Collusion and Speculators in Multi-Object Auctions

Marco Pagnozzi

Università di Napoli Federico II and *CSEF*

pagnozzi@unina.it

- **Speculators** (who have no use value for the auction prize) are attracted to an auction if they can resell to high-value bidders

– *Does the presence of independent speculators and their threat of entry in the auction prevent collusion among bidders?*

- **Speculators** (who have no use value for the auction prize) are attracted to an auction if they can resell to high-value bidders

– *Does the presence of independent speculators and their threat of entry in the auction prevent collusion among bidders?*

"Should the seller encourage speculators, because additional bidders create more competition in the auction?

Or should the seller discourage them, because value captured by speculators must come from someone else's payoff – possibly the seller's?" (Milgrom, 2004)

- *Can speculators win an auction?*
- *Why should a high-value bidder let speculators win?*

- In **single-object** auctions, it is unclear why a high-value bidder should prefer to buy in the resale market

Example

- **Single-object** ascending auction (with full information):
  - 1 bidder (B) with value 8
  - 1 speculator (S) with value 0

Example

- **Single-object** ascending auction (with full information):
  - 1 bidder (B) with value 8
  - 1 speculator (S) with value 0

- With equal sharing of resale surplus, if S wins, he resells at price $\frac{1}{2}(8+0)=4$

$$\Rightarrow \begin{cases} S \text{ is willing to bid up to } 4 \text{ in the auction} \\ B \text{ is willing to bid up to } 4 \text{ in the auction} \end{cases}$$

- B is *indifferent* between winning the auction and buying from the speculator

$\Rightarrow$ Multiple equilibria, but resale is not robust to an (arbitrarily small) resale cost

– *Can speculators win an auction?*

– *Why should a high-value bidder let speculators win?*

- In **single-object** auctions, it is unclear why a high-value bidder should prefer to buy in the resale market

- In **multi-object** auctions, bidders often bid less than value for marginal units to reduce the auction price (*Demand Reduction* ≡ DR)
  (Wilson, 1979; Ausubel & Cramton, 1998)
  (e.g., FCC auctions, German GSM auction, California electricity markets ...)

⇒ A high-value bidder may *strictly* prefer to let speculators win some objects in order to keep the auction price low for the objects she wins
(and then buy from speculators in the resale market)

Example (Cont.)

- **Multi-object** uniform-price auction**:**
  - 2 units, 1 bidder (B), 1 speculator (S)
  - 2 highest bids win and pay 3rd-highest bid

	1st unit	2nd unit
B	8	8
S	0	0

$\overset{\text{resale}}{\Rightarrow}$

	1st unit	2nd unit
B	4	4
S	4	4

- As before, B can win the 2 units in the auction at price 4 ... but ...

Example (**Cont**.)

- **Multi-object** uniform-price auction**:**
  - 2 units, 1 bidder (B), 1 speculator (S)
  - 2 highest bids win and pay 3[rd]-highest bid

	1st unit	2nd unit
B	8	8
S	0	0

$\overset{\text{resale}}{\Rightarrow}$

	1st unit	2nd unit
B	4	4
S	4	4

- As before, B can win the 2 units in the auction at price 4 ... but ...
- With DR $\left\{ \begin{array}{l} B \text{ bids } (4;\ 0) \\ S \text{ bids } (4;\ 0) \end{array} \right\}$, B and S win one unit each and S resells:

$$\pi_B = \underbrace{8-0}_{\text{auction profit}} + \underbrace{8-4}_{\text{resale surplus}}$$

$\Rightarrow$ B *strictly* prefers DR

$$\Rightarrow \left\{ \begin{array}{l} \text{Resale is (the Pareto dominant for } B \text{ and } S\text{) equilibrium} \\ S \text{ wins and the seller's revenue is } 0 \end{array} \right.$$

Example (**Cont**.)

- Resale is an equilibrium even with:

  (i) Different sharing of resale surplus, and even a take-or-leave offer by S

  (ii) Not fully efficient resale market (e.g. if $0 < \Pr[\text{no resale}] \leq \frac{1}{3}$)

  (iii) (Not too large) resale cost (i.e. if $c < \frac{1}{3} \cdot B$'s value)

UK 3.4GHz Auction (June 2003)

- Simultaneous Ascending Auction for 15 licenses for broadband wireless services
  - **PCCW** (a Hong-Kong telecom company) was the highest-value bidder and was expected to win 15 licenses
  - **Red Spectrum** and **Public Hub** were companies created for the auction and chose to be eligible for only 1 license

- As soon as PCCW, RS and PH were the only bidders left, PCCW reduced demand to 13 licenses to end the auction
- PCCW's failure to win all licenses was described as
  - "a surprise, ... a gaffe"
  - "a costly mistake that may cost the chance of offering a nationwide service"

UK 3.4GHz Auction (June 2003)

- Simultaneous Ascending Auction for 15 licenses for broadband wireless services
  - **PCCW** (a Hong-Kong telecom company) was the highest-value bidder and was expected to win 15 licenses
  - **Red Spectrum** and **Public Hub** were companies created for the auction and chose to be eligible for only 1 license

- As soon as PCCW, RS and PH were the only bidders left, PCCW reduced demand to 13 licenses to end the auction
- PCCW's failure to win all licenses was described as
  - "a surprise, ... a gaffe"
  - "a costly mistake that may cost the chance of offering a nationwide service"

But was it really a mistake?

- By March 2004, PCCW took over RS and PH and obtained all licenses

Related Literature

- In **single-object** auctions, resale can take place:
  - if some bidders do not participate in the auction or valuations change (Bikhchandani & Huang *RFS*, 1989; Haile *GEB*, 2003 ...)
  - in 1^{st}-price asymmetric auctions with uncertainty (Hafalir & Krishna *AER*, 2007 ...)
  - if speculators induce bidders to bid 0 by bidding "aggressively" (Garratt & Tröger *Econometrica*, 2006)
  - if the auction price affects bargaining in the resale market (Pagnozzi *RAND*, 2007)

- Resale allows **tacit collusion** in English auctions (Garrat, Tröger & Zheng *Econometrica*, 2009)

- In **multi-object** auctions, demand reduction induces resale (Pagnozzi *AEJ*, forthcoming)

Model

- Uniform-price auction for 2 (identical) units

  (2 highest bids win and pay 3rd-highest bid)

  - 2 **colluding bidders** with values $v_1 > v_2$ (flat demand)
  - 2 **independent speculators** who can pay c to enter the auction

	1st unit	2nd unit
B_1	v_1	v_1
B_2	v_2	v_2
S_1	0	0
S_2	0	0

Model

- Uniform-price auction for 2 (identical) units
  (2 highest bids win and pay 3^{rd}-highest bid)
  - 2 **colluding bidders** with values $v_1 > v_2$ (flat demand)
  - 2 **independent speculators** who can pay c to enter the auction

	1^{st} unit	2^{nd} unit
B_1	v_1	v_1
B_2	v_2	v_2
S_1	0	0
S_2	0	0

- Timing:
  1. Speculators simultaneously choose whether to enter
  2. Bidders observe entry and the auction starts
- Assumptions:

  (i) B_i and S_i know values, seller does not – e.g., Wilson 1979

  (ii) No weakly dominated strategy

  (iii) In the resale market, a unit can be traded only once and players equally share the gains from trade (Nash bargaining)

Bargaining for Resale

– *At what price does S_i resell to B_1?*

- Our qualitative results hold with many alternative assumptions on bargaining:

1. Equal sharing of the gains fro trade: $\frac{1}{2}v_1$

   (without collusion, B_2 resells to B_1 at $\frac{1}{2}(v_1 + v_2)$)

2. Take-it-or-leave-it offer: v_1

3. "Multi-parties" bargaining: $\frac{1}{2}(v_1 + v_2)$

   (when a unit can be sold more than once)

4. Unequal bargaining power: $\alpha_S \cdot v_1, \quad 0 < \alpha_S \leq 1$

...

Benchmark: No Collusion, No Speculator

- If B_2 wins a unit, he resells to B_1 at price $\frac{1}{2}(v_1 + v_2)$
- So B_2 is willing to bid up to $\frac{1}{2}(v_1 + v_2)$ for a unit

(We assume $v_1 > 2v_2$, so there is no "tacit" collusion)

Benchmark: No Collusion, No Speculator

- If B_2 wins a unit, he resells to B_1 at price $\frac{1}{2}(v_1 + v_2)$
- So B_2 is willing to bid up to $\frac{1}{2}(v_1 + v_2)$ for a unit

(We assume $v_1 > 2v_2$, so there is no "tacit" collusion)

$\Rightarrow$ B_1 wins both units at price $\frac{1}{2}(v_1 + v_2)$ and obtains

$$\pi^{NC} = 2v_1 - 2\frac{1}{2}(v_1 + v_2) = v_1 - v_2$$

- Speculators do not enter if bidders do not collude,
  because they cannot obtain positive profit (with 2 bidders and 2 units)

Case (i): No Speculator Enters

- B_1 and B_2 win both units at price 0 and share the collusive profit

$$\pi^* = \underbrace{2\tfrac{1}{2}(v_1 + v_2)}_{\substack{\text{auction price} \\ \text{without collusion}}} - \underbrace{0}_{\substack{\text{auction price} \\ \text{with collusion}}} = v_1 + v_2$$

Case (i): No Speculator Enters

- B_1 and B_2 win both units at price 0 and share the collusive profit

$$\pi^* = \underbrace{2\tfrac{1}{2}(v_1 + v_2)}_{\substack{\text{auction price} \\ \text{without collusion}}} - \underbrace{0}_{\substack{\text{auction price} \\ \text{with collusion}}} = v_1 + v_2$$

- Collusion usually requires bidders to make illegal side payments, but ... (McAfee & McMillan, 1992)

Case (i): No Speculator Enters

- B_1 and B_2 win both units at price 0 and share the collusive profit

$$\pi^* = \underbrace{2\tfrac{1}{2}(v_1 + v_2)}_{\substack{\text{auction price} \\ \text{without collusion}}} - \underbrace{0}_{\substack{\text{auction price} \\ \text{with collusion}}} = v_1 + v_2$$

- Collusion usually requires bidders to make illegal side payments, but ...
  (McAfee & McMillan, 1992)
- Even without side payments, with resale there is an efficient and IC collusive mechanism where each bidder wins one unit at 0 and
  B_2 resells for $r \in [0;\ v_1]$
  ($r = \frac{1}{2}\pi^*$ gives equal sharing of collusive profit)

$\Rightarrow$ Resale allow bidders to share collusive profit *without illegal side payments*

Case (ii): 1 Speculator Enters

- B_1 and B_2 act as a single bidder B
- If S wins, he resells to B at price $\frac{1}{2}v_1$
- B can buy in resale market at price $\frac{1}{2}v_1$

Case (ii): 1 Speculator Enters

- B_1 and B_2 act as a single bidder B

- If S wins, he resells to B at price $\frac{1}{2}v_1$
- B can buy in resale market at price $\frac{1}{2}v_1$

- "Valuations" with resale are

	1st unit	2nd unit
B	v_1	v_1
S	0	0

$\overset{\text{resale}}{\Longrightarrow}$

	1st unit	2nd unit
B	$\frac{1}{2}v_1$	$\frac{1}{2}v_1$
S	$\frac{1}{2}v_1$	$\frac{1}{2}v_1$

Case (ii): 1 Speculator Enters

- B_1 and B_2 act as a single bidder B

- If S wins, he resells to B at price $\frac{1}{2}v_1$
- B can buy in resale market at price $\frac{1}{2}v_1$

- "Valuations" with resale are

	1st unit	2nd unit
B	v_1	v_1
S	0	0

$\overset{\text{resale}}{\Longrightarrow}$

	1st unit	2nd unit
B	$\frac{1}{2}v_1$	$\frac{1}{2}v_1$
S	$\frac{1}{2}v_1$	$\frac{1}{2}v_1$

- Bidding your "valuation" for the 1^{st} unit is a dominant strategy

Def.: In a **(Zero-Price) Demand Reduction equilibrium** $\begin{cases} B \text{ bids } \left(\frac{1}{2}v_1;\ 0\right) \\ S \text{ bids } \left(\frac{1}{2}v_1;\ 0\right) \end{cases}$

(Zero-Price) DR Equilibrium

$\Rightarrow$ In a DR equilibrium, each player wins 1 unit at 0 and S resells for $\frac{1}{2}v_1$:

$$\pi_S^* = \frac{1}{2}v_1 \qquad\qquad \pi_B^* = 2v_1 - \frac{1}{2}v_1 = \frac{3}{2}v_1$$

- S has no incentive to deviate from the DR equilibrium (since to win more units he has to pay $\frac{1}{2}v_1$)
- B has no incentive to deviate from the DR equilibrium (since to win more units he raises the price and obtains $2\left(v_1 - \frac{1}{2}v_1\right) = v_1$)

(Zero-Price) DR Equilibrium

⇒ In a DR equilibrium, each player wins 1 unit at 0 and S resells for $\frac{1}{2}v_1$:

$$\pi_S^* = \frac{1}{2}v_1 \qquad\qquad \pi_B^* = 2v_1 - \frac{1}{2}v_1 = \frac{3}{2}v_1$$

- S has no incentive to deviate from the DR equilibrium (since to win more units he has to pay $\frac{1}{2}v_1$)
- B has no incentive to deviate from the DR equilibrium (since to win more units he raises the price and obtains $2\left(v_1 - \frac{1}{2}v_1\right) = v_1$)

⇒ Colluding bidders reduce demand and accommodate the speculator

- The DR equilibrium is the *Pareto dominant equilibrium* for B and S (among all equilibria in undominated strategies)

Case (iii): 2 Speculators Enter

- Bidders have no incentive to let speculators win both units (because in this case they obtain no profit in the auction)

- "Valuations" with resale are

	1^{st} unit	2^{nd} unit
B	v_1	v_1
S_1	0	0
S_2	0	0

$\overset{\text{resale}}{\Rightarrow}$

	1^{st} unit	2^{nd} unit
B	$\frac{1}{2}v_1$	$\frac{1}{2}v_1$
S_1	$\frac{1}{2}v_1$	$\frac{1}{2}v_1$
S_2	$\frac{1}{2}v_1$	$\frac{1}{2}v_1$

Case (iii): 2 Speculators Enter

- Bidders have no incentive to let speculators win both units (because in this case they obtain no profit in the auction)

- "Valuations" with resale are

	1st unit	2nd unit
B	v_1	v_1
S_1	0	0
S_2	0	0

$\overset{\text{resale}}{\Rightarrow}$

	1st unit	2nd unit
B	$\frac{1}{2}v_1$	$\frac{1}{2}v_1$
S_1	$\frac{1}{2}v_1$	$\frac{1}{2}v_1$
S_2	$\frac{1}{2}v_1$	$\frac{1}{2}v_1$

- B, S_1 and S_2 bid $\frac{1}{2}v_1$ for the first unit

$\Rightarrow$ Competition among speculators raises the auction price to $\frac{1}{2}v_1$ and drives speculators' profit to zero

- With arbitrarily small resale cost or discounting, B wins 2 units

Equilibrium Entry

- Speculators simultaneously choose whether to pay c and enter the auction
- There is no *symmetric* pure-strategy equilibrium:
  - if S_i enters, S_j prefers to stay out
  - if S_i stays out, S_j prefers to enter

Equilibrium Entry

- Speculators simultaneously choose whether to pay c and enter the auction
- There is no *symmetric* pure-strategy equilibrium:
  - if S_i enters, S_j prefers to stay out
  - if S_i stays out, S_j prefers to enter

- In the (symmetric) mixed-strategy equilibrium, S_i enters with probability p that makes him indifferent between entering and staying out:

$$\mathbb{E}\left[\text{auction profit}\right] = \underbrace{(1-p)}_{\Pr[S_j \text{ stays out}]} \times \frac{1}{2}v_1 + \underbrace{p}_{\Pr[S_j \text{ enters}]} \times 0 = \underbrace{c}_{\text{entry cost}}$$

$$\Leftrightarrow \quad p = 1 - \frac{2c}{v_1}$$

$\Rightarrow$ Free entry in the auction drives speculators' profit to zero

Effect of Speculators on Collusion

- The seller's revenue is:

  $= 0$ if speculators do not exist or do not enter (since bidders collude)

  $= 0$ if 1 speculator enters (since players reduce demand)

  $= \frac{1}{2} v_1$ if 2 speculators enter

- **Proposition.** *The presence of speculators who may enter the auction disrupts collusion and increases the seller's revenue iff 2 speculators enter – i.e., with probability*

$$p^2 = \left(1 - \frac{2c}{v_1}\right)^2$$

Collusive Profits

(i) Without speculators:

$$\pi^* = \underbrace{2\tfrac{1}{2}(v_1 + v_2)}_{\text{price without collusion}} - \underbrace{0}_{\text{price with collusion}} = v_1 + v_2$$

$\Rightarrow B_1$ wins 2 units and pays $t \in [0, \pi^*]$ to B_2

Collusive Profits

(i) Without speculators:

$$\pi^* = \underbrace{2\tfrac{1}{2}(v_1 + v_2)}_{\text{price without collusion}} - \underbrace{0}_{\text{price with collusion}} = v_1 + v_2$$

$\Rightarrow B_1$ wins 2 units and pays $t \in [0, \pi^*]$ to B_2

(ii) With 1 speculator:

$$\pi^{**} = 2\tfrac{1}{2}(v_1 + v_2) - \underbrace{\tfrac{1}{2}v_1}_{\text{resale price}} = \tfrac{1}{2}v_1 + v_2 < \pi^*$$

$\Rightarrow B_1$ wins 1 unit, buys the other from S, and pays $t \in [0, \pi^{**}]$ to B_2

Collusive Profits

(i) Without speculators:

$$\pi^* = \underbrace{2\tfrac{1}{2}(v_1 + v_2)}_{\text{price without collusion}} - \underbrace{0}_{\text{price with collusion}} = v_1 + v_2$$

$\Rightarrow B_1$ wins 2 units and pays $t \in [0, \pi^*]$ to B_2

(ii) With 1 speculator:

$$\pi^{**} = 2\tfrac{1}{2}(v_1 + v_2) - \underbrace{\tfrac{1}{2}v_1}_{\text{resale price}} = \tfrac{1}{2}v_1 + v_2 < \pi^*$$

$\Rightarrow B_1$ wins 1 unit, buys the other from S, and pays $t \in [0, \pi^{**}]$ to B_2

(iii) With 2 speculators:

$$\pi^{***} = 2\tfrac{1}{2}(v_1 + v_2) - \underbrace{2\tfrac{1}{2}v_1}_{\text{auction price}} = v_2 < \pi^{**}$$

$\Rightarrow B_1$ wins 2 units and pays $t \in [0, \pi^{***}]$ to B_2

- There is equal sharing of collusive profit if $t = \frac{1}{2}\pi$

Interesting Effects of Speculators?

- Collusive profits depend on the number of speculators in the auction

⇒ Side payments among bidders must depend on the number of speculators

1. High-value bidders have incentive to create *fake speculators* to reduce side payments to other colluding bidders

e.g.: With no real speculators, if B_1 also acts as a speculator he wins 2 units and only pays $\frac{1}{2}\pi^{**} < \frac{1}{2}\pi^{*}$ to B_2

Interesting Effects of Speculators?

- Collusive profits depend on the number of speculators in the auction

⇒ Side payments among bidders must depend on the number of speculators

1. High-value bidders have incentive to create *fake speculators* to reduce side payments to other colluding bidders

e.g.: With no real speculators, if B_1 also acts as a speculator he wins 2 units and only pays $\frac{1}{2}\pi^{**} < \frac{1}{2}\pi^{*}$ to B_2

2. Collusion is harder if side payments must be done ex-ante (because low-value bidders require them to refrain from bidding)

e.g.: With private information on valuations ...

Effect of Resale on Seller's Revenue

– *Should the seller allow resale to attract speculators and reduce collusion?*

Effect of Resale on Seller's Revenue

– *Should the seller allow resale to attract speculators and reduce collusion?*

- Resale has 2 effects on bidders' ability to collude:

(i) **Competition Effect:**

  – Speculators increase the number of competitors,
  so bidders may need to bid higher to win and collusion may be harder

Effect of Resale on Seller's Revenue

– *Should the seller allow resale to attract speculators and reduce collusion?*

- Resale has 2 effects on bidders' ability to collude:

(i) **Competition Effect:**

– Speculators increase the number of competitors,
so bidders may need to bid higher to win and collusion may be harder

(ii) **Profit Sharing:**

– Resale allows bidders to share collusive profit by trading after the auction,
without illegal side payments, so collusion may be easier

Conclusions

- Speculators are attracted by the possibility of resale

- If few speculators enter a multi-object auction, colluding bidders reduce demand and accommodate them in order to keep the auction price low

- Free entry of speculators drives their expected profit to zero, but it does not drive colluding bidders' profit to zero

$\Rightarrow$ The presence of many potential speculators does not always reduce collusion

- Collusion is reduced and the seller's revenue is increased only if sufficiently many speculators enter but eventually lose