owner remains unchanged.*

This definition of a dynamic bargaining game is fairly broad. For example, each period's bargaining protocol is allowed to depend on the identities of the owner and taker: if player 1 is the owner in period 5 then he gets to make a take-it-or-leave-it offer to taker 5. But, if player 2 is the owner in period 5, then taker 5 gets to make the offer. And so on. And,

[11] For example, suppose that in equilibrium the object is taken. Then neither $p_{j,i}$ nor $m_{j,i}$ are paid out; these "prices" are "out of equilibrium" as it were. In this case $p_{j,i}$ and $m_{j,i}$ can be set at arbitrary values which happen to satisfy the equilibrium equations. For example, we may set $p_{j,i} = \infty$ and $m_{j,i} = 0$. Such values, from the viewpoint of price-taking agents, are moot: no taker would buy the object at such a high price $p_{j,i}$, nor would she agree to go away for a bribe of zero. Ergo, the taker must be taking the object in exchange for $D_{j,i}$. This is how the agent's choice in some bargaining game (take the object) is supported in a price-taking environment.

of course, the bargaining protocol in the stage game need not be a take-it-or-leave-it offer either.

We want to focus on bargaining games in which trade is voluntary but subject to the taking rules described in Section 4.2. Voluntariness of the trade can be expressed in terms of the agents' outside options. We define these outside options as follows. (a) If the asset is taken in the equilibrium, then the taker is no worse off than walking away and receiving zero payoff. (b) If the object is traded in the equilibrium, then the taker/buyer is no worse off than he would have been had he taken the asset and paid damages; and also the buyer is no worse off than he would have been had he walked away and received zero payoff; and the owner/seller is no worse off than he would have been if the least favorable of the two following outcomes had materialized - (i) asset is taken and owner/seller gets damages; (ii) owner/seller refuses to sell and keeps the asset. (c) If the taker is bribed away in the equilibrium, then he is no worse off than he would have been had he taken the asset and paid damages; and the owner is no worse off than he would have been if the least favorable of the two following outcomes had materialized - (i) asset is taken and owner/seller gets damages; (ii) owner/seller refuses to sell and keeps the asset.

These outside options express a certain minimal level of protection of one's rights, including the right to take. We now import these outside options into a definition.

Definition 3 *A **consensual** equilibrium is an equilibrium in which a party does not fair worse then her outside option.*

The properties that define a consensual equilibrium seem natural: The equilibria in many bargaining games will be consensual. For example, in a conventional take-it-or-leave-it game in which a buyer makes an offer to a seller (and there is no possibility of taking), the buyer would never offer more than the object is worth to her, and the seller would never accept less than her valuation of the object. Therefore, the equilibrium in such a game is consensual. The equilibrium may fail to be consensual if there is uncertainty about the outcome of the bargaining game. This might happen either due to protocol specifications which create random outcomes, or if equilibrium strategies are mixed. Also, the equilibrium may not be consensual if the bargaining rules do not allow for the protection of rights.

The reader may wonder why we chose to define consensuality of an equilibrium, rather than of a bargaining protocol. The reason is that sometimes the bargaining protocol protects (or undermines) rights "indirectly," for example through the order of moves. Thus, in a take-it-or-leave-it protocol, the first mover (the buyer, say) typically has the freedom to offer a masochistically high price above the value of the asset to him. In equilibrium, however, the buyer chooses not to. Therefore, the equilibrium is consensual even though the rules of the game are not (in that they allow the buyer to select "non-consensual" prices).

Definition 4 *A **Markov-perfect** equilibrium is a subgame-perfect equilibrium of the dynamic game such that equilibrium outcomes in period t do not depend on past actions except through the identity of the owner.*

In a Markov-perfect equilibrium the outcome at time t cannot depend on the size of transfers paid in the past. This restriction can be justified, for example, if the past history of play is not observed by the taker.[12]

The next proposition is the main point of this section: it expresses the connection between the new notion of "bilateral bargaining solution" and Nash equilibria of a class of dynamic games.

Proposition 2 *Every Markov-perfect, consensual equilibrium outcome of any dynamic bargaining game can be supported as a bilateral bargaining solution. And, every bilateral bargaining solution can be supported as a Markov-perfect, consensual equilibrium outcome of some dynamic bargaining game.*

Proof. See the Appendix. ■

This proposition explains our focus on the bilateral bargaining solution.

6 Existence of an Efficient Bilateral Bargaining Solution

We want to see if we can expect the efficient allocation to arise (a first welfare theorem-type result) in a bilateral bargaining environment with weak property rights (i.e., when D is small).

Definition 5 ***(Welfare criterion)*** *An efficient asset allocation is one in which the period-j taker consumes the asset in period j if and only if his valuation exceeds that of the beginning-of-period owner.*

The efficient allocation is for the asset to be owned by the party with the highest per-period value among those who have shown up so far. In other words, when $i < j$ is the current

[12] Markov-perfection does not, however, rule out the possibility that the equilibrium outcome may depend on the owner's identity. This could happen, for example, if there are multiple equilibria in the stage game, and the equilibria are selected based on the owner's identity.

owner and j shows up as the taker, efficiency requires that the asset change hands if and only if $u_j > u_i$.

The following assumption is required for the existence of an efficient solution.

Assumption 1 *For all takers j, and for any two owners $h, i < j$ such that $u_i > u_h$ it must be $D_{j,i} - D_{j,h} > u_h - u_i$.*

This is a mild assumption. It says, roughly, that damages cannot be "too negatively" correlated with owner's valuation. Note that this assumption places no restrictions on whether damages grow or shrink over time (formally: adding the same j-dependent number K_j to $D_{j,i}$ and $D_{j,h}$ leaves the inequality in Assumption 1 unaffected because the K_j's cancel out).[13]

The next lemma establishes the following key technical result: After fixing continuation values $V_{i,j+1}$ which satisfy a certain monotonicity **property**, we can find prices and bribes at time j which are consistent with a solution and are efficient in period j (part a of the lemma). Moreover, some of these prices and bribes (those in part b of the lemma) induce one-period-back continuation values $V_{i,j}$ which also satisfy the monotonicity property (part c of the lemma).

Lemma 1 *Fix j, $\{D_{j,i}\}_{i<j}, \{V_{i,j+1}\}_{i<j+1}$. Assume Assumption 1 holds. Assume that the quantity $u_i + \delta V_{i,j+1}$ is nondecreasing in u_i over all $i < j+1$. Then:*

(a) There exists at least one bi-vector $\{p_{j,i}, m_{j,i}\}_{i<j}$ which solves (6) and (7), and whose asset allocation is efficient.

(b) Among all bi-vectors identified in part (a) there exists at least one bi-vector, which satisfies the following "monotonic selection" condition: $p_{j,i}$ is nondecreasing in u_i for all $u_i < u_j$ and $u_i - m_{j,i}$ is nondecreasing in u_i for all $u_i > u_j$.

(c) All monotonically selected bi-vectors give rise to $u_i + \delta V_{i,j}$ (defined by 4 and 5) which is strictly increasing in u_i over all $i < j$.

Proof. See the Appendix. ■

[13] Assumption 1 is satisfied in all the special cases discussed in footnote 6. In particular, if the court can observe individual use values, and $u_h < u_i$, then in Scenario 1 we have $D_i - D_h = \sum_{t=j}^{T} \delta^{t-j} u_i - \sum_{t=j}^{T} \delta^{t-j} u_h = \sum_{t=j}^{T} \delta^{t-j} (u_i - u_h) > u_h - u_i$; and in Scenario 2 we have $D_i - D_h = (u_i - u_h)/(1-\delta) > u_h - u_i$. If the court cannot observe individual use values and uses a common estimate u for all owners' per-period use values, then in both Scenario 1 and Scenario 2 $D_{j,i} - D_{j,h} = 0 > u_h - u_i$.

Lemma 1 sets up a backward induction argument. If, in the last period, $u_i + \delta V_{i,T+1}$ is nondecreasing in u_i then the lemma affords an induction to all previous periods. Efficiency propagates backwards from period j to period $j-1$ through a choice of monotonically selected prices and bribes $\{p_{j,i}, m_{j,i}\}_{i<j}$. This induction argument proves the existence of an efficient solution.

Theorem 1 ***(Welfare Theorem, ver. 1)*** *Assume Assumption 1 holds. An efficient bilateral bargaining solution exists. Every monotonically selected set* $\{p_{j,i}, m_{j,i}\}_{\substack{i<j \\ j=1,\ldots,T}}$ *which solves (6) and (7) is an efficient bilateral bargaining solution.*

Proof. See the Appendix. ■

The construction in Theorem 1 yields a solution which is efficient not only on the "equilibrium path," but also off equilibrium. What this means is the following. Suppose, for example, that $u_3 < u_4$ and so agent 3 should not be the owner in or after period 4 on the efficient equilibrium path. Suppose nevertheless that agent 3 finds herself the owner in period 5. Then in our solution the efficient allocation of property rights will ensue from period 5 on (efficient conditional on agent 4 not being retrievable, of course). Off-equilibrium path efficiency is usually considered an attractive feature, partly because it shows that efficiency is not sustained by the threat of inefficient punishments. This property is thought to make the equilibrium (or solution, in our case) resistant to renegotiation.

We close this section with an example showing that Assumption 1 cannot be dispensed with.

Example 1 ***(Inefficiency when Assumption 1 fails)*** *Let* $u_0 = 10$, $u_1 = 9$ *and* $u_2 = 5$. *Assume no discounting* ($\delta = 1$)*. Also assume that we are in Scenario 1, namely, the world ends after period 2. The court sets damages* $D_{2,0} = 0$ *and* $D_{2,1} = 7$*. Note that these damages fail Assumption 1; the property rights of agent 0 (the high valuation agent) are weaker than those of agent 1 (the low valuation agent). Given these damages, in any bilateral bargaining solution, in the period-2 subgame in which agent 0 is the owner, owner 0 will be expropriated with no compensation. In the period-2 subgame in which agent 1 is the owner, owner 1 will keep the object and not have to pay taker 2 anything* ($m_{2,1} = 0$)*. This means that, in the period-1 bargaining between owner 0 and taker 1, the former has value 10 from keeping the object, whereas taker 1 has value 18 if he obtains possession of the asset. Hence the object will be either taken or sold to taker 1, depending on the level of* $D_{1,0}$*. In any case the outcome is inefficient.*

7 The Role of Monotonic Selection in Obtaining Efficiency

What is monotonic selection, and why is it important for efficiency? Monotonic selection, intuitively, guarantees that agents with higher valuation for the asset are not treated much worse in the bargaining than agents with lower valuation. This concern would arise, for example, if, in a series of take-it-or-leave-it offer bargaining games, high valuation agents were systematically relegated to "second mover" status, regardless of whether they are owners or takers. In this case high valuation agents would be fully expropriated and would therefore have little incentive to gain control of the asset. This disincentive works against efficiency.

Slightly more formally, and using the language of our bargaining solution: consider an owner i facing a taker j, and suppose $u_i > u_j$. Efficiency requires that i continue to be the owner for at least one more period. If she does, then she receives $u_i + \delta V_{i,j+1}$ (gross of present-period side payments). If j becomes the owner, then he receives $u_j + \delta V_{j,j+1}$ (again gross of side payments). Monotonic selection helps ensure that the first expression is greater than the second. If this property does not hold, then it would be possible for j to experience a greater net present value from taking over the asset, as compared to the current (and efficient) owner i. And so it would be difficult, in a bargaining environment, to prevent j from becoming the owner. This would be inefficient.

Monotonic selection, therefore, is not merely a technical property; rather, it is substantively linked to exchange efficiency. One way to think about monotonic selection is that it guarantees a positive (or, not too negative) correlation between the agents' valuations and their bargaining powers. Similarly, Assumption 1 imposes a condition on the correlation between the agents' valuations and the strength of their property rights. Intuitively, Theorem 1 says that when these correlations are both positive, then efficiency prevails. More precisely, the theorem gives sufficient conditions on the correlation between the agents' valuations and the strength of their property rights, such that it is possible to find (read: monotonically select) an allocation of bargaining powers across agents, which supports efficiency.[14]

The next example illustrates that there can be inefficient bargaining solutions when monotonic selection fails.

Example 2 ***(Inefficiency when monotonic selection fails)*** *Let* $u_0 = 10$, $u_1 = 9$ *and*

[14] Another interpretation of non-monotone selection concerns vulnerability to a taking. Agents can be heterogeneous in their vulnerability to a taking. For example, some agents may keep their assets in more secure locations or invest more in anti-taking security systems. Heterogeneous vulnerability to a taking can result in violation of the monotonic selection condition.

$u_2 = 5$. *Assume no discounting* ($\delta = 1$). *Also assume that we are in Scenario 1, namely, the world ends after period 2. The court, in setting damages, wishes to compensate the owner, whose asset was taken, for lost use value. The court, however, does not observe individual use values; rather it applies a common estimate, in this example a gross underestimate,* $u = 2$. *This means that if the asset is taken in period 1, damages will be* $D_1 = 2 + 2 = 4$, *since the owner loses two periods worth of use; and if the asset is taken in period 2, damages will be* $D_2 = 2$ *(regardless of whether the begining-of-period owner is agent 0 or agent 1).*

For a solution to be efficient in period 2, whoever owns the object at the beginning of period 2 – agent 0 or agent 1 – keeps it. This requires bribing away taker 2. Consider the following allocation of bargaining powers: agent 0 has no bargaining power vis-a-vis agent 2, whereas agent 1 has full bargaining power vis-a-vis agent 2. As we shall see, this allocation of bargaining powers violates monotonic selection, resulting in inefficiency.

Let us first look at the interaction between owner $i = 0$ *and taker* $j = 2$. *This interaction takes place in the subgame which arises after agent 0 retains possession of the asset in period 1. The set of quantities which are efficient and compatible with a solution (i.e., satisfy 6, 7) are found in the proof of Lemma 1. The left-hand column in the next table reproduces this set in the case where* $u_i > u_j$; *the right hand column shows the quantities that obtain given the allocation of bargaining powers that we chose.*

Bilateral bargaining solution correspondence	*Selection for our example*
$p_{j,i} \in [u_i + \delta V_{i,j+1}, \infty)$	$p_{2,0} = \infty$
$m_{j,i} \in [\max[0, u_j + \delta V_{j,j+1} - D], \max[0, u_i + \delta V_{i,j+1} - D]]$	$m_{2,0} = u_i + \delta V_{i,j+1} - D = 10 - 2 = 8$
$V_{i,j} = u_i + \delta V_{i,j+1} - m_{j,i}$	$V_{0,2} = 10 - 8 = 2$

Note that we have used $V_{i,j+1} = 0$ *to translate the left-hand (LH) column into the right-hand (RH) column. This equality holds since* $j + 1$ *is the last period in this example. Let us discuss our selection in the RH column, from the set of bilateral bargaining solutions in the LH column. Our selection of* $p_{2,0}$ *is immaterial because the object is not sold. Our selection of* $m_{2,0}$, *in contrast, is critical because agent 2 would be bribed in this subgame. We selected the highest level of* $m_{2,0}$ *compatible with a bargaining solution. This choice corresponds to agent 2 having all the bargaining power vis-a-vis agent 0.*

Next let us consider the interaction between owner $i = 1$ *and taker* $j = 2$. *This interaction takes place in the subgame which arises after agent 1 takes from agent 0 in period 1. As before,* $u_i > u_j$ *and the bargaining solution quantities are presented in the following table.*

Bilateral bargaining solution correspondence	*Selection for our example*
$p_{j,i} \in [u_i + \delta V_{i,j+1}, \infty)$	$p_{2,1} = \infty$
$m_{j,i} \in [\max[0, u_j + \delta V_{j,j+1} - D], \max[0, u_i + \delta V_{i,j+1} - D]]$	$m_{2,1} = u_j + \delta V_{j,j+1} - D = 5 - 2 = 3$
$V_{i,j} = u_i + \delta V_{i,j+1} - m_{j,i}$	$V_{1,2} = V_{i,j} = 9 - 3 = 6$

Note that we have used $V_{j,j+1} = 0$ to translate the LH column into the RH column. Let us discuss our selection in the RH column. Again the selection of $p_{2,1}$ is immaterial. For $m_{2,1}$, in contrast with the previous case, we select the lowest level of $m_{2,1}$ compatible with a bargaining solution. This choice corresponds to agent 1 having all the bargaining power vis-a-vis agent 2.

Now let's move up one period. For the outcome to be efficient in period 1, agent 1 must be bribed away. For agent 1 this means giving up $u_1 - D + V_{1,2} = 9 - 4 + 6 = 11$. So agent 0 must bribe agent 1 in the amount of at least 11. But how much does agent 0 make if he remains the owner, gross of the bribe? He makes $u_0 + V_{0,2} = 10 + 2 = 12$. And if agent 1 takes, then agent 0 makes 4. So the maximum bribe that agent 0 is willing to pay is $12 - 4 = 8$, which is short of the 11 needed to sway the taker. As a result, agent 1 takes, and the bilateral bargaining solution outcome is inefficient, regardless of agent 0's bargaining power vis-a-vis agent 1.

This example illustrates that there can be inefficiencies if bargaining power is not positively correlated with valuation. In the example, agent 0 has a higher valuation but less bargaining power than agent 1. Formally, the problem is that monotone selection is violated: even though $u_0 > u_1$, we have

$$u_0 - m_{2,0} = 10 - 8 < 9 - 3 = u_1 - m_{2,1}.$$

The feature that interferes with efficiency in the example above can be interpreted, in a noncooperative bargaining game, as a target-specific right to make offers. It is well known that in bargaining games the right to make offers usually confers bargaining power. Our example corresponds to a non-cooperative bargaing game which gives agent 2 the right to make a take-it-or-leave-it offer to agent 0, but not to agent 1. Specifically, agent 2 makes the following offer to agent 0: if you give me $m_{2,0} = 8$, then I will go away. If you give me anything less than 8, I will take. Obviously 8 is the absolute maximum that agent 0 is willing to pay not to be expropriated. Notice that the right to make the offer is valuable to agent 2 (who otherwise might only be able to guarantee himself a payoff of 3 by taking). Conversely, when agent 2 meets agent 1, it is agent 1 who has the right to make the offer to agent 2 (which is why $m_{2,1}$ is so small, in fact equal to agent 2's outside option). The bottom line is that the monotonic selection condition can be violated in a non-cooperative bargaining game when the bargaining protocol favors low-valuation agents.

Although Example 2 assumes a liability rule with low damages, it is the violation of monotonic selection which is responsible for the inefficient outcome, not the specific legal rule. Example 3 in the appendix demonstrates this. In that example possession is afforded property rule protection (i.e., $D = \infty$), and still the violation of monotonic selection leads to inefficiency.

8 Implementation of an Efficient Bilateral Bargaining Solution

The discussion in the previous section suggests that, when Assumption 1 is only marginally satisfied and the property rights of high-valuation agents are relatively unprotected, efficiency is at risk. In such cases, monotone selection compensates by giving high valuation agents enough surplus in the bargaining, such that they are willing to gain control of the asset. Clearly, in such borderline cases monotone selection will have to work hard to achieve the efficient outcome. Translating this idea into a dynamic bargaining game, this means that the bargaining protocols must be stretched significantly in favor of high-valuation agents. This stretching may manifest itself, in a non-cooperative bargaining game, in intuitively implausible bargaining protocols. In this section, we present a sufficient condition, stronger than Assumption 1, such that efficiency can be achieved via a very reasonable bargaining protocol.

Assumption 2 *For all takers j, and for any two owners $h, i < j$ such that $u_i > u_h$ it must be $D_{j,i} \geq D_{j,h}$.*

This assumption, though a strengthening of Assumption 1, is still reasonably mild. It says that damages are non-decreasing in use value. Under this condition, for any given pair of weights $\alpha_1, \alpha_2 \in [0, 1]$, which capture bargaining powers, we will show that the following quantities - equilibrium quantities in a reasonable non-cooperative bargaining game - constitute an efficient bilateral bargaining solution.

Case $u_i < u_j$	$\widehat{p}_{j,i} = \alpha_1 [u_i + \delta V_{i,j+1}] + (1 - \alpha_1) [u_j + \delta V_{j,j+1}]$ $m_{j,i} \in [0, \max\{0, u_i + \delta V_{i,j+1} - D_{j,i}\}]$ $V_{i,j} = \min[D_{j,i}, \widehat{p}_{j,i}]$
Case $u_i > u_j$	$p_{j,i} \in [u_i + \delta V_{i,j+1}, \infty)$ $\widehat{m}_{j,i} = \alpha_2 \max[0, u_j + \delta V_{j,j+1} - D_{j,i}] + (1 - \alpha_2) \max[0, u_i + \delta V_{i,j+1} - D_{j,i}]$ $V_{i,j} = u_i + \delta V_{i,j+1} - \widehat{m}_{j,i}$

Table 1: Equilibrium Quantities

Why do we say that the transfers in Table 1 are "plausible"? These transfers, denoted by $\widehat{p}_{j,i}, \widehat{m}_{j,i}$, are convex combinations of quantities which express the possessory value for the two parties. For example, $\widehat{p}_{j,i}$ is a convex combination of $u_i + \delta V_{i,j+1}$ and $u_j + \delta V_{j,j+1}$, with weights α_1 and $1 - \alpha_1$. This means that if the asset is bought rather than taken (this can happen when damages are very large) and α_1 is large, then the owner gets paid a relatively low price for the asset, close to his point of indifference. The weight α_1 measures taker j's bargaining power in the negotiation that determines the price at which the object is sold, if not taken. Similarly, α_2 measures owner i's bargaining power in the negotiation that determines the bribe required for taker j to go away. The expression for $\widehat{m}_{j,i}$ can be read approximately as saying that $\widehat{m}_{j,i} + D_{j,i}$ is a convex combination of $u_j + \delta V_{j,j+1}$ and $u_i + \delta V_{i,j+1}$, with weights α_2 and $1 - \alpha_2$. Here $\widehat{m}_{j,i} + D_{j,i}$ represents the money transfer that takes place when the taker is bribed away. When α_2 is large this monetary transfer is small.

The noteworthy feature of Table 1 is that the weights α_1 and α_2 are constant over time, which means that there is no weird shift in bargaining power over time. The following theorem states that the quantities in Table 1 give rise to an efficient bilateral bargaining solution. This result shows that an efficient bargaining solution can be implemented with a reasonable bargaining protocol.

Theorem 2 ***(Welfare Theorem, ver. 2)*** *Assume Assumption 2 holds. Fix any* $\alpha_1, \alpha_2 \in [0, 1]$. *The set of prices and bribes in Table 1 is an efficient bilateral bargaining solution.*

Proof. In the Appendix. ■

Summing up, the quantities in Table 1 represent a "plain vanilla" bargaining outcome. This theorem tells us that if property rights are well-behaved (i.e., satisfy Assumption 2), then efficiency can be sustained through reasonable bargaining protocols. If we set $\alpha_1 = \alpha_2 = 1/2$ then we can think of the bargaining outcomes $\widehat{p}_{j,i}, \widehat{m}_{j,i}$ as arising from an alternating offers game a' la Rubinstein (1982) which is played in period j between owner i and taker j.

9 Discussion

9.1 Possessory Regimes as Incentives Schemes

We have shown that, under reasonable conditions, a broad range of generalized liability rules can support exchange efficiency. In other words, the common property rule (captured by a very large D) is just one of many legal rules that can support efficiency at the exchange phase. Accordingly, in justifying the prevalence of the property rule or, more generally, in studying the relative efficiency of different legal rules, the focus may properly shift towards a pre-exchange, investment phase. One might believe that property rules, as opposed to liability rules, provide better incentives to invest in developing the asset prior to the exchange phase. This, however, is not necessarily the case.

Why are property rules believed to induce investment? The basic intuition is that the stronger protection afforded by property rules allows the owner to enjoy the benefits of her investment - by using the asset herself or by selling it at a higher price. Weaker protetction, on the other hand, implies a higher probability of expropriation, which provides a disincentive to invest in the asset. And there are additional arguments for why property rules most efficiently promote ex ante investments. (See Kaplow and Shavell 1996; Bar-Gill and Bebchuk 2010.) But there are other arguments suggesting that property rules can be inferior to liability rules in terms of ex ante investment efficiency. The intuition supporting the investment efficiency of property rules focuses on investments made by the current owner. But potential takers can also make investments that would increase the value of the asset, post-taking. Such investments are especially important in environments, where exchange efficiency requires that the asset change hands often. Liability rules can be better than property rules in inducing investments by potential takers.

More generally, once we recognize the bilateral nature of the investment problem - that both the current owner and the potential taker can invest - it is obvious that property rules can rarely induce optimal investments. The problem is analogous to the hold-up problem studied in the contract theory literature. That literature explores the relative efficiency of allocating property rights to one party or the other. The basic insight is that the party who gets the property right will invest more, while the party who does not get the property right will invest less. (See, e.g., Hart 1995) The contract theory literature, however, (implicitly) assumes that the property rights to be allocated must be protetcted by property rules (i.e., $D = \infty$). Investment efficiency can be improved, when the allocated property rights are protected by liability rules. When property rights are protected by property rules, the transfer in the bargaining game between the current owner and the potential taker will be a function of the asset's value to the parties. The resulting hold-up problem dilutes incentives to invest in increasing the value of the asset. Appendix B, which is not for publication, develops a simple example, where efficient investments obtain under a liability rule, but not under a property rule. Under a liability rule, if we fix damages at a level that is independent of the parties' investments, we can get a lump-sum transfer that does not distort incentives.[15]

The bottom line is that shifting one's focus to investment efficiency need not vindicate property rules as necessarily superior to other rules.

9.2 Transaction Costs and Asymmetric Information

Our analysis implicitly assumes that bargaining is frictionless. But, of course, transaction costs and, specifically, asymmetric information can impede upon successful bargaining. We have shown that liability rules can be as efficient as property rules in a zero transaction

[15] The idea that lump sum transfers may sometimes help induce efficient investment is not new, of course. See, for example, Aghion *et al.* (1994).

costs framework. Do property rules have an advantage when positive transaction costs are introduced? The answer is far from clear. Positive transaction costs can prevent the asset from changing hands through bargaining. In a property rule system, where asset transfers occur only through bargaining, efficient transactions will be prevented. Liability rules allow the asset to change hands without bargaining, through unilateral takings. With ideal compensatory damages - damages equal to the full value of the asset to the owner - a liability regime induces efficient takings and only efficient takings, and is therefore superior to the property regime. If damages are undercompensatory, then a liability regime enables both efficient and inefficient transfers. (See Calabresi and Melamed, 1973; Kaplow and Shavell, 1996)[16]

9.3 Takers Appear Simultaneously

In our framework, takers appear sequentially. What happens if multiple takers appear simultaneously, in the same period? Before directly addressing this question, we note that the lenght of a period is not pre-defined in our model. Our analysis and results apply for arbitrarily short periods.[17] But it does not apply when multiple takers appear in the very same moment. It would be interesting to explore how our setup can be adapted to account for the possibility that multiple takers can try, all at once, to unilaterally take the asset in exchange for a court-determined price, D. Notice, however, that an asset cannot be taken (or held) by more than one person in a single period. So there is a conceptual difficulty in defining the "right" to take. However this difficulty is resolved, we expect that in this environment, as well, efficiency should prevail. When mutliple takers appear simultaneously, and some value the asset more than the beginning-of-period owner, it is natural to think of the owner as auctioning the asset to the multiple takers. Perhaps high-value takers would be willing to pay the owner a price exceeding D, if the owner can help them prevail in the auction. Or perhaps the takers would contract with each other in a cartel-like fashion, suggesting again that the highest-value agent should end up with the asset. If, on the other

[16] Kaplow and Shavell (1996) argue that asymmetric information leads to inefficient outcomes under both property rules and liability rules and conclude: "it may be that either rule is better" (p. 764). Kaplow and Shavell add that if current owners are assumed to enjoy higher values such that asset transfers are rarely efficient, then property rules would be superior, since the cost of failed bargaining, due to asymmetric information, would be small. Under liability rules, bargaining would be needed also to prevent inefficient transfers, so the cost of failed bargaining would be larger.

There is a debate in the literature about whether liability rules facilitate bargaining in the presence of asymmetric information. Compare: Ayres and Talley (1995) to Kaplow and Shavell (1995).

[17] The short period problem raises another concern: Consider a high-value owner who retains possession of the asset across many short periods, but must continuously bargain, and bribe, takers. Can this owner enjoy the asset while (continuously) bargaining with takers? If the answer is no, then our analysis would not apply.

hand, the beginning-of-period owner values the object more than the takers, then the owner might bribe mutliple takers. This expense should be affordable because the magnitude of each bribe will be discounted by the probability that the particular taker will win the takers' competition. In either case, we conjecture that there are strong forces leading to exchange efficiency.

9.4 Reciprocal Takings and Weakly Enforceable Contracts

In our framework, an agent who lost the asset, or failed to gain possession in the first place, exits the game. But this need not be the case. In particular, Kaplow and Shavell (1996) consider also the reciprocal takings case, where, after agent 1 unilaterally takes the asset from agent 0, agent 0 can, in the next period, unilaterally take the asset from agent 1. This process of potentially reciprocal takings is allowed to take place "ad infinitum."

The reciprocal takings setup can also capture an environment in which "weakly enforceable" contracts have a limited power to bind the parties who sign them. Here is what we mean. In our analysis, we assumed that agents can write fully enforceable side contracts. For example, a taker can write a contract which commits her irrevocably to give up the right to take, in exchange for money (a bribe). But in some environments such contracts are only weakly enforced. In particular, we may think of an environment in which, shortly after the contract is signed, its enforceability ceases. In this case, the taker "does not stay bribed" and may show up again. Thus the "weak enforceability" formulation captures the reciprocal takings case.

While a full analysis of the reciprocal takings case (with two or more agents) is beyond the scope of this paper, we note that, once again, there is no reason to believe that liability rules will necessarily lead to inefficient outcomes. Consider a two-player game with a high-value agent 0 and a low-value agent 1. Under a liability rule, agent 0 would bribe agent 1 to prevent an inefficient taking. The magnitude of this bribe will be small, reflecting the understanding that agent 1, if he takes the asset in period 1, will lose it to agent 0 in period 2. (Recall, a similar effect reduces the magnitude of the bribe in the multiple takers case that we study - there agent 1 understands that he might lose the asset to agent 2 in the next period, or pay a bribe to agent 2, and thus settles for a lower bribe from agent 0 in the first period.) When the bribe is sufficiently small, agent 0 would be willing to pay it, and the asset would remain with the high-value agent 0, as is efficient.

9.5 Liability Rules in the Legal System

Most forms of interference with property rights that fall short of dispossessing the owner are protetcted by liability rules. For example, the right to enjoy one's asset free of pollution and other nuisances is often afforded only liability rule protection. Contractual rights are also commonly protetcted by liability rules. See Kaplow and Shavell (1996).

In the increasingly important domain of intellectual property, liability rule protetction is even more common. Copyright law includes eight different compulsory licensing regimes, which are prime examples of liability rules. For instance, under Section 114 of the Copyright Act, webcasters (online radio stations) can publicly perform songs without obtaining prior consent from the song's creator, as long as they pay the statutory fee (currently about 20 cents for every listener that the webcaster has). And under Section 115 of the Copyright Act, if a song has been released, anyone can make, and sell, a cover version, i.e., re-record the song with another performing artist, as long as they pay the copyright owner the statutory fee (currently about 10 cents per copy sold). The fair use doctrine, which excuses certain copyright infringements, can also be viewed as establishing a liability rule, with zero damages. More generally, the Supreme Court, in a recent decision, emphasized that the injunction remedy - or property rule protection - is discretionary in the Intellectual Property domain, and noted categories of cases where liability rule protection may be more appropriate.[18]

These examples make the point that liability rules are ubiquitous. The main difference between many of these applications, specifically the intellectual property applications, and our theory is that we have analyzed the allocation of a good that is fully rival (the "asset"). The field of intellectual property studies the allocation of a good which is only partly rival: an idea or technical innovation can be utilized by many agents simultaneously. However, ideas and technical innovations are, *to some degree*, rival. If a competitor uses my idea to produce a substitute for my product, then my market share will go down. In this sense, there is rivalry in the revenue from ideas, too. Our model can then be seen as a polar case in which only one exploiter of the idea can make any money in the market. This is, of course, an extreme assumption. However, making this assumption ties our hands, and thus strengthens our message. If we show that liability rules can be as exchange-efficient as property rules when the asset is fully rival, then one would conjecture that liability rules might be even better when the asset is only partly rival. Although we do not explore this direction, we feel that our analysis represents a useful first step in the formal exploration of exchange efficiency of liability rules. We hope that future work will further explore the connection with actual

[18] On the eight compulasry licenses - see http://www.copyright.gov/licensing/. The fees for the compulsory licenses are updated periodically by the Copyright Royalty Board. See http://www.loc.gov/crb/. The fair use doctrine is codified in 17 U.S.C. 107. The recent Supreme Court decision is: eBay Inc. v. MercExchange, L.L.C., 547 U.S. 388 (2006).

legal rules.

10 Conclusion

We have shown that the first welfare theorem obtains independently of the strength of property rights protection. In an exchange setting, a large class of legal rules (what we called generalized liability rules) are exchange-efficient. Included in this class are property rules (generalized liability rules with very large damages, Ds), standard liability rules (generalized liability rules with Ds that track the owner's valuation), and even rules which afford possessory interests only very weak protection (generalized liability rules with very small Ds). This result corrects a previous misconception in the literature, and yields the provocative conclusion that strong property rights are not required for exchange efficiency.

What matters for exchange efficiency, we find, is not how much or how little the owner's rights are protected. Rather, what matters is how this protection—whatever its level—*correlates* with the agents' valuations for the asset. If this correlation is not (too) negative, then efficiency can prevail. More precisely, in this case there exist allocations of bargaining power or, from the perspective of non-cooperative games, bargaining protocols, which implement the efficient outcome. If the correlation is too negative, that is, if property rights are too punitive (in a relative sense) of high valuation agents, then efficiency cannot obtain.

Property rules (strong protection of ownership rights) emerge somewhat diminished from this analysis. It is natural to want to rescue property rules. One avenue might be to look at investment efficiency. Property rules are uniquely suited to incentivize an owner's investment in the asset to be traded. This, however, does not per se imply that property rules are efficient from an investment viewpoint; indeed, when investments by potential takers (as opposed to owners) are important, liability rules might be superior. Such considerations, we speculate, might be especially relevant in fields such as intellectual property, in which it may be important to encourage investment by non-current-owners of intellectual property.

We conclude that there is little in the theory of pure exchange that robustly ties strong property rights to efficiency. Having thus cleared the grounds, the question remains open: What is the optimal rule for protecting possessory interests? Investment efficiency, or asymmetric information frictions, might provide the answer. Existing work has begun to explore this possibility, but much more remains to be done.

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Appendices

A Proofs

A.1 Proof of Proposition 2

Every Markov-perfect, consensual equilibrium outcome of any dynamic bargaining game can be supported as a bilateral bargaining solution.

Proof. Case A: The equilibrium outcome in a specific dynamic bargaining game at time j is that the object is taken.

Let $V_{j,j+1}$ and $V_{i,j+1}$ represent the continuation payoffs in the dynamic bargaining game, and $V_{i,T+1} = f(u_i)$ in accordance with condition (8). (Notice that, because we restriction attention to Markov equilibria, these continuation values are not a function of the actions taken in period j.) Given these $V_{j,j+1}$ and $V_{i,j+1}$, can we find a pair $\left(p^*_{j,i}, m^*_{j,i}\right)$ that is a bilateral bargaining solution in which the object is taken? Let's see. If the object is taken then the taker's value must be $V_{j,i} = u_j + \delta V_{j,j+1} - D_{j,i}$. The definition of bilateral bargaining solution then requires, from (4), that these two conditions be satisfied:

$$\begin{aligned} D_{j,i} &\leq p^*_{j,i} \\ m^*_{j,i} &\leq u_j + \delta V_{j,j+1} - D_{j,i}. \end{aligned}$$

Setting $p^*_{j,i} = \infty$ satisfies the first condition. As for the second condition, we know that $u_j + \delta V_{j,j+1} - D_{j,i} \geq 0$ because taking is an equilibrium in the dynamic bargaining game (otherwise, the taker could profitably deviate to walking away for free). Therefore setting $m^*_{j,i} = 0$ satisfies the second condition. Therefore, for this choice of $\left(p^*_{j,i}, m^*_{j,i}\right)$ equation (4) tells us that in this bilateral bargaining solution the object is indeed taken. Equation (5) does not place constraints on $\left(p^*_{j,i}, m^*_{j,i}\right)$; it simply pins down the owner's value $V_{i,j}$ directly from equation (4). Equation (6) is automatically verified by $p^*_{j,i} = \infty$. Equation (7) is automatically verified by $m^*_{j,i} = 0$. Therefore, the pair $\left(p^*_{j,i} = \infty, m^*_{j,i} = 0\right)$ is a bilateral bargaining solution in which the object is taken.

Case B: The equilibrium outcome in a specific dynamic bargaining game at time j is that the object is purchased at a price $p^*_{j,i}$.

Let $V_{j,j+1}$ and $V_{i,j+1}$ be given as in Case A. Can we find an $m^*_{j,i}$ such that the pair $\left(p^*_{j,i}, m^*_{j,i}\right)$ is a bilateral bargaining solution in which the object is purchased? Let's see. If the object is purchased then then the taker's value must be $V_{j,i} = u_j + \delta V_{j,j+1} - p^*_{j,i}$. The definition of bilateral bargaining solution then requires, from (4), that these two conditions be satisfied:

$$\begin{aligned} p^*_{j,i} &\leq D_{j,i} \\ m^*_{j,i} &\leq u_j + \delta V_{j,j+1} - p^*_{j,i}. \end{aligned} \tag{9}$$

The first condition must hold because purchasing is an equilibrium in a game in which the taker could take rather than purchase and this choice has no ramifications in the future (Markov equilibria). Therefore, taking must be more expensive than purchasing. As for the second condition, just as in Case A we know that $u_j + \delta V_{j,j+1} - p^*_{j,i} \geq 0$ because purchasing is an equilibrium in the dynamic bargaining game in which the taker could profitably deviate to walking away for free. Therefore setting $m^*_{j,i} = 0$ satisfies the second condition. Therefore, for this pair $\left(p^*_{j,i}, m^*_{j,i}\right)$ equation (4) tells us that in this bilateral bargaining solution the object is indeed purchased. Equation (5) does not place constraints on $\left(p^*_{j,i}, m^*_{j,i}\right)$; it simply pins down the owner's value $V_{i,j}$ directly from equation (4). To check that equation (6) is satisfied, consider that in the dynamic bargaining game the owner could hold out. What could holding out lead to? Either to keeping the object which would leave the owner with $u_i + \delta V_{i,j+1}$; or to being the subject of a taking, which would leave the owner with $D_{j,i}$. Since the owner chooses to sell, her equilibrium payoff cannot be worse than the least appealing option from holding out. Therefore, it must be that

$$\min \{D_{j,i}, u_i + \delta V_{i,j+1}\} \leq p^*_{j,i}. \tag{10}$$

Using (9) this equation yields

$$\min \{D_{j,i}, u_i + \delta V_{i,j+1}\} \leq p^*_{j,i} \leq D_{j,i}.$$

If all weak inequalities hold as equality then we have $p^*_{j,i} = D_{j,i}$, a degenerate case in which the object is bought at a price exactly equal to the damages. In this case the purchase is equivalent to a taking, and so Case A above applies. If at least one inequality holds strictly then it must be that $u_i + \delta V_{i,j+1} < D_{j,i}$, and then (10) reads

$$u_i + \delta V_{i,j+1} \leq p^*_{j,i}$$

which verifies equation (6). Equation (7) is automatically verified by $m^*_{j,i} = 0$. Therefore, the pair $\left(p^*_{j,i}, m^*_{j,i} = 0\right)$ is a bilateral bargaining solution in which the object is purchased at a price $p^*_{j,i}$.

Case C: The equilibrium outcome in a specific dynamic bargaining game at time j is that the taker is bribed away for a bribe $m^*_{j,i} \geq 0$.

Let $V_{j,j+1}$ and $V_{i,j+1}$ be given as in Case A. Can we find a $p^*_{j,i}$ such that the pair $\left(p^*_{j,i}, m^*_{j,i}\right)$ is a bilateral bargaining solution in which the taker is bribed away for a bribe $m^*_{j,i}$? Let's see. If the taker is bribed away then according to the definition of bilateral bargaining solution we have $V_{j,i} = m^*_{j,i}$, which from (4) is equivalent to verifying the equation:

$$m^*_{j,i} \geq u_j + \delta V_{j,j+1} - \min\left[D_{j,i}, p^*_{j,i}\right].$$

Setting $p^*_{j,i} = \infty$ allows us to rewrite this condition as

$$m^*_{j,i} \geq u_j + \delta V_{j,j+1} - D_{j,i}.$$

This condition must be verified because the taker in equilibrium accepts to be bribed away instead of taking. Therefore, for this pair $\left(p^*_{j,i} = \infty, m^*_{j,i}\right)$ equation (4) tells us that in this bilateral bargaining solution the taker is indeed bribed away for a bribe $m^*_{j,i}$. Equation (5) simply pins down the owner's value $V_{i,j}$ directly from equation (4). Equation (6) is automatically verified by $p^*_{j,i} = \infty$. Equation (7) reads

$$0 \leq m^*_{j,i} \leq \max\left\{u_i + \delta V_{i,j+1} - \min\left[D_{j,i}, p^*_{j,i}\right], 0\right\}.$$

The left hand side is satisfied by assumption, because we restrict bribes to be nonnegative. To see that the right hand inequality must also be satisfied, observe that the owner in equilibrium chooses to bribe rather than the alternative, which can be no worse than $\min\left[D_{j,i}, p^*_{j,i}\right]$. Therefore,

$$u_i + \delta V_{i,j+1} - m^*_{j,i} \geq \min\left[D_{j,i}, p^*_{j,i}\right].$$

Rearrange this equation into

$$u_i + \delta V_{i,j+1} - \min\left[D_{j,i}, p^*_{j,i}\right] \geq m^*_{j,i},$$

and the required condition is implied. Therefore, the pair $\left(p^*_{j,i} = \infty, m^*_{j,i}\right)$ is a bilateral bargaining solution in which the taker is bribed away for a bribe $m^*_{j,i}$. ■

Every bilateral bargaining solution can be supported as a Markov-perfect, consensual equilibrium outcome of some dynamic bargaining game.

Proof. Let $\{p_{j,i}, m_{j,i}\}_{\substack{j=1,\ldots,T \\ i<j}}$ be a bargaining solution. The construction of the bargaining game that supports it is as follows. Suppose the bilateral bargaining outcome of the interaction between i and j (this interaction need not take place on the equilibrium path) is that the asset is sold. Then in the dynamic game we will specify a stage game between i and j in which the taker chooses whether to offer $p_{j,i}$, or zero, or take and pay $D_{j,i}$; and the owner chooses whether to accept the price $p_{j,i}$ or reject it and keep the asset (subject to a possible taking). In this case (6) guarantees that the owner will accept the price. Moreover, since the

bargaining solution outcome is that the object is sold, (4) guarantees that the taker prefers to pay $p_{j,i}$ and get the object rather than taking or walking away (payoff of zero). Therefore, the Markov-perfect equilibrium of the stage game we have constructed supports the bilateral bargaining solution.

Suppose the bilateral bargaining outcome of the interaction between i and j is that the asset is taken. Then the same stage game described previously will have a Markov-perfect equilibrium that supports the bilateral bargaining solution.

Suppose the bilateral bargaining outcome of the interaction between i and j is that the taker is bribed away. Then in the dynamic game we will specify a stage game between i and j in which the owner chooses whether to offer $m_{j,i}$, or zero; and the owner chooses whether to: accept the bribe $m_{j,i}$, reject it and take the asset, or reject it and buy the asset at price $p_{j,i}$. In this case (7) guarantees that the owner will prefer to offer the bribe. Moreover, since the bargaining solution outcome is that the taker is bribed away, (4) guarantees that the taker prefers to be bribed away rather than acquiring the asset. Therefore, the Markov-perfect equilibrium of the stage game we have constructed supports the bilateral bargaining solution. ■

A.2 Proof of Lemma 1

We preface the proof with a convenient restatement of the notion of efficiency. Recall that an efficient bilateral bargaining solution is a set $\{p_{j,i}, m_{j,i}\}_{\substack{i<j \\ j=1,\dots,T}}$ with the property that

$$\begin{array}{ll} V_{j,i} = u_j + \delta V_{j,j+1} - \min\left[D_{j,i}, p_{j,i}\right] & \text{if } u_i < u_j \\ V_{j,i} = m_{j,i} & \text{if } u_i \geq u_j \end{array}$$

Using (4) this condition can be written as

$$u_j + \delta V_{j,j+1} - \min\left[D_{j,i}, p_{j,i}\right] \geq m_{j,i} \text{ if } u_i < u_j \tag{11}$$

$$u_j + \delta V_{j,j+1} - \min\left[D_{j,i}, p_{j,i}\right] \leq m_{j,i} \text{ if } u_i > u_j \tag{12}$$

Proof. (a) Let's first characterize the bi-vector $\{p_{j,i}, m_{j,i}\}_{i<j}$ for those values of i such that $u_i < u_j$. Because of the monotonicity assumption the interval $[u_i + \delta V_{i,j+1}, u_j + \delta V_{j,j+1}]$ is nonempty. Choosing any $p_{j,i}$ in this interval guarantees that (6) is satisfied and that

$$u_j + \delta V_{j,j+1} - \min\left[D_{j,i}, p_{j,i}\right] \geq u_j + \delta V_{j,j+1} - p_{j,i} \geq 0. \tag{13}$$

This equation reads like (11) if we set $m_{j,i} = 0$. Therefore the pair $p_{j,i} \in [u_i + \delta V_{i,j+1}, u_j + \delta V_{j,j+1}]$

coupled with $m_{j,i} = 0$ guarantees that (11), (6), and (7) are satisfied. In fact, all pairs that couple any $p_{j,i} \in [u_i + \delta V_{i,j+1}, u_j + \delta V_{j,j+1}]$ with any $m_{j,i} \in [0, \max\{0, u_i + \delta V_{i,j+1} - \min[D_{j,i}, p_{j,i}]\}]$ satisfy (11), (6), and (7). Some simplification can be achieved in the expression for $m_{j,i}$ by noting that, by choice of $p_{j,i}$,

$$u_i + \delta V_{i,j+1} - p_{j,i} \leq 0,$$

which implies that

$$\begin{aligned}
&\max\{0, u_i + \delta V_{i,j+1} - \min[D_{j,i}, p_{j,i}]\} \\
&= \max\{0, u_i + \delta V_{i,j+1} - D_{j,i}, u_i + \delta V_{i,j+1} - p_{j,i}\} \\
&= \max\{0, u_i + \delta V_{i,j+1} - D_{j,i}\}.
\end{aligned}$$

Therefore $m_{j,i} \in [0, \max\{0, u_i + \delta V_{i,j+1} - D_{j,i}\}]$. Any bi-vector $\{p_{j,i}, m_{j,i}\}_{i<j}$ which belongs to the sets identified above is consistent with an efficient solution. In these solutions we have, from (4) and (5),

$$V_{i,j} = \min[D_{j,i}, p_{j,i}].$$

Let's now characterize the bi-vector $\{p_{j,i}, m_{j,i}\}_{i<j}$ for those values of i such that $u_i > u_j$. The monotonicity assumption guarantees that $u_j + \delta V_{j,j+1} \leq u_i + \delta V_{i,j+1}$. Hence there exists a number $m_{j,i}$ with the property that

$$u_j + \delta V_{j,j+1} - \min[D_{j,i}, p_{j,i}] \leq m_{j,i} \leq u_i + \delta V_{i,j+1} - \min[D_{j,i}, p_{j,i}] \tag{14}$$

The leftmost inequality says that (12) is satisfied. The rightmost inequality is very similar to (7). Choosing $p_{j,i} = u_i + \delta V_{i,j+1}$ ensures that the right hand side of (14) is nonnegative which implies that $m_{j,i}$ can be chosen to be nonnegative, thus satisfying (7) exactly. Finally, notice that this choice of $p_{j,i}$ satisfies (6). Thus the bi-vector $p_{j,i} = u_i + \delta V_{i,j+1}$ and $m_{j,i} = \max[0, u_j + \delta V_{j,j+1} - \min[D_{j,i}, p_{j,i}]]$ satisfies (12), (6), and (7). In fact, all pairs $p_{j,i} \in [u_i + \delta V_{i,j+1}, \infty)$ coupled with

$$m_{j,i} \in [\max[0, u_j + \delta V_{j,j+1} - \min[D_{j,i}, p_{j,i}]], \max[0, u_i + \delta V_{i,j+1} - \min[D_{j,i}, p_{j,i}]]]$$

satisfy(12), (6) and (7). Note that because $p_{j,i} \geq u_i + \delta V_{i,j+1} > u_j + \delta V_{j,j+1}$ we have

$$u_j + \delta V_{j,j+1} - p_{j,i} < 0,$$

hence

$$\begin{aligned}
\max[0, u_j + \delta V_{j,j+1} - \min[D_{j,i}, p_{j,i}]] &= \max[0,\ u_j + \delta V_{j,j+1} - p_{j,i},\ u_j + \delta V_{j,j+1} - D_{j,i}] \\
&= \max[0, u_j + \delta V_{j,j+1} - D_{j,i}].
\end{aligned}$$

By the same logic we have

$$\max\left[0, u_i + \delta V_{i,j+1} - \min\left[D_{j,i}, p_{j,i}\right]\right] = \max\left[0, u_i + \delta V_{i,j+1} - D_{j,i}\right].$$

So we can write the set of bi-vectors more compactly as comprising all pairs $p_{j,i} \in [u_i + \delta V_{i,j+1}, \infty)$ coupled with $m_{j,i} \in \left[\max\left[0, u_j + \delta V_{j,j+1} - D_{j,i}\right], \max\left[0, u_i + \delta V_{i,j+1} - D_{j,i}\right]\right]$. In these solutions we have, from 4 and 5,

$$V_{i,j} = u_i + \delta V_{i,j+1} - m_{j,i}.$$

Summing up, the set of prices, bribes, and associated values that solve (11), (12), (6) and (7) is nonempty and is given in the following table.

$u_i < u_j$	$u_i > u_j$
$p_{j,i} \in [u_i + \delta V_{i,j+1}, u_j + \delta V_{j,j+1}]$ $m_{j,i} \in [0, \max\{0, u_i + \delta V_{i,j+1} - D_{j,i}\}]$ $V_{i,j} = \min[D_{j,i}, p_{j,i}]$	$p_{j,i} \in [u_i + \delta V_{i,j+1}, \infty)$ $m_{j,i} \in \left[\max\left[0, u_j + \delta V_{j,j+1} - D_{j,i}\right], \max\left[0, u_i + \delta V_{i,j+1} - D_{j,i}\right]\right]$ $V_{i,j} = u_i + \delta V_{i,j+1} - m_{j,i}$

(b) Let us treat u_i as a continuous variable which we denote by u. The value of u identifies the owner i by its valuation for the asset. Pick a a differentiable function $D_j(\cdot)$ with the property that $D_j(u)$ equals the damages that taker j is required to pay to an owner with value u. Denote further by $V_{j+1}(u)$ the value $V_{i,j+1}$ for the owner i with $u_i = u$, by $p_j(u)$ the price $p_{j,i}$, and by $m_j(u)$ the bribe $m_{j,i}$, when $u_i = u$. Then the bi-vectors identified in part (a) are such that

$$p_j(u) \in \left[u + \delta V_{j+1}(u), u_j + \delta V_{j,j+1}\right] \stackrel{\text{def}}{=} \Gamma(u) \text{ when } u < u_j.$$

and

$$m_j(u) \in \left[\max\left[0, u_j + \delta V_{j,j+1} - D_j(u)\right], \max\left[0, u + \delta V_{j+1}(u) - D_j(u)\right]\right] \stackrel{\text{def}}{=} \Gamma(u) \text{ when } u > u_j.$$

Since by assumption $u + \delta V_{j+1}(u)$ is nondecreasing in u, the image of the correspondence $\Gamma(u)$ is nonempty for all u's. Moreover, when $u < u_j$ the lower bound of the graph of Γ is nondecreasing in u. Therefore we can select from the correspondence $\Gamma(u)$ a function $p_j(u)$ which is nondecreasing when $u < u_j$. Let us now turn to the case $u > u_j$. Assumption 1 guarantees that $\partial D_j(u)/\partial u \geq -1$, hence

$$\frac{\partial}{\partial u}\left[u_j + \delta V_{j,j+1} - D_j(u)\right] \leq 1$$

which means that the lower bound of the graph of Γ never grows at a faster rate than u. It is therefore possible to select from $\Gamma(u)$ a function $m_j(u)$ with the property that $u - m_j(u)$ is nondecreasing in u for $u > u_j$.

(c) We need to check that $u_i + \delta V_{i,j}$ is strictly increasing in u_i. To this end, rewrite the expression for $V_{i,j}$ obtained in part (a) using the notation developed in part (b):

$$V_j(u) = \begin{cases} \min\left[D_j(u), p_j(u)\right] & \text{if} \quad u < u_j, \quad \text{where} \quad p_j(u) \in \Gamma(u) \\ u + \delta V_{j+1}(u) - m_j(u) & \text{if} \quad u > u_j, \quad \text{where} \quad m_j(u) \in \Gamma(u). \end{cases}$$

Let's start with the case $u < u_j$. We have

$$\frac{\partial}{\partial u}\left[u + \delta V_j(u)\right] = 1 + \delta \frac{\partial}{\partial u} \min\left[D_j(u), p_j(u)\right] \tag{15}$$

The bi-vectors selected in part (b) have the property that $p_j(u)$ is nondecreasing in u, so $\partial p_j(u)/\partial u \geq 0$. Also, Assumption 1 guarantees that $\partial D_j(u)/\partial u \geq -1$. Therefore, expression (15) is nonnegative, which means that $u + \delta V_j(u)$ is nondecreasing for $u < u_j$.

Let's now turn to the case $u > u_j$. We have

$$\begin{aligned} u + \delta V_j(u) &= u + \delta\left[u + \delta V_{j+1}(u) - m_j(u)\right] \\ &= u + \delta^2 V_{j+1}(u) + \delta\left[u - m_j(u)\right] \\ &= \delta\left[\frac{u}{\delta} + \delta V_{j+1}(u)\right] + \delta\left[u - m_j(u)\right]. \end{aligned}$$

By assumption $u + \delta V_{j+1}(u)$ is increasing in u, so *a fortiori* the first term in brackets is increasing in u. As for the second term in brackets, the bi-vectors selected in part (b) have the property that $u - m_j(u)$ is nondecreasing in u. Therefore the whole expression is nondecreasing in u for $u > u_j$.

What happens to the function $V_j(u)$ at $u = u_j$? Denote by $D_J = D_j(u_j)$ the damages that need to be paid when taker j meets an owner with the same value for the asset. Remember that $V_{j+1}(u_j) = V_{j,j+1}$. Then we can write

$$\begin{aligned} V_j\left(u_j^-\right) &= \min\left[D_J, u_j + \delta V_{j,j+1}\right] \\ V_j\left(u_j^+\right) &= u_j + \delta V_{j,j+1} - \max\left[0, u_j + \delta V_{j,j+1} - D_J\right] \\ &= \min\left[u_j + \delta V_{j,j+1}, D_J\right]. \end{aligned}$$

So the function $V_j(u)$ is continuous at u_j. Therefore any bi-vector selected in part (b) generates weak monotonicity of $V_j(u)$ across the entire range of u's. ■

A.3 Proof of Theorem 1

Proof. Apply Lemma 1 to $j = T$. Since $V_{i,T+1} = f(u_i)$, for all i, and $f(\cdot)$ is any nondecreasing function (see 8), $u_i + \delta V_{i,T+1}$ is nondecreasing in u_i. Then there exists a set of monotonically selected bi-vectors $\{p_{T,i}, m_{T,i}\}_{i<T}$ each of which solves (11), (12), (6) and (7); and each of which gives rise through 4 and 5 to a vector of values $\{V_{i,T}\}_{i<T}$ which is nondecreasing in u_i. Pick any of these bi-vectors and associated values (Lemma 1 ensures that at least one exists). Repeat the process for $j = T-1, T-2, ..., 1$ to get a set of monotonically selected $\{p_{j,i}, m_{j,i},\}_{\substack{j=1,...,T \\ i<j}}$ which solves (11), (12), (6) and (7). The bi-vector $\{p_{j,i}, m_{j,i}\}_{\substack{j=1,...,T \\ i<j}}$ satisfies conditions (4) through (8), and so is a bilateral bargaining solution; and satisfies (11), (12), which means it gives rise to an efficient allocation. ■

A.4 Example 3

Example 3 *Let $u_0 = 10$, $u_1 = 9$ and $u_2 = 15$. Assume no discounting ($\delta = 1$). Also assume that we are in Scenario 1, namely, the world ends after period 2. The court applies a property rule, which is analytically equivalent to setting very high damages for all takings, i.e., $D_{j,i} = \infty$ for all j, i.*

For an outcome to be efficient in period 2, agent 2 must gain possession of the asset from whoever owns it at the beginning of period 2 – agent 0 or agent 1. We pick an outcome in which agent 0 has no bargaining power vis-a-vis agent 2, and agent 1 has full bargaining power vis-a-vis agent 2.

Let us first look at the interaction between owner $i = 0$ and taker $j = 2$. This interaction takes place in the subgame which arises after agent 0 retains possession of the asset in period 1. The set of quantities which are efficient and compatible with a solution (i.e., satisfy 6, 7) are found in the proof of Lemma 1. The left-hand column in the next table reproduces this set in the case where $u_i < u_j$; the right hand column shows the quantities that obtain given the allocation of bargaining powers that we chose.

Bilateral bargaining solution correspondence	*Selection for our example*
$p_{j,i} \in [u_i + \delta V_{i,j+1}, u_j + \delta V_{j,j+1}]$	$p_{2,0} = u_i + \delta V_{i,j+1} = 10$
$m_{j,i} \in [0, \max\{0, u_i + \delta V_{i,j+1} - D_{j,i}\}]$	$m_{2,0} = \infty$
$V_{i,j} = \min[D_{j,i}, p_{j,i}]$	$V_{0,2} = p_{2,0} = 10$

Note that we have used $V_{i,j+1} = 0$ to translate the left-hand (LH) column into the right-hand (RH) column. This equality holds since $j+1$ is the last period in this example. Let us discuss our selection in the RH column, from the set of bargaining solutions in the LH column. Our *selection of* $m_{2,0}$ *is immaterial because bribes are not paid in equilibrium (given the very high damages, agent 2 does not have a credible threat to take the asset). Our selection of* $p_{2,0}$*, in contrast, is critical because the asset would be traded in this subgame. We selected the lowest level of* $p_{2,0}$ *compatible with a bargaining solution. This choice corresponds to agent 2 having all the bargaining power vis-a-vis agent 0.*

Next let us consider the interaction between owner $i = 1$ *and taker* $j = 2$*. This interaction takes place in the subgame which arises after agent 1 takes from agent 0 in period 1. As before,* $u_i < u_j$*, and the bargaining solution quantities are presented in the following table.*

Bilateral Bargaining Solution correspondence	*Selection for our example*
$p_{j,i} \in [u_i + \delta V_{i,j+1}, u_j + \delta V_{j,j+1}]$	$p_{2,1} = u_j + \delta V_{j,j+1} = 15$
$m_{j,i} \in [0, \max\{0, u_i + \delta V_{i,j+1} - D_{j,i}\}]$	$m_{2,1} = \infty$
$V_{i,j} = \min[D_{j,i}, p_{j,i}]$	$V_{1,2} = p_{2,1} = 15$

Note that we have used $V_{j,j+1} = 0$ *to translate the LH column into the RH column. Let us discuss our selection in the RH column. Again the selection of* $m_{2,1}$ *is immaterial. For* $p_{2,1}$*, in contrast with the previous case, we select the highest level of* $m_{2,1}$ *compatible with a bargaining solution. This choice corresponds to agent 1 having all the bargaining power vis-a-vis agent 2.*

Now let's move up one period. For the outcome to be efficient in period 1, the asset must remain with agent 0. But this outcome would be difficult to support as a solution since, despite agent 0's higher use value (10 vs. 9), the asset is worth more to agent 1. Specifically, the asset is worth $u_1 + V_{1,2} = 9 + 15 = 24$ *to agent 1, but only* $u_0 + V_{0,2} = 10 + 10 = 20$ *to agent 0. There is a range of prices (*$p_{1,0} \in [20, 24]$*) that support a mutually beneficial, yet socially inefficient, trade in period 1.*

This example illustrates that there can be inefficiencies if bargaining power is not positively correlated with valuation, also when possession is protected with a property rule. In the example, agent 0 has a higher valuation but less bargaining power than agent 1. Formally, the problem is that monotone selection is violated: even though $u_0 > u_1$*, we have*

$$p_{2,0} = 10 < 15 = p_{2,1}.$$

A.5 Proof of Theorem 2

The $\widehat{p}_{j,i}, \widehat{m}_{j,i}$ defined in Table 1 satisfy an induction property analogous to Lemma 1. This property ensures the existence of an efficient solution. The proof is therefore identical to the proof of Theorem 1, except Lemma 1 needs to be replaced by the following lemma:

Lemma 2 *Fix* j, $\{D_{j,i}\}_{i<j}, \{V_{i,j+1}\}_{i<j+1}$, *and any* $\alpha_1, \alpha_2 \in [0,1]$. *Assume Assumption 2 holds. Assume that the quantity* $u_i + \delta V_{i,j+1}$ *is nondecreasing in* u_i *over all* $i < j+1$. *Then:*

(a) The bi-vector $\{\widehat{p}_{j,i}, \widehat{m}_{j,i}\}_{i<j}$ *solves (11), (12), (6) and (7).*

(b) $\widehat{p}_{j,i}$ *is nondecreasing in* u_i *across all* $u_i < u_j$ *and* $u_i - \widehat{m}_{j,i} + \delta V_{i,j+1}$ *is nondecreasing in* u_i *for* $u_i > u_j$.

(c) The bi-vector $\{\widehat{p}_{j,i}, \widehat{m}_{j,i}\}_{i<j}$ *gives rise to* $u_i + \delta V_{i,j}$ *(defined by 4 and 5) which is nondecreasing in* u_i *over all* $i < j$.

Proof. Part (a) This is true by construction, since both $\widehat{p}_{j,i}$ and $\widehat{m}_{j,i}$ are defined as a convex combination of the extremes of the interval $\Gamma(u_i)$ (refer to part b in the proof of Lemma 1.)

Part (b) When $u_i < u_j$ we have

$$\widehat{p}_{j,i} = \alpha_1 [u_i + \delta V_{i,j+1}] + (1-\alpha_1)[u_j + \delta V_{j,j+1}].$$

Since $u_i + \delta V_{i,j+1}$ is nondecreasing in u_i, the desired property is established. When $u_i > u_j$ we have

$$\widehat{m}_{j,i} = \alpha_2 \max[0, u_j + \delta V_{j,j+1} - D_{j,i}] + (1-\alpha_2)\max[0, u_i + \delta V_{i,j+1} - D_{j,i}].$$

Let us rewrite this expression using the notation developed in the proof of Lemma 1.

$$\widehat{m}_j(u) = \alpha_2 \max[0, u_j + \delta V_{j,j+1} - D_j(u)] + (1-\alpha_2)\max[0, u + \delta V_{j+1}(u) - D_j(u)].$$

Note first that $\widehat{m}_j(u)$ is a continuous function of u. Next, since we are in the case $u > u_j$ and, by assumption, $u_i + \delta V_{i,j+1}$ is nondecreasing in u_i, it follows that $u + \delta V_{j+1}(u) \geq u_j + \delta V_{j,j+1}$. Therefore $\widehat{m}_j(u)$ can be written more simply as

$$\begin{array}{cl} \alpha_2 [u_j + \delta V_{j,j+1} - D_j(u)] + (1-\alpha_2)[u + \delta V_{j+1}(u) - D_j(u)] & \text{when } D_j(u) < u_j + \delta V_{j,j+1} \\ (1-\alpha_2)[u + \delta V_{j+1}(u) - D_j(u)] & \text{when } u_j + \delta V_{j,j+1} < D_j(u) < u + \delta V_{j+1}(u) \\ 0 & \text{when } u + \delta V_{j+1}(u) < D_j(u) \end{array}$$

Plugging $\widehat{m}_j(u)$ into the following expression and simplifying yields

$$u - \widehat{m}_j(u) + \delta V_{j+1}(u)$$
$$= \left\{ \begin{array}{cl} \alpha_2 [u + \delta V_{j+1}(u)] + D_j(u) - \alpha_2 [u_j + \delta V_{j,j+1}] & \text{when } D_j(u) < u_j + \delta V_{j,j+1} \\ \alpha_2 [u + \delta V_{j+1}(u)] + (1-\alpha) D_j(u) & \text{when } u_j + \delta V_{j,j+1} < D_j(u) < u + \delta V_{j+1}(u) \\ u + \delta V_{j+1}(u) & \text{when } u + \delta V_{j+1}(u) < D_j(u) \end{array} \right.$$

Since $u+\delta V_{j+1}(u)$ is nondecreasing in u by assumption, and $D_j(u)$ is nondecreasing in light of Assumption 2, it follows that this function is nondecreasing in u over each of its three regions. Therefore, by continuity the function is nondecreasing for all $u > u_j$. Thus the desired property is established.

Part (c) That $u_i+\delta V_{i,j}$ is nondecreasing in u_i when $u_i < u_j$ can be proved directly following the proof of Lemma 1. When $u_i > u_j$ we have, from that same proof,

$$u + \delta V_j(u) = u + \delta\left[u + \delta V_{j+1}(u) - \widehat{m}_j(u)\right].$$

Since the right-hand side is nondecreasing in u by part (b), the result is proved. ■

B Investment Efficiency [Not for Publication]

We have in mind a setup in which, prior to entering the exchange game, agents invest in increasing their private valuations for the asset. The benefit of each agent's investment cannot be transferred to any future possessor. In other words, the investment is agent-specific. However, the benefits of investment are enjoyed by the agent in every period in which he possesses the asset. Formally, each agent i before trade begins, invests $e_i > 0$ and makes his own per-period valuation into $v_i(e_i)$. Note that we allow the valuation functions $v_i(\cdot)$ to vary across agents, and that the choice of having the investment cost be linear is not an assumption—it is simply a choice of scaling.

We assume, for simplicity, that all investments are committed to at time zero rather than undertaken as the game evolves. This assumption simplifies the analysis of the investment game. Allowing the investment of agent i to depend on the investment of previous agents would not substantively alter the incentives in the example we study below, but it would make the analysis more complicated.

Definition 6 *An* ***investment equilibrium with efficient trading*** *is an equilibrium in which each agent i independently chooses e_i to maximizes $u_i + \delta V_{i,i+1}\left(e_i, e^*_{-i}\right) - e_i$, where $u_i = v_i(e_i)$ and, for each $\left(e_i, e^*_{-i}\right)$ the function $V_{i,i+1}$ is generated by an efficient bilateral bargaining solution.*

An investment equilibrium with efficient trading is an equilibrium in the investment game which is played before the trading game begins. By this definition, the allocation arising from trading will be efficient *given the values* $u_i = v_i(e^*_i)$. But, of course, the investments e^*_i themselves need not have desirable properties. In fact, the investments will be a function of the specific legal rule. For most legal rules, we cannot expect the investment level to maximize social welfare.

Definition 7 *An investment equilibrium with efficient trading is said to be* ***investment-efficient*** *if equilibrium investments $\{e^*_i\}_{i=1}^N$ coincide with those of a social planner who chooses both investments and asset allocations in an unconstrained way.*

Example 4 *There are two agents. Agent 0 is endowed with an asset for which he has per-period use value $v_0(e_0) = \sqrt{e_0}$, agent 1 has use value $v_1(e_1) = 2 + \sqrt{e_1}$, and investment $e_i \in [0,1]$. The first observation is that the optimal allocation is for the asset to be traded at the beginning of period 1. Indeed, if agent 0 holds the asset in both periods then social value is given by*

$$\max_{e_0} (1+\delta)\sqrt{e_0} - e_0,$$

which is maximized at $e_0 = (1+\delta)^2/4$. *Substituting into the objective function yields a social value of* $(1+\delta)^2/4$. *If, in constrast, the asset is transferred at the beginning of period 1, then social value is given by*

$$\max_{e_0}\left[\sqrt{e_0} - e_0\right] + \max_{e_1}\left[2 + \delta\sqrt{e_1} - e_1\right] = \frac{1}{4} + \left[2 + \frac{\delta^2}{4}\right].$$

The socially optimal investment levels given this possessory allocation are $e_0^{OPT} = 1/4$, $e_1^{OPT} = \delta^2/4$, *and the resulting social value equals* $2 + (1+\delta)^2/4$. *This social value is larger than what could be achieved if the asset did not change hands. Hence trading the asset at the beginning of period 1 is socially efficient, and* e_0^{OPT}, e_1^{OPT} *represent the socially optimal investment levels.*

Now, suppose the regime is one of liability with $D = 0.1$. *Observe that agent 1 has higher use value than agent 0 no matter their respective investments. Hence, in any efficient solution agent 1 will take the asset. Let us consider the incentives to invest in such an equilibrium. Player 0's payoff is*

$$\max_{e_0} \sqrt{e_0} - e_0 + D, \tag{16}$$

player 1's payoff is

$$\max_{e_1} 2 + \delta\sqrt{e_1} - e_1 - D. \tag{17}$$

In both cases, the solution to the individual's investment problem coincides with the socially optimal one. Hence a liability regime with a small D *gives rise to the socially optimal investment level.*

Consider now a property regime ($D = \infty$)*. In this case at an efficient equilibrium the object must change hands via a negotiation. Agent 1 will have to pay a price* $p \in [u_0, u_1]$ *in order to possess the asset. For example, if the price was determined by an alternating offers game that price would equal* $(u_0 + u_1)/2$. *In general, agent* 1*'s investment problem is*

$$\max_{e_1} 2 + \delta\sqrt{e_1} - e_1 - p(u_0, u_1). \tag{18}$$

As long as the price $p(\cdot,\cdot)$ *depends on its second argument, problem (18) differs from problem (17) and so agent* 1*'s investment level under a property rule cannot be optimal. If the price* $p(\cdot,\cdot)$ *is independent of its second argument then it must be* $p(u_0,\cdot) = u_0$. *In that case agent 1 would have the socially correct incentives to invest, but agent* 0*'s problem would read*

$$\max_{e_0} \sqrt{e_0} - e_0 + p(u_0,\cdot) = 2\sqrt{e_0} - e_0,$$

which differs from problem (16). So even if $p(u_0,\cdot) = u_0$ *we cannot have efficient investment* *in an efficient equilibrium under a property rule.*

This example illustrates that, in some cases, socially optimal investment is promoted by a liability rule with a very small and constant D. We can interpret such a rule as weak property rights. Certain features of this example are not essential. The restriction to two agents is not essential, nor is the specific form of the use-value function, including the feature that investment cannot affect which allocation is optimal. Three features, however, are essential. First, that the optimal allocation entails trade in every period (though there is only one possible period for trading in this example). Second, that damages are independent of use values. Third, damages are set low.