Friendship Network in the Classroom:

Parents Bias and Peer Effects

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Abstract

We interview both parents and their children enrolled in six primary schools in the district of Treviso (Italy). We study the structural differences between the children network of friends reported by children and the one elicited asking their parents. We find that the parents’ network has a bias: parents expect peer effects on school achievement to be stronger than what they really are. Thus, parents of low-performing students report their children to be friends of high-performing students. Our numerical simulations indicate that when this bias is combined with a bias on how some children target friends, then there is a multiplier effect on the expected school achievement.

Keywords: social networks, primary school, friendships, parents’ bias, homophily, peer effects, Bonacich centrality

1. Introduction

The estimation of peer effects on school achievement is one of the most fascinating challenges in economics of education. Several studies have presented convincing evidence about peer effects across race (Angrist and Lang, 2004), gender (Hoxby, 2000; Lavy and Schlosser, 2008), ability (Sacerdote, 2001) and country of origin of immigrants (Gould, Lavy and Pascerman, 2009). More recent studies have investigated if these results hold when we consider a smaller reference groups and they have stressed the crucial role that friendship networks have on peer effects (Babcock, 2008; Carrell et al., 2013; Nathan 2008; Patacchini et al., 2011).\(^1\) In particular, Calvó-Armengol et al. (2009) show that the network structure and the student position within the network is crucial for the intensity of these peer effects.

This vast literature is however far from been conclusive. As in other fields in economics, the identification of peer effects in non-experimental settings may suffer from three different types of problems. First, the famous *reflection problem* pointed out by Manski (1993).\(^2\) If individuals that belong to the same group tend to behave similarly, it is not (always) possible to distinguish if this similar behavior is due to conformity -i.e. individuals tend to conform to the prevalent behavior in the group (endogenous effect) - or it is due to the fact that all individuals face similar environments and/or have similar personal characteristics (exogenous effect). Second, it is rarely possible to have a complete description of the real network of interactions between these peers.
and that take into account the actual friendship network. This lack of information may produce biased estimates that overestimate or underestimate these peer effects.\(^3\) Third, the reference groups (i.e. the network) could be endogenous. In this case, even if we observe that neighboring nodes have similar behavior, and we are able to solve the reflection problem by identifying common exogenous factors, we would still not be able to disentangle the peer effects from the phenomenon of *homophily* (Currrarini et al., 2009, 2010): people with exogenous similar traits will tend to connect together. Under special conditions and with panel data this disentanglement is possible (see Bisin and Ö zgür, 2012; or Shalizi and Thomas, 2011 for a negative result in the most general case).\(^4\)

We contribute to this literature bringing evidence on the importance of the elicitation of friendship network to have a clean estimate of peer effects. From a methodological point of view, this may be difficult and sometimes even tricky. Most studies build directly the network of friends by using data on friendships self-reported by children while others, especially when children are too young, indirectly build the network, by using data reported by adults (typically teachers, as in Gest (2006), or parents, as in Crouter et al. (1990) and Crouter et al. (1999). On one hand, the direct elicitation method is preferable but it may be less reliable and fluctuate more over time. Children (especially if very young) change frequently their friends and every squabble may produce a completely new network structure (Berndt, et al., 1986; Cairns et al. 1995; Gifford-Smith and Brownell, 2003). On the other hand, the indirect elicitation made asking adults may have the problem that both parents and/or teachers are not fully aware of the true

\(^3\) If for instance not all students know (or interact with) all of their classmates the estimated peer effects will be lower than the real (see Carrell et al. 2009). Another possibility is that adults have a biased perception of the network and we rely on parents- or teachers-reported data. This is discussed in detail in Section 5.

\(^4\) What is important to point out, as we will use it in this work, is however that homophily is an assortative matching that could induce an overestimation of the peer effects. However if we find that on some traits there is disassortative matching, so that people with opposite values of that trait will tend to connect together, but still we observe peer effects on the same trait, then the magnitude of this peer effect could be at most underestimated by the endogeneity of the network.
network of friends. For instance, Gest (2006) suggests that teacher reports of children’s friendships and social groups may produce inflated estimates of peer similarity.\(^5\) Parents elicitation also may have the same problem: in fact, as shown in recent studies on parental knowledge,\(^6\) parents are not fully aware of their children life and relationships and, interestingly, this the gap of knowledge can explain child’s probability of exhibiting wide range of antisocial and risk behaviors (e.g. Ary et al., 1999) and it is negatively associated to school achievement (Muller, 1993).\(^7\) Therefore, it remains an open question to understand which elicitation (direct or indirect) produces a more precise representation of the friendship network.

In this paper, we elicit the network of friends of 452 children divided in 33 classes in six primary schools in the district of Treviso (Italy). We ask separately parents and children to report the child friends. Comparing the two networks produced by the direct and indirect elicitations we can study if there exists a structural difference and eventually which network approximate more closely the true network. We find that the parent-reported network presents a bias. This bias can be explained by the tendency of the parents to misreport their children’s friends or by children,\(^5\) Gest (2006) refers to Howes (1988) as the unique study focusing on the validity of the teachers’ reports about reciprocated friendships in class. Howes (1988) compared mutual ‘liking’ nominations by children and teacher reports in a study of children aged 3 to 6 years. According to her results, the teacher reports were correct in 78% percent of the cases. Recent studies use behavioral data by monitoring children behavior in space through technological devices, as, for example, wearable sensors (Stehlé et al., 2013). This last elicitation method, however, depicts the so-called behavioral network of friends, which is somewhat different (even if surely related, as shown in Gest et al, 2013) from the social network of friends elicited (directly or indirectly) through questionnaires.\(^6\) With parental knowledge we mean the parents’ knowledge of their child’s whereabouts, activities and associations or social connections (see Patock-Peckham et al., 2011 and Stattin and Kerr, 2000). The measurement of parental knowledge has been operationalized in many different ways (e.g. some studies measure actual knowledge while others perceived knowledge) and measured on the basis of the information reported by parents, child or both; see Crouter and Head (2002) for a review.\(^7\) Only few of these studies assess the actual parental knowledge by posing to the parents direct questions on aspects of the child’s daily life (e.g. school achievements, homework, friends met, etc.), as, e.g. in Crouter et al. (1990) and Crouter et al. (1999). In most cases, in fact, parental knowledge is assessed indirectly, asking to the parents questions as “do you know i) what does your child do during her/his free time? ii) who does s/he have as friend during her/his free time?, (and similarly to the children/adolescents “do your parents know: i) what do you do during your free time? ii) who do you have as friend during your free time?” etc.), as for example in Stattin and Kerr (2000).
anticipating the expectations of their parents, misreport to their parents their friendships (see e.g. Cumsille et al. 2010; and Smetana et al. 2009).

Interestingly, the structural difference between these two networks cannot be explained by a delay in updating information on friendship ties. On the contrary, our analysis suggests that this difference derive from a peer effect bias: parents expect peer effect to be stronger than they really are. As a result, parents of low-performing students tend to report their children to be friends of high-performing students at a higher rate than what happens in reality. Finally, using numerical simulations we show that if the parent's peer effect bias is combined with a bias on how some children target friends, then there is a multiplier effect on the expected school achievement, with these two biases reinforcing each other and distorting the expectations of parents.

The remainder of this paper is organized as follows. Section 2 provides a description of the dataset and Section 3 discusses the differences between the self-reported and the parents-reported networks, providing evidence that the first ones are more reliable. Section 4 analyses why parents may have a bias in knowing their children’s friends, both proposing a model and finding support against alternative explanations. Section 5 analyzes the bias on peer effects perception due to parents’ biased perception of the network. Section 6 discusses policy implications for education.

2. Data

Our sample consists of 452 children enrolled in six different primary schools from three municipalities in the district of Treviso (Italy). In Italy, children attend the primary school from 6 to 11 year-old, divided in 5 grades. For each class with we elicited the network of friendship
asking separately parents and children to name their friends. More in detail, we asked parents to fill a written questionnaire\(^8\) with questions about i) the family (e.g. marital status and education of the parents; number of children in the family and their age and gender); ii) the work of parents (distinguishing between full-time, part-time and no employment); iii) the strengths and difficulties questionnaire (SDQ), a questionnaire validated by Goodman and Goodman (2010), used to elicit information about the child’s emotional symptoms, conduct problems, hyperactivity/inattention and peer relationship problems. In addition, we asked parents to name (up to) five friends of their child *among the child classmates*. Parents and children could refuse to answer all or part of this questionnaire. Overall, 452 parents returned a questionnaire (response rate was 84\%). Among the latter, 378 questionnaires reported complete information on both parents and children.

One week after distributing the questionnaire, we visit the schools and we asked children to fill a sheet of paper where a table and five chairs are depicted. Each child had to write his/her name on the chair on the head of the table and (up to) five other names of his/her friend in the same class that s/he would like to have seated close to him/her (from the closer to the farther). We informed children that we will keep confidential the names they report and neither the parents nor the teachers or other friends will know what they write. Children received a rubber band for their collaboration. When eliciting the friends’ network from the children we preferred this elicitation method rather than asking directly to name their friends (as done for the parents) in order to present the task in the simplest and intuitive way even for the youngest children present in our database.

When looking at the completeness of the elicitation, we find that 72\% (N=329/452) of children indicated five friends’ name over five, while this percentage is equal to 76\%

\(^8\) Anonymity is ensured using a numerical ID to identify the participants at each step of the data collection.
(N=344/452) for the parents. These two friendship elicitation mechanisms allow us to have a complete picture of the network of friends for the parents and children and to compare them.

3. Self-reported vs. parent-reported networks

Our analysis of the two friendship networks proceeds as follows. First, in Section 3.1, we compare the two networks, testing for the existence of some structural differences. In particular, we compare both the degree of mismatch between the different friendship nominations (from one to five) and some network-level structural measures, such as number of ties, density coefficient and number of reciprocated ties. Then, in Section 3.2, we investigate which network is closest to the “true” one by the mean of two separate tests: a) difference in means test for the percentage of reciprocated ties; and b) estimation of peer effect through indirect friendship ties. In both cases the main idea is to identify which network outline would require the greatest degree of coordination among respondents to be achieved though misreporting alone. Since this coordination is unlikely to be achieved in our setting, we infer that such network is most likely to obtain through the true revealing of friendship ties.

3.1 Comparison of the two networks

Figure 1 shows the structure of friendship networks reported by children while Figure 2 the one reported by parents. In these figures each class is associated with a specific color, each node represents a child, and each edge corresponds to a friendship tie. Children ID number identifies each child. Comparing these two figures one can notice that classes differ in terms of size (i.e. number of nodes), network structure and friendship ties. In particular, we find that:
**Result 1:** There exist a mismatch between the network of friends that is self-reported by students and the network of friends that is reported by parents.

**Figure 1** – Self-reported networks

Note: This picture as well as the following ones are realized with Pajek software ([http://pajek.imfm.si/](http://pajek.imfm.si/)).

**Figure 2** – Parent-reported networks
Result 1 derives from the comparison of different statistics at both the node-level and the network-level. The first node-level statistics that we consider is the percentage of children for which the names reported in each friendship position differ. Specifically, we find that when looking at the first name reported by parents and children (i.e. the closest friend), the two networks differ in the 56% (N=253/452) of the cases. The differences increase to 74% (N=334/452), 78% (N=353/452), when considering the names indicated in the second and third position and they rise up to 84% for the names indicated in the last two positions, N=378/452 and N=380/452, respectively. This suggests that some significant incongruence exists in the way in which friendship ties are reported, especially if we move from close to far friends.

The existence of such a high degree of discrepancy between parent and children for the names reported in each friendship position is revealing, but it is not itself sufficient to signal the existence of a real mismatch. In fact, such discrepancy could simply derive from a different ordering of friends across the five positions available and not from the effective reporting of different friends. For this reason the second node-level statistics that we consider, which we call mismatch, counts the number of friends that are named only by the child or only by the parent. This variable ranges from zero, when the child and the parent named the same friends, to ten, when the child and the parent named five different friends. On average we find that mismatch takes value 2.76 (std. 1.79), i.e. parents and children usually report up to 2.8 different friends. Quite remarkably we find that mismatch takes value zero only in the 12% (N=56/452) of the cases and that for 50% of the cases it takes value greater than 3. These figures suggest the observed discrepancy in friendship nominations between parents and children does not simply derive from a different ordering of friends. Some other factors have to play a relevant role here.
Moving from node-level to network-level statistics, we are also interested in investigating whether the discrepancy in friendship nominations between parents and children translates also in some structural difference of the two networks. On this respect, Table 1 reports summary statistics on total number of ties, density coefficient and number of reciprocated ties for the 33 classes included in our dataset, separately for self-reported and parent-reported networks. The last column reports the result of a standard mean-comparison t-test, comparing the difference between the values reported in the two previous columns. Results reveal that self-reported and parent-reported networks are indeed structurally different. On average self-reported networks are characterized by a greater number of both directed and reciprocated ties than parent-reported networks. In addition, networks that are based on the information self-reported by children exhibit a higher density coefficient than the ones constructed using the information reported by parents.

**Table 1 – Differences in network structure**

<table>
<thead>
<tr>
<th></th>
<th>Self mean (std)</th>
<th>Parent mean (std)</th>
<th>Diff. (S-P) (p-value)</th>
</tr>
</thead>
<tbody>
<tr>
<td># ties</td>
<td>67.87 (4.29)</td>
<td>61.00 (3.67)</td>
<td>6.87*** (0.0002)</td>
</tr>
<tr>
<td>Density</td>
<td>0.27 (0.01)</td>
<td>0.25 (0.01)</td>
<td>0.02*** (0.0009)</td>
</tr>
<tr>
<td># recipr. ties</td>
<td>18.12 (1.24)</td>
<td>16.09 (1.12)</td>
<td>2.03** (0.0167)</td>
</tr>
</tbody>
</table>

Overall, the evidence derived from both node-level and network-level statistics suggests that some significant degree of mismatch exist between self-reported and parent-reported
networks. On this basis, our main aim in the remaining sections of the paper is to find out what the sources of such mismatch are.

3.2 Searching for the “true” network

The first step in studying the nature of the mismatch existing between self-reported and parent-reported networks is to establish which network is actually the best approximation of the “true” one. Given that in both cases we rely on survey data we do not have a direct mean to establish whether the information reported by children is more (or less) reliable than the one reported by parents. In principle, both types of networks can be affected by different sources of bias. Nevertheless, we can exploit some indirect tests to investigate whether one network is most likely to be true, and consequently whether the other is most likely to be false. In particular, we find that:

**Result 2:** The network that most closely approximates the “true” network is the self-reported network.

Result 2 is supported by different analysis. The first analysis that we conduct looks at the percentage of reciprocated ties for each individual. Ties reciprocation (*i.e.* the fact that if *i* names *j* as a friend, also *j* names *i*) requires some degree of coordination in reporting friendships, which is difficult to obtain through misreporting alone. A high proportion of reciprocated ties, therefore, can be considered as a proxy of the fact that respondents are telling the truth.

Table 2 extends the last line of Table 1 and shows the percentage of reciprocated ties for different degrees of mismatch between the two networks. Mismatch is computed at the node-level as described in Section 3.1. The percentage of reciprocated ties is reported separately for the self-reported network and the parent-reported network. The last column of the table reports a
standard mean-comparison t-test, comparing the difference between the values reported in the two previous columns.

**Table 2 – % Reciprocated ties in the two networks**

<table>
<thead>
<tr>
<th></th>
<th>Self mean (std)</th>
<th>Parents mean (std)</th>
<th>Diff. (S-P) (p-value)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full Sample</td>
<td>0.541 (0.01)</td>
<td>0.529 (0.01)</td>
<td>0.012 (0.213)</td>
</tr>
<tr>
<td>Mismatch &gt;0</td>
<td>0.521 (0.02)</td>
<td>0.504 (0.01)</td>
<td>0.017 (0.147)</td>
</tr>
<tr>
<td>Mismatch &gt;1</td>
<td>0.524 (0.02)</td>
<td>0.498 (0.02)</td>
<td>0.027* (0.076)</td>
</tr>
<tr>
<td>Mismatch &gt;2</td>
<td>0.489 (0.02)</td>
<td>0.473 (0.02)</td>
<td>0.015 (0.258)</td>
</tr>
<tr>
<td>Mismatch &gt;3</td>
<td>0.474 (0.02)</td>
<td>0.460 (0.02)</td>
<td>0.013 (0.320)</td>
</tr>
<tr>
<td>Mismatch &gt;4</td>
<td>0.448 (0.04)</td>
<td>0.407 (0.04)</td>
<td>0.041 (0.190)</td>
</tr>
<tr>
<td>Mismatch &gt;5</td>
<td>0.413 (0.05)</td>
<td>0.352 (0.04)</td>
<td>0.060 (0.140)</td>
</tr>
<tr>
<td>Mismatch &gt;6</td>
<td>0.368 (0.09)</td>
<td>0.201 (0.07)</td>
<td>0.167* (0.072)</td>
</tr>
</tbody>
</table>

Overall, we observe that the percentage of reciprocated ties is greater in the self-reported network than in the parent-reported network for any degree of mismatch. This difference is statistically significant for mismatch higher than one and mismatch higher than six. Moreover, for mismatch greater than zero and mismatch greater than five the difference is close to being significant. Therefore, we find that, if anything, in the self-reported network respondents seem more likely to tell the true than in the parent-reported network.

In addition to the percentage of reciprocated ties, a second type of analysis that we conduct to investigate the degree of coordination that would be necessary to support the answers
of respondents consists in estimating peer effects using the methodology developed by Bramoullé et al. (2009). Starting from the crucial assumption that there are no unobserved characteristics that differ among children in a classroom and affect both the likelihood of becoming friends and the dependent variable that is to be estimated (i.e. the network is exogenous), Bramoullé et al. (2009) determine the conditions under which peer effects are identified when individuals interact through social networks that are known by the researcher. The structural model is an extension of the linear-in-means models developed by Manski (1993) and Moffit (2001), with the difference that in this case each individual has his own reference group. By calling $I$ the $N \times N$ identity matrix for the $N$ students in the network, and $G$ the $N \times N$ row-normalized interaction matrix, Bramoullé et al. (2009) show that if $I$, $G$, $G^2$ and $G^3$ are linearly independent social interactions are identified. If that is so, then the characteristics of the friends' friends of a student (and also of friends' friends friends and further) who are not her friends serve as instruments for the outcomes of her own friends, thus solving the so-called reflection problem (Manski, 2000; Brock and Durlauf, 2001).

The intuition behind this result is that the characteristics of friends' friends who are not the student's friends can only have an impact on the student's behavior indirectly by influencing the behavior of her friends. This in turn implies that endogenous effects associated with peers’ behavior can be safely estimated. In the present paper we do not have space to review the full methodology in detail. For a more detailed description with a specific application to education data see Lin (2010) and De Melo (2014).

9 As discussed in the introduction, a bias in the estimation of the peer effect could come from homophily, if similar people tend to connect together. However, if this happens for large clusters, the method of Bramoullé et al. (2009) would be able to instrument this and weaken the bias. So, endogeneity of the network would remain undected by that method only if the network is formed in very specific ways. Suppose for instance that nodes match
endogenously in an assortative way only in couples, with neither of the two having links to other nodes (those that would be used as instruments) with the same characteristic. Under such a very specific endogenous network formation, the method of Bramoullé et al. (2009) would be particularly biased. We come back to this point when commenting Figure 3 below.

The existence of peer effects can provide some useful insights on the truthfulness of the two networks. For peer effects to be significant using the methodology developed by Bramoullé et al. (2009), in fact, social interactions need to be structured in a relatively specific way. For such a structure to obtain through the consistent misreporting of friendship ties, we would require a fairly high degree of coordination among individual respondents, which is unlikely to obtain in our context. Therefore, if there were some evidence on the existence of peer effects in one network but not in the other, it could indeed be taken as sign that the network is likely true.

We estimate peer effects for the six behavioral variables available in our dataset: the five SDQ scores (including peer problems, hyperactivity, prosociality, conduct problem and emotional problem) plus the parent-reported school performance. All peer effects are estimated using the same set of exogenous variables, which include: gender, three dummy variables for parent’s education (completed primary school, completed high school, and university degree – excluded category: completed VIII grade), number of siblings, and two dummies for parent’s work (work full time and unemployed – excluded category: work part time). The endogenous effect is instrumented using the average value of parent’s education (the three dummies) and number of siblings for the friends’ friend and the friends’ friends’ friends. Since our data present some missing values we estimate peer effects on two different samples. In the first case we limit the sample only to those children for whom all information of interest are available (n = 378). In
the second case we extend the sample to all children and introduce dummy variables for missing information \((n = 452)\). The results on peer effects in the two cases are reported in Table 3.

### Table 3 – Peer effects

<table>
<thead>
<tr>
<th></th>
<th>Peer prob.</th>
<th>Hyperactivity</th>
<th>Prosociality</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peer effect (observed)</td>
<td>0.134*</td>
<td>0.048</td>
<td>0.035</td>
</tr>
<tr>
<td></td>
<td>(0.08)</td>
<td>(0.17)</td>
<td>(0.10)</td>
</tr>
<tr>
<td>Peer effect (full sample)</td>
<td>-0.063</td>
<td>-0.14</td>
<td>-0.058</td>
</tr>
<tr>
<td></td>
<td>(0.10)</td>
<td>(0.22)</td>
<td>(0.08)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Conduct prob.</th>
<th>Emotional prob.</th>
<th>School perf.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peer effect (observed)</td>
<td>0.234</td>
<td>-0.060</td>
</tr>
<tr>
<td></td>
<td>(0.20)</td>
<td>(0.20)</td>
</tr>
<tr>
<td>Peer effect (full sample)</td>
<td>-0.237</td>
<td>-0.247</td>
</tr>
<tr>
<td></td>
<td>(0.19)</td>
<td>(0.25)</td>
</tr>
</tbody>
</table>

*Note*: Before estimating peer effect we checked that the matrices \(I, G, G^2\) and \(G^3\) are linearly independent, satisfying the identification condition established by Bramoullé et al. (2009). This was checked by vectorizing matrices \(I, G, G^2\) and \(G^3\), and verifying that the matrix formed by these four vectors is of rank 4. This result holds both in the case of restricted sample \((n = 378)\) and in the case of full sample \((n = 452)\).

We observe that in general peer effects are very difficult to obtain. In the full sample peer effects are significant neither in the self-reported network nor in the parent-reported network. If we restrict the analysis only to the children for whom all information is available, however, we find that some differences between the two networks exist. While peer effects are never significant in the parent network, they are positive and significant for at least one variable (peer problem) in the self-reported network. Figure 3 shows, as an example, the two networks for peer problems are associated with difficulty in making friends and getting along with peers. Although we are not aware of previous studies that investigated the role of peer effects in peer problems, a number of authors find that peer effects are important in explaining other types of negative behavior, such as bullying (Salmivalli et. al 2010), smoking (Fletcher and Ross, 2012; Alexander et al. 2001), risky sexual behaviors, (Santor et al. 2000), drug use (Bauman and Ennettp, 1996) and juvenile delinquency (Bayer et al., 2009; Patacchini and Zenou, 2012).

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10 Peer problems are associated with difficulty in making friends and getting along with peers. Although we are not aware of previous studies that investigated the role of peer effects in peer problems, a number of authors find that peer effects are important in explaining other types of negative behavior, such as bullying (Salmivalli et. al 2010), smoking (Fletcher and Ross, 2012; Alexander et al. 2001), risky sexual behaviors, (Santor et al. 2000), drug use (Bauman and Ennettp, 1996) and juvenile delinquency (Bayer et al., 2009; Patacchini and Zenou, 2012).
two classes taken from Figures 1 and 2 (IDs are the same), where now colors indicate peer problems. Those students whose class-normalized peer problem is greater than 1 are in yellow. Going back to the discussion above on how coordinated an endogenous network formation needs to be in order for the method of Bramoullé et al. (2009) to detect a significant peer effect, we observe that yellow nodes tend to be all clustered together in the parents-reported network, while they just match in isolated couples in the self-reported networks. This result does not provide any definitive evidence on peer effects on peer problems, as the network structure is likely to be endogenous. However, it suggests that if one of the reported networks were ever to approximate the true one, the most likely candidate is the self-reported network.

**Figure 3 – Peer problems and friendship ties**

Combining the evidence derived through the comparison of reciprocated ties and the one obtained from the estimation of peer effects, we are inclined to consider the self-reported
network as a good approximation of the true friendship network. Although the evidence is far from conclusive, both criteria suggest that the structure of friendship ties in the self-reported network exhibit a much higher degree of consistency than the one in the parent-reported network. Such consistency would require a strong coordination mechanism to obtain through simple misreporting. Therefore, this is unlikely to occur. This makes us confident in assuming that the self-reported network is probably true. Obviously, a direct consequence of this assumption is that the parent-reported network is probably biased. On this basis, in the next section we will investigate the reasons why parents may have biased perception of their children’s friends.

4. **Behind the bias: why do parents misreport children friends?**

The analysis of parents’ misreporting of their children’s friends is organized as follows. First, we carry out some tests to reject alternative explanations, such as that parents tend to report the children of their friends rather than the friends of their children (Section 4.1), and that parents misreport their children’s friends because they have delayed information on the network of friends (Section 4.2). Then, in Section 4.3 we propose a mechanism based on the idea that parents have a biased perception of the social network due to an overestimation of peer effects. Because of such bias, parents of low-performing students tend to report their children to be friends of high-performing students at a higher rate than what happens in reality. In Section 4.4, we test our hypothesis on the data, while in Section 4.5 we check for the existence of assortative matching in our networks proving further evidence against those peer effects in the self-reported network.
4.1 Friendship ties among parents

One immediate explanation of the parents’ propensity to misreport the friends of their children could be that parents, lacking information on the families of the children’s classmates, tend to report the children of their friends rather than the friends of their children. In our dataset, however, we tend to reject this hypothesis on several grounds. First, the schools in our dataset are located in relatively small towns where it is plausible to assume that all families know each other.\(^\text{11}\) In addition, the Italian law\(^\text{12}\) forces parents to enroll their children in the closest school to the place of residence or, as an exception, close to the parents’ place of work. In all cases, it is highly likely the most of the parents whose children go to the same school know each other even before the school starts. This in turn reduces the possibility that parents systematically name as friends only the children of the families they know. Secondly, because of explicit request by the Italian Ministry of Education\(^\text{13}\), school managers must arrange the composition of the classes taking into consideration the individual characteristics of children and limiting the chances of discrimination. Such a constraint often leads children with similar characteristics (e.g. same school performance, if foreigners, same place of origin) and whose parents are friends to be assigned to different classes. Since in our questionnaire we ask parents to limit the nominations of their children’s friends only to classmate, it is unlikely that the observed bias derive entirely from the propensity to name only family friends. Finally, even if this possibility were salient and parents indeed had the tendency to consider only the children of their friends, we should observe that the parent-reported network exhibit some degree of assortative-mixing along several dimensions (i.e., preference for network’s nodes to attach to others that are similar in some way).

---

\(^\text{11}\) Our data have been collected in six schools based in three municipalities located in the district of Trevisso. The municipalities are quite homogeneous, they have a population between 7,000 and 12,000 inhabitants (information provided by the ISTAT, The National Institute of Statistics) and a percentage of immigrants of about 9%.

\(^\text{12}\) Decree of the President of the Republic, March 20, 2009, n. 81, art. 5, comma 2 and 3; art. 9, comma 2 and 3.

\(^\text{13}\) Circolare ministeriale n. 2, January 8, 2010.
As discussed below (Section 4.5) with particular reference to school performance, however, this is not the case: the parent-reported network exhibits disassortative-mixing and not assortative-mixing.

4.2 Parents’ delay in updating

Another explanation of the observed mismatch between self-reported and parent-reported networks can be associated with the process through which information on friendship ties is updated. If friendship ties change over time and parents update information on their children’s friends with some delay, it may happen that the structure of the two networks does not coincide. Moreover, if the delay in updating is heterogeneous across parents it may also happen that in any given period the parent-reported network exhibits a lower degree of consistency in the structure of friendship ties than the self-reported network. Both results would indeed be consistent with our data.

On this issue, our main result is that:

**Result 3:** The propensity of parents to misreport their children’s friends cannot be explained by a delay in updating their information on friendship ties.

Several tests support Result 3. First, we check the relationship between mismatch and grade. If the argument associated with the delay in updating is correct we would expect mismatch between self-reported and parent-reported network to be lower in grades where friendships are less volatile, i.e. the higher grades. This however is not the case. Although mismatch tends to decrease with grades, the correlation is not statistically significant. Moreover, standard analysis of variance reveals that the average level of mismatch does not differ among
grades. This is true both considering each grade in isolation and comparing highest grade (fourth and fifth) vs. lowest grades (first, second and third).

Another way to check if the delay in updating affects parents is to look at the difference in the percentage of reciprocated ties according to the grade. If the delay in updating argument is correct, we should expect the difference in the percentage of reciprocated ties between self-reported and parent-reported network to be narrower in high grades than in low grades. In the former, in fact, the relatively low volatility of friendship ties should diminish the propensity of parents to misreport. As shown in Table 4, however, data tend to reject this possibility. While for a degree of mismatch greater than one the difference in the percentage of reciprocated ties is statistically significant for both the first grade and the fifth grade, for a degree of mismatch greater than zero, it is significant only for the fifth grades. Such a result would indeed be difficult to explain in terms of delay in updating alone.

**Table 4 – % Reciprocated ties and grades**

<table>
<thead>
<tr>
<th>Mismatch &gt;0, Grade=5</th>
<th>Self mean (std)</th>
<th>Parents mean (std)</th>
<th>Diff. (S-P) (p-value)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mismatch &gt;0, Grade=1</td>
<td>0.537 (0.04)</td>
<td>0.500 (0.04)</td>
<td>0.037 (0.1929)</td>
</tr>
<tr>
<td>Mismatch &gt;1, Grade=5</td>
<td>0.590 (0.05)</td>
<td>0.524 (0.04)</td>
<td>0.066* (0.0702)</td>
</tr>
<tr>
<td>Mismatch &gt;1, Grade=1</td>
<td>0.567 (0.04)</td>
<td>0.491 (0.04)</td>
<td>0.075** (0.0409)</td>
</tr>
</tbody>
</table>

One final control that we implement to exclude the argument based on delay in updating consists of checking the relationship between mismatch and number of siblings. If the difference between self-reported and parent-reported network were the result of the time that it takes for
parents to adjust information on their children friends, one could expect such time to be longer for parents with many children as opposed to parents with one child only. If this is the case, then, the mismatch between self-reported and parent-reported network should be lower for only child than for child with siblings. Once more, however, data suggest a different story. For child only the average degree of mismatch is higher (3.523) than for child with siblings (2.811), and the difference is statistically significant (p-value = 0.077).

Combining these three different tests together, we tend to reject the argument associated with a delay in updating on the side of parents. Consequently, an alternative explanation for the mismatch between self-reported and parent-reported network should be offered. In the next two sections, we will provide a sketch of this alternative explanation and test its validity on the data.

4.3 Peer effect bias

In the previous subsection, we observed no evidence of peer effects for most of the behavioral variables taken into consideration, as well as for school performance. This, however, does not mean that peer effects play no role in affecting the way in which friendship ties are established. In particular, we argue that the expectations of peer effects (even when they are absent) can indeed be a major source of bias in the parents’ perception of their children’s friend.

To illustrate our argument consider a setting where children can be distinguished by the possession of a trait \( g \), where \( g \) is a trait that is positively associated with good behavioral outcome. For instance, \( g \) could be cognitive abilities that are associated with high school performance. Let us assume that the distribution of \( g \) (or a good proxy of it) is common knowledge among parents and children. What parents do not observe, however, is the real friendship network of their children. In reporting their children friends, therefore, parents rely on a set of heuristics that combine the desire for a specific type of friends for their children and the
information provided by children at home. An alternative or complementary explanation, that delivers the same outcome, is that children, who know that their parents may be unsatisfied by them, try to please them misreporting their true friendships according to the parents’ desire.

Given this information asymmetry, let’s assume that parents expect some peer effects to exist on the possession of trait $g$. Thanks to the network multiplier effect (see Section 5 below), a child without trait $g$ will have greater benefits in connecting with a child with trait $g$ the greater the latter’s number of friends. Now, if the parents of a child without trait $g$ expect the other children without trait $g$ to become friends of children with trait $g$, it will seem like their child is losing an important advantage. Therefore, they will tend to push their child to become friend of $g$-type children. Such a push, however, is the result of an initial bias in the role of peer effects.

The implication of this bias for the process through which parents report their children’s friendships is straightforward. Be either for an explicit intention of affecting the ways in which children establish friendships at school or as consequence of the ex-post information that children report at home to please their parents, parents of children without trait $g$ will tend to report more $g$-type friends than what their children have in reality. This will in turn generate a mismatch between self-reported (where we assume that the peer effect bias is absent) and parent-reported network, which is indeed observed in the data.

The mechanism that we have envisaged in order to explain the mismatch between self-reported and parent-reported networks has some clear implications for the process through friendship ties are expected to form in the two networks. In particular, assuming that school performance is a good proxy for the possession of trait $g$, we should expect that: a) children with high school performance are more popular in the parent-reported network than in the self-reported network; and b) children with low school performance are the ones that exhibit the
greater degree of mismatch between self-reported and parent-reported friends. Both points a) and b) are predictions that follow directly from the argument sketched out above. In the next subsection, we test them on the data.

4.4 Empirical test

To test the predictions outlined in Section 4.4 we estimate two distinct models. In both cases, we use the same set of explanatory variables that we employed in the estimation of peer effects in Section 3.2, which include parents’ education and work, number of siblings and gender. In addition, we include the SDQ scores and school performance among the repressors. To test the effect of school performance on popularity we consider as dependent variable the classroom-normalized in-degree (i.e. the total number of friendship nominations received by each child normalized by the classroom size) computed in the self-reported and in the parent-reported network. To test the effect of school performance on the degree of mismatch between self-reported and parent-reported friends we consider the variable mismatch described above. We estimate the models both with and without classroom fixed effects. In both cases, we estimate these models using a standard OLS.

Our main results can be summarized as follows:

Result 4: 4.1) High school performance is a stronger correlate of children’s in-degree in the parents-reported network than in the self-reported network; 4.2) For boys, the lower school performance, the greater the degree of mismatch between parents-reported and self-reported network. The combination of points 4.1) and 4.2) provides support for the existence of a peer effects bias that affect parents.

Result 4.1 derives from Table 5, which shows the outcomes of the OLS estimates on in-degree. We observe that school performance is positively associated with in-degree in both the
self-reported and the parent-reported network. The magnitude and the significance level of this effect, however, are stronger in the parent-reported network than in the self-reported network. In particular, while in the parent-reported network one standard deviation increase in school performance increases the individual’s in-degree by 53% (42% without fixed effects) of a standard deviation, in the self-reported network it increases only by 42% (39% without fixed effects). This result is in line with our prediction.

Table 5 – In-degree: self-network vs. parent-network

<table>
<thead>
<tr>
<th>Parent’s characteristics</th>
<th>Self</th>
<th>Parent</th>
<th>Self</th>
<th>Parent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sch. Performance</td>
<td>0.390*</td>
<td>0.424**</td>
<td>0.418*</td>
<td>0.530***</td>
</tr>
<tr>
<td></td>
<td>(0.23)</td>
<td>(0.21)</td>
<td>(0.23)</td>
<td>(0.20)</td>
</tr>
<tr>
<td>Male (dummy)</td>
<td>0.560**</td>
<td>0.396*</td>
<td>0.578**</td>
<td>0.459**</td>
</tr>
<tr>
<td></td>
<td>(0.27)</td>
<td>(0.24)</td>
<td>(0.26)</td>
<td>(0.23)</td>
</tr>
<tr>
<td>SDQ Emotional (index)</td>
<td>-0.093</td>
<td>-0.018</td>
<td>-0.1</td>
<td>-0.009</td>
</tr>
<tr>
<td></td>
<td>(0.07)</td>
<td>(0.07)</td>
<td>(0.07)</td>
<td>(0.07)</td>
</tr>
<tr>
<td>SDQ Conduct (index)</td>
<td>-0.119</td>
<td>0.000</td>
<td>0.003</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td>(0.11)</td>
<td>(0.10)</td>
<td>(0.11)</td>
<td>(0.09)</td>
</tr>
<tr>
<td>SDQ Hyperactive (index)</td>
<td>-0.103</td>
<td>-0.166**</td>
<td>-0.158*</td>
<td>-0.166**</td>
</tr>
<tr>
<td></td>
<td>(0.08)</td>
<td>(0.07)</td>
<td>(0.09)</td>
<td>(0.08)</td>
</tr>
<tr>
<td>SDQ Peer Prob. (index)</td>
<td>-0.124</td>
<td>-0.061</td>
<td>-0.019</td>
<td>0.015</td>
</tr>
<tr>
<td></td>
<td>(0.12)</td>
<td>(0.10)</td>
<td>(0.11)</td>
<td>(0.09)</td>
</tr>
<tr>
<td>SDQ Prosocial (index)</td>
<td>0.008</td>
<td>0.047</td>
<td>0.091</td>
<td>0.096</td>
</tr>
<tr>
<td></td>
<td>(0.08)</td>
<td>(0.07)</td>
<td>(0.07)</td>
<td>(0.06)</td>
</tr>
<tr>
<td>Constant</td>
<td>5.376**</td>
<td>2.865</td>
<td>-0.14</td>
<td>-0.133</td>
</tr>
<tr>
<td></td>
<td>(2.31)</td>
<td>(2.03)</td>
<td>(0.18)</td>
<td>(0.16)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Self</th>
<th>Parent</th>
<th>Self</th>
<th>Parent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Obs</td>
<td>452</td>
<td>452</td>
<td>452</td>
<td>452</td>
</tr>
<tr>
<td>LogL</td>
<td>-1069.66</td>
<td>-1012.399</td>
<td>-1057.933</td>
<td>-997.635</td>
</tr>
<tr>
<td>Classroom fixed effect</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>
Result 4.2 derives from Table 6, which shows the outcomes of the OLS estimates on mismatch. We estimate the model considering both the full sample and distinguishing between male and female. We observe that while in the full sample there are not significant effects, in the two subsamples there are some significant results. In particular, we find for male students that school performance is negatively correlated with mismatch (both with and without fixed effects).

<table>
<thead>
<tr>
<th>Parent's characteristics</th>
<th>Full</th>
<th>Male</th>
<th>Female</th>
<th>Full</th>
<th>Male</th>
<th>Female</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sch. Performance</td>
<td>-0.16</td>
<td>-0.495**</td>
<td>-0.036</td>
<td>-0.082</td>
<td>-0.432**</td>
<td>0.131</td>
</tr>
<tr>
<td></td>
<td>(0.16)</td>
<td>(0.23)</td>
<td>(0.22)</td>
<td>(0.15)</td>
<td>(0.21)</td>
<td>(0.21)</td>
</tr>
<tr>
<td>Male (dummy)</td>
<td>0.085</td>
<td>0.252</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.18)</td>
<td>(0.17)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SDQ Emotional (index)</td>
<td>-0.054</td>
<td>0.083</td>
<td>-0.182**</td>
<td>-0.059</td>
<td>0.067</td>
<td>-0.176**</td>
</tr>
<tr>
<td></td>
<td>(0.05)</td>
<td>(0.07)</td>
<td>(0.07)</td>
<td>(0.05)</td>
<td>(0.07)</td>
<td>(0.07)</td>
</tr>
<tr>
<td>SDQ Conduct (index)</td>
<td>-0.021</td>
<td>-0.052</td>
<td>0.05</td>
<td>0.026</td>
<td>-0.027</td>
<td>0.083</td>
</tr>
<tr>
<td></td>
<td>(0.08)</td>
<td>(0.09)</td>
<td>(0.13)</td>
<td>(0.07)</td>
<td>(0.08)</td>
<td>(0.12)</td>
</tr>
<tr>
<td>SDQ Hyperactive (index)</td>
<td>0.03</td>
<td>-0.056</td>
<td>0.189*</td>
<td>-0.01</td>
<td>-0.02</td>
<td>0.03</td>
</tr>
<tr>
<td></td>
<td>(0.07)</td>
<td>(0.08)</td>
<td>(0.11)</td>
<td>(0.06)</td>
<td>(0.08)</td>
<td>(0.09)</td>
</tr>
<tr>
<td>SDQ Peer Prob. (index)</td>
<td>0.063</td>
<td>-0.017</td>
<td>0.16</td>
<td>0.087</td>
<td>-0.05</td>
<td>0.224**</td>
</tr>
<tr>
<td></td>
<td>(0.08)</td>
<td>(0.11)</td>
<td>(0.12)</td>
<td>(0.07)</td>
<td>(0.09)</td>
<td>(0.11)</td>
</tr>
<tr>
<td>SDQ Prosocial (index)</td>
<td>0.057</td>
<td>0.123*</td>
<td>-0.059</td>
<td>0.078*</td>
<td>0.115*</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>(0.05)</td>
<td>(0.07)</td>
<td>(0.10)</td>
<td>(0.05)</td>
<td>(0.06)</td>
<td>(0.08)</td>
</tr>
<tr>
<td>Costant</td>
<td>2.598*</td>
<td>-0.029</td>
<td>3.591</td>
<td>-0.179</td>
<td>-0.006</td>
<td>-0.330*</td>
</tr>
<tr>
<td></td>
<td>(1.54)</td>
<td>(2.37)</td>
<td>(2.30)</td>
<td>(0.12)</td>
<td>(0.17)</td>
<td>(0.17)</td>
</tr>
<tr>
<td>Obs</td>
<td>452</td>
<td>220</td>
<td>232</td>
<td>452</td>
<td>220</td>
<td>232</td>
</tr>
<tr>
<td>LogL</td>
<td>-900.026</td>
<td>-417.905</td>
<td>-468.2</td>
<td>-863.845</td>
<td>-398.056</td>
<td>-453.715</td>
</tr>
<tr>
<td>Classroom fixed effects</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>
This would suggest that parents of boys with low school performance have a greater bias in reporting their children’s friends than parents of boys with high school performance.\textsuperscript{14} This result provides support to our prediction on the sources of network mismatch being based on a biased report of the child to the parents. So, anticipating the expectations of their parents, children report to be friend of other children having good school performance more than in reality. Our dataset does not allow us to detect children lying behavior, however, other studies have found that boys tend to lie more than girls (e.g. Gervais et al. 2000, Keltikangas-Jarvinen and Lindeman, 1997, and Stouthamer-Loeber, 1986). Moreover Crouter et al. (1999) found that mothers know more about daughters and fathers about sons.

In addition, we find that for female emotional problems tend to be negatively associated with mismatch (both with and without fixed effects). This result can derive from another type of parent’ bias in perceiving psychological peer effects and this bias can affect only the parents of girls.

4.5 Peer effects and assortative-mixing

In this section, we check for the existence of some degree of assortative-mixing in our networks. As discussed in footnote 4 above, in fact, the existence of assortative (or disassortative) matching among the nodes of the networks can create a bias in the estimation of peer effects. In particular, the absence of peer effects could derive from the propensity of children with dissimilar values of a trait to connect. If the effect of such trait is stronger than the effect of the variable that we want to test, then peer effects turn out to be not significant.

\textsuperscript{14} There is also a literature showing how the social relations of young boys and girls are actually different. Lindenlaub and Prummer (2013) study gender differences in the self-reported social networks from American High Schools. Stehlé et al. (2013) find the same evidence observing the actual behavior of primary school students in France. Gender differences in the way males and females develop friendship relations is also documented by Vigil, (2007); Lee et al., (2007). Gender differences in the effect of peers effect is reported by Argys and Rees, (2008); Bifulco et al. (2013); and Black et al. (2010).
Table 7 reports a test on the existence of assortative-mixing for school performance. In the parent-reported network, we find some positive degree of disassortative matching among the nodes of the network. This result provide some further evidence in favor of the argument sketched out in the previous sections, according to which the parents of low-performing children tend to over-report friends with high school performance. In the self-reported network, on the contrary, we find evidence neither of assortative-mixing nor of disassortative mixing. It follows that if we don’t observe peer effects on school performance assuming the network as exogenous, and then the network is neutral with respect to that variable, than peer effects are most likely not to exit.

<table>
<thead>
<tr>
<th>Table 7 – School performance and network disassortativity</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Self</strong></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Sch. Performance</td>
</tr>
<tr>
<td>Sch. Performance</td>
</tr>
</tbody>
</table>

On this respect, it is important to notice that the economic literature is still inconclusive on the relevance of peer effects for children of this age. De Melo (2014), for instance, finds evidence of peer effects in math and reading, but not in science. In our data, we cannot distinguish among different dimensions of school performance. Therefore, we cannot exclude that peer effects can exist for at least some type of the school activities.

5. **The multiplier effect of parents’ biased network**

In Section 4, we have supposed that the bias that parents have, in the perception of the friends of their children, is due to a bias in the perception they have about peer effects. The
mechanism of this effect could be due to a desire of the parents for a different network for their children, or even to the fact that they push their children to change friends, and they assume (possibly also because of ex-post misreporting at home from the children) that this pressure has an effect. What would happen in the outcome of the aggregate peer effect process if the network of friendship actually had a bias in the way some children (those whose parents would like them to have different friends) target friends?

In this section, we consider the two biases together. We show, with an exercise of numerical simulation,$^{15}$ that this second bias (in the perception of the network) increases even further the first bias (in the perception of the peer effect).

To do so we generate random networks of $N = 25$ nodes. These nodes are deterministically heterogeneous in terms of some attribute $g$: $G = 5$ nodes are exogenously assigned a value $g_h = 1$, while the other $N - G = 20$ have value $g_l = 0$. The network is generated according to a biased version of the Erdös and Rényi (1960) model. The five $G$ nodes cast a link to each of the other nodes (independently on their type) with i.i.d. probability $p = 0.2$, the other twenty $N - G$ have a heterophilous bias and cast a link to each $G$ node with i.i.d. probability $q \geq p$, and to each of the other $N - 1 - G$ nodes with i.i.d probability $p_0 = \frac{p(N-1)-qG}{N-1-G}$ (so that both types of nodes cast in expectation the same number of links). Then, a link is set between any two nodes of the network if at least one of the two nodes has thrown a link to the other.$^{16}$ Figure 4 shows such a network when there is no bias and $q = p$, while Figure 5 shows a network generated with $q = 0.8$ ($G$ nodes are in light blue).


$^{16}$ The resulting network will have a link between any two $G$ nodes with probability $1 - (1 - p)^2 = p(2 - p)$, a link between any two of the other $N - G$ nodes with probability $p_0(2 - p_0)$, and between any pair of different nodes with probability $p + q - pq$. 
**Figure 4 – Unbiased Random Network**

![Unbiased Random Network Diagram]

**Note:** A realization of an Erdős and Rényi (1960) random network generated between 25 nodes. Here $q = p$, so that each link between any two nodes is present with probability $p(2 - p) = 0.36$. Numbers for each node are the result of $\bar{x}$ in equation (2), with $\beta = 0.05$, $g_h = 1$ for each light-blue node, and $g_l = 0$ for the others.

**Figure 5 – Biased Random Network**

![Biased Random Network Diagram]

**Note:** A realization of a random network generated between 25 nodes, of which 5 (in light-blue) are $G$ nodes. Here $q = 0.8$, so that a link between any two $G$ nodes is present with probability $0.36$, between any two non-$G$ nodes with probability $0.05$, and between a $G$ and a non-$G$ node with probability $\sim 0.84$. As in Figure 4, numbers for each node are the result of $\bar{x}$ in equation (2), with $\beta = 0.05$. 
Once a network is generated, we call $A$ the resulting adjacency matrix, which is symmetric by construction, and we call $D_i$ the set of nodes, which are neighbors of node $i$. Then, we assume that there is a peer effect of intensity $\beta > 0$ between peers on an attribute $x$ that depends on $g$ through the implicit equation

$$x_i = g_i + \beta \sum_{j \in D_i} x_j.$$  

(1)

If we call $I$ the $N \times N$ identity matrix, the above equation is solved explicitly by

$$\bar{x} = (I - \beta A)^{-1} \bar{g},$$

(2)

which gives a stable (in a well-defined sense) and positive (for each node) result only when $\beta$ is small enough (i.e. when the absolute value of the lowest eigenvector of $A$ is less than $\frac{1}{\beta}$, for more details see Bramoullé et al, 2013). Figures 4 and 5 report beside each node also the outcome of equation (2) for the two networks, with $\beta = 0.05$. Before going into the results of the simulations, let us point out that the term $\beta A$ in equation (2) plays exactly the role of a multiplicator, and not only the value of $\beta$, but also the structure of $A$ is important. Equation (2) can be expressed as the result of an infinite series that counts all possible paths between any two nodes, where the length of these paths is discounted by powers of $\beta$ (more informal discussion is provided in Bonacich and Lloyd, 2001): when hubs are present in the network paths will be shorter and will be weighted by smaller powers of $\beta$.

For the simulations we considered 50 different values for the couple $q$ and $\beta$. For each of those 50 values of $q$ and $\beta$ we generate 1000 different networks. Figure 6 reports for each combination of $\beta$ and $q$ the boxplot of the average value of $x_i$ for the $N - G$ nodes for which $g_i = 0$, computed from equation (2) for each of the 1000 networks.
What comes clearly out from Figure 6 is that, the higher the bias $q$ in the generating process of the network, or the higher the factor $\beta$ of the peer effect, then the higher is the aggregate peer effect on the non-$G$ nodes. To put it more formally, the aggregate peer effect on non-$G$ nodes can be written as a function $f(\beta, q)$ that is increasing in both arguments.

Moreover, from the two top plots, $f$ seems linearly increasing in $q$ (not only the means, but also quartiles and outliers seem to be linear, with the same trend). Again from the two top plots of Figure 6, the steepness of this linear trend increases in $\beta$ (from something around 0.025 for $f$ in a range of 0.40 for $q$, to something around 0.160 for $f$ in a range of 0.40 for $q$).\(^{17}\) This means that the second order derivative $\frac{\partial^2 f(\beta, q)}{\partial \beta \partial q}$ is positive (it is actually a positive function of $\beta$ alone – from the two lower plots of Figure 6 it is also evident that $\frac{\partial^2 f(\beta, q)}{\partial \beta^2}$ is positive). To put it in economic terms,

Result 5: the aggregate effects of the two biases $\beta$ (peer effect bias) and $q$ (network bias) are complements, so that they reinforce each other: the perception that one of the two is higher implies that also the effect of the other is perceived to be higher.

\(^{17}\) The main reason why the aggregate peer effect on non-$G$ nodes increases with $\beta$ is clearly that for them connecting with the $G$ nodes increases the likelihood of strong direct peer effects. However, it is not the only reason, and when $\beta$ passes from 0.01 to 0.05 (the two top plots of Figure 6) the outcome from the simulations increases by a factor of almost 8 (and not 5) for each corresponding value of $q$. As discussed above, this happens because, when the $G$ nodes are hubs of the network, they not only provide the direct peer effect of their exogenous characteristic, but they also reduce paths between any two nodes and increase the multiplicator of the aggregate peer effect of the network (see also Bonacich and Lloyd, 2001, for additional intuition).
Figure 6 – Box Plots of the Simulation

Note Box plots from 1000 outcomes for each of 50 different combination of $\beta$ and $q$, for the average result of equation (2) for the non $G$ nodes, in networks generated with bias $q$.

As discussed in the Introduction, this bias falls in the category of those stemming from an erroneous perception of the social network. As long as it is a bias of the parents alone, it will reinforce the mechanism discussed in Section 4.2, as they will perceive the benefits from
selected connections to be higher. If it becomes also a bias of the econometrician, e.g. because the social networks are obtained from parents-reported (or maybe teacher-reported) networks, then it becomes a serious issue that should be taken into account.

6. Discussion and policy implications

In this paper, we find the existence of a structural difference between the network of friends self-reported by students and the network of friends reported by parents. In particular, we find that the network that most closely approximates the true network is the self-reported network, i.e. parents give a biased representation of their children's network of friends. This difference cannot be explained by a delay in updating information on friendship ties. We suggest that this bias derives from a peer effect bias: parents expect peer effects to be stronger than they are in reality. Because of such bias, parents of low-performing students report their children to be friends of high-performing students at a higher rate than what happens in reality. We test our hypothesis on school performance and find support for it. We argue by means of numerical simulations that if the parent's peer effect bias is combined with a bias on how some children target friends, then there is a multiplier effect on the expected school achievement, i.e. the two biases reinforce each other in distorting the expectations of parents.

Our results have several direct policy implications. First, a biased perception of the friend network and of the associated peers effect may influence parents’ choices, for instance on school choice. In fact, one of the most important decisions that parents make regarding their children is the choice of their schools (Bosetti, 2004). Some parents spend considerable resources in terms of time and money to choose the best school for their children but if their information is biased their decision may be biased too. Second, it is important for parents and teacher to know the true
network of friends to prevent and eventually correct negative peer effects. Friendship relationships often provide psychosocial support that positively affect healthy development (Ladd et al., 1996; Harris, 1995) but, at the same time, may produce negative behaviors such as bullying (Salmivalli et al., 2010 for a review), smoking (Fletcher and Ross, 2012; Alexander et al., 2001), risky sexual behaviors, (Santor et al. 2000), drug use (Bauman and Ennett, 1996) and juvenile delinquency (Bayer et al., 2009; Patacchini and Zenou, 2012). Third, the lack of parental knowledge can be a proxy for adolescents risky behaviors and a sign where parents (Stattin and Kerr 2000; Kerr and Stattin, 2000) and teachers have to intervene (Li et al., 2002). For instance, an increasing number of studies have shown that parental monitoring is associated with lower dropping out (Dornbusch et al. 1987; Jimerson et al. 2000, Astone and McLanahan, 1991) and that parents’ knowledge of their adolescent’s friends is positively related to their child’s achievement scores (Muller, 1993); similarly lower levels of parental knowledge is associated with alcohol and illicit drug use (DiClemente, et. al., 2001; Lahey, Van Hulle et al., 2008), cigarette smoking (Lahey et. al., 2008), risky sexual behaviors (DiClemente, et. al., 2001; Sneed, Strachman, Nguyen, and Morisky, 2009) and aggression (Slovak and Singer, 2001).

Finally, it is important to notice that in this paper we find support for a correlation between low-performing children and the bias of their parents in reporting friendship networks. In our framework we explain such correlation on the basis of a causality that runs from the children to their parents. In principle, however, nothing prevents the reverse causality to hold as well, with remarkable implications for policy. Although this reversed causality cannot be tested in our data, it opens very interesting possibilities for future research.
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