ABSTRACT. The housing boom that preceded the Great Recession was due to an increase in credit supply driven by looser lending constraints in the mortgage market. This view on the fundamental drivers of the boom is consistent with four empirical observations: the unprecedented rise in home prices and household debt, the stability of debt relative to house values, and the fall in mortgage rates. These facts are difficult to reconcile with the popular view that attributes the housing boom to looser borrowing constraints associated with lower collateral requirements. In fact, a slackening of collateral constraints at the peak of the lending cycle triggers a fall in home prices in our framework, providing a novel perspective on the possible origins of the bust.

1. INTRODUCTION

The U.S. economy has recently experienced a very severe financial crisis, which precipitated the worst recession since the Great Depression. The behavior of the housing and mortgage markets in the first half of the 2000s was the crucial factor behind these events. Four key empirical facts characterize this behavior in the period leading up to the collapse in house prices and the ensuing financial turmoil.

FACT 1: House prices rose dramatically. Between 2000 and 2006 real home prices increased anywhere between 40 and 70 percent, as shown in Figure 1.1. This boom is unprecedented in U.S. history, and was followed by an equally spectacular bust after 2006.

FACT 2: Households borrowed against the rising value of their real estate, expanding mortgage debt relative to income by a substantial amount. This fact is illustrated in figure 1.2, for both the entire household sector and financially constrained households—the group that is most informative for the parameterization of our model. Both measures of
Figure 1.1. Real house prices. The two measures of house prices are the FHFA (formerly OFHEO) all-transactions house price index and the CoreLogic repeated-sales index. Both indexes are deflated by the consumer price index, and normalized to 100 in 2000:Q1.

indebtedness were stable in the 1990s, but increased by about 30 and 60 percentage points between 2000 and 2007, before beginning to fall during the financial crisis.

**Fact 3:** The increase in mortgage debt kept pace with the rapid appreciation of house prices, leaving the ratio of mortgages to the value of real estate roughly unchanged. This often under-appreciated fact is shown in figure 1.3, where we can also see that this measure of household leverage actually spiked when home values collapsed immediately before the recession.

**Fact 4:** Mortgage rates declined substantially. Figure 1.4 plots the 30-year conventional mortgage rate minus various measures of inflation expectations from the Survey of Professional Forecasters. Real mortgage rates were stable around 5% during the 1990s, but declined substantially afterward, with a fall of about 2.5 percentage points between 2000 and 2005.

We argue that the key factor behind these four stylized facts was the relaxation of *lending constraints* since the late 1990s, which led to a significant expansion in the supply of mortgage loans. This simple mechanism provides a much more successful explanation of the boom phase of the cycle than the relaxation of *borrowing constraints* alone, which have
been the main focus of the literature so far. In fact, a slackening of borrowing constraints that takes place when the credit boom is still in full swing triggers a fall in home prices in our framework, providing a novel perspective on the origins of the bust.

We develop this argument in a very simple model, to make the distinction between credit supply and loan demand as stark as possible. As in the literature spawned by Kiyotaki and Moore (1997), a collateral constraint limits households’ ability to borrow against the value of real estate, thus affecting their demand for credit. In addition, a lending constraint limits the supply of credit to the mortgage market. For simplicity, we impose this constraint
directly on the savers, but we show that a leverage restriction on financial intermediaries would produce the same results.

The lending constraint, which is the key new feature of our model, captures a combination of technological, institutional, and behavioral factors that restrain the flow of funds from savers to mortgage borrowers. Starting in the late 1990s, the diffusion of securitization and market-based financial intermediation lowered many of these barriers, relaxing the constraint. For example, the pooling and tranching of mortgages into mortgage...
backed securities (MBS), with the resulting creation of highly-rated assets, contributed to channel into mortgage products a large pool of savings traditionally directed towards government debt. Thanks to this innovation, institutional investors that face restrictions on the riskiness of their assets—such as money market mutual funds, pension funds and insurance companies—could finance mortgages through their investments in MBS (Brunnermeier, 2009). Furthermore, by allowing banks to convert illiquid loans into liquid funds, securitization reduced banks’ holdings of liquid securities and increased their ability to lend (Loutskina, 2011). On the regulatory side, banks’ capital requirements imply lower charges for agency MBS (and the senior tranches of private-label ones) than for mortgages themselves. The resulting regulatory arbitrage, combined with the rise of off-balance-sheet vehicles, which allowed their sponsors to increase leverage without raising new capital, significantly increased the ability of banks to supply credit (Brunnermeier, 2009, Acharya and Richardson, 2009, Acharya, Schnabl, and Suarez, 2013). Some of the changes in financial intermediation we just described date back to the 1970s, when the Government Sponsored Enterprises created the first mortgage backed securities (e.g. Gerardi, Rosen, and Willen, 2010, Fostel and Geanakoplos, 2012). However, securitization did not take off until the
late 1990s and early 2000s, with the expansion of private-label MBS beyond conforming mortgages and ultimately into subprime products. This is the phase of the credit boom we focus on.

International factors also played an important role in increasing the supply of funds to U.S. mortgage borrowers (Justiniano, Primiceri, and Tambalotti, 2014b). The rapid deterioration of the current account deficit in the first half of the 2000s channeled large pools of international savings, mostly from East Asia and the oil-exporting countries, towards domestic borrowers. Many of these funds found their way into the mortgage market through the purchase of MBS, as documented by Bernanke, Bertaut, DeMarco, and Kamin (2011). In our simple model, this inflow of foreign funds into mortgage products is isomorphic to a slackening of the domestic lending constraint, resulting in a shift in the overall amount of funds available to the borrowers.

We interpret these developments in financial intermediation as sources of more relaxed lending constraints, and use our model to analyze their impact on the rest of the economy, both qualitatively and quantitatively. For the quantitative part of the analysis, we calibrate the model to match some key properties of the balance sheet of U.S. households in the 1990s, using the Survey of Consumer Finances.

A key assumption underlying this quantitative exercise is that the US economy in the 1990s was constrained by a limited supply of funds to the mortgage market, rather than by a scarcity of housing collateral. Starting from this situation, we show that a progressive loosening of the lending constraint in the residential mortgage market increases household debt (fact 2). If the resulting shift in the supply of funds is sufficiently large, the availability of collateral also becomes a binding constraint. As a consequence, a further expansion of the lending limit boosts the collateral value of houses, increasing their price (fact 1), while the interest rate falls (fact 4). The “endogenous” relaxation of the borrowing constraint associated with higher real estate values leads to a parallel increase in household debt, leaving leverage unchanged (fact 3). This is in contrast with the effects of an “exogenous” relaxation of the borrowing constraint through lower down-payments, which would not lead to lower interest rates and would produce an increase in household leverage. When combined with a binding lending constraint, however, a marginal relaxation of collateral requirements produces a decline in house prices, pointing to a potential trigger for the crisis.
This reconstruction of the stylized facts that characterize the recent credit and housing cycle is consistent with the micro-econometric evidence of Mian and Sufi (2009, 2011). They show that an expansion in credit supply was the fundamental driver of the unprecedented boom in household debt, and that borrowing against the increased value of real estate by existing homeowners accounts for a significant fraction of this build-up in debt. Our model, with its emphasis on the role of lending as opposed to borrowing constraints, provides a clean theoretical framework to interpret this evidence. Such a framework is particularly relevant because subsequent work has demonstrated the far-reaching repercussions of the boom and bust in household debt and in real estate values on other macroeconomic outcomes, such as durable consumption and employment, both before and after the crisis (Mian and Sufi, 2010, 2014a,b, Mian, Rao, and Sufi, 2013, Charles, Hurst, and Notowidigdo, 2014a,b).

This paper is related to the recent literature on the causes and consequences of the financial crisis. As Eggertsson and Krugman (2012), Guerrieri and Lorenzoni (2012), Hall (2012), Midrigan and Philippon (2011), Favilukis, Ludvigson, and Nieuwerburgh (2013), Boz and Mendoza (2014), and Justiniano, Primiceri, and Tambalotti (2014a,b), we use a model of household borrowing to analyze the drivers of the boom and bust in credit and house prices that precipitated the Great Recession. As in many of these studies, borrowing is limited by a collateral constraint à la Kiyotaki and Moore (1997), backed by houses as in Iacoviello (2005) and Campbell and Hercowitz (2009b). Our main innovation relative to this literature, which focuses exclusively on borrowing limits, is the introduction of a lending constraint. The interaction between lending and borrowing constraints in our model generates rich patterns of debt and home values that resemble those in the data, both during the boom, as well as around the cycle’s turning point, when the boom turns into bust. This interaction between the constraints also sets our work apart from Adam, Kuang, and Marcet (2012), Garriga, Manuelli, and Peralta-Alva (2012) and Kermani (2012), who study the effects of a reduction in the world interest rate on a small open economy with borrowing constraints. These effects are qualitatively similar to those of a relaxation of the lending constraint in our framework. However, they miss the interaction between collateral and lending constraints that is at the center of our analysis of the boom and of the spark that triggered the bust.
Another important difference is that we interpret the financial liberalization of the early 2000s as a slackening of lending constraints. On the contrary, most of this literature captures variation in the availability of credit only through changes in the tightness of the borrowing constraint. This approach to modeling changes in credit conditions is virtually a default option for these papers, since they do not consider any impediment to lending.\footnote{This modeling device is so common in the literature on household debt that it has also become the foundation of many recent normative studies on macroprudential regulation (e.g. Bianchi, Boz, and Mendoza, 2012, Mendicino, 2012, Bianchi and Mendoza, 2012, 2013, Lambertini, Mendicino, and Punzi, 2013 Farhi and Werning, 2013, Korinek and Simsek, 2014).} We deviate from this practice and model financial liberalization as a relaxation of lending constraints for two reasons. First, the micro-econometric evidence of Ambrose and Thibodeau (2004), Mian and Sufi (2009), Dell'Ariccia, Igan, and Laeven (2012) and Favara and Imbs (2012) attributes the debt expansion of the first half of the 2000s, and the resulting boom in house prices, to a shift in credit supply. This evidence translates directly into a relaxation of lending constraints in our model. Second, in models with a borrowing constraint à la Kiyotaki and Moore (1997), looser collateral requirements lead to an outward shift in the demand for credit. Such a shift is difficult to reconcile with the falling interest rates and stable debt-to-collateral values seen in the data, as we show in our analysis.

The reference to looser collateral requirements as a credit demand shock might sound surprising, since it is common to think of loan-to-value ratios as set by financial intermediaries. If this is the case, it would seem plausible for intermediaries to accept lower down-payments following an increase in their ability to lend. This link between collateral requirements and lending limits is absent in the workhorse model of collateralized borrowing of Kiyotaki and Moore (1997), but it might play a role in practice, resulting in a connection between movements in the supply and demand of credit. Even in this case, however, our results suggest that a satisfactory account of the credit boom requires credit supply to shift more than loan demand in response to their common determinants.

Even though our approach to modeling financial liberalization focuses on lending constraints, it does not rule out a parallel loosening of collateral requirements, which arguably also took place as house prices were rising. Duca, Muellbauer, and Murphy (2011) and Favilukis, Ludvigson, and Nieuwerburgh (2013), for instance, document a reduction in down-payments on new mortgages to first-time home buyers and subprime borrowers during the boom. Our results, however, do imply that the aggregate impact of looser collateral
requirements during the boom was smaller than that of the expansion in credit supply brought about by a progressive erosion of the existing barriers to lending.

Our study also builds on a vast literature that studies the microeconomic foundations of leverage restrictions on financial intermediaries, in environments with agency or informational frictions (e.g. Holmstrom and Tirole, 1997, Adrian and Shin, 2008, Geanakoplos, 2010, Gertler and Kiyotaki, 2010, Gertler and Karadi, 2011, Christiano and Ikeda, 2013, Bigio, 2013, Simsek, 2013). As Adrian and Shin (2010a), Gertler, Kiyotaki, and Queralto (2012), Adrian and Boyarchenko (2012, 2013), Dewachter and Wouters (2012), He and Krishnamurthy (2013), and Brunnermeier and Sannikov (2014), we take these restrictions as given. These papers focus on risk as the fundamental determinant of credit supply through its effects on asset prices and intermediaries’ leverage, on their fragility when leverage rises in tranquil times, and on the consequences of this fragility when tranquillity gives way to turbulence. In contrast, we abstract from risk entirely, to concentrate on the link between the availability of credit, household debt and home prices. The result is a very simple model of the causes of the credit and housing boom and of a possible trigger of its demise. Central to these results is the interaction between lending and borrowing constraints, which is instead absent in these papers.

The paper closest to ours is Landvoigt (2014), who also stresses the interaction between supply and demand of mortgage debt. He proposes a very rich model of borrowing and lending with intermediation, mostly focused on the effects of securitization on mortgage finance over the past several decades. In this model, mortgages can default, and securitization allows to transfer this risk from leverage-constrained intermediaries to savers with low risk aversion. The final section of the paper studies the boom and bust of the 2000s, as we do here. In this experiment, the credit cycle is driven by a slackening of collateral requirements, along with a perceived decline in the riskiness of mortgages, which turns out to be incorrect. Qualitatively, this combination of shocks generates a plausible boom and bust in debt and real estate values. However, the response of house prices is small, partly because the MBS yield rises during the boom. This counterfactual effect of mortgage rates is presumably due to the slackening of the collateral constraint, which increases the demand for credit.

The rest of the paper is organized as follows. Section 2 presents the simple model of lending and borrowing with houses as collateral. Section 3 analyzes the properties of this
model and characterizes its equilibrium. In section 4, we conduct a number of quantitative experiments to study the impact of looser lending and collateral constraints. Section 5 concludes.

2. The model

This section presents a simple model with heterogeneous households that borrow from each other, using houses as collateral. We use the model to establish that the crucial factor behind the boom in house prices and mortgage debt of the early 2000s was an outward shift in the supply of funds to borrowers, rather than an increase in the demand for funds driven by lower collateral requirements, as mostly assumed by the literature so far. We illustrate this point in the simplest possible endowment economy, abstracting from the complications arising from production and capital accumulation.

2.1. Objectives and constraints. The economy is populated by two types of households, with different discount rates, as in Kiyotaki and Moore (1997), Iacoviello (2005), Campbell and Hercowitz (2009b) and our own previous work (Justiniano, Primiceri, and Tambalotti, 2014b,a). Patient households are denoted by \( l \), since in equilibrium they save and lend. Their discount factor is \( \beta_l > \beta_b \), where \( \beta_b \) is the discount factor of the impatient households, who borrow in equilibrium.

At time 0, representative household \( j = \{b, l\} \) maximizes utility

\[
E_0 \sum_{t=0}^{\infty} \beta_j^t \left[ u(c_{j,t}) + v_j(h_{j,t}) \right],
\]

where \( c_{j,t} \) denotes consumption of non-durable goods, and \( v_j(h_{j,t}) \) is the utility of the service flow derived from a stock of houses \( h_{j,t} \) owned at the beginning of the period. The function \( v(\cdot) \) is indexed by \( j \) for reasons that will become clear in section 2.3. Utility maximization is subject to the flow budget constraint

\[
c_{j,t} + p_t \left[ h_{j,t+1} - (1 - \delta) h_{j,t} \right] + R_{t-1} D_{j,t-1} \leq y_{j,t} + D_{j,t},
\]

where \( p_t \) is the price of houses in terms of the consumption good, \( \delta \) is their depreciation rate, and \( y_{j,t} \) is an exogenous endowment of consumption goods and new houses. \( D_{j,t} \) is the amount of one period debt accumulated by the end of period \( t \), and carried into period \( t + 1 \), with gross interest rate \( R_t \). In equilibrium, debt is positive for the impatient
borrowers and it is negative for the patient lenders, representing loans that the latter extend to the former. Therefore, borrowers can use their endowment, together with loans, to buy non-durable consumption goods and new houses, and to repay old loans with interest.

Households’ decisions are subject to two more constraints. First, on the liability side of their balance sheet, a collateral constraint limits debt to a fraction $\theta$ of the value of the borrowers’ housing stock, along the lines of Kiyotaki and Moore (1997). This constraint takes the form

$$(2.1) \quad D_{j,t} \leq \theta p_t h_{j,t+1},$$

where $\theta$ is the maximum admissible loan-to-value (LTV) ratio. Higher values of $\theta$ capture looser collateral requirements, such as those brought about by mortgages with higher initial LTVs, multiple mortgages on the same property (so-called piggy back loans), and home equity lines of credit. Together, they contribute to enhance households’ ability to borrow against a given value of their property. An emerging consensus in the literature identifies these looser credit conditions and their reversal as the fundamental force behind the credit cycle of the 2000s. Recent papers based on this hypothesis include Eggertsson and Krugman (2012), Guerrieri and Lorenzoni (2012), Hall (2012), Midrigan and Philippon (2011), Garriga, Manuelli, and Peralta-Alva (2012), Favilukis, Ludvigson, and Nieuwerburgh (2013), Boz and Mendoza (2014), and Justiniano, Primiceri, and Tambalotti (2014a).

The second constraint on households’ decisions applies to the asset side of their balance sheet. This constraint consists in an upper bound on the total amount of mortgage lending they can engage in, as in

$$(2.2) \quad -D_{j,t} \leq \bar{L}.$$ 

Expression (2.2) is meant to capture all implicit or explicit regulatory and technological constraints on the economy’s ability to channel funds towards the mortgage market. In this simple model without banks, we impose this constraint directly on the lenders. However, in

---

2 This type of constraint is often stated as a requirement that contracted debt repayments (i.e. principal plus interest) do not exceed the future expected value of the collateral. The choice to focus on a contemporaneous constraint is done for simplicity and it is inconsequential for the results, since most of the results pertain to steady state equilibria.

3 In our extremely stylized economy, this constraint results into a limit on households’ overall ability to save. But this is simply an artifact of the simplifying assumption that mortgages are the only financial assets in the economy, and is not important for the results.
appendix B we show that it can be interpreted as stemming from a leverage (or regulatory-capital, or “skin in the game”) constraint on financial intermediaries. If raising capital is costly—as typically assumed in the literature, e.g. Jermann and Quadrini (2012)—these leverage restrictions translate into a restraint on overall mortgage lending. In the limit in which equity adjustments become very onerous, these leverage restrictions on financial intermediaries would produce the same results of (2.2).

We focus on this stark formulation of the constraint to create the cleanest possible contrast with the more familiar collateral constraint imposed on the borrowers. From a macroeconomic perspective, this lending limit produces an upward sloping supply of funds in the mortgage market, which mirrors the downward sloping demand for credit generated by the borrowing constraint. We illustrate this point in the next section, which characterizes the equilibrium of the model. In section 4, we will use this equilibrium, and its observable implications, to argue that the boom in credit and house prices of the early 2000s is best understood as the consequence of the relaxation of constraints on lending, rather than on borrowing, as most of the literature has been assuming (i.e. as an increase in \( \bar{L} \), rather than in \( \theta \)).

2.2. *Equilibrium conditions*. Given their lower propensity to save, impatient households borrow from the patient in equilibrium, and the lending constraint (2.2) does not influence their decisions. As a consequence, their optimality conditions are

\[
(2.3) \quad (1 - \mu_t) u'(c_{b,t}) = \beta_b R_t E_t u'(c_{b,t+1})
\]

\[
(2.4) \quad (1 - \mu_t \theta) u'(c_{b,t}) p_t = \beta_b v_b'(h_{b,t+1}) + \beta_b (1 - \delta) E_t [u'(c_{b,t+1}) p_{t+1}]
\]

\[
(2.5) \quad c_{b,t} + p_t [h_{b,t+1} - (1 - \delta) h_{b,t}] + R_{t-1} D_{b,t-1} = y_{b,t} + D_{b,t}
\]

\[
(2.6) \quad \mu_t (D_{b,t} - \theta p_t h_{b,t+1}) = 0, \quad \mu_t \geq 0, \quad D_{b,t} \leq \theta p_t h_{b,t+1},
\]

where \( u'(c_{b,t}) \cdot \mu_t \) is the Lagrange multiplier on the collateral constraint.

Equation (2.3) is a standard Euler equation weighting the marginal benefit of higher consumption today against the marginal cost of lower consumption tomorrow. Relative to the case of an unconstrained consumer, the benefit of higher current consumption is reduced
by the cost of a tighter borrowing constraint. Equation (2.4) characterizes the borrowers’ housing demand: the cost of the forgone consumption used to purchase an additional unit of housing today must be equal to the benefit of enjoying this house tomorrow, and then selling it (after depreciation) in exchange for goods. The term \((1 - \mu_t \theta)\) on the left-hand side of (2.4) reduces the cost of foregone consumption, because the newly purchased unit of housing slackens the borrowing constraint when posted as collateral. Equation (2.4) makes it clear that the value of a house to a borrower is higher when the borrowing constraint is tighter, and when the ability to borrow against it (i.e. the maximum loan-to-value ratio) is higher. Finally, equation (2.5) is the flow budget constraint of the borrowers, and expressions (2.6) summarize the collateral constraint and the associated complementary slackness conditions.

Since the patient households lend in equilibrium, the lending constraint is the relevant one in their decisions. Their equilibrium conditions are

\[
(1 + \xi_t) u'(c_{l,t}) = \beta_t R_t E_t u'(c_{l,t+1}) 
\]

\[
u'(c_{l,t}) p_t = \beta_t u'(h_{lt+1}) + \beta_t (1 - \delta) E_t \left[ u'(c_{l,t+1}) p_{t+1} \right] 
\]

\[
c_{l,t} + p_t \left[ h_{lt+1} - (1 - \delta) h_{lt} \right] + R_{t-1} D_{l,t-1} = y_{l,t} + D_{l,t} 
\]

\[
\xi_t \left( -D_{l,t} - \bar{L} \right) = 0, \quad \xi_t \geq 0, \quad -D_{l,t} \leq \bar{L},
\]

where \(u'(c_{l,t}) \cdot \xi_t\) is the Lagrange multiplier on the lending constraint. When this constraint is binding, the lenders would like to save more at the prevailing interest rate, but they cannot. To make it optimal for them to consume instead, the multiplier \(\xi_t\) boosts the marginal benefit of current consumption in their Euler equation (2.7), or equivalently reduces their perceived rate of return from postponing it. Contrary to what happens with the borrowers, who must be enticed to consume less today not to violate their constraint, the lenders must be driven to tilt their consumption profile towards the present when their constraint is binding. Unlike the collateral constraint, though, the lending constraint does not affect the demand for houses, since it does not depend on their value. Otherwise, equations (2.3)-(2.6) have a similar interpretation to (2.7)-(2.10).
The model is closed by imposing that borrowing is equal to lending

\[ D_{b,t} + D_{l,t} = 0, \]

and that the housing market clears

\[ h_{b,t} + h_{l,t} = \bar{h}, \]

where \( \bar{h} \) represents a fixed supply of houses.

2.3. Functional forms. To characterize the equilibrium of the model, we make two convenient functional form assumptions. First, we assume that the lenders’ utility function implies a rigid demand for houses at the level \( \bar{h}_l \). As a consequence, we replace equation (2.8) with

\[ h_{l,t} = \bar{h}_l. \]

This assumption implies that houses are priced by the borrowers, which amplifies the potential effects of collateral constraints on house prices, since these agents face a fixed supply equal to \( \bar{h}_b \equiv \bar{h} - \bar{h}_l \), and they are leveraged. This assumption is appealing for several reasons. First, housing markets are highly segmented in practice (e.g. Landvoigt, Piazzesi, and Schneider (2013)) and the reallocation of houses between rich and poor, lenders and borrowers, is minimal. By assuming a rigid demand by the lenders, we shut down all reallocation of houses between the two groups of agents, thus approximating reality. In addition, this simple modeling device captures the idea that houses are priced by the most leveraged individuals, as in Geanakoplos (2010).

The second simplifying assumption is that utility is linear in consumption. As a consequence, the marginal rate of substitution between houses and non-durables does not depend on the latter, and the level and distribution of income do not matter for the equilibrium in the housing and debt markets. Therefore, the determination of house prices becomes very simple and transparent, as we can see by re-writing equation (2.4) as

\[ p_t = \frac{\beta_b}{(1 - \mu \theta)} [mrs + (1 - \delta) E_t p_{t+1}], \]

where \( mrs = v' (\bar{h} - \bar{h}_l) \), with the constant marginal utility of consumption normalized to one.
According to this expression, house prices are the discounted sum of two components: the marginal rate of substitution between houses and consumption, which represents the “dividend” from living in the house, and the expected selling price of the undepreciated portion of the house. The discount factor depends on the maximum LTV ratio $\theta$—prices are higher if a larger fraction of the house can be used as collateral—and on the tightness of the borrowing constraint $\mu_t$. The tighter the constraint, the more valuable is every unit of collateral.

Although extreme, the assumption of linear utility has the virtue of simplifying the mathematical structure of the model, as well as its economics. With a constant marginal rate of substitution, the only source of variation in house prices is the tightness of the borrowing constraint, as captured by the multiplier $\mu_t$. To the extent that, in the data, part of the movement in prices is driven by changes in the marginal utility of housing services relative to other consumption, this simplification stacks the deck against our goal of explaining the behavior of the housing market in the 2000s.

3. Characterization of the Equilibrium

The model we presented in the previous section features two balance sheet constraints, both limiting the equilibrium level of debt in the economy. The collateral constraint on the liability side of the borrowers’ balance sheet—a standard tool to introduce financial frictions in the literature—limits directly the amount of borrowing by impatient households to a fraction of the value of their houses ($D_{b,t} \leq \theta p_t \bar{h}_b$). The lending constraint, instead, puts an upper bound on the ability of patient households to extend mortgage credit. But in our closed economy, where borrowing must equal lending, this lending limit also turns into a constraint on borrowing ($D_{b,t} \leq \bar{L}$).

Which of the two constraints binds at any given point in time depends on the parameters $\theta$ and $\bar{L}$, but also on house prices, and is therefore endogenous. Moreover, both constraints bind simultaneously when $\theta p_t \bar{h}_b = \bar{L}$, a restriction that is not as knife-edge as one might think, as illustrated below.

To illustrate the interaction between the two balance sheet constraints, start from the standard case with only a borrowing limit, which is depicted in figure 3.1. The supply of funds is perfectly elastic at the interest rate represented by the (inverse of the) lenders’

---

In an open economy model with borrowing from abroad, such as Justiniano, Primiceri, and Tambalotti (2014b), this constraint would become $D_{b,t} \leq \bar{L} + L_{f,t}$, where $L_{f,t}$ denotes the amount of foreign borrowing. Therefore, in such a model, $L_{f,t}$ plays a similar role to $\bar{L}$ in relaxing or tightening the constraint.
Figure 3.1. Demand and supply of funds in a model with collateral constraints.

demand of funds. The demand of funds is also flat, at a higher interest rate determined by the borrowers’ discount factor, but only up to the borrowing limit, where it becomes vertical. The equilibrium is at the (gross) interest rate $1/\beta_t$, where demand meets supply and the borrowing constraint is binding. This implies a positive value of the multiplier of the collateral constraint ($\mu_t$) and a house price determined by equation (2.12), which in turn pins down the location of the kink in the demand of funds in figure (3.1).

Consider now the case of a model with a lending constraint as well, depicted in figure 3.2. With a lending constraint, the supply of funds also has a kink, at the value $\bar{L}$. Whether this constraint binds in equilibrium depends on the relative magnitude of $\bar{L}$ and $\theta p_t \bar{h}_b$. If $\bar{L} > \theta p_t \bar{h}_b$, as in the figure, the equilibrium is the same as in figure 3.1.\(^5\) In particular, small variations of the lending limit $\bar{L}$ do not affect equilibrium interest rates and house prices.

If instead $\bar{L} < \theta p_t \bar{h}_b$, the model behaves differently, as shown in figure 3.3. In this case, it is the lending limit to be binding and the interest rate settles at the level $1/\beta_b$, higher than before. Borrowers are limited in their ability to anticipate consumption not by the value of their collateral, but by the scarcity of funds that the savers are allowed to lend in the mortgage market. At the going rate of return, savers would be happy to expand their mortgage lending, but they cannot. As a result, the economy experiences a dearth of

\(^5\)For this to be an equilibrium, the resulting house price must be such that $\bar{L} > \theta p_t \bar{h}_b$. 
lending and a high interest rate. House prices are again determined by equation (2.12), but with $\mu_t = 0$, which puts them below those in the scenario of figures 3.1 and 3.2.

Qualitatively, the transition from a steady state with a low $\bar{L}$ (figure 3.2) to one with a higher $\bar{L}$ (figure 3.3), in which the lending constraint no longer binds, matches well what happened in the U.S. in the first half of the 2000s, with interest rates falling and household debt and house prices rising. Section 4 shows that this parallelism also works quantitatively,
and that a slackening of the constraint on mortgage lending is also consistent with other patterns in the data.

In contrast, a slackening of the borrowing constraint caused by an increase in the LTV parameter $\theta$ would have opposite effects, making it an unlikely source of the boom observed in the U.S. in the 2000s. Assuming that the borrowing constraint binds initially, as in figure 3.2, an increase in $\theta$ that is sufficiently large leads to an increase in interest rates from $1/\beta_l$ to $1/\beta_b$, as the vertical “arm” of the demand for funds crosses over the lending limit $\bar{L}$, causing the latter to become binding. Moreover, as the borrowing constraint ceases to bind, the multiplier $\mu_t$ falls to zero, putting downward pressure on house prices.\footnote{Starting instead from a situation in which the lending constraint is binding, as in figure 3.3, an increase in $\theta$ would leave the equilibrium unchanged.}

Intuitively, changes in $\theta$ affect the demand for credit by its ultimate users, so that an increase in this parameter increases credit demand, driving its price (the interest rate) higher. And with higher mortgage rates, house prices fall. On the contrary, the lending limit $\bar{L}$ controls the supply of funds from lenders, so that an increase in the limit drives the prices of mortgage credit down and house prices up, also leading to more household debt, at roughly unchanged levels of leverage.

Before moving on, it is useful to consider the case in which $\bar{L} = \theta p_t \bar{h}_b$, when the vertical arms of the supply and demand for funds exactly overlap. This situation is not an unimportant knife-edge case, as the equality might suggest, due to the endogeneity of home prices. In fact, there is a large region of the parameter space in which both constraints bind, so that $p_t = \frac{L}{\theta \bar{h}_b}$. Given $p_t$, equation (2.12) pins down the value of the multiplier $\mu_t$, which, in turn, determines a unique interest rate

$$R_t = \frac{1 - \mu_t}{\beta_b}$$

via equation (2.3). This is an equilibrium as long as the implied value of $\mu_t$ is positive, and the interest rate lies in the interval $[1/\beta_l, 1/\beta_b]$.

We formalize these intuitive arguments through the following proposition.

**Proposition 1.** In the model of section 2 there exist two threshold house prices, $p \equiv \frac{\beta_h \text{mrs}}{1 - \beta_b(1 - \delta)}$ and $\hat{p}(\theta) \equiv \frac{\beta(\theta) \text{mrs}}{1 - \beta(\theta)(1 - \delta)}$, such that:
(i) if \( \bar{L} < \theta \bar{p} \hat{h}_b \), the lending constraint is binding and

\[ p_t = \bar{p}, \quad D_{b,t} = \bar{L} \quad \text{and} \quad R_t = \frac{1}{\tilde{\beta}_b}; \]

(ii) if \( \bar{L} > \theta \bar{p}(\theta) \hat{h}_b \), the borrowing constraint is binding and

\[ p_t = \bar{p}(\theta), \quad D_{b,t} = \theta \bar{p}(\theta) \hat{h}_b \quad \text{and} \quad R_t = \frac{1}{\tilde{\beta}_l}; \]

(iii) if \( \theta \bar{p} \hat{h}_b \leq \bar{L} \leq \theta \bar{p}(\theta) \hat{h}_b \), both constraints are binding and

\[ p_t = \frac{\bar{L}}{\theta \hat{h}_b}, \quad D_{b,t} = \bar{L} \quad \text{and} \quad R_t = \frac{1}{\tilde{\beta}_b} \left[ 1 - \frac{1 - \beta_h (1 - \delta) - mrs \cdot \beta_b \theta \hat{h}_b / \bar{L}}{\theta} \right]; \]

where \( mrs \equiv v' (\hat{h} - \hat{h}_l) \), \( \tilde{\beta}(\theta) \equiv \frac{\beta_b \beta_l}{\theta \beta_b + (1 - \theta) \beta_l} \) and \( \bar{p}(\theta) \geq \bar{p} \) for every \( \theta \geq 0 \).

Proof. See appendix A. \( \square \)

To further illustrate Proposition 1, figure 3.4 plots the equilibrium value of house prices, debt and interest rates, as a function of the lending limit \( \bar{L} \), for a given value of the loan-to-value ratio \( \theta \). On the left of the three panels, tight lending constraints are associated with high interest rates, low house prices and low levels of indebtedness. As \( \bar{L} \) rises and lending constraints become looser, interest rates fall, boosting house prices and allowing people to borrow more against the increased value of their home. However, the relation between lending limits and house prices is not strictly monotonic. When lending constraints become very relaxed, the model is akin a standard model with collateral constraints, and lending limits become irrelevant for the equilibrium.

The qualitative consequences of a transition towards looser lending constraints—a reduction in interest rates, an increase in house prices and debt, with a stable debt-to-collateral ratio—square very well with the stylized fact of the period preceding the Great Recession that we have outlined in the introduction. In the next section we will use a calibrated version of our model to analyze its performance from a quantitative standpoint.

4. Quantitative analysis

This section provides a quantitative perspective on the simple model illustrated above. The model is parametrized so that its steady state matches key statistics for the period of relative stability of the 1990s. We interpret this steady state as associated with a binding lending constraint, as in figure 3.3 above. This assumption seems appropriate for a period
in which mortgage finance was still relatively unsophisticated, securitization was a nascent phenomenon, and as a result savers faced relatively high barriers to accessing mortgage loans.

We then analyze the extent to which a lowering of these barriers, in the form of a progressive increase in the lending limit \( \bar{L} \), generates a surge in debt and house prices and a fall in interest rates comparable to those observed in the early 2000s. The main conclusion we draw from this experiment is that looser lending constraints are key to understanding the dynamics of debt, house prices and interest rates in the period leading up to the financial crisis. In contrast, a slackening of borrowing constraints, which we also consider as an alternative source of those dynamics, has entirely counterfactual implications for the boom.
phase of the credit and housing cycle. In fact, in our framework, a relaxation of collateral requirements at the peak of the boom triggers a fall in house prices.

4.1. Parameter values. Table 1 summarizes the model’s calibration, which is based on U.S. macro and micro data.

<table>
<thead>
<tr>
<th>$\delta$</th>
<th>$\beta_b$</th>
<th>$\beta_l$</th>
<th>$\theta$</th>
<th>$\rho$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.003</td>
<td>0.9879</td>
<td>0.9938</td>
<td>0.80</td>
<td>0.0056</td>
</tr>
</tbody>
</table>

Table 1. Calibration of the model.

Time is in quarters. We set the depreciation rate of houses ($\delta$) equal to 0.003, based on the Fixed Asset Tables. Real mortgage rates are computed by subtracting 10-year-ahead inflation expectations from the Survey of Professional Forecasters from the 30-year nominal conventional mortgage rate published by the Federal Reserve Board. The resulting series is plotted in figure 1.4. The average real rate in the 1990s is slightly less than 5% (4.63%) and it falls by about 2.5% between 2000 and 2005. Therefore, we set the discount factor of the borrowers to match a 5% real rate in the initial steady state, and we calibrate the lenders’ discount factor to generate a fall in interest rates of 2.5 percentage points following the relaxation of the lending constraint. The heterogeneity in discount factors consistent with this decline, $\beta_b = 0.9879$ and $\beta_l = 0.9938$, is in line with that chosen by Krusell and Smith (1998) or Carroll, Slacalek, and Tokuoka (2013) to match the distribution of household wealth in the US.

For the calibration of the remaining parameter—the maximum allowed LTV ratio ($\theta$)—we face two main challenges, due to some aspects of the theoretical model that are stark simplifications of reality. First, the model assumes a collateral constraint with a constant loan-to-value ratio over the entire life of a loan. This simple specification, which is by far the most popular in the literature, works well to provide intuition about the working of the model. However, calibrating $\theta$ to the initial loan-to-value ratio of the typical mortgage, say around 0.8, would overstate the aggregate debt-to-real estate ratio in the economy.\footnote{In this case, the effects of looser lending constraints would be even larger than in the baseline calibration.} This is because, in reality, mortgage contracts require a gradual repayment of the principal, leading to the accumulation of equity in the house and therefore to average loan-to-value ratios that are lower than the initial one.
To capture this feature of reality in our quantitative exercises, we follow Campbell and Hercowitz (2009b) and generalize the model by replacing the collateral constraint (2.1) with

\[ D_{b,t} \leq \theta p_t H_{b,t+1} \] (4.1)

\[ H_{b,t+1} = \sum_{j=0}^{\infty} (1 - \rho)^j [h_{t+1-j} - (1 - \delta) h_{t-j}] , \] (4.2)

where the last expression can be written recursively as

\[ H_{b,t+1} = (1 - \rho) H_{b,t} + [h_{b,t+1} - (1 - \delta) h_{b,t}] . \] (4.3)

The variable \( H_{b,t+1} \) denotes the share of the housing stock that can be used as collateral, which does not necessarily coincide with the physical stock of houses \( H_{b,t+1} \). Equation (4.2) describes the evolution and composition of \( H_{b,t+1} \). The housing stock put in place in the latest period \((h_{t+1} - (1 - \delta) h_t)\) can all be used as collateral, and hence it can “sustain” an amount of borrowing equal to a fraction \( \theta \) of its market value. However, only a fraction \((1 - \rho)^j\) of the houses purchased in \(t - j\) is collateralizable, with the remaining share representing the amortization of the loan and the associated accumulation of equity by the borrowers. If \( \rho = \delta \), amortization and depreciation of the housing stock proceed in parallel, so that the entire housing stock can always be used as collateral. As a result, the new collateral constraint is identical to (2.1). If \( \rho > \delta \), however, amortization is faster than depreciation, reducing the borrowing potential of the housing stock and the average debt-to-real estate ratio in the economy, for any given value of the initial LTV \( \theta \). Appendix C characterizes the solution of the model with this generalized version of the collateral constraint.

To calibrate \( \theta \), we should look at the mortgages of households that resemble the borrowers in the model. One option would be identify as borrowers all households with mortgage debt in the micro data. However, in the data many borrowers also own a substantial amount of financial assets (Campbell and Hercowitz, 2009a, Iacoviello and Pavan, 2013, Kaplan and Violante, 2014). Our model is not equipped to capture this feature of the data, since in it agents only borrow because they are impatient, and as a result they do not own any asset, aside from their house. In the data, instead, people might choose to borrow against their houses (instead of running down their financial assets) for a variety of other reasons that
are not explicitly considered in our model. To avoid including in the group of borrowers also many individuals possibly with the characteristics of a lender, an alternative, more prudent choice is to identify as borrowers all households with little financial assets.

More specifically, we separate the borrowers from the lenders using the Survey of Consumer Finances (SCF), which is a triennial cross-sectional survey of the assets and liabilities of U.S. households. We identify the borrowers as the households that appear to be liquidity constrained, namely those with liquid assets whose value is less than two months of their total income. Following Kaplan and Violante, we compute the value of liquid assets as the sum of money market, checking, savings and call accounts, directly held mutual funds, stocks, bonds, and T-Bills, net of credit card debt. We apply this procedure to the 1992, 1995 and 1998 SCF.

To calibrate the initial loan-to-value ratio $\theta$, we take an average of the loan-to-value ratios of the mortgages of all borrowers who purchased their home (or refinanced their mortgage) in the year immediately before the surveys. An average of these ratios computed over the three surveys of 1992, 1995 and 1998 yields a value for $\theta$ of 0.8, a very common initial LTV for typical mortgages, which is also broadly in line with the cumulative loan-to-value ratio of first-time home buyers estimated by Duca, Muellbauer, and Murphy (2011) for the 1990s. As for $\rho$, the parameter that governs the amortization speed on the loans, we pick a value of 0.0056, so as to match the average ratio of debt to real estate for the borrowers, which is 0.43 in the 1990s SCFs. Finally, the lending limit $\bar{L}$ is chosen in the context of the experiments described in the next subsection.

4.2. **An expansion in credit supply.** In this subsection we study the quantitative effects of a relaxation of lending constraints in the mortgage market. The premise for this exercise is that, at the end of the 1990s, the U.S. economy was constrained by a limited supply of credit, as in figure 3.3 above. Starting in 2000, the lending constraint is gradually lifted, following the linear path depicted in figure 4.1. The experiment is timed so that the lending constraint is no longer binding in 2006. The dotted part of the line in figure 4.1 corresponds to the time periods in which the lending constraint becomes irrelevant for the equilibrium.

---

8Following Campbell and Hercowitz (2009b), among these households, we restrict attention to the ones who borrow at least half of their home value, since those with lower initial LTVs are probably not very informative on the credit conditions experienced by the marginal buyers.
In the barebones model presented so far, an increase in $\bar{L}$ affects the equilibrium house prices and interest rates only in the region in which both the lending and borrowing constraints are binding, as shown in the proposition 1. This is the region in which our numerical experiments put the economy between 2000 and 2006, because the dynamics associated with a relaxation of $\bar{L}$ in this region are very consistent with the empirical facts we highlighted in the introduction. However, we think that it is plausible that the relaxation of lending constraints was in fact an ongoing phenomenon, that probably started in the 1990s, and possibly earlier. However, the model suggests that this process would have had relatively modest effects as long as the maximum volume of lending was far enough below the borrowing limit, which is why we ignore this earlier period in the simulations.

What real world phenomena are behind the relaxation of the lending limit in the model? Broadly speaking, we think of an increase in $\bar{L}$ as capturing technological and regulatory developments that made it easier for savings to flow towards the mortgage market. Securitization, for instance, turned mortgages into MBS. These standardized products could easily be traded on financial markets, and their higher-rated tranches could be acquired by institutional and other investors whose portfolios are restricted by regulation and their own statutes to include only safe assets. This innovation, in turn, opened up the possibility
for a large pool of savings previously directed towards Government debt to get exposed to mortgage products, tapping a source of funds that was previously unavailable. Similarly, the capital requirements imposed by regulation on commercial banks imply lower charges for agency MBS (and the senior tranches of private label ones) than for the mortgages themselves. In addition, the rise of highly levered off-balance-sheet special purpose vehicles allowed banks to bypass capital regulations altogether, further increasing the amount of funds directly intermediated towards mortgage lending. Finally, the rapid deterioration of the current account deficit in the first half of the 2000s channeled large pools of international savings, mostly from East Asia and the oil-exporting countries, towards domestic borrowers. Many of these funds found their way into the mortgage market through the purchase of MBS, as documented by Bernanke, Bertaut, DeMarco, and Kamin (2011).

While some of these developments date back at least to the 1970s, they really took off in the late 1990s and early 2000s. From the perspective of the model, this expansion in credit starts affecting the economy only when the lending limit becomes loose enough to approach the borrowing limit. Circumstantially, it seems plausible to assume that this interaction between the two constraints started in the early 2000s. This is around the time when enough funds had become available to mortgage finance that borrowers’ collateral (and creditworthiness) begun to represent a material impediment to further credit expansion. The fact that these borrower-side factors, more than the ability of lenders to direct their savings towards the mortgage market, started to represent the relevant constraint is confirmed by the pressure to looser credit standards that lead to the boom in subprime, high-LTV, piggy back and other risky mortgages that sowed the seeds of the subsequent financial crisis.

Figure 4.2 plots the response of the key variables in the model to the loosening of the lending constraints described above. The expansion in credit supply lowers mortgage rates by 2.5 percentage points. This decline reflects the gradual transition from a credit-supply-constrained economy, where the interest rate equals $\frac{1}{\beta_b}$, to a demand-constrained economy, with interest rate $\frac{1}{\beta_l}$. This permanent fall in mortgage rates is a distinctive feature of our environment with lending constraints, and it would be difficult to obtain in the context of more standard models with only collateral constraints, in which the steady state interest rate is always pinned down by the discount factor of the lenders. The magnitude of the
Figure 4.2. The response to a loosening of lending constraints

decline is in line with the evidence presented in the introduction, but this is just a function of our calibration of the discount factors of the two sets of households.

When the supply of credit is tight, the borrowers are unconstrained, and the interest rate is equal to the inverse of their discount factor. However, when lending constraints become looser and mortgage rates fall below $\frac{1}{\beta_b}$, the impatient households increase their demand for credit up to the limit allowed by their collateral constraint, which becomes binding. The lower the mortgage rate, the more desirable is borrowing, and the higher is the shadow value $\mu_t$ of the collateral constraint. According to equation (2.12), a rise in $\mu_t$ increases the value of houses to the borrowers, who are the agents pricing them, because their collateral services become more valuable. In our calibration, house prices increase by almost 40 percent in real terms following a lending liberalization, a magnitude that is comparable to the U.S. experience depicted in figure 1.1.

The substantial increase in house prices then relaxes the collateral constraint of the impatient households, allowing them to borrow more against the higher value of their homes. In the model, mortgage debt raises by approximately 30 percentage points of GDP.
However, the debt-to-real estate ratio remains unchanged, with debt and home values rising in parallel. This is precisely what happened in the data through 2006, as shown in figure 1.3.

4.3. **Looser collateral requirements.** In this section, we contrast the implications of the loosening of lending constraints described above to those of a slackening of the borrowing constraint, either through a higher initial LTV, or through slower amortization. This comparison is important, because most of the literature focuses on an increase in borrowing limits as the trigger of the boom in household debt and, to a certain extent, in house prices. Our results suggest that this view is difficult to reconcile with some of the key stylized facts discussed in the introduction. On the contrary, we show that a relaxation of collateral requirements can even generate a decline in home values, rather than a rapid increase. This is an appealing feature of our model because it suggests a possible trigger for the turnaround in house prices that started in the fall of 2006, at a time when financial liberalization was still in full swing.

We start this analysis by slackening the collateral constraint in an economy without lending limits, which is the case typically considered in the literature. The model is parameterized to match the same targets used in section 4.1, which produces the same values for most parameters, except for $\beta_l$ and $\beta_b$. In particular, $\beta_l$ is set equal to 0.9879 to match the 5 percent average real mortgage rate in the 1990s, since the equilibrium interest rate in this model corresponds to $\frac{1}{\beta_l}$. As for $\beta_b$, we choose the value 0.9820 to maintain the same gap from the discount factor of the lenders as in the previous calibration.

Given this parametrization, we study the effects of a gradual increase in the maximum LTV from 0.8, the baseline value of $\theta$, to 1.02, representing a situation in which borrowers can borrow up to the entire amount of their house (panel a in figure 4.3). This change in $\theta$ generates the same increase in household debt as in the previous experiment, making the two simulations easily comparable.

Figure 4.4 plots the behavior of debt, interest rates and house prices in response to this change in $\theta$, contrasting them to the responses of these variables to the relaxation in the lending constraint described above. Even though the model is extremely stylized, the contrast between the responses highlights the remarkable success of the first experiment in generating the stylized facts we are focusing on. In comparison, not much happens when $\theta$ increases.
First, interest rates remain unchanged as a result of the looser collateral requirements, since lenders are unconstrained and their discount factor pins down the interest rate. In a model with short-run dynamics, interest rates would even increase, so as to convince the patient households to lend additional funds to the now less constrained borrowers (Justinoano, Primiceri, and Tambalotti, 2014a). Second, house prices move very little in response to an increase in the maximum LTV. As a result, the increase in household debt stems from a combination of slightly higher house prices and a higher debt-to-collateral ratio, as shown in the lower-right panel. This increase in leverage is counterfactual, as we already stressed, because the debt-to-real estate ratio was essentially flat over this period.

Results are very similar if we drive an increase in household debt via a reduction in the speed of amortization $\rho$, rather than via a rise in $\theta$. This experiment is depicted by the green dashed-dotted line in figure 4.4. In this case, the change in $\rho$ is calibrated to generate the same dynamics of household debt as in the other two experiments, which requires gradually decreasing $\rho$ from the initial value of 0.0056 to a value of 0.0041, as shown in panel b of figure 4.3. The resulting dynamics of debt and house prices are similar to those generated by the previous experiment, and they are equally counterfactual.

4.4. Why house prices started to fall: a potential trigger for the bust. The previous two experiments analyze the consequences of looser collateral constraints when lending constraints are absent, or so slack that they are irrelevant for the equilibrium. In this subsection, we show that the same relaxation of collateral constraints has substantially different effects if lending constraints are in fact present, as in our model, and eventually become binding. In this scenario, an increase in $\theta$ lifts interest rates and the debt-to-collateral

**Figure 4.3.** Increase in LTV and reduction in the speed of repayment

![Graph showing increase in LTV and reduction in speed of repayment](image)
ratio, and depresses house prices. These outcomes are consistent with the data between 2006 and 2008, when the mature phase of the credit boom gave way to the bust. This novel account of the turnaround in the cycle is appealing because it does not rely on a reversal of the forces behind the boom, unlike most of the literature (see Burnside, Eichenbaum, and Rebelo, 2013 for an exception to this statement, in a model with houses, but no credit).

To illustrate this point, we modify the baseline experiment of section 4.2 and assume that the increase in $\bar{L}$ between 2000 and the end of 2005 is followed by a rise in $\theta$ from 0.8 in 2006 to 1.02 at the end of 2008. The results of this combined experiment are depicted in figure 4.4. These simulations are identical to those of figure 4.2 until the end of 2005. At that point, following the expansion in $\bar{L}$, the lending constraint is no longer binding and the equilibrium is determined by the collateral constraint, as in figure 3.2. Starting in 2006, however, the relaxation of the collateral constraint shifts the kink of the demand for funds to the right, to the point where the collateral and the lending constraint are both binding again.
At this point, a further increase in $\theta$ reduces house prices, as in case (iii) of Proposition 1, since $\bar{L} = \theta p_t \bar{h}_b$. Intuitively, a slackening of the borrowing constraint (i.e. a higher $\theta$) reduces its shadow value ($\mu_t$) by more when the amount of borrowing is constrained by the supply of funds rather than by their demand. Moreover, mortgage rates increase, debt is stable and the debt-to-collateral ratio rises, as shown in figure 4.4.

In the experiment of this subsection, we increased $\bar{L}$ and $\theta$ sequentially, to isolate their relative role in the boom-bust episode of the 2000s. In reality, the relaxation of lending and borrowing constraints probably proceeded in parallel, rather than sequentially, since they are both manifestations of a broader process of financial liberalization. However, our model’s key insight is that an increase in $\theta$ will trigger a fall in house prices even in these more complex and realistic environments, as long as the resulting expansion in credit demand eventually exceeds that in supply.

Finally, we have no ambition to capture with such a simple model the intricate dynamics of the acute phase of the financial and economic crisis that followed the collapse of Lehman Brothers at the end of 2008. The model does suggest, however, that a successful account of this phase of the bust must feature a sharp contraction in the demand for credit, relative to...
its supply, to simultaneously match the decline in mortgage rates and house prices observed during this period.

5. Concluding Remarks

An unprecedented boom and bust in house prices and household debt have been among the defining features of the U.S. macroeconomic landscape since the turn of the millennium. Most accounts of this cycle in credit and collateral values—and of the Great Recession and slow recovery that accompanied it—have pointed to changes in the tightness of borrowing constraints, and the consequent change in the demand for credit, as its key drivers. In this paper, we argued that the focus of this discussion should shift from constraints on borrowing to impediments to lending, in particular when it comes to understanding the boom phase of the cycle.

We make this point in a stylized model of borrowing and lending among households, which features both a collateral constraint on the borrowing side, and a constraint on households’ ability to lend in the mortgage market. A progressive loosening of this lending constraint is consistent with four key empirical facts characterizing the boom—the large increase in house prices and in mortgage debt, the stability of the ratio between mortgages and the value of the real estate collateralizing it, and the fall in mortgage interest rates. The empirical success of the model depends on the interaction between the borrowing and lending constraints, but it cannot be reproduced with either of the two constraints in isolation. In fact, the interaction of the two constraints produces rich dynamics of interest rates, debt and house prices, which might account for both the boom and bust phases of the cycle. A fuller analysis of these dynamics, in a model in which the tightening and loosening of the two constraints is intertwined, is on our research agenda.

Appendix A. Proof of Proposition 1

To prove part (i) of the proposition, consider first the case in which the lending constraint is binding, but the collateral constraint is not, so that \( D_{b,t} = \bar{L} < \theta p_t \bar{h}_b, \xi_t > 0 \) and \( \mu_t = 0 \).

With linear utility in consumption, \( R_t = 1 / \beta_b \) follows from equation (2.3), and equation (2.4) implies \( p_t = \frac{\beta_h m_{rs}}{1 - \beta_h(1 - \delta)} \equiv p \). For this to be an equilibrium, it must be verified that the collateral constraint is not binding, as assumed initially. This requires \( \bar{L} < \theta p \bar{h}_b \).
To prove part (ii) of the proposition, consider the opposite case in which the collateral constraint is binding, but the lending constraint is not. It follows that $D_{b,t} = \theta p_t \bar{h}_b < \bar{L}$, $\xi_t = 0$ and $\mu_t > 0$. We can now derive $R_t = 1/\beta_t$ from equation (2.7), while equation (2.3) implies $\mu_t = \beta_b/\beta_l - 1$. Substituting the expression for $\mu_t$ into equation (2.4) yields $p_t = \tilde{\beta}(\theta) \frac{mrs}{1-\tilde{\beta}(1-\delta)} \equiv \tilde{p}(\theta)$, where $\tilde{\beta}(\theta) \equiv \frac{\beta_b \beta_l}{\theta \beta_b + (1-\theta) \beta_l}$. This is an equilibrium, provided that $\bar{L} > \theta \tilde{p}(\theta) \bar{h}_b$.

To prove part (iii) of the proposition, we must find the equilibrium in the region of the parameter space in which $\theta p_t \bar{h}_b \leq \bar{L} \leq \theta \tilde{p}(\theta) \bar{h}_b$. Equations (2.3) and (2.7) together imply that at least one of the two constraints must be binding in this region, but parts (i) and (ii) of the proposition imply that we cannot have only one of them binding in this region of the parameter space. It follows that both constraints must be binding simultaneously, implying $D_{b,t} = \bar{L} = \theta p_t \bar{h}_b$ and $p_t = \frac{\bar{L}}{\theta \bar{h}_b}$. Substituting the expression for $p_t$ into equation (2.4), we can compute $\mu_t = \frac{1-\beta_b(1-\delta)-mrs \beta_l \bar{h}_b / L}{\bar{L}}$ and, using (2.3), $R_t = \frac{1}{\beta_t} \left[ 1 - \frac{1-\beta_b(1-\delta)-mrs \beta_l \bar{h}_b / L}{\bar{L}} \right]$. Finally, $\mu_t$ satisfies $\mu_t \geq 0$ as long as $\theta p_t \bar{h}_b \leq \bar{L} \leq \theta \tilde{p}(\theta) \bar{h}_b$, which concludes the proof.

**Appendix B. A Simple Model with Financial Intermediaries and Capital Requirements**

This appendix shows that our simple baseline model with a parametric lending limit $\bar{L}$ is equivalent to the limiting case of a more realistic model with financial intermediation. In this model, intermediaries face a capital requirement that their equity be above a certain fraction of their assets, i.e. a “skin in the game” constraint, as in He and Krishnamurthy (2013), and Brunnermeier and Sannikov (2014). Intermediaries finance mortgages by collecting savings from the patient households in the form of either debt (i.e. deposits) or equity, where the latter can only be adjusted by paying a convex cost, similar to Jermann and Quadrini (2012). In the limit in which the marginal cost of adjustment tends to infinity, so that equity is fixed in equilibrium, the capital requirement becomes a hard constraint on the funds supplied to the borrowers, exactly as in the baseline model.

Although this case with infinite adjustment costs is extreme, it is qualitatively consistent with the evidence on the stickiness of intermediaries’ equity first uncovered by Adrian and Shin (2010b). If the marginal cost of adjusting the intermediaries’ capital were not prohibitively large, as assumed here, the resulting supply of funds would be differentiable,
rather than having a kink, but it would still be upward sloping. This property of the supply of mortgage finance is the key driver of our results.

In the model with intermediaries, competitive “banks” finance mortgages with a mix of equity and deposits collected from the savers. Although the borrowers receive funds from the intermediaries, rather than directly from the savers, their optimization problem is identical to the one in section 2. The lenders, in contrast, maximize the same utility function of section 2, but subject to the flow budget constraint

\[ c_{l,t} + p_t [h_{l,t+1} - (1 - \delta) h_{l,t}] - D_{l,t} + E_t \leq y_{l,t} - R^D_{l,t-1} D_{l,t-1} + R^E_{l-1} E_{l-1}, \]

where \(-D_{l,t}\) represents “deposits”, which pay a gross interest rate \(R^D_t\), and \(E_t\) represents equity capital, with rate of return \(R^E_t\). These interest rates can differ from the borrowing rate \(R_t\).

With linear utility in consumption, the first order conditions of the problem of the lenders become

\[ R^D_t = R^E_t = \frac{1}{\beta_l}, \]

together with the condition \(h_{l,t} = \bar{h}_l\) following from our maintained assumption that the lenders’ demand for houses is rigid.

The competitive financial intermediaries maximize profits

\[ R_t D_{b,t} + R^D_t D_{l,t} - R^E_t [1 + f(E_t)] E_t \]

subject to the constraints that assets must equal liabilities,

\[ D_{b,t} + D_{l,t} = E_t, \]

and to a “capital requirement” that limits lending to a multiple of equity,

\[ D_{b,t} \leq \chi E_t. \]

The function \(f(E_t)\) represents a convex cost of issuing equity. As in Jermann and Quadrini (2012), this cost is positive, creating a pecking order of liabilities whereby debt is preferred to equity. We parametrize it as

\[ f \left( \frac{E_t}{E} \right) = \tau \left( \frac{E_t}{E} \right)^\gamma \]
so that the bank’s first order conditions become

\[(B.5) \quad R_t - R^D_t = \phi_t\]

\[(B.6) \quad R^E_t \left[ 1 + \tau (1 + \gamma) \left( \frac{E_t}{E} \right)^\gamma \right] - R^D_t = \chi \phi_t,\]

where \(\phi_t\) is the Lagrange multiplier on the capital requirement.

Combining these two conditions with the fact that \(R^D_t = R^E_t\), we find that the interest rate on loans is a weighted average of the cost of funding those loans with a combination of equity and deposits

\[(B.7) \quad R_t = \frac{1}{\chi} R^D_t \left[ 1 + \tau (1 + \gamma) \left( \frac{E_t}{E} \right)^\gamma \right] + \frac{\chi - 1}{\chi} R^D_t.\]

In this expression, \(1/\chi\) is the share of bank liabilities held as equity when the capital requirement is binding. Its cost is a markup over the interest rate on deposits \(R^D_t\), which reflects the marginal cost of issuing equity.

Since this marginal cost is everywhere positive, debt is always preferable to equity, making the capital requirement constraint always binding for the financial intermediary. Therefore, we can turn equation (B.7) into the supply of funds by substituting \(E_t = \frac{D_{b,t}}{\chi}\) to obtain

\[
R_t = \frac{1}{\bar{\beta}} \left[ 1 + \frac{\tau (1 + \gamma)}{\chi} \left( \frac{D_{b,t}}{\chi E} \right)^\gamma \right].
\]

This supply function is increasing and convex for \(\gamma > 1\). When \(\gamma \to \infty\), the function exhibits a kink at \(D_{b,t} = \chi \bar{E}\), thus establishing the equivalence between this model with intermediation and the simple model with a lending constraint, as long as \(\bar{L} = \chi \bar{E}\). This equivalence furthermore provides an interpretation for changes in the lending limit \(\bar{L}\), as stemming from changes in the leverage ratio of intermediaries \(\chi\), or in their cost of issuing equity.

**Appendix C. Solution of the model with home equity accumulation**

The model used in section 4 to generate the quantitative results differs from the baseline specification because the collateral constraint allows for the gradual repayment of the mortgage principal. This generalization involves replacing expression (2.1) with (4.1) and
The optimality conditions of the problem of the borrowers become
\[(1 - \mu_t) u'(c_{b,t}) = \beta_b R_t E_t u'(c_{b,t+1})\]
\[(1 - \zeta_t) u'(c_{b,t}) p_t = \beta_b u'(h_{b,t+1}) + \beta_b (1 - \delta) E_t [(1 - \zeta_{t+1}) u'(c_{b,t+1}) p_{t+1}]\]
\[(\zeta_t - \theta \mu_t) u'(c_{b,t}) p_t = \beta_b E_t [(1 - \rho) \zeta_{t+1} u'(c_{b,t+1}) p_{t+1}]\]
\[c_{b,t} + p_t [h_{b,t+1} - (1 - \delta) h_{b,t}] + R_{t-1} D_{b,t-1} = y_{b,t} + D_{b,t}\]
\[\mu_t (D_{b,t} - \theta p_t \mathcal{H}_{b,t+1}) = 0, \quad \mu_t \geq 0, \quad D_{b,t} \leq \theta p_t \mathcal{H}_{b,t+1},\]
where \(u'(c_{b,t}) \cdot \mu_t\) and \(u'(c_{b,t}) \cdot p_t \cdot \zeta_t\) are the Lagrange multipliers on the constraint \(D_{b,t} \leq \theta p_t \mathcal{H}_{b,t+1}\) and the evolution of \(\mathcal{H}_{b,t+1}\) respectively. The optimality conditions of the problem of the lenders and the market clearing conditions are the same as in the baseline.

To solve this model, first note that
\[\mathcal{H}_{b,t+1} = (1 - \rho) \mathcal{H}_{b,t} + [h_{b,t+1} - (1 - \delta) h_{b,t}]\]
where \(u'(c_{b,t}) \cdot \mu_t\) and \(u'(c_{b,t}) \cdot p_t \cdot \zeta_t\) are the Lagrange multipliers on the constraint \(D_{b,t} \leq \theta p_t \mathcal{H}_{b,t+1}\) and the evolution of \(\mathcal{H}_{b,t+1}\) respectively. The optimality conditions of the problem of the lenders and the market clearing conditions are the same as in the baseline.

Suppose now that the lending constraint is binding and the collateral constraint is not, so that \(D_{b,t} = L < \theta p_t \bar{h}_b, \xi_t > 0\) and \(\mu_t = 0\). With linear utility in consumption, \(R_t = 1/\beta_b\) follows from equation (C.1), and equations (C.2) and (C.3) imply \(p_t = \beta_b m_{rs} \frac{1}{1 - \beta_b (1 - \delta)} \equiv \bar{p}\). For this to be an equilibrium, the collateral constraint must actually not be binding, as assumed above. This requires \(\bar{L} < \theta \bar{p} \bar{h}_b\).

Suppose now to be in the opposite situation in which the collateral constraint is binding, while the lending constraint is not. It follows that \(D_{b,t} = \theta p_t \bar{h}_b < \bar{L}, \xi_t = 0\) and \(\mu_t > 0\). We can now derive \(R_t = 1/\beta_t\) from equation (2.7), while equation (C.1) implies \(\mu_t = \beta_b / \beta_t - 1\). Substituting the expression for \(\mu_t\) into equation (C.3) and combining it with (C.2) yields
\[p_t = \frac{\beta_b m_{rs}}{1 - \beta_b (1 - \delta)} \cdot \frac{1 - \beta_b (1 - \rho)}{1 - \beta_b (1 - \rho) - \theta (1 - \beta_b / \beta_t)} \equiv \bar{p} (\theta, \rho) > \bar{p}.
This is an equilibrium, provided that \(\bar{L} > \theta \bar{p} (\theta) \bar{h}_b\).
Finally, we must find the equilibrium of the model in the region of the parameter space in which \( \theta p \beta \bar{h}_b \leq \bar{L} \leq \theta \bar{p} (\theta) \beta \bar{h}_b \). Combining equations (C.1) and (2.7) implies that at least one of the two constraints must be binding, and the results above show that the value of the parameters in this region is inconsistent with only one of them being binding. It follows that both constraints must bind at the same time, implying \( D_{b,t} = \bar{L} = \theta p_t \beta \bar{h}_b \) and \( p_t = \frac{\rho \bar{L}}{\delta \bar{h}_b} \). Substituting the expression for \( p_t \) into equations (C.2) and (C.3), we can compute the equilibrium value of \( \mu_t = \frac{1-\beta_b(1-\delta)-mrs \beta_b \delta \bar{h}_b/(\rho L)}{\theta} \frac{1-\beta_b(1-\rho)}{1-\beta_b(1-\delta)} \), and verify that it is positive if \( \theta p \beta \bar{h}_b \leq \bar{L} \leq \theta \bar{p} (\theta) \beta \bar{h}_b \). We can then obtain \( R_t = \frac{1}{\beta_b} \left[ 1 - \frac{1-\beta_b(1-\delta)-mrs \beta_b \delta \bar{h}_b/(\rho L)}{\theta} \frac{1-\beta_b(1-\rho)}{1-\beta_b(1-\delta)} \right] \) using (C.1).

These results can be summarized in the following proposition.

**Proposition 2.** In the model of section 4 there exist two threshold house prices, \( \underline{p} \equiv \frac{\beta_b \cdot mrs}{1-\beta_b(1-\delta)} \) and \( \bar{p} (\theta, \rho) \equiv \frac{\beta_b \cdot mrs}{1-\beta_b(1-\delta)} \cdot \frac{1-\beta_b(1-\rho)}{1-\beta_b(1-\delta)} \), such that:

(i) if \( \bar{L} < \theta p \beta \bar{h}_b \), the lending constraint is binding and

\[
p_t = \underline{p}, \quad D_{b,t} = \bar{L} \quad \text{and} \quad R_t = \frac{1}{\beta_b};
\]

(ii) if \( \bar{L} > \theta \bar{p} (\theta, \rho) \beta \bar{h}_b \), the borrowing constraint is binding and

\[
p_t = \bar{p} (\theta, \rho), \quad D_{b,t} = \theta \bar{p} (\theta, \rho) \beta \bar{h}_b \quad \text{and} \quad R_t = \frac{1}{\beta_t};
\]

(iii) if \( \theta p \beta \bar{h}_b \leq \bar{L} \leq \theta \bar{p} (\theta, \rho) \beta \bar{h}_b \), both constraints are binding and

\[
p_t = \frac{\rho}{\delta \bar{h}_b} \bar{L}, \quad D_{b,t} = \bar{L} \quad \text{and}
\]

\[
R_t = \frac{1}{\beta_b} \left[ 1 - \frac{1-\beta_b(1-\delta)-mrs \beta_b \delta \bar{h}_b/(\rho \bar{L})}{\theta} \frac{1-\beta_b(1-\rho)}{1-\beta_b(1-\delta)} \right];
\]

where \( mrs \equiv v' (\bar{h} - \bar{h}_t) \) and \( \bar{p} (\theta) \geq \underline{p} \) for every \( 0 \leq \theta \leq 1 \).

**References**


Landvoigt, T., M. Piazzesi, and M. Schneider (2013): “The Housing Market(s) of San Diego,” Stanford University, mimeo.


Federal Reserve Bank of Chicago
E-mail address: ajustiniano@frbchi.org

Northwestern University, CEPR, and NBER
E-mail address: g-primiceri@northwestern.edu

Federal Reserve Bank of New York
E-mail address: a.tambalotti@gmail.com