Entry by takeover: Auctions vs. bilateral negotiations

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Abstract

Firms often enter new markets by taking over an incumbent. We analyze a potential entrant’s choice of target under two (exogenously given) takeover mechanisms: (i) auctions, where other incumbents can bid for the target against the entrant, and (ii) bilateral negotiations between the entrant and the target. The entrant’s choice of target depends on the mechanism, and it may not maximize its ex-post profit (nor consumer welfare). In an auction, the entrant pays a higher price to take over a target with higher synergies, because these impose stronger negative externalities on incumbents and increase their willingness to pay for preventing entry. Auctions increase the price obtained by the target, but reduce welfare compared to negotiations because they may discourage the entrant from acquiring a target with higher synergies.

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“...The aim is to buy good businesses at fair prices rather than fair businesses at good prices”

[–Warren Buffett]
of the takeover deal and whose terms cannot be observed by outsiders, while auctions are a form of open, multilateral negotiations. Indeed, as argued by Che and Lewis (2007), an auction is a reasonable depiction of the sequential negotiation/bidding arising among multiple rival buyers.

We assume that the takeover mechanism is exogenously fixed. For example, auctions may be unfeasible in the presence of high bidding costs for incumbents, or in regulated industries where incumbents are not allowed to merge. Alternatively, if relationship-specific investments are necessary for the takeover to be profitable, the target may have to enter an exclusive-dealing arrangement with the entrant in order to induce it to submit a serious takeover offer (e.g., by using break-up fees or stock lockups; see Che and Lewis, 2007). Moreover, potential buyers often have an incentive to commit not to participate in auctions — as Warren Buffett famously states in his annual report: “We don’t want to waste our time ... We don’t participate in auctions.” By contrast, an auction may be the only possible mechanism when a takeover target is legally required to solicit offers from all potentially interested acquirers. Both mechanisms are commonly used for takeovers in the real world: Boone and Mulherin (2007) show that, in a sample of 400 takeovers of major U.S. corporations in the 1990s, half of the targets were auctioned among multiple bidders, while the remaining negotiations were with a single buyer (see also Andrade et al., 2001).

We identify three factors that affect the entrant’s choice of which incumbent to acquire: (i) the incumbents’ market shares before entry, (ii) the level of synergies that the entrant can realize with the incumbents, and (iii) the price that the entrant has to pay to acquire an incumbent. While the first two factors depend solely on the primitives of the model (the number of incumbents, their marginal costs, and the level of synergies), the third one depends on the specific takeover mechanism. Hence, the choice of which incumbent to acquire depends on the takeaway mechanism.

With bargaining, the choice of a takeover target is determined by a trade-off between efficiency gains (that depend on the synergies) and the incumbents’ market shares (that depend on their costs), which determine their reservation values — i.e., the minimum prices that the entrant has to pay to acquire the incumbent. On the one hand, if all incumbents had the same market share, the entrant would always take over the one with the highest synergies. On the other hand, if it experienced the same synergies with all incumbents, the entrant would always take over the one with the larger market share. With asymmetric incumbents and target-specific synergies, the choice of the incumbent to acquire depends on which of these two effects dominate.

In an auction, instead, the entrant may have to pay a price higher than the target’s reservation value in order to outbid other incumbents. As a takeover results in a new firm producing at a lower marginal cost, entry imposes a negative externality on other incumbents and reduces their profits. Hence, other incumbents are willing to bid more than the target’s intrinsic value in order to prevent entry. This provides a justification for “takeover premia” that raiders pay for targets: in the presence of negative externalities, with an auction the takeover target may be paid a price higher than its intrinsic value.

Our main result is that an auction may induce the entrant to select a less efficient target, resulting in a takeover that generates a lower consumer surplus. The reason is that takeovers that generate higher consumer surplus (by creating a more efficient firm ex-post) also generate stronger negative externalities on other incumbents. Hence, other incumbents are willing to bid more aggressively to prevent entry, so that these takeovers are especially costly for the entrant with auctions. By contrast, when the takeover takes place through a bilateral negotiation between the entrant and the targeted incumbent, the negative externalities that the takeover imposes on other incumbents do not affect the price that the entrant has to pay. Indeed, we show that the entrant may select the target that maximizes consumer surplus with bargaining and a different target in an auction, but not vice versa. Hence, takeovers by bargaining always result in a (weakly) higher consumer surplus than takeovers by auctions.

Furthermore, a novel empirical prediction of our model is that compared to bargaining, in a takeover by auction the entrant is more likely to select a target that has a higher initial market share but with which it realizes smaller synergies. Hence, a more competitive takeover mechanism is likely to skew the target selection towards “bigger” firms.

In addition, since entry increases welfare because of synergies, takeovers by auctions also generate inefficiencies if they allow incumbents to outbid the entrant and acquire the target. In this case, it reduces consumer surplus by preventing entry and increases market concentration by reducing the number of firms. We show that, in a takeover by auction, incumbents can prevent entry if they bid jointly against the entrant or if they have a direct incentive to merge even without an entry threat. By contrast, entry always takes place with bargaining.

So the takeover mechanism affects the entrant’s choice of target and consumer surplus. Which takeover mechanism is likely to prevail in the real world? Since the entrant always prefers bilateral negotiations, these are likely to be used when the entrant is able to impose the takeover mechanism. By contrast, a target always prefers auctions, because they result in a higher takeover price so that auctions are likely to be used when a target is in a strong bargaining position compared to the entrant. Similarly, other incumbents also prefer auctions, because auctions may allow them to prevent entry. But our analysis suggests that a regulator should not necessarily prefer auctions.

Under Delaware law (one of the principal bodies of corporate law in the US), when a potential buyer makes a serious bid for a target, the target’s board of directors is required to act as would “auctioneers charged with getting the best price for the stock-holders at a sale of the company” (Cramton, 1998). Indeed, auctions are not only advised (see, for example, Cramton and Schwartz, 1991; Bulow and Klemperer, 1996, 2009; Brusco et al., 2007), but also widely used in takeovers. However, we uncover a trade-off between maximizing the target shareholders’ surplus from the takeover, which is achieved through auctions, and maximizing consumers’ welfare in the market, which is achieved through bargaining. Therefore, forcing the acquisition market to be more competitive may result in inferior outcomes for incumbents.

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5 In the real world, there is a variety of possible takeover mechanisms that are influenced to different degrees by the externalities created by entry on other incumbents, depending on timing of negotiations, outside options, legal frameworks, lobbying abilities, etc. To show how the entrant’s choice of takeover mechanism may vary, we choose to focus on two extreme cases — a mechanism in which externalities have no effect on the takeover price and a mechanism in which the price fully reflects externalities because all parties affected by the takeover can directly influence the outcome.

6 For simplicity, in our model we assume that there are no synergies among incumbents but our qualitative results also hold when mergers among incumbents generate synergies. In particular, depending on the size of the synergies, incumbents may manage to prevent entry in a takeover by auctions even when consumer surplus would be higher with entry.

7 Since the takeover price is higher with auctions, entry may also be prevented in takeovers by auction but not with bargaining in the presence of a fixed entry cost that makes the takeover unprofitable only with auctions.

8 Notice however that while Bulow and Klemperer (1996) consider the mechanism preferred by the seller, our focus is on the mechanism that maximizes consumers’ surplus (which in our model turns out to be the same as the one preferred by the buyer). For an analysis of when a seller might prefer bilateral negotiations to auctions, see Herweg and Schmidt (2015).

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2 In an alternative interpretation, bilateral negotiations are friendly acquisitions, whereas auctions are hostile takeovers. Indeed, the target of a hostile takeover often solicits offers from additional bidders, who may have not been interested in the target without the takeover attempt, so that the price is bid up to a point well above the initial offer. By contrast, friendly takeovers tend to be consummated at lower prices.

3 In the recent acquisition of KPN’s German unit by Telefónica, KPN would have had to pay a €50 billion break-up fee had its shareholders rejected the deal.

4 Molnar (2002) shows that two firms competing in an auction to acquire a competitor may bid more than the competitor’s (intrinsic) value if synergies are large enough. In contrast to our paper, he considers an environment with a fixed takeover target and symmetric firms and synergies, and does not consider entry and bargaining.
consumers. Given that antitrust authorities in the U.S., the European Union, and many other jurisdictions apply a consumer-welfare standard when evaluating potential mergers and acquisitions, our analysis suggests that they may want to favor takeovers by bargaining.

With respect to merger control, our analysis suggests that an antitrust authority should never block a merger between a new entrant and an incumbent, since this does not affect the market structure in our model and only results in efficiency gains, regardless of the takeover mechanism. By contrast, an antitrust authority should block preemptive mergers among incumbents aimed at preventing the entry of a new firm into the market.

Our paper is related to the work by Jehiel and Moldovanu (2000), Das Varma (2002), Hoppe et al. (2006) and Hu et al. (2013) who analyze auctions with allocative externalities created by ex-post interaction among bidders. Several papers discuss how externalities may arise in takeover auctions. Specifically, Inderst and Wey (2004) show that a takeover is more likely to succeed under Bertrand (resp. Cournot) competition if goods are substitutes (resp. complements); Ding et al. (2013) compare cash and profit-share auctions with bidder-specific synergies. In contrast to ours, these papers only consider mergers among incumbents with an exogenous target and do not analyze bilateral negotiations.

Our paper is also related to the literature on endogenous mergers. Fridolfsson and Stenner (2005) show that, with negative externalities on outsiders, an unprofitable merger may occur to prevent the target from merging with a rival. Similarly, Qiu and Zhou (2007) find that merger waves may arise because firms which merge early free-ride on subsequent increases in the market price caused by future mergers. In an environment where a “pivotal” firm chooses to propose one among several mutually exclusive mergers, Nocke and Whinston (2013) show that, in order to maximize consumer surplus, an antitrust authority commits to imposing tougher standards on mergers involving firms with larger market shares.

Finally, there is a large empirical literature in corporate finance documenting that stockholders of target firms receive large takeover premia. Theoretical explanations of this empirical anomaly include Roll’s (1986) hubris hypothesis, Jensen’s (1986) theory of free cash flows, Shleifer and Vishny’s (1990) managerial entrenchment hypothesis, Shleifer and Vishny’s (2003) and Rhodes-Kropf and Viswanathan’s (2004) models of market misvaluation, Jovanovic and Bravinsky’s (2004) theory of learning about investment opportunities, and Malmendier and Tate’s (2008) theory of overconfident CEOs. By contrast, like in Molnar (2002), takeover premia in our model arise because of the negative externality that the takeover imposes on other firms in the market.

The rest of the paper is organized as follows. Section 2 presents the model. In Section 3 we consider the difference between the profit-maximizing target for an entrant and the efficient one. Sections 4 and 5 analyze takeovers by bargaining and auction, respectively. In Section 6 we show how the choice of a takeover target depends on the takeover mechanism. Section 7 considers various extensions of the main model and Section 8 concludes. All proofs are in Appendix A.

2. The model

2.1. Players and environment

Consider a market with \( n \geq 3 \) incumbent firms producing a homogeneous good and competing à la Cournot. The marginal cost of firm \( i \) is \( c_i \), \( i = 1, \ldots, n \). Fixed costs of production are equal to zero. We assume that firms \( 2, \ldots, n \) are symmetric and have the same marginal cost \( c_2 = \ldots = c_n \), while firm 1 is a dominant firm that produces at a lower marginal cost \( c_1 < c_2 \). This can be thought of as a market in which there is a dominant firm with a technological advantage and \( n - 1 \) smaller competitors with (approximately) equal market shares.

The inverse linear demand function is \( P(Q) = A - Q \) where \( Q \) is the total quantity produced in the market. Therefore, firm \( i \)’s initial equilibrium profit is

\[
\pi_i(c_1; \sum_{k\neq i} c_k) = \left( A - n c_i + \sum_{k\neq i} c_k \right)^2, \quad i = 1, \ldots, n, 
\]

where (slightly abusing notation) the subscript \( n \) indicates the number of firms active in the market. Notice that the function \( \pi_i(\cdot) \) is decreasing in the first argument and increasing in the second one.

There is a potential entrant \( E \) that can enter the market only by taking over an incumbent, for example because of legal or technological reasons (e.g., \( E \) lacks a necessary input for production or there is a fixed number of licenses to operate in the market, as in the telecommunications industry), or because of high fixed costs to enter the market as a new competitor. Without loss of generality, we assume that \( E \) can choose to take over either firm 1 or firm 2 (since all other incumbents are identical to firm 2). There are firm-specific synergies: if \( E \) takes over firm 1, the resulting firm has marginal cost \( c_1 - s_i, i = 1, 2 \), where \( s_1 \in [0, c_1] \) represents the strength of the synergy between the entrant and firm 1. Marginal costs and synergies are common knowledge among players.

Let

\[
\Phi_i = A - n c_i + \sum_{k\neq i} c_k.
\]

To ensure that all firms produce positive quantities in equilibrium, we assume that \( \Phi_i > s_j \) for all \( i, j \).

2.2. Takeover

We consider two different (exogenously fixed) takeover mechanisms:

- **Bargaining**: \( E \) makes a take-it-or-leave-it offer to its chosen target.
- **Auction**: \( E \) competes against the other incumbents in an ascending auction for its chosen target.

When the takeover takes place through an auction, once \( E \) selects a takeover target, the other incumbents can react and compete to acquire it. Hence, an incumbent can prevent \( E \)’s takeover by merging with the target itself. By contrast, when the takeover takes place through bargaining, other incumbents cannot prevent the takeover — e.g., because they are not allowed to acquire the target for antitrust reasons (since a merger between incumbents reduces the number of competitors in the market) or because of the presence of an exclusive dealing arrangement between the entrant and the target.

In our main model, we assume that \( E \) has full bargaining power in a takeover by bargaining. In Section 7.1, we show that all our results also hold with a more general bargaining mechanism in which the takeover

\footnote{The model could be easily extended to accommodate for more than two levels of marginal cost; however, this would increase the number of cases to consider, complicating the analysis without providing additional insight.}

\footnote{We consider a linear demand function for simplicity, in order to obtain closed-form solutions. In Appendix A we show that most of our qualitative results continue to hold under alternative demand specifications.}

\footnote{Our qualitative results also hold in an environment where the entrant is privately informed about synergies at the takeover stage, and synergies with different incumbents are drawn from different distributions (see footnote 31).}

\footnote{This implies that, with bargaining, the target’s outside option does not depend on other incumbents’ willingness to pay to acquire the target. This is natural because, as we are going to show, other incumbents have no incentive to acquire the target after its negotiations with the entrant break down. See also Section 7.2.}

\footnote{For other analyses of endogenous merger waves, see also Gowrisrinathan (1999), Faule-Oller (2000) and Nocke and Whinston (2010). These papers endogenize merger decisions in a dynamic framework, but they do not address the question of “with whom” to merge.}
target obtains some positive share of the gains from trade generated by entry (even, in the limit, as the target is arbitrarily close to having full bargaining power), and in Section 7.2 we consider sequential bargaining.

Takeover contests are typically modeled as ascending auctions because companies are unable to commit to sealed-bid auctions and to not accepting higher offers after the end of the auction; see, for instance, Bulow et al. (1999), Bulow and Klemperer (1996, 2009), and McAdams and Schwartz (2007).14 However, our all bids also hold with sealed-bid second-price auctions, and with sealed-bid first-price auctions (see the proof of Lemma 1).

Following most of the literature, we restrict attention to equilibria in pure strategies. To characterize the price paid by E, we assume that there is an arbitrarily small probability ε that E bids up to a random price that is lower than its valuation.15 Hence, for any of their bids, incumbents assign a positive probability to the entrant dropping out at a lower price. This simply eliminates trivial equilibria (in which all incumbents drop out at prices at which they would be happy to win against E) and ensures that at least one incumbent has an incentive to participate in an auction against the entrant and bid up to its willingness to pay for preventing the takeover (otherwise, in our model with complete information, an incumbent would be indifferent between participating or not, and between any possible bid, when E has a valuation higher than the incumbent’s willingness to pay). This is equivalent to eliminating dominated strategies when only one incumbent and E are left in the auction. We let ε → 0 and neglect it in the description of the players’ profits.

2.3. Timing

The timing of the game is as follows:

• **Period 1.** E selects the takeover target.
• **Period 2.** The takeover occurs either by auction or by bidding.
• **Period 3.** Market competition among the remaining firms.

For simplicity, we assume that if E fails to take over its chosen target, then it cannot take over another incumbent (for example, because there are high fixed costs associated with each takeover attempt that make it unprofitable for the entrant to make more than one attempt).

2.4. Period 3’s profits

If there is no takeover, in period 3 firm i continues to earn its current profit. Therefore, if firm i is selected by E as the takeover target in period 1, its reservation value is

\[ \pi_i = \pi(n - 1) \min(c_i, c_j) \sum_k c_k \]

both with bargaining and in an auction. If E takes over firm i, then in period 3 its profit is \( \pi(n - 1) (c_i - c_j) \sum_k c_k \) and the profit of an incumbent that is not taken over is \( \pi(n - 1) (c_i - c_j) \sum_k c_k \) if \( j \neq i \).

If two incumbents merge, we assume that the resulting firm’s marginal cost is the minimum of the two incumbents’ initial marginal costs (see Fauli-Oller, 2000; Qiu and Zhou, 2007, and Stennek, 2003).16 Hence, if firm i and firm j merge, their joint profit in period 3 is

\[ \pi(n - 1)(\min(c_i, c_j)) \sum_k c_k \]

This condition is satisfied either if firm i’s synergy is sufficiently larger than firm j’s synergy, or if firm i’s marginal cost is sufficiently smaller than firm j’s marginal cost, or both. In this case, we say that firm i is the (gross) profit-maximizing target. Notice that, in order for firm 2 to be the profit-maximizing target, the difference between the two firms’ synergies must exceed the difference between their costs. Moreover, firm 1 may be the profit-maximizing target even if E would be able to produce at a lower ex-post marginal cost by taking over firm 2 (while the converse is impossible) – i.e., E may prefer to produce at a higher marginal cost but face less efficient competitors.20

If E takes over firm i, total output in the market is

\[ \frac{n}{n-1} (\sum c_k + s_i) \]

Therefore, in order to maximize consumer surplus, E should take over the firm with the strongest synergies – i.e., with the highest \( s_i \). We define this firm as the efficient target. Although the efficient target is not necessarily the one that maximizes total welfare (Farrell and Shapiro, 1990), the “consumer-welfare standard” adopted by competition authorities requires that, when evaluating potential mergers and acquisitions, efficiency gains are only taken into account to the extent that they are passed on to consumers as lower prices.

14 In an ascending auction the price raises continuously and bidders who wish to be active at the current price keep a button pressed. When a bidder releases the button, he is withdrawn from the auction. The auction ends when only one active bidder is left.

15 See Jehiel and Moldovanu (2000) for a discussion of the problems arising when constructing equilibria in auctions with negative externalities.

16 Even without synergies, horizontal mergers may increase social welfare through production rationalization – e.g., if the resulting firm relocates production from less efficient plants to more efficient ones. None of our qualitative results hinge on the assumption that a merger between incumbents does not create synergies.
In this section, we analyze \( E \)'s choice of target when the takeover takes place through bargaining and \( E \) makes a take-it-or-leave-it offer to the selected target. In this case, the price that \( E \) has to pay in order to acquire incumbent \( i \) is equal to the reservation value \( r_i \) and, by taking over firm \( i \), \( E \) obtains surplus
\[
\pi_i \left( c_i - s_i; \sum_{k \neq i} c_k \right) - \pi_i \left( c_i; \sum_{k \neq i} c_k \right).
\]

Since this is strictly positive whenever \( s_i > 0 \), \( E \) has an incentive to take over any of the incumbents with bargaining and, hence, it always enters the market.

Of course, \( E \)'s preferred target is the one that allows it to obtain the highest surplus. This choice is determined by a trade-off between efficiency gains (that depend on the synergies) and the incumbents' pre-merger market shares, which determine their reservation values and depend on the difference between the incumbents' initial costs.

**Proposition 1.** When the takeover takes place through bargaining, \( E \) takes over firm 1 if
\[
s_1^2 - s_2^2 > \frac{2}{n} (s_2 - s_1) - \frac{2}{n} (s_2 - s_1) t_{maximizing} \quad (4.1)
\]
and firm 2 otherwise.

Condition (4.1) requires \( s_2 \) to be sufficiently low.\(^{24}\) If all incumbents had the same market share \((q_i = q_2)\), the entrant would take over the one with the highest synergies. On the other hand, if it experienced the same synergies with all incumbents \((s_1 = s_2)\), the entrant would take over the one with the larger pre-merger market share. With asymmetric incumbents and target-specific synergies, the choice of target depends on which of these two effects dominate.

If \( s_1 > s_2 \), then condition (4.1) holds, so that \( E \) takes over firm 1 when it is both the profit-maximizing and the efficient target; if \( s_1 \leq s_2 \leq \frac{s_1}{c_2-c_1} \), then condition (4.1) does not hold (see the proof of Corollary 1), so that \( E \) takes over firm 2 when it is both the profit-maximizing and the efficient target. Hence, we have the following result.

**Corollary 1.** In a takeover by bargaining, if a firm is both the profit-maximizing and the efficient target, then \( E \) always takes over that firm.

The intuition for the above result is straightforward. If firm 1 is both the profit-maximizing and the efficient target, it provides \( E \) with the larger synergies as well as the larger market share. Both these effects induce \( E \) to acquire firm 1. If firm 2 is both the profit-maximizing and the efficient target, it provides \( E \) with extremely large synergies, even if it has a lower pre-merger market share than firm 1. Hence, the efficiency-gain effect dominates and induces \( E \) to acquire firm 2. By contrast, when \( s_2 > s_1 > \frac{s_1}{c_2-c_1} \) so that firm 1 is the profit-maximizing target while firm 2 is the efficient one, \( E \) prefers to take over the profit-maximizing (resp. efficient) firm 1 (resp. 2) if condition (4.1) holds (resp. fails); that is, \( E \) takes over the more efficient firm if and only if the efficiency-gain effect dominates the market-share effect.

**Fig. 4.1.** Displays condition (4.1), represented by the dashed curve: \( E \) takes over firm 1 (resp. 2) if \( s_2 \) is below (resp. above) the dashed curve.\(^{25}\) Therefore, for values of \( s_1 \) and \( s_2 \) between the top line and the dashed curve, \( E \) takes over firm 2 that is the efficient but not the profit-maximizing target; for values of \( s_1 \) and \( s_2 \) between the dashed

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\(^{21}\) In fact, if firm 1 has a lower marginal cost and is the efficient target, then it is also the profit-maximizing one: \( c_1 < c_i \) and \( s_1 > s_i \) imply (3.1). It is also easy to verify that in this case firm 1 is the target with the lowest ex-post marginal cost.

\(^{22}\) When \( s_j > s_i \) and (3.1) holds, firm \( j \) is the efficient target while firm \( i \) the profit-maximizing one. Notice that it is not possible that firm 1 is the efficient target but firm 2 is the profit-maximizing one.

\(^{23}\) In fact, if firm \( j \) has a higher marginal cost but is the profitable target, then it is also the efficient one: \( c_j < c_i \) and \( m(s_j - s_i) > (n+1)(s_i - c_i) \) imply that \( s_j > s_i \). It is also easy to verify that in this case firm 2 is the target with the lowest ex-post marginal cost.

\(^{24}\) Specifically, condition (4.1) requires that \( s_j < \frac{1}{2} \sqrt{n^2 s_j^2 + 2 n s_j + \Phi_2 - \Phi_1} \).

\(^{25}\) As in Fig. 3.1, the top line represents condition (3.1). It is straightforward to show that the dashed curve lies between this line and the 45-degree line.
curve and the 45-degree line, \( E \) takes over firm 1 that is the profit-maximizing but not the efficient target.\(^{26}\)

Notice that, as \( c_2 = c_1 \), the dashed curve moves towards the 45-degree line, and if \( c_2 = c_1 \), the dashed curve coincides with the 45-degree line so that \( E \) takes over firm i if and only if \( s_i > s_p \). Therefore, the smaller the asymmetry between incumbents, the less likely it is that the entrant does not take over the efficient firm. On the other hand, as the number of symmetric firms increases, it can be shown that the dashed curve becomes steeper while the top line shifts downward, so that the area between the two shrinks. Therefore, in less concentrated markets, it is less likely that the profit-maximizing and efficient targets differ; however, if they do differ, then it is more likely that the entrant does not take over the efficient target.

5. Takeover by auction

In this section, we analyze \( E \)'s choice of target when the takeover takes place through an ascending auction. Hence, other incumbents can bid for the target against \( E \) and, if they are successful, prevent \( E \)'s entry into the market.

In an auction for firm \( i, i = 1, 2 \), firm \( j \)'s willingness to pay for blocking \( E \)'s takeover and merging with firm \( i \) is

\[ v_j = \pi_{n-1} \left( \min \{ c_i, c_j \} \sum_{k \neq i, j} c_k - \pi_n \left( c_j \sum_{k \neq i} c_k - s_i \right) \right). \]

This is increasing in firm \( j \)'s profit if it merges with firm \( i \), and decreasing in firm \( j \)'s profit if firm \( i \) is taken over by \( E \). Hence, firm \( j \)'s willingness to pay depends on two effects:

1. The increase in firm \( j \)'s profit with respect to its current profit if it merges with firm \( i \) i.e., \( \pi_{n-1} (\min \{ c_i, c_j \} \sum_{k \neq i, j} c_k - \pi_n (c_j \sum_{k \neq i} c_k - s_i) \).

2. The reduction in firm \( j \)'s profit with respect to its current profit if \( E \) takes over firm \( j \) i.e., \( \pi_n (c_j \sum_{k \neq j} c_k - s_j) - \pi_n (c_j \sum_{k \neq j} c_k - s_i) \).

The second effect is a negative externality created by \( E \)'s takeover of firm \( i \); following the takeover, firm \( j \) faces a more efficient competitor in period 3 and earns a lower profit. This externality is increasing in \( s_i \). The larger is the externality, the higher is the price that firm \( j \) is willing to pay to prevent the takeover.

In order to acquire an incumbent, \( E \) has to pay the highest between the other incumbents’ bids and the reservation value. Because of the presence of externalities, however, an incumbent's bid in an ascending auction is not necessarily equal to its willingness to pay for blocking the takeover. In fact, an incumbent may prefer the other incumbents to win the auction and merge with the target, rather than winning itself.

The next lemma characterizes the highest bid by an incumbent depending on the takeover target, and compares it with the target's reservation value.

**Lemma 1.** In order to acquire firm 2 in an auction, \( E \) pays a price equal to \( \max\{v_2, r^2\} \). In order to acquire firm 1 in an auction, \( E \) pays a price equal to \( \max\{v_1, r^1\} \).

In the proof of *Lemma 1*, we show that \( v_2 > v_j \), for \( j \neq 1 \), and that the highest bid by an incumbent in an auction for firm 2 is \( v_2 \), which is the bid by the dominant firm 1. The reason is that, since only firm 1 obtains a positive profit by winning against \( E \) at a price higher than \( v_2 \), it is a weakly dominant strategy for firm 1 to bid up to \( v_2 \) once the auction price reaches \( v_2 \) and \( E \) is the only other bidder active in the auction. For example, it is an equilibrium for firm 1 to bid \( v_2 \) and for all other incumbents to bid 0.

In an auction for firm 1, all other incumbents have the same willingness to pay to block \( E \)'s takeover (because they have the same marginal cost) i.e., \( v_2 \). In the proof of *Lemma 1*, we show that in any equilibrium of the auction the highest bid by an incumbent is equal to \( v_2 \). For example, it is an equilibrium for one incumbent to bid \( v_2 \) and for all other incumbents to bid 0.

Firm \( j \)'s willingness to pay for firm \( i \), \( v_j \), is higher than the reservation value, \( r \), if and only if

\[ \pi_{n-1} \left( \min \{ c_i, c_j \} \sum_{k \neq i, j} c_k - \pi_n (c_j \sum_{k \neq i} c_k - s_i) \right) > \pi_n (c_j \sum_{k \neq j} c_k - s_j) - \pi_n (c_j \sum_{k \neq j} c_k - s_i). \]

When this condition is not satisfied, the reservation value of firm \( i \) is binding and the price that \( E \) has to pay to take it over in an auction is equal to the price with bargaining. It is easy to verify that \( r^1 > v_2 \) i.e., firm 1's reservation value is binding if and only if

\[ s_1 < \Phi_2 - \frac{1}{n} \sqrt{\Phi_2 (\Phi_2 + 2n\Phi_1)} \neq s_1, \]

and that \( r^2 > v_2 \) if and only if

\[ s_2 < \Phi_1 - \frac{1}{n} \sqrt{n^2 (\Phi_1^2 - \Phi_2^2) + 2n\Phi_1 \Phi_2 + \Phi_2^2} \neq s_2. \]

Intuitively, the reservation value of firm \( i \) is binding if \( s_i \) is sufficiently low, because the incumbents' willingness to pay is increasing in the negative externality of the takeover and, hence, in the level of \( E \)'s synergies with firm \( i \). Moreover, since firm 1's pre-merger profit is higher than firm 2's, the threshold on synergies for firm 1's reservation value to bind is higher than for firm 2's (see condition (5.1)) — that is, \( s_1 > s_2 \).

Of course, \( E \)'s preferred target is the one that allows it to obtain the highest net surplus. The next proposition characterizes \( E \)'s choice of which incumbent to target.

**Proposition 2.** If the takeover takes place through an auction:

(i) When both firms’ reservation values are binding, \( E \) takes over firm 1 if condition (4.1) holds, and firm 2 otherwise.
(ii) When only firm 1’s reservation value is binding, $E$ takes over firm 1 if
\[ s_1^* - s_2^* > \frac{2}{n} (s_2 - s_1) + \frac{s_2}{n} (s_2 - 2\phi_1) - \frac{1}{n^2} (\phi_2 + n(2\phi_1 - n\phi_2)), \] and firm 2 otherwise.

(iii) When no firm’s reservation value is binding, $E$ takes over firm 1 if
\[ s_1^* - s_2^* > 2 \left( \frac{n^2 s_2 + s_1}{n^2 + 1} \right) \phi_2 - 2 \left( \frac{n^2 s_1 + s_2}{n^2 + 1} \right) \phi_1, \] and firm 2 otherwise.

If both reservation values bind, the choice of the target is the same as with bargaining. When at least one reservation value does not bind, the entrant takes over firm 2 if $s_2$ is sufficiently high. Therefore, although firm 1 has a lower marginal cost than firm 2, the entrant takes over firm 2 when its synergies are sufficiently higher than firm 1’s.

When an incumbent is both the profit-maximizing and the efficient target, we have the following result.

**Corollary 2.** In a takeover by auction, if a firm is both the profit-maximizing and the efficient target, then $E$ always takes over that firm.

By Corollaries 1 and 2, when a firm is both the profit-maximizing and the efficient target, the takeover mechanism is irrelevant for $E$’s choice of target. The next section, however, shows that this is not the case when the profit-maximizing target differs from the efficient one.

Fig. 5.1 displays conditions (4.1), (5.2) and (5.3), represented by the dotted curve: $E$ takes over firm 1 (resp. 2) if $s_2$ is below (resp. above) the dotted curve.\(^{27}\) Therefore, for values of $s_1$ and $s_2$ between the top line and the dotted curve, $E$ takes over firm 2 that is the efficient but not the profit-maximizing target; for values of $s_1$ and $s_2$ between the dotted curve and the 45-degree line, $E$ takes over firm 1 that is the profit-maximizing but not the efficient target.

As $c_2 < c_1$, the dotted curve moves towards the 45-degree line, and if $c_2 = c_1$, the dotted curve coincides with the 45-degree line so that $E$ takes over firm 1 if and only if $s_1 > s_2$. Therefore, the smaller the asymmetry between incumbents, the less likely it is that the entrant does not take over the efficient firm.

Notice that, in a takeover auction, the entrant’s willingness to pay for the target is always higher than the other incumbents’ one, so that if (almost) always $E$ outbids them.\(^{28}\) Hence, as with bargaining, the entrant always acquires its preferred target in a takeover by auction. This happens in our model because of the presence of complete information and because of the assumption that a merger between incumbents is not profitable ex-ante (see Section 7.4). Of course, in a richer model with incomplete information, an incumbent may outbid the entrant in an auction for the target.\(^{29}\)

6. Auction vs. bargaining

In this section, we compare $E$’s choice of target in a takeover by auction, when incumbents can react and acquire the target, with its choice in a takeover by bargaining. We show that these choices may differ (Proposition 3) and that takeovers by auction yield a (weakly) lower consumer surplus than takeovers by bargaining (Corollary 3 of Proposition 4).

The next result shows that the entrant may choose a different target depending on the takeover mechanism.

**Proposition 3.** There exist values of $s_1$ and $s_2$ such that $E$ takes over firm 1 in an auction and firm 2 with bargaining. However, it can never happen that $E$ takes over firm 2 in an auction and firm 1 with bargaining.

Hence, $E$ may prefer to take over the incumbent with the highest marginal cost if it can make a take-it-or-leave-it offer, without the other incumbents reacting, and the incumbent with the lowest marginal cost if it has to compete against other incumbents, but not vice versa. In other words, when the choice of a takeover target differs in the two mechanisms, the entrant always chooses firm 1, the incumbent with the lowest marginal cost, in an auction.

The conditions for the choice of takeover target to differ in the two mechanisms require that $s_1$ is higher than $s_2$, but not too much so.\(^{30}\) Indeed, $s_2$ has to be relatively high for the entrant to take over firm 2 with bargaining, but not too high for the entrant to take over firm 1, rather than firm 2, in an auction.

Recall that, when a firm is the efficient and the profit-maximizing target, $E$ takes it over both in an auction and with bargaining (Corollaries 1 and 2). The following proposition shows that $E$ may choose a different target depending on the takeover mechanism when the profit-maximizing and efficient targets differ.

**Proposition 4.** When firm 2 is the efficient target but firm 1 the profit-maximizing target, $E$ may take over the profit-maximizing target in an auction and the efficient target with bargaining, but $E$ never takes over the efficient target in an auction and the profit-maximizing target with bargaining.

The intuition for this result is that if incumbents are allowed to react to a takeover attempt by an entrant, their willingness to pay to block the takeover is increasing in the production efficiency of the firm resulting from the takeover. And the higher is the incumbents’ willingness to pay, the more likely the entrant is to prefer a different target. Hence,

\(^{27}\) As in Fig. 3.1, the top line represents condition (3.1). It is straightforward to show that the dotted curve lies between this line and the 45-degree line.

\(^{28}\) Recall that with an exogenous and arbitrarily small probability $E$ drops out of the auction at a price lower than its valuation, so that incumbents still have an incentive to participate and bid up to a price at which they are happy to win and prevent $E$ from entering the market.

\(^{29}\) For example, an incumbent may win a takeover auction if there is uncertainty about the level of synergies, and only the entrant learns its synergy with an incumbent once it has committed to bid for it.

\(^{30}\) In the proof of Proposition 3, we show that $E$ takes over firm 1 in an auction and firm 2 with bargaining if and only if: either (i) $s_1 > s_2$ and both conditions (5.3) and (4.1) hold; or (ii) $s_1 \leq s_2$ and both conditions (5.2) and (4.1) hold.
in a takeover by auction the entrant is less likely to select a target with whom it has higher synergies than in a takeover by bargaining.31

By Proposition 4, when the entrant does not take over the efficient target with bargaining, it does not take over the efficient target in an auction either. However, when the entrant does not take over the efficient target in an auction, it may do so with bargaining. Hence, we have the following result.

**Corollary 3.** Takeovers by auctions always result in a (weakly) lower consumer surplus compared to takeovers by bargaining.

**Fig. 6.1** displays how E’s choice of target depends on the takeovers mechanism. If a firm is both the profit-maximizing and the efficient target — that is, for values of \( s_1 \) and \( s_2 \) below the 45-degree line or above the top line — the actual takeover mechanism is irrelevant, because \( E \) always takes over this target with both mechanisms. However, for values of \( s_1 \) and \( s_2 \) between the top line and the 45-degree line (so that firm 2 is the efficient target but firm 1 is the profit-maximizing one), \( E \) takes over the efficient target in an auction above the dotted curve, whereas \( E \) takes over the efficient target with bargaining above the dashed curve. Therefore, \( E \) takes over a different firm depending on the takeover mechanism when \( s_1 \) and \( s_2 \) lie between the dotted and dashed curves, and in this case it takes over the efficient target with bargaining and the profit-maximizing target in an auction.

Our analysis suggests that an antitrust authority that can control takeover mechanisms and aims to maximize consumer surplus should favor bargaining mechanisms, in which incumbents cannot bid for the target against the entrant, when the takeover generates efficiency gains. Of course, \( E \) always prefers takeovers by bargaining as they allow it to acquire a target at its reservation value, while in an auction the target price is weakly higher than its reservation value. For the same reason, potential targets, conditional on being acquired, always prefer takeovers by auctions. In the next section we show that also non-targeted incumbents prefer takeovers by auctions since they may allow them to prevent entry.

**7. Extensions**

In our model, the negative externality that entry imposes on incumbents induces them to bid aggressively in an auction for a takeover target, and this may force the entrant to choose a different target than the one that it would choose with bargaining, when the entrant makes a take-it-or-leave-it offer for the target. One may wonder whether this result depends on the specific bargaining mechanism that we have considered. We now show that our results also hold with different bargaining mechanisms: Section 7.1 considers a more general Nash bargaining mechanism and Section 7.2 considers sequential bargaining.

Moreover, we also highlight other sources of inefficiency of takeovers by auctions. Since an incumbent may win a takeover auction and acquire a competitor, takeovers by auction may reduce welfare by preventing entry in the market and increasing market concentration. We discuss two cases in which this may happen. First, even though a single incumbent may be unable to outbid the entrant in an auction, incumbents may profitably collude to block entry by jointly bidding more than the entrant’s willingness to pay for the target. Hence, the threat of entry may induce incumbents to merge, even when incumbents have no incentive to merge in the absence of a potential entrant. Second, an incumbent may outbid the entrant in “small markets,” where incumbents have incentive to merge even without the threat of entry. These sources of inefficiency never arise with bargaining, when entry always occurs because the entrant obtains a positive surplus from entering the market and other incumbents cannot block the takeover.

**7.1. Bargaining weights**

Assume that, in a takeover by bargaining, the entrant has bargaining power \( (1−β) \) and the target has bargaining power \( β \), where \( β \in [0, 1] \). The outcome of bargaining between the entrant and the target is given by the Generalized Nash Bargaining Solution, where the disagreement point is represented by players’ current profits.32

Therefore, the entrant obtains a share \( (1−β) \) of the gains from trade if it takes over an incumbent, while the incumbent obtains a share \( β \). If the entrant makes a take-it-or-leave-it offer as in our main model, \( β = 0 \). In a random proposer take-it-or-leave-it offer game, each player is chosen to make a take-it-or-leave-it offer with equal probability, \( β = \frac{1}{2} \). When \( β = 1 \), the entrant obtains no surplus from taking over an incumbent and, hence, it is indifferent among all potential targets.

In a takeover by bargaining of firm i: firm i’s disagreement point is \( π_i(c_i; \sum_{k=1}^n c_k) \); E’s disagreement point is 0; the gains from trade are \( π_i(c_i; \sum_{k=1}^n c_k) − π_i(c_i; \sum_{k=1}^n c_k) \). Hence, if it takes over firm i, E pays a price equal to

\[
π_i(c_i; \sum_{k=1}^n c_k) + β \left[ π_i(c_i; \sum_{k=1}^n c_k) − π_i(c_i; \sum_{k=1}^n c_k) \right],
\]

and obtains surplus

\[
(1−β) \left[ π_i(c_i; \sum_{k=1}^n c_k) − π_i(c_i; \sum_{k=1}^n c_k) \right].
\]

By inspection, as in our main model, E takes over firm 1 rather than firm 2 with bargaining if and only if condition (4.1) is satisfied — i.e., the

---

31 If the entrant is privately informed about synergies at the takeover stage, and synergies with different incumbents are drawn from different distributions, an incumbent’s willingness to pay to prevent entry is based on expected synergies. In this environment, taking over firm 2 in an auction is even more costly since this choice signals relatively higher synergies with the target and, hence, increases other incumbents’ willingness to pay by a larger amount. Therefore, we expect the entrant to be even more likely to take over firm 1 in an auction and firm 2 with bargaining.

32 In a strategic model of alternating offers, players’ current profits can be interpreted as their “impasse points” or “inside options” in case bargaining continues forever without agreement being reached or the negotiation being abandoned — see, e.g., Binmore et al. (1986, 1989).
dashed curve in Fig. 4.1 that determines \( E \)’s target choice with bargaining remains unchanged.

Therefore, all our results from Sections 4 and 6 continue to hold. Basically, bargaining weights do affect players’ profits in negotiations, but they do not affect \( E \)’s choice of target.

7.2. Sequential bargaining

In our main model we assumed that, while bargaining with the entrant, the outside option of the target is equal to its current profits — i.e., the profits that the target would obtain without any entry or merger among incumbents. Alternatively, the target could have the option to seek an agreement to merge with another incumbent, in case negotiations with the entrant break down. One may think that, in this case, the entrant should pay a higher price with bargaining than in our model, because the outside option of the target should be higher.

However, we now argue that this not the case. The reason is that, if negotiations with the entrant break down the other incumbents have no incentive to acquire the target anymore and, hence, the target’s outside option when negotiating with the entrant is still equal to its current profit.

Specifically, consider a sequential bargaining model in which the entrant bargains with the chosen target first and then, if negotiations break down, the chosen target can bargain with any other incumbent of its choice in order to reach an agreement to merge. By backward induction, if bargaining with the entrant fails, no other incumbent will be willing to merge with the target, and the target will have no reason to merge with the other incumbents anyway. In other words, a target’s threat to merge with another incumbent in case negotiations with the entrant fail would not be credible. Hence, the possibility of bargaining with other incumbents after the entrant does not affect the target’s outside option and the outcome of this form of sequential bargaining is the same as in our model.  

7.3. Collusion among incumbents

In this section we analyze the possibility that, in a takeover by auction, non-targeted incumbents form a bidding ring and jointly bid against the entrant. We show that the ring’s willingness to pay to block the takeover may be higher than the entrant’s willingness to pay for the target. In this case, incumbents jointly manage to block entry, even though no single incumbent would be able to do so on its own.

If the takeover of firm 1 is successful, the total profits of non-targeted incumbents are

\[
(n-1)n_1 \left( c_2 + \sum_{k=2}^{N} c_k - s_1 \right).
\]

If instead one of the incumbents wins the auction for firm 1, so that \( E \)’s entry is blocked, the total industry profits are

\[
\pi_{n-1} \left( c_1 + \sum_{k=1}^{N} c_k \right) + \left( (n-2)n_{n-1} \right) \left( c_2 + \sum_{k=2}^{N} c_k \right).
\]

Hence, the total willingness to pay of all non-targeted incumbents to block the takeover of firm 1 is equal to the difference between these two profits, and non-targeted incumbents can prevent entry if and only if this is higher than \( E \)’s willingness to pay for firm 1, which requires that \( s_1 \) is sufficiently low. Similarly, the total willingness to pay of non-targeted incumbents to block the takeover of firm 2 is higher than \( E \)’s willingness to pay for firm 2 if and only if \( s_2 \) is sufficiently low.

If the entrant knows that non-targeted incumbents will form a ring, the choice of the takeover target also depends on which of the incumbents can be acquired by the entrant, if any, because colluding incumbents may outbid the entrant. When the entrant can acquire both incumbents, it chooses the one that yields a higher surplus, taking into account the willingness to pay of the ring.

**Proposition 5.** If non-targeted incumbents can collude in a takeover by auction, there exist two thresholds \( s_1^* \) and \( s_2^* \) such that:

(i) When either \( s_1 > s_1^* \) or \( s_2 > s_2^* \), \( E \) takes over firm 2 if and only if \( s_2 \) is sufficiently high.

(ii) When \( s_1 \leq s_1^* \) and \( s_2 \leq s_2^* \), \( E \) does not enter the market.

Fig. 7.1 displays how \( E \)’s choice of target depends on the takeovers mechanism: with bargaining \( E \) takes over firm 1 (resp. 2) if \( s_1 \) is below (resp. above) the dashed curve (as in our model); with auctions \( E \) prefers to take over firm 1 (resp. 2) if \( s_1 \) is below (resp. above) the dotted curve. Therefore, similarly to our main model: for values of \( s_1 \) and \( s_2 \) between the dotted curve and the 45-degree line, in an auction with collusion \( E \) takes over firm 1 that is the profit-maximizing but not the efficient target; for values of \( s_1 \) and \( s_2 \) between the dotted and dashed curves, \( E \) takes over the efficient target with bargaining, but not with auctions.

Compared to our main model, when non-targeted incumbents collude takeovers by auctions create two additional inefficiencies. First, when \( s_1 \leq s_1^* \) and \( s_2 \leq s_2^* \), \( E \) does not enter the market with auctions (but it does enter with bargaining). This is inefficient because \( E \)’s entry always increases consumer surplus. Second, when \( s_1 > s_1^* \) and \( s_2 > s_2^* \), takeovers by auctions are more likely to reduce consumer surplus.

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33 Of course, if the target may solicit offers by other incumbents and make them simultaneously compete with the entrant, the outcome of this form of negotiation would be different. Indeed, this is precisely the type of mechanism that we interpret as an auction.

34 See Appendix A for details. In Appendix A, we also show how firms can design side payments that support collusion to prevent entry by making it individually rational for all non-targeted incumbents to join a ring. In contrast to standard models of collusion in auctions, where the designated auction winner compensates other colluding bidders, in our context it is the designated bidder that has to be compensated by other non-targeted incumbents in order to induce it to merge with the target.

35 See the proof of Lemma 5 for the equation that defines the dotted curve.
than in our main model. The reason is that, when non-targeted incumbents collude, $E$ is more likely to take over the efficient firm 2 with bargaining and the profit-maximizing firm 1 in an auction — that is, the dotted curve in Fig. 7.1 is strictly higher than the dotted curve in Fig. 6.1. The intuition is that the presence of colluding incumbents increases the price that $E$ has to pay in an auction for firm 2 more than in an auction for firm 1. Hence, it is more likely that $E$ is discouraged from acquiring firm 2.

7.4 Small markets

In this section we consider a situation in which incumbents have an incentive to merge even if $E$ does not attempt to enter the market. Therefore, we assume that condition (2.2) is not satisfied, so that a merger between firm 1 and one of the symmetric incumbents is profit maximizing in period 1. This is more likely to happen in small markets — i.e., if $n$ is small — and with more asymmetric firms — i.e., if the difference between $C_2$ and $C_1$ is large.

Because condition (2.2) does not affect a target’s reservation value, the analysis of takeovers by bargaining in small markets is the same as in our main model. In a takeover by auction, instead, when condition (2.2) is not satisfied an incumbent’s willingness to pay to block $E$’s entry may be higher than $E$’s willingness to pay to target, so that the incumbent may outbid $E$ and prevent entry. Specifically, in an auction for firm $i$, $E$’s entry is blocked if and only if

$$v_j \geq \pi_n\left(c_i - s_j; \sum_{k \neq i} c_k\right), \quad i, j = 1, 2, \ i \neq j.$$

This condition requires that $s_j$ is sufficiently low. Therefore, if incumbents cannot outbid the entrant, the choice of the takeover target in an auction is determined by the same condition as in our main model when reservation values do not bind (i.e., when $s_1 > s_2$).

**Proposition 6.** In a takeover by auction in small markets, there exist two thresholds $s_1$ and $s_2 > s_1$ such that:

(i) When either $s_1 > s_1$ or $s_2 > s_2$, $E$ takes over firm 1 if condition (5.3) holds, and firm 2 otherwise.

(ii) When $s_1 \leq s_1$ and $s_2 \leq s_2$, $E$ does not enter the market.

Fig. 7.2 displays how $E$’s choice of target depends on the takeover mechanism: with bargaining $E$ takes over firm 1 (resp. 2) if $S_2$ is below (resp. above) the dashed curve (as in our main model); with auctions $E$ prefers to take over firm 1 (resp. 2) if $S_2$ is below (resp. above) the dotted curve. Therefore, similarly to our main model, for values of $s_1$ and $s_2$ between the dotted and the 45-degree line, $E$ takes over firm 1 that is the profit-maximizing but not the efficient target in an auction; for values of $s_1$ and $s_2$ between the dotted and dashed curves, $E$ takes over the efficient target with bargaining, but not with auctions.

As for collusion, in small markets takeovers by auctions create two additional inefficiencies: (i) when $s_1 \leq s_1$ and $s_2 \leq s_2$, $E$ does not enter the market with auctions (but it does enter with bargaining); (ii) when $s_1 > s_1$ and $s_2 > s_2$, takeovers by auctions are more likely to reduce consumer surplus than in our main model — that is, the dotted curve in Fig. 7.2 is weakly higher than the dotted curve in Fig. 6.1.

8. Conclusions

We have analyzed a model of entry by takeover with endogenous target choice and compared two alternative takeover mechanisms: (i) bilateral bargaining between the entrant and the selected target, and (ii) an auction for the selected target in which the entrant competes against other incumbents. With bargaining, the entrant pays the target’s reservation value. By contrast, because entry imposes negative externalities on non-targeted incumbents, in an auction they bid aggressively to prevent entry and the entrant may pay more than the target’s reservation value. This provides a justification for takeover premia observed in the real world.

The choice of which incumbent to acquire depends on the takeover mechanism. Specifically, an auction may induce the entrant to choose a less efficient target than the one chosen with bargaining, resulting in a takeover that generates a lower consumer surplus. The reason is that takeovers that generate higher consumer surplus also generate stronger negative externalities on other incumbents and, hence, they are especially costly for the entrant with auctions. Therefore, forcing the acquisition market to be more competitive via requiring an auction may result in inferior outcomes for consumers.

While our model is one of quantity competition with homogeneous products, the effects we have identified also apply to other forms of competition. For instance, even with differentiated products and Bertrand competition, entry imposes a negative externality on non-targeted incumbents who, therefore, raise the price that the entrant has to pay in an auction, and may induce the entrant to acquire a target with lower synergies (than with bargaining).

Takeovers by auction also reduce consumer welfare when incumbents outbid the entrant by merging with the target, since this increases industry concentration and prevents the entry of a more efficient firm. Therefore, our analysis suggests that antitrust authorities should be especially careful in evaluating mergers between incumbents when one of the merging firms is a target for a potential entrant (e.g., when national champions attempt to block takeovers by foreign firms), since these mergers may be dictated by the desire to prevent entry of a more efficient competitor rather than by the presence of superior efficiency gains.

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36 A possible interpretation is that of an industry where firms face technology demand shocks that result in the possibility of a profitable merger among incumbents and, at the same time, of profitable entry by an outsider.

37 See the proof of Proposition 6 for details.

38 When condition (2.1) is not satisfied, $v_j - \pi_n-1(c_i; \sum_{k \neq i} c_k) = \pi_n(c; \sum_{k \neq i} c_k - s_j) > -\pi_n(c_i)$. See, for example, Mayo and Sappington (2015) who show that an auction among Hotelling duopolists for an input that reduces production costs does not necessarily yield the allocation that maximizes welfare, precisely because of the presence of negative externalities.
Appendix A. Proofs

Proof of Proposition 1. When the takeover takes place through bargaining, $E$ prefers to take over firm 1 rather than firm 2 if and only if

$$
\pi_1\left(c_1 - s_1: \sum_{k=1}^n c_k\right) - \pi_n\left(c_1: \sum_{k=1}^n c_k\right) > \pi_n\left(c_2 - s_2: \sum_{k=2}^n c_k\right) - \pi_1\left(c_2: \sum_{k=2}^n c_k\right)
\implies \left(\frac{\Phi_1 + ns_1}{n+1}\right)^2 - \left(\frac{\Phi_1}{n+1}\right)^2 > \left(\frac{\Phi_2 + ns_2}{n+1}\right)^2 - \left(\frac{\Phi_2}{n+1}\right)^2.
$$

Rearranging yields the statement. ■

Proof of Corollary 1. First, if $s_1 > s_2$, condition (4.1) is satisfied since the left-hand side is positive while the right-hand side is negative.

Second, if $s_1 = s_2 = \frac{n+1}{n} (c_2 - c_1)$, using the fact that $\Phi_1 - \Phi_2 = (n+1)(c_2 - c_1)$ condition (4.1) simplifies to

$$
\left[s_2 - \frac{n+1}{n} (c_2 - c_1)\right]^2 - \frac{2}{n} s_2 \Phi_2 + \frac{2}{n} \Phi_1 \left[s_2 - \frac{n+1}{n} (c_2 - c_1)\right] > 0 \iff 2\Phi_1 < (n+1)(c_2 - c_1),
$$

which is never satisfied since

$$
\frac{A-n c_1 + (n-1)c_2}{\Phi_1} > (n+1)(c_2 - c_1) \iff \frac{A-2c_2 + c_3}{\Phi_2} > 0.
$$

And if condition (4.1) is not satisfied when $s_1 = s_2 = \frac{n+1}{n} (c_2 - c_1)$, it is not satisfied for smaller values of $s_1$ either. ■

Proof of Lemma 1. We first derive the highest equilibrium bid by an incumbent when $E$ attempts to take over firm $i = 1, 2$ in an auction, and then we compare it with firm's reservation value.

First, consider an auction for firm 2. Firm 1's willingness to pay for blocking the takeover of firm 2 is

$$
v_1^2 = \frac{A-(n-1)c_1 + (n-2)c_2}{n^2} - \frac{A-n c_1 + (n-1)c_2 - s_2}{(n+1)^2}.
$$

Firm $j$'s willingness to pay, $j > 2$, for blocking the takeover of firm 2 is

$$
v_j^2 = \frac{A-(n-1)c_2 + (n-3)c_2 + c_1}{n^2} - \frac{A-n c_2 + (n-2)c_2 + c_1 - s_2}{(n+1)^2}.
$$

Therefore,

$$
v_1^2 - v_j^2 = \frac{2c_2-c_1}{n+1} (A-2c_2 + c_1 + ns_2),
$$

which is strictly positive since $\Phi_2 = A - 2c_2 + c_1 > 0$ by assumption.

Therefore, only firm 1 can obtain a positive profit by winning the auction for firm 2 and blocking the takeover at a price higher than $v_1^2$, and no other incumbent is willing to bid higher than $v_1^2$. So it is a weakly dominant strategy for firm 1 to bid up to $v_1^2$, his willingness to pay in an ascending auction for firm 2, once the auction price reaches $v_1^2$ and $E$ is the only other bidder active in the auction, since by dropping out at a lower price firm 1 only gives up the possibility of winning and obtaining a positive profit (because of the assumption that $E$ drops out with probability $e$). It is straightforward to see that it is an equilibrium for firm 1 to bid $v_1^2$ and for the other incumbents to bid 0.

Comparing firm 1's bid with the reservation value of firm 2,

$$
v_1^2 \leq r^2 \iff \frac{A-(n-1)c_1 + (n-2)c_2}{n^2} - \frac{A-n c_1 + (n-1)c_2 - s_2}{(n+1)^2} \leq \frac{A-2c_2 + c_1}{(n+1)^2}.
$$

The relevant solution of this inequality is

$$
s_2 \leq \Phi_1 - \frac{1}{n} \sqrt{n^2 \left(\Phi_1^2 - \Phi_2^2\right) + 2n \Phi_1 \Phi_2 + \Phi_2^2} = \Phi_2.
$$

Consider now an auction for firm 1. Since all other incumbents have the same willingness to pay to block the takeover of firm 1, there is no pure-strategy equilibrium in which $E$ wins the auction at a price lower than $v_1^2$ — i.e., in which no incumbent bids up to his willingness to pay. To see this, suppose by contradiction that, in equilibrium, the last incumbent active in the auction should drop out when the auction reaches a price $p < v_1^2$ (possibly simultaneously with other incumbents). In this case, this incumbent would have an incentive to deviate and remain active at price $p$, since it would obtain a higher profit by winning (because of the assumption that $E$ drops out with probability $e$) rather than losing at any price
lower than \( v_j^2 \). Moreover, it is straightforward to see that no incumbent has an incentive to bid more than \( v_j^2 \), and that it is an equilibrium for one incumbent to bid \( v_j^2 \) and for the other incumbents to bid 0.

Equilibria with the properties that we have characterized are the only ones that satisfy a “no ex-post regret” property for incumbents: an incumbent never allows another bidder to win at a price that he would have been happy to pay to outbid the winner and win the auction.

Notice that in our model there is no symmetric mixed-strategy equilibrium in which incumbents randomize between bidding a positive price and bidding zero (as in the equilibria characterized by Hoppe et al., 2006 for sealed-bid second-price auctions). The reason is that, in an ascending auction, an incumbent would have an incentive to deviate and never drop out at zero. First, if at least one other incumbent does not drop out at zero, the deviating incumbent can drop out at a positive but arbitrarily low price and the deviation has no effect. Second, if all other incumbents drop out at zero, the deviation increases the incumbent’s profit because it gives him a chance to bid up to his willingness to pay to block the entrant.

Clearly, our results for the ascending auction also hold in all pure-strategy equilibria of a sealed-bid second-price auction. Moreover, the equilibrium that we have characterized for the ascending auction is also equivalent (in terms of auction price, seller’s revenue and bidders’ profit) to the “natural” equilibrium of a sealed-bid first-price auction (in which the second-highest bidder, an incumbent, bids up to his willingness to pay for winning the auction, and \( E \) outbids him by an arbitrarily small amount).

Comparing firm \( j \)'s bid, \( j > 2 \), with the reservation value of firm 1,

\[
v_j^1 > r^1 \quad \iff \quad \frac{[A-(n-1)c_2+(n-2)c_1]}{n^2} \leq \frac{[A-nc_2+(n-2)c_1+c_1-s_1]}{(n+1)^2} \leq \frac{[A-nc_1+(n-1)c_2]}{(n+1)^2}.
\]

The relevant solution of this inequality is

\[ s_1 \leq \Phi_2 - \frac{1}{n} \sqrt{\Phi_2(\Phi_2 + 2n\Phi_1)} \Phi_1. \]

Finally,

\[ \Phi_1 - \Phi_2 \iff \Phi_1 - \Phi_2 \leq \left( 2\Phi_1 + \Phi_2 - \Phi_2 n^2 \right) < 0 \iff A > \frac{2c_2(2n^2-n-1)-c_1(3n^2-1)}{n^2-2n-1}, \]

which is satisfied by condition (2.2). ■

**Proof of Proposition 2.**

(i) Assume that \( s_1 \leq \delta_1 \) and \( s_2 \leq \delta_2 \). Since reserve prices are binding for both targets, \( E \)'s choice of target is the same as with bargaining.

(ii) Assume that \( s_1 \leq \delta_1 \) and \( s_2 > \delta_2 \): firm 1’s reservation value is binding, whereas firm 2’s is not. Therefore, \( E \) takes over firm 1 if and only if

\[
\pi_n(c_1-s_1; (n-1)c_2) - r^1 > \pi_n(c_2-s_2; (n-2)c_2 + c_1) - v_j^1 \iff \pi_n(c_1-s_1; (n-1)c_2) + \pi_n(c_2-s_2; (n-2)c_2 + c_1) > \pi_n(c_1-s_1; (n-1)c_2) + \pi_n(c_2-s_2; (n-2)c_2 + c_1) + \pi_n(c_2-s_2; (n-2)c_2 + c_1) + \pi_n(c_2-s_2; (n-2)c_2 + c_1).
\]

Substituting and rearranging yield the statement. Notice that condition (5.2) requires that

\[
s_2 < \frac{\Phi_1 - \Phi_2 + \sqrt{\Phi_1^2 + \Phi_2^2 + n^2\Phi_1^2 + n^4\Phi_1^2 + 2\Phi_1\Phi_2 + 2n\Phi_1\Phi_2 + 2n\Phi_1\Phi_2 + \Phi_1^2 - 2\Phi_2\Phi_2 + \Phi_2^2}}{n^2 + 1}.
\]

(iii) Assume that \( s_1 > \delta_1 \): reserve prices are not binding in an auction. In order to take over firm 2, \( E \) has to pay \( v_j^2 \) by Lemma 1. In order to take over firm 1, \( E \) has to pay firm \( j \)'s willingness to pay, \( j > 2 \), for blocking the takeover, \( v_j^2 \). Therefore, \( E \) takes over firm 1 if and only if

\[
\pi_n(c_1-s_1; (n-1)c_2) - v_j^1 > \pi_n(c_2-s_2; (n-2)c_2 + c_1) - v_j^1 \iff \pi_n(c_1-s_1; (n-1)c_2) + \pi_n(c_2-s_2; (n-2)c_2 + c_1) > \pi_n(c_1-s_1; (n-1)c_2) + \pi_n(c_2-s_2; (n-2)c_2 + c_1) + \pi_n(c_2-s_2; (n-2)c_2 + c_1) + \pi_n(c_2-s_2; (n-2)c_2 + c_1).
\]

Substituting and rearranging yield the statement. Notice that condition (5.2) requires that

\[
s_2 < \frac{\Phi_1 - \Phi_2 + \sqrt{n^4\Phi_1^2 + 2n^2\Phi_1\Phi_2 + n^4\Phi_1^2 + n^2\Phi_1^2 - 2n^2\Phi_2\Phi_1 + 2n^2\Phi_1^2 - 2n^2\Phi_2\Phi_1 + \Phi_1^2 - 2\Phi_2\Phi_2 + \Phi_2^2}}{n^2 + 1}.
\]

Finally, it can be shown that conditions (5.2) and (5.3) coincide if \( s_1 = \delta_1 \) and conditions (4.1) and (5.2) coincide if \( s_2 = \delta_2 \) (this implies that the red curve in Fig. 3 is continuous). ■

**Proof of Corollary 2.** Assume that \( s_1 > s_2 \), so that firm 1 is both the profit-maximizing and the efficient target. Conditions (4.1) and (5.3) hold by inspection. Condition (5.2) holds as well because, when (4.1) holds, \( E \) takes over firm 1 with bargaining and firm 1’s price in an auction is the same as in bargaining, whereas firm 2’s price is higher. Hence, \( E \) takes over firm 1.
Assume that \( s_2 > s_1 + \frac{n}{n^2+1}(c_2-c_1) \), so that firm 2 is both the profit-maximizing and the efficient target. In this case, conditions (4.1), (5.2) and (5.3) are not satisfied, so that \( E \) takes over firm 2. First, condition (4.1) is not satisfied by Lemma 1. Second, when \( s_2 > s_2 \) condition (5.2) is not satisfied since

\[
\frac{s_2^2-s_1^2}{n} < \frac{2}{n^2} \left( \frac{s_2^2-2s_1^2+2n(s_2-s_1)(c_2-c_1)}{n^2} \right)
\]

where \( H < K \) (because condition (4.1) is not satisfied) and \( J > 0 \) (using the definition of \( \delta \) in (A.1) and the assumption that \( s_2 > s_2 \)). Third, condition (5.3) requires that

\[
s_2 < s_1 + \frac{n}{n^2+1}(c_2-c_1)
\]

which is always satisfied because of the assumption that \( s_1 < \Phi_2 \). Hence, \( s_2 > s_2 \) when \( s_2 > s_1 + \frac{n}{n^2+1}(c_2-c_1) \), as we have assumed.

### Proof of Proposition 3

If \( s_1 > \delta_1 \), conditions (5.3) and (4.1) require that

\[
\frac{2}{1+n^2}(\Phi_1s_2 - \Phi_2s_1) > \frac{2}{n^2+1}(s_1(\Phi_1s_2 - \Phi_2s_2)) > \frac{s_2^2-s_1^2}{n} > 0.
\]

(A.2)

If \( s_1 < \delta_1 \), conditions (5.2) and (4.1) require that

\[
\frac{s_2^2}{n^2} (2(\Phi_1s_2 - \Phi_2s_1)) > \frac{s_2^2-s_1^2}{n} > 0.
\]

(A.3)

Finally, we prove that \( E \) never takes over firm 2 in an auction and firm 1 with bargaining. In order to do this, we show that when \( s_2 = \frac{n}{n^2+1}(c_2-c_1) > s_2 \) (so that the choices of target in auction and bargaining may differ): (i) if \( s_1 > \delta_1 \), the opposite of condition (A.2) cannot hold; (ii) if \( s_1 < \delta_1 \), the opposite of condition (10) cannot hold.

First, the opposite of condition (A.2) requires that

\[
\frac{s_2}{n^2} (2(\Phi_1s_2 - \Phi_2s_1)) > \frac{s_2^2-s_1^2}{n} > 0.
\]

Since \( s_2 > s_1 \),

\[
\frac{(s_2-s_1)(\Phi_1s_2 - \Phi_2s_1)}{n^2} > 0.
\]

which contradicts (A.4).

Similarly, the opposite of condition (A.3) — i.e.,

\[
\frac{s_2}{n^2} (2(\Phi_1s_2 - \Phi_2s_1)) > \frac{s_2^2-s_1^2}{n} > 0.
\]

does not hold since

\[
\frac{s_2}{n^2} (2(\Phi_1s_2 - \Phi_2s_1)) > 0.
\]

is strictly positive when \( s_2 > \delta_2 \) (using the definition of \( \delta_2 \) in (A.1)).

### A.1 Non-linear demand

Consider a general demand function \( P(Q) \) such that \( P(Q) + \Phi P(Q) < 0 \) and \( \lim_{Q \to \infty} P(Q) = 0 \). A firm’s equilibrium quantity is determined by the following first order condition:

\[
P(Q) + P(Q)\Phi_1 = c_1.
\]

\(\Phi\) These are sufficient conditions for the Cournot game to be “stable” and have “well-behaved” comparative statics.
It is easy to see that \( q_i > q_j \iff c_i < c_j \): in equilibrium firms with lower marginal costs have larger market shares and earn higher profits. Summing over all the \( n \) firms, we obtain

\[
nP(Q) + P'(Q)Q = \sum_{i=1}^{n} c_i.
\]

Since total quantity in the market is decreasing in the sum of firms’ marginal costs, our definition of efficient target — the one that maximizes consumer surplus — still applies.

As in our model with linear demand, the profit-maximizing target may differ from the efficient one. In fact, the profit-maximizing target is the one that provides the entrant with the highest downstream profits, which in turn depends on the synergies, the cost of the target, and the costs of other incumbents. The condition for a target to be profit-maximizing varies with the shape of \( P(Q) \) and may not be linear, but it always depends both on the synergies and on the difference between the incumbents’ costs. By contrast, the efficient target is determined solely by the size of the synergies. Hence, for \( c_2 > c_1 \), the condition for firm 2 to be the profit-maximizing target will always be strictly above the one for firm 2 to be the efficient target.

Our main result about auctions vs. bilateral negotiations does not hinge on the assumption of linear demand. Regardless of the shape of \( P(Q) \), with bilateral negotiations \( E \) takes over the incumbent with the highest efficiency gains over the status quo — i.e., it chooses the incumbent that solves

\[
\max_{i \in \{1,2\}} n_s \left(c_i - s_i; \sum_{j \neq i} c_j\right) - n_s \left(c_i; \sum_{j \neq i} c_j\right).
\]

In particular, \( E \) takes over firm 2 if and only if

\[
\pi_s(c_2 - s_2; (n-2)c_2 + c_1) - \pi_n(c_1 - s_1; (n-1)c_2) > \pi_s(c_2; (n-2)c_2 + c_1) - \pi_n(c_1; (n-1)c_2 - s_2).
\]

When no firm’s reserve price is binding (the case when only one reserve price binds is similar), in an auction \( E \) takes over the incumbent that solves

\[
\max_{i \in \{1,2\}} n_s \left(c_i - s_i; \sum_{j \neq i} c_j\right) - \left[\pi_n \left(\min\{c_i, c_j\}; \sum_{k \neq i,j} c_k\right) - \pi_n \left(c_i; \sum_{k \neq i} c_k - s_i\right)\right].
\]

In particular, \( E \) takes over firm 1 if and only if

\[
\pi_n(c_1 - s_1; (n-1)c_2) + \pi_n(c_2; (n-2)c_2 + c_1 - s_1) > \pi_n(c_2 - s_2; (n-2)c_2 + c_1) + \pi_n(c_1; (n-1)c_2 - s_2).
\]

Combining (A.5) and (A.6), \( E \) chooses firm 2 with bargaining and firm 1 with auctions if and only if

\[
\pi_n(c_2; (n-2)c_2 + c_1 - s_1) - \pi_n(c_1; (n-1)c_2 - s_2) > \pi_n(c_2 - s_2; (n-2)c_2 + c_1) - \pi_n(c_1 - s_1; (n-1)c_2).
\]

This requires that

\[
\pi_n(c_1; (n-1)c_2) - \pi_n(c_1; (n-1)c_2 - s_2) > \pi_n(c_2; (n-2)c_2 + c_1) - \pi_n(c_2; (n-2)c_2 + c_1 - s_1)
\]

i.e., that firm 1’s change in profits (compared to the status quo) when \( E \) takes over firm 2 is larger than firm 2’s change in profits when \( E \) takes over firm 1. It is easy to verify that, if \( s_2 - s_1 > c_2 - c_1 \) and \( n_s \left(\cdot; \sum \cdot\right) \) satisfies increasing differences in \((-c_i, \sum \cdot)\),\(^{41}\) then condition (A.7) is satisfied. Moreover, since \( c_2 - c_1 > 0 \), if \( s_2 - s_1 > c_2 - c_1 \), then firm 2 is the efficient target. Hence, there exist values of \( s_1 \) and \( s_2 \), with \( s_2 > s_1 \), such that \( E \) takes over firm 2 with bargaining and firm 1 in an auction, but not vice versa, so that our qualitative results hold.

**Proof of Proposition 4.** Suppose that \( s_1 + \frac{n_s}{n} (c_2 - c_1) > s_2 > s_1 \), so that firm 1 is the profit-maximizing target but firm 2 is the efficient one. By Proposition 3, \( E \) cannot take over firm 2 in an auction and firm 1 with bargaining, but may take over firm 1 in an auction and firm 2 with bargaining.

\[\blacksquare\]

### A.2. Collusion among incumbents

The total willingness to pay of non-targeted incumbents to block the takeover of firm 1 is

\[
\Delta(s_1) \equiv \pi_n \left(c_1; \sum_{k=1} c_k\right) - (n-2)\pi_n \left(c_2; \sum_{k=2} c_k\right) \geq \left(\frac{\Delta_x^2 + n(n-1)c_1^2}{n^2}\right) + (n-2) \frac{\Delta y^2}{n^2} - (n-1) \frac{(\Delta y - s_1)^2}{(n+1)^2}.
\]

Non-targeted incumbents can prevent entry if and only if their total willingness to pay is higher than \( E \)'s willingness to pay for firm 1 — i.e.,

\[
\Delta(s_1) \geq \pi_n \left(c_1 - s_1; \sum_{k=1} c_k\right) \iff s_1 \leq s_1^*,
\]

where

\[
s_1^* = \frac{n(n-1)\Delta_x^2 + n(n-1)c_1^2 + \sqrt{n^2\Delta_x^2 + 2n(n-1)\Delta_x^2 + 2n(n-1)c_1^2 + 4n^2\Delta y^2 - 4n^2\Delta y - 2n^2 - 2n + 1}}{n^2 + n - n^2 - n - 1}.
\]

\(^{41}\) It is well-known that the Cournot model with linear demand satisfies this condition.
The total willingness to pay of non-targeted incumbents to block the takeover of firm 2 is\textsuperscript{42}

\[
\Delta(s_2) \equiv \frac{[\Phi_2 + n(c_2 - c_1)]^2}{n^2} + (n-2)\frac{\Phi_2^2}{n^2} - \frac{(\Phi_1 - s_2)^2}{(n + 1)^2} - (n-2)\frac{(\Phi_2 - s_2)^2}{(n + 1)^2}
\]

Non-targeted incumbents can prevent entry if and only if

\[
\Delta(s_2) \geq \pi_n \left( c_2 - s_2; \sum_{k \neq 2} c_k \right) \iff s_2 \leq s_2^*.
\] (A.9)

where \( s_2^* \equiv \frac{2n\Phi_1 - 2n(c_2 - c_1)n^2 - 1}{n^2} \).

We assume that a ring is formed if and only if all non-targeted incumbents join it. Following the literature on collusion in auctions, we introduce a “ring center” that implements the collusive mechanism and designates a non-targeted incumbent to bid against the entrant (e.g., Graham and Marshall, 1987). Before the auction, the ring center collects payments from all non-targeted incumbents except the designated bidder and, after the auction, transfers these payments to the designated bidder if it acquires the target, and returns them otherwise.

We show how incumbents can design side payments to support collusion when conditions (A.8) and (A.9) are satisfied. Let firm \( i \) be the designated bidder and let \( t_i' \) be the transfer between incumbent \( j \) and the designated bidder. We consider symmetric transfers, so that \( t_i' = t_j \equiv t, \forall j \). Let \( T = \sum_j t_j = (n-2)t \). In this case, firm \( i \)'s surplus is

\[
\frac{[\Phi_2 + n(c_2 - c_1)]^2}{n^2} - \frac{(\Phi_1 + n s_1)^2}{(n + 1)^2} + T_i,
\]

and firm \( j \)'s surplus, \( j \neq i \), is

\[
\frac{\Phi_2^2}{n^2} - t_i.
\]

Consider side payments such that all non-targeted incumbents obtain the same surplus — i.e.,

\[
\frac{[\Phi_2 + n(c_2 - c_1)]^2}{n^2} - \frac{(\Phi_1 + n s_1)^2}{(n + 1)^2} + T_i = \frac{\Phi_2^2}{n^2} \iff t_i' = \frac{(\Phi_1 + n s_1)^2}{(n + 1)^2} - \frac{[\Phi_2 + n(c_2 - c_1)]^2}{n^2} + \frac{\Phi_2^2}{n^2}.
\]

If a non-targeted incumbents does not join the ring, collusion fails and \( E \) acquires firm 1. The ring is stable if and only if non-targeted incumbents prefer to join the ring — i.e.,

\[
\Phi_2^2 - t_i' > \pi_n \left( c_2; \sum_{k \neq 2} c_k - s_1 \right).
\]

Substituting and re-arranging yield condition (A.8).

Second, consider an auction for firm 2 and suppose that firm 1 is the designated bidder. (A similar analysis applies to the case in which a different incumbent is the designated bidder.) In order to outbid \( E \) and block entry, firm 1 has to bid at least \( \frac{(\Phi_1 + n s_1)^2}{(n+1)^2} \). The ring is stable if and only if non-targeted incumbents prefer to join the ring (rather than let \( E \) acquire firm 2). This individual rationality constraint for firm 1 is

\[
\frac{[\Phi_2 + n(c_2 - c_1)]^2}{n^2} - \frac{(\Phi_2 + n s_2)^2}{(n + 1)^2} + T_1 \geq \pi_n \left( c_1; \sum_{k \neq 1} c_k - s_2 \right), \tag{A.10}
\]

and for firm \( j \) is

\[
\Phi_2^2 - t_j \geq \pi_n \left( c_2; \sum_{k \neq 2} c_k - s_2 \right), \quad j = 3, ..., n. \tag{A.11}
\]

Adding up these constraints yields condition (A.9). Hence, there exist symmetric transfers between other non-targeted incumbents and firm 1 that support collusion (although in this case firm 1 may have to obtain a larger share of the collusive profits than other incumbents because of its higher outside option). \textsuperscript{42]

\textsuperscript{42} Notice that the total industry profits if the takeover is blocked do not depend on the identity of the non-targeted incumbent that wins the auction because, in this case, the industry always includes \( n - 2 \) firms with cost \( c_2 \) and 1 firm with cost \( c_e \).
Proof of Proposition 5. If \( s_1 > s_1^* \) and \( s_2 > s_2^* \), non-targeted incumbents cannot block the takeover of any target. Hence, \( E \) takes over firm 1 if and only if

\[
\pi_n(c_1 - s_1; (n-1)c_2) - \Delta(s_1) > \pi_n(c_2 - s_2; (n-2)c_2 + c_1) - \Delta(s_2)
\]

\[
\iff \pi_n(c_1 - s_1; (n-1)c_2) + (n-1)\pi_n(c_2; c_1 + (n-2)c_2 - s_2) + \pi_n(c_1; (n-1)c_2 - s_2).
\]

\[
\implies \Phi_n(c_1 - s_1; (n-1)c_2) > \Phi_n(c_2 - s_2; (n-2)c_2 + c_1) > \Phi_n(c_2 - s_2; (n-2)c_2 + c_1)
\]

\[
\iff \frac{s_1^2 - s_2^2}{\Phi_n(2b_2(2s_2 + (n-1)s_1) - \Phi_n(2b_2(2s_2 + (n-1)s_1)) > 0.
\]

The other statement follows directly from the definition of \( s_1^* \) and \( s_2^* \).

Finally, \( s_2 > s_1 \iff 2n(b_1 - b_2)(n + 1)(2n^2b_1 - 3nb_2 + 2n^2b_2) > 0 \),

which holds since \( b_1 > b_2 \) and \( n > 2 \).

Proof of Proposition 6. We first prove part (ii) of the statement. In an auction for firm 1, \( E \)'s entry is blocked if and only if

\[
\Phi_v^2 \equiv \Phi_v^2 \left( \begin{array}{c} c_1 - s_1; \sum_{k=1}^{K_k} k_x \end{array} \right) \iff S_1 \leq \tilde{S}_1,
\]

where \( \tilde{S}_1 = \frac{n^2b_1 - n^2d_1 + \sqrt{n^2b_1 - n^2d_1 \Phi_n(2b_2(2s_2 + (n-1)s_1) - \Phi_n(2b_2(2s_2 + (n-1)s_1))}}}{\Phi_n} \).

In an auction for firm 2, \( E \)'s entry is blocked if and only if

\[
\Phi_v^2 \equiv \Phi_v^2 \left( \begin{array}{c} c_2 - s_2; \sum_{k=1}^{K_k} k_x \end{array} \right) \iff S_2 \leq \tilde{S}_2,
\]

where \( \tilde{S}_2 = \frac{n^2b_2 - n^2d_2 + \sqrt{n^2b_2 - n^2d_2 \Phi_n(2b_2(2s_2 + (n-1)s_1) - \Phi_n(2b_2(2s_2 + (n-1)s_1))}}}{\Phi_n} \).

Part (i) of the statement follows directly from the definition of \( \tilde{S}_1 \) and \( \tilde{S}_2 \) and part (iii) of Proposition 2 (since reserve prices are never binding, \( \tilde{S}_1 < 0 \)). We now show that \( S_2 > S_1 \) . Using the definitions of \( \tilde{S}_1 \) and \( S_2 \),

\[
C + 2(nv_1 - b_1)S_1 + S_1^2(1 + n^2) = 0
\]

and

\[
C + 2(nv_2 - b_2)S_2 + S_2^2(1 + n^2) = 0,
\]

where \( C = \Phi_v^2 - \Phi_v^2 - (n + 1)^2\Phi_n(1; (n - 2)c_2) < 0 \) by assumption. The result follows since \( nv_1 - b_1 > nv_2 - b_2 \).

References


