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Are Speculators Unwelcome in Multi-Object Auctions?†

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I consider a uniform-price auction under complete information. The possibility of resale attracts speculators who have no use value for the objects on sale. A high-value bidder may strictly prefer to let a speculator win some of the objects and then buy in the resale market, in order to keep the auction price low. Although resale induces entry by speculators and therefore increases the number of competitors, high-value bidders' incentives to "reduce demand" are also affected. Allowing resale to attract speculators reduces the seller's revenue when bidders' valuations are dispersed. Speculators increase the seller's revenue only when they are outbid. (*JEL* D44, D83)

When resale after an auction is allowed, speculators—players who have no use value for the object on sale—may participate in the hope of winning the auction and then reselling to a bidder who has a high use value for the object.[1] However, it is not clear why a bidder with a high use value should let a speculator win the auction, only then to buy from him in the resale market. Indeed, in a single-object auction with complete information, a bidder is indifferent between buying in the resale market and winning the auction (at the same price at which he can buy in the resale market).[2] If there is an arbitrarily small cost to trade in the resale market, or if players discount the future surplus from resale, then the bidder strictly prefers to outbid the speculator and win the auction.[3]

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[1] A speculator can also be defined as a player who has a positive, but low use value, and only participates in the auction to resell to a player with a higher use value.

[2] Consider, for example, an ascending auction with one bidder who has value v for the object on sale and one speculator who has value 0. If the speculator wins the auction and players equally share the gains from trade in the resale market, the speculator resells to the bidder at price $½v$. Therefore, the speculator is willing to bid up to $½v$ in the auction. Similarly, the bidder is also willing to bid up to $½v$ because this is the price at which she can buy in the resale market. So there are multiple equilibria, but the bidder has no compelling reason to let the speculator win the auction. (If the bidder bids up to her willingness to pay, the speculator should expect to obtain no surplus and so has no reason to participate in the auction.)

[3] Nevertheless, a recent literature shows how resale may arise in richer models of single-object auctions. Rod Garratt and Thomas Tröger (2006) show that, if a bidder is privately informed about her use value, in second-price auctions there are also equilibria in undominated strategies in which a speculator wins by inducing the bidder to bid zero. Pagnozzi (2007) shows that a high-value bidder may prefer to let a low-value bidder win if

I analyze how the possibility of resale and the presence of speculators affect the seller's revenue in multi-object auctions. I show that, in contrast to single-object auctions, under complete information it is natural to expect speculators to win in multi-object auctions: bidders with positive use values may *strictly* prefer to let a speculator acquire some of the objects on sale, and then purchase those objects in the resale market. By letting a speculator win, bidders may keep the auction price low, and hence pay a lower price for the objects that they acquire in the auction. Therefore, it can be common knowledge that speculators will win an auction, even when competing against bidders with higher use values, and even if there are (small) resale costs and discounting.

Speculators can also win when some bidders with positive use values cannot participate in the auction and can only acquire the objects in the resale market, as with treasury bill auctions and large real-estate auctions (Sushil Bikhchandani and Chi-fu Huang 1989; Subir Bose and George Deltas 1999). But I show that speculators may win even if all bidders with positive use values participate in the auction. Moreover, in my model, resale is not caused by uncertainty in valuations or by changes in bidders' valuations after the auction (as in Haile 2000, 2003).

As an example, consider the 2003 UK 3.4 GHz auction, a simultaneous ascending auction for 15 licenses to offer broadband wireless services. Pacific Century Cyberworks (PCCW) was widely considered the bidder with the highest value and was expected to win all 15 licenses. Red Spectrum and Public Hub were two small companies created explicitly to participate in the auction, and neither company ever bid for more than one license. As soon as PCCW, Red Spectrum, and Public Hub were the only three bidders left in the auction, PCCW reduced its demand to 13 licenses, thus allowing Red Spectrum and Public Hub to win one license each and preventing the auction price from rising any further.[4] A few months after the auction, PCCW obtained the final two licenses by taking over Red Spectrum and Public Hub. My analysis provides one plausible interpretation of the bidders' behavior (although other explanations cannot be ruled out, of course).

It is often argued that speculators always increase the seller's revenue, because their participation increases the number of bidders, and hence enhances competition. So, it is argued, the seller should always allow resale and welcome speculators. As the previous example suggests, however, attracting speculators also affects bidders' strategies. Speculators can induce bidders with positive use values to bid more aggressively in order to win the auction and, so, increase the seller's revenue. But the possibility of resale also affects bidders' incentives to "reduce demand"—i.e., to bid for fewer objects than they want in order to pay a lower price for the objects they win—which can reduce the seller's revenue (Robert Wilson 1979).[5] There are three

the auction price affects bargaining in the resale market because of wealth effects. Resale can also take place if bidders' valuations change after the auction (Philip A. Haile 2000, 2003), if additional buyers appear after the auction (Paul R. Milgrom 1987), and in first-price auctions with asymmetric bidders (Madhurima Gupta and Bernard Lebrun 1999; Isa Hafalir and Vijay Krishna 2008).

[4] See "UK Operators Miss Out in Wireless Broadband Auction" (available at http://news.zdnet.co.uk/communications/0,39020336,2136110,00.htm), where PCCW's failure to win all 15 licenses was described as a "surprise," a "gaffe," and "a costly mistake [that] may have cost the chance of offering a nationwide service."

[5] The literature on demand reduction also includes Lawrence M. Ausubel and Peter Cramton (1998), Kerry Back and Jaime F. Zender (1993), Richard Engelbrecht-Wiggans and Charles M. Kahn (1998), Paul Klemperer

effects. First, resale can correct an inefficient allocation, and makes demand reduction less costly for the high-value bidders, because they can buy in the resale market the object they lose to a speculator in the auction. Second, it is more costly for a bidder to outbid low-value competitors, who bid more aggressively when they can resell. These two effects make demand reduction more profitable for bidders. However, by attracting speculators, resale may make demand reduction less profitable for some bidders with positive use values, if they have to share the objects with speculators.

Speculators do increase competition in the auction. But to attract speculators the seller has to allow resale, and this may induce an accommodating strategy by bidders with positive use values, and thus reduce the seller's revenue.[6]

To analyze the effects described above, I consider a simple model of a uniform-price auction in which bidders have constant marginal valuations, and these valuations are common knowledge. This implies that players always exploit profitable trade opportunities after the auction, and that, when they participate in the auction, they can predict the price at which they may trade in the resale market. Nonetheless, I show that the qualitative results of the analysis also hold in an example with downward sloping demand, and they do not require that the resale market is always efficient. I also assume that the number of speculators who participate in the auction is not so large that competition among them always reduces their profit to zero, and, whenever possible, I focus on the Pareto dominant equilibrium for bidders and speculators.

In this context, the net effect on the seller's revenue of allowing resale and attracting speculators depends on the bidders' relative valuations: if bidders' valuations are relatively similar (i.e., clustered), speculators increase the seller's revenue; if bidders' valuations are sufficiently different (i.e., dispersed), speculators reduce the seller's revenue. Suppose that the valuations of bidders who are going to buy in the resale market are not too much higher than the valuations of bidders who are going to sell. After winning an object in the auction, bidders with lower valuations have a higher outside option (relative to the highest valuations) than speculators in the resale market, and hence they obtain a higher profit than speculators from reselling. In this case, the presence of speculators induces low-value bidders to bid more aggressively, and there is a strong competitive effect. In contrast, if bidders' valuations are dispersed, all bidders with positive use values reduce demand, because it is now too costly for low-value bidders to outbid speculators.

In order to increase revenue, the seller should design an auction that induces bidders to compete aggressively. In my model, the seller should allow resale to attract speculators if bidders are relatively symmetric. However, if it is credible, the seller should

and Margaret A. Meyer (1989), Ilan Kremer and Kjell G. Nyborg (2004), and Charles Noussair (1995). John H. Kagel and Dan Levin (2001) and John A. List and David Lucking-Reiley (2000) provide experimental evidence of demand reduction; Robert J. Weber (1997), Catherine D. Wolfram (1998), and Frank A. Wolak (2003) argue that demand reduction affected several FCC spectrum auctions, as well as the United Kingdom and the California electricity markets; Klemperer (2004) describes demand reduction in the 1999 GSM spectrum auction in Germany and in the 2000 3G spectrum auction in Austria. According to Milgrom (2004), "[t]he issue of extreme price equilibria is plainly of great practical importance."

[6] For an analysis of the effects of resale on the seller's revenue in single-object auctions see Charles Zhoucheng Zheng (2002), Giacomo Calzolari and Alessandro Pavan (2006), Garratt and Tröger (2006), Pagnozzi (2007), and Hafalir and Krishna (2008).

forbid resale if he knows bidders are asymmetric, even though this excludes speculators and reduces the number of competitors. When bidders are asymmetric, attracting speculators by allowing resale only induces high-value bidders to reduce demand.

The presence of speculators increases the seller's revenue only if speculators are eventually outbid by bidders with positive use values. If bidders accommodate speculators and allow them to win by reducing demand, the seller's revenue is (weakly) reduced. Winning bids by speculators, or resale trade, are bad news for the seller, because they imply that bidders with high use values have allowed speculators to win in order to pay a lower price.[7]

However, if the seller cannot prevent resale, the presence of speculators always weakly increases the seller's revenue. If resale is possible, low-value bidders can resell to high-value bidders, and high-value bidders have an incentive to reduce demand and then buy in the resale market even without speculators. In this case, speculators only increase competition in the auction.

Finally, when there is no speculator who may participate in the auction, allowing resale unambiguously facilitates demand reduction because resale makes it more profitable for high-value bidders to reduce demand and less profitable for other bidders to deviate from a demand reduction equilibrium. Without speculators there is no countervailing effect on the number of players who participate in the auction. In this case, allowing resale reduces the seller's revenue when it induces bidders to reduce demand.

A compelling reason to allow resale after an auction is to obtain an efficient final allocation. But my analysis suggests that a revenue-maximizing seller may want to prevent resale in order to increase the auction price, even if this may reduce efficiency.

The rest of the paper is organized as follows. Section I presents the model and Section II defines the two types of equilibria that are analyzed. Section III discusses bidding strategies and shows that speculators may win an auction against high-value bidders. The effects of resale and speculators on the seller's revenue and the strategies that the seller can adopt to increase his revenue are analyzed in Section IV. Section IVA compares an auction with resale and speculators to one in which resale is not allowed and hence speculators do not participate. Section IVB compares an auction with resale and speculators to one in which resale is allowed, but speculators cannot participate. Finally, Section V analyzes how resale affects the seller's revenue when there is no speculator, and Section VI concludes. All proofs are in the Appendix.

I. The Model

Consider a (sealed-bid) uniform-price auction for k units of the same good, that may be followed by a resale market. The reserve price in the auction is normalized to zero.[8] Each player who participates in the auction submits k nonnegative bids, one for each of the units. The k highest bids are awarded the units, and the winner(s) pay for each unit won a price equal to the $(k+1)^{\text{th}}$-highest bid. Uniform-price auctions

[7] My analysis predicts that prices are always higher in the resale market than in the auction, because players trade in the resale market only if they reduced demand in the auction. In the New Zealand auctions for import quota licenses held from 1981 to 1991, the prices at which licenses were traded in the secondary market were 26 percent higher, on average, than the auction prices (R. Preston McAfee, Wendy Takacs, and Daniel R. Vincent 1999).

[8] Footnote 16 and Section IVC discuss the effects of a positive reserve price.

are often used to allocate multiple identical objects—for example, in online IPOs (including the one of Google in August 2004), electricity markets, markets for emission permits, and by the US Treasury Department to issue new securities.

I analyze a uniform-price auction for simplicity, because this is the auction mechanism in which the incentive to reduce demand arises more clearly (Ausubel and Cramton 1998). But the qualitative results of the analysis also hold for simultaneous ascending auctions (in which bidding remains open on all units on sale until no one wants to bid any more on any unit), and indeed for any mechanism to allocate multiple units in which players face a trade-off between winning more units and paying lower prices.

Players and Information.—There are $n \geq 2$ female bidders, called $B_1,\dots,B_n$, who have positive use values for the units on sale. To make the model interesting, I assume that $n < k$. If the number of bidders is greater than or equal to the number of units on sale, competition among bidders crowds out speculators because the auction price is at least equal to the highest price that a bidder who does not win any unit is willing to pay. Hence, speculators cannot possibly make positive profit. To simplify the analysis, I assume that bidders have flat demand, i.e., that each bidder B_i has the same use value v_i for each of the units on sale. But the qualitative results of the analysis also hold with downward-sloping demand (i.e., if use values for additional units are decreasing) as long as the trading procedure in the resale market is known before the auction, so that players know whether they will buy or sell in the resale market, and the price at which they will trade (see the Appendix for an example).[9] Without loss of generality, $v_1 > v_2 > \dots > v_n$.

There are $(k - n)$ male speculators who have no use value for the units on sale. (Speculators can also be interpreted as players whose use value is lower than the opportunity cost to participate in the auction.) Therefore, speculators are willing to participate in the auction only if resale is allowed. The relevant difference between bidders and speculators is that speculators can *only* obtain a positive profit by reselling, while bidders can also obtain a positive profit by owning the units on sale. Hence, bidders participate in the auction even if resale is not allowed. The number of speculators can be endogenized by assuming that a speculator pays an entry cost to participate in the auction, and hence he does so if and only if he expects to make positive profit.[10] I generically refer to a "player" when I want to indicate either a

[9] For example, this is the case if trades take place sequentially, starting from the one that generates the largest surplus.

[10] Specifically, suppose speculators arrive sequentially at the auction and observe only the number of players who have entered. A speculator can pay an arbitrarily small cost to learn bidders' use values and enter the auction, or walk away and obtain zero profit. Bidding is costless, so a speculator who enters always bids. (See, e.g., Jeremy Bulow and Klemperer 2009.) It follows that a speculator enters the auction if and only if he has some positive probability of winning (before knowing bidders' use values). If more than $(k - n)$ speculators have entered, no speculator can obtain positive profit (see footnote 18). While if there are no more than $(k - n)$ speculators, speculators win the auction and obtain positive profit for some bidders' use values (Proposition 1). Therefore, only the first $(k - n)$ speculators pay the entry cost and participate in the auction.

If, instead, speculators pay a bidding cost after learning bidders' use values, then they only participate in the auction if they know they are going to win, and speculators cannot increase the seller's revenue (see Section IV). But the main result still holds: allowing resale to attract speculators reduces the seller's revenue when they participate in the auction because they induce bidders to reduce demand.

bidder or a speculator; player S indicates a speculator and player i indicates bidder B_i, $i = 1, \ldots, n$.

I make the following assumption on valuations, which is often used in the literature on demand reduction (e.g., Wilson 1979).

ASSUMPTION 1: *Use values are common knowledge among bidders and speculators, but the seller does not know bidders' use values.*

Hence, players know the ex post efficient allocation of the units on sale before the auction. Assumption 1 allows me to abstract from the effects of information transmission between the auction and the resale market. So the analysis can be focused on the strategic interaction between equally informed players, in order to show how resale and speculators affect bidders' strategies and the seller's revenue, even when bidders' strategies do not influence the information and the behavior of speculators in the resale market.

Resale Market.—When resale is allowed, players always trade in the resale market if there are gains from trade. For simplicity, I assume that a unit can be traded only once and that, if two players start bargaining on the terms of trade, they cannot then trade with any other player (i.e., two players have to commit to trade with each other before bargaining on the terms of trade). Hence, in the resale market the outside option of the player who has won the unit in the auction is equal to his use value, while the outside option of the player who is trying to acquire the unit is normalized to zero.[11] It follows that the gains from trade between two players are equal to the difference between their use values and, if any player apart from bidder B_1 wins a unit in the auction, to maximize the gains from trade he always resells to bidder B_1, the player with the highest use value.

Assumption 1 implies that the resale market is always efficient, because players are capable of exploiting all profitable trade opportunities after the auction. However, the qualitative results of the analysis do not hinge on this feature of the model. For example, even if with a strictly positive, but not too large, probability players are unable to trade after the auction, speculators may still win the auction, and resale and speculators may still reduce the seller's revenue (see footnote 30).

I make the following assumption on players' sharing of the gains from trade.

ASSUMPTION 2: *When two players trade a unit in the resale market, they equally share the gains from trade.*

[11] I model the bargaining procedure in the resale market in the simplest possible way. But the results of the analysis are robust to many alternative assumptions about bargaining. For example, if in the resale market the auction winner can threaten to trade with other bidders, then his outside option is higher than his use value, and he obtains a larger share of the gains from trade. So speculators are willing to pay a higher price in the auction. This changes the specific equilibrium conditions of Sections III and IV, but not their qualitative interpretation, and it actually reinforces the results because it makes demand reduction even more attractive for bidders when resale is allowed. Similarly, if a unit can be traded more than once and there is "multi-parties" bargaining (with alternated offers and a risk of breakdown of negotiation) as in David de Meza and Mariano Selvaggi (2007), then in the resale market every player can sell to bidder B_1 at price $\frac{1}{2}(v_1 + v_2)$. In this case, demand reduction is always an equilibrium when resale is allowed, and resale and speculators always (weakly) reduce the seller's revenue.

Therefore, the outcome of bargaining between two players in the resale market is given by the Nash bargaining solution, where the disagreement point is represented by players' outside options. The resale price at which two players trade is "half way" between the two players' use values.

This assumption is made for simplicity, but all the qualitative results hold for *any* given sharing of the gains from trade in the resale market (as long as the resale price is higher than the use value of the auction winner and, hence, he obtains some of the gains from trade) and, in particular, even if the auction winner makes a take-it-or-leave-it offer in the resale market, hence obtaining the whole resale surplus.[12] Moreover, giving different bargaining powers to different players (e.g., by assuming that speculators can obtain a smaller share of the gains from trade than low-value bidders when trading with bidder B_1, or vice versa) also does not affect any of the qualitative results.

"Willingness to Pay."—When resale is allowed, a player's "willingness to pay" for a unit in the auction, which I define as the highest auction price that a player is happy to pay for a unit, is represented by the price at which he can buy or sell a unit in the resale market (e.g., Milgrom 1987).

If a speculator wins a unit in the auction, he resells to bidder B_1 at price $\frac{1}{2}v_1$; and if bidder B_i, $i \neq 1$, wins a unit in the auction, he resells to bidder B_1 at price $\frac{1}{2}(v_1 + v_i)$. Therefore, during the auction, the highest price a speculator is happy to pay for one unit is $\frac{1}{2}v_1$ (i.e., the resale price he can obtain in the aftermarket); and the highest price bidder B_i, $i \neq 1$, is happy to pay for one unit is $\frac{1}{2}(v_1 + v_i)$ (i.e., the resale price she can obtain in the aftermarket). And since bidder B_1 can buy a unit in the resale market at a price that is at most equal to $\frac{1}{2}(v_1 + v_2)$, this is also the highest price she may be happy to pay in the auction.

Hence, taking into account the resale market, bidder B_1 is willing to pay a lower price in the auction because of the possibility of purchasing the units she loses from the auction winners in the aftermarket, while all other bidders and speculators are willing to pay a higher price in the auction because of the possibility of reselling in the aftermarket to bidder B_1.[13]

When resale does not take place, the profit of a player who wins the auction is given by the difference between his use value for the unit(s) he acquires and the auction price. When resale takes place, the profit of a player who wins the auction and resells is given by the difference between the resale price and the auction price, while the profit of a player who buys in the resale market is given by the difference between his use value for the unit(s) he acquires and the resale price. The profit of a player who loses the auction and does not trade in the resale market is normalized to zero.

[12] Indeed, when the auction winner makes a take-it-or-leave-it offer, all bidders strictly prefer to reduce demand and allow other players to win the auction. This is because all players are willing to pay up to v_1 in the auction; hence no bidder can outbid her competitors and make positive profit. In this case, allowing resale always (weakly) reduces the seller's revenue.

[13] In the terminology of Haile (2003), bidder B_1 is willing to pay a lower price in the auction because of the "resale buyer effect" and the other players are willing to pay a higher price in the auction because of the "resale seller effect." See also Philippe Jehiel and Benny Moldovanu (2000) who analyze the externalities created by various types of interaction among players after an auction.

Bidding Strategies.—In the auction, a strategy for player i is a k-element vector:

$$\mathbf{b}_i = (b_i^1; b_i^2; \ldots; b_i^k), \quad i = 1, \ldots, n, S,$$

where b_i^j is player i's bid for the j^{th} unit. Bids must be such that $b_i^j \geq b_i^{j+1}$ (i.e., a player's demand must be nonincreasing in price). There is *demand reduction* if a player's bid is lower than his willingness to pay for a unit. I assume that players do not play weakly dominated strategies. This implies that no player bids more than his willingness to pay for a unit. Moreover, without resale, it is a weakly dominant strategy for each bidder i to bid v_i for the first unit and, with resale, it is a weakly dominant strategy for bidder B_i, $i \neq 1$, to bid $\frac{1}{2}(v_1 + v_i)$ for the first unit, and for each speculator to bid $\frac{1}{2}v_1$ for the first unit.[14] To simplify the exposition, I also assume that, with resale, bidder B_1 bids $\frac{1}{2}(v_1 + v_2)$ for the first unit—i.e., she makes the highest bid that is not weakly dominated.

An equilibrium is Pareto dominated by another equilibrium from the players' point of view if in the second equilibrium at least one player is strictly better off and no player is worse off than in the first equilibrium.

ASSUMPTION 3: *Players do not play an equilibrium that is Pareto dominated, from the players' point of view, by another equilibrium* (*in undominated strategies*).

Assumption 3 is used to select among multiple equilibria. Finally, I also make the following simplifying assumption.

ASSUMPTION 4: *If two players submit the same bid for a unit, the unit is assigned to the player with the highest use value.*

This assumption simplifies the description of equilibrium strategies, but none of the results hinges on it.

II. Definition of Equilibria

Players may find it profitable to reduce demand and bid less than their willingness to pay for some units other than the first one, in order to pay a lower price for the units they win and so obtain a higher profit (Wilson 1979; Ausubel and Cramton 1998). The logic is the same as the standard textbook logic for a monopsonist withholding demand: buying an additional unit increases the price paid for the first, inframarginal units.

I consider two types of equilibria (in undominated strategies) of the auction. In one type of equilibria, which I call Demand Reduction equilibria, speculators win if

[14] In a uniform-price auction, it is a weakly dominant strategy for a player to bid his willingness to pay for the first unit (see, e.g., Milgrom 2004). The reason is that a player's first-unit bid affects the auction price only when it is the $(k + 1)^{\text{th}}$-highest bid, in which case the player wins no unit and the price is irrelevant to him. Therefore, exactly as in a single-unit second-price auction, the first-unit bid is chosen to allow the player to win whenever it is profitable for him to do so (i.e., when his willingness to pay is no lower than the auction price).

they participate. In the other type of equilibria, which I call Positive Price equilibria, all speculators, and possibly also some of the bidders, lose. In order for speculators to win the auction, it is necessary that bidders reduce demand, because bidders have a higher willingness to pay for all units than do speculators, and speculators do not bid more than their willingness to pay.

DEFINITION 1: *A* (*zero-price*) *Demand Reduction* (*DR*) *equilibrium is a Nash equilibrium of the auction in which each player who participates in the auction wins at least one of the units on sale, and the auction price is zero.*

There can be other equilibria in which each player wins at least one of the units on sale, but the auction price is strictly positive. However, any such equilibrium with a positive auction price is Pareto dominated for the players by the zero-price DR equilibrium in which each player wins exactly the same number of units as in the first equilibrium (because the final allocation is the same in both equilibria, but the auction price that players pay is lower in the zero-price DR equilibrium). Moreover, whenever the auction has an equilibrium in which each player wins at least one unit and the auction price is positive, it also has a zero-price DR equilibrium.

DEFINITION 2: *A Positive Price* (*PP*) *equilibrium is a Nash equilibrium of the auction in which the auction price is strictly positive and:* (*i*) *if speculators participate in the auction, no speculator wins any unit;* (*ii*) *if speculators do not participate in the auction, at least one of the bidders does not win any unit.*

There are many possible types of PP equilibria in which speculators lose when they participate in the auction. For example, bidders may outbid speculators and share the units on sale, so that each bidder wins at least one unit.[15] Or some higher-value bidders may outbid all other players (including lower-value bidders) and win all units. Similarly, when speculators do not participate in the auction, there are many possible types of PP equilibria in which some of the bidders lose.

I distinguish between DR and PP equilibria in order to analyze when bidders reduce demand and allow speculators to win. In this case, the auction price may be zero, and resale and speculators may reduce the seller's revenue.

III. Successful Speculators

Assume that resale is allowed and speculators participate in the auction. I consider a zero-price DR equilibrium in which each player bids the highest price he is happy to pay for one unit and zero for all other units. Precisely, bidder B_1 bids

$$\mathbf{b}_1 = (\tfrac{1}{2}(v_1 + v_2); \underbrace{0;\ldots;0}_{k-1}),$$

[15] Although in a PP equilibrium some players do not win any unit, bidders do not necessarily bid the highest price they are willing to pay for all units and may still reduce demand.

bidder B_i bids

$$\mathbf{b}_i = (\tfrac{1}{2}(v_1 + v_i); \underbrace{0;\ldots;0}_{k-1}), \quad i = 2,\ldots,n,$$

and each speculator bids

$$\mathbf{b}_S = (\tfrac{1}{2}v_1; \underbrace{0;\ldots;0}_{k-1}).$$

So each player wins exactly one unit and the seller's revenue is zero. All bidders, apart from B_1, and all speculators resell the units they buy in the auction to bidder B_1 in the aftermarket.

I investigate under which condition there is a DR equilibrium in which speculators win the auction.[16] Define the following $(n - 1)$ conditions "Dispersed Top Values" (DTV):

$$\text{(DTV)} \qquad \begin{cases} v_1 + (k + 2 - i)v_i > (k + 1 - i)v_2, & i = 3,\ldots,n, \\ v_1 > (k - n)v_2. \end{cases}$$

LEMMA 1: *When resale is allowed and speculators participate, the auction has a (zero-price) DR equilibrium if and only if conditions (DTV) are satisfied. Moreover, if conditions (DTV) are satisfied, the (zero-price) DR equilibrium is the Pareto dominant equilibrium for players. If, instead, one or more of conditions (DTV) is not satisfied, the auction only has PP equilibria.*

As shown in the proof of Lemma 1, if conditions (DTV) are satisfied, no players want to deviate from the DR equilibrium described. In other words, when his competitors reduce demand to one unit, each player prefers to win a single unit in the auction at price zero, rather than try to obtain more units by outbidding his competitors. (And deviating by winning less than one unit is clearly not profitable.) I now provide intuition for why this is the case.

Bidder B_1 and the speculators have no incentive to deviate from demand reduction. To see this notice that, in order to win more than one unit, a speculator has to at least outbid another speculator. But this raises the auction price to at least $\tfrac{1}{2}v_1$, a price at which the speculator can obtain no profit. Similarly, if bidder B_1 wins more than one unit in the auction, she only increases the auction price that she pays for the first unit (since after reducing demand to one unit, she still buys all other units in the resale market, at the same prices she pays to win them in the auction).

[16] If the auction has a positive reserve price, it is less profitable for players to reduce demand, because they have at least to pay the reserve price to win a unit. In this case, an equilibrium in which all players reduce demand requires more restrictive conditions than the ones we derive for a (zero-price) DR equilibrium. However, the qualitative results on the effects of resale and speculators hold even with a positive reserve price.

Bidder B_2 is the one who has the strongest incentive to deviate from demand reduction, because she gains the most from outbidding low-value players and winning more units to resell in the aftermarket. So, if bidder B_2 prefers not to deviate from the DR equilibrium, all other bidders with lower values also prefer not to deviate. Bidder B_2 does not deviate if and only if conditions (DTV) hold.[17]

Conditions (DTV) require that the two highest use values are sufficiently dissimilar or, in other words, that the valuation of the bidder who is going to buy in the resale market is sufficiently higher than the valuations of the bidders who are going to sell. For example, all conditions (DTV) are satisfied if $v_1 > (k-2)v_2$. The reason is that, if bidder B_1's use value is sufficiently higher than bidder B_2's use value, it is too costly for bidder B_2 to outbid low-value bidders and/or speculators. Precisely, if v_1 is high, low-value bidders and speculators can resell at a high price to bidder B_1, and thus they bid a high price for at least one unit in the auction. And if v_2 is low, bidder B_2 has a low outside option in the resale market; hence, she can only resell at a relatively low price and she is not willing to pay a high price in the auction.

When conditions (DTV) are satisfied, there may also be PP equilibria. However, as shown in the proof of Lemma 1, when a bidder prefers not to deviate from the DR equilibrium, she also obtains a higher profit in the DR equilibrium than in any equilibrium with a positive auction price. Therefore, when conditions (DTV) are satisfied, the zero-price DR equilibrium is the Pareto-dominant equilibrium for players and, by Assumption 3, players select the zero-price DR equilibrium. When one or more of conditions (DTV) is not satisfied, at least one bidder (bidder B_2) prefers to outbid the speculators (by bidding at least $\frac{1}{2}v_1$ for $(k-n+1)$ units). So speculators cannot win the auction and there are only PP equilibria with an auction price at least equal to $\frac{1}{2}v_1$.

By Lemma 1, all bidders with positive use values may strictly prefer to let speculators win some of the units on sale in equilibrium.

PROPOSITION 1: *If and only if conditions (DTV) are satisfied, in the Pareto dominant equilibrium each speculator wins one of the units on sale and resells to bidder B_1 in the aftermarket.*

Therefore, speculators may successfully participate in the auction, and resell in the aftermarket to the highest value bidder, thus obtaining a strictly positive profit.[18] The speculators' success is due to their ability to exploit in the resale market their superior information compared to the auction seller. In order to obtain the qualitative result of Proposition 1, however, it is only necessary to assume that speculators have enough information to be able to obtain some surplus in the resale market—i.e., that they are able to resell to bidder B_1 at any price different from zero. It is straightforward to show that the same result holds even if: players have to pay a (not too large) cost to trade in

[17] For example, bidder B_2 prefers to win one unit at price 0 rather than outbid bidder B_3 (together with low-value bidders and speculators) and win $(k-1)$ units to resell to bidder B_1 if and only if $\frac{1}{2}(v_1+v_2) > (k-1)[\frac{1}{2}(v_1+v_2) - \frac{1}{2}(v_1+v_3)] \Leftrightarrow (k-1)v_3 + v_1 > (k-2)v_2$. Similarly, bidder B_2 prefers to win one unit at price 0 rather than outbid speculators and win $(k-n+1)$ units if and only if $\frac{1}{2}(v_1+v_2) > (k-n+1)[\frac{1}{2}(v_1+v_2) - \frac{1}{2}v_1] \Leftrightarrow v_1 > (k-n)v_2$.

[18] If more than $(k - \mathrm{n})$ speculators participate in the auction, competition among speculators drives their profit to zero because each speculator bids $\frac{1}{2}v_1$ for at least one unit (and bidders bid even higher for at least one unit) and, therefore, the auction price is no lower than $\frac{1}{2}v_1$, which is the profit a speculator can obtain by reselling.

the resale market, bidders discount the surplus obtained in the resale market, or there is a (not too large) probability that resale fails after the auction, so that the resale market is not necessarily efficient. All these assumptions make trading in the resale market less profitable for players, because they increase bidder B_1's willingness to pay in the auction, and reduce all other players' willingness to pay. This may reduce, but does not eliminate, bidders' incentive to accommodate speculators.

As showed in Proposition 1, speculators are not always able to win the auction. The reason is that the presence of speculators in the auction has two contrasting effects for bidders:

- *Competition Effect*: speculators increase the number of competitors in the auction;
- *Demand Reduction Effect*: resale and speculators affect bidders' incentive to reduce demand.

The demand reduction effect induces bidder B_1 to bid less aggressively and accommodate speculators because she can always buy in the resale market the units she loses in the auction. Therefore, outbidding speculators has the only effect of increasing the price of the units that bidder B_1 wins in the auction. Moreover, the demand reduction effect may also induce other bidders to bid less aggressively, in order to pay a lower auction price.[19] On the other hand, the competition effect may induce bidders different from B_1 to bid more aggressively in order to outbid speculators, because these bidders are directly competing with speculators for a chance to resell to bidder B_1. If these bidders lose a unit in the auction, they have no chance of buying it in the resale market. Speculators win the auction if the demand reduction effect is stronger than the competition effect.

In other words, even when resale is allowed, it is the possibility that bidders reduce demand that really attracts speculators to the auction. But speculators win the auction only if bidders actually prefer to reduce demand rather than outbid them.[20]

IV. Seller's Revenue

In this section, I analyze how the presence of speculators in an auction affects the seller's revenue. Notice that the two effects of speculators described in the previous section may affect the seller's revenue in opposite directions: the competition effect tends to increase the seller's revenue, while the demand reduction effect may reduce the seller's revenue.

There are two different reasons why speculators may not participate in an auction: (i) resale is not allowed, and so speculators have no incentive to participate in the auction; (ii) the seller prevents speculators from participating, even if resale is allowed, and so speculators would like to participate in the auction. To analyze the

[19] Bidders have an incentive to reduce demand even without speculators. But allowing resale and attracting speculators in the auction can increase this incentive. See Section IV.

[20] This suggests that, in a single-unit auction, the reason why it is less natural that speculators win is that there is no scope for profitable demand reduction by a bidder with a positive use value, because if the bidder loses the single unit on sale, she obtains no profit in the auction.

effects of the presence of speculators in each of these cases, I consider the seller's revenue in three different scenarios:

- Resale is allowed and speculators are allowed to participate in the auction.
- Resale is not allowed and hence speculators do not participate in the auction.
- Resale is allowed but speculators are not allowed to participate in the auction.

The first scenario (in which speculators participate in the auction) was analyzed in Section III. In the next two sections, I analyze the seller's revenue in the other two scenarios (in which, for different reasons, speculators do not participate in the auction) and compare it with the seller's revenue in the first scenario.

A. *Should Resale Be Allowed to Attract Speculators?*

Assume that resale is not allowed, so that speculators do not participate in the auction. In this case, we have a standard auction with a fixed number of bidders,[21] and the highest price a bidder is happy to pay in the auction is equal to her use value. Define a *sharing of the units* on sale among bidders as a vector $\boldsymbol{\alpha} \equiv (\alpha_1,\dots,\alpha_n)$ such that $1 \leq \alpha_i \leq k - n + 1$ and $\sum_{i=1}^{n} \alpha_i = k$, where $\alpha_i \in \mathbb{N}_0$ indicates the number of units obtained by bidder B_i. A special role in the analysis is played by the sharing $\boldsymbol{\alpha}^* \equiv (1,\dots,1,k-n+1)$, in which bidder B_n obtains $(k-n+1)$ units and all other bidders obtain one unit each.

Even without speculators, there may be DR equilibria in which each bidder wins a number of units between 1 and $(k-n+1)$ and the auction price is zero. Specifically, consider a zero-price DR equilibrium with sharing $\boldsymbol{\alpha}$, in which bidder B_i bids her valuation for α_i units and zero for all other units, i.e.,

$$\mathbf{b}_i = (\underbrace{v_i;\dots;v_i}_{\alpha_i};\underbrace{0;\dots;0}_{k-\alpha_i}), \quad i = 1,\dots,n.$$

In this equilibrium, each bidder B_i wins α_i units, and the seller obtains no revenue.

I investigate under which condition the auction has a DR equilibrium. Define the following conditions "Clustered Values" (CV):

$$\text{(CV)} \quad \alpha_i v_j > (\alpha_j + \alpha_{j+1} + \dots + \alpha_n)(v_i - v_j), \quad i = 1,\dots,n-1, \quad j = i+1,\dots,n.$$

LEMMA 2: *When resale is not allowed, the auction has a* (*zero-price*) *DR equilibrium with sharing* $\boldsymbol{\alpha}$ *if and only if conditions* (*CV*) *are satisfied. Moreover, if conditions* (*CV*) *are satisfied for the sharing* $\boldsymbol{\alpha}^*$ *and* $i = 1$, *then conditions* (*CV*) *are satisfied for every sharing* $\boldsymbol{\alpha}$ *and every i, and any equilibrium with a positive auction price is Pareto dominated, from the bidders' point of view, by a* (*zero-price*)

[21] In contrast to speculators, low-value bidders have an incentive to participate in the auction even when resale is not allowed (and even with a small entry cost) because they can win and obtain their use value if their higher-value competitors reduce demand.

DR equilibrium. If, instead, there is no $\boldsymbol{\alpha}$ *such that conditions* (*CV*) *are satisfied, the auction only has PP equilibria.*

Each bidder may have an incentive to deviate from a DR equilibrium, because she may prefer to outbid lower value bidders and win more units. As shown in the proof of Lemma 2, for a given $\boldsymbol{\alpha}$, conditions (CV) imply that each bidder B_i does not want to deviate from the DR equilibrium with sharing $\boldsymbol{\alpha}$ by outbidding a low-value bidder.[22] Therefore, if conditions (CV) are satisfied, each bidder prefers to reduce demand and maintain the auction price at 0 when her competitors also reduce demand, rather than obtain more units by outbidding her competitors.

Conditions (CV) require that the use values of high-value bidders are not too much higher than the use values of low-value bidders. For example, conditions (CV) are satisfied for the sharing $\boldsymbol{\alpha}^*$ if and only if $(k-j+2)v_j > (k-j+1)v_1, j = 2, \ldots, n$; and these conditions are all satisfied if $kv_n > (k-1)v_1$. Similarly, for $\alpha_1 = \ldots = \alpha_n = k/n$, conditions (CV) are satisfied if and only if $(n-j+2)v_j > (n-j+1)v_1, j = 2, \ldots, n$; and these conditions are all satisfied if $nv_n > (n-1)v_1$. The intuition is that, if bidders' use values are sufficiently close to each other, low-value bidders are willing to pay a relatively high price in the auction. Therefore, it is too costly for high-value bidders to outbid low-value competitors, and bidders prefer to keep the auction price low by reducing demand.

A DR equilibrium with sharing $\boldsymbol{\alpha}^*$ requires more restrictive conditions than any other DR equilibrium, since in a DR equilibrium with sharing $\boldsymbol{\alpha}^*$ the incentive to deviate is stronger (because by outbidding her competitors a bidder can win a higher number of units than in any other DR equilibrium). So, if the auction has a DR equilibrium with sharing $\boldsymbol{\alpha}^*$, then it also has a DR equilibrium with any other possible sharing. Moreover, because bidder B_1 has a stronger incentive to deviate from a DR equilibrium with sharing $\boldsymbol{\alpha}^*$ than any other bidder (because she wins the same number of units as all her competitors apart from bidder B_n, but she has a higher use value and hence obtains a higher profit from winning more units), if it is not profitable for bidder B_1 to deviate, then it is also not profitable to deviate for all other bidders. Finally, as shown in the proof of Lemma 2, the fact that bidders do not want to deviate from a DR equilibrium with sharing $\boldsymbol{\alpha}^*$ also implies that, whenever the auction has another equilibrium with a positive auction price, this equilibrium is Pareto dominated by a DR equilibrium and, by Assumption 3, it is never chosen by bidders.

Now consider the seller's revenue. Allowing resale increases the maximum price that low-value bidders are willing to pay and, by attracting speculators, it increases the number of competitors. These effects tend to increase the seller's revenue. However, as discussed in Section III, allowing resale also affects bidders' incentive to reduce demand. On the one hand, in a DR equilibrium with speculators, bidders can win less units (because speculators have to win too). This makes demand reduction less attractive. On the other hand, when resale is allowed, demand reduction is more attractive for bidder B_1 because she can buy in the resale market the units she

[22] For example, bidder B_1 prefers not to deviate from the DR equilibrium by outbidding bidder B_j (and all lower-value bidders) and winning $(\alpha_1 + \alpha_j + \alpha_{j+1} \ldots + \alpha_n)$ units if and only if $\alpha_1 v_1 > (\alpha_1 + \alpha_j + \alpha_{j+1} \ldots + \alpha_n) \times (v_1 - v_j) \Leftrightarrow \alpha_1 v_j > (\alpha_j + \alpha_{j+1} \ldots + \alpha_n)(v_1 - v_j)$.

does not win in the auction; and deviating from demand reduction is less attractive for all other bidders, because they have to pay a higher auction price to outbid their competitors. So allowing resale and attracting speculators may induce players to choose a DR equilibrium and reduce the seller's revenue. By contrast, the presence of speculators increases the seller's revenue if it eliminates a DR equilibrium or if it induces bidders to bid more aggressively in a PP equilibrium.

PROPOSITION 2: *Assume speculators participate in the auction if and only if resale is allowed.*

(i) *If conditions* (*DTV*) *are satisfied and, for every sharing* α, *one or more of conditions* (*CV*) *is not satisfied, then the auction has a Pareto dominant* (*zero-price*) *DR equilibrium with resale, but it only has PP equilibria without resale. Therefore, allowing resale to attract speculators reduces the seller's revenue.*

(ii) *If conditions* (*CV*) *are satisfied for the sharing* α^* and $i = 1$, *and one or more of conditions* (*DTV*) *is not satisfied, then the auction has a Pareto dominant* (*zero-price*) *DR equilibrium without resale, but it only has PP equilibria with resale. Therefore, allowing resale to attract speculators increases the seller's revenue.*

(iii) *Allowing resale to attract speculators can increase the seller's revenue only if one or more of conditions* (*DTV*) *is not satisfied.*

The effect of resale and speculators on the seller's revenue depends on bidders' relative valuations. If bidders are asymmetric (i.e., if bidders' use values are sufficiently dispersed), conditions (DTV) are satisfied and conditions (CV) are not satisfied. Therefore, with resale, bidders maintain the auction price low by bidding for less units than they actually want. In this case, when resale is allowed the demand reduction effect prevails and reduces the seller's revenue. By contrast, if bidders are symmetric (i.e., if the use values of the two highest-value bidders are sufficiently similar), one or more of conditions (DTV) is not satisfied and, with resale, some bidders bid more aggressively in order to outbid speculators. When conditions (CV) are satisfied (i.e., when bidders' use values are sufficiently clustered), this eliminates a DR equilibrium (compared to an auction without resale). In this case, when resale is allowed the competition effect prevails and increases the seller's revenue.[23]

Example 1: Let $k = 4$ and $n = 2$ (i.e., there are 4 units on sale, 2 bidders and 2 speculators). When resale is allowed, the seller's revenue is equal to 0 if and only if $v_1 > 2v_2$. When resale is not allowed, the seller's revenue is equal to 0 if and only if

[23] When both conditions (DTV) and conditions (CV) (for $\alpha = \alpha^*$ and $i = 1$) are satisfied, allowing resale and attracting speculators do not affect the seller's revenue because the auction has a DR equilibrium both with and without resale. When there is no DR equilibrium regardless of whether resale is allowed or not, the effect of allowing resale on the seller's revenue depends on whether resale induces more bidders to reduce demand or to compete more aggressively in a PP equilibrium.

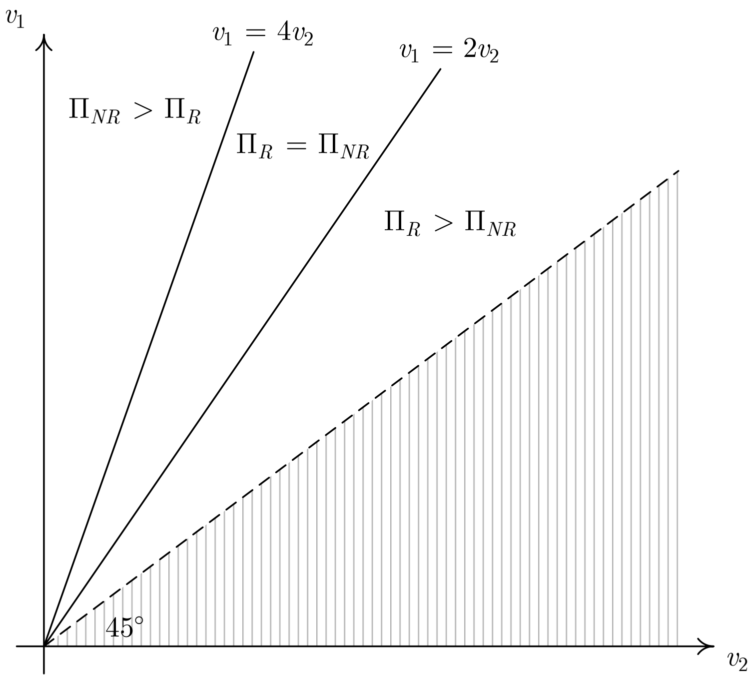

Figure 1. The Effect of Allowing Resale on the Seller's Revenue as a Function of Bidders' Valuations

$v_1 < 4v_2$.[24] Therefore, allowing resale and attracting speculators reduces the seller's revenue if and only if $v_1 > 4v_2$, and increases the seller's revenue if and only if $v_1 < 2v_2$. (See Figure 1, where Π_R indicates the seller's revenue with resale and Π_{NR} indicates the seller's revenue without resale.)

By Proposition 2, the presence of speculators can only increase the seller's revenue if the auction has no (zero-price) DR equilibrium when resale is allowed, so that speculators are eventually outbid by bidders with positive use values.

COROLLARY 1: *Allowing resale and attracting speculators increase the seller's revenue only if speculators do not win any unit in the auction.*

The presence of speculators in the auction is not per se good news for the seller, because speculators may be accommodated by bidders in order to keep the auction price low. It is the fact that speculators participate in the auction, but eventually lose, that indicates an actual increase in competition, and therefore a higher seller's revenue.[25]

In order to compare the seller's revenue with and without resale, I have used Assumption 3 to select among multiple equilibria. Alternatively, without needing to

[24] See the Appendix for more details on this example.

[25] In my model, speculators never win the auction at a positive price. Speculators may win at a positive price in the presence of uncertainty. For example, in a simultaneous ascending auction bidders may reduce demand only

select among equilibria, I would have obtained the same results of Proposition 2 by arguing that allowing resale and attracting speculators reduce the seller's revenue when the lowest auction price supported in an equilibrium with resale is lower than the lowest auction price supported in an equilibrium without resale.

B. *Should Speculators Be Allowed to Participate?*

Assume now that resale is allowed, but that the seller prevents speculators from participating in the auction. In this case, the highest price a bidder is happy to pay in the auction is equal to the price at which she can trade in the resale market.

Consider a zero-price DR equilibrium with sharing $\boldsymbol{\alpha}$, in which each bidder B_i bids a positive price for α_i units and zero for all other units. Precisely, bidder B_1 bids

$$\mathbf{b}_1 = (\underbrace{\tfrac{1}{2}(v_1 + v_2); \ldots ; \tfrac{1}{2}(v_1 + v_2)}_{\alpha_1}; \underbrace{0; \ldots ; 0}_{k-\alpha_1}),$$

and bidder B_i bids

$$\mathbf{b}_i = (\underbrace{\tfrac{1}{2}(v_1 + v_i); \ldots ; \tfrac{1}{2}(v_1 + v_i)}_{\alpha_i}; \underbrace{0; \ldots ; 0}_{k-\alpha_i}), \quad i = 2, \ldots, n.$$

In this equilibrium, bidder B_i wins α_i units, $i = 1,\ldots, n$, and the seller's revenue is zero. All bidders, apart from B_1, resell the units they win to bidder B_1.

I investigate under which condition the auction has a DR equilibrium, and therefore the seller can obtain no revenue. Define the following conditions "Dispersed Top Values or Clustered Bottom Values" (DCV):

$$\text{(DCV)} \qquad \alpha_i(v_1 + v_j) > (\alpha_j + \alpha_{j+1} + \ldots + \alpha_n)(v_i - v_j),$$

$$i = 2,\ldots,n-1, \quad j = i+1,\ldots, n.$$

LEMMA 3: *When resale is allowed, but speculators cannot participate, the auction has a (zero-price) DR equilibrium with sharing* $\boldsymbol{\alpha}$ *if and only if conditions (DCV) are satisfied. Moreover, if conditions (DCV) are satisfied for the sharing* $\boldsymbol{\alpha}^*$ *and* $i = 2$*, then they are also satisfied for every sharing* $\boldsymbol{\alpha}$ *and every i, and any equilibrium with a positive auction price is Pareto dominated, from the bidders' point of view, by a (zero-price) DR equilibrium. If, instead, there is no* $\boldsymbol{\alpha}$ *such that conditions (DCV) are satisfied, the auction only has PP equilibria.*

after raising the auction price to test the credibility of speculators. In this case, the seller's revenue may be higher if resale is allowed even when speculators win. However, it remains true that bidders accommodating speculators rather than competing aggressively harms the seller.

As shown in the proof of Lemma 3, if conditions (DCV) are satisfied, no players want to deviate from the DR equilibrium described. Exactly as with speculators, bidder B_1 has no incentive to deviate from demand reduction and bidder B_2 has the strongest incentive to deviate from demand reduction. So if bidder B_2 does not deviate, the other bidders with lower values do not deviate either.

Conditions (DCV) require that bidder B_1's use value is sufficiently higher than bidder B_2's use value or, alternatively, that the use values of low-value bidders are sufficiently high and close to each other.[26] For example, all conditions (DCV) are satisfied for every α if $v_1 + (k-1)v_n > (k-2)v_2$. The intuition is that, when bidder B_2's use value is closer to lower value bidders' use values than to bidder B_1's use value, it is too costly for bidder B_2 to outbid lower value bidders (whose bids for the first unit are increasing in bidder B_1's use value and in their own use values). So bidder B_2 prefers to keep the auction price low by reducing demand, rather than win more units by outbidding her competitors.

Now consider the seller's revenue. If there is no DR equilibrium when speculators participate in the auction, the presence of speculators only increases the number of competitors without affecting bidders' willingness to pay; hence, it cannot reduce the seller's revenue. Speculators reduce the seller's revenue if and only if they induce bidders to choose a DR equilibrium.

PROPOSITION 3: *If resale cannot be prevented, the presence of speculators in the auction* (*weakly*) *increases the seller's revenue.*

As shown in the proof of Proposition 3, when resale is allowed, if the auction has a Pareto dominant (zero-price) DR equilibrium with speculators, then it also has a Pareto dominant (zero-price) DR equilibrium without speculators.[27] Therefore, the presence of speculators never reduces the seller's revenue. Moreover, the presence of speculators strictly increases the seller's revenue when the auction has a Pareto dominant (zero-price) DR equilibrium without speculators, but it only has equilibria with positive prices with speculators.

The intuition for this result is that, if resale is possible, bidders have a stronger incentive to reduce demand when speculators do not participate in the auction. Bidder B_1 has an incentive to reduce demand even without speculators, because any bidder who wins a unit resells it to bidder B_1. Without speculators, low-value bidders have a stronger incentive to reduce demand, because they can win more units in a DR equilibrium and, even if they win the same number of units, they pay a (weakly) higher price. Hence, in contrast to the case in which the seller can prevent resale, if resale is always possible demand reduction cannot possibly be easier when speculators participate in the auction, and speculators only increase competition in the auction.

[26] Conditions (DCV) can also be written as $\alpha_i v_1 + (\alpha_i + \alpha_j + \ldots + \alpha_n)\, v_j > (\alpha_j + \ldots + \alpha_n) v_i$, $i = 2, \ldots, n-1$, $j = i+1, \ldots, n$.

[27] I show this by proving that if conditions (DTV) (the necessary and sufficient conditions for a DR equilibrium when speculators participate in the auction) are satisfied, then conditions (DCV) (the sufficient conditions for a DR equilibrium when resale is allowed but speculators do not participate in the auction) are also satisfied, for every possible sharing of the units, but the converse is not true.

C. *Seller's Strategy*

Consider a seller who is committed to run a uniform price auction.[28] The results of the last two sections suggest that, if the seller cannot prevent resale, he should always welcome speculators in the auction. But if the seller can credibly forbid resale and has an estimate of bidders' *relative* valuations,[29] he should forbid resale if bidders are asymmetric (i.e., if their use values are dispersed), while he should allow resale and attract speculators to the auction if bidders are symmetric (i.e., if their use values are clustered).

Therefore, knowing bidders' relative valuations and being able to prevent resale can help the seller to increase his revenue. By contrast, being able to distinguish speculators from bidders is not useful for the seller, because excluding speculators from the auction can increase the seller's revenue only if it is achieved through forbidding resale. And if the seller wants to induce enough speculators to participate, he can simply allow resale and not restrict entry in the auction.

In the model, when resale is allowed the final allocation of the units is always efficient, even if bidders reduce demand during the auction (and so the allocation at the end of the auction is not efficient). But if the seller prevents resale to increase his revenue, the highest value bidder may still prefer to reduce demand and let other bidders win, in which case the final allocation of the units is inefficient. So the seller may face a trade-off between increasing revenue and maximizing efficiency.

However, if the resale market is not necessarily efficient—because, for example, bidders may not be able to exploit profitable trade opportunities after the auction—allowing resale may actually reduce efficiency and result in an inefficient final allocation of the units on sale. The reason is that resale may still induce high-value bidders to reduce demand during the auction, only then to find themselves unable to trade with low-value bidders and/or speculators in the aftermarket.[30]

The seller may also want to impose a higher reserve price in order to make demand reduction less attractive for bidders. While it is typically argued that a reserve price reduces efficiency because it may lead to no sale, in my model a higher reserve price can increase the efficiency of the initial allocation achieved by the auction, because it can eliminate a DR equilibrium and crowd out speculators if resale is allowed.

[28] This is not an optimal selling mechanism, neither with resale nor without resale (see Eric Maskin and John Riley 1989). When bidders have flat demand, but not when they have downward sloping demand, the seller maximizes his revenue by bundling the units on sale (see also Pagnozzi 2009) or by selling them sequentially because both these mechanisms eliminate demand reduction. However, in actual practice, sellers are often unable or unwilling to bundle multiple units—perhaps because they are unsure of the number of units that each bidder is willing to acquire—or sell them sequentially—perhaps for fear of selling identical units at different prices to different bidders.

[29] Even if he does not know the exact bidders' valuations, the seller may have a good estimate of how heterogeneous those valuations are. For example, in spectrum auctions incumbents typically have substantially higher use values for a license than new entrants, and the seller may know whether the incumbents' values are more or less than, say, twice the entrants' values, even though he does not know the exact amount of any of these values. The same is true if the value of a license for a mobile-phone operator is proportional to the number of its existing subscribers.

[30] For example, suppose that there are only two bidders and two speculators, and that with probability one-fifth players are unable to trade in the resale market. Then it can be shown that, if $10v_2 > v_1 > 3v_2$, the auction has a DR equilibrium if resale is allowed, but not if resale is not allowed. In this case, allowing resale generates an inefficient allocation of the units on sale with probability one-fifth. (This point is further discussed in Pagnozzi 2009.)

Moreover, if resale is not allowed, a reserve price that eliminates a DR equilibrium also increases the efficiency of the final allocation of the units on sale.

V. Resale without Speculators

What is the effect on the seller's revenue of allowing resale, when the number of competitors in the auction is fixed and there is no speculator who is willing to participate in the auction? To answer this question, I compare an auction with n bidders and no speculator in which resale is not allowed, to an auction with the same number of bidders and no speculator in which resale is allowed. (I basically compare the second and third scenarios of Section IV.)

The effect on the seller's revenue of allowing resale depends on whether it induces bidders to reduce demand (compared to an auction without resale) or it induces bidders to bid more aggressively when they do not reduce demand.

PROPOSITION 4: *Assume there is no speculator who may participate in the auction.*

(i) *If conditions (DCV) are satisfied for $\alpha = \alpha^*$ and $i = 1$ and, for every sharing α, one or more of conditions (CV) is not satisfied, then the auction has a Pareto dominant (zero-price) DR equilibrium with resale, but it only has PP equilibria without resale. Therefore, allowing resale reduces the seller's revenue.*

(ii) *If bidder B_2 outbids all other lower value bidders when resale is allowed and, for every sharing α, one or more of conditions (DCV) is not satisfied, then the auction price in all equilibria with resale is higher than the auction price in any Pareto undominated equilibrium without resale. Therefore, allowing resale increases the seller's revenue.*

Recall from Section IVA that, without resale, there is no DR equilibrium if bidders are asymmetric (i.e., if bidder B_1's use value is much higher than the other bidders' use values) because in this case bidder B_1 prefers to win more units in the auction. But resale makes bidders' willingness to pay in the auction closer to each other, because bidder B_1 is willing to pay a price lower than her use value due to the option to buy in the resale market, while all other bidders are willing to pay a price higher than their use values due to the option to sell in the resale market. This unambiguously makes demand reduction more attractive for bidders.[31]

Specifically, allowing resale makes it more likely that the auction has a DR equilibrium because, as with speculators, resale makes it more profitable for bidder B_1 to reduce demand and more costly for other bidders to deviate from a DR equilibrium and, without speculators, it has no countervailing effect on the number of competitors in the auction (which could otherwise make demand reduction less profitable). Resale reduces the seller's revenue if bidders choose a DR equilibrium when resale is allowed.

[31] In the proof of Proposition 4, I show that, for any given sharing α, conditions (CV) imply conditions (DCV), but the converse is not true.

However, allowing resale also increases the highest prices that all bidders, apart from B_1, are willing to pay for the first unit in the auction. When bidders never choose a DR equilibrium (i.e., when there are only PP equilibria both when resale is allowed and when resale is not allowed), resale induces bidders to bid more aggressively for at least some units and may increase the seller's revenue (even though resale still makes it relatively more profitable for bidders to reduce demand). For example, allowing resale increases the seller's revenue when conditions (DCV) are not satisfied and bidder B_2 prefers to outbid all her lower value competitors rather than reduce demand when resale is allowed.

Allowing resale ensures an efficient final allocation of the units on sale. By contrast, if resale is not allowed and bidders reduce demand, the final allocation of the units is inefficient. Therefore, as when there are speculators who may participate in the auction, in choosing whether to allow resale the seller may face a trade-off between increasing revenue and maximizing efficiency.

VI. Conclusions

Although speculators are attracted by the possibility of resale, in single-object auctions, it is unclear why high-value bidders should let speculators win and then buy in the resale market, rather than simply outbid speculators during the auction.

By analyzing a simple model of a uniform-price auction with complete information, I have made three main points. First, I have shown that high-value bidders may *strictly* prefer to let speculators win when multiple objects are on sale, in order to keep the price low and acquire some of the objects more cheaply in the auction.

Second, it is not true that the only effect of allowing resale is to increase competition in the auction by attracting speculators. I have shown that, when bidders' valuations are dispersed, the possibility of resale induces an accommodating strategy by high-value bidders and, hence, allowing resale reduces the seller's revenue even though it attracts speculators. In fact, when high-value bidders allow speculators to win the auction, they do so to avoid raising the auction price. So resale and speculators increase the seller's revenue only if their effect on competition is stronger than their effect on bidders' incentives to reduce demand.

Third, when it does not attract speculators, it is even more likely that resale reduces the seller's revenue in multi-object auctions, because allowing resale increases the incentive for all bidders to reduce demand, and without speculators it has no countervailing effect on the number of competitors in the auction.

It is often argued that resale after an auction should never be forbidden because, by allowing bidders to exploit gains from trade in the aftermarket, resale ensures an efficient final allocation of the units on sale.[32] But my analysis shows that the possibility of resale, through its effect on bidding strategies during the auction, may

[32] For example, the 2002 "Cave Report," which was commissioned by the UK government to review its spectrum policies, recommends allowing trading of the spectrum licenses auctioned by the government to increase efficiency. And since 2003, the US Federal Communications Commission allows leasing and trading of the spectrum licenses it sells by auctions.

yield a lower revenue for the seller, even when additional bidders participate in the auction when resale is allowed. Therefore, when bidders' valuations are dispersed, a revenue-maximizing seller should commit to prevent resale, even though this may reduce efficiency.

Appendix

PROOF OF LEMMA 1:

Let π_i^* denote the profit that player i obtains in equilibrium. In a DR equilibrium, each player wins one unit in the auction at price zero. Therefore, each speculator obtains a profit equal to the price at which he resells in the aftermarket, i.e.,

$$\pi_S^* = \frac{1}{2}v_1,$$

bidder B_i, $i \neq 1$, obtains a profit equal to the price at which she resells in the aftermarket, i.e.,

$$\pi_i^* = \frac{1}{2}(v_1 + v_i), \quad i = 2,\ldots,n,$$

and bidder B_1 obtains a profit equal to the difference between her valuation for k units, and the price she has to pay to acquire $(k-1)$ units in the aftermarket ($(n-1)$ units from other bidders and $(k-n)$ units from speculators), i.e.,

$$\pi_1^* = kv_1 - \frac{1}{2}\sum_{i=2}^{n}(v_1 + v_i) - (k-n)\frac{1}{2}v_1$$
$$= (k+1)\frac{1}{2}v_1 - \frac{1}{2}\sum_{i=2}^{n}v_i.$$

First, notice that no player has an incentive to deviate from a (zero-price) DR equilibrium by winning less units, because this does not affect the auction price, and hence can only reduce the player's profit. And speculators have no incentive to deviate from a DR equilibrium at all because, in order to win more than one unit, a speculator has to raise the auction price at least up to $\frac{1}{2}v_1$, in which case he obtains no profit.

Similarly, bidder B_1 has no incentive to deviate from a DR equilibrium because, after reducing demand, bidder B_1 still buys in the resale market all the units she does not win in the auction, at the same price she would have to pay to win them in the auction. So the only effect of winning more than one unit in the auction, when other players reduce demand, is to increase the auction price that bidder B_1 pays for the first unit she wins.

By contrast, other bidders may want to deviate from the DR equilibrium described and outbid lower value bidders and/or speculators. (It is clearly never profitable for a bidder to outbid the first-unit bid of a competitor with a higher value.) Consider bidder B_2. If bidder B_2 outbids speculators, she outbids all of them. By Assumption 4, bidder B_2 can outbid all speculators by bidding $\frac{1}{2}v_1$ for

$(k - n + 1)$ units. In this case, she wins $(k - n + 1)$ units in the auction that she can resell at price ½$(v_1 + v_2)$ to bidder B_1 in the aftermarket. This deviation is not profitable if and only if

$$\text{(A1)} \quad \pi_2^* = \frac{1}{2}(v_1 + v_2) > (k - n + 1)\left[\frac{1}{2}(v_1 + v_2) - \frac{1}{2}v_1\right] \Leftrightarrow v_1 > (k - n)v_2.$$

Bidder B_2 may also want to outbid low-value bidders. If it is profitable for bidder B_2 to outbid bidder B_j, $j \geq 3$, then it is also profitable to outbid all other bidders with a use value lower than v_j. Indeed, by Assumption 4, by bidding ½$(v_1 + v_j)$ for $(k - j + 2)$ units and a lower price for all other units, bidder B_2 outbids bidders $B_j, \ldots, B_n$ (in addition to all speculators) and wins $(k - j + 2)$ units in the auction that she can resell to bidder B_1 in the aftermarket. This deviation is not profitable if and only if, for $j = 3, \ldots, n$,

$$\text{(A2)} \quad \pi_2^* = \frac{1}{2}(v_1 + v_2) > (k - j + 2)\left[\frac{1}{2}(v_1 + v_2) - \frac{1}{2}(v_1 + v_j)\right]$$
$$\Leftrightarrow (k - j + 2)v_j + v_1 > (k - j + 1)v_2.$$

Summing up conditions (A1) and (A2), bidder B_2 does not want to deviate from the DR equilibrium described if and only if the following $(n - 1)$ conditions are satisfied:

$$\text{(DTV)} \quad \begin{cases} v_1 + (k - 1)v_3 > (k - 2)v_2, \\ v_1 + (k - 2)v_4 > (k - 3)v_2, \\ \vdots \\ v_1 + (k - n + 3)v_{n-1} > (k - n + 2)v_2, \\ v_1 + (k - n + 2)v_n > (k - n + 1)v_2, \\ v_1 > (k - n)v_2. \end{cases}$$

If bidder B_2 does not want to deviate from the DR equilibrium, then no other lower-value bidder wants to deviate either. To see this, consider bidder B_i, $i \neq 1, 2$. Bidder B_i prefers to reduce demand and win one unit at price 0 rather than outbid bidder B_j, $j > i$, and win $(k - j + 1)$ units if and only if

$$\pi_i^* = \frac{1}{2}(v_1 + v_i) > (k - j + 1)\left[\frac{1}{2}(v_1 + v_i) - \frac{1}{2}(v_1 + v_j)\right]$$
$$\Leftrightarrow (k - j + 1)v_j + v_1 > (k - j)v_i.$$

Clearly, this condition is implied by (A2), the condition for bidder B_2 not wanting to outbid bidder B_j. Similarly, the condition for bidder B_i not wanting to deviate from the DR equilibrium by outbidding speculators is implied by condition (A1).

Notice that, for bidder B_1, only bids for the first unit higher than $½(v_1 + v_2)$, the highest price she can pay in the resale market, or lower than $½v_1$, the lowest price she can pay in the resale market, are weakly dominated. So there may be other (zero-price) DR equilibria in undominated strategies that are identical to the one described, except that bidder B_1 bids $b_1^1 \in [½v_1; ½(v_1 + v_2))$ rather than $½(v_1 + v_2)$ for the first unit. However, these equilibria require conditions that are more restrictive than conditions (DTV), because bidder B_2 has a stronger incentive to deviate from demand reduction when bidder B_1 bids less than $½(v_1 + v_2)$ for the first unit, since she can also obtain positive profit by outbidding bidder B_1. Therefore, whenever the auction has a (zero-price) DR equilibrium in which bidder B_1 bids less than $½(v_1 + v_2)$ for the first unit, it also has a (zero-price) DR equilibrium in which bidder B_1 bids $½(v_1 + v_2)$ for the first unit.

In conclusion, if and only if conditions (DTV) are satisfied, all players prefer to reduce demand and win one unit at price zero when their opponents are reducing demand to one unit, rather then deviate and win more units. Hence, there is a zero-price DR equilibrium.

When conditions (DTV) are satisfied, there may be other equilibria with demand reduction in which each player wins one unit, but the auction price is strictly positive.[33] However, these equilibria are Pareto dominated for all players by the zero-price DR equilibrium because the final allocation is the same in all these equilibria, and only the auction price that players pay is different. Moreover, it is straightforward to verify that equilibria with demand reduction and a strictly positive auction price require conditions that are more restrictive than conditions (DTV). Therefore, whenever the auction has an equilibrium with demand reduction and a strictly positive price, it also has a zero-price DR equilibrium.

When conditions (DTV) are satisfied, there may also be PP equilibria. However, these equilibria are Pareto dominated for all players by the zero-price DR equilibrium. To see this, notice that in a PP equilibrium the auction price is at least $½v_1$ if only speculators lose, and is at least $½(v_1 + v_i)$ if bidder B_i loses. But, when conditions (DTV) are satisfied, each bidder prefers to win one unit at price zero, rather than win more units by outbidding speculators and paying a price equal to $½v_1$, or by outbidding a lower-value bidder and paying a price equal to her willingness to pay. Therefore, each bidder obtains a strictly higher profit in a DR equilibrium than in a PP equilibrium. And, clearly, also speculators are strictly better off in a DR equilibrium.

When one or more of conditions (DTV) is not satisfied, at least one bidder wants to outbid all speculators, even if all other players reduce demand. So speculators cannot win the auction. And in order to outbid speculators, this bidder has to raise the auction price at least up to $½v_1$ (because each speculator bids $½v_1$ for the first unit). So the only possible equilibria of the auction are PP equilibria.

[33] For example, it may be an equilibrium for each player to bid the highest price he is happy to pay for one unit, and price p such that $0 < p < ½v_1$ for all other units. In this case, each player wins one unit and the auction price is p.

PROOF OF PROPOSITION 1:

The "if" part of the statement follows from Lemma 1. The "only if" part follows because, in order for all speculators to win in equilibrium, each bidder must prefer to reduce demand and bid less than $\frac{1}{2}v_1$ for all units apart from one, rather than outbid speculators in order to win more units. As shown in the proof of Lemma 1, if this is the case, then the auction also has a Pareto dominant (zero-price) DR equilibrium.

PROOF OF LEMMA 2:

In the zero-price DR equilibrium with sharing $\boldsymbol{\alpha}$ that we have considered (in which each bidder bids her use value for the units she wins), bidder B_i wins α_i units at price zero, where $1 \leq \alpha_i \leq k - n + 1$ and $\sum_{i=1}^{n} \alpha_i = k$, and obtains a profit equal to

$$\pi_i^* = \alpha_i v_i, \quad i = 1, \ldots, n.$$

This is indeed an equilibrium if and only if no bidder prefers to deviate, outbid her competitors, and win more units. First, notice that it is never profitable for a bidder to outbid a competitor with a higher value. Moreover, if for a bidder B_i it is profitable to outbid a bidder B_j, $j > i$, then it is also profitable to outbid all other bidders with a use value lower than v_j. Bidder B_i, $i = 1, \ldots, n-1$, prefers to win α_i units at price 0, rather than outbid bidders $B_j, \ldots, B_n$, for $j = i+1, \ldots, n$, and win $(\alpha_i + \alpha_j + \alpha_{j+1} + \ldots + \alpha_n)$ units by bidding at least v_j for those units, if and only if

$$\text{(CV)} \qquad \pi_i^* = \alpha_i v_i > \left(\alpha_i + \sum_{k=j}^{n} \alpha_k\right)(v_i - v_j) \Leftrightarrow \left(\alpha_i + \sum_{k=j}^{n} \alpha_k\right) v_j > \sum_{k=j}^{n} \alpha_k v_i.$$

Therefore, for a given sharing $\boldsymbol{\alpha}$, if and only if conditions (CV) are satisfied for every $i \neq n$ and $j > i$, each bidder B_i prefers to reduce demand to α_i units and bid zero for all other units when her competitors reduce demand according to the sharing $\boldsymbol{\alpha}$ (i.e., when each competitor B_j bids v_j for α_j units and 0 for all other units), rather than deviate and outbid any other bidder to win more units. In this case, the auction has a zero-price DR equilibrium sharing $\boldsymbol{\alpha}$.[34]

There may be other equilibria (in undominated strategies) that are exactly equivalent to the zero-price DR equilibrium with sharing $\boldsymbol{\alpha}$ that we consider. Specifically there may be a zero-price DR equilibrium with sharing $\boldsymbol{\alpha}$ in which each bidder B_i bids v_i for the first unit and any price b_i^j such that $v_{i+1} \leq b_i^j \leq v_i$, $j = 2, \ldots, \alpha_i$, for the other $\alpha_i - 1$ units that she wins. But the conditions that have to be satisfied for these strategies to be an equilibrium are more restrictive than the conditions for the zero-price DR equilibrium that we consider (in which bidders bid their use values for all the units they win), because deviating by outbidding low-value bidders is less costly if those bidders bid less than their valuation for some units. This proves the first part of the statement.

[34] Notice that, when in a DR equilibrium with sharing $\boldsymbol{\alpha}$ a bidder B_i wins a number of units higher than or equal to the number of units won by a lower value bidder B_l (i.e., $\alpha_i \geq \alpha_l$ for $i < l$), if bidder B_i does not want to deviate from the DR equilibrium, then bidder B_l does not want to deviate either. Therefore, for a given sharing $\boldsymbol{\alpha}$, if conditions (CV) are satisfied for bidder B_i, then they are also satisfied for bidder B_l.

To prove the second part of the statement, I proceed as follows. (1) I first show that if conditions (CV) are satisfied for $\alpha = \alpha^*$ and $i = 1$, then conditions (CV) are also satisfied for $\alpha = \alpha^*$ and $i > 1$. (2) I then show that if conditions (CV) are satisfied for sharing α^* (and every i), then conditions (CV) are also satisfied for every possible sharing $\alpha \neq \alpha^*$ (and every i). (3) Finally, I show that if conditions (CV) are satisfied for sharing α^* (and every i), then in any possible equilibrium with a positive auction price each bidder obtains a lower profit than in the DR equilibrium with sharing α^*.

(1) Suppose that conditions (CV) are satisfied for $\alpha = \alpha^*$ and $i = 1$, i.e.,

(A3) $$(\underbrace{1 + \ldots + 1}_{n-j+1} + k - n + 1)\, v_j > (\underbrace{1 + \ldots + 1}_{n-j} + k - n + 1)\, v_1$$

$$\Leftrightarrow (k - j + 2)v_j > (k - j + 1)v_1, \quad j = 2,\ldots,n.$$

This implies that bidder B_1 does not want to deviate from a zero-price DR equilibrium with sharing α^*, because she prefers to win 1 unit at price 0, rather than outbid any bidder B_j and win $(k - j + 2)$ units at price v_j. The inequalities (A3) clearly imply that, for $i > 1$,

$$(k - j + 2)v_j > (k - j + 1)v_i, \quad j = i + 1,\ldots,n.$$

In other words, if bidder B_1 does not want to deviate from a DR equilibrium with sharing α^*, then no other bidder wants to deviate either. Therefore, the auction has a zero-price DR equilibrium with sharing α^* if and only if conditions (CV) are satisfied for $\alpha = \alpha^*$ and $i = 1$.

(2) Conditions (CV) for a DR equilibrium with sharing α^* can be written as

(A4) $$v_j + (k - j + 1)(v_j - v_i) > 0, \quad i = 1,\ldots,n - 1, \quad j = i + 1,\ldots,n.$$

Because $\alpha_i \geq 1$, $(v_j - v_i) < 0$, and $(\alpha_j + \alpha_{j+1} + \ldots + \alpha_n) \leq (k - n + 1)$ (since in any sharing α bidders $B_1,\ldots, B_{j-1}$ have to win at least one unit each), the inequalities (A4) imply that

$$\alpha_i v_j + (\alpha_j + \alpha_{j+1} + \ldots + \alpha_n)(v_j - v_i) > 0, \quad i = 1,\ldots,n - 1, \quad j = i + 1,\ldots,n.$$

These last inequalities represent conditions (CV) for a DR equilibrium with sharing α. Therefore, if the auction has a zero-price DR equilibrium with sharing α^*, then it also has a zero-price DR equilibrium with any possible sharing α.

(3) Suppose the auction has a zero-price DR equilibrium with sharing α^*. Clearly, a DR equilibrium in which each bidder wins at least one unit and the auction price is strictly positive is Pareto dominated for all bidders by the zero-price DR equilibrium in which each bidder wins the same number of units, but the auction price is equal to 0.

But the auction may also have PP equilibria in which some bidders do not win any unit and the auction price is positive. However, in a PP equilibrium the auction price is at least equal to the valuation of the highest value bidder who loses the auction, because that bidder bids her valuation for the first unit. Moreover, if bidder B_j does not win any unit in a PP equilibrium, then no bidder with a lower use value than bidder B_j wins any unit either. The reason is that, if bidder B_j does not win any unit, the auction price is $\geq v_j$; hence, higher than the valuation of any bidder B_l such that $l > j$.

So consider a generic PP equilibrium in which bidders $B_j, \ldots, B_n$ do not win any unit $(j \geq 2)$, and the other $j - 1$ bidders win at least one unit each. Specifically, let β_i be the number of units won by bidder B_i, $i = 1, \ldots, j - 1$. Notice that $1 \leq \beta_i \leq k - j + 2$. In this equilibrium, the auction price must be $\geq v_j$. But if conditions (CV) are satisfied for sharing α^*, bidder B_i, $i = 1, \ldots, j - 1$, prefers to win 1 unit at price 0, rather than $(k - j + 2)$ units at price v_j. So each bidder B_i obtains a strictly higher profit in the DR equilibrium with sharing α^* than in any PP equilibrium. (And, clearly, bidders who do not win any unit in a PP equilibrium also obtain a strictly higher profit in the DR equilibrium with sharing α^*.)

Summing up, when the auction has a zero-price DR equilibrium with sharing α^*, all other equilibria with a positive auction price are Pareto dominated, from the bidders' point of view, by the zero-price DR equilibrium with sharing α^*.

Finally, the last part of the statement follows because, when there is no sharing α such that all conditions (CV) are satisfied, at least one bidder prefers to outbid a competitor with a lower value and raise the auction price in order to win more units, when all other bidders reduce demand according to any sharing α. Therefore, the auction has no zero-price DR equilibrium, and there are only PP equilibria in which some bidders do not win any unit and the auction price is strictly positive.

PROOF OF PROPOSITION 2:

First notice that conditions (DTV) and conditions (CV) are not necessarily mutually exclusive, because the two sets of conditions are not disjoint and may be satisfied together.[35] Moreover, none of the two sets of conditions implies the other;[36] and conditions in both sets may simultaneously not be satisfied. Whether resale reduces or increases the seller's revenue depends on whether demand reduction is an equilibrium when resale is allowed.

If conditions (DTV) are satisfied and, $\forall \alpha$, one or more of conditions (CV) is not satisfied, the auction has a unique Pareto dominant zero-price DR equilibrium if and only if resale is allowed. Therefore, allowing resale induces bidders to choose a DR equilibrium and strictly reduces the seller's revenue. This proves part (i) of Proposition 2.

If one or more of conditions (DTV) is not satisfied and conditions (CV) are satisfied for $\alpha = \alpha^*$ and $i = 1$, the auction has a unique Pareto dominant zero-price DR equilibrium if and only if resale is not allowed. Therefore, allowing resale induces

[35] For example, conditions (DTV) and conditions (CV) (for every α) are all satisfied if $v_1 > (k - 2)\, v_2$ and $kv_n > (k - 1)v_1$— i.e., $kv_n > (k - 1)v_1 > (k - 2)\,(v_1 + v_2)$.

[36] If conditions (CV) are satisfied for $\alpha = \alpha^*$ and $i = 1$, then the first $(n - 2)$ conditions (DTV) are also satisfied. However, the last condition (DTV) is not necessarily satisfied.

bidders to choose an equilibrium with a positive auction price and increases the seller's revenue. This proves part (ii) of Proposition 2.

If both conditions (DTV) and conditions (CV) are satisfied for $\alpha = \alpha^*$ and $i = 1$, there is a Pareto dominant zero-price DR equilibrium both with and without resale. Therefore, allowing resale has no effect on the seller's revenue.

Finally, suppose that one or more of conditions (DTV) is not satisfied and, for at least one α, one or more of conditions (CV) is not satisfied either. Hence, when resale is allowed the auction only has PP equilibria. By contrast, when resale is not allowed the auction may have both PP equilibria and DR equilibria, but the latter do not necessarily Pareto dominate the former; hence, they are not necessarily played by bidders. Allowing resale induces all bidders apart from bidder B_1 to bid more aggressively for the first unit than when resale is not allowed because of the possibility of reselling to bidder B_1, and induces some bidders (but not bidder B_1) to also bid aggressively for more than one unit, in order to outbid speculators and other lower value bidders. This tends to increase the seller's revenue. However, allowing resale also affects bidders incentive to reduce demand, and there may be a PP equilibrium in which bidders reduce demand, even if they outbid all speculators. This may reduce the seller's revenue when bidders do not play a DR equilibrium, if one exists, when resale is not allowed. So a PP equilibrium when resale is allowed may have both a higher price or a lower price than a PP equilibrium when resale is not allowed. Hence, the effect of allowing resale on the seller's revenue is more ambiguous and depends on whether resale induces high-value bidders to reduce demand or to bid more aggressively to beat their competitors, compared to an auction without resale.

Nonetheless, in order for resale to strictly increase the seller's revenue, it is necessary that the auction has no Pareto dominant (zero-price) DR equilibrium when resale is allowed; hence, that one or more of conditions (DTV) is not satisfied. (Otherwise, the seller's revenue is equal to zero when resale is allowed.) This proves part (iii) of Proposition 2.

ANALYSIS OF EXAMPLE 1:

When resale is allowed, by Lemma 1, the auction has a Pareto-dominant zero-price DR equilibrium, and the seller's revenue is equal to 0, if and only if $v_1 > 2v_2$. Otherwise, the auction only has PP equilibria. When resale is not allowed, by Lemma 2: the auction has a zero-price DR equilibrium with sharing $\alpha' = (3, 1)$ if and only if $v_1 < 4v_2$; the auction has a zero-price DR equilibrium with sharing $\alpha'' = (2, 2)$ if and only if $v_1 < 2v_2$; and the auction has a zero-price DR equilibrium with sharing $\alpha''' = (1, 3)$ if and only if $v_1 < (4/3)v_2$. The auction may also have equilibria with a positive auction price. But if there is a DR equilibrium with a positive auction price and sharing α', α'' or α''', then there is also a zero-price DR equilibrium with the same sharing, and the former equilibrium is Pareto dominated by the latter. Finally, the auction may have a PP equilibrium in which bidder B_1 wins all four units, and the auction price is $\geq v_2$. But this equilibrium is Pareto dominated by a zero-price DR equilibrium with sharing $\alpha' = (3, 1)$ when the latter equilibrium exists (because in this case bidder B_1 obtains a strictly higher profit by winning 3 units at price 0, rather than 4 units at price v_2). Summing up, when resale is not allowed, if and only if $v_1 < 4v_2$, the auction has a zero-price DR equilibrium and, even if it also has an equilibrium with a positive auction price, this equilibrium is Pareto dominated by a zero-price DR equilibrium.

PROOF OF LEMMA 3:

In a zero-price DR equilibrium with sharing $\boldsymbol{\alpha}$, bidder B_i, $i = 1, \ldots n$, wins α_i units in the auction at price zero, where $1 \le \alpha_i \le k - n + 1$ and $\sum_{i=1}^{n} \alpha_i = k$. Each bidder B_i, $i \neq 1$, resells to bidder B_1 and obtains a profit equal to the resale price for α_i units, i.e.,

$$\pi_i^* = \frac{1}{2}\alpha_i(v_1 + v_i), \quad i = 2, \ldots, n.$$

While bidder B_1 obtains a profit equal to the difference between her valuation for the k units, and the price she has to pay to acquire $(k - \alpha_1)$ units in the resale market, i.e.,

$$\pi_1^* = kv_1 - \frac{1}{2}\sum_{i=2}^{n} \alpha_i(v_1 + v_i).$$

First notice that bidder B_1 has no incentive to deviate from the DR equilibrium that we have considered (in which each bidder bids the same price for all the units she wins), because the only effect of outbidding a low-value competitor and winning more units in the auction is to increase the auction price that bidder B_1 pays for the α_1 units she wins in the DR equilibrium. Bidder B_n has no incentive to deviate from a DR equilibrium either, because to outbid a high-value competitor she has to pay more than her willingness to pay.

Now consider bidder B_i, $i = 2, \ldots, n - 1$. Clearly, it is never profitable for bidder B_i to outbid a higher value bidder. Moreover, bidder B_i prefers to win α_i units at price 0 rather than outbid bidders $B_j, \ldots, B_n$, for $j = i + 1, \ldots, n$, and win $(\alpha_i + \alpha_j + \alpha_{j+1} + \ldots + \alpha_n)$ units if and only if

$$\pi_2^* = \alpha_i \frac{1}{2}(v_1 + v_i) > (\alpha_i + \alpha_j + \alpha_{j+1} + \ldots + \alpha_n)\left[\frac{1}{2}(v_1 + v_i) - \frac{1}{2}(v_1 + v_j)\right]$$
$$\Leftrightarrow \quad (\alpha_i + \alpha_j + \alpha_{j+1} + \ldots + \alpha_n)v_j + \alpha_i v_1 > (\alpha_j + \alpha_{j+1} + \ldots + \alpha_n)v_i.$$

So bidder B_i does not deviate from the DR equilibrium with sharing $\boldsymbol{\alpha}$ if and only if the following $(n - i)$ conditions are satisfied:

$$\text{(DVC)} \qquad \begin{cases} \alpha_i v_1 + (\alpha_i + \alpha_{i+1} + \ldots + \alpha_n)v_{i+1} > (\alpha_{i+1} + \ldots + \alpha_n)v_i, \\ \alpha_i v_1 + (\alpha_i + \alpha_{i+2} + \ldots + \alpha_n)v_{i+2} > (\alpha_{i+2} + \ldots + \alpha_n)v_i, \\ \vdots \\ \alpha_i v_1 + (\alpha_i + \alpha_n)v_n > \alpha_n v_i. \end{cases}$$

And if and only if these conditions are satisfied for every $i \neq 1, n$, all bidders prefer to reduce demand when their opponents reduce demand, rather then deviate and outbid any other bidder to win more units.

Of course, there may be other equilibria in undominated strategies that are exactly equivalent to the zero-price DR equilibrium with sharing α that we consider, and in which each bidder B_i bids less than $\frac{1}{2}(v_1 + v_i)$ for some of the units she wins. But the conditions that have to be satisfied for these strategies to be an equilibrium are more restrictive than the conditions for the zero-price DR equilibrium that we consider, because deviating by outbidding low-value bidders is less costly if those bidders bid less than their willingness to pay.

By the argument in the proof of the second part of Lemma 2, when conditions (DCV) are satisfied for $\alpha = \alpha^*$ and $i = 2$, conditions (DCV) are also satisfied for every sharing α and every i (because, when resale is allowed, bidder B_2 has a stronger incentive to deviate from a DR equilibrium than any other bidder, and her incentive to deviate is stronger from a DR equilibrium with sharing α^*, than from any other DR equilibrium, since by outbidding her competitors bidder B_2 can win the highest number of units consistent with a DR equilibrium). And when conditions (DCV) are satisfied for every sharing α, all other equilibria with a positive auction price are Pareto dominated, from the bidders' point of view, by a zero-price DR equilibrium (because each bidder prefers to win even only one unit at price 0, rather than more units at the price that is necessary to outbid her competitors). Finally, when there is no sharing α such that conditions (DCV) are satisfied, at least one bidder prefers to outbid a lower value competitor when all other bidders reduce demand according to any sharing α. Therefore, the auction has only PP equilibria and no zero-price DR equilibrium.

PROOF OF PROPOSITION 3:

I am going to show that, if conditions (DTV) are satisfied, then conditions (DCV) are also satisfied for every α, but the converse is not true. In other words, it is easier to have a Pareto dominant DR equilibrium without speculators than with speculators; hence, speculators cannot reduce the seller's revenue by inducing bidders to reduce demand. To see this, notice that the first $(n - 2)$ conditions (DTV) can be written as

$$v_1 + v_j - (k - j + 1)(v_2 - v_j) > 0, \quad j \geq 3. \tag{A5}$$

And because $(k - j + 1) \geq k - (\alpha_1 + \ldots + \alpha_{j-1}) \equiv (\alpha_j + \ldots + \alpha_n)$ and $v_2 > v_i$ for $i > 2$, if the inequalities (A5) are satisfied, then the following inequalities are also satisfied:

$$v_1 + v_j - (\alpha_j + \ldots + \alpha_n)(v_i - v_j) > 0, \quad i \geq 2, \quad j \geq 3. \tag{A6}$$

Finally, because $\alpha_i \geq 1$, if the inequalities (A6) are satisfied, then the following inequalities are also satisfied:

$$\alpha_i(v_1 + v_j) - (\alpha_j + \ldots + \alpha_n)(v_i - v_j) > 0, \quad i \geq 2, \quad j \geq 3.$$

These last inequalities represent conditions (DCV). Moreover, conditions (DTV) may not be satisfied even if conditions (DCV) are satisfied for every α, because conditions (DTV) also require that $v_1 > (k - n)v_2$ and, regardless of α, this is not implied by conditions (DCV).

Therefore, when resale is allowed, if there is a Pareto dominant zero-price DR equilibrium when speculators participate in the auction, there are also DR equilibria with every sharing α when speculators do not participate in the auction, and other equilibria with positive prices are Pareto dominated by a DR equilibrium. But the converse is not true. So the presence of speculators eliminates a DR equilibrium and increases the seller's revenue when conditions (DCV) are satisfied for every α and one or more of conditions (DTV) is not satisfied.

Finally, if there is no DR equilibrium when speculators participate in the auction, the presence of speculators can never reduce the seller's revenue because, in this case, speculators increase the number of competitors in the auction and can only induce bidders to bid more aggressively to outbid speculators, thus raising the auction price.

PROOF OF PROPOSITION 4:

Conditions (CV) are more restrictive than conditions (DCV). Indeed, for any given sharing α, conditions (CV) imply conditions (DCV), but the converse is not true:

$$\alpha_i v_j > (\alpha_j + \ldots + \alpha_n)(v_i - v_j) \quad \underset{\not\Leftarrow}{\Rightarrow} \quad \alpha_i(v_1 + v_j) > (\alpha_j + \ldots + \alpha_n)(v_i - v_j), \quad i > 2.$$

Hence, without speculators, if there is a Pareto dominant DR equilibrium when resale is not allowed, there is also a Pareto dominant DR equilibrium when resale is allowed. So the possibility of resale facilitates demand reduction and, therefore, it may reduce the seller's revenue. This happens when conditions (DCV) are satisfied for $\alpha = \alpha^*$ and $i = 1$, so that there is a Pareto dominant zero-price DR equilibrium with resale, and for every sharing α one or more of conditions (CV) is not satisfied, so that there is no zero-price DR equilibrium without resale. This proves part (i) of Proposition 4.

On the other hand, if both conditions (DCV) and conditions (CV) are satisfied for $\alpha = \alpha^*$ and $i = 1$, resale does not affect the seller's revenue, because there is a Pareto dominant zero-price DR equilibrium both with and without resale.

If for every α one or more of conditions (DCV) is not satisfied, there is no DR equilibrium when resale is allowed. And since one or more of conditions (CV) is not satisfied either, there is also no DR equilibrium when resale is not allowed. So the auction price is positive both with resale and without resale. But when resale is allowed, all bidders, apart from bidder B_1, bid more aggressively for the first unit than when resale is not allowed because of the option to resell. This tends to increase the seller's revenue. However, allowing resale also affects bidders' incentive to reduce demand in a PP equilibrium, even if the auction price is positive. Therefore, the effect of allowing resale on the auction price is more ambiguous.

But suppose that bidder B_2 outbids all other lower value bidders when resale is allowed. Then in any PP equilibrium the auction price p^* is $\geq \frac{1}{2}(v_1 + v_3)$. By contrast, when resale is not allowed, there can be a PP equilibrium with an auction price at most equal to v_2, which is bidder B_2's bid for the first unit. Notice that there is no PP equilibrium with a price $> v_3$ and $< v_2$ that is not Pareto dominated (by a PP equilibrium with the same allocation and price v_3). Hence, if without resale the auction price is lower than v_2, allowing resale increases the seller's revenue. So suppose that there is a PP equilibrium in which bidder B_1 wins all units and the auction price is equal to v_2, which may be higher than p^*. (This is the only case in which the price is equal to v_2 in an equilibrium that is not Pareto dominated.) Then it is at least necessary that bidder B_1 prefers to outbid all other bidders and win n units at price v_2, rather than win $(n-1)$ units only at price v_3. Otherwise, by Assumption 3, an equilibrium in which bidder B_1 wins all units and the auction price is v_2 would not be played because it would be Pareto dominated by an equilibrium in which bidder B_1 reduces demand and wins $(n-1)$ units, bidder B_2 wins 1 unit, and the auction price is v_3. Therefore, in order for the auction price to be equal to v_2 when resale is not allowed, it is necessary that

$$n(v_1 - v_2) > (n-1)(v_1 - v_3) \quad \Leftrightarrow \quad v_1 + (n-1)v_3 > nv_2. \tag{A7}$$

And rearranging inequality (A7):

$$v_1 + v_3 - (n-2)(v_2 - v_3) > 2v_2 \quad \Rightarrow \quad v_1 + v_3 > 2v_2 \quad \Leftrightarrow \quad p^* > v_2.$$

Hence, in any equilibrium that is not Pareto dominated without resale, the auction price is lower than p^*, which is the auction price when resale is allowed and bidder B_2 outbids all other lower value bidders. This proves part (ii) of Proposition 4.

AN EXAMPLE WITH DOWNWARD SLOPING DEMAND:

Suppose that $n = 3$ and $k = 5$ (i.e., there are three bidders, two speculators and five units on sale), and that bidders demand at most four units. Specifically, each bidder B_i, $i = 1, 2, 3$, has a use value equal to v_i for the first four units, and zero for the fifth unit.

I assume that, if resale is allowed, trades in the aftermarket take place sequentially, starting from the one that generates the largest surplus: first the bidder who has the highest use value among those who are going to buy in the resale market trades with the player who has the lowest use value among those who are going to sell, then the bidder who has the second highest use value trades with the player who has the second lowest use value, and so on. (This procedure ensures that, given a fixed sharing of the gains from trade, with each trade the bidder who has the highest use value for one of the units that remain to be traded obtains the largest possible surplus.) Therefore, if the two speculators and bidders B_2 and B_3 win one unit each, only the speculators and bidder B_3 resell to bidder B_1, at price $½v_1$ and $½(v_1 + v_3)$, respectively.

Assume that resale is allowed and consider a zero-price DR equilibrium in which each player bids a positive price for the first unit on sale, and zero for all other units. Specifically, the speculators bids $½v_1$ for the first unit and bidder B_3 bids $½(v_1 + v_3)$ for the first unit, since these are the prices at which they can sell to bidder B_1 in the resale market. Notice that, for the usual reasons, neither the speculators nor bidder B_1 wants to deviate from the DR equilibrium. By contrast, bidders B_2 and B_3 may want to outbid the speculator and/or a lower value bidder in order to win more units.

Bidder B_2 does not want to deviate from the DR equilibrium by outbidding the speculators if and only if she obtains a higher profit by winning one unit at price zero, rather than by raising the price up to ½v_1 and winning two more units to resell to bidder B_1, i.e.,

$$v_2 > v_2 + 2\frac{1}{2}(v_1 + v_2) - 3\frac{1}{2}v_1 \quad \Leftrightarrow \quad v_1 > 2v_2. \tag{A8}$$

Clearly, if condition (A8) is satisfied, then bidder B_3 does not want to deviate from the DR equilibrium by outbidding the speculators either. Moreover, bidder B_2 does not want to deviate from the DR equilibrium by outbidding bidder B_3 (and the speculators) if and only if she obtains a higher profit by winning one unit at price zero, rather than by winning four units at price ½$(v_1 + v_3)$ and reselling three of them to bidder B_1, i.e.,

$$v_2 > v_2 + 3\frac{1}{2}(v_1 + v_2) - 4\frac{1}{2}(v_1 + v_3) \quad \Leftrightarrow \quad v_1 + 4v_3 > 3v_2. \tag{A9}$$

Therefore, if and only if both conditions (A8) and (A9) are satisfied, the auction has a zero-price DR equilibrium when resale is allowed and speculators win. These conditions are analogous to the ones of Lemma 1.

Assume now that resale is not allowed and speculators do not participate in the auction. The auction has a zero-price DR equilibrium with sharing $\boldsymbol{\alpha} \equiv (\alpha_1, \alpha_2, \alpha_3)$ such that $\alpha_i \geq 1$ and $\alpha_1 + \alpha_2 + \alpha_3 = 5$ (in which each bidder bids her use value for the units she wins), if and only if each bidder B_i obtains a higher profit by winning α_i units at price 0 than by outbidding her lower-value competitors, i.e.,[37]

$$\begin{cases} (\alpha_1 + \alpha_2 + \alpha_3)v_2 > (\alpha_2 + \alpha_3)v_1, \\ (\alpha_1 + \alpha_3)v_3 > \alpha_3 v_1, \\ (\alpha_2 + \alpha_3)v_3 > \alpha_3 v_2. \end{cases} \tag{A10}$$

By the argument of Lemma 2, if conditions (A10) are satisfied for the sharing $\boldsymbol{\alpha}^* \equiv (1, 1, 3)$ (i.e., if $4v_3 > 3v_1$), then they are also satisfied for any other sharing $\boldsymbol{\alpha}$, and any equilibrium with a positive auction price is Pareto dominated by a zero-price DR equilibrium.

[37] These conditions are analogous to conditions (CV). The first condition ensures that bidder B_1 does not want to outbid bidder B_2; the second condition ensures that bidder B_1 does not want to outbid bidder B_3; and the third condition ensures that bidder B_2 does not want to outbid bidder B_3. Of course, since bidder B_1 does not want to win more than four units, when $\alpha_1 + \alpha_3 = 4$ the first condition is not required, since bidder B_1 never wants to outbid both bidder B_3 and bidder B_2.

Summing up, if for every sharing α one or more of conditions (A10) is not satisfied and conditions (A8) and (A9) are satisfied, allowing resale reduces the seller's revenue. By contrast, if $4v_3 > 3v_1$ and condition (A8) is not satisfied—i.e., if $3v_1 < \max\{4v_3; 2(v_1 + v_2)\}$—allowing resale increases the seller's revenue. These results are analogous to the ones of Proposition 2.

REFERENCES

Ausubel, Lawrence M., and Peter Cramton. 1998. "Demand Reductions and Inefficiency in Multi-Unit Auctions." http://www.ausubel.com/auction-papers/demand-reduction-r.pdf.

Back, Kerry, and Jaime F. Zender. 1993. "Auctions of Divisible Goods: On the Rationale for the Treasury Experiment." *Review of Financial Studies*, 6(4): 733–64.

Bikhchandani, Sushil, and Chi-fu Huang. 1989. "Auctions with Resale Markets: An Exploratory Model of Treasury Bill Markets." *Review of Financial Studies*, 2(3): 311–39.

Bose, Subir, and George Deltas. 1999. "Welcoming the Middlemen: Restricting Competition in Auctions by Excluding the Consumers." In *Current Trends in Economics: Theory and Applications*, ed. Ahmet Alkan, Charalambos D. Aliprantis, and Nicholas C. Yannelis, 119–31. Berlin: Springer.

Bulow, Jeremy, and Paul Klemperer. 2009. "Why Do Sellers (Usually) Prefer Auctions?" *American Economic Review*, 99(4): 1544–75.

Calzolari, Giacomo, and Alessandro Pavan. 2006. "Monopoly with Resale." *RAND Journal of Economics*, 37(2): 362–75.

de Meza, David, and Mariano Selvaggi. 2007. "Exclusive Contracts Foster Relationship-Specific Investment." *RAND Journal of Economics*, 38(1): 85–97.

Engelbrecht-Wiggans, Richard, and Charles M. Kahn. 1998. "Multi-Unit Auctions with Uniform Prices." *Economic Theory*, 12(2): 227–58.

Garratt, Rod, and Thomas Tröger. 2006. "Speculation in Standard Auctions with Resale." *Econometrica*, 74(3): 753–69.

Gupta, Madhurima, and Bernard Lebrun. 1999. "First Price Auctions with Resale." *Economics Letters*, 64(2): 181–85.

Hafalir, Isa, and Vijay Krishna. 2008. "Asymmetric Auctions with Resale." *American Economic Review*, 98(1): 87–112.

Haile, Philip A. 2000. "Partial Pooling at the Reserve Price in Auctions with Resale Opportunities." *Games and Economic Behavior*, 33(2): 231–48.

Haile, Philip A. 2003. "Auctions with Private Uncertainty and Resale Opportunities." *Journal of Economic Theory*, 108(1): 72–110.

Jehiel, Philippe, and Benny Moldovanu. 2000. "Auctions with Downstream Interaction among Buyers." *RAND Journal of Economics*, 31(4): 768–91.

Kagel, John H., and Dan Levin. 2001. "Behavior in Multi-Unit Demand Auctions: Experiments with Uniform Price and Dynamic Vickrey Auctions." *Econometrica*, 69(2): 413–54.

Klemperer, Paul. 2004. *Auctions: Theory and Practice*. Toulouse Lectures in Economics. Princeton and Oxford: Princeton University Press.

Klemperer, Paul, and Margaret A. Meyer. 1989. "Supply Function Equilibria in Oligopoly under Uncertainty." *Econometrica*, 57(6): 1243–77.

Kremer, Ilan, and Kjell G. Nyborg. 2004. "Underpricing and Market Power in Uniform Price Auctions." *Review of Financial Studies*, 17(3): 849–77.

List, John A., and David Lucking-Reiley. 2000. "Demand Reduction in Multiunit Auctions: Evidence from a Sportscard Field Experiment." *American Economic Review*, 90(4): 961–72.

Maskin, Eric, and John Riley. 1989. "Optimal Multi-Unit Auctions." In *The Economics of Missing Markets, Information, and Games*, ed. Frank Hahn, 312–35. Oxford: Oxford University Press.

McAfee, R. Preston, Wendy Takacs, and Daniel R. Vincent. 1999. "Tariffying Auctions." *RAND Journal of Economics*, 30(1): 158–79.

Milgrom, Paul R. 1987. "Auction Theory." In *Advances in Economic Theory: Fifth World Congress*, ed. Truman F. Bewley, 1–32. Cambridge, UK: Cambridge University Press.

Milgrom, Paul R. 2004. *Putting Auction Theory to Work*. Cambridge, UK: Cambridge University Press.

Noussair, Charles. 1995. "Equilibria in a Multi-Object Uniform Price Sealed Bid Auction with Multi-Unit Demands." *Economic Theory*, 5(2): 337–51.

Pagnozzi, Marco. 2007. "Bidding to Lose? Auctions with Resale." *RAND Journal of Economics*, 38(4): 1090–112.

Pagnozzi, Marco. 2009. "Resale and Bundling in Auctions." *International Journal of Industrial Organization*, 27(6): 667–78.
Weber, Robert J. 1997. "Making More from Less: Strategic Demand Reduction in the FCC Spectrum Auctions." *Journal of Economics and Management Strategy*, 6(3): 529–48.
Wilson, Robert. 1979. "Auctions of Shares." *Quarterly Journal of Economics*, 93(4): 675–89.
Wolak, Frank A. 2003. "Measuring Unilateral Market Power in Wholesale Electricity Markets: The California Market, 1998–2000." *American Economic Review*, 93(2): 425–30.
Wolfram, Catherine D. 1998. "Strategic Bidding in a Multiunit Auction: An Empirical Analysis of Bids to Supply Electricity in England and Wales." *RAND Journal of Economics*, 29(4): 703–25.
Zheng, Charles Zhoucheng. 2002. "Optimal Auction with Resale." *Econometrica*, 70(6): 2197–224.