A Theory of Venture Capital Fund Size with Directed Search*

Francesco Sannino†

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Abstract

I develop a theory of fund size and structure in venture capital where fund managers - the VCs - add value to the projects they finance, but their human capital is scarce. I propose a matching model where VCs span their nurturing activity over more projects, and entrepreneurs, who own the projects, direct their search to VCs based on their projects' quality. VCs differ in the ability to scale up their human capital. I derive necessary and sufficient conditions for positive and negative assortative matching over VC attention and project quality to emerge. Anticipating positive sorting, VCs shrink fund size below the efficient level. Entry of unskilled VCs feeds back into equilibrium sorting, increases returns at the top of the distribution - consistently with empirical evidence - and always results in a Pareto-improvement. This offers a new angle to think about policies encouraging entry in the venture capital industry. When extended to a dynamic setting, the model illustrates a novel advantage of closed, finite-horizon funds, which emerge in equilibrium even when they are socially undesirable: they attract the best entrepreneurs, who value the most the exclusive relationship that only a closed-end fund can guarantee. VCs benefit from committing to a size in the first place.

JEL codes: G24, G31, D82, D83

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†London School of Economics. E-mail: f.sannino@lse.ac.uk. Website: francescosannino.wixsite.com/sannino

1 Introduction

Venture capital has been undoubtedly a successful model of financing entrepreneurship. The common view among practitioners and academics is that venture capitalists (henceforth VCs) create value through a number of activities such as monitoring, selecting top management, and experimenting innovative business strategies. There is evidence that VCs differ considerably in their ability to generate returns.[1] In light of the significant role they play in boosting growth, it is therefore important to understand how capital is allocated across, and used by, these scarce, and differently skilled VCs. This paper argues that self-selection of different entrepreneurs seeking VC finance into different VC funds is responsible for an inefficient choice of fund size by VCs, and can explain some of the regularities observed in this industry.

The success of the venture capital model has motivated many governments to try and stimulate the provision of VC financing in various ways. This has generated a debate, and some scepticism among academics, on the role of the public in improving private VC activity. In a thorough analysis of the subject, Lerner (2009) argues that public measures encouraging VC investments may favour only the less efficient VCs, and even crowd out investments from the most knowledgeable ones.[2] But does allowing less sophisticated VCs in the economy necessarily result in bad outcomes?

I tackle these issues by developing a matching model of fund management in venture capital. There are two sets of agents in the economy: VCs, and entrepreneurs. To capture scarcity in the quality and quantity of a VC's human capital and expertise, I assume that VCs value added - or *attention* - to each investment dilutes as the number of projects they finance increases.[3] VCs differ in skill, which governs how efficient they are at increasing the size of their portfolio: the combination of VC skill and size ultimately determines the level of attention the VC can provide to each project under their management; more skilled VCs are those that can provide higher attention for any given portfolio size. For them, the diseconomies of scale are less severe. On the other side of the market, each entrepreneur owns one project. Projects are heterogeneous in quality. A projects needs the input of a VC to become profitable. The return from a project is a deterministic function of its own quality and of the VC's attention.

In the model VCs move first and choose a fund size - or capacity - to which they commit. Entrepreneurs move after VCs. They first decide whether to enter the market and, if they do

[1]See for example the recent findings in Korteweg and Sorensen (2017). For a survey of research in private equity, see Da Rin et al. (2011). The most relevant empirical findings that I will refer to are in Kaplan and Schoar (2005), Harris et al. (2014) and Robinson and Sensoy (2016). In particular, it appears that 1) in the cross section, there is a positive size-returns relationship at the fund level and 2) accounting for fund managers fixed effects, average returns to investors are decreasing in fund size.

[2]See in particular the discussion of the Canadian Labor Fund Program in Chapter 6.

[3]This is arguably one of the most significant drivers of the diseconomies of scale observed in the industry. For direct evidence of this, see for example Cumming and Dai (2011).

so, they observe their projects' quality; finally, they search for a suitable VC. Once a match is formed, returns are produced and shared exogenously between the VC and the entrepreneur. The focus on directed search is motivated by the application: one major distinction between the activity of VCs compared to that of other fund managers (e.g. buyouts, mutual funds) is that the former invest in targets that are in turn interested in their ability to add value; after all, entrepreneurs remain owners of a significant fraction of the firm they grow with the VC. The idea that entrepreneurs seeking venture capital money discriminate among VCs based on their reputation and perceived quality is supported by compelling evidence.[4]

Once entrepreneurs have directed their search, as many entrepreneurs as vacancies available are matched at random in a given VC skill-size combination, which defines a submarket. Since the measure of VCs in the economy, and the capacity they commit to, are limited, entrepreneurs in a given submarket may get rationed. Hence, when choosing which VC to search for, entrepreneurs trade off matching with VCs that can devote more attention to their projects, against the lower search frictions in markets where VCs attention is lower. Complementarity between the two inputs of the returns function mean that for the best entrepreneurs, the first force - the value attached to higher attention - is relatively more salient. This generates positive sorting between VCs' attention and entrepreneurial quality.

In turn, this has effects at the initial stage of the game, since VCs anticipate that managing a fund of larger size attracts low quality entrepreneurs. In equilibrium, some unskilled VCs shrink the size of their funds below what the welfare maximizing solution prescribes. The inefficiency arises because VCs don't internalize the effect their choice induces on the equilibrium assignment: what drives the separation among entrepreneurs is the increase in search frictions in markets where attention is higher compared to where it is lower. But if too many VCs offer high attention, this increase is too small, and entrepreneurs' separation is suboptimal. That is, some entrepreneurs whose quality is relatively low search for high-attention funds, lowering average quality in those submarkets. In addition, multiple equilibria may generally emerge, with Pareto-dominated equilibria being those characterized by smaller funds size.

In this environment, subsidizing entry of low skilled VCs that are inactive - for example because their ability to generate returns is not sufficient to cover the fixed costs of starting operations - always results in net aggregate gains. The reason is that these agents will absorb low quality entrepreneurs; those efficient VCs who choose to provide higher attention will attract even better projects, because only the worse entrepreneurs they were originally matched to will find it worthwhile to switch in the now larger market associated to low attention. In some cases, the total measure of projects funded by incumbent VCs will also increase. This offers a new angle to think about public intervention in this market, and a more optimistic point

[4] Hsu (2004) finds that entrepreneurs are willing to accept worse terms in order to affiliate with VCs that can provide greater value added. Recent empirical studies, starting from the seminal contribution by Sørensen (2007), show that there exist positive sorting in the industry between better VCs and start-up firms with greater potential.

of view on policies that encourage fundraising devoted to venture investments. Interestingly, Brander et al. (2014) find evidence that the presence of government-sponsored VCs does not crowd out, but rather increases investments from private VCs at the aggregate level.[5]

The model provides novel implications from entry of new VCs on the whole returns distribution. Specifically, when more unskilled VCs enter the market, while a larger share of funded projects end up in the lower side of the returns distribution, those at the top deliver higher returns. This is consistent with the findings in Nanda and Rhodes-Kropf (2013), who document that investment made "hot" periods are more likely to fail and give higher returns conditional on not failing, and in Kaplan and Schoar (2005), who find that, in times characterized by more intense activity in the industry, capital flows disproportionately to worse performing funds.

A benchmark model with random matching, or with homogeneous entrepreneurs would not produce the inefficiency in equilibrium fund size, nor the beneficial effect of entry of new VCs and its effect on the shape of the returns distribution described above.

In the second part of the paper, I use the machinery developed in the first part to study one aspect of the typical VC fund structure: differently from, for example, mutual funds, private equity funds - of which VC funds are an example - are finitely lived: VCs' activity is restricted by a clear deadline when their investments must be exited. Investors and VCs form a limited partnership. This arrangement can have the negative consequence of forcing VCs to give up investment opportunities that are discovered too late in the fund's life. Kandel et al. (2011) find suggestive evidence that being closer to the end of the fund induces myopic behavior by VCs. Barrot (2016) finds that the length of the investment horizon is associated to selection of different startups, meaning that it has real effects on the VCs' investment strategy. The common understanding is that such fund configuration, despite introducing some potential distortions, helps mitigating agency problems between limited partners - the investors - and the VCs. But one could ask whether in absence of such problems, a different arrangement would emerge. In other words, is a finite fund life also in the VC's best interest, or is it just an unavoidable cost?

To answer this question I accommodate the model to a dynamic setting where projects don't realize returns immediately, VCs can match to one entrepreneur every period, and follow the projects until they are ready to produce returns. I allow VCs to choose between a short-term contract and a long-lasting, open credit relationship with the investors. In the former case, VCs are forced to wait until the current project has realized its returns before they can get to manage a new fund, and go back to the market for entrepreneurs. In the latter, they have access to investors' money and can add a new project to the fund while the first investment is still ongoing. Projects under management of a VC that is in a short-term contract with

[5] An empirical assesment of the effects of subsidized funds activity on the profitability of investments made by incumbent, non-subsidized VCs is still missing in the literature.

investors won't overlap.[6] Thus, such contract allows the VC to commit its attention to the current project.

I show that there is no equilibrium where every VC is in a long-lasting credit relationship with investors, even when this is the most efficient arrangement. This happens because a deviating VC, by choosing the short-term contract, will be able to skim the market and attract the very best entrepreneurs, being them those who are willing to pay the highest search friction in order to match to a "committed" VC. This provides a new rationale for the prevalence of closed, finite-horizon funds in venture capital, as opposed to the open funds we observe in other contexts where fund managers invest in public securities and are not subject to a two-sided matching problem.

Relation to the Literature. The paper directly contributes to the literature focusing on size determination in fund management, with particular application to the venture capital asset class. One natural reference is Berk and Green (2004), who derive several predictions concerning fund flows in the mutual fund industry; like in that paper, fund managers in my model possess scarce skills, and therefore receive all the rents from investors by choosing fund size and fees appropriately. However, while in Berk and Green (2004) this results in an efficient allocation of money across managers, adding entrepreneurs self selection in my model produces: 1) a generically inefficient outcome, 2) multiple equilibria that are not welfare equivalent and 3) a feedback effect of entry of unskilled managers on returns at the top of the distribution. Fulghieri and Sevilir (2009) model the optimal investment strategy of a VC who trades off the higher value added from a small portfolio, with the diversification gains from a large one. Inderst et al. (2006) hold portfolio size constant, and model the beneficial effect - through stronger competition among entrepreneurs - of having limited capital at the refinancing stage. I share with the first paper the view that VC's human capital dilutes with a larger portfolio, and with the second the idea that the amount of capital a VC raises affects the type of projects funded. But in my model the distribution of VCs size and structure affects the sorting; I study the equilibria that result from the interaction among VCs that anticipate this effect.

In terms of the entrepreneur-VC relationship, in my economy matches form between two parties whose payoffs are asymmetrically affected by the current match: while the entrepreneur is solely interested in the return from his project, the VC cares about the total fund's returns. The VC faces a typical quality-quantity of matches trade-off. This approach to modelling the venture capital environment, and the essential tension implicit to it, is shared with several recent works. In Michelacci and Suarez (2004), the focus is on identifying institutional market characteristics that increase total welfare by alleviating this trade-off and allowing VCs to free

[6] This is a stylized representation. Nonetheless, while it is true that a fund manager can open new funds in parallel, fundraising is typically time consuming, which strongly limits the extent to which VCs can put projects "on hold" until enough money is raised; the practice is also limited by contractual restrictions that are meant to protect current investors, so new funds can't be raised before the current has been substantially invested.

up their human capital quicker, without destroying too much of the monitored firm's value; Jovanovic and Szentes (2013) find conditions under which the optimal contractual arrangement in presence of moral hazard on the entrepreneurs' side takes the form of an equity contract. They also explain the returns premium to VC-backed firms; Silveira and Wright (2015) study project selection on the VC's side and optimal fund size when start-up costs are random but committing funds entails opportunity costs. Contrary to mine, none of the aforementioned models analyse sorting of different entrepreneurs with different VCs in presence of these forces. More importantly, while I also assume diseconomies of scale, I don't restrict intermediaries to run one project at a time. This more realistic assumption allows to study 1) the equilibrium choice of span of control and 2) the choice of how frequently go back to the market and actively search for new investments, possibly before the current one has produced returns. The VC-entrepreneur relationship has been also the subject of a large strand of literature focusing on the inherent agency problems associated to venture capital financing, and the contractual arrangements aimed at solving this problems: notable examples are Cornelli and Yosha (2003) and Repullo and Suarez (2004), both analysing optimal security design when new information is produced about the investment at an intermediate stage, which is an essential characteristic of this environment; in Schmidt (2003) the double moral hazard problem between the two parties justifies the use of convertible preferred equity, while Hellmann (2006) extends this analysis to allow for a distinction between exit via IPO and via private acquisition and finds that automatic conversion is only triggered under exit via IPO in the optimal contract; finally, Casamatta (2003) studies the endogenous emergence of external financing from venture capitalist who also provide human capital, and shows that the optimality of common stocks versus preferred equity depends on the relative amount invested by the venture capitalist. I abstract from these issues and take a reduced form approach to the determination of returns to a project, and assume an exogenous equity contract between the two parties. However, project's quality in my model could be interpreted as a (negative) measure of the severity of the moral hazard problem on the entrepreneur's side, naturally affecting total surplus from a match.

Like this paper, Marquez et al. (2014) builds upon the fundamental observation that investments in venture capital are special in that they are subject to a two-sided matching problem. Marquez et al. (2014) develop a signal-jamming model where VCs with differential ability to produce returns manipulate fund size in order to affect entrepreneurs' learning; this, coupled with rigidity in fees adjustment ex-post, prevents them from extracting the full surplus from investors. In my model instead, the VCs ability is common knowledge. Moreover, while Marquez et al. (2014) take a reduced form approach to the determination of a fund's portfolio quality, I study and characterize sorting explicitly; since relative gains from committing higher attention are endogenous, I can derive conditions under which an equilibrium where every VC chooses a certain fund structure might unravel; plus, modelling sorting allows me to study efficiency of the funds allocation across VCs, and study the effects of entry of VCs on the entire allocation and returns distribution.

The paper also contributes to a literature focusing on the most observed features and contractual arrangements at the basis of investment funds: in Stein (2005) open-ended fund structure emerges because mutual fund managers compete for money flows and the best ones can credibly signal their ability by offering an open-end structure that can prevent them from fully exploiting arbitrage opportunities; Axelson et al. (2009) explain why buyout funds exhibit a mix of outside debt and equity financing in a setting where the key tension is between imposing discipline to privately informed managers while at the same time making efficient use of their superior screening ability.

On a more abstract level, my paper provides conditions for sorting in a matching environment with non-transferable utilities and search frictions. Eeckhout and Kircher (2010) derive general results on the consequences of search frictions in an assignment problem where sellers commit on posted prices. Requirements on the match-value function for positive and negative sorting are found to depend on the elasticity of substitution in the matching technology. In my model, where utilities are non-transferable, the strongest form of supermodularity (and submodularity) is needed to guarantee sorting, under any specification of the matching function. More results related to my setting are in Eeckhout and Kircher (2016) who study the interaction between the choice of span of control and the sorting pattern in an assignment economy; they look at competitive equilibria where types are observable on both sides, and the allocation is not limited to one-to-one. In my model there will be no direct type complementarity, hence what will govern sorting is the interaction between the diseconomies of scale, the span-of-control complementarity and the managerial resource complementarity.

Roadmap: Section 2 introduces the setup, followed by the characterization of the equilibria; Equilibria are ranked in terms of welfare achieved and compared to a second best solution in Section 3; Section 4 explores the effects of entry of new VCs in the economy; Section 5 uses results in previous sections to analyse the choice between short and long-term investors-VC relationships, in an appropriately accommodated setup; Section 6 concludes; All proofs are relegated to the Appendix.

2 Model

Agents. The economy consists of heterogeneous venture capitalists (henceforth VCs), identical investors and ex-ante identical entrepreneurs. There is an arbitrarily large measure of investors. Each investor is endowed with money, which they can invest into funds, each managed by a single VC. VCs are exogenously endowed with ability, denoted x, according to the measure G, that admits a continuous density g with full support $[\underline{x}, \overline{x}] \subset \mathbb{R}_+$. The measure of VCs in the economy is fixed. Entrepreneurs are in large supply, and can enter the market upon paying startup cost c. If they do, they draw a type λ, the quality of the project they own, from a continuous distribution f strictly positive on the entire support $\left[\underline{\lambda}, \overline{\lambda}\right] \subset \mathbb{R}_{+}$. An higher λ is a better project in a way specified in the next paragraph. Entrepreneurs need money and the VCs' input to make their projects turn into profitable firms.[7]

Projects. All projects need only one unit of money to become a firm. Call m the measure of projects a given VC is matched to in equilibrium. Define a the attention the VC devotes to each project. Assume $a \in \{a_0, a_1, ..., a_N\}$, with $a_i > a_{i-1}$. VC's attention, or managerial input, is a function of his ability and the number of firms he is matched to, $a \coloneqq a\left(m, x\right)$. In particular $a\left(m, x\right)$ is the step function:

$$a\left(m, x\right) \;\; = \begin{cases} a_N & \forall m \in [0,\, m_N^x] \\ a_i & \forall m \in \left(m_{i+1}^x,\, m_i^x\right] \end{cases}$$

with $m_i^x - m_{i+1}^x = \Delta > 0$ for all x and i, and $\partial m_i^x / \partial x > 0$ for all i and all x. In words, the two conditions mean that 1) VCs' input gets diluted when working on more projects in parallel, 2) better managers can run more projects at a given level of attention. A manager with ability x can be matched to a maximum of m_0^x projects.[8,9] Each project's return, R, is assumed to be a function of attention, a, and of the project's quality, λ. Call this function $R\left(a, \lambda\right)$.[10] It is natural to have $R_a\left(a, \lambda\right)$ and $R_\lambda\left(a, \lambda\right) > 0$. I further assume that $R\left(a, \lambda\right)$ is twice continuously differentiable in its arguments.

Matching and Information. While VCs' size and ability are common knowledge, the entrepreneur's type, λ, is his private information. Therefore, I study directed search from the long and informed side of the market, the entrepreneurs. Each VC's combination of size and ability, (w, x), will therefore form a submarket where entrepreneurs will select into, possibly depending on their type. Finally, assume that as many matches as possible are formed in each submarket; that is, the number of matches as a function of the measure of entrepreneurs searching, q_e, and the measure of money available (or "vacancies"), q_k, is given by $M\left(q_k, q_e\right) = \min\left\{q_k, q_e\right\}$.

[7]A natural interpretation - which fits the common view of the role of venture capitalists - is that young firms need to be constantly monitored, be it because entrepreneurs are unexperienced, or because the lack of collaterals makes it impossible to find alternative sources of financing.

[8]The assumption that attention jumps discontinuously with m is of no consequence in terms of the qualitative results, but allows to guarantee existence of equilibria when size is the VC's choice.

[9]A more general setting could allow for Δ to be a function of x . In which case, to ensure separation of VCs in equilibrium, I would need to impose the single crossing condition $\partial\left(\frac{m_0^x}{m_0^x - \Delta(x)}\right)/\partial x < 0$, which is satisfied when Δ is constant across xs.

[10]The direct implication is that a is all that matters to a given type of entrepreneur. In other words, project's quality does not interact with VC's ability or fund size *per se.* This separability will greatly simplify the analysis.

Payoffs, Strategies and Timing. In the first stage of the game, each VCs offers investors a contract (w, p) , which specifies the size of the fund, w, and fixed fee p that the VC receives from the investors for every dollar invested.[11] As all projects require one unit of money, I will refer to fund size w as the fund's capacity, that is the maximum measure of entrepreneurs the VC can be matched to. Investors can accept the contract, and provide the VC with w dollars, or reject and invest in an alternative technology delivering constant returns R_0. When investing in a certain VC, they will get a fixed share $\alpha \in (0,1)$ of the VC's average returns from the fund. In the second stage, entrepreneurs observe the joint distribution of (w, x) induced by the first stage, and choose whether or not to pay the startup cost. Those who do, can direct their search towards different VCs. Conditional on being matched, they receive the residual - $(1-\alpha)$ - share of the returns from their projects. All agents are risk neutral and maximize expected returns.

2.1 The Entry and Sorting Subgames

Market Tightness. Let me first study the subgame where entrepreneurs make the entry decision and direct their search into different VCs. Assume that the allocation of investors' money generates fund size between $\underline{w}$ and $\overline{w}$ with $\overline{w} > \underline{w}$. Denote $H(w, x)$ the measure of venture capitalists with fund size below w and ability below x.[12] Upon entry, the search strategy for an entrepreneur is described by a distribution over $[\underline{w}, \overline{w}] \times [\underline{x}, \overline{x}]$. Formally, the entrepreneur strategy is a mapping

$$s: \left[\underline{\lambda}, \overline{\lambda}\right] \rightarrow \triangle\left([\underline{w}, \overline{w}] \times [\underline{x}, \overline{x}]\right).$$

The strategy generates for every λ a cumulative density function $S(w, x; \lambda)$. Calling E the measure of entrepreneurs who decide to enter. Define $\tilde{S}(w, x, E)$ the measure of entrepreneurs searching in market with size below w and ability below x, given E. This is given by summing the search strategy over all the entrepreneurs, so $\tilde{S}(w, x; E) = \int_{\lambda} ES(w, x; \lambda)\, dF(\lambda)$. On the other side of the market, as a VC managing fund of size w can follow up to w projects in parallel, the amount of vacancies in submarkets below (w, x) is given by $\int_{-\infty}^{x} \int_{-\infty}^{w} \hat{w} dH(\hat{w}, \hat{x})$. To define expected payoffs properly, let $\theta(w, x; E)$ be the expected ratio of vacancies to entrepreneurs in submarket (w, x), when a measure of E entrepreneurs has entered. I will refer to $\theta(w, x; E)$ as

[11] As it will be clear when studying size determination, the assumption that VCs receive no performance-based compensation is without loss of generality. This is due to: 1) the fact that there is no agency conflict between investors and VCs, nor uncertainty about the VC's ability, and 2) the presence of a large measure of investors, which implies that investors' participation constraint will bind in all equilibria.

[12] This is endogenous, as it is determined by the investors and VCs equilibrium choice. Hence no assumption on H is made at this stage.

market tightness. The function will solve:

$$\int_{-\infty}^{x}\int_{-\infty}^{w}\hat{w}dH\left(\hat{w},\hat{x}\right)=\int_{-\infty}^{x}\int_{-\infty}^{w}\theta\left(\hat{w},\hat{x};E\right)d\tilde{S}\left(\hat{w},\hat{x};E\right).$$

Finally, define $Q(w,x;E)$ the probability an entrepreneurs finds a match when searching in market (w,x). Given that the matching function is Leontief, this is:[13]

$$Q\left(w,x;E\right)\coloneqq\min\left\{\theta\left(w,x;E\right),1\right\}.$$

I can now write type-λ entrepreneur's expected payoff from choosing to search in market (w,x) as:

$$\left(1-\alpha\right)Q\left(w,x;E\right)R\left(a\left(m\left(w,x;E\right),x\right),\lambda\right)$$

where $m(w,x;E)$ is the measure of projects per VC in market (w,x). Note that $m(w,x;E)\leq w$, but the condition may, in principle, not bind. To save on notation, I will denote $\pi_\lambda(E,s^*)$ the *equilibrium* value of type-λ entrepreneur's expected payoff. I can now describe what is an equilibrium of this subgame.

Definition 1. ***(Equilibrium in the Subgame).*** *An equilibrium in the entry and sorting subgame is characterized by a vector (E,s^*) such that:*

$$(i)\qquad s^*\left(\lambda\right)=\arg\max\nolimits_s\mathbb{E}_{w,x}\left[Q\left(w,x;E,s^*\right)\left(1-\alpha\right)R\left(a\left(m\left(w,x;E,s^*\right),x\right),\lambda\right)\right]$$

$$(ii)\qquad \int_\lambda\pi_\lambda\left(E,s^*\right)dF\left(\lambda\right)=c$$

Part (i) imposes optimality. Part (ii) from the unlimited number of entrepreneurs: it states that, ex-ante, entrepreneurs must be indifferent between entering the market and staying out.

An immediate observation to make is that in this model, not only the entrepreneur's search strategy imposes an externality to each other entrepreneur through its usual effect on search frictions, it also does by affecting VCs attention. In principle, this can generate a multitude of equilibria where the value of a VC is ultimately determined by the measure of entrepreneurs searching in a given submarket. However, one additional assumption can be shown to substantially simplify the sorting game. The assumption requires that lower VC's attention is not *too* detrimental to the average type, as formalized below.

Assumption A1. $(1-\alpha)\mathbb{E}_\lambda R(a_0,\lambda)>c\qquad\forall\lambda.$

[13] The assumption that the matching function is Leontief does not affect the equilibrium characterization. However, it is relevant in the welfare analysis. By assuming that as many matches as possible are formed in every submarket, I can abstract from inefficiencies that might arise from matching frictions within the submarket, and focus on those coming from the directed search assumption alone.

A1 states that, ex-ante, an entrepreneur would strictly benefit from paying the startup cost and match to a VC in absence of search frictions, even when the VC's attention is fully diluted (at its lowest level it is given by a_0). When A1 holds, because entrepreneurs are in large supply, new ones will enter the market until search frictions kick in. This also implies that a situation where some VCs attract no entrepreneur can not be an equilibrium of the subgame, since those VCs would be able to provide the highest attention at no search friction, offering a strict incentive to deviate to entrepreneurs.

Lemma 1. *Under A1, in any equilibrium, in each submarket there are more entrepreneurs than vacancies. That is,* $Q(w, x, E) < 1$ *and* $m(w, x, E) = w$, $\forall (w, x)$

The implication of Lemma 1 is that all VCs operate at full capacity. The next result is a direct consequence of Lemma 1, and will help characterize the equilibrium strategies in the sorting subgame.

Lemma 2. *Given* E, *in any equilibrium,* $Q(w, x; E)$ *is a function of* $a(w, x)$ *only.*

Intuitively, because VCs must operate at full capacity in every equilibrium, attention in market (w, x) is given by $a(w, x)$. As returns are only a function of attention and project's quality, an entrepreneur must be indifferent between searching in two markets where attention is the same. This suggests that, in essence, the entrepreneur's strategy reduces to which attention levels a to seek matching with.

Lemma 3. *For a given* E, *any equilibrium of the sorting subgame is mirrored by one from a game where entrepreneurs can only direct their search to different attention levels, and are then matched with VCs that are at the chosen attention, in proportion to each VC's size.*

In words, because entrepreneurs must be indifferent between searching in any market where attention is the same, any equilibrium can equivalently be represented by one where their strategy is to simply choose to search over different levels of attention, which in this reduced model is a fixed, predetermined characteristic of the VC. The distribution of vacancies will reflect total size summed across all VCs at a given iso-attention locus in the original model.

Lemma 3 turns useful because it allows to focus on a particular type of sorting equilibrium, where the sole characteristic of a VC, hence what defines a sub-market to search in, is attention. The interest is then to study what requirements should the return function obey to, so that in a general setting, independently of the distribution of types, sorting would emerge. If such conditions are identified, one can conclude that the same sorting pattern would emerge in the original model, once mixed strategies are adjusted accordingly.

Let $\Lambda^s(a)$ be the set of entrepreneurs applying to market a under strategy s, $\Lambda^s(a) := \{\lambda\colon s(a; \lambda) > 0\}$.

Definition 2. *An equilbrium exhibits positive (negative) assortative matching if* $\forall a, a^{'}$ with $a > a^{'}$

$$\lambda \in \Lambda^{s}(a) \cap \lambda^{'} \in \Lambda^{s}\left(a^{'}\right) > 0 \Rightarrow \lambda > (<) \lambda^{'}.$$

Intuitively, under positive assortative matching (henceforth PAM), higher attention can not be associated with a worse entrepreneur; however, pooling of more entrepreneurs into a given attention level is allowed. I can now state the main result of this section, that estabilishes necessary and sufficient conditions for equilibria to exhibit PAM or NAM.

Proposition 1. ***(Sorting).*** *All equilibria exhibit PAM (NAM) if and only if* $R(a, \lambda)$ *is everywhere logsuper(sub)modular.*

Notice that logsuper(sub)modularity implies super(sub)modularity, while the opposite does not hold. To build intuition why a stronger form of supermodularity is necessary for PAM, notice that, as emphasized by Eeckhout and Kircher (2010), when allowing for search frictions in matching models, two forces drive the sorting pattern, in opposite directions: the "trading security motive", which motivates higher types to select into less crowded markets, and the "match value motive", which is related to the value of being matched to better types. In this setting, the latter motive corresponds to the value of the VC's attention, which is a bigger concern when λ is high[14]. This trade-off becomes evident if one looks at the difference in expected payoff from searching in any two markets, a and $a^{'}$, with $a > a^{'}$, and differentiates it with respect to λ. This difference is increasing in λ when:

$$\underbrace{\frac{-\left(Q(a) - Q\left(a^{'}\right)\right)}{Q(a^{'})} R_{\lambda}(a, \lambda)}_{\text{trading security motive}} < \underbrace{\frac{\partial\left(R(a, \lambda) - R\left(a^{'}, \lambda\right)\right)}{\partial \lambda}}_{\text{match value motive}}.$$

In words, only when complementarities in the returns function between attention and quality are sufficiently strong does a higher-λ entrepreneur prefer to search for higher attention and face the larger search frictions in this, more crowded, market.

To understand why the logsupermodularity is sufficient, notice that a function $R(a, \lambda)$ is log-supermodular if and only if, for any $\left(a, a^{'}\right)$ with $a^{'} > a$, the ratio $R\left(a^{'}, \lambda\right)/R(a, \lambda)$ is strictly increasing in λ. This means that, if for some type $\tilde{\lambda}$, $Q\left(a^{'}\right) R\left(a^{'}, \tilde{\lambda}\right) > Q(a) R\left(a, \tilde{\lambda}\right)$, the same would be true for all $\lambda > \tilde{\lambda}$. This ensures separation.

[14] It should be noted that the condition in Proposition 1 is particularly strong because utilities are non-transferable. In the framework proposed by Eeckhout and Kircher (2010), where sellers can commit on posted prices, it is shown that, although supermodularity per se is generally not sufficient, the requirements for PAM to emerge are milder. In particular, the degree of supermodularity depends on the elasticity of substitution in the matching function. Notably here, with directed search and non-transferable utilities, the result that R must be logsupermodular holds true under any specification of the matching function.

The rest of the analysis focuses on the case when $R(a,\lambda)$ is logsupermodular. [15]

Assumption A2. $R(a,\lambda)$ *is everywhere logsupermodular.*

2.2 Choice of Fund Size and Equilibrium in the Supergame

In this section I study the VCs' choice at the initial stage, when contracting on size and fees with investors. Therefore, I endogenize the distribution $H(w,x)$, and hence will characterize equilibria of the entire game. I will restrict attention to equilibria where both VCs and entrepreneurs play symmetric, pure strategies.

As in Berk and Green (2004), VCs contract with competing investors over the fund's size and a per-dollar fee. Notice that, for every unit of money invested in the fund, investors' participation constraint gives:

$$\alpha\mathbb{E}\left[R\left(a\left(w,x\right),\lambda\right)|\lambda\in\Lambda^{s}\left(a\left(w,x\right)\right)\right]-p\geq R_{0}. \tag{1}$$

Since VCs have all bargaining power, it must be that the net return to investors equals their outside option, R_0. In other words (1) has to bind. It follows that VCs will choose w to maximize total excess returns, and then set p in such a way that investors' participation constraint binds, so to extract the full surplus and maximize total fees.

VC Strategy. The VC's decision can be further simplified by noting that, as entrepreneur's selection is affected by fund size only through its effect on attention, a VC will never set a size strictly in one region where the function $a(m,x)$ is constant. It follows that the relevant strategic choice from a VC is which attention a_i to offer. The VC will consequently propose investors the maximum size conditional on a_i, that is m_i^x. VC's strategy is therefore fully described by a mapping $\sigma:\ [\underline{x},\overline{x}]\rightarrow\{a_0,a_1,...,a_N\}$. I will sometimes refer to funds associated with higher attention as to more "focused" funds, although it should be emphasized that a more focused fund could well be of larger size than a less focused one, if it is managed by a more efficient VC. Define the set of VCs types choosing to offer a_i given σ, $X_i^\sigma := \{x:\ \sigma(x)=a_i\}$. Finally, define the set of attention levels offered in equilibrium $I^* := \{i:\ X_i^\sigma\neq\emptyset\}$. At any a_i with $i\in I^*$, and given s, E, and σ, one can then compute the probability for an entrepreneur

[15] Focusing on the case when R leads to assortative matching is motivated by the fact that, as it will be clear in the next section, this will guarantee all equilibria exhibit positive sorting between firms and managers, which is consistent with the evidence started by Sørensen (2007). Interestingly, the idea that better entrepreneurs are those that gain more by receiving VCs' advise appears to be at the core of the following quote by Fred Wilson, managing partner at Union Square Ventures: *"When it's clear the founder only wants your money and has no interest in your advice, it is hard to get excited about the investment. When it seems that all the founder wants is your advice and isn't worried about getting money, it makes you want to work with that founder".*

to find a match, or, equivalently, market tightness as:

$$Q(a_i) = \frac{\int_{x \in X_i^\sigma} m_i^x dG(x)}{E \int_{\lambda \in \Lambda^s(a)} dF(\lambda)}.$$

Before I define what is an equilibrium in the entire game, it is necessary to specify how VCs beliefs about the composition of entrepreneurs in a given market are formed. The notion of Weak Perfect Bayesian Equilbrium only disciplines beliefs on the equilibrium path, by restricting these to be computed via Bayes rule.[16] Formally, the belief β is a mapping:

$$\beta: \{a_0, .., a_N\} \to \triangle\left(\left[\underline{\lambda}, \overline{\lambda}\right]\right)$$

and, using Bayes rule, we have that, for $i \in I^*$,

$$\beta_\lambda(a_i) = \frac{f(\lambda)}{\int_{\lambda \in \Lambda^s(a)} dF(\lambda)}$$

where $\beta_\lambda(a_i)$ is the pdf $\beta(a_i)$ evaluated at λ. What about beliefs for markets where no VC is positioned, that is for any $j \notin I^*$? I am going to impose a restriction on these beliefs. The approach I follow is based on the same argument adopted by Guerrieri et al. (2010) in a similar setting. Let me first state the restriction, and then explain the intuition behind it.

Requirement 1. *Given a subgame equilibrium (s^*, E) and associated entrepreneurs expected payoff π_λ^*, the belief $\beta_\lambda(a_j)$ is strictly positive if and only if the set:*

$$Q(\lambda; a_j) := \{Q \in [0,1] \mid Q(1-\alpha) R(a_j, \lambda) \geq \pi_\lambda\}$$

is maximal.[17] *If $Q(\lambda)$ is empty for all λ, the VC expects no entrepreneur to search in market a_j.*

Essentially, for every λ, one can construct the set of Qs such that the entrepreneur would (weakly) benefit from deviating and search in market a_j. A VC that is contemplating to offer such level of attention must believe that this offer would attract the type(s) that are willing to face the highest search friction, that is, to deviate at the lowest level of Q.[18]

[16] For a formal definition of Weak Perfect Bayesian Equilibrium see definition 9.C.3 in Mas-Colell et al. (1995)

[17] For a given collection of sets $Q(\lambda; a_j)$, $\lambda \in [\underline{\lambda}, \overline{\lambda}]$, $Q\left(\hat{\lambda}; a_j\right)$ is said to be maximal if it is not a subset of any other $Q(\lambda; a_j)$.

[18] Note that the value of $Q(\lambda; a_j)$ can come from the VCs off-equilibrium behavior, the vacancies posted at attention a_j. Requirement 1 can be then interpreted as follows: "the type that is expected to search in a_j is the one for which there is a larger set of VCs actions that would make this deviation profitable". In this sense, Requirement 1 is an adaptation of condition D1 introduced by Cho and Kreps (1987) for signaling games.

In comparing equilibria, I will sometime need to compute market tightness in empty markets. To do this, I will use the lowest Q such that the type(s) selected by Requirement 1 (weakly) benefits from the deviation. Armed with the definitions above, I can now formally state what is an equilibrium of the game.

Definition 3. ***(Equilibrium).*** *An Equilibrium is a vector $(E, s^*, \sigma^*, \beta)$ constituting a Weak Perfect Bayesian Equilibrium, with the restriction that β satisfies Requirement 1 off the equilibrium path.*

In what follows, I characterize all equilibria of the game. The main message will be that better VCs will necessarily match with higher quality entrepreneurs. This comes directly from the result in the previous section, together with the properties of the function $a(m, x)$, ensuring that the best VCs have to give up fewer projects in order to provide higher levels of attention.

Proposition 2. ***(Partitional Equilibria).*** *All equilibria are described by a partition of the set $[\underline{x}, \bar{x}]$ defined by cutoffs $\{\underline{x} = x_{-1}, ..x_i, .., x_N = \overline{x}\}$ and a partition of $\left[\underline{\lambda}, \overline{\lambda}\right]$ defined by cutoffs $\left\{\underline{\lambda} = \lambda_{-1}, ..\lambda_i, .., \lambda_N = \overline{\lambda}\right\}$ such that for any $i \in I^*$, $\Lambda_i^{s^*} = [\lambda_{i-1}, \lambda_i]$ and $X_i^{\sigma^*} = [x_{i-1}, x_i]$. If $i \notin I^*$,$\lambda_i = \lambda_{i-1}$ and $x_i = x_{i-1}$. For all adjacent $i, j \in I^*$ with $i > j$:*

$$(i) \quad m_j^{x_j}\left(\alpha \mathbb{E}\left[R(a_j, \lambda) \mid \lambda \in \mathit{\Lambda}_j^{s^*}\right] - R_0\right) = m_i^{x_j}\left(\alpha \mathbb{E}\left[R(a_i, \lambda) \mid \lambda \in \mathit{\Lambda}_i^{s^*}\right] - R_0\right).$$

$$(ii) \qquad Q(a_j)(1-\alpha) R(a_j, \lambda_j) = Q(a_i)(1-\alpha) R(a_i, \lambda_j).$$

$$(iii) \qquad \textit{For any } j \notin I^*,$$

$$m_i^{x_i}\left(\alpha \mathbb{E}\left[R(a_i, \lambda_i) \,|\lambda \in \mathit{\Lambda}_i^{s^*}\right] - R_0\right) \geq m_j^{x_i}(\alpha R(a_j, \lambda_j) - R_0) \qquad \forall i \in I^*.$$

In words the proposition states that all equilibria have the following form: entrepreneurs and VCs select into different attention levels according to their type, with successive subintervals of the equilibrium partitions $\mathit{\Lambda}_i^{s^*}$ and $X_i^{\sigma^*}$ corresponding to set of VCs and entrepreneurs selecting higher attention. Conditions (i) and (ii) impose that type at the limit of each subinterval are indifferent between the two adjacent attention levels where types right below and above are assigned to. Condition (iii) is where the requirement on off-equilibrium beliefs kicks in. Notice that, if $j \notin I^*$, $\lambda_j = \lambda_{j-1}$. Hence, condition (iii) is requiring that no VC finds it profitable to deviate to an off equilibrium a_j, given that this deviation would attract the highest entrepreneur in the set of those who select the closest lower a_i among those $i \in I^*$. Notice that this also implies that, when offering some out-of-equilibrium attention higher than in any non-empty market, a VC must expect to attract (if any) only type $\overline{\lambda}$, the type most willing to switch to that market. Similarly, offering attention lower than in any non-empty market can only attract the lowest type, $\underline{\lambda}$. Figure 1 provides a graphical representation of an equilibrium.

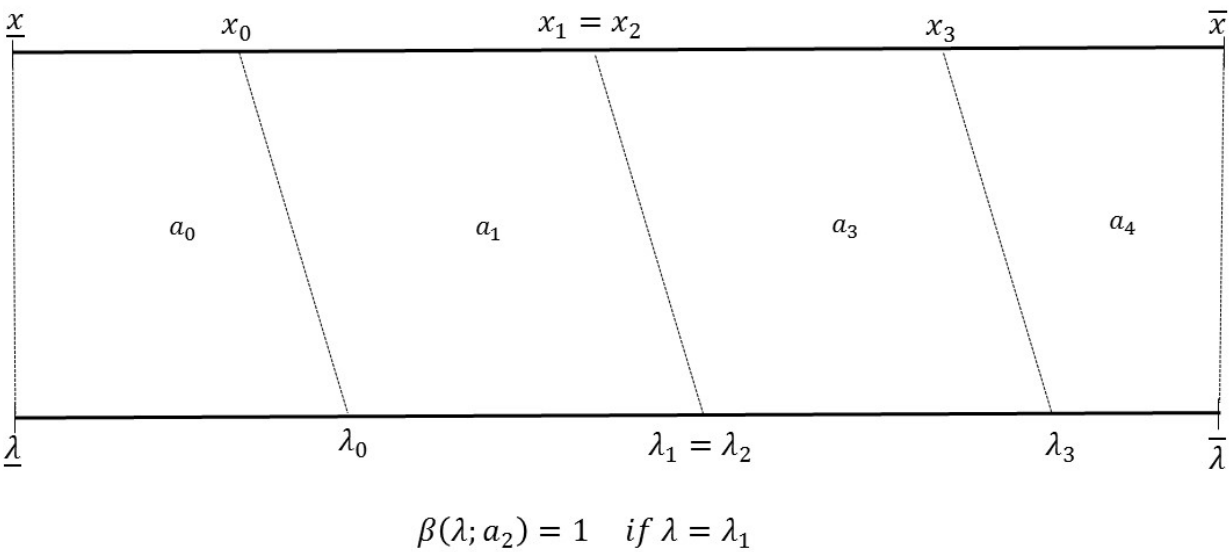

Figure 1: In this example, there are four non-empty submarkets in equilibrium. By offering attention a_2 - that no VC chooses in this example - a VC must believe to attract type λ_1, being the highest type searching in a_1 in equilibrium.

3 Efficiency

Ex-ante, total welfare in the economy amounts to the expected fees VCs receive from the investors. This is due to investors perfectly competing for VCs, and the entrepreneurs' free entry condition. In expectation, VCs are the only agents extracting rents. Denote total vacancies in a given market i, $W_i^{\sigma^*}$, which obviously depends on the particular equilibrium strategy profile that is examined. Since the fees VCs get equal the total excess returns to investors, for a given equilibrium, aggregate welfare is then given by:

$$V(E, s^*, \sigma^*) = \sum_{i \in \{1,..,N\}} W_i^{\sigma^*} \left(\alpha \mathbb{E} \left[R(a_i, \lambda) \,|\, \lambda \in \Lambda_i^{\sigma^*} \right] - R_0 \right).$$

Generally, equilibria need not be unique. A first question one can ask is whether some equilibria are more desirable then others, from an ex-ante point of view. The next proposition states that some type of equilibria can be unambiguously ranked. Interestingly, the undesirable equilibria are those where markets for higher level of attention are thicker, relatively to those for lower attention.

Proposition 3. ***(Ranking Equilibria).*** (i). *An equilibrium of the game induces higher welfare than any another equilibrium where markets for higher attention are thicker, that is* Q_i/Q_j *is bigger for all* (i,j) *and* $i>j$. (ii). *An equilibrium of the game induces higher welfare than any another equilibrium where the ratio* W_i/W_j *is bigger for all* (i,j) *and* $i>j$.

The reason why equilibria where markets for higher level of attention are thicker are Pareto inferior is that, when increases in search frictions for any two adjacent market are small, the resulting assignment is characterized by worse selection at the top, that is, each cutoff λ_i is lower, leading to lower average quality at each attention level. The second part of the Proposition is a consequence of this, and the fact that, whenever W_i/W_j is larger, entrepreneurs' search adjust so that the relative search friction between market i and j , Q_i/Q_j, is also larger. The emergence of Pareto dominated equilibria is due to a typical coordination failure on the VCs side: when many choose to raise more focused funds, it is relatively easy for entrepreneurs to find a match in the associated markets; as a result, only very low quality entrepreneurs are willing to give up the higher attention, and go for a less crowded market. In these equilibria, this exacerbates the adverse selection associated to setting a larger fund capacity, and the economy is stuck in a situation where even relatively inefficient VCs choose to raise a focused fund.

I now study what would be the welfare maximizing allocation of VCs into fund sizes when the induced aggregate effect on sorting is taken into account. Below I define a Second Best Allocation as a solution to this problem. Because for a given profile of VCs strategies the sorting equilibrium need not be unique, call $\Lambda^{s^*}(\sigma)$ the collection of equilibrium partitions of the set $\left[\underline{\lambda},\overline{\lambda}\right]$ associated to a strategy profile σ. Call $\Lambda^{s^*}(\sigma;n)$ one element of this set. By Proposition 2, $\Lambda^{s^*}(\sigma;n)$ is composed of successive intervals, each associated to a submarket a_i, and denoted $\Lambda_i^{s^*}(\sigma;n)$.

Definition 4. *A* ***Second Best Allocation*** *is a mapping* $\tilde{\sigma}:[\underline{x},\overline{x}]\rightarrow\{a_0,a_1,...,a_N\}$ *that solves:*

$$\tilde{\sigma} \quad = \quad \arg\max_{\sigma} \qquad \textstyle\sum_{i\in\{1,..,N\}} \qquad W_i^{\sigma}\left(\alpha\mathbb{E}\left[R\left(a_i,\lambda\right)|\lambda\in\Lambda_i\right]-R_0\right)$$

$$s.t. \qquad \forall i,\ \Lambda_i=\Lambda_i^{s^*}(\sigma;n) \qquad\qquad \textit{for some } n$$

It is easy to observe that a Second Best allocation must be characterized by a partition of $[\underline{x},\overline{x}]$, with more skilled VCs being assigned to higher levels of attention. Call x_i^{sb} the limits of this partition. The next result compares the equilibrium with the second best, in an environment when attention can only be high are low.

Proposition 4. ***(Inefficiently small funds).*** *When* $a\in\{a_0,a_1\}$, $x_0^{sb}>x_0$: *in equilibrium, too many VCs choose high attention compared to the second-best solution.*

There is a simple intuition behind this result. A solution to the Second Best problem involves a tradeoff between allocating VCs to their optimal size, and the motive to increase relative search frictions so to induce a higher cutoff, and hence higher average quality in both markets; however, starting from any equilibrium - including the Pareto superior one - a marginal increase in x_0 come at a negligible (close to zero) cost in terms of the misallocation of VCs to a larger fund size, but has a strictly positive impact on the sorting outcome through the increase in λ_0.

4 Entry and Comparative Statics

The analysis so far has focused on an economy where the measure and distribution of VCs is fixed. As introduced in Section 1, however, one object of interest of my analysis is to study the effect of entry of new VCs on the equilibrium allocation of investors money and projects to VCs. This is mainly motivated by the debate around the effectiveness of policies that encourage VC investments, and by the recent finding that government sponsored VC has not crowded out investments by private VCs at the aggregate level. Moreover, there exists evidence that money committed in the venture capital industry is highly volatile, that it is subject to booms and busts and that the number of funds dedicated to this asset class vary across time, sometimes in response to the business cycle. Determining the reason why these cycles occur is beyond the scope of this paper. However, the model can offer predictions on how the distribution of returns is affected by the inclusion of new VCs in the economy.

Stable Equilibria. Let me restrict the analysis of this section to the case where attention can be either high or low, so that VCs and entrepreneurs can sort into two submarkets only. That is, $a \in \{a_0, a_1\}$. The main advantage is that, for a given equilibrium cutoff x_0, the induced equilibrium sorting is unique. This allows to make comparative statics around a candidate equilibrium. First, it is convenient to define the function

$$\phi\left(a, a', \tilde{\lambda}\right) := \frac{\alpha \mathbb{E}\left[R\left(a, \lambda\right) | \lambda \geq \tilde{\lambda}\right] - R_0}{\alpha \mathbb{E}\left[R\left(a', \lambda\right) | \lambda \leq \tilde{\lambda}\right] - R_0}.$$

The function $\phi\left(a, a', \tilde{\lambda}\right)$ is the expected *per dollar* excess return from choosing attention, a, and attract entrepreneurs above some $\tilde{\lambda}$, *relative* to the excess return from choosing attention a' and attract entrepreneurs below the same threshold. The function $\phi\left(a, a', \tilde{\lambda}\right)$ need not be monotone in $\tilde{\lambda}$. Below is an example where it is always decreasing.

Example 1. Assume quality λ is uniformly distributed over the support $[0,1]$. Returns are given by $R(a,\lambda) = a + (a-k)\rho(\lambda)$ with $a > k > 0$.[19] If $\rho(.)$ is any increasing linear function, it can be verified that the ratio $\phi\left(a, a^{'}, \tilde{\lambda}\right)$ is decreasing in $\tilde{\lambda}$ for any $a > a^{'}$ and any k, $R_0 > 0$.

Unless the function is increasing everywhere, equilibria may not be unique. I introduce below one appealing property of a candidate equilibrium, that will help identify the comparative statics of this section. The property is based on a stability argument and will refine the set of equilibria. Notice that the equations identifying the equilibrium vector (x_0, λ_0) are

$$m_1^{x_0}(\alpha\mathbb{E}\left[R(a_1,\lambda) \mid \lambda \geq \lambda_0\right] - R_0) - m_0^{x_0}(\alpha\mathbb{E}\left[R(a_0,\lambda) \mid \lambda \leq \lambda_0\right] - R_0) = 0 \tag{2}$$

and

$$\frac{W_1(x_0)}{1-F(\lambda_0)}(1-\alpha)R(a_1,\lambda_0) - \frac{W_0(x_0)}{F(\lambda_0)}(1-\alpha)R(a_0,\lambda_0) = 0. \tag{3}$$

Call $\eta(x_0,\lambda_0)$ the left hand side of (2) and $\mu(\lambda_0,x_0)$ the left hand side of (3).

Definition 5. ***(Stable Equilibria).*** *An Equilibrium* $\left(\tilde{x_0}, \tilde{\lambda_0}\right)$ *is stable if it is an attracting fixed point of the vector function:*

$$\Theta(x_0,\lambda_0) = \begin{bmatrix} \eta(x_0,\lambda_0) + x_0 \\ \mu(\lambda_0,x_0) + \lambda_0 \end{bmatrix}$$

In words, a stable equilibrium is one that, after a small perturbation that forces some agents' strategies away from it, will eventually converge back to itself.[20]

[19] This function is logsupermodular whenever $\rho^{'} > 0$.

[20] In the Appendix, it is shown that this is equivalent to requiring stability of the costant solution (x_0,λ_0) to a system of differential equations where x_0 is assumed to increase (decrease) proportially to the marginal benefit (loss) to type x_0 from chossing attention a_0 rather than a_1, given λ_0, and the same is assumed for the differential equation governing the changes of λ_0 for a given x_0.

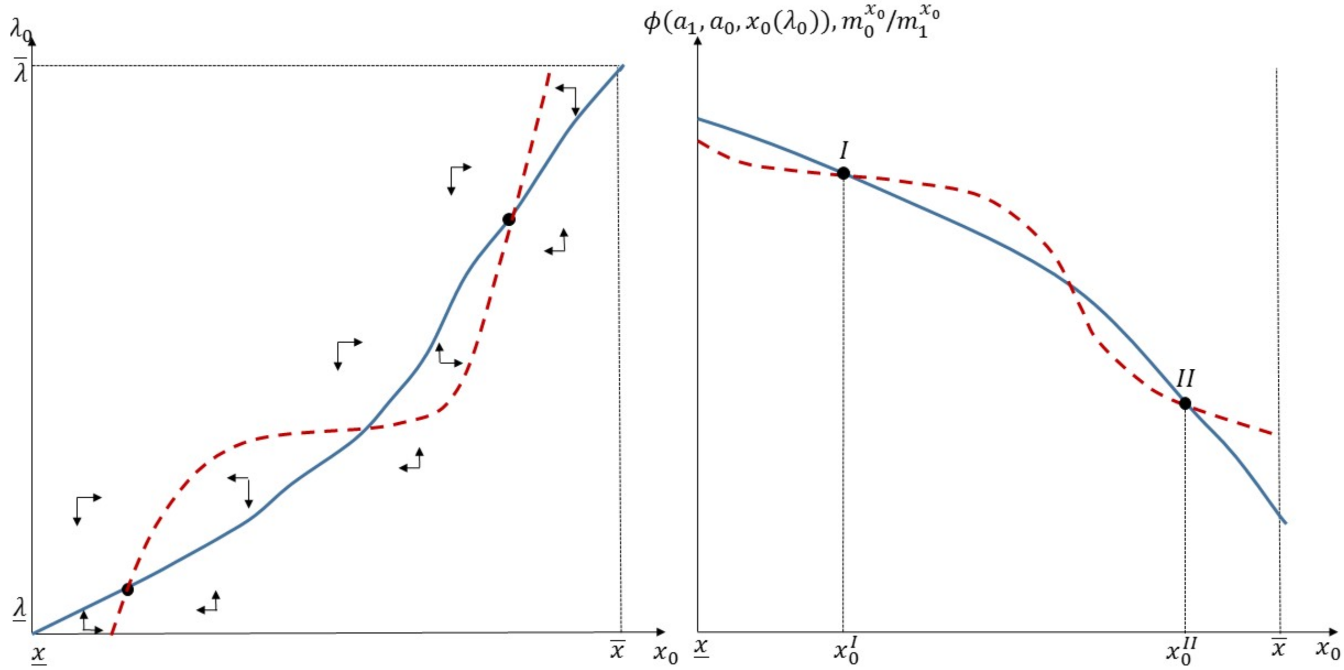

Figure 2: Left: The solid line is the solution to the entrepreneur's indifference condition for each level of x_0. Arrows above (below) this line point upwards (downwards) because if the population cutoff was type λ, he would strictly benefit (lose) from moving to market a_0. The dotted line connects all the indifferent VCs, for each λ_0. Arrows at the west (east) of the line point to the right (left) because if the population cutoff was type x, he would strictly benefit (lose) from moving to market a_0. The stable equilibria are the two intersection at the bottom-left and top-right of the picture. Right: The solid line is the function $m_0^{x_0}/m_1^{x_0}$, decreasing because the relative difference between m_0 and m_1 is smaller for better VCs. The dotted line is $\phi\left(a_1, a_0, \lambda_0\left(x_0\right)\right)$ which moves with x_0 through its effect on λ_0 and is decreasing because when x_0 increases, λ_0 increases, and $\phi_{\tilde{\lambda}} < 0$ by assumption. An equilibrium is an intersection of this two curves, and stable equilibria (denoted I and II) are those where ϕ is flatter than $m_0^{x_0}/m_1^{x_0}$ at the intersection. In this example, there are three equilibria. I is the worse equilibrium, while II is the welfare maximizing equilibrium.

I will now conduct comparative statics around a stable equilibrium.

Comparative Statics. One interesting exercise is to study what happens when new unskilled VCs enter the market. More precisely, imagine the distribution of skills g is defined on a support larger than $[\underline{x}, \overline{x}]$. Initially, only VCs in $[\underline{x}, \overline{x}]$ operate. What will happen if some of the worse VCs previously excluded decide to enter? In other words, what are the consequences of a decrease in $\underline{x}$? Notice that the exclusion of some VCs from the market could be resulting from the presence of barriers to entry. Since VCs expected payoff in equilibrium is strictly increasing in x, if being active in the market requires a fixed investment κ, the ex-ante payoff to the marginal VC - $\underline{x}$ - would be given, in an interior equilibrium, by:

$$m_0^{\underline{x}}\alpha\left(\mathbb{E}\left[R\left(a_0,\lambda\right)\mid\lambda\leq\lambda_0\right]-R_0\right)=\kappa.$$

In this environment, subsidizing the investment κ to the highest type outside the market would be equivalent to induce a marginal decrease in $\underline{x}$ in the venture capital market. The result below states what is the induced effect of this change on all stable equilibria.

Proposition 5. ***(Entry of unskilled VCs)***. *For every stable equilibrium* (x_0,λ_0)*:* (i). $\frac{\partial\lambda_0}{\partial\underline{x}}<0$. (ii) *. As* $\underline{x}$ *decreases, welfare increases.* (iii) *.* $\frac{\partial x_0}{\partial\underline{x}}>0$ *if and only if* $\phi_{\tilde{\lambda}}\left(a_1,a_0,\lambda_0\right)>0$.

In words, the inflow of unskilled VCs leads some entrepreneurs to switch to the low attention market: the indifferent entrepreneur's quality is higher; total welfare increases; when the function $\phi\left(a,a',\tilde{\lambda}\right)$ is decreasing in the cutoff $\tilde{\lambda}$ - hence the relative gain in attracting entrepreneurs above versus those below $\tilde{\lambda}$ is lower the higher is $\tilde{\lambda}$ - some VCs originally raising a small fund, opt for a large fund.

The intuition is simple. The relatively unskilled VCs who enter the market will select a_0. The larger number of vacancies in the market for low attention pushes the cutoff λ_0 up. This implies that those VCs who will keep raising relatively smaller funds will select better projects. Essentially, the now larger market for unfocused funds absorbs some of the low quality entrepreneurs from the economy. Because the increase in λ_0 increases average quality in both submarkets, total welfare increases. When the function $\phi\left(a,a',\tilde{\lambda}\right)$ is decreasing in $\tilde{\lambda}$, the adverse selection problem associated to managing a larger fund is less severe, inducing more VCs to raise one. In this circumstance, the inflow of unsophisticated venture capitalists increases average fund size and aggregate investments in the market. This is consistent with the findings in Brander et al. (2014).

Think now of the distribution of returns of funded projects in the industry. Returns will necessarily take values $R\in\left[R\left(a_0,\underline{\lambda}\right),R\left(a_1,\overline{\lambda}\right)\right]$. The shape of the returns distribution will depend on that of the distribution of projects quality - f - *and* on the equilibrium choices of VCs and entrepreneurs.

I show that the returns distribution is also affected by entry of new, unskilled VCs. In absence of sorting, the effect one should expect is mechanic: relatively more VC funds would now end up delivering low returns. When sorting is taken into account though, the positive externality induced to those incumbents VCs who keep raising focused fund results in higher returns at the top of the distribution. The corollary below formalizes this observation.

Corollary 1. ***(Entry and returns distribution)***. *For each equilibrium, as* $\underline{x}$ *decreases, there exists a point* $\tilde{R}$ *in the new distribution of returns, such that* $\forall R>\tilde{R}$ *returns are higher conditional on being above* R *and are more likely to be below* $\tilde{R}$.

The effect is consistent with the findings in Nanda and Rhodes-Kropf (2013), who document that investment made in "hot" periods are more likely to fail and give higher returns conditional on not failing, and in Kaplan and Schoar (2005), who find that, in times characterized by more intense activity in the industry, capital flows disproportionately to worse funds.

5 Finite Fund's Life

Contrary to standard firms and organizations, private equity funds are finitely lived. Fund managers' activity is restricted by a clear deadline when their investments must be exited so that limited partners get the returns from them. This can generate a number of inefficiencies. Perhaps the most obvious of these - which does not have to do with any asymmetric information and conflict of interest between fund managers and investors - is the fact that managers constrained by capital committed in the vintage year don't approach investors before returns from (a good share of) the first investments have materialised, and hence might be forced to give up new investment opportunities that show up too late in the fund's life. While it is true that a manager can in principle open new funds in parallel, fundraising is typically time consuming and this strongly limits the extent to which VCs can put projects "on hold" until enough money is raised; the practice is also limited by contractual restrictions that are meant to protect current investors, so that fund managers can't form successor funds before the existing one is substantially invested or has completed its investment period. Typically, this configuration is justified by agency problems between limited partners and general partners. But one could ask whether in absence of such problems, a different arrangement would emerge. In other words, is a finite fund life also in the VC's best interest? The main point of this section is to show this is the case. I will argue that this structure arises in equilibrium due to the incentive that entrepreneurs' sorting provides.

5.1 A Dynamic Setting

Time - denoted t - is countably infinite. To make the analysis more accessible, I assume in this section that all VCs are identical. They are infinitely lived and, at each period in time, maximise the sum of expected fees, with common discount factor $\delta \in (0, 1)$. Investors are in large supply and are endowed with one unit of money per period. In this section I normalize their outside option - R_0 - to zero. For simplicity, assume now VCs can only find up to one entrepreneur in each round of matching they participate to. It takes two periods for each project, independently on its quality, to develop and produce returns. Returns are again a deterministic function of project's quality and attention, $R(a, \lambda)$, which in this section is further assumed to be log-supermodular everywhere. Once a project has produced returns, the match expires. It follows that a VC searching for an entrepreneur can be in either of two states: he can be unmatched, or already be dealing with a project that is currently at its intermediate stage.

Diseconomies of Scale. To capture the same quality-quantity trade-off as in the previous sections, I assume VCs attention into a particular project is determined by whether he has dealt with another one in any period of the project's life span. Denote the levels of a with and without overlapping projects a^h and a^l respectively, with $a^h > a^l$.

As before, on the other side of the market there are entrepreneurs. Assume at each t a new generation of entrepreneurs is born. They make the irreversible entry choice, and, conditional on entry, draw a type and direct their search. Those who don't get matched die. Those who do, and receive the share $(1-\alpha)$ of the returns, and those who don't enter the market at all, leave the economy forever. Finally, assume that at any time t a measure of new VCs enter the market; at random, an equal measure of VCs dies.

5.2 Choice of Fund Structure

At the beginning of each period, managers approaching investors can opt for either of two fund structures. In one case, they can choose an open credit line that allows them, at any time t to have the necessary cash to finance a new project. Alternatively, they can form a closed fund, with a finite, two periods long horizon. In the latter case, a fund consists essentially of a single investment, that matures returns two periods from the initial formation. This structure can be interpreted as a short-term contract between the investor and the VC. One can imagine that an investor writing this type contract will have its wealth at the intermediate period invested on the alternative asset. If approached by the VC in the intermediate period, the investor wouldn't have the liquidity to provide the VC with the money to start a new project. This is realistic: pension funds (representing a large share of investors in venture capital) usually meet capital calls by selling positions in liquid indexes[21]. Whichever is the interpretation, all that matters is that a short-term contract creates an endogenous commitment not to start a new project before the original has produced its returns.

Note that, as clarified in Section 2, managers' choice maximizes the fund's total returns, because they can set fees so to hold investors to their participation constraint. Therefore a fund manager's objective is to maximize the discounted sum of expected excess returns.

Summary of the Timing. Let me now summarize the timing of the game:

- At each time t newborn VCs and pennyless ones approach investors, choose a fund structure, and contract over the fee p that investors pay them upon realization of each project's

[21] See Robinson and Sensoy (2016).

returns. Managers opting for a finite-horizon fund can't approach investors before the project they are currently financing has produced returns.

- Entrepreneurs observe investors' strategy and make the entry choice. Those who enter the market privately observe their type λ and direct their search.
- At $t+1$ managers who chose a open credit line have the money to search for a new project. Every match formed in t generates returns $R(a, \lambda)$ at time $t+2$. By then, t-generation entrepreneurs who matched leave the market forever.

Assumption A3. $(1+\delta) R\left(a^l, \lambda\right) > R\left(a^h, \lambda\right) \qquad \forall\lambda.$

The assumption above limits the extent to which a manager's human capital is destroyed when working on parallel projects. Under A3, keeping quality fixed, it is always optimal to start a new project every period. In turn this means that a manager that expects to attract the same type of projects is always going to search actively for new ones. It is worth noting that, if the inequality in A3 was reversed, the choice of which contract to enter with the investors would be inconsequential: managers would never start a new project before the current has reached termination; the fact that they can't access liquidity at any point in time would not constrain their choices. When A3 holds instead, entrepreneurs that are searching for a match will effectively face the choice between two distinct markets: one where "uncommitted" VCs will provide attention a^l, and the other where attention is at a^h, because agents managing a closed, finite-horizon fund will not be able to search before the original investment has matured.

The Sorting Subgame. I first derive sorting behavior when a positive measure of vacancies is available in both markets. Normalize the mass of VCs to one and denote γ_t the share of managers running a finite-horizon fund at time t. It is immediate from results in previous sections to observe that equilibrium search behavior at time t is then characterized by a threshold λ^* such that entrepreneurs search in the high-attention market if and only if $\lambda \geq \lambda^*$. This is due to log-supermodularity of $R(a, \lambda)$. The threshold is implicitly defined by the equation:

$$\frac{\gamma_t}{1-\lambda^*} R\left(a^h, \lambda^*\right) = \frac{1-\gamma_t}{\lambda^*} R\left(a^l, \lambda^*\right). \tag{4}$$

Lemma 4. *The solution to* (4) *is unique: given a share of managers with finite-horizon funds* γ_t*, there is a unique equilibrium of the sorting subgame.*

In principle, VCs might choose different contracts at different times of their lives, and I allow for that. However, for the sake of simplicity, I restrict attention to equilibria where, whenever VCs are indifferent which fund structure to choose, the share of VCs going for either option stays constant. Such equilibria are always possible to construct, provided at a certain time t agents are indifferent about which contract to choose, thanks to the assumption that new VCs enter the market at each t.

Definition 6. *A stationary equilibrium is an equilibrium in which the shares of VCs choosing either fund structure is independent on t.*

Let me focus on the case when selecting the best entrepreneur is appealing to the VC. Formally, this means imposing the following restriction.

Assumption A4. $R\left(a^h, \bar{\lambda}\right) > (1+\delta)\,\mathbb{E}\left[R\left(a^l, \lambda\right)\right].$

Under A4 the returns from following the best entrepreneur exclusively are higher than those from financing two average projects in two subsequent periods. The main result of this section can now be stated.

Proposition 7. ***(Equilibrium Fund Structure).*** *(i). There is no stationary equilibrium where all VC choose the open credit line. (ii). The equilibrium's measure of VCs with finite-horizon funds, γ, is the solution to the equation:*

$$\mathbb{E}\left[R\left(a^h, \lambda\right)|\lambda \geq \lambda^*(\gamma)\right] = (1+\delta)\,\mathbb{E}\left[R\left(a^l, \lambda\right)|\lambda \leq \lambda^*(\gamma)\right]$$

whenever it exists. (iii). When $\mathbb{E}\left[R\left(a^h, \lambda\right)\right] \geq (1+\delta)\,R\left(a^l, \underline{\lambda}\right)$, there is a stationary equilibrium where every VC chooses the finite-horizon fund.

Notice that, because of A4, if the function $\phi\left(a^h, a^l, \tilde{\lambda}\right)$ was decreasing in $\tilde{\lambda}$ - as it is in Example 1 - the condition in (ii) would have no solution, while condition (iii) would always hold. This means that a situation where every VC chooses the finite-horizon fund would be the unique equilibrium.

Similarly to how established in the general static model, the motive to attract better entrepreneurs generates an inefficiency, as VCs don't internalize the aggregate effect - due to equilibrium sorting - of their choices. In particular, one natural and policy relevant question would be whether allowing these short-term contracts is desirable. It turns out that, under A3, banning the short-term contracts is always beneficial.

Proposition 8. ***(Banning finite-horizon funds).*** *Every equilibrium of the game delivers lower welfare than the case where every VC chooses the open credit line. That is, banning finite-horizon funds improves welfare.*

It is easy to see why the "corner" equilibrium where every VC chooses the short-term contract is welfare detrimental. Infact, notice that under A3, the choice of which contract to sign involves a simple trade-off: on the one hand, starting a new project every period allows the VC to make the best use of his human capital, as the dilution in attention is assumed to be small; on the other, committing to an exclusive relation helps the VC attract the best entrepreneurs. However, *in equilibrium* this commitment confers no benefit at all, since every VC will look alike. In all interior equilibria, VCs that opt for the infinite-horizon fund are attracting a negatively selected subset of entrepreneurs. Since expected returns are ultimately the same to all VCs - as they have to be indifferent which contract to choose - it follows that expected aggregate returns would be higher if all VCs would choose the long-term contract and get matched with the average entrepreneur.

6 Conclusion

I have introduced a matching model of fund management where the two key ingredients are scarcity in fund managers human capital and directed search from entrepreneurs who are heterogeneous in their projects' quality. The two features are inspired by several stylized facts and empirical findings about the venture capital industry. In the model, entrepreneurs trade off matching with better VCs - those who opt for relatively smaller funds, and hence can devote more attention to their projects - against the lower search frictions associated to worse VCs. VCs are different in ability to scale up their human capital, and select fund size to maximise total returns. Anticipating that, due to complementarities in the returns function, higher quality entrepreneurs will sort into funds where attention is higher, VCs tend to shrink fund size excessively. In this environment, subsidizing entry of VCs that are inactive - for example because their ability to generate returns is not sufficient to cover the fixed costs of starting operations - always results in net aggregate gains. The reason is that these agents will absorb low quality entrepreneurs, and hence alleviate the adverse selection problem incumbent VCs are facing. This offers a more optimistic point of view on policies that encourage fundraising devoted to venture investments. Since the distribution of fund sizes feeds back into the equilibrium sorting, entry of VCs at the bottom of the skills distribution increases returns at the top. The effects of entry, as well as the inefficiency and the emergence of multiple, Pareto-dominated equilibria are a consequence of entrepreneurs self-selection and would not result from a model with random matching or homogenous entrepreneurs. In the second part of the paper, I have used the results from the first part to analyse the equilibrium choice between a closed, finite-horizon fund, versus an open, firm-like investors-VC relationship, in a simple dynamic version of the model. From the VC's point of view, finite-horizon funds come at the cost of giving up investment opportunities arriving when the current fund is still ongoing; entrepreneurs value this commitment, because it guarantees exclusive attention. A situation where all VCs opt for the open fund unravels, even when this would be the welfare maximising solution, due to the incentive to skim the market and attract the best entrepreneurs, who are the most willing to pay the highest search friction and get exclusive attention. Similarly, one where every VC has the closed fund is sustainable as an equilibrium outcome, because a deviation to the open-end fund would attract the worse entrepreneurs. This result suggests another reason why VCs raise funds with a finite horizon and with explicit limits on the investments that they can make while the current fund is still ongoing. They might benefit from committing to a size in the first place.

Appendix

Proof of Lemma 1. Consider first those (w,x) for which $\tilde{S}(w,x;E)>0$. Take one submarket $(\tilde{w},\tilde{x})$ where $Q(\tilde{w},\tilde{x})=1$. By entering and searching in it, an entrepreneur that is outside the market gets in expectation:

$$(1-\alpha)\mathbb{E}\left(a\left(m\left(\tilde{x},\tilde{w},E\right),\tilde{x}\right),\lambda\right)-c\geq(1-\alpha)\mathbb{E}R\left(a_0,\lambda\right)-c>0.$$

Hence, when $\tilde{S}(w,x;E)>0$, it must be that $Q(\tilde{w},\tilde{x})<1$. To show that $\tilde{S}(w,x;E)>0$ for all (w,x), assume not and denote $(\hat{w},\hat{x})$ the submarket where no entrepreneur searches. Then, any type λ who entered the market would like to deviate and search in $(\hat{w},\hat{x})$, as this would give:

$$(1-\alpha)R\left(a_N,\lambda\right)>Q\left(w,x;E\right)(1-\alpha)R\left(a\left(m\left(w,x\right),x\right),\lambda\right)\qquad\forall\left(w,x\right).$$

■

Proof of Lemma 2. By Lemma 1, returns to type λ in market (w,x) conditional on matching are given by $R\left(\left(a\left(w,x\right)\right),\lambda\right)$. Take two markets (w,x) and $\left(w^{'},x^{'}\right)$, with associated attention levels a and $a^{'}$, with $a=a^{'}$. Assume that $Q\left(w,x,E\right)>Q\left(w,x^{'},E\right)$. Then, any entrepreneur searching in $\left(w^{'},x^{'}\right)$ could deviate to (w,x) and get:

$$Q\left(w,x,E\right)R\left(a,\lambda\right)>Q\left(w^{'},x^{'},E\right)R\left(a^{'},\lambda\right).$$

■

Proof of Lemma 3. In the original model entrepreneurs maximize $Q\left(w,x,E\right)R\left(a\left(w,x\right),\lambda\right)$, and, by Lemma 2, $Q\left(w,x,E\right)=\theta\left(w,x,E\right)$. Because $R\left(a\left(w,x\right),\lambda\right)$ is costant across an iso-attention locus, and since Lemma 2 must apply to the transformed model, all that remains to show is that market tightness is the same in submarket a as it is at any point in the iso-attention locus in the original model. That is, formally, $\theta\left(a\right)=\theta\left(w,x,E\right)\ \forall\left(w,x\right):\ a\left(w,x\right)=a$. Call $\mathit{\Gamma}\left(a\right)$ the sum of vacancies across all VCs at a given iso-attention locus. From the definition of $\theta\left(a\right)$, one can write $\theta\left(a\right)=\frac{d\mathit{\Gamma}(a)}{dS(a)}$ with

$$S\left(a\right):=\int_{\tilde{a}\leq a}\int_{\{(w,x):a(w,x)=\tilde{a}\}}\int_{\lambda}EdS\left(w,x;\lambda\right)d\tilde{a}.$$

Recall that

$$\mathit{\Gamma}\left(a\right):=\int_{\tilde{a}\leq a}\int_{\{(w,x):a(w,x)=\tilde{a}\}}wdH\left(w,x\right)d\tilde{a}.$$

Therefore,

$$d\mathit{\Gamma}\left(a\right)=\int_{\{(w,x):a(w,x)=a\}}wdH\left(w,x\right)$$

and

$$dS(a) = \int_\lambda \int_{\{(w,x):a(w,x)=a\}} EdS(w,x;\lambda).$$

Notice that

$$d\tilde{S}(w,x,E)\,\theta(w,x,E) = d\tilde{H}(w,x).$$

Integrating on both sides over a given iso-attention locus, and taking $\theta(w,x,E)$ outside of the integral by Lemma 2, it follows that

$$\theta(w,x,E) \int_{\{(w,x):a(w,x)=\tilde{a}\}} d\tilde{S}(w,x,E) = \int_{\{(w,x):a(w,x)=\tilde{a}\}} d\tilde{H}(w,x).$$

Therefore,

$$\theta(w,x,E) = \left(\int_{\{(w,x):a(w,x)=a\}} d\tilde{H}(w,x) \Big/ \int_{\{(w,x):a(w,x)=a\}} d\tilde{S}(w,x,E) \right) = \theta(a).$$

■

Proof of Proposition 1. (Sufficiency). Assume $R(a,\lambda)$ is logsupermodular everywhere. If there is an equilibrium that does not exhibit PAM everywhere, then there must exist at least two markets a_i , a_j with $a_i > a_j$, and two types λ' λ with $\lambda' > \lambda$ such that $\lambda \in \Lambda_i$ and $\lambda' \in \Lambda_j$. Optimality of the search strategy requires that type λ is at least as well off searching in a_i rather that in a_j and similarly λ' (weakly) prefers a_j to a_i, that is:

$$Q(a_i)\,R(a_i,\lambda) \;\geq\; Q(a_j)\,R(a_j,\lambda) \tag{5}$$

$$Q(a_j)\,R\left(a_j,\lambda'\right) \;\geq\; Q(a_i)\,R\left(a_i,\lambda'\right) \tag{6}$$

The two inequalities imply

$$\frac{R(a_i,\lambda)}{R(a_j,\lambda)} \geq \frac{R\left(a_i,\lambda'\right)}{R(a_j,\lambda')}$$

which contradicts the fact that $R(a,\lambda)$ is logsupermodular[22].

(Necessity). Assume $R(a,\lambda)$ is not logsupermodular at some point $\left(\hat{a},\hat{\lambda}\right)$. The continuity properties of $R(a,\lambda)$ (see Section 2) imply that there exists a number $\varepsilon > 0$, s.t. the function is not logsupermodular anywhere in $[\hat{a}-\varepsilon,\hat{a}+\varepsilon] \times \left[\hat{\lambda}-\varepsilon,\hat{\lambda}+\varepsilon\right]$. I construct an economy where NAM could be supported, hence a contradiction arises. Let F be defined on $\left[\hat{\lambda}-\varepsilon,\hat{\lambda}+\varepsilon\right]$, and $a_i \in [\hat{a}-\varepsilon,\hat{a}+\varepsilon]$, for all i. By construction, all matches will be in $[\hat{a}-\varepsilon,\hat{a}+\varepsilon] \times \left[\hat{\lambda}-\varepsilon,\hat{\lambda}+\varepsilon\right]$. In order for only PAM sorting patterns to emerge, a necessary condition is that for at least two

[22] Logsupermodularity of $R(a,\lambda)$ implies that for any $\left(a,a'\right)$ with $a' > a$, the ratio $\frac{R\left(a',\lambda\right)}{R(a,\lambda)}$ is strictly increasing in λ.

$\left(\lambda,\lambda'\right)$ with $\lambda>\lambda'$, and two $\left(a,a'\right)$, with $a>a'$,

$$Q\left(a\right)R\left(a,\lambda\right)\quad\geq\quad Q\left(a'\right)R\left(a',\lambda\right) \tag{7}$$

$$Q\left(a'\right)R\left(a',\lambda'\right)\quad\geq\quad Q\left(a\right)R\left(a,\lambda'\right) \tag{8}$$

and, crucially, at least one of the two inequalities is strict[23]. When either (7) or (8) or both are satisfied with strict inequality, it holds that

$$\frac{R\left(a,\lambda\right)}{R\left(a',\lambda\right)}>\frac{R\left(a,\lambda'\right)}{R\left(a',\lambda'\right)}$$

which means $R\left(a,\lambda\right)$ is logsupermodular somewhere in $\left[\hat{a}-\varepsilon,\hat{a}+\varepsilon\right]\times\left[\hat{\lambda}-\varepsilon,\hat{\lambda}+\varepsilon\right]$, a contradiction. ■

Proof Proposition 2. By Proposition 1, sorting in the subgame must exhibit PAM. Each point in the sequence that forms the equilibrium partition of the set $\left[\underline{\lambda},\bar{\lambda}\right]$ is a type that must be indifferent between searching in the two markets where types just above and just below are assigned.

Consider a VC with ability x that is choosing between a_i (and associated size, m_i^x) and a_j (with associated size m_j^x), with $i>j$ and $i,j\in I^*$. This VC will prefer the first option if and only if:

$$m_i^x\mathbb{E}\left[R\left(a_i,\lambda\right)|\lambda\in\Lambda_i^{s^*}\right]>m_j^x\mathbb{E}\left[R\left(a_j,\lambda\right)|\lambda\in\Lambda_j^{s^*}\right].$$

Which can be rewritten as:

$$\frac{m_i^x}{m_j^x}>\frac{\mathbb{E}\left[R\left(a_j,\lambda\right)|\lambda\in\Lambda_j^{s^*}\right]}{\mathbb{E}\left[R\left(a_i,\lambda\right)|\lambda\in\Lambda_i^{s^*}\right]}. \tag{9}$$

The right hand side of (9) is independent on x. The left hand side is continuous and *increasing* in x. Therefore, if (9) holds for some x, it will hold for any VC with ability above x. Moreover, if the same inequality is reverted for some $x'<x$, then, by the Intermediate Value Theorem, there exist a level of ability $\tilde{x}\in\left[x',x\right]$ such that the payoff from a_i and a_j is the same.

Finally, consider a deviation to some a_j with $j\notin I^*$. Take the closest smaller market in I^* to a_j, call it a_i.

$$i:=\arg\max_{h\in I^*\backslash\{h\geq j\}}h$$

It can be argued that the belief $\beta\left(a_j\right)$ has to place all support on type λ_i, which, since all markets in between a_i and a_j are empty, is also equal to λ_j. Assume not, and first, say that $\beta\left(a_j\right)$ is supported on another type $\lambda\in\Lambda_h$ with $h\in I^*$ and $h>j$. From the partitional

[23] Otherwise, it would be possible to support an equilibrium with NAM, and the contradiction would immediately arise.

equilibrium, $\lambda > \lambda_i$. If this type finds a deviation to a_j weakly beneficial for at least some Q, then, the set of $Q \in [0,1]$ such that this is true is an interval $[Q_\lambda, 1]$, with Q_λ

$$Q(a_h) R(a_h, \lambda) = Q_\lambda R(a_j, \lambda)$$

However, the fact that $R(a_h, \lambda) / R(a_j, \lambda)$ is increasing in λ - by logsupermodularity - implies that, $\forall \lambda' \in \Lambda_h$ and $\lambda' < \lambda$,

$$Q(a_h) R\left(a_h, \lambda'\right) < Q_\lambda R\left(a_j, \lambda'\right).$$

That is, all types in Λ_h *lower* than λ would strictly benefit from deviating at Q_λ, and, similarly for some $Q > Q_\lambda$. Hence the set of Qs such that λ would deviate is a subset of the set of Qs at which these types would deviate. Because this set not maximal, β should place no density at λ. A contradiction.

Assume instead $\beta(a_j)$ is supported on some $\lambda \in \Lambda_h$ with $h \in I^*$ and $h < j$. A similar argument applies. In this case, $\forall \lambda' \in \Lambda_h$ and $\lambda' > \lambda$,

$$Q(a_h) R\left(a_h, \lambda'\right) < Q_\lambda R\left(a_j, \lambda'\right).$$

Given what the off-equilibrium deviation attracts, condition (iii) from the Proposition guarantees that no VC offers a_j. ■

Proof Proposition 3. *(i).* The proof of part (i) proceeds in two steps.

(Step 1). First, call I the equilibrium exhibiting lower ratio Q_i/Q_j for any two (i,j) with $i > j$, with II being the other equilibrium. Use the superscripts I and II to denote the limits of the equilibrium partitions X_i and Λ_i under equilibrium I and II. It can be shown that, for any i, $\lambda_i^I > \lambda_i^{II}$. This is easily seen by looking at the entrepreneur's λ_i indifference condition. Rewrite it as:

$$\frac{R(a_{i+1}, \lambda_i)}{R(a_i, \lambda_i)} = \frac{Q_i}{Q_{i+1}} \tag{10}$$

Condition (10) has to hold under both equilibrium values λ_i^I and λ_i^{II}. The left hand side is increasing in λ_i by logsupermodularity. The right hand side is assumed to be larger under equilibrium I. Therefore, $\lambda_i^I > \lambda_i^{II}$ for all i.

(Step 2). Given $\lambda_i^I > \lambda_i^{II}$, it can be proven that welfare is higher under equilibrium I. Notice that, for any i, average quality is higher under I. That is:

$$\mathbb{E}\left[R(a_i, \lambda) | \lambda_{i-1}^I \le \lambda \le \lambda_{i+1}^I\right] > \mathbb{E}\left[R(a_i, \lambda) | \lambda_{i-1}^{II} \le \lambda \le \lambda_{i+1}^{II}\right].$$

Since equilibrium requires that VCs select a_i to maximise total returns, it must be that each one is strictly better off under I compared to II.

(ii). For part (ii), similarly call I the equilibrium exhibiting lower ratio W_i/W_j for any two (i,j) with $i>j$, with II being the other equilibrium. Use the superscripts I and II to denote the limits of the equilibrium partitions X_i and Λ_i under equilibrium I and II. It can be shown that, for any i, $\lambda_i^I > \lambda_i^{II}$. To prove this, assume this is not the case. That is, assume that, for at least some i , $\lambda_i^I \leq \lambda_i^{II}$. First focus on the case where the inequality is strict for some i. Take the *largest* i such that this holds. Rewrite the indifference condition for the indifferent type, λ_i , as:

$$\frac{R\left(a_{i+1},\lambda_i\right)}{R\left(a_i,\lambda_i\right)}=\frac{W_i}{W_{i+1}}\frac{F\left(\lambda_{i+1}\right)-F\left(\lambda_i\right)}{F\left(\lambda_i\right)-F\left(\lambda_{i-1}\right)}. \tag{11}$$

When λ_i is lower, the left hand side of (11) decreases (due to logsupermodularity). W_i/W_{i+1} is higher in equilibrium I by assumption. Hence, $\left(F\left(\lambda_{i+1}\right)-F\left(\lambda_i\right)\right)/\left(F\left(\lambda_i\right)-F\left(\lambda_{i-1}\right)\right)$ is lower under I. The numerator is higher under I , since i is the largest submarket for which $\lambda_i^I \leq \lambda_i^{II}$ (this is also true in case $i=N-1$ and hence $\lambda_{i+1}=\bar{\lambda}$). Therefore, it must be that $\lambda_{i-1}^I < \lambda_{i-1}^{II}$, giving a contradiction. It remains to show that it is impossible that $\lambda_i^I = \lambda_i^{II}$ for all i. Assume this is the case. This would imply that under the two equilibria, the left hand side of (16) stays constant, as well as the ratio $\left(F\left(\lambda_{i+1}\right)-F\left(\lambda_i\right)\right)/\left(F\left(\lambda_i\right)-F\left(\lambda_{i-1}\right)\right)$. Because W_i/W_{i+1} is not the same under the two equilibria, the desired contradiction arises.

Given $\lambda_i^I > \lambda_i^{II}$, welfare is higher under equilibrium I, as proven for part (i). This completes the proof. ■

Proof Proposition 4. First, observe that, for any allocation described by a cutoff x_0, so that VCs are assigned to the high attention market if and only if their ability is above x_0, the sorting outcome is described by a unique, increasing, and continuously differentiable cutoff $\lambda_0\left(x_0\right)$. To see why, rewrite the entrepreneur's indifference condition as:

$$\frac{R\left(a_1,\lambda_0\right)}{R\left(a_0,\lambda_0\right)}=\frac{W_0\left(x_0\right)}{W_1(x_0)}\frac{\left(1-F\left(\lambda_0\right)\right)}{F\left(\lambda_0\right)}. \tag{12}$$

The left hand side of (12) is continuous and strictly increasing in λ_0 by assumption (as R is continuous in both arguments and assumed to be log-supermodular in this section). The right hand side is continuous and strictly decreasing in λ_0. Recall that $W_0=\int_{\underline{x}}^{x_0} m_0^x dG\left(x\right)$, and $W_1=\int_{x_0}^{\overline{x}} m_1^x dG\left(x\right)$. Therefore, the ratio $W_0\left(x_0\right)/W_1(x_0)$ is continuous and decreasing in x_0, as $\partial m_i^x/\partial x$ is positive and continuous, and the distribution g is continuous.

Denote x_0^* the largest equilibrium cutoff x_0, which is, by Proposition 2, the Pareto superior equilibrium. The proof proceeds in two steps.

(Step 1). First, I show that any allocation $\tilde{x}_0 < x_0^*$ delivers lower welfare than x_0^*. Notice that welfare induced by an allocation $\tilde{x}_0$ is bounded above by what total returns would be if, given the sorting subgame, VCs would optimally select fund size. Formally:

$$V(\tilde{x_0}) \leq \int_x \max\left\{m_0^x \mathbb{E}\left[R(a_0,\lambda)|\lambda \leq \lambda_0(\tilde{x_0})\right], m_1^x \mathbb{E}\left[R(a_1,\lambda)|\lambda \geq \lambda_0(\tilde{x_0})\right]\right\} dG(x).$$

Hence, for all $\tilde{x}_0 < x_0^*$:

$$V(x_0^*) = \int_x \max\left\{m_0^x \mathbb{E}\left[R(a_0,\lambda)|\lambda \leq \lambda_0(x_0^*)\right], m_1^x \mathbb{E}\left[R(a_1,\lambda)|\lambda \geq \lambda_0(x_0^*)\right]\right\} dG(x) > V(\tilde{x_0}).$$

(Step 2). Second, I show that the Second Best Problem can be improved by a marginal increase in x_0, starting from x_0^*. To see why, write the objective function:

$$V(x_0) = W_0(x_0)\mathbb{E}\left[R(a_0,\lambda)|\lambda \leq \lambda_0(x_0)\right] + W_1(x_0)\mathbb{E}\left[R(a_1,\lambda)|\lambda \geq \lambda_0(x_0)\right].$$

So,

$$\begin{aligned} \frac{\partial V(x_0)}{\partial x_0}\Big|_{x_0=x_0^*} &= \underbrace{\left(m_0^{x_0}\mathbb{E}\left[R(a_0,\lambda)|\lambda \leq \lambda_0(x_0)\right] - m_1^{x_0}\mathbb{E}\left[R(a_1,\lambda)|\lambda \geq \lambda_0(x_0)\right]\right)g(x_0)}_{=0} \\ &+ \left(W_0(x_0^*)\frac{\partial\left(\mathbb{E}\left[R(a_0,\lambda)|\lambda\leq\lambda_0\left(x_0^*\right)\right]\right)}{\partial\lambda_0} + W_1(x_0^*)\frac{\partial\left(\mathbb{E}\left[R(a_1,\lambda)|\lambda\geq\lambda_0\left(x_0^*\right)\right]\right)}{\partial\lambda_0}\right)\frac{\partial\lambda_0\left(x_0^*\right)}{\partial x_0} > 0, \end{aligned}$$

where the term in the first bracket is zero because x_0^* is indifferent in equilibrium. ■

Stable Equilibria. A fixed point of the vector function Θ - defined in Section 4.1 - is attracting if and only if all eigenvalues of the the Jacobian of Θ - denoted $J(\Theta)$ - are smaller than one in absolute value. Therefore, in this context a necessary condition for $\left(\tilde{x_0}, \tilde{\lambda_0}\right)$ to be an attracting fixed point is that the determinant of the 2x2 matrix $J(\Theta)$ is smaller than one in absolute value. Formally, the condition is:

$$\left|\det\begin{bmatrix} \eta_{x_0}\left(\tilde{x_0},\tilde{\lambda_0}\right)+1 & \eta_{\lambda_0}\left(\tilde{x_0},\tilde{\lambda_0}\right) \\ \mu_{x_0}\left(\tilde{x_0},\tilde{\lambda_0}\right) & \mu_{\lambda_0}\left(\tilde{x_0},\tilde{\lambda_0}\right)+1\end{bmatrix}\right| < 1.$$

Since η_{x_0} and μ_{λ_0} are always positive, for all stable equilibria it has to be the case that:

$$\eta_{\lambda_0}\left(\tilde{x_0},\tilde{\lambda_0}\right)\mu_{x_0}\left(\tilde{x_0},\tilde{\lambda_0}\right) < \eta_{x_0}\left(\tilde{x_0},\tilde{\lambda_0}\right)\mu_{\lambda_0}\left(\tilde{x_0},\tilde{\lambda_0}\right). \tag{13}$$

An adjustment process. Consider the following dynamic adjustment process. Take an initial (x_0,λ_0). Impose that, starting from it, the cutoff x_0 increases (decreases) proportionally to the benefit (loss) from selecting a large fund size against a small size, given the rest of the agents are following the strategy described by the two cutoffs (x_0,λ_0). Similarly, impose that the cutoff λ_0 increases (decreases) proportionally to the benefit (loss) from searching in the low attention market versus searching for high attention. This process defines a system of autonomous differential equations as below:

$$\begin{cases} \dot{x}_0(t) = & -b\left[m_1^{x_0(t)}\left(\mathbb{E}\left[R\left(a_1,\lambda\right) \mid \lambda \geq \lambda_0(t)\right] - R_0\right) - m_0^{x_0(t)}\left(\mathbb{E}\left[R\left(a_0,\lambda\right) \mid \lambda \leq \lambda_0(t)\right] - R_0\right)\right] \\ \dot{\lambda}_0(t) = & -b\left[\frac{W_1(x_0(t))}{1-F(\lambda_0(t))}R\left(a_1,\lambda_0(t)\right) - \frac{W_0(x_0(t))}{F(\lambda_0(t))}R\left(a_0,\lambda_0(t)\right)\right] \end{cases} \tag{14}$$

for some $b > 0$. Notice the right hand side of the first part of (14) is $-b\eta\left(x_0,\lambda_0\right)$ and the right hand side of the second part is $-b\mu\left(x_0,\lambda_0\right)$. One interpretation is that at each point in time a fraction of the population readjusts their strategies, starting from a state where all agents are following cutoff strategies and taking those strategies as given. An equilibrium of the game - $\left(\tilde{x_0},\tilde{\lambda}_0\right)$ - is clearly a constant solution to the system. One can then study local stability of such equilibria.

An equilibrium $\left(\tilde{x_0},\tilde{\lambda}_0\right)$ is locally asymptotically stable if all eigenvalues of the Jacobian:

$$-b\begin{bmatrix} \eta_{x_0}\left(\tilde{x}_0,\tilde{\lambda}_0\right) & \eta_{\lambda_0}\left(\tilde{x}_0,\tilde{\lambda}_0\right) \\ \mu_{x_0}\left(\tilde{x}_0,\tilde{\lambda}_0\right) & \mu_{\lambda_0}\left(\tilde{x}_0,\tilde{\lambda}_0\right) \end{bmatrix}$$

have negative real parts.[24] Since the trace of this matrix is:

$$-b\left(\eta_{x_0}\left(\tilde{x}_0,\tilde{\lambda}_0\right) + \mu_{\lambda_0}\left(\tilde{x}_0,\tilde{\lambda}_0\right)\right) < 0$$

local asymptotic stability of $\left(\tilde{x_0},\tilde{\lambda}_0\right)$ is implied by:

$$\eta_{\lambda_0}\left(\tilde{x_0},\tilde{\lambda_0}\right)\mu_{x_0}\left(\tilde{x_0},\tilde{\lambda}_0\right) < \eta_{x_0}\left(\tilde{x_0},\tilde{\lambda_0}\right)\mu_{\lambda_0}\left(\tilde{x_0},\tilde{\lambda}_0\right)$$

which is exactly equation (13).

Proof of Proposition 5. First, rewrite the system characterizing the vector of equilbrium cutoffs (x_0,λ_0) as*:*

$$m_1^{x_0}\left(\mathbb{E}\left[R\left(a_1,\lambda\right) \mid \lambda \geq \lambda_0\right] - R_0\right) - m_0^{x_0}\left(\mathbb{E}\left[R\left(a_0,\lambda\right) \mid \lambda \leq \lambda_0\right] - R_0\right) = 0 \tag{15}$$

$$\frac{W_1\left(x_0\right)}{1-F\left(\lambda_0\right)}R\left(a_1,\lambda_0\right) - \frac{W_0\left(x_0,\underline{x}\right)}{F\left(\lambda_0\right)}R\left(a_0,\lambda_0\right) = 0 \tag{16}$$

where I have made explicit in (16) the dependence of W_0 on $\underline{x}$. Infact, notice that $W_0 = \int_{\underline{x}}^{x_0} m_0^x dG(x)$, so $\frac{\partial W_0}{\partial \underline{x}} = -m_0^{\underline{x}} g\left(\underline{x}\right) < 0$. Call $\Phi\left(x_0,\lambda_0,\underline{x}\right)$ the left-hand side of (15) and

[24] For details see for example Theorem 2.5 from Acemoglu (2008).

$\Psi(x_0, \lambda_0, \underline{x})$ the left-hand side of (16). Using the Implicit Function Theorem, one gets that:

$$\frac{\partial \lambda_0}{\partial \underline{x}} < 0 \iff \Phi_{\lambda_0}(x_0, \lambda_0, \underline{x}) \Psi_{x_0}(x_0, \lambda_0, \underline{x}) < \Phi_{x_0}(x_0, \lambda_0, \underline{x}) \Psi_{\lambda_0}(x_0, \lambda_0, \underline{x})$$

which is implied by condition (13). Similarly, for x_0 one gets that:

$$\frac{\partial x_0}{\partial \underline{x}} > 0 \iff \frac{\Phi_{x_0}(x_0, \lambda_0, \underline{x})}{\Phi_{\lambda_0}(x_0, \lambda_0, \underline{x})} \Psi_{\lambda_0}(x_0, \lambda_0, \underline{x}) - \Psi_{\lambda_0}(x_0, \lambda_0, \underline{x}) > 0$$

and, using condition (13) because we are looking at stable equilibria, it follows that the sign of $\partial x_0 / \partial \underline{x}$ is the same as that $\Phi_{\lambda_0}(x_0, \lambda_0, \underline{x})$, which is positive if and only if $\phi_{\bar{\lambda}}(a_1, a_0, \lambda_0)$ is positive.

The effect on welfare directly follows from the fact that, as λ_0 increases, average quality in both submarkets increases. Since VCs choose size to maximize returns, it must be that all of them are better off after the change in λ_0. Hence, for all equilibria $V(E, s^*, \sigma^*)$ increases. ■

Proof of Corollary 1. Consider the distribution of returns for a given equilbrium $(x_0(\underline{x}), \lambda_0(\underline{x}))$. The Cdf of the returns is a step function, denoted Y, taking the form:

$$Y(r; \underline{x}) = \begin{cases} \frac{W_0(x_0(\underline{x}),\underline{x})}{W_0(x_0(\underline{x}),\underline{x})+W_1(x_0(\underline{x}))} \frac{F\left(R^{-1}(r)\right)}{F(R^{-1}(R(a_1,\lambda_0(\underline{x}))))} & \text{if } r \leq R(a_1, \lambda_0(\underline{x})) \\ \\ \frac{W_0(x_0(\underline{x}))}{W_0(x_0(\underline{x}),\underline{x})+W_1(x_0(\underline{x}))} + \frac{W_1(x_0(\underline{x}))}{W_0(x_0(\underline{x}),\underline{x})+W_1(x_0(\underline{x}))} \frac{F\left(R^{-1}(r)\right)-F\left(R^{-1}(R(a_1,\lambda_0(\underline{x})))\right)}{F\left(R^{-1}\left(R\left(a_1,\bar{\lambda}\right)\right)\right)-F(R^{-1}(R(a_1,\lambda_0(\underline{x}))))} & o.w. \end{cases}$$

where $W_0(x_0(\underline{x}), \underline{x}) = \int_{\underline{x}}^{x_0(\underline{x})} m_0^x dG(x)$, and $W_1(x_0(\underline{x})) = \int_{x_0(\underline{x})}^{\bar{x}} m_1^x dG(x)$. The associated Pdf - y - is given by:

$$y(R; \underline{x}) = \begin{cases} \frac{W_0(x_0(\underline{x}),\underline{x})}{W_0(x_0(\underline{x}),\underline{x})+W_1(x_0(\underline{x}))} \frac{f\left(R^{-1}(\lambda)\right)}{F(R^{-1}(R(a_1,\lambda_0(\underline{x}))))} & \text{if } r \leq R(a_0, \lambda_0(\underline{x})) \\ \\ 0 \qquad \text{if } r \in (R(a_0, \lambda_0(\underline{x})), R(a_1, \lambda_0(\underline{x}))) \\ \\ \frac{W_1(x_0(\underline{x}))}{W_0(x_0(\underline{x}),\underline{x})+W_1(x_0(\underline{x}))} \frac{f\left(R^{-1}(\lambda)\right)}{F\left(R^{-1}\left(R\left(a_1,\bar{\lambda}\right)\right)\right)-F(R^{-1}(R(a_1,\lambda_0(\underline{x}))))} & o.w. \end{cases}$$

Since $\partial \lambda_0 / \partial \underline{x} < 0$, it has to be the case that W_0/W_1 increases, so to satisfy the entrepreneur's indifference condition. Therefore, $W_0/(W_0 + W_1)$ is now higher. Hence, at $r = R(a_0, \lambda_0(\underline{x}))$, $Y(r; \underline{x})$ has increased.

Consider now the expectation of r conditional on being larger than $R(a_0, \lambda_0(\underline{x}))$. This is

$$\int_{R(a_1,\lambda_0(\underline{x}))}^{R\left(a_1,\bar{\lambda}\right)} rdY(r;\underline{x}) / (1 - Y(R(a_1, \lambda_0(\underline{x})); \underline{x})),$$

which is higher the higher is $\lambda_0(\underline{x})$. The same is true when conditioning on r being above any number in the set $(R(a_0, \lambda_0(\underline{x})), R(a_1, \lambda_0(\underline{x})))$. The conditional expectation of given that r is above some $\hat{r} > R(a_1, \lambda_0(\underline{x}))$ is instead given by

$$\int_{\hat{r}}^{R\left(a_1,\bar{\lambda}\right)} rdY(r;\underline{x}) / (1 - Y(\hat{r};\underline{x})),$$

which is unaffected by the change in $\underline{x}$ because

$$(1 - Y(\hat{r};\underline{x})) = \frac{W_1(x_0(\underline{x}))}{W_0(x_0(\underline{x}), \underline{x}) + W_1(x_0(\underline{x}))} \int_{\hat{r}}^{R\left(a_1,\bar{\lambda}\right)} rdY(r;\underline{x}).$$

■

Proof of Lemma 4. It is convenient to rewrite (2) as:

$$\frac{R\left(a^h, \lambda^*\right)}{R(a^l, \lambda^*)} = \frac{1-\gamma_t}{\gamma_t} \frac{(1 - F(\lambda^*))}{F(\lambda^*)} \tag{17}$$

The left-hand side of (17) is continuous and strictly increasing in λ^* by assumption (as R is continuous in both arguments and assumed to be log-supermodular in this section). The right hand side is continuous and strictly decreasing in λ^*. In particular, notice the left-hand side is positive and finite for all $\lambda^* \in \left[\underline{\lambda}, \bar{\lambda}\right]$. The right hand side is zero as $\lambda^* \to \bar{\lambda}$ and tends to infinity as $\lambda^* \to \underline{\lambda}$ ■

Proof of Proposition 7. (i). I first show that there is no equilibrium where every manager has the open credit line. Take an equilibrium where $\gamma = 1$ and consider a manager who deviates to a finite-horizon structure. If there is a $Q\left(a^h\right)$ for which some λ benefits from deviating to a^h, all types above λ would strictly deviate. This means the manager must expect to attract type $\bar{\lambda}$. This is profitable as long as:

$$R\left(a^h, \bar{\lambda}\right) > (1+\delta)\,\mathbb{E}\left[R\left(a^l, \lambda\right)\right]$$

which is true by assumption.

(ii). The second part of the Proposition follows from the fact that all VCs are identical, therefore they must be indifferent in an interior equilibrium.

(*iii*). Finally, by an argument similar to that for part (*i*), when all managers are in a short-term contract with investors, a deviation to the open credit line must necesserily attract $\underline{\lambda}$. This is not profitable as long as:

$$\mathbb{E}\left[R\left(a^h, \lambda\right)\right] \geq (1+\delta)\, R\left(a^l, \underline{\lambda}\right)$$

■

Proof of Proposition 8. Recall that the mass of VCs is normalized to one in this section. Since all VCs are alike, ex-ante welfare, V, in the economy is the expected equilibrium payoff to the VC. Consider first an equilibrium where every VC opts for the finite-horizon fund. Welfare is:

$$V = \mathbb{E}\left[R\left(a^h, \lambda\right)\right] < (1+\delta)\, \mathbb{E}\left[R\left(a^l, \lambda\right)\right]$$

where the second inequality is a consequence of A3 once expectations are taken on both sides. Hence this equilibrium is dominated by a situation where every VC is forced to choose the open credit line. Second, consider welfare from any interior equilibrium. This is given by:

$$V = \mathbb{E}\left[R\left(a^h, \lambda\right) | \lambda \geq \lambda^*(\gamma)\right] = (1+\delta)\, \mathbb{E}\left[R\left(a^l, \lambda\right) | \lambda \leq \lambda^*(\gamma)\right] < (1+\delta)\, \mathbb{E}\left[R\left(a^l, \lambda\right)\right]$$

where the second inequality trivially holds since $\lambda^*(\gamma) < \bar{\lambda}$, for any $\gamma \in (0,1)$. ■

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