Bidding to Lose?
Auctions with Resale

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  FT quit and let Orange win
  FT took over Orange after the auction.
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  FT quit and let Orange win
  FT took over Orange after the auction.

• Telia did not bid in 3G auctions
  Sonera won licenses in Spain, Germany, Italy and Norway
  Telia took over Sonera after the auctions.
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(i) A weak bidder may bid aggressively to win and sell to a strong bidder.

(ii) A strong bidder may:

   either overbid competitor and win the auction,

   or drop out and buy from competitor.
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\[
\begin{align*}
\text{Resale takes place in equilibrium,} \\
\Rightarrow \begin{cases} \\
gives weak bidders reason to participate and increases seller’s revenue.
\end{cases}
\end{align*}
\]
Model

- Ascending auction (for a spectrum license).
  - 2 bidders: \( A \) is strong, \( B \) is weak and wealth constrained
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- We assume:
  - \( v_A > v_B; \)
  - \( w_B < v_B; \)
  - \( w_A = \infty; \)
  - \( v_i \) and \( w_i \) are common knowledge.
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  (Typical reason for resale is uncertainty about relative valuation before the auction; Milgrom ’87; Bikhchandani & Huang ’89; Haile ’99 ...)

• To obtain \( v_i \), owner pays \textit{operating} cost \( c \sim U [0, 1] \).

• To pay more than \( w_B \), \( B \) borrows at cost \( \beta \)
  (e.g. bank interest rate).

• \( B \)’s liability is limited by \( w_B \)
  (cannot end up with negative wealth).
Timing

1. \( i \) wins the auction at price \( p \) (paid in stage 4).

2. *Resale* can take place.

3. Operating cost \( c \) is realized.

4. Owner of the project, \( j \), can:
   
   (i) pay \( p + c \) to obtain \( v_j \), or
   
   (ii) go bankrupt and liquidate \( w_j \).
Profit without Wealth Constraint

\[ \pi_B(c) \]

\[ v_B - p \]

\[ v_B - (p + c) \]

\[ 0 \]

\[ v_B - p - 1 \]

\[ 1 \]
\[ \pi_B(c) \]

\[ v_B - p \]

\[ v_B - (p + c) \]

\[ v_B + \beta w_B - (1 + \beta)(p + c) \]

+ Borrowing Cost

Fig. A2
$B$'s Profit with a Wealth Constraint

\[ \pi_B(c) \]

\[ v_B - p \rightarrow v_B - (p + c) \]

\[ v_B + \beta w_B - (1 + \beta)(p + c) \]

+ Borrowing Cost

+ Limited Liability
$B$'s Expected Profit

\[ E[\pi_B] \]

\[ v_B - \frac{1}{2} \]

\[ v_B - p - \frac{1}{2} \]

\[ 0 \]

\[ -w_B \]
Resale

- If $B$ wins the auction, gains from trade are:

$$\mathbb{E}[\pi_A] - \mathbb{E}[\pi_B].$$
Resale

- If $B$ wins the auction, \textit{gains from trade} are:

\[ E[\pi_A] - E[\pi_B]. \]

- Bidders equally share gains from trade.
Resale

• If $B$ wins the auction, gains from trade are:

$$\mathbb{E}[\pi_A] - \mathbb{E}[\pi_B].$$

• Bidders equally share gains from trade.

⇒ Resale price is $\frac{1}{2} \left( \mathbb{E}[\pi_A] + \mathbb{E}[\pi_B] \right)$. 
Bidding by B

- **Lemma**  \[ B \text{ bids more aggressively with resale.} \]
Bidding by B

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**Proof.** Without resale, \( B \) bids up to \( p' \) s.t.

\[
\mathbb{E}[\pi_B (p')] = 0.
\]

- With resale, \( B \) bids up to \( p_B \) s.t.

\[
\mathbb{E}[\pi_A (p_B)] + \mathbb{E}[\pi_B (p_B)] = 0.
\]

\( \Rightarrow p_B > p' \). \blacksquare
Bidding by B

• Lemma  

*B bids more aggressively with resale.*

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\( \Rightarrow p_B > p' \). ■

\( \Rightarrow \) A weak bidder does not drop out of the auction as soon as price reaches his valuation.
Bidding by A

• With resale, raising $p$ makes $A$ better off iff

$$\frac{\partial}{\partial p} (\mathbb{E} [\pi_A] - \mathbb{E} [\pi_B]) > 0 \iff \left| \frac{\partial \mathbb{E} [\pi_B]}{\partial p} \right| > \left| \frac{\partial \mathbb{E} [\pi_A]}{\partial p} \right|$$
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$$\Rightarrow$$ To buy from $B$, $A$ drops out at $p^*$ because increasing price above $p^*$ reduces gains from trade
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(i) **Borrowing Cost Effect**: raises “cost” of bidding
⇒ lower profit on good projects (low $c$)
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       ⇒ lower losses on bad projects (high $c$)

• For high $p$, **Limited Liability Effect** dominates
  ⇒ increase in $p$ improves $B$’s bargaining position
$B'$s Expected Profit

![Graph showing $E[\pi_B]$ and $v_B - \frac{1}{2}$ against $p$ with a dashed line at $v_B - p - \frac{1}{2}$ and a point $p^*$.]
Resale Equilibrium

• **Lemma**  \[ A \text{ prefers to drop out at } p^* \text{ and buy from } B \]
  
  \[ \text{iff } B \text{ bids more than } p^*. \]
Lemma A prefers to drop out at $p^*$ and buy from $B$ iff $B$ bids more than $p^*$.

Proof. $B$ bids up to $p_B$ s.t.:

$$\mathbb{E}[\pi_A (p_B)] + \mathbb{E}[\pi_B (p_B)] = 0$$
**Resale Equilibrium**

- **Lemma**  
  A prefers to drop out at $p^*$ and buy from B iff B bids more than $p^*$.

**Proof.** B bids up to $p_B$ s.t.:

$$\mathbb{E}[\pi_A (p_B)] + \mathbb{E}[\pi_B (p_B)] = 0$$

$$\Rightarrow \begin{cases} 
A’s \text{ resale surplus} = \\
\quad = \frac{1}{2} (\mathbb{E}[\pi_A] - \mathbb{E}[\pi_B]) = \mathbb{E}[\pi_A (p_B)] \\
\quad = A’s \text{ auction profit}
\end{cases}$$
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⇒ At lower price, resale surplus is higher. □
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$\Rightarrow$ At lower price, resale surplus is higher. ■

• Prop. 1 Resale is the unique equilibrium iff:

$(i)$ $w_B < w^*$ and $\beta > \beta^*$, or

$(ii)$ $w_B > w^*$ and $\beta < \beta^*$.

($Where$ $w^* = \frac{1}{2} (v_A - 1)$ and $\beta^* = \frac{v_B - v_A + 2w_B - 1}{v_A - 2w_B - \frac{1}{2}}$.)$
Interpretation

- $B$ bids over $p^*$ (price at which $A$ drops out) iff:

$$\mathbb{E}[\pi_B(p^*)] + \frac{1}{2} (\mathbb{E}[\pi_A(p^*)] - \mathbb{E}[\pi_B(p^*)]) > 0$$

outside option \quad \text{gains from trade}
Interpretation

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⇒ Resale happens if the following are large:

(i) $\mathbb{E}[\pi_A(p^*)]$ — joint bidders’ surplus,

(ii) $\mathbb{E}[\pi_B(p^*)]$ — $B$’s outside option
    (which determines $B$’s share of joint surplus).
Effects of High $\beta$

Direct Effect

- High $\beta$ reduces $\mathbb{E}[\pi_B]$ (outside option).

$\Rightarrow$ High $\beta$ makes resale *harder*
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- High $\beta$ increases limited liability effect
  (bankruptcy is more likely)
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- High $\beta$ increases *limited liability effect* (bankruptcy is more likely)

$\Rightarrow$ $A$ drops out sooner ($p^* \downarrow$) and \[ \begin{align*}
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$\Rightarrow$ High $\beta$ makes resale easier

• For large $w_B$:
  – Direct effect is stronger (bankruptcy is more costly)
  – Indirect effect is weaker ($p^*$ is higher)
Effects of High $\beta$

**Direct Effect**
- High $\beta$ reduces $\mathbb{E}[\pi_B]$ (outside option)

$\implies$ High $\beta$ makes resale **harder**

**Indirect Effect**
- High $\beta$ increases *limited liability effect* (bankruptcy is more likely)

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- For **large** $w_B$:
  - Direct effect is stronger (bankruptcy is more costly)
  - Indirect effect is weaker ($p^*$ is higher)

$\implies$ Resale happens for \[ \begin{cases} 
\text{low } \beta \text{ if } w_B \text{ is large} \\
\text{high } \beta \text{ if } w_B \text{ is small}
\end{cases} \]
Seller’s Strategy

• With resale, weak bidder participates in the auction.
  \[\Rightarrow \text{Resale raises the auction price.}\]
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\[ \Rightarrow \text{Prop. 2 When resale takes place,} \]
\[ \text{seller reduces } \beta \text{ and increases } w_B \text{ to raise } p^*. \]
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⇒ Prop. 2 When resale takes place,
seller reduces \( \beta \) and increases \( w_B \) to raise \( p^* \).

But to induce resale, seller \( \begin{cases} \text{reduces } \beta \text{ if } w_B \text{ is high,} \\ \text{increases } \beta \text{ if } w_B \text{ is low.} \end{cases} \)
Extensions

- No entry cost
- More bidders
- Different bargaining
- Auction price paid before resale
- Returning the prize
Conclusions

• When prize value is uncertain and weak bidder enjoys limited liability strong bidder may prefer to drop out.
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⇒ Resale takes place and:

– Weak bidders participate and bid aggressively (even if valuations are common knowledge),
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– Seller manipulates borrowing cost (to induce resale).
Conclusions

• When prize value is uncertain and weak bidder enjoys limited liability strong bidder may prefer to drop out.

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– Weak bidders participate and bid aggressively (even if valuations are common knowledge),
– Seller’s revenue is higher,
– Seller manipulates borrowing cost (to induce resale).

• In EU 3G auctions resale was difficult, this may have discouraged weak bidders from entering and resulted in lower governments’ revenue.
No Entry Cost

- Assume weak bidder always participates (bidding is costless).

⇒ Without resale, auction price is $p'$ s.t.:

$$
\mathbb{E}[\pi_B (p')] = 0.
$$

⇒ Auction price is higher with resale if $p^* > p'$.

- **Proposition 3**  
  If weak bidder always enters, resale raises auction price iff $2w_B (1 + \beta) > 1$.

  - High $w_B$ reduces limited liability effect ⇒ $p^* \uparrow$.

  - High $\beta$ reduces $B$’s profit ⇒ $p' \downarrow$. 
Example

• 2 bidders:  \( A \) has value \( v_A = 10 \)
  
  \( B \) has value \( v_B = \begin{cases} 5 \text{ pr. } \frac{1}{2} \\ 3 \text{ pr. } \frac{1}{2} \end{cases} \)

• \( B \)’s wealth is \( w_B = 0 \) — limited liability

⇒ After winning at price \( p \), \( B \)’s profit is:

\[
\mathbb{E} [\pi_B] = \frac{1}{2} \max \{ 5 - p; 0 \} + \frac{1}{2} \max \{ 3 - p; 0 \}.
\]

• \( B \) resells to \( A \) after winning the auction,
  
  and bidders share resale surplus \( \pi_A - \mathbb{E} [\pi_B] \).

• At what price does \( A \) drop out?

<table>
<thead>
<tr>
<th>( p )</th>
<th>( \pi_A = v_A - p )</th>
<th>( \mathbb{E} [\pi_B] )</th>
<th>( \pi_A - \mathbb{E} [\pi_B] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>10</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>1</td>
<td>9</td>
<td>3</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>2</td>
<td>6</td>
</tr>
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<td>3</td>
<td>7</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>( \frac{1}{2} )</td>
<td>( \frac{5}{2} )</td>
</tr>
<tr>
<td>5</td>
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<td>0</td>
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</tr>
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<td>0</td>
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</tr>
</tbody>
</table>

⇒ \( A \) never bids more than 3.
• Does $A$ prefer any price $\leq 3$?
• Assume $B$ pays $\beta p$ to bid above $w_B$, $0 < \beta < \frac{1}{4}$.

\[
\begin{array}{c|c|c|c}
 p & \pi_A = v_A - p & \mathbb{E} [\pi_B] & \pi_A - \mathbb{E} [\pi_B] \\
0 & 10 & 4 & 6 \\
1 & 9 & 3 - \beta & 6 + \beta \\
2 & 8 & 2 - 2\beta & 6 + 2\beta \\
3 & 7 & \frac{1}{2} (2 - 3\beta) & 6 + \frac{3}{2}\beta \\
4 & 6 & \frac{1}{2} (1 - 4\beta) & 5\frac{1}{2} + 2\beta \\
5 & 5 & 0 & 5 \\
6 & 4 & 0 & 4 \\
\end{array}
\]

$\Rightarrow A$ prefers price 2.

• Questions:
  – Does $B$ bid up to 2?
  – Does $A$ prefer resale to winning the auction?
  – What if $w_B > 0$?
  – Seller’s revenue?
Auction Price Paid before Resale

• $v_A = 10$; $w_B = 5$; $v_B = \begin{cases} 2 & \text{pr. } \frac{1}{2} \\ -2 & \text{pr. } \frac{1}{2} \end{cases}$

⇒ After winning at price $p$, $B$’s value is:

$$
\mathbb{E}[v_B] = \frac{1}{2} \cdot 2 + \frac{1}{2} \cdot \begin{cases} \max \{-2; -w_B\} & \text{if } p < w_B \\ 0 & \text{if } p \geq w_B \end{cases}
$$

• $B$ cannot bid more than $w_B$.

<table>
<thead>
<tr>
<th>$p$</th>
<th>$w_B - p$</th>
<th>$\mathbb{E}[v_B]$</th>
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<tr>
<td>4</td>
<td>1</td>
<td>$1 - \frac{1}{2} = \frac{1}{2}$</td>
<td>$9\frac{1}{2}$</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>1</td>
<td>9</td>
</tr>
<tr>
<td>6</td>
<td>X</td>
<td>X</td>
<td></td>
</tr>
</tbody>
</table>

• $B$ bids up to 6
  (after winning at 5, $B$’s surplus is $1 + \frac{1}{2}9 > 5$).

• $A$ can win at price 6 and obtain 4 or
  drop out at price 3 and obtain $\frac{1}{2}10$ with resale.

⇒ $B$ wins at price 3.
Resale with Fixed Mark-up

- $v_A = 5; \quad v_B = 0.$

- $B$’s managers resell at price $p + k$, say $k = 1$ (e.g. to justify strategy with shareholders).

<table>
<thead>
<tr>
<th>$p$</th>
<th>$\pi_A = v_A - p$</th>
<th>$S_A = v_A - (p + 1)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>$-1$</td>
</tr>
</tbody>
</table>

- $B$ bids up to $v_A - 1 = 4$ (max $p$ at which he resells).
- $A$ can win at price 4 and obtain 1 or drop out earlier and buy in resale market.

$\Rightarrow A$ drops out at $p = 0.$
### European 3G Ascending Auctions
(in date order)

<table>
<thead>
<tr>
<th>Country</th>
<th>Number of Bidders</th>
</tr>
</thead>
<tbody>
<tr>
<td>UK</td>
<td>13</td>
</tr>
<tr>
<td>Netherlands</td>
<td>6</td>
</tr>
<tr>
<td>Germany</td>
<td>7</td>
</tr>
<tr>
<td>Italy</td>
<td>6</td>
</tr>
<tr>
<td>Austria</td>
<td>6</td>
</tr>
<tr>
<td>Switzerland</td>
<td>4</td>
</tr>
<tr>
<td>Belgium</td>
<td>3</td>
</tr>
<tr>
<td>Greece</td>
<td>3</td>
</tr>
</tbody>
</table>

- After UK auction, bidders learned relative valuations.

⇒ If bidders expect to lose, they do not participate.