On the Interaction between Unemployment and Inter-regional Mobility

Raquel Fonseca

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Abstract

This paper aims at examining the interaction between unemployment and inter-regional mobility in the presence of asymmetric productivity shocks. We present an intertemporal two-regions equilibrium model where unemployed workers can migrate from one region to another. The hiring process is represented by a matching function à la Pissarides. We also analyse the wage setting procedure by introducing Nash bargaining between firms and employees. We compare two extremely different scenarios. The case where there is no mobility with unemployment persistence versus the same economy where perfect mobility is assumed. This paper also studied different types of asymmetries such as changes in “unemployment benefits” and/or the cost of posting vacancies. We show the importance of these changes to explain regional unemployment disparities. Finally, we calibrate the model for Spain and we analyse the relevance of the model in explaining regional disparities and inter-regional labour force mobility.

JEL classification: J51, J61, R23.

Keywords: Migration, Unemployment, Nash bargaining

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1 Introduction

Despite persistent differences of unemployment rates across European regions, European inter-regional migration rates are relatively low in comparison with the US, ranging from 0.6% in countries such as Italy and Spain and 1% -2.5% in other European community countries, to 3% in the US during the nineties (see Overman and Puga, (1999)).

In particular, inter-regional migration rates have been low in Spain during the last decades (see Bover and Velilla (1999), and Bentolila (2001)). During the late 1970’s and early 1980’s there was a decline in the inter-regional migration, 0.32% in 1982. Even if until the mid 1990’s this rate has increased, for 1995 the gross inter-regional rate in Spain was 0.62% (see Bover and Velilla (1999)). Spanish regional unemployment disparities are very pronounced. Some regions are persistently above the average (e.g. Andalusia 29.64% in 1985 and 24.48% in 2000), while some other regions persistently remain below the national average (e.g. La Rioja 17.29% and 8.14% respectively). A great deal of empirical literature has shown that the low inter-regional mobility as well as the unemployment persistence in Spanish regions has been comparatively higher than in other European countries in the past (see Bentolila and Dolado, (1992), Ródenas (1994), Antolín and Bover (1997), Bover and Velilla (1999) and Bentolila S. (2001)).

The aim of this paper is to shed some light on the role and consequences of inter-regional mobility. We mainly address the following issues: i) to what extent does mobility affect unemployment? and ii) to what extent can labour force mobility contribute to diminishing regional disparities?

We develop a RBC model with two regions. This paper aims at analysing the labour market with migration. We compare two extremely different scenarios: the case where there is no mobility with unemployment persistence versus the same economy where costless mobility is allowed. In particular we focus on unemployment, inter-regional mobility, wages and employment dynamics.

Theoretical literature on the interactions between migration and unemployment begins with Harris and Todaro (1970). They present an economy with two regions in a static framework. Workers decide to migrate depending on their income expectations, however, wages are exogenously determined in the region that receives the migration. They conclude that migration has a negative effect on the region receiving the migrants because the number of available jobs then have to be divided among a larger number of workers. A positive relationship between unemployment rates and wages arises. The model does not take into account the different employment opportunities in each region. Other models such as Schmidt et al. (1994), use the same framework with wage bargaining concluding that there is a positive effect of migration, however, their analysis is limited by an exogenous migration.


\footnote{The inter-regional rate is defined as the ratio of migrant persons to population.}

\footnote{One important factor to explain the lack of mobility in Spain is a rigid housing market (see Antolín and Bover (1997) for empirical evidence).}
show that migrations can decrease regional unemployment disparities. These papers link the regional unemployment rate with labour supply, labour demand and wage-setting factors in an econometric model. These studies debate the macroeconomic adjustments to achieve stabilisation and coordination in order to absorb asymmetric shocks in the different regions. A policy of mobility is studied looking for stability, a better assimilation of asymmetric shocks in the economy or as an alternative to share shocks among regions. They observe a negative relationship between unemployment rates and wages when the economy faces asymmetric shocks.

More recently papers based on matching models study the relationship between migration and unemployment. In this framework we find several empirical studies such as Pissarides and Wadsworth (1989), Pissarides and McMaster (1990), and Jackman and Savouri (1992). They use the job-matching approach to consider the probability for unemployed workers to find a job. More theoretical matching models are constructed by Ortega (2000) and Chéron (1999). Ortega (2000) studies the properties of the matching process in a model with two countries where wage is determined in a Nash bargaining process. Workers decide whether to search in their country or look for a job abroad. Ortega attributes additional costs to the migrants. He shows migration as Pareto optimum and the positive effect of migration given that immigration leads to an increase in job creation in the country receiving workers. However, this paper is limited to a static framework. Chéron (1999), builds a dynamic equilibrium matching model for two countries. In his model, the migration is made endogenous through a migration cost, which depends on one exogenous parameter, while wages are exogenously determined. These assumptions imply that there is an immediate adjustment of employment and migration has then a negative effect generated by a positive relationship between unemployment and wages.

As we can observe in migration literature, the relationship between unemployment and wages seems very important for the interpretation of mobility in the dynamic of labour market variables. The difference between our model and the ones above mentioned is that in a dynamic equilibrium model with matching, we consider wages are endogenously determined by Nash bargaining between firms and workers in each region. In this model labour force mobility depends on the wages of both regions and the labour market tightness. Wages depend not only on productivity but also on the value of a new job and the probability of finding a job for unemployed workers (that depends itself on workers in the other region and hirings). Workers adjust according to their values, which take this into account.

There are different opportunities for employment across regions and unemployment is subject to asymmetric shocks. Each region faces a specific productivity shock, which introduces regional heterogeneity. Mobility can be studied from three different viewpoints: mobility of firms, mobility of the employed workers and mobility of the unemployed workers. The mobility of the firms is not explicitly modelled, however, is implied in the specification of capital adjustment. The mobility of the employed workers is not considered in this study in that we wanted to concentrate on the importance of unemployment persistence
rather than on the mobility from job to job. The mobility of the unemployed workers is then studied. Only the unemployed workers are therefore able to migrate from one region to the other.

Hiring is a matching process between vacancies and unemployment in each region. This mechanism allows us to observe the influence of migration on unemployment dynamics when there are coordination failures in the labour market. A regional-specific negative shock lowers productivity in a region, heightening the incentive to move to another region. This allocative process occurs because of endogenous probabilities of finding a job and endogenous wages. The choice of localisation by unemployed workers determines the dynamics of the labour force through the wage setting procedure, which induces migration by introducing Nash bargaining.

We apply the model to the Spanish economy. We introduce asymmetries in some parameters of the model in order to shed some light on explaining regional disparities and the role of mobility in the unemployment dynamic without taking into account migration costs. We observe that the regional disparities are important when we introduce differences in certain parameters such as the cost of posting a vacancy and benefits of staying in a specific region.

We introduce asymmetric shocks for both scenarios and we conclude that mobility may allow the labour market to adjust under asymmetric shocks, causing positive effects in both regions, and a reduction in the unemployment growth compared to no mobility. In our model, wages relate negatively to unemployment. This paper is organised as follows: First, we present the model and second by presenting the data and calibration results. We subsequently study a steady state and transitional dynamic analysis of the main variables. We first study the persistent unemployment disparities and we then study the effect of asymmetric productivity shocks. The results are analysed in the two alternative cases: when there is no mobility and when mobility is occurs between regions. Finally, we conclude.

2 The model

We first describe the matching model with two regions. Secondly, we present the maximisation problems of firms and workers. Thirdly, we describe the labour contract. Finally, we show the mobility of workers.

2.1 Trade in the labour market

The economy is divided into two regions. The size of the labour force is normalised to one: $\ell_{1,t} + \ell_{2,t} = 1$, where $\ell_{j,t}$ denotes the labour force in region $j \in \{1, 2\}$. At any time $t$, workers can be unemployed or employed (denoted $u_{j,t}$ and $n_{j,t}$ respectively, for region $j$). Unemployment in each region can be expressed in terms of employment $n_{j,t}$ and labour force $\ell_{j,t}$:
\[
\begin{align*}
    u_{1,t} &= \ell_{1,t} - n_{1,t} \\
    u_{2,t} &= 1 - \ell_{1,t} - n_{2,t}.
\end{align*}
\]

Firms are region-specific such as different sectors: they cannot transfer their vacancies from one region to another. Finally, \( v_{j,t} \) is the vacancy rate in the region \( j \) at period \( t \). We assume that only unemployed workers can apply to a vacancy post.

Following Pissarides (1990), we assume that trade in the labour market is an uncoordinated and costly activity. Each region is described by a collection of workers and employers: meetings take place according to a matching technology (see Saint Paul, 1992). We further assume that only the unemployed workers can migrate. The flow from unemployment to employment is represented in each region by a Cobb-Douglas constant returns to scale matching function. Coordination failures imply that the match \( h_{j,t} \), which associates vacancies \( v_{j,t} \) and unemployed workers \( u_{j,t} \) is not perfect. In particular

\[
h_{j,t} = h(u_{j,t}, v_{j,t}) = \overline{h}_{j} v_{j,t}^\gamma u_{j,t}^{1-\gamma},
\]

where \( \gamma \) represents the elasticity ofhirings with respect to vacancies and \( \overline{h}_{j} \) is a scale factor for region \( j \).

The transition probabilities between these different states are assumed to be Poisson rates. The probability for a vacancy to be filled \( q_{j,t}(\theta_{j,t}) = h_{j,t}/v_{j,t} \) is a function of \( \theta_{j,t} \), the ratio of vacancies to unemployment in region \( j \). This ratio describes the labour market tightness. Hence, the transition probability for an unemployed worker (to become employed) in region \( j \) is \( p_{j,t}(\theta_{j}) = h_{j,t}/u_{j,t} = \theta_{j} q_{j,t}(\theta_{j}) \). The evolution of employment in each region is given by

\[
n_{j,t+1} = q_{j,t} v_{j,t} + (1 - s_{j}) n_{j,t},
\]

where \( s_{j} \) is the exogenous separation rate Equation (4) shows that the employment dynamic depends on two factors: vacancies in the labour market (associated with the probability of filling a vacancy) and the employment from the previous period (associated with the probability of remaining in the same firm, \( 1 - s_{j} \)).

Figure 1 illustrates the dynamics of regional migration in this model. In each region, the employed workers can become unemployed with an exogenous separation rate \( s_{j} \). The unemployed workers can find a job in their region with a probability \( p_{j,t} \) or consider the possibility of moving to the other region.

Aggregate variables are defined as follows: aggregate employment is \( n_t = \sum_j n_{j,t} \), unemployment is \( u_t = \sum_j u_{j,t} \) while vacancies are \( v_t = \sum_j v_{j,t} \). Consequently, since each regional labour force is \( l_{j,t} = n_{j,t} + u_{j,t} \), the aggregate labour is \( l_t = n_t + u_t \).
2.2 Firms

Firms\(^3\) in region \(j\) have access to a constant returns to scale (CRS) production function\(^4\) with only employment as productive factor,

\[
y_{j,t} = (a + r_j)n_{j,t}.
\]  
\(^5\)

We introduce exogenous shocks, \(a\) and \(r_j\): an aggregate-specific shock, \(a\), and \(r_j\) is a regional-specific shock. The latter can be interpreted as different sectorial specialisation and the former as an aggregated shock.

Each firm maximises her profits flows over the control variable \(v_{j,t}\). Employment, \(n_{j,t}\), is the firm’s state variable. The discount factor of the firm is defined by \(0 < \beta < 1\). Then, the firm’s program can be written as a recursive problem

\[
\Upsilon^F(n_{j,t}) = \max \{ \Pi_{j,t} + \beta E_t \Upsilon^F(n_{j,t+1}) \}
\]  
\(^6\)

that the maximisation is constrained by

\[
\Pi_{j,t} = (a + r_j)n_{j,t} - w_{j,t}n_{j,t} - \bar{c}v_{j,t}
\]  
\(^7\)

\[
n_{j,t+1} = q_{j,t}v_{j,t} + (1 - s_j)n_{j,t}
\]  
\(^8\)

\(^3\)To simplify, we suppose that all firms of region \(j\) are similar.

\(^4\)Alternatively, equation (4.5) can be interpreted as coming of a more general Cobb-Douglas function with CRS, exogenous interest rate and instantaneous adjustment of the capital stock.

\(^5\)\(\beta = \frac{r}{1+r}\), where \(r\) is the exogenous interest rate.
where equation (7) defines how the present profit flow of the firm \( \Pi_{j,t} \) depends on the production function (5) and her costs: the real wage times the employment in region \( j \), and the cost of having vacancies, \( \varpi_j \), times vacancies posted on the labour market. The constraint (8) describes the evolution of employment. Denoting \( X_{j,t}^\alpha \) the Lagrange multiplier associated to the budget constraint. The first order condition\(^6\) of (6), implies

\[
X_{j,t}^\alpha = \frac{\varpi_j}{q_{j,t}}. \tag{9}
\]

Hence, following (9), the firm will post a vacancy when the discounted marginal revenue from filling a vacant job in the region \( j \) is equal to the marginal cost to create a new job.

### 2.3 Workers

Workers in each region are infinitely lived and supply inelastically one unit of employment. Workers are risk neutral. We assume that the unemployed workers can migrate and look for a job in a different region.

The expected value of an employee

\[
V_{j,t}^E = w_{j,t} + \beta E_t \left[(1 - s_j)V_{j,t+1}^E + s_j V_{j,t+1}^U\right]. \tag{10}
\]

It depends on the current wage \( w_{j,t} \) as well as on the discounted future value. We denote by \( s_j \) the exogenous separation rate, the probability of becoming unemployed, while \( V_{j,t+1}^U \) is the expected value of being unemployed at period \( t + 1 \) in region \( j \). Then, \( 1 - s_j \) is the probability to continue working in the firm the next period, with the associated expected value \( V_{j,t+1}^E \).

Unemployed workers are allowed to migrate: their decision stems from the comparison between the utility value of being unemployed in their region \( V_{1,t}^U \) and the utility value to migrate \( V_{1 \rightarrow 2,t}^M \). For instance, considering workers in region 1,

\[
V_{1,t}^U = \max \{ V_{1,t}^U, V_{1 \rightarrow 2,t}^M \}.
\]

We suppose that the value of migrating is equal to the value of being unemployed worker in the other region, i.e., there are no mobility costs, as an extreme case of lack of mobility. Hence \( V_{1 \rightarrow 2,t}^M = V_{2,t}^U \). Workers have rational expectations and they can anticipate the equilibrium of migrations. At equilibrium, the value of being unemployed in region 1 is equal to the value of being unemployed in region 2, \( V_{1,t}^U = V_{2,t}^U \), such that no additional unemployed wish to migrate. At that point, the value of being unemployed in region \( j \) is:

The value of being unemployed in region \( j \) is:

\(^6\)More detailed information of the firm optimization in the Appendix 1 in Section 6.1.
\[ V_{j,t}^U = b_j + \beta E_t \left[ p_{j,t} V^E_{j,t+1} + (1 - p_{j,t}) V_{j,t+1}^U \right] \]

where \( b_j \) is the instantaneous utility for an unemployed worker of being in a region \( j \). This can be interpreted as unemployment benefits and other utilities specific to the region.\(^7\)

The future expected value depends on the probability of being employed in the following period versus being unemployed. Notice that the incentive to migrate will depend on the probabilities of finding a job in another region, hence it will depend on population and unemployed workers in the other region compared to her own region. More precisely, the non-arbitrage condition\(^8\):

\[ V_{1,t}^U = V_{2,t}^U = V_t^U, \quad \forall \ t \]

i.e.,

\[ V_{1,t}^U = b_1 + \beta E_t \left[ p_{1,t} V^E_{1,t+1} + (1 - p_{1,t}) V_{1,t+1}^U \right] \]
\[ = b_2 + \beta E_t \left[ p_{2,t} V^E_{2,t+1} + (1 - p_{2,t}) V_{2,t+1}^U \right]. \]

Therefore, unemployed workers who decide to migrate must be compensated by the probabilities of finding a job \( p_{j,t} \) in both regions and the future values of being employed and unemployed workers.

### 2.4 Wage determination

Wages are determined by a bargaining process between the firm and the workers. In each period, employees and firms simultaneously re-negotiate wages. Otherwise there would be as many wages as workers. The solution criterion is the Nash solution as proposed by Pissarides (1990).

Both parts take into account their marginal values when negotiating. Marginal value for the worker is the difference of earnings obtained when being employed and unemployed. Then, the marginal value of employment for a worker may be expressed as:

\[ \Upsilon_{j,t}^W = V_{j,t}^E - V_{j,t}^U. \]

The marginal value of current employment for a representative firm is\(^9\)

\[ \Upsilon^F(n_{j,t}) = (a_t + r_{j,t}) - w_{j,t} + (1 - s_j) \beta E_t \Upsilon^F(n_{j,t+1}). \]

Let \( 0 < \xi < 1 \) denotes the worker’s share of the marginal value of a new job that it is equivalent to the bargaining power of the worker. The Nash

\(^7\)For instance, housing, weather, quality of life and so on. We take “unemployment benefits” as exogenously given.

\(^8\)This condition is checked at steady state.

\(^9\)Notice that we have denoted \( \Upsilon^F(n_{j,t}) = \frac{\partial \Upsilon^F(n_{j,t})}{\partial n_{j,t}} \).
bargaining criterion maximises over wages the marginal values value of a match
for a representative firm and a representative worker:

\[
\max_{w_j} \left\{ \left( T^F_n(n_{j,t}) \right)^{1-\xi} \left( T^W_{j,t} \right)^\xi \right\},
\]

(15)

where the first order condition states that:

\[
T^F_n(S^F_{j,t}) = \frac{1-\xi}{\xi} T^W_{j,t}.
\]

(16)

If we replace by the value of \( V^W_{j,t} \) (from (13)) in equation above, considering
that workers’ non-arbitrage condition implies \( V^U_t = V^U_{1,t} = V^U_{2,t}, \ \forall \ t \), we have
the following expression:10

\[
\xi T^F_n(n_{j,t}) = (1 - \xi) \left[ w_{j,t} - b_j + \beta E_t \left\{ (1 - s_j - p_{j,t})(V^F_{j,t+1} - V^U_{j,t+1}) \right\} \right].
\]

(17)

Sustituing the value of \( T^F_n(n_{j,t}) \) from equation (14) as well as the Euler equation
into (17) and rearranging, we have the wage evolution equation:

\[
w_{j,t} = \xi \left[ (a_j + r_{j,t}) + \theta_{j,t} w_{j,t} \right] + (1 - \xi) b_j.
\]

(18)

The wage is determined by a rent sharing mechanism, which depends on the
bargaining power of each agent. The part of the firm are the gains in labour
productivity and the tightness of the labour market associated to the cost of
posting a vacancy (notice that this expression is the marginal value of a new job
associated to the probability of finding a job). The share the employee retains
is the probability for a worker of having unemployment benefits. In Pissarides’
model when wages are determined by Nash bargain mechanism do not depend
only on marginal productivity of employment as in neoclassical models, but
also positively on the discounted valued of filling a job and the labour market
tightness.

2.5 Mobility of workers

As we have seen in the previous sections, only the unemployed can migrate.
From the non-arbitrage condition, unemployed workers migrate when the present
value of an unemployed worker in region 1 is equal to the present value of an
unemployed worker in region 2, given by the equation (11):

\[
V^U_{1,t} = V^U_{2,t}.
\]

(19)

We solve equation (19) and we obtain the following expression:11

\[\text{\footnotesize{\cite{11}}}\]

---

10For more details see Appendix 2 in Section 6.2.

11More details in Appendix 3 in section 6.3.
\[ b_1 - b_2 = \frac{\xi}{1 - \xi} \left( p_{2,t} \frac{\omega_2}{q_{2,t}} - p_{1,t} \frac{\omega_1}{q_{1,t}} \right). \] 

or rewritten as

\[ b_1 - b_2 = \frac{\xi}{1 - \xi} \left( \omega_2 \theta_{2,t} - \omega_1 \theta_{1,t} \right). \]

where \( \theta_{j,t} = \frac{1}{\omega_j} \). This non-arbitrage condition (21) shows that in the particular case where \( b_1 = b_2 \) and \( \omega_2 = \omega_1 \), the equilibrium labour market tightness rates will be equal in both regions (\( \theta_{2,t} = \theta_{1,t} \)). This implication of the non-arbitrage will remain satisfied in all circumstances, as long as \( b_1 = b_2 \), \( \omega_2 = \omega_1 \) and there is perfect labour mobility. The equality between \( \theta_{1,t} = \frac{1}{\omega_1} \) and \( \theta_{2,t} = \frac{1}{\omega_2} \) will be the result from workers’ mobility across regions, which in case of perfect mobility implies the assumption (19). Every period workers move to satisfy the non-arbitrage condition. This takes place until there is no longer any incentive to move. This occurs when the probabilities of finding a job and wages in both regions are equal.

In case of asymmetric regions the labour market tightness are not necessarily equal (\( \theta_{2,t} \neq \theta_{1,t} \)). This occurs when \( b_1 \neq b_2 \) (“unemployment benefit” is different across regions) and/or \( \omega_2 \neq \omega_1 \) (cost of posting vacant jobs is different across regions).

Another case of asymmetric regions takes place when \( \bar{\eta}_1 \neq \bar{\eta}_2 \) (the efficiency of matching is different across regions), which makes differences between unemployment rates while leaving \( \theta_{2,t} = \theta_{1,t} \).\(^\text{12}\) If there is a change in productivity, workers will move to satisfy the non-arbitrage condition (21) every period and they will stop moving when probabilities of labour market and wages become equal.

From the definition of \( q_{j,t} \), we substitute

\[ \frac{\omega_j}{q_{j,t}} = \frac{\omega_j v_{j,t}}{p_{j,t}(\ell_{j,t} - n_{j,t})} \]

in expression (20), to determine labour force in region 1:

\[ \ell_{1,t} = n_{1,t} + \frac{\omega_1 v_{1,t}}{1 - \xi (b_2 - b_1) + p_{2,t} \frac{\omega_2}{q_{2,t}}} \]  

As we know that \( \ell_{1,t} + \ell_{2,t} = 1 \), we observe movements in the population in the whole economy. Labour force mobility depends on employment in its own region \( (n_{1,t}) \), on the ratio between the vacancies posted in this region times the cost \( (\omega_1 v_{1,t}) \), on the difference in the “unemployment benefits” in both regions, on the worker’s bargaining power \( \left( \frac{1 - \xi}{\xi} (b_2 - b_1) \right) \) and finally on the discounted value for the firm to create a new job in the other region, associated with the probability for an unemployed worker to find a job in the other region \( (p_{2,t} \frac{\omega_2}{q_{2,t}}) \).

\(^\text{12}\)Take into account in equation (4.20) the \( p_{j,t} = \frac{\omega_j}{\omega_{j,t}} \) depends on \( \bar{\eta}_j \).
This equation is essential to understand mobility in the mechanism of the model. In each period, the non-arbitrage condition is satisfied. However, if a change occurs, each time the movement in population changes depending on the probabilities of finding a job and on wages until transversality conditions are satisfied.

3 Data and calibration

We analyse the response of some key variables of labour market to exogenous permanent shocks: specific regional shocks. We solve the equilibrium model\textsuperscript{13} taking into account the rational expectations hypothesis. The model has not any analytical solution, we then simulate the model using the software DYNARE developed by Juillard (1996).\textsuperscript{14}

We first perform a static exercise assessing the steady state implications of such changes in the model. We then study the transitional dynamics delivered by the model facing the same shocks. This comparative analysis enables us to offer some insights on the role of regional mobility to explain unemployment dynamics and regional disparities. Sensibility analysis will be carried out for certain parameters.

Labour market variables and probabilities of the model are reported in Table 2. Unemployment, employment, and vacancy rates are reproduced in the model to replicate the yearly average for Spanish data for the period 1977-1999. We assume two baseline regions with the same unemployment rates.\textsuperscript{15}

Table 1 reports the behavioural parameters. As Hosios (1990), we set the elasticity of the matching function with respect to the vacancies and the firms’ bargaining power in the Nash bargaining process as $\gamma = 1 - \xi = 0.4$. The parameter of the efficiency of the matching function $\tilde{m}$, is set to 0.3. The separation rate $s$ is computed in steady state with equation (4.8), such as $s = \frac{1}{\tilde{m}}$. The value is consistent with yearly estimation of separations for the Spanish economy 1977-1996 in Antolín (1997), about 4% in average of the labour force. We determine the unemployment benefits $b = 0.34$ such that the model reproduces a replacement ratio of 70.55%, which corresponds to the Spanish replacement ratio for 1994/95 in OECD data (see Martin (1996)). The common discount rate, $\beta$, is set equal to 0.99.

The ratio of recruiting expenditures to output $\omega V/Y$ is chosen to 3\% (Andolfatto (1996) set this value to 1\%, Merz (1995) about 2\% and Abowd and Kramarz (1998) to 5\%). The cost of posting a vacancy is equal to 3\% of the regional production. The cost of posting $\omega$ is to get from the steady state using the first order condition equation of the firm (equation 9). The total factor productivity is set in such a way that model solves it in steady state and the regional

\textsuperscript{13}See equilibrium equations in Appendix 4 in section 6.4.
\textsuperscript{14}More about DYNARE and resolution of forward-looking models can be found in Laflargue (1990) and Boucekkine (1995).
\textsuperscript{15}Taking into account that labour force is normalised to one, we assume that half of the population lives in each region.
<table>
<thead>
<tr>
<th>Behavioural Parameters</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>elasticity of hiring function to vacancies</td>
<td>γ</td>
</tr>
<tr>
<td>bargaining power</td>
<td>ξ</td>
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<tr>
<td>discount factor</td>
<td>β</td>
</tr>
<tr>
<td>hiring costs relative to output</td>
<td>ωv/y</td>
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<tr>
<td>cost of posting vacancies</td>
<td>ω</td>
</tr>
<tr>
<td>separation rate</td>
<td>s</td>
</tr>
<tr>
<td>unemployment benefits</td>
<td>b</td>
</tr>
<tr>
<td>efficiency factor of matching function</td>
<td>k</td>
</tr>
<tr>
<td>total factor productivity</td>
<td>a</td>
</tr>
<tr>
<td>regional factor productivity</td>
<td>r</td>
</tr>
</tbody>
</table>

Table 1: Behavioural Parameters

factor productivity is set to zero in our benchmark case. Yet, changes in this parameter will be considered to introduce asymmetries in the regions.

Table 2 indicates labour market variables, endogenous probabilities and wages. Endogenous probabilities to fill a vacancy, q and to find a job, p, are settled in steady state with \( q = h/v \) and \( p = h/u \) respectively. The hiring rate is set in steady state using the matching function, equation (3). The endogenous wage is evaluated in steady state with equation (18).

<table>
<thead>
<tr>
<th>Labour market variables, Probabilities and wages</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>unemployment rate</td>
<td>u/l</td>
</tr>
<tr>
<td>employment rate</td>
<td>n/l</td>
</tr>
<tr>
<td>vacancy rate</td>
<td>v/l</td>
</tr>
<tr>
<td>prob. of a firm of filling a vacancy</td>
<td>q</td>
</tr>
<tr>
<td>prob. of an unemployed person of finding a job</td>
<td>p</td>
</tr>
<tr>
<td>hiring rate</td>
<td>h/l</td>
</tr>
<tr>
<td>wages</td>
<td>w</td>
</tr>
</tbody>
</table>

Table 2: Probabilities and wages

4 Analysis of results

The aim of this section is to examine the effects of labour mobility in explaining regional disparities. More precisely the contribution of mobility to explain
regional unemployment differences when no migration costs are presented. The model applies to two different scenarios. First, we introduce the case where workers do not move from one region to another (the case of infinite migration costs). Then, we study the case where workers have the possibility of moving across regions (the case of no migration costs). Finally, we compare both cases and analyse their results. In both instances, we will first assume that regions are asymmetric and then symmetric to understand the mechanism of the model.

In our exercise we have first assumed asymmetric regions and parameters, which implies ex-ante regional unemployment disparities. Some examples of these disparities may be differing “unemployment benefits”, efficiency of the matching functions and cost of vacancies in each region. First, we analyse the quantitative analysis of steady state effects for (i) the case of no mobility and then (ii) the case of perfect mobility. Secondly, we study the steady state and the dynamic adjustment of the main variables of the symmetric model in which the mobility is affected through expected changes in productivity for both cases a no mobility economy and a mobility economy.

4.1 Persistent unemployment disparities

Firstly, we examine the steady state effects in our model when both regions are asymmetric. Regional disparities would be larger if we assume certain features are different across regions. (i) Unemployment benefits can be greater in one of the regions. In order to represent regional disparities in the economy we interpret the unemployment benefits not only as a payment, but also as all the possible utilities of staying in a given region (i.e. weather, housing, quality of life) The second feature (ii) is different costs of posting vacancies across regions (i.e. more information in job centers in some regions than others, less cost of advertisement). Both cases (as we have seen in Section 4.2.5) influence the labour market tightness. The last feature (iii) is varying matching efficiency of regional markers (see Ibourk et al. (2001) and Petrongolo and Wasmer (1999)). In this case the initial unemployment rates for both regions are different. However, the labour market tightness variables remains equal. This change can be interpreted as the search efficiency is large for workers in their own region because there may exist transport or location costs.

We compare (i) ex-ante no regional unemployment disparities (where labour market tightness are equal) and (ii) with regional unemployment disparities after a change of 10% in different parameters.

4.1.1 Unemployment benefits

We first introduce asymmetries in the unemployment benefits after a positive change of 10% in the unemployment benefits of the region 1. In Table 3, we observe positive relationship between the unemployment and wages for the region with more unemployment benefits and a negative relationship for the region in the case of the mobility economy. This occurs because the labour force in region 1 increases, affecting endogenous probabilities and wages. This situation
increases incentives to workers to go to region 1, however, the unemployment increases while decreases in the region 2.

Moreover, we introduced regional disparities in unemployment rates and differences in the unemployment benefits to the mobility case at the same time. In fact, we can see through the difference between unemployment benefits, we can explain unemployment disparities, interpreted here as unemployment benefits not only as such, but also as other kinds of benefits of staying in the region: weather, housing, quality of life, which may affect workers. For the group of Spanish regions with less unemployment rates (an average of 13%, i.e. Aragon) for the period studied and for the group of Spanish regions with more unemployment rates (an average of 27%, i.e. Andalusia), we obtain that the difference in unemployment benefits is of 22% more in the region with high unemployment rates.

\[
\begin{array}{cccccc}
10\% & b_1 > b_2 & \omega_1 > \omega_2 \\
& \text{NM} & \text{M} & \text{NM} & \text{M} \\
\Delta U_j & 9.54\% & 0 & 7\% & -1.41\% & 13\% & 0 & 3.32\% & 0.20\% \\
\Delta \omega_j & 0.64\% & 0 & 0.03\% & 1.20\% & 0.55\% & 0 & 0.09\% & -0.17\% \\
\end{array}
\]

Table 3: Steady state effects in the asymmetric case (a change of 10% in each parameter)

4.1.2 Cost of posting a vacancy job

If we now consider a change in the cost of posting of 10% in the region 1, the effect is different because the unemployment increases in both regions comparing with the symmetric case, in which labour market tightness is equal. Firms in region 1 post less vacancies and there is a change in the probabilities with an increase in wages in region 1 and a decrease in wages in region 2. This situation provides incentives to workers in region 1 to move to region 2 (see Table 3).

4.1.3 Efficiency of matching vacancies and unemployment

As we can see in Table 4 results for an increase of 10% of the matching efficiency in region 1. More efficiency of matching in one of the region implies that workers move to this region. At the same time the relationship between wages and unemployment is negative for this region while positive for the region with less matching efficiency.
4.2 Asymmetric productivity shocks

Secondly, we want to observe the impact of a shock in one of the regions. For this purpose we introduce a negative permanent regional shock of 1% \((r_j)\) in region 1. Now we will assume that regions are symmetric. This assumption will enable us to study the impact of an asymmetric shock in one region for each case in steady state and then compare both results in order to solve the dynamic adjustment of main variables. Thus, only the labour force mobility assumption will differ from one case to another. Symmetric regions imply regional unemployment rates, as unemployment benefits, efficiency of the matching functions and cost of vacancies in each region are equal. In this case, mobility takes place with expected changes in productivity.

4.2.1 Unemployment persistence and no mobility

For the case of no mobility, each region is isolated so that the shock has no economic effect on the other region. One example of a likely permanent negative regional shock could be illustrated by a regional industrial restructuring as it has been the case in northern Spain in the late seventies with the mining industry. (A positive shock would lead to the opposite conclusions).

<table>
<thead>
<tr>
<th>10% symmetric case</th>
<th>No mobility</th>
<th>Mobility</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Delta U_j)</td>
<td>+2.41%</td>
<td>0</td>
</tr>
<tr>
<td>(\Delta w_j)</td>
<td>-2.08%</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 5: Steady state effects in the symmetric case after a shock of 1% in region 1

A permanent decrease in \(r_j\) reduces the marginal value of labour of region \(j\), making it less worthy for firms to post vacancies in region \(j\). Hence the employment level is lower and the labour market tightness decreases (the ratio \(v/u\)). This leads to two effects for the region which is affected by the negative shock: (i) it decreases the competition among firms and congestion effect, which increases the probability of filling a vacancy and (ii) a negative trade externality
that decreases the probability for an unemployed worker of finding a job. As a result, the unemployment rate increases. For instance, a 1% permanent decrease in the productivity factor of one region leads to a growth of 2.41% in the regional unemployment rate. Finally, the reduction of the marginal productivity of labour lowers the value of the surplus accrued by workers, which makes the real wage to fall, as we show in Table 5. In this Table, we show the changes in unemployment and wages after the shock at regional and at country level.

At country level, we observe that between both regions there is a regional disparity. We observe 2.41% more of unemployment rate compared with the region not affected by the shock. We can interpret it as an increase in regional unemployment rates persistence in a country without possibility to move between regions.

The transitional dynamics of the aforementioned variables are reported in Figure 2. Then the dynamics of employment and unemployment are monotonic towards their new steady state levels. According to the wage setting mechanism, wages are closely related to both the marginal productivity of employment and the labour market tightness. The dynamic adjustment described explains the fall in wages.

4.2.2 The role of the inter-regional mobility

We now analyse the regional model with mobility across regions described in the second section. We have examined the model without mobility in our framework to the baseline case of the Spanish economy. We then introduce a negative shock
of 1% in one of the regions, introducing asymmetries and we analyse what occurs if unemployed workers can move without any restriction in this economy. As in the case of no mobility we analyses a permanent decrease of the productivity of a region, which might be seen as a recession with a permanent effect in this region, or a pure labour demand shock.

A first effect of a permanent decrease of productivity is a downward of the probability of workers to find a job in this region. Indeed, the effect is the same in both regions though the reason is different. In the affected region by the negative shock, a lower marginal productivity of employment and a high cost of posting vacancies make the firm to hire less workers in this region, decreasing employment. As a result the probability of finding a job decreases. However, a significant quantity of the unemployed workers move to the other region with better employment opportunities. In the second region, without negative shock, the labour force increases as well as the number of unemployed workers. In both cases labour creates a negative trade externality on the supply side which, via the matching process, makes it much harder for unemployed workers to find a job.

On the other hand, it decreases the congestion effect that benefits firms and increases the probability of filling a vacancy. Firms post less vacancies and the number of unemployed increases. In Table 5, we show that unemployment increases in both regions but, as probabilities by different reasons. For instance, a 1% permanent decrease in the productivity factor of one region leads to growth of 0.33% in the unemployment rate of this region (lower than the regional rate in the model with no mobility 2.41%).

In Table 5, we also observe the negative relationship between wages and unemployment, larger when we are in the no mobility case. The total output per capita in the economy with mobility is larger than in the economy without mobility.

Figures 3 and 4 show the transitional dynamics. Mobility across regions adjusts labour force through wages due to different marginal productivity of employment and employment opportunities. We observe that workers adjust in one period. The shock occurs and workers move adjusting until to the new steady state. There is no mobility cost and capital accumulation. Therefore, once the shock takes place the migrants have incentives to move to another region due to the change in their initial situation. In this case, we can observe that the introduction of the endogenous wages through the non-arbitrage conditions allows labour mobility until the moment that probabilities and wages are the same in both regions and the workers have no incentive to move.

On the one hand, workers move from region 1 to region 2 increasing labour force in the region 2 (the one without productivity change). The same decrease occurs in labour force of region 1. On the other hand, there is a symmetric growth of the number of vacancies posted in region 2.

In the short term, unemployment rates of region 2 increase because of a growth their labour force. However, there is a decrease in unemployment rate in region 1 due to the labour force mobility and the decrease of probabilities of finding a job in the short term. Figure 4 shows wage reduction in region 1 and
Figure 3: Impulse-response function of unemployment and employment variables

region 2. We also observe the wage reduction is larger at in region 1. Region 2 absorbs migration from region 1 reducing wages in this region, but less than in region 1. Both regions come back to a new steady state. In the long term, Region 2 shares the negative productive shock of region 1, and as a result both unemployment rates end up at the same level.

Figure 4 shows changes immediately the probability of filling a vacancy. The dynamic of the probabilities will change in the opposite side in region 1. The impact in unemployment growth is lower than the no mobility scenario and the endogenous wages work as stabilised factors of adjustment. Hence the output in the region without any shock increases due to the positive effect of migration through probabilities.

Thirdly, we also compare an asymmetric case facing a productivity shock in the region 1. When we introduce in asymmetric regions with different unemployment benefits the changes are similar to the symmetric case. However, in particular a productivity change in the region with more unemployment benefits has also more growth in unemployment rates, but decrease in wages. Wages are lower than the region with more unemployment benefits and output per capita lower too. In total economy and comparing to no mobility case, the unemployment growth is lower when the economy share asymmetric shocks through mobility as well as the output per capita slightly larger with mobility than without mobility. If we introduce a productivity shock in region with more cost of posting a vacancy and in the region with more matching efficiency, the implica-
tions are similar to the increase in unemployment benefits. And the relationship between wages and unemployment is negative for both regions.

In conclusion, mobility in the whole economy can represent unemployment disparities and show an increase in the unemployment rates when there is a deterioration in the economy. We also observe the effect of mobility face to different changes. In general, we observe an improvement compared to the no mobility case.

Depending on which parameter there is the change there are less or more regional disparities. When there is a change in unemployment benefits seems to be more unemployment disparities, however, in relative terms between no mobility case and mobility one, there are more differences when there is a change in the efficiency of matching and or a change in the cost of posting a vacancy.

The asymmetric productivity shocks causes less disparities than the other changes. Even if we evaluate the mobility without migration cost, we can see the role the mobility in order to shed some light on unemployment disparities. One of the results, the growth of unemployment is large when there is no mobility.

Another important result is the different relationship between wages and unemployment in each case. When there is a negative productivity shock in one of the regions, the relationship is always negative. However, when we have done different sensitivity analysis in the parameters, the wages-unemployment variables move in the same direction when we are in the region affected by an economic deterioration. And the other region has a negative relationship with wages. We can also show that the output per capita is always higher in the case
of mobility than in the opposite case.

In fact we can also observe a depopulation effect and geographic concentration. In this model, when there is a change in the different conditions, workers have the incentive to move to the other region with better conditions. However, there is not a negative effect in the whole of the economy because workers take into account the probabilities of finding a job and wages. In the literature there exist some explanations of geographic concentration to explain unemployment disparities (see de la Fuente (2001) and Overman and Puga, (1999)), which we have not developed.

5 Conclusion

This paper aims to study regional unemployment disparities and the interaction between unemployment and inter-regional mobility in a country against asymmetric shocks. We build a model with inter-regional migration of the labour force with the matching model in a framework of dynamic equilibrium. Workers measure this discounted value when faced with different job opportunities in different regions. Wage setting mechanisms are determined with a Nash bargaining criterion. We compare the case of an economy characterised by no mobility and unemployment persistence as the extreme case of Spain; and we compare this case when workers are permitted to move.

When mobility of the labour force is allowed, asymmetric shocks have effects in both regions. With a permanent negative shock, unemployment rates of both regions increase. There exists an adjustment that both regions must support in the short-term. However, both regions share the shock in the long-term. Mobility is important because the regional disparities between two regions would reduce compared with no mobility.

Moreover, the welfare measured as the output per capita in the economy with mobility is slightly larger than in the economy without mobility. We also find a negative correlation between wages and unemployment.

This paper also studied different types of asymmetries such as changes in “unemployment benefits” and/or the cost of posting vacancies. This was done with the purpose of explaining regional disparities and above all the contribution of mobility without taking into account migration costs. We show the importance of these changes to explain regional unemployment disparities.

We conclude that inter-regional mobility seems to be an alternative and/or a complement to other policies against unemployment. Although this is a baseline model, it can shed light on how an economy like Spain with regional unemployment persistence and low labour force mobility could, through mobility, reduce its regional disparities and explain how different factors (i.e. differences in utilities of staying in a region) can contribute to these disparities.
References


6 Appendix

6.1 Appendix 1: The firm.

The firm maximises the value function taking into account their discounted future profits with respect to the vacancies, \( v_{i,j,t} \), with \( 0 < \beta < 1 \):

\[
\Upsilon^F(n_{i,j,t}) = \max_{v_{i,j,t}} \left\{ \Pi_{i,j,t} + \beta \Upsilon^F(n_{i,j,t+1}) \right\}
\]  

(24)

where the maximisation is constrained by

\[
\Pi_{i,j,t} = (a_t + r_{i,j,t})n_{i,j,t} - w_{j,t}n_{i,j,t} - \pi_j v_{i,j,t}
\]  

(25)

\[
n_{i,j,t+1} = q_{i,j,t}v_{i,j,t} + (1 - s_j)n_{i,j,t}
\]  

(26)

where equation (25) the present profit flow of the firm, which depends on the production function minus the real wage times the employment in region \( j \), and minus the cost of having vacancies times vacancies. The other constraint is equation (26), the evolution of employment. \( X^n_{i,j,t} \) the Lagrange multiplier associated to the employment. The firm solves the recursive problem 24 subject to (25)-(26). The first order condition of this optimisation with respect to \( v_{i,j,t} \), implies

\[
X^n_{i,j,t} = \frac{\omega_j}{q_{j,t}}
\]  

(27)

The firm will post a vacancy when the left part of equation (27), the marginal revenue from filling a vacant job in the region \( j \) is equal to the marginal cost to post a vacancy. The marginal value of the employment for the firm is given by the envelope theorem:

\[
\frac{\partial \Upsilon^F(n_{i,j,t})}{\partial n_{i,j,t}} = (a_t + r_{i,j,t}) - w_{j,t} + (1 - s_j)X^n_{i,j,t}.
\]  

(28)

Both envelope condition and the first order condition yield Euler equation related to employment:

\[
X^n_{i,j,t} = \beta \frac{\partial \Upsilon^F(n_{i,j,t+1})}{\partial n_{i,j,t+1}}.
\]  

(29)

6.2 Appendix 2: Wage determination

Both parts take into account their net marginal values when negotiate. The net marginal value of employment for a worker may be expressed as:

\[
\Upsilon^W_{j,t} = V^E_{j,t} - V^U_{j,t}.
\]  

(30)
The net marginal value of current employment for a representative firm is\(^{16}\)

\[ \Upsilon_n^F(n_{j,t}) = (a_t + r_{j,t}) - w_{j,t} + (1 - s_j)\beta E_t \Upsilon_n^F(n_{j,t+1}). \]  
(31)

Let 0 < \(\xi\) < 1 denotes the bargaining power of the worker. The Nash bargaining criterion maximises over wages their net marginal values:

\[ \max_{w_j} \left\{ (\Upsilon_n^F(n_{j,t}))^{1-\xi} (\Upsilon_{j,t}^W)^{\xi} \right\}, \]

(32)

where the first order condition states that:

\[ \Upsilon_n^F(n_{j,t}) = \frac{1-\xi}{\xi} \Upsilon_{j,t}^W. \]

(33)

We substitute (30) in equation (33) to obtain:

\[ \xi \Upsilon_n^F(n_{j,t}) = (1 - \xi) [V_{j,t}^E - V_{j,t}^U]. \]

(34)

If we replace by the values of \(V_{j,t}^E\) and \(V_{j,t}^U\) from equations 10 and 11, considering the workers’ non-arbitrage condition implies \(V_{j,t}^U = V_{1,t}^U = V_{2,t}^U, \forall t\), we have the following expression:

\[ \xi \Upsilon_n^F(n_{j,t}) = (1 - \xi) \left[ w_{j,t} - b_j + \beta E_t \left\{ (1 - s_j - p_{j,t})(V_{j,t+1}^E - V_{1,t+1}^U) \right\} \right]. \]

(35)

Rearranging, we get

\[ \Upsilon_n^F(n_{j,t}) = \frac{(1-\xi)}{\xi} (w_{j,t} - b_j) + (1 - s_j - p_{j,t})\beta E_t \Upsilon_n^F(n_{j,t+1}). \]

(36)

As we know from equation (29), \(X_{j,t}^n = \beta \Upsilon_n^F(n_{j,t+1})\), we can rewrite the equation above as:

\[ \Upsilon_n^F(n_{j,t}) = \frac{(1-\xi)}{\xi} (w_{j,t} - b_j) + (1 - s_j - p_{j,t})X_{j,t}^n. \]

(37)

Substituting the value of \(\Upsilon_n^F(n_{j,t})\) from equation (31) and rearranging, we have the wage evolution equation:

\[ w_{j,t} = \xi \left\{ (a_j + r_{j,t}) + p_{j,t}X_{j,t}^n \right\} + (1 - \xi)b_j. \]

(38)

\(^{16}\)Recall \(\Upsilon_n^F(n_{j,t}) = \frac{\partial \Upsilon_n^F(n_{j,t})}{\partial n_{j,t}}\).
6.3 Appendix 3: Mobility

The adjustment of the mobility\(^ {17} \) these non arbitrage conditions will occur when the present value of an unemployed in region 1 is equal to the present value of an unemployed in region 2:

\[
V_{1,t}^U = V_{2,t}^U.
\]  

(39)

After solving it we obtain from equation 11,

\[
b_1 - b_2 = \beta E_t \left[ p_{2,t} \left[ V_{2,t+1}^W - V_{1,t+1}^W \right] + [V_{2,t+1}^U - V_{1,t+1}^U] \right].
\]  

(40)

Considering the non-arbitrage condition \( E_t V_{2,t+1}^U = E_t V_{1,t+1}^U \), the last term disappears. Then, we substitute the first order condition of the Nash program, equation (16), in the expression \( V_{j,t+1}^E - V_{j,t+1}^U \). Taking into account that the first order condition of the firm \( \beta Y_n^F (S^F_{j,t+1}) = X_{j,t}^n \), we obtain the net value of migration for an unemployed worker to move from region 1 to region 2 is given by the following expression:

\[
b_1 - b_2 = \frac{\xi}{1 - \xi} \left\{ p_{2,t} X_{2,t}^n - p_{1,t} X_{1,t}^n \right\}.
\]  

(41)

\[
b_1 - b_2 = \frac{\xi}{1 - \xi} \left\{ \varpi_2 \theta_{2,t} - \varpi \theta_{1,t} \right\}.
\]  

(42)

When symmetric regions, the non-arbitrage condition is \( \theta_{2,t} = \theta_{1,t} \), only asymmetric shocks can difference both regions, and they will be equal at steady state. However, if unemployment benefits and cost of posting vacancies are different across regions, we are facing of asymmetric regions and the non arbitrage condition is the equation (41). There is not symmetric effects, with regional disparities are larger. (i.e. if \( \varpi_2 < \varpi_1 \) then \( \theta_{1,t} = \frac{\varpi_2 \theta_{2,t}}{\varpi_1} \)). Workers will move until that transversality conditions of the model are satisfied.

By the first order condition of the firm \( X_{j,t}^n = \frac{\omega_j}{q_j,t} \), we substitute above in equation (41)and we get

\[
b_1 - b_2 = \frac{\xi}{1 - \xi} \left\{ p_{2,t} \frac{\omega_2}{q_2,t} - p_{1,t} \frac{\omega_1}{q_1,t} \right\}.
\]  

(43)

From the definition of \( q_{j,t} \), we substitute

\[
\frac{\omega_j}{q_{j,t}} = \frac{\omega_j v_{j,t}}{p_{j,t} (\ell_{j,t} - n_{j,t})}
\]  

(44)

\(^{17}\)When there is no mobility the model converge to the stationary equilibrium in each region as in the standard matching model. When we assume perfect mobility assumption, there exists a restriction on the probabilities and the tightness of labour market coming from the non arbitrage conditions of the workers and the firm until both endogenous probabilities and wages are equal.
in expression (43), to determine labour force in region 1:

\[
\ell_{1,t} = n_{1,t} + \frac{\omega_1 v_{1,t}}{\frac{1-\xi}{\xi} (b_2 - b_1) + p_{2,t} \frac{u}{q_{2,t}}}.
\]

(45)

6.4 Appendix 4: Equilibrium

\[
u_{1,t} = \ell_{1,t} - n_{1,t}
\]

(46)

\[
u_{2,t} = 1 - \ell_{1,t} - n_{2,t}.
\]

(47)

\[
h_{j,t} = h(u_{j,t}, v_{j,t}) = \mathbb{H}_j v_{j,t}^{\gamma} u_{j,t}^{1-\gamma},
\]

(48)

\[
p_{j,t} = \frac{h_{j,t}}{u_{j,t}}
\]

(49)

\[
q_{j,t} = \frac{h_{j,t}}{v_{j,t}}
\]

(50)

\[
n_{j,t+1} = q_{j,t} v_{j,t} + (1 - \eta_j) n_{j,t},
\]

(51)

\[
y_{i,j,t} = (a + r_j) n_{i,j,t},
\]

(52)

\[
X_{i,j,t}^n = \frac{w_j}{q_{i,j,t}}
\]

(53)

\[
X_{i,j,t}^{n+1} = (a_{t+1} + r_{j,t+1}) - w_{j,t+1} + (1 - \eta_j) \beta X_{i,j,t+1}^n
\]

(54)

\[
w_{j,t} = \xi \left[ (a_j + r_{j,t}) + p_{j,t} \frac{w_j}{q_{i,j,t}} \right] + (1 - \xi) b_j
\]

(55)

\[
\ell_{1,t} = n_{1,t} + \frac{\omega_1 v_{1,t}}{\frac{1-\xi}{\xi} (b_2 - b_1) + p_{2,t} \frac{u}{q_{2,t}}}.
\]

(56)

The endogenous variables: \( u_1, u_2, h_j, q_j, p_j, n_j, X_j, v_j, w_j, y_i, \ell_1. \)

\[\text{Both envelope condition and the first order condition yield Euler equation related to employment. We get this equation with both equations (4.28) and (4.29).}\]