Resale and Bundling in Multi-Object Auctions

Marco Pagnozzi

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• Allowing resale after an auction helps to achieve an efficient allocation

  “Spectrum trading should be implemented as soon as possible”
  “Spectrum users should be allowed to sell or lease any unused spectrum”

(2002 Cave Report for UK Government on post-auction spectrum policy)
• Allowing resale after an auction helps to achieve an efficient allocation

  “Spectrum trading should be implemented as soon as possible”
  “Spectrum users should be allowed to sell or lease any unused spectrum”
(2002 Cave Report for UK Government on post-auction spectrum policy)

• What is the effect of resale on bidders’ strategies in multi-object auctions?
• With resale, a losing bidder can purchase the prize from the auction winner

• In **single-object** auctions, resale induces weak bidders to participate and bid aggressively, thus increasing the seller’s revenue (Pagnozzi, *RAND* 07)
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• In **multi-object** auctions, bidders often bid less than value for marginal units to reduce the auction price (*Demand Reduction ≡ DR*) (Wilson, *QJE 79*; Ausubel & Cramton, 98) (e.g., FCC auctions, California electricity markets, German GSM auction ...)

  \[ \Rightarrow \begin{cases} 
  \text{low seller’s revenue} \\
  \text{inefficient allocation} 
  \end{cases} \]
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• In **single-object** auctions, resale induces weak bidders to participate and bid aggressively, thus increasing the seller’s revenue (Pagnozzi, *RAND* 07)

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  \[ \Rightarrow \begin{cases} 
  \text{low seller’s revenue} \\
  \text{inefficient allocation} 
  \end{cases} \]

• But **resale** corrects an inefficient allocation ...

  \[ \Rightarrow \text{Resale may induce bidders to reduce demand, thus reducing the seller’s revenue} \]

  (Pagnozzi, *AEJ: Micro*)
• **Bundling** is a natural reaction by the seller to the risk of demand reduction, because it forces bidders to win all objects, or none (Anton & Yao, *QJE* 92)

• Bundling increases the seller’s revenue when resale is allowed (while its effect is ambiguous when resale is not allowed)
• **Bundling** is a natural reaction by the seller to the risk of demand reduction, because it forces bidders to win all objects, or none (Anton & Yao, *QJE* 92)

- Bundling increases the seller’s revenue when resale is allowed
  (while its effect is ambiguous when resale is not allowed)

- Bundling *and* allowing resale are **complement strategies** for the seller:
  - revenue is higher with bundling *and* resale than
    without bundling and/or without resale (if bidders are not too asymmetric)
  - resale reduces revenue without bundling but
    increases revenue with bundling
  - bundling may reduce revenue without resale but
    increases revenue with resale (and by larger amount)
Model

- Uniform-price auction, 2 bidders, 2 (identical) units

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<th>1\textsuperscript{st} unit</th>
<th>2\textsuperscript{nd} unit</th>
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<tr>
<td>Strong</td>
<td>$v^1_S$</td>
<td>$v^2_S$</td>
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<td>Weak</td>
<td>$v^1_W$</td>
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- Assumptions:
  - Bidders know values, seller does not
  - $v^1_i \geq v^2_i$ (and $v^2_S \geq v^1_W$ for today)
  - No weakly dominated strategies
  - In the resale market, $W$ obtains a share $\alpha$ and $S$ obtains a share $(1 - \alpha)$ of the gains from trade, $0 < \alpha \leq 1$
Model

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- Assumptions:
  - Bidders know values, seller does not
  - \(v_{1}^{1} \geq v_{2}^{2}\) (and \(v_{S}^{2} \geq v_{W}^{1}\) for today)
  - No weakly dominated strategies
  - In the resale market, \(W\) obtains a share \(\alpha\) and \(S\) obtains a share \((1 - \alpha)\) of the gains from trade, \(0 < \alpha \leq 1\)

**Def.** In a *DR equilibrium* bidders bid:

(i) “willingness to pay” for 1\(^{st}\) unit (weakly dominant strategy)
(ii) 0 for 2\(^{nd}\) unit

⇒ Each bidder wins 1 unit and the seller’s revenue is 0
DR without Resale

- Assume resale is not allowed
  - $i$ bids $v_i^1$ for the 1st unit (weakly dominant strategy)
  - $W$ bids 0 for the 2nd unit (since that bid can only affect the auction price)
**DR without Resale**

- Assume resale is not allowed
  - \( i \) bids \( v_i^1 \) for the 1st unit (weakly dominant strategy)
  - \( W \) bids 0 for the 2nd unit (since that bid can only affect the auction price)

\((i)\) \( S \) can win 2 units at price \( v_W^1 \) and obtain
\[
\pi_S (\text{No DR}) = v_S^1 + v_S^2 - 2v_W^1
\]

\((ii)\) \( S \) can bid 0 for the 2nd unit and obtain
\[
\pi_S (\text{DR}) = v_S^1 - 0
\]

\( \Rightarrow \) There is no DR equilibrium iff:

\[
\pi_S (\text{No DR}) > \pi_S (\text{DR}) \quad \Leftrightarrow \quad v_S^2 > 2v_W^1 \quad \text{(i.e., bidders are asymmetric)}
\]
**DR without Resale**

- Assume resale is not allowed
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\[(i)\] \(S\) can win 2 units at price \(v_W^1\) and obtain \(\pi_S (\text{No DR}) = v_S^1 + v_S^2 - 2v_W^1\)

\[(ii)\] \(S\) can bid 0 for the 2\textsuperscript{nd} unit and obtain \(\pi_S (\text{DR}) = v_S^1 - 0\)

\[\Rightarrow\] There is no DR equilibrium iff:

\[
\pi_S (\text{No DR}) > \pi_S (\text{DR}) \iff \boxed{v_S^2 > 2v_W^1} \quad \text{(i.e., bidders are asymmetric)}
\]

- If \(v_S^2 < 2v_W^1\), \(S\) prefers to win 1 unit at price 0, since outbidding \(W\) is costly
  (DR is the unique Pareto dominant equilibrium)

- With DR, the allocation is inefficient and bidders would like to trade ...
“Willingness to Pay” with Resale

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- If $W$ wins 1 unit, he resells it to $S$ at price $v^1_W + \alpha (v^2_S - v^1_W)$
  ⇒ In the auction, $S$ is willing to pay $\alpha v^2_S + (1 - \alpha) v^1_W$ for the 2nd unit

- If $W$ wins 2 units, he resells the second to $S$ at price $v^2_W + \alpha (v^1_S - v^2_W)$
  ⇒ In the auction, $S$ is willing to pay $\alpha v^1_S + (1 - \alpha) v^2_W$ for the 1st unit
Resale and DR

• **Without resale** there is no DR if bidders are asymmetric (i.e., $v_S^2 > 2v_W^1$)
Resale and DR

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- **Resale** makes bidders more “symmetric”:
  - $W$’s willingness to pay is higher (due to option to sell)
  - $S$’s willingness to pay is lower (due to option to buy)
  and $S$ can buy in the resale market after reducing demand
Resale and DR

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and $S$ can buy in the resale market after reducing demand

$\Rightarrow$ Bidders always prefer to reduce demand

\[ \text{e.g., let } b_W^1 \text{ be } W's \text{ bid for first unit, } \pi_S(\text{DR}) > \pi_S(\text{No DR}) \quad \Leftrightarrow \]

\[ v_S^1 + (1 - \alpha) (v_S^2 - v_W^1) > v_S^1 + v_S^2 - 2b_W^1 \quad \Leftrightarrow \quad b_W^1 > \frac{1}{2} [\alpha v_S^2 + (1 - \alpha) v_W^1] \]

- **Lemma:** *With resale, DR is the unique Pareto dominant equilibrium*
Seller’s Revenue

- The seller’s revenue is \( \begin{cases} 0 & \text{with DR} \\ > 0 & \text{without DR} \end{cases} \)
  - **With resale** there is always DR
  - **Without resale** there is DR iff \( v^2_S < 2v^1_W \)
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⇒ **Proposition 1**: *In a multi-unit uniform-price auction,* allowing resale reduces the seller’s revenue

- \( W \) bids more aggressively with resale (like in a single-unit auction) but this induces \( S \) to reduce demand and lowers revenue
Bundling

• It is weakly dominant to bid one’s willingness to pay for the bundle

⇒ Bundling makes it impossible for bidders to profitably reduce demand but it makes the auction price depend on the lowest valuation of the losing bidder
Bundling

- It is weakly dominant to bid one’s willingness to pay for the bundle
  \[\Rightarrow\] Bundling makes it impossible for bidders to profitably reduce demand \textit{but}
  it makes the auction price depend on the lowest valuation of the losing bidder

- If resale is not allowed, the seller’s revenue is:
  \[(a)\] With bundling: \(\Pi_{NR}^B = v_1^W + v_2^W\)
  \[(b)\] Without bundling: \(\Pi_{NR}^{NB} = \begin{cases} 
  2v_1^W & \text{if } v_2^S > 2v_1^W \text{ (No DR)} \\
  0 & \text{if } v_2^S < 2v_1^W \text{ (DR)}
\end{cases}\)

  \[\Rightarrow\] Without resale, bundling increases the seller’s revenue iff \(v_2^S < 2v_1^W\)
Bundling (Cont.)

• If resale is allowed, the seller’s revenue is:

  (a) With bundling: \( \Pi_R^B = \alpha (v^1_S + v^2_S) + (1 - \alpha) (v^1_W + v^2_W) \equiv \text{resale price} \)

  (b) Without bundling: \( \Pi_R^{NB} = 0 \) (due to DR)
Bundling (Cont.)

• If resale is allowed, the seller’s revenue is:

  \[(a) \text{ With bundling: } \Pi^B_R = \alpha (v^1_S + v^2_S) + (1 - \alpha) (v^1_W + v^2_W) \equiv \text{resale price} \]
  \[(b) \text{ Without bundling: } \Pi^{NB}_R = 0 \text{ (due to DR)} \]

⇒ **Proposition 2:** *With resale, bundling increases the seller’s revenue*

− DR always takes place with resale, and bundling eliminates DR.
Bundling + Resale

- Bundling and allowing resale are **complement strategies** for the seller

- The seller’s revenue with bundling and resale is:

\[
\Pi^B_R = \alpha (v^1_S + v^2_S) + (1 - \alpha) (v^1_W + v^2_W)
\]

e.g., in the resale market \( S \) can buy the 1\textsuperscript{st} unit for \( \alpha v^1_S + (1 - \alpha) v^2_W \) and the 2\textsuperscript{nd} unit for \( \alpha v^2_S + (1 - \alpha) v^1_W \), so she bids the sum of these prices in the auction
Bundling + Resale

- Bundling and allowing resale are complement strategies for the seller.

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  \[ \Pi^B_R = \alpha (v^1_S + v^2_S) + (1 - \alpha) (v^1_W + v^2_W) \]

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  and the 2nd unit for \( \alpha v^2_S + (1 - \alpha) v^1_W \),
  so she bids the sum of these prices in the auction.

- **Prop. 3:** Bundling and allowing resale yields a higher seller’s revenue than
  (i) not bundling and allowing resale
  (ii) bundling and not allowing resale
  (iii) not bundling and not allowing resale, if bidders are not too asymmetric

  \( (i) \Pi^B_R > \Pi^{NB}_R \) by Proposition 2
  \( (ii) \Pi^B_R > \Pi^B_{NR} \) like in single-object auctions.
(iii) \[ \Pi^B_R = \alpha (v_S^1 + v_S^2) + (1 - \alpha) (v_W^1 + v_W^2) \] > \[ \Pi^{NB}_R = \begin{cases} 2v_W^1 & \text{if } v_S^2 > 2v_W^1 \\ 0 & \text{if } v_S^2 < 2v_W^1 \end{cases} \]

\[ \Leftrightarrow 2v_W^1 > v_S^2 \text{ or } \alpha > \frac{v_W^1 - v_W^2}{v_S^1 + v_S^2 - v_W^1 - v_W^2} \]

(i.e., W’s value or W’s share of the resale surplus is not too low compared to S’s)
Bundling + Resale (Cont.)

(iii) \( \Pi^B_R = \alpha (v^1_S + v^2_S) + (1 - \alpha) (v^1_W + v^2_W) > \Pi^N_{NR} = \begin{cases} 
2v^1_W & \text{if } v^2_S > 2v^1_W \\
0 & \text{if } v^2_S < 2v^1_W 
\end{cases} \)

\[ \iff 2v^1_W > v^2_S \quad \text{or} \quad \alpha > \frac{v^1_W - v^2_W}{v^1_S + v^2_S - v^1_W - v^2_W} \]

(i.e., \( W \)'s value or \( W \)'s share of the resale surplus is not too low compared to \( S \)'s)

- Resale induces \( W \) to bid more aggressively
  and bundling prevents \( S \) from reacting by reducing demand

- This always increases revenue if there is DR without resale (i.e., \( 2v^1_W > v^2_S \))

- If there is no DR without resale (i.e., \( 2v^1_W < v^2_S \)),
  bundling makes the auction price depends on \( W \)'s lowest value,
  but resale increases \( W \)'s values if \( \alpha \) is sufficiently high (e.g., \( \alpha = \frac{1}{2} \))
Bundling + Resale (Cont.)

- Moreover, each strategy increases the effect of the other:
  - The effect of resale on revenue is stronger with bundling than without it:

\[
\underbrace{\Pi^B_R - \Pi^B_{NR}}_{>0} > \underbrace{\Pi^{NB}_R - \Pi^{NB}_{NR}}_{\leq 0}
\]
Bundling + Resale (Cont.)

• Moreover, each strategy increases the effect of the other:
  – The effect of resale on revenue is stronger with bundling than without it:
    \[
    \frac{\Pi^B_R - \Pi^B_{NR}}{>0} > \frac{\Pi^{NB}_R - \Pi^{NB}_{NR}}{\leq0}
    \]
  – The effect of bundling on revenue is stronger with resale than without it:
    \[
    \Pi^B_R - \Pi^{NB}_R = \alpha (v^1_S + v^2_S) + (1 - \alpha) (v^1_W + v^2_W) - 0 > \\
    \Pi^B_{NR} - \Pi^{NB}_{NR} = \begin{cases} 
    v^1_W + v^2_W - 0 & \text{if } v^2_S > 2v^1_W \\
    v^1_W + v^2_W - 2v^1_W & \text{if } v^2_S < 2v^1_W
    \end{cases}
    \]
Inefficient Resale Market

- Assume bidders are unable to trade with probability \((1 - p) > 0\)

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\[ S \text{ resale} \Rightarrow S \]

- Compared to \(p = 1\), \(W\) is willing to pay a lower price and \(S\) a higher price
  but there is still a DR equilibrium if \(p\) is not too low

\[
\begin{align*}
1^{\text{st}} & \quad 2^{\text{nd}} \\
S & \quad v_S^1 - p(1 - \alpha)(v_S^1 - v_W^2) \quad v_S^2 - p(1 - \alpha)(v_S^2 - v_W^1) \\
W & \quad v_W^1 + p\alpha(v_S^2 - v_W^1) \quad v_W^2 + p\alpha(v_S^1 - v_W^2)
\end{align*}
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\[
\begin{array}{c|c|c|c|c}
& 1^{\text{st}} & 2^{\text{nd}} \backslash \text{resale} & 1^{\text{st}} & 2^{\text{nd}} \\
\hline
S & v^1_S - p(1 - \alpha)(v^1_S - v^1_W) & v^2_S - p(1 - \alpha)(v^2_S - v^1_W) \\
W & v^1_W + p\alpha(v^2_S - v^1_W) & v^2_W + p\alpha(v^1_S - v^2_W) \\
\end{array}
\]

- Compared to \(p = 1\), \(W\) is willing to pay a lower price and \(S\) a higher price but there is still a DR equilibrium if \(p\) is not too low

- Without resale there is no DR equilibrium if \(v^2_S > 2v^1_W\)

⇒ **Proposition 4:** If \(v^2_S > 2v^1_W\) and \(p > \frac{v^2_S - 2v^1_W}{(1 + \alpha)(v^2_S + v^1_W)}\), allowing resale reduces efficiency with probability \((1 - p)\)

- The possibility of resale increases efficiency ex post, but it may induce demand reduction even if bidders may be unable to trade
Summary

- The possibility of resale affects bidders’ strategies

- In multi-object auctions, resale increases bidders’ incentives to reduce demand (by increasing the willingness to pay of low-value bidders and reducing the willingness to pay of high-value bidders)

- So resale may reduce both revenue and efficiency

- Allowing resale and bundling (often) increase the seller’s revenue