Online Appendix
"Selling Information to Competitive Firms"

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A Numerical Example

To gain further insights on the optimal policy, consider a linear example that allows to obtain closed form solutions. Let $\underline{\theta} = 0$, $\overline{\theta} = 1$, and let the utility function be

$$u(x) = 2x - b\frac{x^2}{2},$$

so that $\varepsilon(1) = \frac{2}{b} - 1$ is decreasing in b.[31] By equation (4),

$$\tilde{\alpha} \equiv \frac{1 - \nu(2 + b(1 - \nu))}{b(1 - \nu)^2},$$

which is decreasing in ν and b (when it is positive).

First, if $b \leq \frac{1-2\nu}{1-\nu}$, then $\tilde{\alpha} > 1$ and

$$\rho(\alpha, 1) = \nu - (1 - \alpha)(1 - \nu)[1 - b(\nu + (1 - \nu)\alpha)] \quad \forall \alpha \in [0, 1].$$

Since this function is increasing in α, the optimal experiment is fully informative in this case.

Second, if $b > \frac{1-2\nu}{\nu(1-\nu)}$, then $\tilde{\alpha} \leq 0$ and

$$\rho(\alpha, 1) = (1 - \nu)\alpha[1 - b(\nu + (1 - \nu)\alpha)] \quad \forall \alpha \in [0, 1].$$

This function is maximized at

$$\hat{\alpha} \equiv \min\left\{1, \frac{1 - b\nu}{2b(1 - \nu)}\right\},$$

which is (weakly) increasing in ν and decreasing in b. Hence,

$$\hat{\alpha} < 1 \quad \Leftrightarrow \quad b > b^*(\nu) \equiv \frac{1}{2 - \nu}.$$

[31] Assumption 1 requires $b < 1$ while Assumption 2 is always satisfied.

Third, if $\frac{1-2\nu}{1-\nu} < b < \frac{1-2\nu}{\nu(1-\nu)}$, then $\tilde{\alpha} \in (0,1)$ and the optimal experiment features

$$\alpha^* = \begin{cases} \max\{\hat{\alpha}, \tilde{\alpha}\} & \text{if } \hat{\alpha} < 1 \\ 1 & \text{if } \hat{\alpha} = 1. \end{cases}$$

Moreover,

$$\hat{\alpha} - \tilde{\alpha} = \frac{b\nu(1-\nu) + 3\nu - 1}{2b(1-\nu)^2} \geq 0 \quad \Leftrightarrow \quad b \geq \frac{1-3\nu}{\nu(1-\nu)}.$$

Finally: if $\nu > 1 - \frac{1}{\sqrt{2}}$, then $b^*(\nu) > \frac{1-2\nu}{1-\nu}$ and $b^*(\nu) > \frac{1-3\nu}{\nu(1-\nu)}$; if $\nu \leq 1 - \frac{1}{\sqrt{2}}$, then $b^*(\nu) \leq \frac{1-3\nu}{\nu(1-\nu)}$.

Summing up, if $\nu \leq 1 - \frac{1}{\sqrt{2}}$, then the optimal experiment features

$$\alpha^* = \begin{cases} 1 & \text{if } b \leq \frac{1-2\nu}{\nu(1-\nu)} \\ \tilde{\alpha} & \text{if } \frac{1-2\nu}{\nu(1-\nu)} < b \leq \frac{1-3\nu}{\nu(1-\nu)} \\ \hat{\alpha} & \text{if } b > \frac{1-3\nu}{\nu(1-\nu)}; \end{cases}$$

whereas if $\nu > 1 - \frac{1}{\sqrt{2}}$, then the optimal experiment features

$$\alpha^* = \begin{cases} 1 & \text{if } b \leq b^*(\nu) \\ \hat{\alpha} & \text{if } b > b^*(\nu). \end{cases}$$

Hence, when demand is relatively unresponsive to price, the information provider offers an experiment that does not fully reveal the agent's cost in order to increase the market price. The optimal informativeness of the experiment α^* is decreasing in b, whereas it is decreasing (resp. increasing) in ν for intermediate (resp. large) values of b.

B Stochastic Rationing

Consider an information provider who sells the fully informative experiment to a mass $x < 1$ of principals, through a stochastic rationing rule. What is the optimal choice of x for the provider?

Suppose, for simplicity, that a principal who does not acquire information shuts down production in the high cost state — i.e., $u'(v) < \frac{\nu}{1-\nu}\Delta\theta$. Since a principal who acquires information always produces by Assumption 1, in equilibrium aggregate supply is

$$y(x) = x + (1-x)\nu = \nu + (1-\nu)x,$$

so that the equilibrium market price is

$$p = u'(\nu + (1-\nu)x).$$

Therefore, a principal's willingness to pay for information is

$$\rho(x) \equiv (1-\nu)\left[u'\left(\nu+(1-\nu)\,x\right)-\overline{\theta}\right],$$

and the information provider's profit is

$$x\rho(x) = (1-\nu)\,x\left[u'\left(\nu+(1-\nu)\,x\right)-\overline{\theta}\right].$$

This is equivalent to the provider's profit in our model when $\alpha = x$ (see equation (5) when $\beta = 1$).

Maximizing this function with respect to x yields

$$u'\left(\nu+(1-\nu)\,x\right)-\overline{\theta}+x\,(1-\nu)\,u''\left(\nu+(1-\nu)\,x\right)=0,$$

which is identical to condition (6) that characterizes the optimal choice of α when the provider cannot discriminate principals. Hence, compared to our main model, when the provider can discriminate principals the optimal choice of x is identical to the optimal choice of α (so that the provider has the same incentive to restrict the information provided to principals in order to reduce aggregate supply) and the provider obtains the same profit.

A similar result can be obtained when $u'(v) > \frac{\nu}{1-\nu}\Delta\theta$ and when the provider offers an experiment that is not fully informative to some of the principals only (because principals always produce when agents have low cost).

C Three or More Types

In order to show that our main results do not hinge on the assumption of two types, we analyze an example with more than two types in which the information provider still finds it profitable to restrict information in order to increase the market price. Suppose that $\theta \in \left\{\underline{\theta}, \hat{\theta}, \overline{\theta}\right\}$, with $\Pr\left[\theta=\overline{\theta}\right] = \Pr\left[\theta=\hat{\theta}\right] = \Pr\left[\theta=\underline{\theta}\right] = \frac{1}{3}$ and $\overline{\theta}-\hat{\theta} = \hat{\theta}-\underline{\theta} = \Delta\theta > 0$, for simplicity. Note that the monotone hazard rate property is satisfied — i.e.,

$$\frac{\Pr\left[\theta<\hat{\theta}\right]}{\Pr\left[\theta=\hat{\theta}\right]} = 1 < \frac{\Pr\left[\theta<\overline{\theta}\right]}{\Pr\left[\theta=\overline{\theta}\right]} = 2.$$

Hence, for a given market price p, the optimal contract offered by an uninformed principal is such that only local incentive constraints bind — i.e., $q_i(\underline{\theta}, \varnothing) = 1$ and

$$q_i(\hat{\theta}, \varnothing) = 1 \quad \Leftrightarrow \quad \Gamma(\hat{\theta}, p) = p - \hat{\theta} - \Delta\theta \geq 0,$$

$$q_i(\overline{\theta}, \varnothing) = 1 \quad \Leftrightarrow \quad \Gamma(\overline{\theta}, p) = p - \overline{\theta} - 2\Delta\theta \geq 0,$$

so that $q_i(\underline{\theta}, \varnothing) \geq q_i(\hat{\theta}, \varnothing) \geq q_i(\overline{\theta}, \varnothing)$ as required by the monotonicity condition (see, e.g., Laffont and Martimort, 2002, Ch. 3).

Without loss of generality, consider the fully informative experiment with $S_E = \{\underline{s}, \hat{s}, \overline{s}\}$ and associated probabilities

	$\overline{s}$	$\hat{s}$	$\underline{s}$
$\overline{\theta}$	1	0	0
$\hat{\theta}$	0	1	0
$\underline{\theta}$	0	0	1

Assuming that principals buy the experiment in equilibrium, the provider's profit is

$$\rho^{FI} = \frac{p - \sum_{\theta=\overline{\theta},\hat{\theta}} \theta - \sum_{\theta=\overline{\theta},\hat{\theta}} \max\{0, \Gamma(\theta, p)\}}{3}.$$

Now consider an experiment E' with the same signals and probabilities

	$\overline{s}$	$\hat{s}$	$\underline{s}$
$\overline{\theta}$	α	$\frac{1-\alpha}{2}$	$\frac{1-\alpha}{2}$
$\hat{\theta}$	$\frac{1-\beta}{2}$	β	$\frac{1-\beta}{2}$
$\underline{\theta}$	0	0	1

Similarly to the binary case, without loss of generality, assume as convention that $2\alpha > 1-\beta$ and $2\beta > 1-\alpha$.

A principal's willingness to pay for experiment E' is

$$\rho(\alpha, \beta) = \sum_{\theta=\overline{\theta},\hat{\theta}} \Pr[\theta] \left\{ \sum_{s=\underline{s},\hat{s},\overline{s}} \Pr[s|\theta] \max\{0, \Gamma_{\alpha,\beta}(\theta, s, p)\} - \max\{0, \Gamma(\theta, p)\} \right\},$$

where, if only local incentive constraints bind, virtual surpluses are

$$\Gamma_{\alpha,\beta}(\hat{\theta}, s, p) = p - \hat{\theta} - \frac{\Pr[s|\underline{\theta}]}{\Pr[s|\hat{\theta}]}\Delta\theta = \begin{cases} p - \hat{\theta} & \text{if } s = \overline{s}, \hat{s} \\ p - \hat{\theta} - \frac{2}{1-\beta}\Delta\theta & \text{if } s = \underline{s} \end{cases}$$

$$\Gamma_{\alpha,\beta}(\overline{\theta}, s, p) = p - \overline{\theta} - \frac{\Pr[s|\underline{\theta}] + \Pr[s|\hat{\theta}]}{\Pr\left[s|\overline{\theta}\right]}\Delta\theta = \begin{cases} p - \overline{\theta} - \frac{3-\beta}{1-\alpha}\Delta\theta & \text{if } s = \underline{s} \\ p - \overline{\theta} - \frac{2\beta}{1-\alpha}\Delta\theta & \text{if } s = \hat{s} \\ p - \overline{\theta} - \frac{1-\beta}{2\alpha}\Delta\theta & \text{if } s = \overline{s} \end{cases}$$

When $2\alpha > 1-\beta$ and $2\beta > 1-\alpha$,

$$\frac{\Pr[s|\underline{\theta}]}{\Pr[s|\hat{\theta}]} < \frac{\Pr[s|\underline{\theta}] + \Pr[s|\hat{\theta}]}{\Pr\left[s|\overline{\theta}\right]} \quad \forall s,$$

monotonicity is preserved, and $q(\hat{\theta}, s) \geq q(\overline{\theta}, s)$ for every s.

Suppose, only for simplicity, that an agent produces only in the low-cost state when the principal does not acquire information — i.e.,

$$u'\left(\frac{1}{3}\right) < \hat{\theta} + \Delta\theta < \overline{\theta} + 2\Delta\theta \quad \Rightarrow \quad \max\left\{0, \Gamma(\hat{\theta}, p)\right\} = \max\left\{0, \Gamma(\overline{\theta}, p)\right\} = 0.$$

Restricting attention to $\beta = 1$, which may be suboptimal, the provider's profit is

$$\rho\left(\alpha, 1\right) = \frac{p - \hat{\theta}}{3} + \frac{1-\alpha}{3}\max\left\{0, p - \overline{\theta} - \frac{2}{1-\alpha}\Delta\theta\right\} + \frac{\alpha\left(p - \overline{\theta}\right)}{3}.$$

If $\alpha = 1$, the experiment is fully informative and $\rho\left(1, 1\right) = \rho^{FI}$. If $\alpha < 1$, aggregate supply is

$$y\left(\alpha\right) = \begin{cases} \frac{2}{3} + \frac{\alpha}{3} & \text{if } p - \overline{\theta} < \frac{2}{1-\alpha}\Delta\theta, \\ 1 & \text{if } p - \overline{\theta} \geq \frac{2}{1-\alpha}\Delta\theta. \end{cases}$$

Therefore, if $p - \overline{\theta} < \frac{2}{1-\alpha}\Delta\theta$, the equilibrium market price is

$$p^{*}(\alpha) = u'\left(\frac{2+\alpha}{3}\right),$$

and the provider solves

$$\max_{\alpha\in(\overline{\alpha},1]} \rho\left(\alpha, 1\right) = \max_{\alpha\in(\overline{\alpha},1]} \left\{\left(1+\alpha\right)u'\left(\frac{2+\alpha}{3}\right) - \hat{\theta} - \alpha\overline{\theta}\right\}.$$

where $\overline{\alpha}$ is uniquely determined by

$$u'\left(\frac{2+\overline{\alpha}}{3}\right) - \overline{\theta} = \frac{2}{1-\overline{\alpha}}\Delta\theta.$$

Neglecting the constraint $\alpha > \overline{\alpha}$, the first-order condition for an interior solution α^{*} is

$$u'\left(\frac{2+\alpha^{*}}{3}\right) - \overline{\theta} + \frac{1+\alpha^{*}}{3}u''\left(\frac{2+\alpha^{*}}{3}\right) = 0.$$

This solution is lower than 1 if and only if

$$u'\left(1\right) - \overline{\theta} + \frac{2}{3}u''\left(1\right) < 0 \quad \Leftrightarrow \quad \varepsilon\left(1\right) \leq \overline{\varepsilon}(\overline{\theta}) \equiv \frac{2}{3} + \frac{\overline{\theta}}{|u''\left(1\right)|}.$$

This condition, whose interpretation is similar to the one in Proposition 3, guarantees that the provider never chooses the fully informative experiment because there always exists an $\alpha < 1$ that yields a profit higher than $\rho\left(1, 1\right)$.

It can be shown that a similar argument extends to the case with a finite number of types.