Oil Price Shocks: Testing for Non-linearity

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March 2004
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Abstract

This paper presents evidence of a non-linear relationship between GDP growth and oil price changes in the US economy. We also argue that this non-linearity is not merely due to the use of data from the mid-1980s onwards, as most authors, so far, seem to believe. In fact, we find the existence of non-linearity with the use of data earlier than 1984, and even before 1977. Furthermore, we question that the non-linear transformations of oil prices proposed in the literature are the most appropriate indicators for reflecting such non-linearity.

Keywords: Macroeconomic fluctuations; Oil price shock; Non-linearity.

JEL codes: E32.

Acknowledgement: I am grateful to Arielle Beyaert, Gonzalo Camba, George Kapetanios, Juan Mora, Gabriel Pérez-Quiros, and Juan Toro for their helpful comments. I also acknowledge the financial support from IVIE and from a Marie Curie Fellowship of the European Community programme Improving Human Potential (contract number HPMD-CT-2001-00069).

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References
1 Introduction

From the middle of twentieth century onwards, crude oil has become one of the main indicators of economic activity worldwide, due to its outstanding importance in the supply of the world’s energy demands.

Nowadays, the importance of crude oil as the main source of energy has waned somewhat, due to the appearance of alternative forms of energy (such as wind, water, and solar power). Notwithstanding this, the importance of oil exceeds economic aspects and affects social life in general. One of the issues that the public has been particularly concerned about is oil-price fluctuations, so that these fluctuations have become one of the current affairs published on the front pages by the vast majority of the world’s newspapers, mainly from the Yom Kippur War (October 5, 1973) on. Thus, the prevailing view among economists is that there is a strong relationship between the growth rate of a country and oil-price changes.\(^1\) Precisely what form this relationship takes, and how it might be modified, and other such questions are issues of outstanding value.

As such, the relationship between the macroeconomic variables and the oil-price shocks has been extensively analyzed in the literature, but especially so over the last twenty-five years. Hamilton (1983), Burbidge and Harrison (1984), Gisser and Goodwin (1986), Mork (1989), Hamilton (1996), Bernanke et al. (1997), Hamilton and Herrera (2001), Hamilton (2003), and several others, have concluded that there is a negative correlation between increases in oil prices and the subsequent economic downturns in the United States.\(^2\) The relationship seems weaker, however, when data from 1985 onwards is included.\(^3\) Nevertheless, the role of the break-date, 1985-86, has been only considered by a very few researchers, most of whom argue that the instability observed in the relationship may well be due to a mis-specification of the functional form employed. The linear specification\(^4\) might well mis-represent the relationship between GDP growth and oil prices.

This mis-representation of the linear specification has led to different attempts to re-define the measure


\(^2\)Figure 1 shows the historical behaviour of the US GDP growth and the percentage changes in nominal oil price (See Data Appendix). Recessions, as dated by National Bureau of Economic Research, are shaded.

\(^3\)It is worth noting that there was a decline in the price of oil of more than 50% in 1986:I.

\(^4\)We should emphasise that all of these authors, with the exception of Hamilton (2003), consider the GDP-Oil price relationship within a linear multivariate context.
of the oil-price changes. These attempts are based on non-linear transformations of the oil prices, in an effort to re-establish the correlation between GDP growth and oil prices. In fact, they are, actually, attempts to restore the Granger-causality between oil prices and GDP, which disappears when data from 1985 onwards (i.e., periods of oil-price declines) is included. On the one hand, Mork (1989) finds asymmetry between the responses of the GDP to oil-price increases and decreases, concluding that the decreases are not statistically significant. Thus, his results confirm that the negative correlation between GDP and increases in oil-price is persistent when data from 1985 onwards is included. Lee et al. (1995), on the other hand, report that the response of the GDP to an oil-price shock depends greatly on the environment of oil-price stability. An oil shock in a price stable environment is more likely to have greater effects on GDP than one in a price volatile environment. These authors thus propose a measure that takes the volatility of oil prices into account.\(^5\) They find asymmetry in the effects of positive and negative oil-price shocks, but they also manage to re-establish the above-mentioned negative correlation. In the same way, Hamilton (1996) claims that it seems more appropriate to compare the prevailing price of oil with what it was during the previous year, rather than during the previous quarter. He therefore proposes defining a new measure, the NOPI,\(^6\) which also restores the negative correlation between GDP and oil-price increases.

In such a context, Hooker (1996a) perceives the existence of a break-point in 1973:III,\(^7\) observing the existence of Granger-causality in the first subsample (1948:I-73:III), although not so in the second one (1973:IV-94:II) nor in the full sample (1948:I-1994:II) either. He thus concludes that the oil price-GDP relationship changes when data from the mid-1980s is considered, since a simple oil-price increase/decrease asymmetry is not enough to represent it accurately. Likewise, Hooker (1999) argues that Lee-Ni-Ratti’s (1995) and Hamilton’s (1996) transformations do not, in fact, Granger-cause GDP in post-1980 data,\(^8\) but that their apparent success is due to an improved fit in the 1950s data. Finally, Hamilton (2003) reports evidence of a non-linear representation and states that the functional form that relates GDP to oil prices looks very much like what has been suggested in earlier parametric studies. More specifically, he analyzes

\(^5\)Specifically, they capture these features through a GARCH model based on an oil-price transformation that scales estimated oil-price shocks by their conditional variance.

\(^6\)Net Oil Price Increase (NOPI) is defined as the percentage increase in oil price if the quarter’s price exceeds the previous year’s maximum, and zero otherwise.

\(^7\)He argues that 1973 marks the beginning of the productivity slowdown, the period of the floating exchange rate, and several years of unusually low real interest rates. Furthermore, there have been different institutional regimes that have been determining oil prices since 1973.

\(^8\)He now considers the existence of a break-point around 1980.
the non-linear transformations of oil prices proposed in the literature, and he points out that, on the basis of the non-linearity test (Hamilton, 2001), the Lee-Ni-Ratti’s formulation does the best job of summarizing the non-linearity.

The aim of our study, therefore, is threefold: to analyze whether the relationship between oil prices and GDP growth is linear or not, to study whether the above-mentioned non-linear transformations are appropriate for representing the non-linearity, and to identify the dating of the non-linearity.

This paper presents evidence of a non-linear relationship between GDP growth and fluctuations in the price of crude oil and argues that despite the fact that the above non-linear specifications do not take oil-price decreases into account they continue to give problems with the out-of-sample forecast. We question the non-linear transformations previously mentioned, since they are rather ad hoc and only consider oil-price increases. There seems, therefore, to be some form of data-mining.

This paper also argues that the non-linearity observed is not merely due to the use of data from the mid-1980s onwards, as many authors have been suggesting up to now. Indeed, we find the existence of non-linearity with the use of data from before 1984, and even earlier than 1977.

We develop the paper on three different parts. We first take the traditional linear approach as a starting point, summarizing economic activity through a seven-variable system, in particular, a VAR specification. With this model, we verify the accuracy of the out-of-sample forecasting. We also discuss the results of the Granger-causality analysis in both a bivariate and a multivariate context, as well as presenting the results of the parameter stability analysis. Moreover, we consider the effects of a positive oil shock through the orthogonalized impulse-response functions. Secondly, we challenge that the non-linear transformations are the most appropriate indicators for summarizing the non-linearity. Finally, we test the linear specification and such transformations with the non-linearity test proposed by Hamilton (2001), and we also study the dating of the non-linearity.

The paper is organized as follows. Section 2 describes the linear approach. Section 3 presents the non-linear transformations. Section 4 presents evidence of non-linearity, and the dating of such non-linearity. Concluding remarks are offered in Section 5.
2 First Approach: Linear Model

2.1 Previous Considerations

We begin by modelling the economy of the United States, considering financial, output, and price variables, which summarize economic activity. Our aim is to analyze the relationship between output variables and oil-price changes. One of the main problems of using this sort of modelling, however, is the difficulty in choosing of the specific variables that should be included. We obviously choose the ones that we consider to be most relevant for our goals.

We consider the “chain-weighted real GDP”, $gdp_t$, and the unemployment rate, $ur_t$, as output variables; the long-run interest rate, $lr_t$, and the Federal funds rate, $fed_t$, as financial variables; wage, $w_t$, consumer price index, $cpi_t$, and a measure of oil-price change, $oil_t$, as price variables (See Data Appendix).

It is our belief that an oil-price shock has both direct and indirect effects on macroeconomic variables. The indirect effects might come from the responses of the monetary policy to the shock, so that we have included two monetary variables. Our belief in the existence of indirect effects of an oil-price shock through monetary responses is based on the movements observed in the monetary variables after the shocks, especially after increases, as well as on the fact that there are several papers that support this belief. Bohi (1989), among others, argues that the economic downturns observed after oil-price shocks are caused by the price-shocks themselves and by the monetary responses to them. Along these lines of thought we find Bernanke et al. (1997), who state that the effects of an oil shock in isolation (i.e., without responses from monetary policies) are considerably smaller than when monetary responses are considered. Hamilton and Herrera (2001), on the other hand, challenge the Bernanke-Getler-Watson conclusion on two basic grounds: (a) the feasibility of the monetary policy proposed, and (b) the short monthly lag length used in their specification. They conclude that the potential of monetary policy to avert the contractionary consequences of an oil-price shock is not as great as Bernanke et al. (1997) suggest, although they could not disregard the Bernanke-Getler-Watson conclusions on the effects of the monetary policies undertaken.

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9 At the beginning, we considered a bivariate model with GDP growth and oil-price changes as variables. Figure 2 shows that this specification forecasts important decreases in GDP for the mid-1970s and notable increases in GDP for the mid-1980s, which is not observed in the GDP. We perform the omitted variables test and observe that more variables are required to improve the model.

10 We shall also refer to oil-price changes as $o_t$.

11 We have chosen these variables, considering the six-variable dynamic system developed by Sims (1980), as a reduced-form of macroeconomic reality.
after oil shocks. We do not extend more about the indirect effects, leaving it as an open question, since it can be an outstanding issue for future research.

2.2 Linear Macroeconomic Model

We denote \( y_{m}^{t} = (y_{1}^{m}, y_{2}^{m}, y_{3}^{m}, y_{4}^{m}, y_{5}^{m}, y_{6}^{m}, y_{7}^{m})' \) = \((\text{gdp}_{t}, \text{ur}_{t}, \text{oil}_{t}, \text{cpi}_{t}, \text{fed}_{t}, \text{lr}_{t}, \text{w}_{t})'\), which is a \((7 \times 1)\) vector.

One way of summarizing the economic activity is to represent it through a seven-variable system. Specifically, we model it as a \(p\)th-order vector autoregression, \(\text{VAR}(p)\):

\[
y_{m}^{t} = c + \sum_{i=1}^{p} \Phi_{i}y_{m}^{t-i} + \xi_{t},
\]

where \(c = (c_{1}, ..., c_{7})'\) is the \((7 \times 1)\) intercept vector of the \(\text{VAR}\), \(\Phi_{i}\) is the \(i^{th}\) \((7 \times 7)\) matrix of autoregressive coefficients for \(i = 1, 2, ..., p\), and \(\xi_{t} = (\xi_{1t}, ..., \xi_{7t})'\) is the \((7 \times 1)\) generalization of a white noise.

Assuming that \(\xi_{t}\) is a Gaussian white noise process, the \(\text{VAR}\) can then be estimated by Maximum Likelihood.\(^{12}\) The estimate sample used (including the lagged initial values) runs from 1960:I to 2000:III,\(^{13}\) for a total of \(T = 163\) useful quarterly observations. We choose a lag-length of four periods on the basis of the Information Criteria and the Sims’ modification (1980) of the Likelihood Ratio Test. Hereafter we consider a \(\text{fourth-order VAR}\).

We now briefly comment on the results obtained from the different tests performed in the \(\text{VAR}\) context. We first observe that oil prices do not appear to be significant variables in the GDP equation of the multivariate \(\text{VAR}\) (See Table 1), although when we consider the bivariate \(\text{VAR}\) the fourth lag of oil prices is statistically significant in the GDP equation. Secondly, the Wald test, whose null hypothesis is that all lags of oil-price changes are zero in the GDP equation, gives us a \(\chi^{2}\) statistic of 1.188 with an accompanying p-value of 0.8799, indicating that all lags of oil-price changes are not statistically significant as a whole in the GDP equation of the multivariate \(\text{VAR}\). Finally, all of the equations, except for the one for oil prices, are jointly significant in explaining the dependent variable (See Table 2). There is a clear intuitive explanation for this: oil prices are fixed\(^{14}\) on the worldwide crude oil market, which considers

\(^{12}\)It is well known that it is enough to estimate the system by OLS, equation by equation, to get such estimates.

\(^{13}\)We have used this sample size because the available sample for the unemployment rate starts in 1960:I.

\(^{14}\)Right up to 1973, oil prices were controlled mainly by the Texas Railroad Commission and other institutions. From this date onwards, however, and right through to the 1980s, the OPEC countries began to dominate the worldwide petroleum market, and, from then on, the forces of the free market have been establishing the price of crude oil.
both the demand and the supply. As such, although the US might be an important part of that demand, it is no longer able to fix oil prices as it wishes.

Figures 3 presents the one-period-ahead out-of-sample forecasting of GDP growth in the VAR. As can be observed, the “problem” of the 1980s was not very important, but the linear out-of-sample forecasting has not been very accurate. As such, we can tend to believe that there is a structural change in the GDP equation of the multivariate VAR. But can we verify this? We find the answer by analyzing the existence of a structural change in both the oil-price coefficients and in all of the regression coefficients. Figure 4 presents the p-values for a test of the null hypothesis that all oil-price coefficients are stable (in the Chow’s sense) against the alternative hypothesis that these coefficients change on a given date on the horizontal axis. We look for the existence of a break-point in the period that runs from 1970:IV to 1990:III. We note that there is no evidence of a structural change for any given date. This result is confirmed when we consider the optimal tests: Andrews’ test (1993), and Andrews and Ploberger’s (1994) tests (both the average and exponential specifications) (See Table 3). Nevertheless, when we consider the possibility of a structural change in all of the regression coefficients (See Figure 5), the results of the Chow test indicate the possible existence of a break-point, suggesting that the possible instability might come from other variables and not from oil prices. This is confirmed when we look at the asymptotic p-values of the optimal tests (See Table 4), although this is not the case when we consider the bootstrap p-values of such tests. Specifically, the bootstrap p-values indicate that there is stability in all of the coefficients at a 5% critical level.

Finally, as we are interested in verifying whether an increase in the price of crude oil Granger-causes the recession, and also whether a decrease Granger-causes the economic boom, we shall discuss the results of the Granger-causality analysis within both a bivariate and a multivariate framework. We first perform the bivariate Granger-causality test for each variable of the VAR with respect to oil prices for the full sample (See Table 5), and for GDP growth with respect to oil prices for different subsamples (See Figures 6.1

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15 These tests are asymptotically optimal tests for parameter instability and structural change with an unknown break-point, which is a nuisance parameter that exists under the alternative hypothesis but not under the null.

16 This Table reports the asymptotic p-values developed by Hansen (1997), and the bootstrap p-values suggested by Hansen (2000) under the assumptions of homoskedastic and heteroskedastic disturbances.

17 If we consider the bivariate VAR, we obtain that neither the oil price coefficients nor all of the regression coefficients have changed at any date. These results were confirmed by Andrews’ test and Andrews-Ploberger’s tests, using $\pi = 0.15$. (The results are available from the author upon request).

18 To do so, we consider the longest available sample for these two variables, i.e. 1947:II-2001:III.
In the full sample, the oil price only Granger-causes the unemployment rate at a 5% critical level and CPI at a 10% critical level. Moreover, if we consider, on the one hand, the first subsample that runs from 1947:II to the date indicated \( t_1 \) in the horizontal axis (Figure 6.1), we obtain that oil-price changes Granger-cause GDP growth when \( t_1 \) is any date between 1974:III and 1986:III. On the other hand, if we consider the second subsample that runs from the date indicated \( t_1 \) in the horizontal axis to 2001:III (Figure 6.2), we obtain that oil-price changes do not Granger-cause GDP growth on any date at all. It is clear, therefore, that oil-price changes do not Granger-cause (in the bivariate sense) GDP growth either in the full sample or in the second subsample, although causality appears when we consider subsamples that end before 1986:III. Secondly, we carry out a Granger-causality analysis in a multivariate context. In this context, the concept of Granger-causality is assessed in terms of both a Wald test and a so-called test of block exogeneity. Thus, the Wald test considers the null hypothesis that all of the oil-price coefficients are jointly zero in the GDP equation of the VAR model. Figures 7.1 and 7.2 plot the p-values for the first and second subsamples, respectively. With regard to block-exogeneity test, we consider three different aspects. We first test for whether oil-price changes are Granger-caused by the remaining variables of the system. The results are reported in the first line of Table 6. Second, we verify whether oil-price changes Granger-cause the remaining variables of the system and present the results in the second line of Table 6. Finally, we also consider here the test for the lack of any relationship between oil-price changes and the rest of the system, the results of which are reported in the third line of Table 6.

The results of the Wald test, with a \( \chi^2 - \text{Statistic} \) of 1.188 (\( p-value = 0.8799 \)), indicate that oil-price changes do not Granger-cause GDP growth in the full sample. On observing different subsamples, the first one, which runs from 1960:I to the date indicated in the horizontal axis (See Figure 7.1), shows us that oil-price changes do not Granger-cause GDP growth. Likewise, the second subsample, which runs from the date indicated in the horizontal axis to 2000:III (See Figure 7.2), illustrates that oil-price changes only Granger-cause GDP growth if the subsample starts in any quarter of 1980, otherwise the Granger-causality disappears. On considering the three aspects of the block-exogeneity test (See Table 6), we obtain that

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19 The results with the sample employed in the multivariate VAR are as follows: on the one hand, if we consider the first subsample, which runs from 1960:I to any date \( t_1 \) beyond 1969:I, we obtain that oil-price changes Granger-cause GDP growth when \( t_1 \) is any date between 1970:I and 1982:II (with exceptions) or any date between 1983:II and 1986:III. On the other hand, if we consider the second subsample, which runs from any date \( t_1 \) between 1960:I and 1991:III up to 2000:III, we obtain that oil-price changes do not Granger-cause GDP growth on any date at all. (These results are available from the author upon request).
oil-price changes are not Granger-caused by the remaining variables of the system.\textsuperscript{20} However, oil prices Granger-cause the rest of the system and we observe a relationship among the variables considered.

The results therefore indicate that the interaction between oil-price changes and macroeconomic variables is significant, with oil-price changes Granger-causing the rest of the system. Notwithstanding this, oil prices do not directly Granger-cause GDP growth, either in the multivariate context or in the bivariate context, for the full sample.\textsuperscript{21} Furthermore, the analysis of both bivariate and multivariate Granger-causality indicates that bivariate Granger-causality appears if we consider subsamples that start in 1947:II (or 1960:I) and end before 1986:III,\textsuperscript{22} otherwise causality disappears. In addition, there is no bivariate Granger-causality in the second subsample.\textsuperscript{23} This analysis also indicates that multivariate Granger-causality is not found in either the first or the second subsample,\textsuperscript{24} but only when a subsample that starts in any quarter of 1980 and ends in 2000:III is considered.\textsuperscript{25}

2.3 The effects of an oil price shock

In order to appreciate the effects of an oil-price shock in the VAR context, we examine the orthogonalized impulse-response functions, using a Cholesky decomposition. This orthogonalization method involves the assignment of contemporaneous correlation to specific series. As such, we place the oil-price variable at the top of the following ordering of the variables: \((oil, gdp, cpi, fed, lr, w, ur)\).\textsuperscript{26}

Figure 8 shows the response of GDP growth, over 24 quarters, to one standard deviation oil-price shock. We only comment on the response of GDP growth to oil-price innovations. An oil-price innovation has a negative influence on GDP growth, and its greatest negative effect occurs during the fourth quarter.

\textsuperscript{20}See reasoning discussed with regard to Footnote 14.
\textsuperscript{21}Mork (1989), Lee et al. (1995), Hooker (1996a), and Hooker (1999) find similar results.
\textsuperscript{22}Lee et al. (1995) obtain that real oil price Granger-causes GNP growth considering data from 1950:III to 1986:I.
\textsuperscript{23}In this regard, Hooker (1999) finds that Granger-causality is dissipated for second subsamples that start around 1980 and end in 1998:IV.
\textsuperscript{24}Hamilton (1983), Lee et al. (1995), Hooker (1996a), and Hooker (1999) observe the existence of Granger-causality (in the multivariate sense) for samples that end in 1972:IV, 1986:I, 1973:III, and before 1980:I, respectively. The latter two studies, however, indicate that such causality does not appear with the second subsample.
\textsuperscript{25}Hooker (1999) observes a similar exception when he analyses the second subsample in the multivariate model with a 4-quarter lag.
\textsuperscript{26}We are considering the contemporaneous influence of oil-price innovation on GDP growth with this ordering of the variables. We have verified that the impulse-responses do not substantially change when we consider a different sort of ordering. The contemporaneous effect is all that changes, being zero when the oil-price variable is not placed at the top of the ordering.
following it. This is entirely consistent with the result obtained by most of studies carried out on the topic.

We have observed that the linear model creates some problems, basically, in out-of-sample forecasting. Furthermore, this model indicates that oil-price changes do not Granger-cause GDP growth in the full sample, although there is an interaction between oil-price changes and macroeconomic variables with oil-price changes Granger-causing the remaining variables of the system. In trying to solve such problems, different non-linear transformations of oil prices have appeared. In the following section, we briefly present the main non-linear transformations proposed in the literature.

3 Non-linear transformations

The literature offers evidence of a non-linear relationship between GDP growth and oil-price changes. Mork (1989), Lee et al. (1995), Hamilton (1996), Hooker (1996a), Hamilton (2003), among others, “found” evidence against the linear specification. Mork (1989), Lee et al. (1995), and Hamilton (1996), all propose non-linear transformations of oil-price data to capture such non-linearity. Hamilton (2003) verified the existence of a non-linear relationship, offering more evidence against linearity and identifying non-linearity with some of the above non-linear specifications. Hooker (1996a, 1996b, 1999) also reports non-linear evidence, and although he criticizes the specifications previously mentioned, he has not been able to find the “right” transformation for oil prices. The common conclusion is, therefore, that increases in oil prices affect GDP growth, whereas declines do not. Furthermore, oil-price increases after a long period of stability in the price had more dramatic consequences than those that were merely corrections to greater oil-price declines during the previous quarter.

We now look at the non-linear transformations proposed in the literature.

Mork (1989) shows asymmetry between the GDP’s responses to oil-price increases and decreases. He concludes that oil-price decreases are not statistically significant. We refer to Mork’s specification as one in which only increases are considered.

\[ o_t^+ = \begin{cases} 
  o_t & \text{if } o_t > 0 \\
  0 & \text{otherwise} 
\end{cases} \]  

(2)

Lee et al. (1995), and Hamilton (1996), observe that oil-price increases after long periods of price stability have more dramatic consequences than those that are merely corrections to greater oil-price
decreases during the previous quarter. Thus, the first authors consider a GARCH representation of oil-prices to reflect the above fact. We refer to Lee, Ni and Ratti specification as SOPI (scaled oil price increase).\textsuperscript{27}

\[ o_t' = \alpha_0 + \alpha_1 o_{t-1}' + \alpha_2 o_{t-2}' + \alpha_3 o_{t-3}' + \alpha_4 o_{t-4}' + e_t \]  

\[ e_t | I_{t-1} \sim N(0, h_t) \]  

\[ h_t = \gamma_0 + \gamma_1 e_{t-1}^2 + \gamma_2 h_{t-1} \]  

\[ SOPI_t = \max(0, \hat{e}_t / \sqrt{h_t}) \]  

where \( o_t' \) is the real oil-price changes.\textsuperscript{28}

Hamilton (1996) proposes the non-linear transformation, known as net oil price increase (NOPI). We refer to Hamilton’s specification as NOPI (i.e., the amount by which the log of oil prices in quarter \( t, p_t \), exceeds the maximum value over the previous 4 quarters; and 0 otherwise).

\[ NOPI_t = \max \{0, p_t - \max \{p_{t-1}, p_{t-2}, p_{t-3}, p_{t-4}\}\} \]  

There is a variation of the above-mentioned measure that considers the previous 12 quarters. We refer to this specification as NOPI3 (i.e., the amount by which the log of oil prices in quarter \( t \) exceeds the maximum value over the previous 12 quarters; and 0 otherwise).

\[ NOPI3_t = \max \{0, p_t - \max \{p_{t-1}, p_{t-2}, ..., p_{t-12}\}\} \]  

Note that all of these non-linear transformations are the ones that have been proposed in the literature for restoring Granger-causality and avoiding the forecasting of a non-existent GDP increase when oil-prices decrease. But these specifications are rather ad hoc, and ignore the effects of oil-price decreases.

We observe that the bivariate Granger-causality is re-established in the full sample (1947:II-2001:III) when these specifications are employed (See Table 7). We also note, however, that when we split the sample, the above result does not hold (See Figure 9). To be more specific, if the subsample runs from 1947:II to any date beyond 1974:II, there is bivariate Granger-causality. On the other hand, if the subsample is

\textsuperscript{27}We also consider, as Hamilton (2003) did, \( o_t' / \sqrt{h_t} \) rather than \( \hat{e}_t / \sqrt{h_t} \) for comparability with the other results presented in the paper.

\textsuperscript{28}The real oil price is defined as the nominal oil price deflated by the GDP deflator (See Data Appendix).
from any date beyond 1974:I to 2001:III, the bivariate Granger-causality disappears, suggesting that the success of the bivariate Granger-causality is due merely to the first dates considered. This consideration is confirmed when we consider the multivariate Granger-causality (Wald test), given that none of the non-linear transformations are able to Granger-cause GDP growth either in the full sample (1960:I-2000:III) or in any subsample at a 5% critical level (See Table 8 and Figure 10, respectively).\footnote{Hooker (1999) finds that SOPI and NOPI do not Granger-cause GDP growth for samples that start around 1980 and end in 1998:IV. Hamilton (2003), for his part, carries out a bivariate Granger-causality analysis for second subsamples that start on any date between 1948:II and 1989:IV and end in 2001:III, considering all of the non-linear transformations above-mentioned. He obtains that the Mork’s and NOPI specifications cannot Granger-cause GDP growth for subsamples that start beyond 1960 and 1974, respectively. He also finds that there is no such causality for SOPI and NOPIII specifications when subsamples start after 1981.} In addition, the block-exogeneity test indicates that none of the non-linear measures of oil prices Granger-cause the remaining variables of the system (See Table 9, line 2), although an interaction between these measures and macroeconomic variables is found.

Furthermore, none of these transformations succeed in solving the problem of the linear specification in out-of-sample forecasting (See Figure 11).\footnote{In Figure 11 we make a direct comparison between the SOPI specification, which is the one with the lowest MSFE, and the linear specification.}

Table 10 presents the results of the Diebold and Mariano (1995) test, whose null hypothesis is that there is equal forecast accuracy. We set up this statistic such that a positive value means that the linear specification fits better than the other specifications considered. We then find that in-sample and out-of-sample the non-linear specifications considered have a smaller MSE/MSFE, but we cannot reject the null hypothesis of the DM test.

Furthermore, to verify that the problem is not one of a structural change, we have performed different tests for stability of coefficients on oil prices and for stability of all of the regression coefficients in the GDP equation of the multivariate model with the non-linear specifications. The results of the Andrews’ test and those of Andrews and Ploberger’s tests indicate that all of the specifications are essentially stable (See Tables 11 and 12).

It seems only natural, therefore, that doubts should arise with regard to the ability of these non-linear transformations to accurately reflect non-linearity.
4 Non-linearity test

As we have seen in the previous section, the literature “offers” evidence of a non-linear relationship between GDP growth and oil-price changes, but the greatest contribution to this evidence has been the results of the non-linearity test \(^{31}\) proposed by Hamilton (2001).\(^{32}\)

Hamilton (2003) has already performed this test for the full sample (1947:II-2001:III) and for the different specifications mentioned above. We, in contrast to Hamilton, have performed this test for different subsamples, in an effort to identify where the non-linearity appears.\(^{33}\) We have also established different window sizes in an effort to pinpoint the dating of the non-linearity.

4.1 Test description

We have followed Hamilton’s indications in testing the null hypothesis that the true relationship between GDP growth and oil-price changes is linear, considering a non-linear regression model of the following form:

\[
y_t = \mu(x_t) + \delta z_t + \varepsilon_t
\]

where \(y_t\) is the real GDP growth; \(x_t\) is a \(k\)-dimensional vector that contains lags in oil-price changes, with \(k = 4\), \(x_t = (\omega_{t-1}, \omega_{t-2}, \omega_{t-3}, \omega_{t-4})^\prime\), for which linearity is not assumed; \(\mu(.)\) is a function, whose form is unknown;\(^{34}\) \(z_t\) is a \(p\)-dimensional vector with lags in GDP growth, with \(p = 4\), \(z_t = (y_{t-1}, y_{t-2}, y_{t-3}, y_{t-4})^\prime\), for which linearity is assumed; and \(\varepsilon_t\) is an error term.\(^{35}\) To implement the test, \(g_i\) is defined on the basis of the sample standard deviation of the \(i\)th explanatory variable as follows:

\[
g_i = 2\left[k(T^{-1} \sum_{t=1}^{T} (x_{it} - \bar{x}_i))^\prime\right]^{-1/2},\]

\(^{31}\)There are several tests for neglected non-linearity, among which we find: the Regression Error Specification Test, also called the Ramsey’s Reset test (Ramsey, 1969), Tsay’s test (Tsay, 1986), the V23 test (Terasvirta et al., 1993), the neural network test (White, 1989, and Lee et al., 1993), and others.

\(^{32}\)Dahl (1999) finds that this test performs well in finite samples and that, in general, it has good size and power properties as compared to some of the most popular and powerful tests commonly cited in the literature (most of which are mentioned in the previous footnote).

\(^{33}\)It is noteworthy that practically all of the authors referred to above, attribute the non-linearity to declines in oil prices during the mid-1980s.

\(^{34}\)Hamilton’s (2001) approach considers the function \(\mu(.)\) itself as being the outcome of a random field. He uses the generalization of the finite-differenced Brownian motion.

\(^{35}\)The sample period employed runs from 1947:II to 2001:III.
governing so the variability of the non-linear component with respect to the $i$th explanatory variable, $x_i$. Thus $g_i = 0$ implies that $\mu(x_t)$ is linear with respect to $x_i$. Using these values for $g_i$, we calculate $h_{st} = (1/2)[\sum_{i=1}^{4} g_i^2(x_{it} - x_{is})^2]^{1/2}$ and construct the $(T \times T)$ matrix $H$ whose row $t$, column $s$ element is given by

$$H\{h_{st}\} = 1 - (2/\pi)\{(2/3)h_{st}(1 - h_{st}^2)^{(3/2)}h_{st}(1 - h_{st}^2)^{(1/2)} + \sin^{-1}(h_{st})\}$$

when $0 \leq h_{st} \leq 1$ and by zero when $h_{st} > 1$.

The non-linear regression model (7) can be written in the following form:

$$y_t = \alpha_0 + \alpha'x_t + \delta'z_t + u_t,$$

with $u_t = \lambda m(x_t) + \varepsilon_t$, where $m(.)$ is the realization of a scalar-valued Gaussian random field with mean zero, unit variance, and covariance function given by (9), and $\lambda$ is the parameter that governs the overall importance of the non-linear component. Thus $\lambda = 0$ implies that the relationship between GDP growth and oil-price changes is linear.

We perform an OLS linear regression\(^{37}\) of $y_t$ on $x_t$, $z_t$ and a constant, $y = X\beta + \varepsilon$:

$$y_t = \begin{bmatrix} 0.747219 - 0.004826 \alpha_{t-1} - 0.006458 \alpha_{t-2} \\ (0.116232) \quad (0.096531) \quad (0.096603) \\
-0.006482 \alpha_{t-3} - 0.011968 \alpha_{t-4} + 0.275762 y_{t-1} \\ (0.066622) \quad (0.06574) \quad (0.069062) \\
+0.121160 y_{t-2} - 0.077809 y_{t-3} - 0.125788 y_{t-4} \\ (0.071475) \quad (0.071332) \quad (0.068425) \end{bmatrix}$$

Calculate the OLS residuals, $\hat{\varepsilon}$, regression squared standard error, $\hat{\sigma}^2 = (T - k - p - 1)^{-1}\hat{\varepsilon}'\hat{\varepsilon}$, and $(T \times T)$ projection matrix $M = I_T - X(X'X)^{-1}X'$.

We then calculate the Lagrange multiplier statistic for neglected non-linearity:

$$\nu^2 = \frac{\hat{\varepsilon}'H\hat{\varepsilon} - \hat{\sigma}^2 tr(MHM)^2}{\hat{\sigma}^4 \{2 tr\{(MHM - (T - k - p - 1)Mtr(MHM)^2)\}\}}.$$  

Hamilton (2001) shows that this statistic has an asymptotic $\chi^2(1)$ distribution under the null hypothesis of linearity.

---

\(^{36}\)Hamilton (2001) provides closed-form expressions for $H\{h_{st}\}$ in his Table I when $k$ is equal to any number between 1 and 5.

\(^{37}\)Standard errors are in parentheses.
4.2 Empirical results

We carried out this test twice, first with our own data set and then with Hamilton’s (2003),\(^{38}\) so that we could make a direct comparison.

Table 13 shows the results of the non-linearity test performed with both sets of data (See Appendix A). When we consider the full sample, we observe that the null hypothesis that the relationship between oil prices and GDP growth is linear is rejected with either set of data. We also observe, again with the full sample, the acceptance of the null hypothesis that any of the non-linear transformations considered in the previous section is a correct representation of the non-linearity\(^{39}\) with either set of data.\(^{40}\)

We now wish to see what happens when we consider different subsamples. Figure 12 plots the p-values of the non-linearity test for the first subsample, which runs from 1947:II to the date indicated in the horizontal axis (ending in 2001:III), and also plots the corresponding p-values for the second subsample, which runs from the date indicated in the horizontal axis (ending in 1989:I) to 2001:III, for all of the specifications considered. With regard to the first subsample, we observe linearity for subsamples that end before 1974:III. However, non-linearity appears when we extend the subsample to dates beyond 1974:III. It is worth noting, at this point, that all of the previous authors attribute the non-linearity of the relationship between GDP growth and oil-price changes to the use of the mid-1980s data. We, however, find non-linearity in samples that do not contain such data. Furthermore, we find that the non-linear specifications are not correct representations of the non-linearity when we consider subsamples that end before 1983:IV, and even before 1977:I. Regarding the second subsample, we find the paradoxical result that despite the fact that we accept the existence of linearity for any starting-date, we also observe that any of the non-linear transformations is a correct representation of non-linearity.\(^{41}\)

\(^{38}\)We use a different set of data for oil prices, because we do not have access to Citibase.

\(^{39}\)It is noteworthy that this is the Hamilton’s (2003) interpretation when he applies the non-linearity test to non-linear transformations.

\(^{40}\)We have also investigated, as Hamilton (2003) did, the sensitivity of the results to possible outliers. To do so, we drop observation \(t_{0} \in \{1, ..., T\}\) from the sample and we perform the non-linearity test without it. When we consider our own data set, we always reject linearity, except when we exclude 1950:I (\(p\)-value = 0.11). Likewise, we always accept that non-linear transformations are correct representations of non-linearity with the only exception of the NOPI3 specification, in which we reject the null hypothesis if we exclude 1949:IV. Considering Hamilton’s data set, we always reject linearity, and we also reject the hypothesis that non-linear specifications, with the exception of SOPI and NOPI3, are correct representations of non-linearity for certain dates that were excluded (e.g., 1949:IV, 1960:II, 1960:IV, 1970:IV, 1971:I, 1976:I, 1978:II, 1990:IV, and 1998:IV for both Mork and NOPI). (All of these results are available from the author upon request).

\(^{41}\)When we work with Hamilton’s data set (See Figure 13), we should consider first subsamples that run beyond 1974:IV to
Note that we have considered different subsamples of different sizes, so that we should perform this test with different subsamples, although they might all be of the same size. We first establish a window with a fixed number of observations. We consider different window sizes, \( T = 55, 60, 70, 80, 90, 100, 110, 120, 130, 140, 150, \) and 160 observations.\(^{42}\) We then displace this window over time and perform the non-linearity test.

a) For the linear case (See Figure 14.1), the results are as follows:

- For window sizes of less than 100 observations, and for observations that do not contain data beyond 1973:I,\(^{43}\) we obtain the acceptance of the linear relationship between GDP growth and oil-price changes.

- For window sizes of less than 110 observations, and for observations that do not contain data beyond 1976, we observe the rejection of the above linear relationship. To attribute the non-linearity of the GDP-Oil price relationship to the mid-1980s data, as the previous authors have done, would therefore seem inappropriate on the basis of this test. Furthermore, when we consider window sizes of 120, 130 or 140, and observations that do not contain data beyond 1984, we obtain the existence of non-linearity again.\(^{44}\)

b) In the Mork case (See Figure 14.2), for a window size of less than 70 observations, and for any observations contained therein (with 2 exceptions), we obtain the acceptance of the null hypothesis that \textit{the specification proposed is a correct representation of the non-linearity}. Moreover, for window sizes of less than 110 observations, and for observations that do not contain data beyond 1977, we obtain the rejection of the null hypothesis. Furthermore, we reject the null hypothesis for window sizes of 120 and 140, and for observations that do not contain data beyond 1984. We accept the null hypothesis, however, for all of the non-linear specifications are not so when we consider subsamples that end on any date in the interval [1974:IV-1977:I]. We also observe that NOPI3 does not adequately represent the non-linearity for first subsamples that end in either 1978:I or on any date in the interval [1995:II-1997:I]. Furthermore, we obtain non-linearity for second subsamples that start on any date of either [1961:IV-1965:I] or [1970:I-1975:III] but 1973:I. We also find that all of the non-linear transformations are correct representations of non-linearity for any starting-date (with the exception of 1971:I, 1971:IV, and 1972:I in the NOPI3 case).\(^{42}\) Dahl (1999) shows through Monte Carlo experiments that this test has good small-sample size and strong power properties.\(^{43}\) It is noteworthy that oil-price changes were not very important before 1973:I.

\(^{42}\) Dahl (1999) shows through Monte Carlo experiments that this test has good small-sample size and strong power properties.
\(^{43}\) It is noteworthy that oil-price changes were not very important before 1973:I.
\(^{44}\) With the window size of 160 observations, it is worth noting that when we consider the first 160 observations (including only two years of oil-price decreases, [1947:II-1987:I]) and perform the non-linearity test, the null hypothesis of linearity is rejected. When we consider, however, the last 160 observations (forgetting the 1950s where the movements of oil prices were low frequency, and considering observations that include dates in the 1970s, the 1980s, and the 1990s, [1961:IV-2001:III]), we accept the existence of linearity.
the other window sizes considered.

c) In the SOPI case (See Figure 14.3), we accept the null hypothesis that \textit{the specification proposed is a correct representation of the non-linearity} for window sizes of less than 90 observations, and for any observations contained therein, and for those that are longer than 120 observations (with some exceptions). We reject the null hypothesis, however, for window sizes of 100 and 110, and for observations that do not contain data beyond 1976.

d) In the NOPI and NOPI3 cases (See Figures 14.4 and 14.5), for window sizes of less than 110 observations, and for observations that do not contain data beyond 1976, we reject the null hypothesis that \textit{the specification proposed is a correct representation of the non-linearity}. Furthermore, for window sizes of less than 140 observations, and for observations that do not contain data beyond 1984, the null hypothesis is rejected. It is also rejected even when we consider observations that include both the 1970s and the 1980s.\footnote{The results of Hamilton’s data set are quite similar to those given in the text. We, however, reject the null hypothesis of linearity when we consider observations that include dates in both the 1970s and the 1980s, and for windows of any size. Moreover, we accept that SOPI is a correct specification of non-linearity for windows of any size and for any observations contained therein. (These results are available from the author upon request).}

We conclude, therefore, that the belief that \textit{“the non-linearity of the relationship between GDP growth and oil-price changes is only due to the use of data from the mid-80s onwards”} is not entirely clear to us, as we observe the existence of a non-linear relationship in subsamples that do not contain such data. Moreover, although we reject linearity in the full sample, the non-linear transformations that ignore the oil-price declines do not solve the problem. It must be remembered that these specifications attribute the non-linearity to the oil-price declines, and that is basically why they are ignored. Notwithstanding this, the SOPI specification seems to be the most appropriate of the non-linear specifications under study on the basis of both MSFE and the results of the non-linearity test.\footnote{This consideration is valuable because there are several studies that have considered the NOPI as an accurate oil-price measure (Bernanke \textit{et al.}, 1997; Raymond and Rich, 1997; Hamilton and Herrera, 2001; Lee and Ni, 2002; among others), and their conclusions might change when the SOPI variable is taken into account.} The latter result is in agreement with what Hamilton (2003) suggests: \textit{“the transformation proposed by Lee \textit{et al.} (1995) seems to do the best job of the measures explored in this paper”}.
5 Conclusions

In this paper, we have presented evidence of a non-linear relationship between GDP growth and changes in the price of crude oil. We argue that this non-linearity is not solely due to the use of data from the mid-1980s onwards, as many authors have been suggesting up to now. In particular, we find the existence of non-linearity with the use of data earlier than 1984, and indeed, even before 1977.

This paper also questions that the non-linear transformations of oil prices proposed in the literature are the most appropriate ones for reflecting such non-linearity. We show that these transformations still do not solve the forecasting of a spurious increase in GDP growth for the mid-1980s. Furthermore, when we consider data earlier than 1977, the non-linearity test shows that these specifications are not the most accurate in summarizing the non-linearity. It should be remembered, as well, that these transformations ignore oil-price declines, treating them as if nothing had happened, which is, at the very least, questionable. There would seem to be some sort of data-mining.
Appendix A

The distribution theory of Hamilton’s (2001) test is asymptotic and has been derived under the assumption that the regressors are stationary, which excludes any structural change in the marginal distribution of regressors. We, however, observe that there is a structural change in the variance of oil-price regressor variable.47 Table 14 reports on Andrews’ (1993) and Andrews and Ploberger’s (1994) tests, and shows that the variance of oil price changes in 1973:1.48 We therefore realize that the results of the non-linearity test may change. When we performed this test in the full sample, we rejected the null hypothesis of linearity with a p-value of 0.00625. We now consider a bootstrap by block with and without fixed regressors49 referred to GDP regressors.

We perform a bootstrap with 10.000 replications and blocks of six elements:

Step 1: Perform an OLS regression of $y_t$ on $x_t$, $z_t$ and a constant,$^{50}$ $y = X\beta + \zeta$.

Step 2: Calculate the OLS residuals, $\hat{\zeta}_t$.\(^{51}\)

Step 3: Conduct a bootstrap by block re-sampling residuals, $\hat{\zeta}_t^*$ (with and without a seed).

Step 4: Generate 10.000 \{y_t^*\}:

- fixed regressor bootstrap:
  \begin{equation}
  y_t^* = \hat{\beta}_0 + \hat{\beta}_1 y_{t-1} + ... + \hat{\beta}_4 y_{t-4} + \hat{\gamma}_1 o_{t-1} + ... + \hat{\gamma}_4 o_{t-4} + \hat{\zeta}_t^*
  \end{equation}

- non-fixed regressor bootstrap:
  \begin{equation}
  y_t^* = \hat{\beta}_0 + \hat{\beta}_1 y_{t-1} + ... + \hat{\beta}_4 y_{t-4}^* + \hat{\gamma}_1 o_{t-1} + ... + \hat{\gamma}_4 o_{t-4}^* + \hat{\zeta}_t^*
  \end{equation}

Step 5: Calculate the Lagrange Multiplier statistic for each \{y_t^*\}.

We then calculate the percentage of times we accept the null hypothesis at a 5% critical level. On observing Table 15, we find that we accept the null hypothesis of linearity at a high percentage, indicating the fact that the asymptotic distribution of this test might change when there are changes in the marginal distribution of regressors. For this reason, we should look at the results of this test with caution.

---

47 We have followed the method employed by McConnell and Pérez-Quirós (2000) in order to test for a structural break in the volatility of oil-price changes.

48 We look for a structural change from 1955:I to 1993:II, using $\pi = 0.15$.

49 We always consider the lags of oil prices as fixed regressors.

50 Notice that $y_t$ is the real GDP growth, $x_t$ is a 4-dimensional vector which contains lags in oil-price changes, and $z_t$ is a 4-dimensional vector with lags in GDP growth.

51 The estimated residuals should capture the non-linearity.
Data Appendix

The data, sources, and transformations used in this study were taken from the first period of 1947, 1959, and 1960, and up to either 2000:III or 2001:III, depending on the case.

The United States:

**GDP:** Gross Domestic Product; Billions of chained 1996 dollars SAAR; NIPA; (Quarterly data); downloaded from the Bureau of Economic Analysis web page (http://www.bea.doc.gov/bea/dn/gdplev.htm); entered in first log-differences.

**ur:** Standardized unemployment rate; Quarterly S.A., Percent; downloaded from the OECD Main Economic Indicators CD-ROM 2001.

**poil:** Price of West Texas Intermediate Crude, Monthly N.S.A., Dollars Per Barrel; from www.economagic.com; aggregated from monthly to quarterly using the monthly-average value of the quarter; entered in first log-differences.

**deflator:** Gross Domestic Product Implicit Price Deflator; (1996=100) S.A. (Quarterly data); from www.economagic.com.

**cpi:** All Urban Consumers-(CPI-U): U.S. city average: All items: 1982-84=100 (Monthly data); from www.economagic.com; aggregated from monthly to quarterly using the monthly-average value of the quarter; entered in first log-differences.

**lr:** Ten-year Treasury Constant Maturity (Monthly data); from www.economagic.com; aggregated from monthly to quarterly using the monthly-average value of the quarter.

**fed:** Federal Funds Rate (Monthly data); downloaded from www.economagic.com; aggregated from monthly to quarterly using the monthly-average value of the quarter.

**w:** Average hourly earnings of production workers; (Monthly data); downloaded from the Bureau of Labor Statistics (National Employment, Hours, and Earnings) web page (http://stats.bls.gov/datahome.htm); Seasonally Adjusted; aggregated from monthly to quarterly using the monthly-average value of the quarter; entered in first log-differences.
References


Table 1
Individual significance of oil-price coefficients in the GDP equation (Multivariate model) (1960:I-2000:III)

<table>
<thead>
<tr>
<th>Oil Price Change</th>
<th>Lag Coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>t - Statistic</td>
<td>1</td>
</tr>
<tr>
<td>(p-value)</td>
<td>(0.8874)</td>
</tr>
<tr>
<td></td>
<td>2</td>
</tr>
<tr>
<td>(p-value)</td>
<td>(0.4458)</td>
</tr>
<tr>
<td></td>
<td>3</td>
</tr>
<tr>
<td>(p-value)</td>
<td>(0.4882)</td>
</tr>
<tr>
<td></td>
<td>4</td>
</tr>
<tr>
<td>(p-value)</td>
<td>(0.7959)</td>
</tr>
</tbody>
</table>

Note.- This Table reports the t-statistic values and the p-values for individual significance of oil-price lag coefficients in the GDP equation of the VAR(4) model (1960:I-2000:III). One/two/three asterisks mean a p-value of less than 10%/5%/1%.

Table 2

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>GDP</th>
<th>UR</th>
<th>Oil</th>
<th>CPI</th>
<th>Fed</th>
<th>LR</th>
<th>W</th>
</tr>
</thead>
<tbody>
<tr>
<td>F - Statistic</td>
<td>3.526***</td>
<td>218.9***</td>
<td>1.249</td>
<td>28.73***</td>
<td>79.04***</td>
<td>158.1***</td>
<td>14.75***</td>
</tr>
<tr>
<td>(p-value)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.202)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
</tbody>
</table>

Note.- These tests are performed equation by equation in the VAR(4) framework (1960:I-2000:III). The null hypothesis of this F-statistic is that “all of the regression coefficients, except the constant term, are zero”. p-values appear in parentheses. One/two/three asterisks mean a p-value of less than 10%/5%/1%.

Table 3
Test for stability of coefficients on oil prices in the GDP equation (Multivariate model) (1960:I-2000:III)

<table>
<thead>
<tr>
<th></th>
<th>Sup F (date)</th>
<th>Avg F</th>
<th>Exp F</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>6.672 (1972:1)</td>
<td>3.908</td>
<td>2.429</td>
</tr>
</tbody>
</table>

Asymptotic 5% critical values | 15.34 | 8.09 | 5.11 |

Note.- We have performed these tests using \( \pi = 0.25 \) and four restrictions. Critical values were taken from Andrews (1993) and Andrews and Ploberger (1994). One/two/three asterisks mean a p-value of less than 10%/5%/1%.

Table 4
Test for stability of all coefficients in the GDP equation (Multivariate model) (1960:I-2000:III)

<table>
<thead>
<tr>
<th>Test statistic</th>
<th>Asymptotic p-value</th>
<th>Homoskedastic bootstrap p-value</th>
<th>Heteroskedastic bootstrap p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sup F</td>
<td>0.016**</td>
<td>0.076*</td>
<td>0.246</td>
</tr>
<tr>
<td>Exp F</td>
<td>0.024**</td>
<td>0.087*</td>
<td>0.263</td>
</tr>
<tr>
<td>Avg F</td>
<td>0.273</td>
<td>0.279</td>
<td>0.121</td>
</tr>
</tbody>
</table>

Note.- We have performed these tests using \( \pi = 0.25 \) and 29 restrictions. Asymptotic and bootstrap p-values were calculated as in Hansen (1997) and Hansen (2000), respectively. One/two/three asterisks mean a p-value of less than 10%/5%/1%.
Table 5
Bivariate Granger-causality test (Linear case)

<table>
<thead>
<tr>
<th></th>
<th>Oil</th>
<th>GDP</th>
<th>Oil</th>
<th>UR</th>
<th>Oil</th>
<th>CPI</th>
<th>Oil</th>
<th>Fed</th>
<th>Oil</th>
<th>LR</th>
<th>Oil</th>
<th>W</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F - Statistic$</td>
<td>1.659</td>
<td>3.297**</td>
<td>2.260*</td>
<td>1.075</td>
<td>0.705</td>
<td>1.867</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(p-value)$</td>
<td>(0.182)</td>
<td>(0.012)</td>
<td>(0.065)</td>
<td>(0.370)</td>
<td>(0.589)</td>
<td>(0.119)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note.- 9 denotes 'does not Granger-cause'. p-values appear in parentheses. One/two/three asterisks mean a p-value of less than 10%/5%/1%.

Table 6
Granger-causality in a multivariate context
(Block-exogeneity test)
($\chi^2 - statistic$)

<table>
<thead>
<tr>
<th>Linear Specification</th>
<th>$y_1$</th>
<th>$y_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$null Hypothesis$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$A_2 = 0$</td>
<td>30.916</td>
<td></td>
</tr>
<tr>
<td>(0.18444)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$B_1 = 0$</td>
<td>39.269</td>
<td></td>
</tr>
<tr>
<td>(0.02538)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$A_2 = 0$, $B_1 = 0$, and $\Omega_{21} = 0$</td>
<td>105.489</td>
<td></td>
</tr>
<tr>
<td>(3.5E−005)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note.- We categorize the variables of the VAR in two groups, as represented by the ($n_1 \times 1$) vector $y_1$ and the ($n_2 \times 1$) vector $y_2$. We rewrite the $p$th-order VAR as follows:

$$y_{1t} = c_1 + A_{11}x_{1t} + A_{12}x_{2t} + \varepsilon_{1t}$$  \hspace{1cm} (T.1)

$$y_{2t} = c_2 + B_{11}x_{1t} + B_{22}x_{2t} + \varepsilon_{2t}$$  \hspace{1cm} (T.2)

where $x_{1t}$ is an ($n_1 \times 1$) vector containing lags of $y_{1t}$, and $x_{2t}$ is an ($n_2 \times 1$) vector containing lags of $y_{2t}$.

$y_1$ ($y_2$) is block-exogenous in the time series sense with respect to $y_2$ ($y_1$) when $A_2 = 0$ ($B_1 = 0$) (See Hamilton, 1994).

The statistic for testing the null hypothesis $A_2 = 0$ is the following:

$$T \times \{log|\Omega_{11}(0)| - log|\Omega_{11}|\} \sim \chi^2(n_1 n_2 p)$$

where $\Omega_{11}$ is the variance-covariance matrix of the residuals from the OLS estimation of (T.1) and $\Omega_{11}(0)$ that of the residuals from the OLS estimation of (T.1) when $A_2 = 0$.

The test statistic for $H_0$: $B_1 = 0$ can be constructed analogously. Likewise, there is no relation at all between $y_1$ and $y_2$ when $A_2 = 0$, $B_1 = 0$, and $\Omega_{21} = 0$.

The statistic values and the p-values of these three tests are reported. One/two/three asterisks mean a p-value of less than 10%/5%/1%. 
### Table 7
Bivariate Granger-causality test (F-statistic)  
(Longest available sample: 1947:II-2001:III)

<table>
<thead>
<tr>
<th>Oil Price Measure</th>
<th>Linear</th>
<th>Mork</th>
<th>SOPI</th>
<th>NOPI</th>
<th>NOPI3</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP</td>
<td>1.815</td>
<td>3.329**</td>
<td>4.436**</td>
<td>3.509***</td>
<td>3.961***</td>
</tr>
<tr>
<td></td>
<td>(0.127)</td>
<td>(0.013)</td>
<td>(0.001)</td>
<td>(0.008)</td>
<td>(0.004)</td>
</tr>
</tbody>
</table>

Note.- We consider Mork’s specification, “Mork”, to be one in which only increases are considered. We refer to Lee-Ni-Ratti specification, SOPI, as one in which estimated positive oil-price shocks are scaled by their conditional variance. We consider Hamilton’s specification, NOPI, to be the amount by which the log of oil prices in quarter t exceeds the maximum value during the previous 4 quarters, and 0 otherwise. And we refer to NOPI3 as a variation of the NOPI that considers the previous 12 quarters. 9 denotes ‘does not Granger-cause’. p-values appear in parentheses. One/two/three asterisks mean a p-value of less than 10%/5%/1%.

### Table 8
Multivariate Granger-causality test  
(Wald test ($\chi^2$-statistic)  

<table>
<thead>
<tr>
<th>Oil Price Measure</th>
<th>Linear</th>
<th>Mork</th>
<th>SOPI</th>
<th>NOPI</th>
<th>NOPI3</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP</td>
<td>1.188</td>
<td>1.266</td>
<td>1.731</td>
<td>1.880</td>
<td>3.389</td>
</tr>
<tr>
<td></td>
<td>(0.8799)</td>
<td>(0.9670)</td>
<td>(0.7851)</td>
<td>(0.7577)</td>
<td>(0.4949)</td>
</tr>
</tbody>
</table>

Note.- “Mork”, SOPI, NOPI, and NOPI3 are as described in Table 7. The null hypothesis is that “all lags of oil-price measure considered in GDP equation of the VAR(4) are zero”. p-values appear in parentheses. One/two/three asterisks mean a p-value of less than 10%/5%/1%.

### Table 9
Granger-causality in a multivariate context  
(Block-exogeneity test)  
($\chi^2$ - statistic)

<table>
<thead>
<tr>
<th>Specifications</th>
<th>Mork</th>
<th>SOPI</th>
<th>NOPI</th>
<th>NOPI3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_t$</td>
<td>31.605</td>
<td>26.726</td>
<td>46.156***</td>
<td>56.414***</td>
</tr>
<tr>
<td></td>
<td>(0.13710)</td>
<td>(0.31737)</td>
<td>(0.00424)</td>
<td>(0.00020)</td>
</tr>
<tr>
<td>$B_1 = 0$</td>
<td>26.303</td>
<td>35.527*</td>
<td>27.805</td>
<td>35.539*</td>
</tr>
<tr>
<td></td>
<td>(0.33797)</td>
<td>(0.06096)</td>
<td>(0.2835)</td>
<td>(0.06079)</td>
</tr>
<tr>
<td>$A_2 = 0, B_1 = 0,$ and $\Omega_{21} = 0$</td>
<td>84.98***</td>
<td>76.302**</td>
<td>97.229***</td>
<td>115.411***</td>
</tr>
<tr>
<td></td>
<td>(0.00452)</td>
<td>(0.02451)</td>
<td>(0.00028)</td>
<td>(2.48E-006)</td>
</tr>
</tbody>
</table>

Note.- “Mork”, SOPI, NOPI, and NOPI3 are as described in Table 7. The block-exogeneity test is as described in Table 6. p-values appear in parentheses. One/two/three asterisks mean a p-value of less than 10%/5%/1%.
<table>
<thead>
<tr>
<th>Diebold-Mariano Test</th>
<th>DM-S test (relative to Linear model)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_0$: equal forecast accuracy</td>
<td>Statistics ($p$-value)</td>
</tr>
<tr>
<td>Linear</td>
<td></td>
</tr>
<tr>
<td>IN (MSE: 0.4618)</td>
<td>-</td>
</tr>
<tr>
<td>OUT (MSFE: 0.9698)</td>
<td>-</td>
</tr>
<tr>
<td>Mork</td>
<td></td>
</tr>
<tr>
<td>IN (MSE: 0.4615)</td>
<td>-0.0467 (0.962)</td>
</tr>
<tr>
<td>OUT (MSFE: 0.9651)</td>
<td>-0.2078 (0.835)</td>
</tr>
<tr>
<td>SOPI</td>
<td></td>
</tr>
<tr>
<td>IN (MSE: 0.4599)</td>
<td>-0.2104 (0.833)</td>
</tr>
<tr>
<td>OUT (MSFE: 0.8970)</td>
<td>-0.9168 (0.359)</td>
</tr>
<tr>
<td>NOPI</td>
<td></td>
</tr>
<tr>
<td>IN (MSE: 0.4594)</td>
<td>-0.4028 (0.687)</td>
</tr>
<tr>
<td>OUT (MSFE: 0.9550)</td>
<td>-0.6937 (0.488)</td>
</tr>
<tr>
<td>NOPI3</td>
<td></td>
</tr>
<tr>
<td>IN (MSE: 0.4542)</td>
<td>-0.8444 (0.399)</td>
</tr>
<tr>
<td>OUT (MSFE: 0.9431)</td>
<td>-1.1798 (0.238)</td>
</tr>
</tbody>
</table>

Note.- “Mork”, SOPI, NOPI, and NOPI3 are as described in Table 7. Mean-Square Error and Mean-Square Forecast Error are defined as follows: $MSE = E[(y_T - \hat{y}_T)^2 | I_T]$ and $MSFE = E[(y_{T+1} - \hat{y}_{T+1})^2 | I_T]$, respectively, where $\hat{y}_T$ is the in-sample estimation, $\hat{y}_{T+1}$ is the one-period-ahead out-of-sample forecasting, and $I_T$ is the available information in $T$. In-sample refers to the period that runs from 1961:I to 2000:III. Out-of-sample refers to the period that runs from 1975:II to 2000:III. The DM statistic tests the null hypothesis that there is not any statistically significant difference between the non-linear specifications and the linear one. This statistic is set up such that a positive value means that the linear specification fits better than the other specifications considered. $p$-values based on two-sided tests appear in parentheses. One/two/three asterisks mean a $p$-value of less than 10%/5%/1%.
Table 11
Test for stability of coefficients on oil prices in the GDP equation (Multivariate model) (1960:I-2000:III)

<table>
<thead>
<tr>
<th>Oil price measure</th>
<th>Sup F (date)</th>
<th>Avg F</th>
<th>Exp F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mork specification</td>
<td>5.719 (1978:III)</td>
<td>2.840</td>
<td>1.750</td>
</tr>
<tr>
<td>SOPI specification</td>
<td>7.645 (1980:II)</td>
<td>5.157</td>
<td>2.775</td>
</tr>
<tr>
<td>NOPI specification</td>
<td>4.946 (1980:III)</td>
<td>2.445</td>
<td>1.487</td>
</tr>
<tr>
<td>NOPI3 specification</td>
<td>4.661 (1975:I)</td>
<td>2.405</td>
<td>1.340</td>
</tr>
</tbody>
</table>

Asymptotic 5% critical values 15.34 8.09 5.11

Note.- “Mork”, SOPI, NOPI, and NOPI3 are as described in Table 7. We have performed these tests using $\pi = 0.25$ and four restrictions. Critical values were taken from Andrews (1993) and Andrews and Ploberger (1994). One/two/three asterisks mean a p-value of less than 10%/5%/1%.

Table 12
Test for stability of all coefficients in the GDP equation (Multivariate model) (1960:I-2000:III)

<table>
<thead>
<tr>
<th>Oil price measure</th>
<th>Test statistic</th>
<th>Asymptotic p-value</th>
<th>Homoskedastic bootstrap p-value</th>
<th>Heteroskedastic bootstrap p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mork</td>
<td>Sup F</td>
<td>0.033**</td>
<td>0.122</td>
<td>0.289</td>
</tr>
<tr>
<td>specification</td>
<td>Exp F</td>
<td>0.048**</td>
<td>0.155</td>
<td>0.310</td>
</tr>
<tr>
<td></td>
<td>Avg F</td>
<td>0.411</td>
<td>0.444</td>
<td>0.229</td>
</tr>
<tr>
<td>NOPI</td>
<td>Sup F</td>
<td>0.019**</td>
<td>0.100</td>
<td>0.251</td>
</tr>
<tr>
<td>specification</td>
<td>Exp F</td>
<td>0.018**</td>
<td>0.100</td>
<td>0.247</td>
</tr>
<tr>
<td></td>
<td>Avg F</td>
<td>0.171</td>
<td>0.219</td>
<td>0.080*</td>
</tr>
<tr>
<td>SOPI</td>
<td>Sup F</td>
<td>0.047**</td>
<td>0.143</td>
<td>0.294</td>
</tr>
<tr>
<td>specification</td>
<td>Exp F</td>
<td>0.067*</td>
<td>0.174</td>
<td>0.324</td>
</tr>
<tr>
<td></td>
<td>Avg F</td>
<td>0.428</td>
<td>0.443</td>
<td>0.249</td>
</tr>
<tr>
<td>NOPI3</td>
<td>Sup F</td>
<td>0.098*</td>
<td>0.225</td>
<td>0.361</td>
</tr>
<tr>
<td>specification</td>
<td>Exp F</td>
<td>0.129</td>
<td>0.290</td>
<td>0.389</td>
</tr>
<tr>
<td></td>
<td>Avg F</td>
<td>0.501</td>
<td>0.535</td>
<td>0.286</td>
</tr>
</tbody>
</table>

Note.- “Mork”, SOPI, NOPI, and NOPI3 are as described in Table 7. We have performed these tests using $\pi = 0.25$ and 29 restrictions. Asymptotic and bootstrap p-values were calculated as in Hansen (1997) and Hansen (2000), respectively. One/two/three asterisks mean a p-value of less than 10%/5%/1%.
Table 13
Non-linearity test

<table>
<thead>
<tr>
<th>Specification</th>
<th>Our data set</th>
<th>Hamilton's data set</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear specification</td>
<td>0.0084</td>
<td>0.51</td>
</tr>
<tr>
<td>(Asymptotic p-value)</td>
<td>(0.92681)</td>
<td>(0.47652)</td>
</tr>
<tr>
<td>Mork specification</td>
<td>0.21</td>
<td>3.84*</td>
</tr>
<tr>
<td>(Asymptotic p-value)</td>
<td>(0.64823)</td>
<td>(0.05008)</td>
</tr>
<tr>
<td>SOPI specification</td>
<td>0.0084</td>
<td>0.51</td>
</tr>
<tr>
<td>(Asymptotic p-value)</td>
<td>(0.92681)</td>
<td>(0.47652)</td>
</tr>
<tr>
<td>NOPI specification</td>
<td>0.81</td>
<td>1.58</td>
</tr>
<tr>
<td>(Asymptotic p-value)</td>
<td>(0.36798)</td>
<td>(0.20826)</td>
</tr>
<tr>
<td>NOPI3 specification</td>
<td>0.81</td>
<td>1.58</td>
</tr>
<tr>
<td>(Asymptotic p-value)</td>
<td>(0.36798)</td>
<td>(0.20826)</td>
</tr>
</tbody>
</table>

Note.- “Mork”, SOPI, NOPI, and NOPI3 are as described in Table 7. This Table reports the statistic value and the p-value of the non-linearity test performed in the full sample (1947:II-2001:III). p-values appear in parentheses. One/two/three asterisks mean a p-value of less than 10%/5%/1%.

Table 14
Test for structural stability of the residual variance of oil-price changes

<table>
<thead>
<tr>
<th>Estimated breakpoint: 1973:I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variance</td>
</tr>
<tr>
<td>1.9161 10.5232</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Structural Break Tests</th>
</tr>
</thead>
<tbody>
<tr>
<td>Statistic</td>
</tr>
<tr>
<td>(Asymptotic p-value)</td>
</tr>
<tr>
<td>15.00 6.01 10.17</td>
</tr>
<tr>
<td>(0.004) (0.000) (0.000)</td>
</tr>
</tbody>
</table>

Note.- This Table reports the statistic values and the asymptotic p-values calculated as in Hansen (1997) for Andrews’ (1993) and Andrews and Ploberger’s (1994) tests. One/two/three asterisks mean a p-value of less than 10%/5%/1%.

Table 15
Bootstrap results

<table>
<thead>
<tr>
<th>Bootstrap results</th>
<th>Fixed regressor bootstrap</th>
<th>Non-fixed regressor bootstrap</th>
</tr>
</thead>
<tbody>
<tr>
<td>With Seed</td>
<td>96.44 %</td>
<td>96.56 %</td>
</tr>
<tr>
<td>Without Seed</td>
<td>93.84 %</td>
<td>94.03 %</td>
</tr>
<tr>
<td>Number of replications</td>
<td></td>
<td></td>
</tr>
<tr>
<td>100 x</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note.- This Table reports the percentage of times we accept the null hypothesis of linearity at a 5% critical level.
Figure 1
GDP Growth and Changes in Oil Price
1947:II-2001:III

Note: This figure plots both the US GDP growth and the percentage changes in nominal oil price from 1947:II to 2001:III. Shaded bands correspond to recessions as dated by the National Bureau of Economic Research (NBER).
Figure 2
One-period-ahead out-of-sample linear forecasting
(Bivariate model)

Note: This figure plots the one-period-ahead out-of-sample forecasting for GDP growth in a bivariate model with GDP growth and oil-price changes as variables. The forecast runs from 1957:II to 2001:III.

Figure 3
One-period-ahead out-of-sample linear forecasting
(Multivariate model: Seven-variable system)

Note: This figure plots the one-period-ahead out-of-sample forecasting for GDP growth in a seven-variable system. The forecast runs from 1975:II to 2000:III.
Figure 4
Chow test for stability of coefficients on oil prices
(Multivariate model) (Ho: Stability)

Note: This figure represents the p-values for a test of the null hypothesis that all oil-price coefficients in the GDP equation (in the multivariate model) are stable (in the Chow's sense) against the alternative hypothesis that these coefficients change on indicated date in the horizontal axis.

Figure 5
Chow test for stability of all regression coefficients
(Multivariate model) (Ho: Stability)

Note: This figure represents the p-values for a test of the null hypothesis that all of the regression coefficients in the GDP equation (in the multivariate model) are stable (in the Chow's sense) against the alternative hypothesis that these coefficients change on indicated date in the horizontal axis.
Figure 6.1
Bivariate Granger-causality test: First subsample
(Ho: Oil prices do not Granger-cause GDP)

Note: This figure presents the p-values for the Granger-causality test of the null hypothesis that oil-price changes do not Granger-cause (in the bivariate sense) GDP growth in the first subsample, which runs from 1947:II to the date indicated in the horizontal axis that starts in 1959:IV.

Figure 6.2
Bivariate Granger-causality test: Second subsample
(Ho: Oil prices do not Granger-cause GDP)

Note: This figure presents the p-values for the Granger-causality test of the null hypothesis that oil-price changes do not Granger-cause (in the bivariate sense) GDP growth in the second subsample, which runs from the date indicated in the horizontal axis that ends in 1989:I to 2001:III.
Figure 7.1
Multivariate Granger-causality test (Wald Test): First subsample
(Ho: Oil prices do not Granger-cause GDP)

Note: This figure presents the p-values for the Granger-causality test of the null hypothesis that oil-price changes do not Granger-cause (in the multivariate sense) GDP growth in the first subsample, which runs from 1960:I to the date indicated in the horizontal axis that starts in 1973:IV.

Figure 7.2
Multivariate Granger-causality test (Wald Test): Second subsample
(Ho: Oil prices do not Granger-cause GDP)

Note: This figure presents the p-values for the Granger-causality test of the null hypothesis that oil-price changes do not Granger-cause (in the multivariate sense) GDP growth in the second subsample, which runs from the date indicated in the horizontal axis that ends in 1986:IV to 2000:III.
Note: This figure plots the orthogonalized impulse-response function of GDP growth, over 24 quarters, to one standard deviation oil-price innovation in the multivariate model.
Figure 9
Bivariate Granger-causality test
(Ho: Non-linear measure of oil price does not Granger-cause GDP)

Note: These figures present the p-values for the Granger-causality test of the null hypothesis that the corresponding non-linear measure of oil price does not Granger-cause (in the bivariate sense) GDP growth in the first subsample, which runs from 1947:II to the date indicated in the horizontal axis that starts in 1959:IV, and in the second subsample, which runs from the date indicated in the horizontal axis that ends in 1989:I to 2001:III.
Figure 10
Multivariate Granger-causality test (Wald test)
(Ho: Non-linear measure of oil-price does not Granger-cause GDP)

Note: These figures present the p-values for the Granger-causality test of the null hypothesis that the corresponding non-linear measure of oil price does not Granger-cause (in the multivariate sense) GDP growth in the first subsample, which runs from 1960:1 to the date indicated in the horizontal axis that starts in 1973:IV, and in the second subsample, which runs from the date indicated in the horizontal axis that ends in 1986:IV to 2000:III.
Note: This figure plots the one-period-ahead out-of-sample forecasting for GDP growth in the linear and SOPI cases. The forecast runs from 1975:II to 2000:III.
Figure 12
p-values of non-linearity test for different oil-price measures and for different subsamples (Our data set)

Note: These figures represent the p-values of the non-linearity test for different oil-price measures and for different subsamples. The first chart of each specification represents the p-values for the first subsample, which runs from 1947:II to the date indicated in the horizontal axis (starting in 1962:1 and ending in 2001:III). The second chart plots the p-values for the second subsample, which runs from the date indicated in the horizontal axis (starting in 1962:1 and ending in 1989:I) up to 2001:III.
Figure 13
p-values of non-linearity test for different oil-price measures and for different subsamples (Hamilton data set)

Note: These figures represent the p-values of the non-linearity test for different oil-price measures and for different subsamples. The first chart of each specification represents the p-values for the first subsample, which runs from 1947:II to the date indicated in the horizontal axis (starting in 1962:1 and ending in 2001:III). The second chart plots the p-values for the second subsample, which runs from the date indicated in the horizontal axis (starting in 1962:1 and ending in 1989:I) up to 2001:III.
Note: These figures represent the p-values of the non-linearity test for different window sizes ($T = 55, 60, 70, \ldots, 160$). We first establish a window with a fixed number of observations. We then displace this window over time and perform the non-linearity test. For instance, the first p-value represented in any chart plots the p-value of the non-linearity test for any sample that starts in 1947:II and ends $T$ quarters later.
Figure 14.2
p-values of non-linearity test: Different window sizes
(Mork specification)
(Our data set)

Note: These figures represent the p-values of the non-linearity test for different window sizes ($T=55, 60, 70, \ldots, 160$).
Figure 14.3
p-values of non-linearity test: Different window sizes
(SOPI specification)
(Our data set)

Note: These figures represent the p-values of the non-linearity test for different window sizes ($T=55, 60, 70, ..., 160$).
Figure 14.4  
*p-values of non-linearity test: Different window sizes*  
(NOPI specification)  
(Our data set)  

Note: These figures represent the p-values of the non-linearity test for different window sizes ($T=55, 60, 70, \ldots, 160$).
Figure 14.5
p-values of non-linearity test: Different window sizes
(NOPI3 specification)
(Our data set)

Note: These figures represent the p-values of the non-linearity test for different window sizes ($T=55, 60, 70, \ldots, 160$).