Vertical Separation with Private Contracts

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Introduction

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- By charging a high wholesale price, a manufacturer can induce a retailer to sell at a high price.
- With **public contracts** (and price competition), a high wholesale price induces rival retailers to increase prices too, thus reducing competition – strategic effect (Bonanno and Vickers, 1988)
- With **private contracts**, a manufacturer’s wholesale price does not affect the strategy of rival retailers, but a retailer’s strategy depends on its conjecture about the wholesale price paid by rival retailers.
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If retailers conjecture that identical manufacturers always choose identical contracts (symmetric beliefs), manufacturers delegate and earn higher profit (with both price and quantity competition).
Results

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- When manufacturers delegate, their profit may be higher with private than with public contracts
Results

- The effect of delegation depends on retailers’ beliefs.
- If retailers conjecture that identical manufacturers always choose identical contracts (symmetric beliefs), manufacturers delegate and earn higher profit (with both price and quantity competition).
- When manufacturers delegate, their profit may be higher with private than with public contracts.
- The results do not hinge on beliefs being perfectly symmetric.
Related Literature

- **Vertical separation with public contracts**
  (Fershtman and Judd, 1987; Bonanno and Vickers, 1988; Vickers 1995; Rey and Stiglitz 1995)

- **Neutrality result** with private contracts and passive beliefs
  (Coughlan and Wernerfelt, 1989; Katz 1991; Caillaud and Rey 1995)

- Beliefs with a single manufacturer and **multiple retailers**
  (Horn and Wolinsky 1988; Hart and Tirole 1990; McAfee and Schwartz 1994; Rey and Vergè 2004)

- Vertical separation with **asymmetric information**
  (Katz 1991; Caillaud, Jullien and Picard 1995)
Model

- 2 manufacturers: $M_1$ and $M_2$ produce substitute goods
- 2 exclusive retailers: $R_1$ and $R_2$
- $D^i(p_i, p_j) = \text{(smooth, symmetric) demand for good } i, i = 1, 2$
- Marginal cost = 0
Each manufacturer simultaneously and publicly chooses the organizational structure: **vertical integration** or **vertical separation**
Timing

1. Each manufacturer simultaneously and publicly chooses the organizational structure: 
   **vertical integration** or **vertical separation**

2. If $M_i$ is separated, it privately offers $R_i$ a **two-part contract**

   $\left( \begin{array}{c} T_i \\ w_i \end{array} \right)$

   franchise fee  wholesale price
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\hline
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franchise fee \hspace{1cm} \text{wholesale price}

3. Competition: firms simultaneously choose retail prices $p_1$, $p_2$

4. $R_i$ observes demand and pays $w_i \cdot D_i(p_i, p_j)$
Assumptions

\[ \frac{\partial D^i(p_i, p_j)}{\partial p_i} < 0; \quad \frac{\partial^2 D^i(p_i, p_j)}{\partial p_i^2} \leq 0 \]

\[ \frac{\partial D^i(p_i, p_j)}{\partial p_j} \geq 0: \text{ substitute goods} \]

Let \( \Pi_i(p_i, p_j) = D^i(p_i, p_j)(p_i - w_i) \) (retailer’s profit)

\[ \frac{\partial^2 \Pi_i(p_i, p_j)}{\partial p_i \partial p_j} > 0: \text{ strategic complements} \]

\[ \frac{\partial^2 \Pi_i(p_i, p_j)}{\partial p_i^2} + \frac{\partial^2 \Pi_i(p_i, p_j)}{\partial p_i \partial p_j} < 0: \text{ stability} \]
Off-Equilibrium Beliefs

- (Weak) **PBE**: no restriction on beliefs off the equilibrium path
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**Passive Beliefs**: if $R_i$ is offered $w_i \neq w_i^*$, he does not revise its beliefs about $w_j$ — i.e., $\tilde{w}_j (w_i) = w_j^*$. 

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1. **Passive Beliefs**: if $R_i$ is offered $w_i \neq w_i^*$, he does not revise its beliefs about $w_j$ — i.e., $\tilde{w}_j(w_i) = w_j^*$

2. **Symmetric Beliefs**: $R_i$ believes that $M_i$ and $M_j$ always offer the same contract — i.e., $\tilde{w}_j(w_i) = w_i$ 
   (Hart and Tirole 1990; McAfee and Schwartz 1994)
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   (Hart and Tirole 1990; McAfee and Schwartz 1994)

3. **Mixed Beliefs**: if $R_i$ is offered $w_i \neq w_i^*$, he believes that, with probability $\alpha$, $R_j$ is offered $w_i$ and, with probability $(1 - \alpha)$, $R_j$ is offered $w_j^*$
Passive beliefs may not be the most natural assumption:

“If a manufacturer wants to change its contract, why should a competing identical manufacturer not want to do the same?”
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Incomplete Information: retailers’ are uninformed about some (correlated) characteristics of manufacturers that affects their contracts

e.g., Symmetric beliefs arise in a Hotelling model in which manufacturers are privately informed about their correlated costs of production, and costs have full support
Bounded Rationality: symmetric beliefs are simple
Interpretation of Symmetric Beliefs II

- **Bounded Rationality**: symmetric beliefs are simple
- With passive beliefs, a retailer must compute manufacturers’ equilibrium contracts, given retailers’ strategies, to make a conjecture about opponents’ wholesale prices
• **Bounded Rationality**: symmetric beliefs are simple

• With passive beliefs, a retailer must compute manufacturers’ equilibrium contracts, given retailers’ strategies, to make a conjecture about opponents’ wholesale prices

⇒ symmetric beliefs are a “rule of thumb”: a retailer bases his conjecture on the manufacturer’s offer and only computes its own best strategy
Beliefs:
- Passive
- Symmetric beliefs
- Mixed

Uncertainty about manufacturers’ costs

Extensions:
- Private vs. public contracts
- Quantity competition
Passive Beliefs

- **Passive beliefs**: \( R_i \)'s conjecture about \( w_j \) is independent of \( w_i \).
Passive Beliefs

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**Lemma**

*With passive beliefs, $w_1 = w_2 = 0$ and the retail price is $p^e$ s.t.*

$$\frac{\partial D_i(p^e,p^e)}{\partial p_i} p^e + D_i (p^e, p^e) = 0$$

*Neutrality result*: Manufacturers’ profit does not depend on their organizational structure (Katz, 1991)
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marginal revenue

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**Neutrality result**: Manufacturers’ profit does not depend on their organizational structure (Katz, 1991)

- Separated manufacturers act as if integrated with retailers
  ⇒ Manufacturers have no incentive to sell through retailers
Symmetric Beliefs

- 3 subgames:
  1. Both manufacturers choose vertical integration
  2. Both manufacturers choose vertical separation
  3. One manufacturer chooses separation, the other manufacturer chooses integration
Symmetric Beliefs

- 3 subgames:
  1. Both manufacturers choose vertical integration
  2. Both manufacturers choose vertical separation
  3. One manufacturer chooses separation,
     the other manufacturer chooses integration

- With integrated manufacturers, the retail price is $p_e$
  (the same as with passive beliefs)
2 Separated Manufacturers

- Given $w_i$, $R_i$ maximizes expected profit

$$\max_{p_i} \ (p_i - w_i) \ D^i(p_i, p_j(\tilde{w}_j(w_i)))$$

where $p_j(\tilde{w}_j(w_i))$ is $R_i$’s expectation about $p_j$
2 Separated Manufacturers

- Given $w_i$, $R_i$ maximizes expected profit
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  \max_{p_i} (p_i - w_i) D^i(p_i, p_j(\tilde{w}_j(w_i)))
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  where $p_j(\tilde{w}_j(w_i))$ is $R_i$’s expectation about $p_j$
- When $R_i$ is offered $w_i$, he conjectures that $R_j$ is also offered $w_i$
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- The price chosen by $R_j$ when he is offered $w_i$ is
  \[
  \hat{p}(w_i) \in \arg\max_{p_j} (p_j - w_i) D^j(p_j, \hat{p}(w_i))
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- The price chosen by $R_j$ when he is offered $w_i$ is
  \[
  \hat{p}(w_i) \in \arg \max_{p_j} (p_j - w_i) D_j(p_j, \hat{p}(w_i))
  \]
  \[\Rightarrow \] When $R_i$ is offered $w_i$, he chooses $\hat{p}(w_i)$ and expects $R_j$ to choose $\hat{p}(w_i)$ too
2 Separated Manufacturers

- $M_i$ maximizes profit subject to $R_i$’s participation constraint
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- $M_i$ maximizes profit subject to $R_i$’s participation constraint

$\Rightarrow$ With symmetric beliefs, separated manufacturers choose

$$
\begin{align*}
w^* & \in \arg \max_{w_i} \left[ w_i D_i (\hat{p}(w_i), \hat{p}(w^*)) + T_i \right] \\
& \text{wholesale revenue}
\end{align*}
$$

$$
\begin{align*}
s.t. \quad T_i &= D_i (\hat{p}(w_i), \hat{p}(w_i)) (\hat{p}(w_i) - w_i) \\
& \text{$R_i$'s expected profit}
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(\text{IR})
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$R_i$’s expected profit

- $M_i$ expects $R_j$ to choose $\hat{p}(w^*)$ in equilibrium
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\[s.t. \quad T_i = D_i (\hat{p}(w_i), \hat{p}(w_i)) (\hat{p}(w_i) - w_i) \quad (IR)\]

- $M_i$ expects $R_j$ to choose $\hat{p}(w^*)$ in equilibrium
- $R_i$ believes that $R_j$ chooses $\hat{p}(w_i)$
Lemma

With separated manufacturers, the wholesale price is \( w^* > 0 \) s.t.

\[
\frac{\partial D_i(\hat{p}(w^*),\hat{p}(w^*))}{\partial p_i} w^* + \frac{\partial D_i(\hat{p}(w^*),\hat{p}(w^*))}{\partial p_j} (\hat{p}(w^*) - w^*) \equiv 0
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\text{belief effect} > 0
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and the retail price is \( p^* \equiv \hat{p}(w^*) > p^e \)
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- A high $w_i$ has 2 effects:
  1. it reduces the wholesale revenue by increasing $\hat{p}(w_i)$
  2. it increases $R_i$'s expected profit (and hence $T_i$) by inducing $R_i$ to believe that $R_j$ pays a high $w_j$ and charges a high $p_j$ — belief effect
Lemma

With separated manufacturers, the wholesale price is \( w^* > 0 \) s.t.

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  1. it reduces the wholesale revenue by increasing \( \hat{p}(w_i) \)
  2. it increases \( R_i \)'s expected profit (and hence \( T_i \)) by inducing \( R_i \) to believe that \( R_j \) pays a high \( w_j \) and charges a high \( p_j \) — belief effect

\( \Rightarrow \) \( M_i \)s charge \( w^* > 0 \) and reduce competition among retailers
Intuition

- How can manufacturers sustain high wholesale prices?
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- *How can manufacturers sustain high wholesale prices?*

- With **passive beliefs**, if $M_i$ chooses a high wholesale price, $M_j$ has an incentive to undercut it, since $R_j$ expects this to increase profit
Intuition

- How can manufacturers sustain high wholesale prices?

- With **passive beliefs**, if $M_i$ chooses a high wholesale price $M_j$ has an incentive to undercut it, since $R_j$ expects this to increase profit

- With **symmetric beliefs**, if $M_j$ undercuts $M_i$’s wholesale price $R_j$ does not expect $M_i$ to maintain a high wholesale price, so $R_j$ expects lower profit and pays a lower franchise fee
Asymmetric Hierarchies

- In subgame 3:

**Lemma**

*If* $M_i$ *is separated while* $M_j$ *is integrated, $M_i$ chooses* $w_i = 0$ *and the retail price is* $p^e$ *s.t.*

$$\frac{\partial D^i(p^e, p^e)}{\partial p_i} p^e + D^i(p^e, p^e) = 0$$
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- $M_i$ has no incentive to increase $w_i$
- The retail price is the same as with integrated manufacturers
- $M_i$ and $M_j$ obtain the same profit
Equilibrium in Period 1

Since manufacturers extract the whole surplus, $M_i$’s profit is

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**Theorem**

There are two equilibria: $(I; I)$ and $(S; S)$. The equilibrium with separation Pareto dominates (and risk dominates) the equilibrium with integration.
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**Theorem**

There are two equilibria: (I; I) and (S; S). The equilibrium with separation Pareto dominates (and risk dominates) the equilibrium with integration.

- Separation is a **weakly dominant strategy** (since $p^* > p^e$):
  
  by charging a high $w_i$, $M_i$ induces $R_i$ to pay a high fee and sell at a high price, thus increasing profit.
Mixed beliefs: if $w_i \neq w_i^*$, $R_i$ believes that with probability $\alpha$, $R_j$ is offered $w_i$ and with probability $(1 - \alpha)$, $R_j$ is offered $w_j^*$
Mixed Beliefs

- **Mixed beliefs**: if $w_i \neq w_i^*$, $R_i$ believes that with probability $\alpha$, $R_j$ is offered $w_i$ and with probability $(1 - \alpha)$, $R_j$ is offered $w_j^*$
  - $\alpha = 0 \Rightarrow$ passive beliefs
Mixed beliefs: if \( w_i \neq w_i^* \), \( R_i \) believes that with probability \( \alpha \), \( R_j \) is offered \( w_i \) and with probability \( (1 - \alpha) \), \( R_j \) is offered \( w_j^* \)

- \( \alpha = 0 \Rightarrow \) passive beliefs
- \( \alpha = 1 \Rightarrow \) symmetric beliefs
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- **Mixed beliefs**: if \( w_i \neq w_i^* \), \( R_i \) believes that with probability \( \alpha \), \( R_j \) is offered \( w_i \) and with probability \( (1 - \alpha) \), \( R_j \) is offered \( w_j^* \)
  - \( \alpha = 0 \Rightarrow \text{passive beliefs} \)
  - \( \alpha = 1 \Rightarrow \text{symmetric beliefs} \)

- Let \( p^*_\alpha \) be the equilibrium retail price
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**Mixed beliefs:** if $w_i \neq w_i^*$, $R_i$ believes that with probability $\alpha$, $R_j$ is offered $w_i$ and with probability $(1 - \alpha)$, $R_j$ is offered $w_j^*$

- $\alpha = 0 \Rightarrow$ passive beliefs
- $\alpha = 1 \Rightarrow$ symmetric beliefs

Let $p_\alpha^*$ be the equilibrium retail price

Let $\tilde{p}_j(w_i)$ be the price that $R_i$ expects $R_j$ to choose, when $R_j$ is offered $w_i$
Mixed Beliefs

- **Mixed beliefs**: if \( w_i \neq w_i^* \), \( R_i \) believes that with probability \( \alpha \), \( R_j \) is offered \( w_i \) and with probability \( (1 - \alpha) \), \( R_j \) is offered \( w_j^* \)
  - \( \alpha = 0 \Rightarrow \) passive beliefs
  - \( \alpha = 1 \Rightarrow \) symmetric beliefs

- Let \( p_{\alpha}^* \) be the equilibrium retail price
- Let \( \tilde{p}_j(w_i) \) be the price that \( R_i \) expects \( R_j \) to choose, when \( R_j \) is offered \( w_i \)
- \( R_i \) chooses the retail price

\[
\hat{p}_\alpha(w_i) \in \arg \max_{p_i} (p_i - w_i) \times \left[ \alpha D^i(p_i, \tilde{p}_j(w_i)) + (1 - \alpha) D^i(p_i, p_{\alpha}^*) \right]
\]
Solving manufacturers’ problem:

**Lemma**

*With mixed beliefs, the wholesale price $w^*_\alpha$ is s.t.*

\[
\frac{\partial D^i(\hat{p}_\alpha(w^*_\alpha), \hat{p}_\alpha(w^*_\alpha))}{\partial p_i} w^*_\alpha + \alpha \frac{\partial D^i(\hat{p}_\alpha(w^*_\alpha), \hat{p}_\alpha(w^*_\alpha))}{\partial p_j} (\hat{p}_\alpha(w^*_\alpha) - w^*_\alpha) \equiv 0
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\[
\text{belief effect} > 0
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Mixed Beliefs

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*belief effect $> 0$*

- The belief effect is *weaker* than with symmetric beliefs
Mixed Beliefs

Theorem

With separated manufacturers, when \( \alpha \in (0; 1) \)
the wholesale price \( w_{\alpha}^* \) is s.t. \( 0 < w_{\alpha}^* < w^* \) and
the retail price \( p_{\alpha}^* \) is s.t. \( p^e < p_{\alpha}^* < p^* \).
Separation is a weakly dominant strategy \( \forall \alpha \neq 0 \).
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- When $\alpha = 0$: $w_{\alpha}^* = 0$ and $p_{\alpha}^* = p^e$ (passive beliefs)
- When $\alpha = 1$: $w_{\alpha}^* = w^*$ and $p_{\alpha}^* = p^*$ (symmetric beliefs)
Uncertainty about Manufacturers’ Costs

- Standard **Hotelling model** of differentiated products:

\[
D^i(p_i, p_j) = \frac{p_j - p_i + t}{2t},
\]

where \( t \) is the transport cost.
Uncertainty about Manufacturers’ Costs

- **Standard Hotelling model** of differentiated products:

  \[ D^i(p_i, p_j) = \frac{p_j - p_i + t}{2t} \]

  where \( t \) is the transport cost.

- \( M_i \) has **private information** about his marginal cost \( c_i \) and
  - with probability \( \beta \), \( c_1 = c_2 \);
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  \( (c_i \sim (-\infty, +\infty) \text{ and } \mathbb{E}[c_i] = 0) \)
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- Interpretation:
  - \( \beta = 1 \Rightarrow \) manufacturers face a **common cost shock**
  - \( \beta = 0 \Rightarrow \) manufacturers face **idiosyncratic cost shocks**
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In the separating PBE, \( M_i \) offers \( w^*(c_i) = t \beta + \frac{2 - \beta}{2 - \beta^2} c_i \)
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Uncertainty about Manufacturers’ Costs

**Lemma**

*In the separating PBE, $M_i$ offers $w^*(c_i) = t \beta + \frac{2-\beta}{2-\beta^2} c_i$ and $R_i$ chooses $p^*(c_i) = \frac{1+\beta}{2} (c_i + 2t) - \frac{\beta(1-\beta)(2+\beta)}{2(2-\beta^2)} c_i$.*

- Given $w_i$, $R_i$ believes that
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\( \Rightarrow \) Partly symmetric beliefs naturally arise in equilibrium since \( R_i \) uses \( w_i \) to infer information about \( c_j \) and hence \( w_j \)
Choice of Organizational Structure

- Let $\beta = 1$, so that manufacturers have common cost $c$
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Theorem

If $\beta \approx 1$, vertical separation is a strictly dominant strategy for manufacturers.
Asymmetric Manufacturers

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- **Asymmetric manufacturers:**
  - $M_1$ has cost $c$ and $M_2$ has cost $c + k$
  - ($c$ is private information to manufacturers)
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  \[ M_1 \text{ has cost } c \text{ and } M_2 \text{ has cost } c + k \]
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- In a linear separating equilibrium,
  - when \( R_1 \) is offered \( w_1 \), he expects \( R_2 \) to be offered \( w_1 + \frac{3}{5}k \),
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  ⇒ **Partly symmetric beliefs** naturally arise since
  - $R_i$ uses $w_i$ to infer information about $c$ and hence $w_j$
Private vs. Public Contracts

- *Do manufacturers prefer private or public contracts?*
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With symmetric beliefs, prices and manufacturers’ profits 
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e.g., With linear demand, profits are higher with private contracts
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**Lemma**

*With separated manufacturers, the wholesale price is* $w^* = -P'(2q^*)q^* > 0$ *and each retailer produces* $q^*$ *s.t.* $2q^*P'(2q^*) + P(2q^*) = 0$
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- $2q^*$ *is the monopoly quantity*
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*With separated manufacturers, the wholesale price is* \( w^* = -P'(2q^*) q^* > 0 \) *and each retailer produces* \( q^* \) *s.t.* \( 2q^* P'(2q^*) + P(2q^*) = 0 \)

- \( 2q^* \) is the monopoly quantity
- \( \Rightarrow \) With symmetric beliefs, the *belief effect* allows separated manufacturers to maximize joint profit (by extracting the whole surplus ex ante)
Separation is a \textbf{weakly dominant strategy} for manufacturers (as with price competition)
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With public contracts, the *strategic effect* induces manufacturers to choose vertical separation

but separated manufacturers charge lower wholesale prices and obtain lower profit (Fershtman and Judd, 1987)
Separation is a weakly dominant strategy for manufacturers (as with price competition)

With public contracts, the strategic effect induces manufacturers to choose vertical separation but separated manufacturers charge lower wholesale prices and obtain lower profit (Fershtman and Judd, 1987)

⇒ With quantity competition, prices and manufacturers’ profits are always higher with private than with public contracts
With private contracts and not completely passive beliefs, manufacturers prefer to sell through retailers, both with price and quantity competition.

By charging high wholesale prices, manufacturers earn high fees and reduce competition among retailers (by affecting retailers’ beliefs about rivals’ strategies).

Manufacturers may agree to keep contracts private.

Symmetric beliefs naturally arise when manufacturers are privately informed about their correlated costs.