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Abstract

During an English auction, bidders' behaviour conveys information on their valuation of the prize. So whenever valuations are not independent, a bidder's strategy depends on the price at which his competitors drop out before he does. A ring of bidders can strategically manipulate the information reported through its members' bids, in order to mislead other bidders into bidding less aggressively and so allow a ring member to bid more aggressively. Collusion increases the probability that a ring bidder wins the auction and reduces the price he pays. The presence of a ring harms other bidders (as well as the seller) and reduces efficiency.

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Contents

1. Introduction

In the real world, the possibility that bidders collude during an auction is a crucial concern for the seller because collusive behaviour typically results in substantial loss of revenue.[1] There is considerable evidence that collusion in auctions is a widespread phenomenon.[2] For example, many observers argued that the outcome of the recent auctions for 3G mobile-phone licenses in Europe was affected by collusion and antitrust agencies investigated bidders' behaviour in Italy, the Netherlands and Switzerland (Klemperer, 2002b).

This paper aims to extend the theoretical literature on collusion to English (or ascending) auctions in which bidders' valuations are not independent. In this context, a ring of bidders can exploit the characteristics of the bidding process in order to win more often and pay a lower price. In particular, ring bidders can strategically modify their behaviour in order to send misleading signals that affect the strategies of their competitors: bidders use their bids as a vehicle of misinformation.

Most of the existing literature on collusion assumes that a ring designates a single bidder who participates in the auction on behalf of all colluding bidders, while other ring members have no active task and do not participate in the auction at all.[3] So the ring reduces competition in the auction by reducing the number of active bidders and, hence, tries to reduce the price paid by the winner (hoping he is a ring member), but cannot influence the probability that a ring member wins the auction.[4] However, when valuations are not independent, the ring can do better than simply eliminating competition among its members — the ring can reduce the aggressiveness of its competitors, thus biasing the outcome of the auction to its advantage.

Consider, as an example, an auction for the extraction rights of offshore oil leases. The value of the rights is determined by the amount of oil contained in the tract and, arguably, is approximately the same for every bidder. Before the auction, bidders are very uncertain about their valuation and have access to different information, such as different seismic data and/or estimation of drilling costs. Therefore, a bidder's valuation is affected by the information possessed by his competitors and knowing this information would allow a bidder to better

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[1] For a discussion of the relevance of collusion in auctions see Klemperer (2002a). The author argues that trying to prevent collusive behaviour is one of the main challenges faced by the auction designer.

[2] Graham and Marshall (1987) report that a retired auctioneer noted that, in 40 years of work, he had never attended an auction at which a ring was not present. According to Hendricks and Porter (1989), 81% of the 319 Sherman Act Section 1 criminal cases filed by the U.S. Department of Justice from November 1979 to May 1988 were in auction markets. The U.S. Department of Justice's antitrust chief (as quoted by McAfee and McMillan, 1992) reports that collusive behaviour among bidders in auctions for highway contracts increased the cost of building roads for the government by at least 10%.

[3] See, for example, Robinson (1985), Graham and Marshall (1987), McAfee and McMillan (1992), and Mailath and Zemsky (1991). (However, Graham and Marshall, 1987, argue that, when a ring includes all bidders, non-designated bidders can place random bids in order to conceal the presence of the ring and induce the seller not to raise the reserve price. See also Porter and Zona, 1993.)

[4] In the words of Graham and Marshall (1987): "a coalition [...], which contains K of the N bidders at an auction, gains in expected terms by removing $K-1$ bidders from the competitive bidding. If the coalition does not contain the two bidders with the two highest valuations from the N bidders, then the ring realizes no gain beyond what each member could have obtained acting non-cooperatively".

estimate his valuation.

During an English auction, each bidder can observe the prices bid by his competitors and infer the information they posses. Therefore, a bidder's strategy depends on his competitors' bids, as these convey information on the value of the rights. But then ring bidders may strategically manipulate the information they report through their bids, in order to influence the bidding strategies of their opponents. If all but one ring bidder drop out of the auction at a low price, pretending their estimate of the rights value is low, then non-ring bidders are misled into reducing their own value estimate and into bidding less aggressively. As a consequence, the remaining ring bidder can bid more aggressively since he suffers a lower "winner's curse", and the ring can share the enhanced profit.

Unlike in other models analyzed in the literature, the collusive strategy described here requires all ring members to participate actively in the auction. Collusion both reduces competition among ring bidders and misleads the behaviour of non-ring bidders. The existing literature on collusion has concentrated on the first effect while neglecting the second one. Moreover, the collusive strategy is aimed both at reducing the price paid by the ring and at increasing its probability of winning. By contrast, in other analyses of collusion the probability of a ring bidder winning the auction is not affected by collusion. The presence of a ring also reduces the efficiency of the English auction, because a ring bidder can obtain the prize even if he does not have the highest valuation.

Standard economic analysis (and previous models of collusion in auctions) suggests that all firms in a market, even non-colluding ones, should (weakly) benefit from collusion, since competition and prices are lower and profit higher in a market where players collude. By contrast, in our model the presence of a ring makes other bidders strictly worse off because colluding bidders strategically affect the behaviour of outsiders. Indeed, non-ring bidders are induced to bid less aggressively and, compared to an auction without collusion, they win less often. Moreover, when they win against a ring bidder, they pay higher prices. Therefore, our model provides an explanation of why bidders are hurt by a collusive agreement among their competitors and typically try to prevent it.

After the 3G mobile-phone licenses auctions in the UK and Germany (which raised considerable revenue for the governments), various potential buyers failed to enter other European auctions.[5] This was possibly a consequence of a genuine concern about licenses' profitability or deteriorating bidders' credit rating. However, by failing to bid firms caused a drastic reduction in markets' estimate of the licenses' value and in the auction prices in many European countries.[6] Our analysis of the signalling effects of firms' bidding behaviour suggests that failure to bid may have been an explicit strategic choice.

Following the literature, we assume that non-ring bidders are unaware of the presence of a ring in the auction. However, we argue in Section 7.3 that, at least in a common-value

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[5] There were 13 bidders (for 5 licenses) in the UK auction but, for example, only 6 (for 5 licenses) in Italy and the Netherlands and 4 (for 4 licenses) in Switzerland (Klemperer, 2002b).

[6] Compared to a per-capita price of 630 euros in the UK, prices in Italy, the Netherlands and Switzerland were 240, 170 and 20 respectively.

model, our assumption is without loss of generality because the outcome of the auction is the same when non-ring bidders know they are facing a ring, as it is when non-ring bidders are unaware of the presence of a ring.[7] And even if non-ring bidders only place some positive probability on the existence of a ring in the auction, the ring can always credibly signal its presence and obtain the same outcome as it does under our assumption.

In the next section, we review the theoretical literature on collusion in auctions. Section 2 discusses a simple example, based on a pure common-value model, to introduce the main ideas of the paper. In Section 3 we present the model of an English auction with affiliated valuations and, following Milgrom and Weber (1982), we describe the equilibrium bidding strategies of potential buyers in the absence of collusion. Section 4 presents a collusive mechanism that results in all ring members truthfully reporting their signal and following the bidding strategies set by a risk-neutral ring center. Section 5 analyzes the consequences of the ring strategy on the bidding behaviour of both ring and non-ring bidders and Section 6 describes the profit obtained by colluding bidders and the effects of collusion. Section 7 reconsiders the simpler model of Section 2 and extends the analysis to almost common-value auctions and to sequential private-value auctions. The seller's strategy is discussed in Section 7.1. The last section concludes. Omitted proofs are contained in the Appendix.

1.1. Related Literature

The theoretical literature on collusion in auctions is still rather limited.[8] Robinson (1985) assumes all bidders join a ring and credibly reveal their valuation to each other. The ring selects one bidder who is designated to win the auction. Other bidders should remain inactive, but they may find it profitable to cheat on the collusive agreement and try to win the auction. The author argues that collusion is easier to sustain in a second-price auction than in a first-price auction. Indeed, in a second-price auction the designated winner can bid infinitely high and other bidders have no incentive to deviate. On the other hand, to win a first-price auction at a low price, the designated winner must bid a low price and bidders who should remain inactive have an incentive to deviate and win the auction.

While Robinson (1985) assumes that ring members reveal their valuation to one another, one of the main problems faced by a ring is how to induce its members to report their information truthfully. This problem arises because the division of the ring profit depends on bidders' valuations and, hence, ring members have an incentive to misreport their valuations to obtain a higher share of profit. So the ring has to find a mechanism that efficiently and incentive-compatibly designates the winner and divides the collusive profit.

McAfee and McMillan (1992) analyze a ring including all bidders in an auction with

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[7] When non-ring bidders know they are facing a ring, a common-value English auction has a continuum of equilibria (Bikhchandani and Riley, 1991). However, by analyzing a pure common-value auction as the limit of an almost common-value auction, we prove that it is natural to select a unique equilibrium in which bidding strategies are the same as when non-ring bidders are unaware of the presence of a ring.

[8] The (growing) empirical literature on collusion includes Baldwin, Marshall and Richard (1997), Hendricks and Porter (1992), Hendricks et al. (1998), Pesendorfer (2000), and Porter and Zona (1993).

independent and private valuations and introduce the following efficient and ex-post budget balanced mechanism. Ring bidders (jointly) win the main auction at the reserve price and allocate the object among themselves by a first-price "knockout" auction, the winner of which pays each ring bidder (including himself) an equal share of the difference between his bid in the "knockout" and the reserve price. The mechanism is incentive-compatible since a losing bidder's payoff does not depend on his bid and, hence, as in a standard first-price auction, in the "knockout" each bidder bids up to the expected second-highest valuation, given that his valuation is the highest.[9,10] (This is a special case of the mechanism proposed by Cramton, Gibbons and Klemperer (1987) to assign an object jointly owned by a group of agents.) Notice that, in this case, revenue equivalence in the main auction holds, but each bidder's surplus is higher by an equal amount.

Graham and Marshall (1987) show that, with independent and private valuations, ring bidders can efficiently allocate the object among themselves in dominant strategies by running a second-price knockout auction, the winner of which pays (the second-highest bid to) a risk-neutral "ring center" who previously paid all ring bidders an equal share of the expected second-highest of their valuations (again a loser's payoff does not depend on his bid, so each ring bidder bids his valuation). However, this mechanism is only budget balanced in expectation.

When the main auction is a second-price or an English auction (and valuations are private and independent), Graham and Marshall (1987) extend the result to cases when not all bidders collude. The ring center selects the designated bidder by a second-price knockout auction and, if the designated bidder wins the main auction (in which other ring bidders do not participate), he pays the ring center the difference between the price in the knockout and the main auction price, if this difference is positive. Before the knockout, the ring center pays each ring bidder an equal share of the expected payment by the designated bidder. Therefore, because a colluding bidder pays the second-highest bid among all bidders when he obtains the prize, bidding his valuation in the knockout (and in the main) auction is a dominant strategy. Mailath and Zemsky (1991) extend the result of Graham and Marshall (1987) to auctions in which bidders' valuations are drawn from different supports.[11]

Note, however, that neither the mechanism of Graham and Marshall (1987) nor the one of McAfee and McMillan (1992) is incentive-compatible if bidders' valuations are not independent because, to share the ring profit, side-payments to ring bidders who lose the knockout have to depend on the information they report to the ring, because this information affects the prize value. This would induce ring bidders to deviate and not reveal their information truthfully, in order to obtain a higher share of profit. We extend the analysis of Graham

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[9] McAfee and McMillan (1992) also prove that (under an "increasing hazard rate" condition), when the ring cannot make side-payments, it cannot extract any useful information from its members and can do no better than randomize the right to bid in the main auction among its members.

[10] However, ring bidders may want to cheat at the main auction.

[11] Mailath and Zemsky (1991) also show that efficient collusion can be achieved without the need for an outside agent, but using a more complex mechanism.

and Marshall (1987) and propose an incentive compatible mechanism for the case of non-independent valuations.

The use of bids as a signalling device has already been underlined by Bikhchandani (1988) and Brusco and Lopomo (2002). Bikhchandani analyses a series of second-price common-value auctions without collusion and shows that a bidder can use his bids to establish a reputation for bidding aggressively, hence forcing his competitors to bid less aggressively in future auctions. Brusco and Lopomo (2002) analyze (tacit) collusion in multi-unit ascending auctions and prove that a bidder can use his bid to truthfully signal to his competitors which object has the highest value for him.[12] Bidders agree on a division of the objects and end the auction at low prices. Cramton and Schwartz (2000) argue that this type of signalling strategy was adopted during the FCC spectrum auctions in the USA. By contrast, we prove how bidders can use their bids to communicate false information regarding their valuations.

2. Example: Common Values

Consider an English auction for a prize whose value is exactly the same for all bidders.[13] There are three bidders, called $1, 2$ and 3, and each bidder i receives a non-negative private signal x_i about the value of the prize. Signals are independently distributed. As in the "wallet game" of Klemperer (1998) and Bulow and Klemperer (2002), the common value of the object is:[14]

$$V = x_1 + x_2 + x_3.$$

At the beginning of the auction, the unique symmetric equilibrium is for each bidder i to bid up to:[15]

$$\alpha_0^i(x_i) = 3x_i, \quad i = 1, 2, 3. \tag{2.1}$$

That is, each bidder bids up to the value at which he makes no money if he wins the auction when all other bidders have his same signal (and, therefore, he is indifferent between winning or losing).[16]

Once the bidder with the lowest signal quits the auction, he reveals his signal to the two remaining bidders, who update their bidding strategies using the new information. Suppose

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[12] Other papers analyzing collusive outcomes in multi-unit ascending auctions are Weber (1997), Ausubel and Cramton (1998), Englebrecht-Wiggans and Kahn (1998), and Grimm et al. (2002).

[13] In an English auction the price is raised continuously by the auctioneer until only one active bidder is left. A bidder who wishes to be active at the current price depresses a button and, when he releases it, he is withdrawn from the auction (and cannot become active again). The price level and the number of active bidders are continuously displayed.

[14] The "wallet game" is a classroom example in which three students are selected and each privately checks how much money is in his wallet. A prize equal to the total content of the wallets of the three students is auctioned using an ascending auction.

[15] The functions $\alpha_k^i(.)$ and $\beta(.)$ are defined more generally in Section 4.

[16] To see that this is an equilibrium, assume all bidders bid according to (2.1). Suppose bidder i deviates, stays longer in the auction and wins at price $3x_i + \varepsilon$. Then, however, the value of the prize is $x_i + \frac{1}{3}(3x_i + \varepsilon) + \frac{1}{3}(3x_i + \varepsilon) < 3x_i + \varepsilon$. Therefore, i pays the prize more than it is worth and, hence, he would have done better by quitting earlier. A symmetric argument applies if bidder i loses at price $3x_i - \varepsilon$. Hence, no bidder can increase his payoff by deviating from the strategy (2.1).

that bidder j has the lowest signal and quits the auction at price $3x_j$. Then the unique symmetric equilibrium is for the two remaining bidders to bid up to:[17]

$$\alpha_1^i(x_i, x_j) = 2x_i + x_j, \quad i \neq j. \tag{2.2}$$

Basically, the auction proceeds in two phases. In the first one, the bidder with the lowest signal drops out and reveals his private information. In the second phase, the two remaining bidders engage in a second-price auction using the information acquired in the first phase. In each phase, a bidder bids up to his estimate of the prize value, conditional on all the information he has and on winning against opponent(s) with his same signal. Hence, to update their estimate of the prize value, bidders infer their competitors' private information from their bidding behaviour.

Suppose now that two bidders, say 1 and 2, join a ring and that the third one does not know they do (nor does she suspect it)[18] — we are going to relax this assumption in Sections 7.2 and 7.3. Since the quitting prices of the two bidders with the highest signals depend on the price at which the bidder with the lowest signal drops out, the ring can influence the strategy of bidder 3 to its advantage. In particular, the ring can induce bidder 3 to bid less aggressively, thus enabling a ring bidder to bid more aggressively and increasing the ring profit.

Assume, without loss of generality, that $x_1 > x_2$ and assume that ring members know each other signals.[19] The bidder with the highest signal (i.e., bidder 1) is the *designated* bidder while the bidder with the lowest signal (i.e., bidder 2) drops out of the auction at price zero. This misleads bidder 3 into thinking that bidder 2 has a very bad signal of the prize value (i.e., $x_2 = 0$). Therefore, bidder 3 reduces her own estimate of the prize value and, hence, bids less aggressively. Precisely, by equation (2.2), she bids up to $2x_3$.

As a result, bidder 1 suffers a lower winner's curse and can bid more aggressively. Indeed, given that bidder 3 bids up to $2x_3$, if bidder 1 wins at price p he knows the prize is worth $x_1 + x_2 + \frac{p}{2}$ (since 3's signal is equal to half the price at which he drops out). Because this value is greater than p if and only if $p < 2(x_1 + x_2)$, bidder 1 bids up to:

$$\beta(x_1, x_2) = 2(x_1 + x_2).$$

The ring achieves two objectives:

(*i*) it reduces competition in the auction by eliminating one "serious" bidder;

(*ii*) it reduces the aggressiveness of the non-ring bidder.

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[17] The described equilibrium is independent of the signals' distributions and does not require the distributions to be symmetric.

[18] We adopt the convention of using feminine pronouns for the non-ring bidder.

[19] In Section 4 we are going to prove that the ring can design a mechanism such that it is incentive compatible for each colluding bidder to truthfully reveal his signal.

Collusion increases the probability that the designated bidder wins the auction because, even if bidder 3 has the highest signal, the designated bidder can still win.[20] Moreover, even when he would have won anyway, with collusion the designated bidder pays a lower price ($2x_3$ rather than $\max\{2x_2 + x_3;\ 2x_3 + x_2\}$).[21] The extra profit obtained by the ring is given by the difference between the price the designated bidder would have paid without collusion and the price he actually pays, when he has got the highest signal, and by the difference between the prize value and the price the designed bidder pays, when he hasn't got the highest signal and wins the auction.

We are now going to generalize the example to an auction with affiliated signals and n bidders.

3. The Model

Consider an English auction for a single object with n risk-neutral bidders. Each bidder i receives a (private) signal $x_i \geq 0$ of the value of the prize, which is the realization of a random variable X_i. Let $X = (X_1, \ldots, X_n)$ and let S be an additional random variable which influence the value of the prize and is independent of X. The random elements of X have joint probability density function $f(x)$. We assume that $f(.)$ is symmetric in all its arguments and, therefore, that bidders' signals are identically distributed. Following Milgrom and Weber (1982), we assume that the variables $X_1, \ldots, X_n$ are *affiliated* in the sense of the following definition.

Definition 1. *Let x, $x' \in \mathbb{R}^n$ be two realizations of X and let $x \vee x'$ and $x \wedge x'$ denote respectively the component-wise maximum and minimum of x and x'. The random variables $X_1, \ldots, X_n$ are affiliated if, for all x and x',*

$$f\left(x \vee x'\right) f\left(x \wedge x'\right) \geq f(x)\, f\left(x'\right).$$

Roughly, random variables are said to be affiliated when higher values for some of the variables make the other variables more likely to be high than low.

The value of the prize to bidder i is:

$$V_i = u\left(X_i; \{X_j\}_{j \neq i}; S\right),$$

where $u : \mathbb{R}^{n+1} \to \mathbb{R}_+^0$ and $\{X_j\}_{j \neq i}$ represents the *unordered* set of signals different from X_i. Hence, each bidder's valuation is a symmetric function of the other bidders' signals and all bidders' valuations depend on S in the same way. We assume that u is continuous and (weakly) increasing in each of its arguments, which implies that bidders' valuations are

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[20] For example, if signals are uniformly distributed on $[0, 1]$, the ring wins the auction with probability $\frac{5}{6}$ while, without collusion, each bidder wins with probability $\frac{1}{3}$.

[21] If signals are uniformly distributed on $[0, 1]$, before the auction the expected price the designated bidder pays conditional on winning is equal to $\frac{9}{10}$ while the expected price he pays conditional on winning without collusion is equal to $\frac{5}{4}$.

affiliated too (Milgrom and Weber, 1982, Theorem 3). Moreover, we assume that in his valuation each bidder "weights" his own signal no less than any of the other bidders' signals. Formally, we require a *single crossing condition* that $\partial V_i(X)/\partial X_i \geq \partial V_i(X)/\partial X_j$, for every X and $j \neq i$. This assumption ensures that, if bidder i's signal is higher than bidder j's one, then bidder i values the prize more than bidder j.[22] If all bidders' signals are equal to zero, than in expectation the prize is worthless, that is $\mathbb{E}[u(0;0,\ldots,0;S)] = 0$.

Note that the value function $u(.)$ includes both the pure common value and the private value as special cases. Indeed, for the pure common value case, $u\left(X_i; \{X_j\}_{j\neq i}; S\right) = u\left(X_k; \{X_j\}_{j\neq k}; S\right)$ for every i and k while, for the private value case, $u\left(X_i; \{X_j\}_{j\neq i}; S\right) = u(X_i)$.[23]

Let $Y_1, \ldots, Y_{n-1}$ denote respectively the smallest,..., largest signal from among $\{X_j\}_{j\neq i}$. Then, bidder i's valuation is given by $V_i = u(X_i; Y_{n-1}, \ldots, Y_1; S)$. Since the variables $X_1, \ldots, X_n$ are affiliated and their joint density function is a symmetric function of its arguments, the variables $X_i, Y_1, \ldots, Y_{n-1}$ are affiliated too (Milgrom and Weber, 1982, Theorem 2).

In an English auction, a strategy for a bidder specifies whether, at any price level, he remains active or he drops out, as a function of his value estimate, of the number of bidders who quit the auction, and of the price at which they quit. So if k bidders dropped out at prices $p_1 \leq \ldots \leq p_k$, bidder i's strategy can be described by a function $\alpha_k^i(x_i; p_1, \ldots, p_k)$ which specify the price at which he drops out. Milgrom and Weber (1982) prove the following result.

Proposition 1. *The (symmetric) strategies* $\alpha^i = \left(\alpha_0^i, \ldots, \alpha_{n-2}^i\right)$, $i = 1, \ldots, n$, *defined iteratively by:*

$$\alpha_0^i(x_i) = \mathbb{E}[V_i \,| X_i = Y_{n-1} = \ldots = Y_1 = x_i], \tag{3.1}$$

$$\alpha_k^i(x_i; p_1, \ldots, p_k) = \mathbb{E}\left[V_i \left| \begin{array}{l} X_i = Y_{n-1} = \ldots = Y_{k+1} = x_i, \\ \alpha_{k-1}^i(Y_k \,| p_1, \ldots, p_{k-1}) = p_k, \ldots, \alpha_0^i(Y_1) = p_1 \end{array} \right.\right], \tag{3.2}$$

$k = 1, \ldots, n-2$, *are equilibrium bidding strategies.*

Note that the bidding strategy (3.2) is equivalent to:

$$\alpha_k^i(x_i; y_1, \ldots, y_k) = \mathbb{E}[V_i \,| X_i = Y_{n-1} = \ldots = Y_{k+1} = x_i, Y_k = y_k, \ldots, Y_1 = y_1],$$

where $y_1, \ldots, y_k$ are the realizations of the random variables $Y_1, \ldots, Y_k$. Therefore, in equilibrium each bidder remains active until the price reaches to the point where he is just indifferent between winning and losing, if all active bidders have his same signal. A bidder uses the quitting prices of bidders who drop out of the auction to infer their signal and update his estimate

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[22] This assumption is not necessary for our results, but it simplifies the analysis since it ensures that the bidder with the highest signal is also the one with the highest valuation.

[23] In the pure common-value example of Section 2, signals are independent (and hence affiliated), the prize value is "symmetrically" increasing in all signals, and there is no additional random variable S.

of the prize value. Colluding bidders can exploit this feature to mislead their competitors and modify the outcome of the auction to their advantage.

Milgrom and Weber (1982) also prove the following result.

Lemma 1. *Let* $X_1, \ldots, X_n$ *be affiliated random variables and let* $x_1, \ldots, x_n$ *be their realizations. Let* $V(.)$ *be a non-decreasing function. Then the functions:*

$$\alpha_k (x_1, \ldots, x_k) = \mathbb{E}\left[V (X_1, \ldots, X_n; S) | x_1, \ldots, x_k\right], \quad k = 1, \ldots, n,$$

are non-decreasing in all of its arguments.

As in Milgrom and Weber (1982), to simplify the analysis we assume that these functions are strictly increasing. So the equilibrium bidding strategy α_k^i is increasing in bidder i's signal, x_i, and in each of his opponents' signals, $y_1, \ldots, y_k$. This implies that, the higher his opponents' signals, the higher bidder i's estimate of prize value and, hence, the higher the price at which he drops out of the auction.

We assume m bidders join a ring, $2 \leq m < n$. It is not possible for all potential buyers to join the ring because, for example, legal considerations force the ring to limit membership in order to avoid detection by antirust authorities. The set of colluding bidders is exogenously given. Let $W_1, \ldots, W_m$ be respectively the lowest,..., highest signal of the prize value received by ring members. Let $Z_1, \ldots, Z_{n-m}$ be respectively the lowest, ..., highest signal received by non-ring bidders. We denote the realizations of W_i and Z_i by w_i and x_i respectively. We assume that bidders who are not ring members do not know that they are facing a ring.[24]

4. Collusive Mechanism

There is a risk-neutral agent called *ring center* who acts as mediator and banker for the ring. The ring center must design a mechanism to regulate the behavior of ring bidders. We will prove that there is a mechanism that results in all ring bidders revealing their true signal of the prize value and that allows the ring to increase its probability of winning the auction and its expected profit.

Consider a mechanism that requires each ring bidder to report his private information. For any vector of reports, the mechanism must determine: (i) the strategy of each bidder in the auction, (ii) the *designated* bidder who receives the prize if it is won by the ring, and (iii) the payments each ring bidder makes/receives. The mechanism is *incentive-compatible* if it is an equilibrium for each ring bidder to report his private information truthfully and to follow the bidding strategy set by the ring. The mechanism is (ex-ante) *budget-balanced* if the side-payments sum to zero in expectation.

The following mechanism, called (M), generalizes the one proposed by Graham and Marshall (1987) that handled the special case of independent private values.

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[24] This simplifying assumption is widely used in literature (see, for example, Assumption 3 in Graham and Marshall, 1987). The assumption will be relaxed in Sections 7.2 and 7.3

1. Each ring bidder receives from the ring center a fixed side-payment of:

   $$\frac{1}{m}\mathbb{E}\left[\pi_C^m\left(W_m = W_{m-1}\right)\right],$$

   that is, an equal share of the expected collusive profit of the ring bidder with the highest signal, if he has a signal equal to the expected second-highest signal among ring bidders.[25]

2. Each ring bidder reports his signal to the ring center. Let $w_1, \ldots, w_m$ be respectively the lowest, ..., highest reported signal.

3. The ring member who reported the highest signal (and, hence, the highest valuation) is the designated bidder. He pays the ring center $\mathbb{E}\left[\pi_C^m\left(W_m = w_{m-1}\right)\right]$, his expected collusive profit if he had a signal equal to the second-highest reported signal.

4. The $m-1$ ring bidders who reported the lowest signals drop out of the auction immediately at price zero.[26]

5. The designated bidder bids according to the strategy $\beta = \left(\beta_0, \ldots, \beta_{n-m-1}\right)$ where β_k is the price at which he drops out given that k non-ring bidders have dropped out. This strategy is (implicitly) defined by:[27]

   $$\beta_k = \mathbb{E}\left[V_m \,\middle|\, Z_{n-m} = \ldots = Z_{k+1} = \psi_k^{-1}\left(\beta_k\right);\ w_m, \ldots, w_1, z_k, \ldots, z_1\right], \tag{4.1}$$

   $k = 0, \ldots, n-m-1$, where:[28]

   $$\psi_k\left(x_i\right) = \alpha_{k+m-1}^i\left(x_i; 0, \ldots, 0, z_1, \ldots, z_k\right), \quad k = 0, \ldots, n-m-1.$$

6. If the designated bidder wins the auction, he retains the prize.

Proposition 2. *The mechanism (M) is incentive-compatible and (ex-ante) budget-balanced.*

Proof. We need to prove that the truthful revelation of their signals is an equilibrium for ring bidders. Notice that the side-payment received by a ring bidder does not depend on the signal he reports and, hence, cannot affect incentives. Therefore, a ring bidder's report

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[25] The ring's profit and the function $\pi_C^m(.)$ are derived in Section 6.

[26] To simplify the analysis, we assume all ring bidders apart the designated one drop out exactly at price zero, but this can be interpreted as each bidder dropping out at a different very low price (arbitrarily close to zero).

[27] In Section 5, we are going to prove that β is an equilibrium bidding strategy and that, when the ring uses the mechanism (M), ψ_k is the strategy adopted by a non-ring bidder after $k+m-1$ bidders have quit the auction.

Note that the strategy β simply calls on the designated bidder to remain active up to the price at which he would be just indifferent between winning and losing the auction, given his valuation and the reports of other ring members and given all the information regarding the signals of non-ring bidders' he can infer from their behaviour in the auction.

[28] The function $\alpha_k^i(.)$ is defined in Proposition 1.

depends only on his expected payment and his expected profit if he is chosen as the designated bidder.

In mechanism (M), ring bidders actually participate in a second-price sealed-bid knockout auction whose prize is the right to be chosen as the designated bidder and retain the auction prize if it is won by the ring. So the value of winning the knockout for a ring bidder is the expected collusive profit if he is the designated bidder, which is increasing in a bidder's signal since, other things being equal, a bidder with a higher signal has a higher valuation and, hence, expects to obtain a higher collusive profit. If he wins the knockout, a bidder pays the expected collusive profit if he had a signal equal to the second-highest reported signal, which does not depend on his report. This payment is lower than his expected collusive profit as the designated bidder if and only if his signals is higher than the second-highest reported signal. Therefore, if other ring bidders report their true signals, a bidder is pleased to win the knockout if and only if he has the highest signal among all ring bidders. This implies that it is an equilibrium for each ring bidder to report his signal truthfully.

In Section 5 we are going to prove that it is an equilibrium for the designated bidder to bid in the auction according to the strategy β — i.e., up to his expected valuation conditional on winning, given the ring information and the information he infers from the behaviour of non-ring bidders. Other ring bidders drop out of the auction at price zero and cannot gain by deviating because they cannot win at a price lower than the expected valuation of the designated bidder, which is higher than their valuation (since the designated bidder has the highest signal).

It follows that mechanism (M) is incentive-compatible. The fact that (M) is (ex-ante) budget-balanced in expectation follows from the definitions of the side-payments made and received by the ring center. ■

Notice that mechanism (M) implies that, before the auction, each ring bidder receives from the ring center an equal share of the expected payment by the designated bidder. In addition, the designated bidder retains the auction prize if he wins it and any additional profit (or losses) he obtains during the auction.

5. How Collusion Affects Bidding Strategies

Since the ring can design a mechanism to make each bidder truthfully report his signal, it can be assumed that the ring knows its members' signals. We now analyze the bidding behavior of non-ring bidders when the ring adopts mechanism (M) and compare it to their behaviour in an auction without collusion.

Definition 2. *A buyer bids more (less) aggressively in auction A than in auction B if, given any set of signals, the price at which he drops out in auction A is not higher (lower) than the price at which he drops out in auction B.*

Lemma 2. *When the ring adopts mechanism* (M)*, after* k *non-ring bidders have dropped out of the auction, a non-ring bidder with signal* x_i *bids up to:*

$$\psi_k(x_i) \equiv \mathbb{E}\left[V_i \left| X_i = Y_{n-1} = \ldots = Y_{k+m} = x_i;\ z_k, \ldots, z_1, 0, \ldots, 0\right.\right].$$

Non-ring bidders bid less aggressively than in an auction without collusion.

The intuition for this result is straightforward. When non-ring bidders observe potential buyers dropping out at price zero, they infer that their signal is equal to zero. Therefore, they update their information and reduce the value estimate of the prize, which induces them to bid less aggressively.

As the next lemma shows, when the ring adopts mechanism (M), bidding according to the strategy β defined in Section 4 is an equilibrium for the designated bidder, given the bidding strategies of non-ring bidders.

Lemma 3. *When the ring adopts mechanism* (M)*,* β *is an equilibrium bidding strategy for the designated bidder. The designated bidder bids more aggressively than in an auction without collusion.*

The intuition for this result is that, given that non-ring bidders bid less aggressively, the designated bidder suffers a lower winner's curse if he wins the auction and, hence, he can bid more aggressively. Precisely, the designated bidder bids up to the price at which he is exactly indifferent between winning and losing, that is up to his expected valuation conditional on winning, given all ring bidders' signals and the information revealed by non-ring bidders' behaviour.

6. Collusion Pays

By adopting mechanism (M), the ring allows the designated bidder to win the auction more often and pay a lower price. This follows from Section 5, because due to collusion non-ring bidders bid less aggressively while the designated bidder bids more aggressively.

Lemma 4. *When the ring adopts mechanism* (M)*, the probability that the designated bidder wins the auction is higher than in an auction without collusion.*

Without collusion, the designated bidder wins the auction if and only if he has the highest valuation. Therefore, collusion may lead to an inefficient allocation of the auction prize, because the designated bidder can win even against a bidder who has a higher signal and, hence, a higher valuation.

Lemma 5. *When the ring adopts mechanism* (M)*, if the designated bidder wins an auction he would have won even without collusion, he pays a lower price.*

The total profit ring bidders expects to obtain by adopting mechanism (M) is:

$$\mathbb{E}\left[\pi_C^m\right] = \mathbb{E}\left[\left(V_m - \psi_{n-m-1}\left(Z_{n-m}\right)\right) \cdot \mathbf{1}_{\left\{\beta_{n-m-1} > \psi_{n-m-1}\left(Z_{n-m}\right)\right\}} \middle| w_1, \ldots, w_m\right], \tag{6.1}$$

where $\mathbf{1}_{\{.\}}$ is the indicator function. With mechanism (M), the ring increases its expected profit both by increasing the probability of winning the auction and by reducing the price paid.

Proposition 3. *By adopting mechanism (M), the ring increases its expected profit (compared to an auction without collusion).*

The extra profit obtained by collusion depends on two different effects:

1. The *reduced competition* effect due to the fact that $m-1$ ring bidders do not bid positive prices.[29]
2. The *signalling* effect due to the strategic behavior of ring bidders who drop out at price zero, making non-ring bidders bid less aggressively and the designated bidder bid more aggressively.[30]

The signalling effect is peculiar to the English auction with affiliated valuations. In fact, if bidders collude when different types of auctions are used by the seller (for example, sealed-bid or Dutch auctions), they cannot exploit this effect since their competitors cannot observe their bid and hence infer their valuations. And when bidders' valuations are independent, even if bidders can infer their competitors' valuations from their bid, they find this information useless. In both cases, bids lose their signalling content.[31]

The reduced competition effect does not affect the probability that the designated bidder wins the auction, it only increases his payoff, given that he wins. The previous literature on collusion in auctions concentrated on this first effect and neglected the potential advantage for ring bidders of strategically manipulating their bids. Moreover, by contrast to standard analysis that suggest that all players benefit from (or at least are not hurt by) collusion (because collusion reduces competition), in our model non-ring bidders are made worse off by collusion because, compared to an auction without collusion, they are induced to bid less aggressively and this reduces their probability of winning the auction.

The actual (ex-post) extra profit of the ring is the given by the extra profit the designated bidder obtains by collusion, which depends on bidders' signals. There are three cases. Firstly,

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[29] This effect reduces the expected price paid by the designated bidder from $\mathbb{E}\left[\alpha_{n-2}^i\left(Y_{n-1}\right)\right]$ to $\mathbb{E}\left[\alpha_{n-m-1}^i\left(Z_{n-m}\right)\right]$ and increases the expected bidding price of the designated bidder from $\mathbb{E}\left[\alpha_{n-2}^i\left(W_m\right)\right]$ to $\mathbb{E}\left[\alpha_{n-m-1}^i\left(W_m, \ldots, W_1\right)\right]$.

[30] This effect further reduces the expected price paid by the designated bidder from $\mathbb{E}\left[\alpha_{n-m-1}^i\left(Z_{n-m}\right)\right]$ to $\mathbb{E}\left[\psi_{n-m-1}\left(Z_{n-m}\right)\right]$ and increases the expected bidding price of the designated bidder from $\mathbb{E}\left[\alpha_{n-m-1}^i\left(W_m, \ldots, W_1\right)\right]$ to $\mathbb{E}\left[\beta_{n-m-1}\left(W_m, \ldots, W_1\right)\right]$.

[31] However, in Section 7.4 we argue that with sequential auctions bids have a signalling content that can be exploited by colluding bidders even if valuations are independent.

when the designated bidder has the highest among all signals (i.e., when $w_m > z_{n-m}$), the (extra) ring profit is:

$$\alpha^i_{n-2}\left(\max\left\{z;\ w_{m-2}\right\}\right) - \psi_{n-m-1}\left(z_{n-m}\right).$$

This is, the difference between the price up to which the bidder with the second-highest signal (among all signals) would have bid in the absence of collusion and the price the designated bidder pays.

Secondly, even when the designated bidder does not have the highest signal, with collusion he can still win the auction because he bids more aggressively. Indeed, when $w_m < z_{n-m}$ and $\beta_{n-m-1} > \psi_{n-m-1}\left(z_{n-m}\right)$, the (extra) ring profit is:

$$V_m - \psi_{n-m-1}\left(z_{n-m}\right).$$

That is, the whole difference between the designated bidder's valuation and the price he pays. Finally, when $w_m < z_{n-m}$ but $\beta_{n-m-1} < \psi_{n-m-1}\left(z_{n-m}\right)$, the designated bidder loses the auction, as he would have done without collusion, and, hence, the ring profit is equal to zero.

Summing up, when the designated bidder has the highest signal among all potential buyers, the ring gains by reducing the price paid for the object, and when the designated bidder does not have the highest signal, the ring gains by giving him a chance to win the auction anyway.

7. Extensions

In this section, we reconsider the simple model of Section 2 (with common values and independent signals) and discuss some extensions of our analysis.

7.1. Seller's Strategy

Collusion reduces the efficiency of the auction because the prize can be assigned to the designated ring bidder even when he does not have the highest valuation. Moreover, with independent signals, collusion also reduces the expected auction price and the expected seller's revenue. To see this, notice that, with independent signals (and downward sloping *marginal revenues*),[32] an English auction with an appropriate reserve price maximizes the seller's revenue if bidders bid independently, because it sells to the bidder with the highest *marginal revenue* (Myerson, 1981, and Bulow and Klemperer, 1996). But collusion among bidders modifies the allocation achieved by the auction since the prize need not be assigned to the bidder with the highest *marginal revenue*, and this reduces the expected seller's revenue.[33]

So a seller who wants to achieve an efficient allocation and maximize his revenue should try to prevent bidders from joining a ring and, if he cannot do so, he should try to prevent

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[32] Letting $h_i\left(x_i\right)$ be bidder i's hazard rate, the *marginal revenue* of bidder i is defined as $V_i - \frac{1}{h_i} \cdot \frac{\partial V_i}{\partial x_i}$.

[33] For example, if signals are uniformly distributed on $[0,1]$, the expected seller's revenue of the pure common-value auction with three bidders without collusion is equal to $\frac{5}{4}$, while the expected seller's revenue when two bidders collude is equal to $\frac{11}{12}$.

colluding bidders from signalling to their opponents. For example, the seller could choose an auction mechanism in which bids are unobservable, like a second-price sealed-bid auction.

7.2. Non-Secret Ring

We have assumed that a non-ring bidder is completely unaware of the presence of a ring in the auction. It is natural to ask what happens if a non-ring bidder knows she is facing a ring and, hence, a low bid made by a ring bidder does not fool the non-ring bidder into believing he has a low signal.

Consider again the pure common-value example of Section 2 but assume bidder 3 knows that bidders 1 and 2 joined a ring. Then bidder 3 knows she is bidding against a ring who has shared all the information of its members and is bidding accordingly, and the bid of the first ring bidder who drops out loses its information content. So this is like an auction with two bidders who have signals x_3 and $x_1 + x_2$ respectively. The problem is that, in a pure common-value auction, there is a continuum of equilibria and, typically, a single equilibrium is only pinned down by assuming symmetry among bidders (Bikhchandani and Riley, 1991). But when a bidder knows she is facing a ring, there is an intrinsic asymmetry between the ring's information and bidder 3's information on the prize value.[34] Indeed, bidder 3 bidding up to kx_3 and a ring bidder bidding up to $\frac{k}{k-1}(x_1 + x_2)$ is an equilibrium of the auction, for every $k > 1$. So the outcome of the auction appears extremely uncertain. However, in the next section we argue that there is a natural way to select one of these equilibria.

7.3. Almost Common Values

In order to overcome the problem of multiple equilibria, we consider an almost common-value auctions with three bidders, similar to the one in Bulow and Klemperer (2002). Assume bidders 1 and 2 join a ring and learn each other signals. Bidders' valuations are:

$$\begin{cases} V_1 = V_2 = (1 + \varepsilon)(x_1 + x_2) + x_3, \\ V_3 = (1 + \varepsilon) x_3 + x_1 + x_2, \end{cases}$$

where $\varepsilon \approx 0$. This represents a situation where a bidder places a slightly higher weight to a signal he knows before the auction starts.

An interpretation of these value functions is that information known before the auction starts is more valuable than information obtained after the auction (like a competitor's signal), because bidders are better able to exploit information they obtain earlier, and act upon it in order to earn higher profit. Another interpretation is that, if bidders choose what particular type of information to collect after joining a ring, they can focus on information that is better suited to their specific use of the auction prize and hence is more valuable then their competitor's information. For example, before an auction for a mobile-phone license, telecom firms usually conduct surveys of costumers in order to forecast future demand, and each firm's

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[34] Levin (2004) analyzes joint bidding in a second-price auction by symmetric groups of bidders (i.e., groups composed by the same number of bidders).

survey is also valuable for its competitors. But firms with different business plans conduct different surveys and attach different weights to their competitors' surveys: a firm that plans to focus on business customers will conduct a survey of those customers and will attach a lower weight to a survey made by another firm focused on residential customers. And if the firm learns before the auction that demand from business customers is low, then it can start refocusing its business plan and marketing strategy straight away, thus increasing its ability to extract profit from the license.

Assume first that bidder 3 thinks she is not facing a ring. It is straightforward that, in equilibrium, bidder 3 starts bidding up to $(3+\varepsilon)\,x_3$ and, after a bidder drops out at price p, he bids up to $(2+\varepsilon)\,x_3+\frac{p}{3+\varepsilon}$. Therefore, if the ring adopts mechanism (M) and a ring bidder drops out at price zero, then bidder 3 bids up to $(2+\varepsilon)\,x_3$ while the remaining ring bidder bids up to:[35]

$$\beta(x_1,x_2)=(2+\varepsilon)(x_1+x_2)\,.$$

For $\varepsilon\rightarrow 0$, these bidding strategies converge to the equilibrium bidding strategies of the pure common-value example of Section 2.[36]

Suppose now that bidder 3 knows her opponents joined a ring for sure. The next lemma describes equilibrium bidding strategies.

Lemma 6. *When bidder 3 knows she is facing a ring, in the unique equilibrium of the almost common-value auction bidder 3 bids up to $(2+\varepsilon)\,x_3$ and one ring bidder bids up to $(2+\varepsilon)(x_1+x_2)$ (while the other ring bidder does not participate in the auction).*

Notice that the equilibrium involves exactly the same bidding strategies as in the case in which bidder 3 does not know she is facing a ring. For $\varepsilon\rightarrow 0$, the almost common-value model selects a "natural" equilibrium for the pure common-value case (equivalent to $k=2$ in Section 7.2).[37]

The intuition for this result is the following. If bidder 3 believes she is not facing a ring (but a ring is active), then after a bidder drops out at a low price she believes that bidder has a low signal, which is bad news for the prize value. However, bidder 3 also believes that the other remaining active bidder is choosing to stay in the auction notwithstanding the fact that he knows the bidder who dropped out has a low signal. This means that the remaining active bidder has a high signal and this is good news for the prize value. On the other hand,

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[35] To see that $\beta(.)$ is an equilibrium, notice that if bidder 3 bids up to $(2+\varepsilon)\,x_3$, then when bidder 1 wins the auction at price p he knows the prize is worth $(1+\varepsilon)(x_1+x_2)+\frac{p}{(2+\varepsilon)}$ and, hence, he is willing to stay in the auction up to price p^* such that $p^*=(1+\varepsilon)(x_1+x_2)+\frac{p^*}{(2+\varepsilon)}$.

[36] From the seller's point of view, even if allowing bidders to join a ring in this almost common-value setting slightly increases their valuation, it can still reduce revenue because it induces a non-ring bidder to bid less aggressively.

[37] In this equilibrium, bidder 3 bids relatively cautiously and the ring bidder can bid quite aggressively. An interpretation is that, when the presence of a ring is common knowledge, bidder 3 knows she is competing against a bidder who is "advantaged" (since, on average, his valuation is $\varepsilon\cdot\mathbb{E}[x]$ higher) and, hence, has to bid cautiously to avoid the winner's curse.

if bidder 3 knows she is facing a ring, she makes none of the two inferences.[38] In our simple example of an almost common-value auction, the bad and good news exactly cancel out, so that the outcome of the auction is the same whether bidder 3 believes there is a ring with probability 0 (but a ring is active) or with probability 1.

Therefore, the ring can do just as well when bidder 3 knows she is facing a ring as it can when bidder 3 does not suspect his opponents joined a ring. And even if bidder 3 is uncertain whether she is facing a ring or not, but only places some positive probability on the presence of a ring, the ring can credibly signal its presence by, for instance, having all but one ring bidders drop out at a predetermined price (which is a zero probability event).

7.4. Sequential Private-Value Auctions

In sequential private-value auctions, a ring of bidders can adopt a strategy similar to the one we have described for a single auction with affiliated values. Consider, as a simple example, a sequence of two English auctions for two identical prizes with three bidders. Each bidder i demands exactly one prize and has a privately known valuation v_i for each prize, $i = 1, 2, 3$.

Suppose there is no collusion. In the second auction, it is a dominant strategy for the two bidders who did not win the first auction to bid up to their valuation. In the first auction, bidders start bidding up to their valuation. After a bidder drops out at price p, the two remaining bidders learn that their opponent's value is p and, hence, know they can win the second auction at price p. So they both drop out immediately of the first auction (and the prize is assigned randomly to one of them).

Suppose now that bidders 1 and 2 join a ring and that bidder 3 does not know they do. Assume that ring bidders know each other valuation and, without loss of generality, $v_1 > v_2$. Then bidder 1 drops out at price zero in the first action and this induces bidder 3 to bid less aggressively, because she expects to win the second auction at price zero if she loses the first one. So bidder 3 drops out at zero (immediately after bidder 1) and bidder 2 wins the first auction. In the second auction, bidder 2 does not participate and it is a dominant strategy for bidder 1 and bidder 3 to bid up to their valuations (even if bidder 3 is "surprised" to see bidder 1 bidding more than zero).

Therefore, as in a common-value auction, the collusive strategy induces the non-ring bidder to bid less aggressively and this increases the probability that the ring wins the auctions and reduces the price it pays. In contrast to a common-value auction, by dropping out of the first auction immediately, a ring bidder sends a misleading signal to the non-ring bidder about the intensity of competition in the second auction, rather than the prize value for the non-ring bidder.[39]

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[38] Of course, in both cases bidder 3 is worse off than in an auction without a ring, since in this last case bidder 3 makes the two inferences described (after a bidder drops out) *and* the remaining active bidder bids less aggressively than he does when he is part of a ring.

[39] Sequential (private-value) auctions have a common-value element given by the value of losing the first auction and winning the second one. As in our main model, with sequential auctions, by dropping out early of the first auction, a ring bidder signals to his opponent that the value of winning the first auction is low and,

Our analysis suggests that, when some (but not all) bidders collude in sequential auctions, prices should be increasing. Collusion reduces the efficiency of sequential auctions (and the seller's revenue). So the seller should use simultaneous auctions rather than sequential ones to sell multiple objects.[40]

8. Conclusions

Collusive behaviour in auctions is arguably the main concern of auction designers and sellers. We have described how colluding bidders may strategically use bids to mislead their competitors (and the auctioneer) into believing that their valuation of the prize is very low. Collusion hurts outsiders and reduces the efficiency of an English auction.

During recent European 3G auctions, some bidders managed to convince governments and competitors that the licenses on sale were not profitable by bidding extremely low prices or by failing to participate altogether. Perhaps firms were trying to reduce competition in future auctions, improve their bargaining power with sellers, or induce more favorable trading conditions with suppliers or a more benevolent attitude from regulators. Many telecom firms are now trying to induce governments to relax rules that prevent them from owning two licenses or from sharing a 3G network.

But when bidders drop out of an auction at a very low price, they may not necessarily do it because they believe the prize is not worth it.

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hence, induces his opponent to bid less aggressively in the first auction.

[40] Pagnozzi (2003) also argues that, when bidders are unequally informed, a simultaneous auction may raise higher revenue (than sequential ones) because it reduces the information asymmetry among bidders and, hence, the winner's curse of less informed bidders, inducing them to bid more aggressively.

Appendix: Omitted Proofs

Proof of Lemma 2. Since non-ring bidders are unaware of the presence of a ring, their bidding strategy is defined by equations (3.1) and (3.2). Therefore, after the $m-1$ ring bidders with the lowest signals drop out at price zero and k non-ring bidders drop out at prices $p_m \leq \ldots \leq p_{k+m-1}$, a non-ring bidder bids up to:

$$\begin{aligned} \psi_k(x_i) &= \alpha^i_{k+m-1}(x_i; 0, \ldots, 0, p_m, \ldots, p_{k+m-1}) \\ &= \mathbb{E}\left[V_i \left| \begin{array}{l} X_i = Y_{n-1} = \ldots = Y_{k+m} = x_i, \\ Y_{k+m-1} = \ z_k, \ldots, Y_m = z_1, Y_{m-1} = \ldots = Y_1 = 0 \end{array} \right. \right]. \end{aligned}$$

By Lemma 1, this is lower than the price at which she drops out when there is no collusion, that is if $m-1$ ring bidders do not drop out at price zero. ■

Proof of Lemma 3. After k non-ring bidder dropped out, if the last $n-m-k$ non-ring bidders all drop out at price p and the designated bidder wins the auction, his expected valuation is:

$$\mathbb{E}\left[V_m \left| Z_{n-m} = \ldots = Z_{k+1} = \psi_k^{-1}(p);\ w_m, \ldots, w_1, z_k, \ldots, z_1 \right. \right],$$

because, by Lemma 2, each of the $n-m-k$ non-ring bidder has signal $\psi_k^{-1}(p)$. Therefore, after winning at price p^*, the designated bidder's profit is positive if and only if:

$$p^* \leq \mathbb{E}\left[V_m \left| Z_{n-m} = \ldots = Z_{k+1} = \psi_k^{-1}(p^*);\ w_m, \ldots, w_1, z_k, \ldots, z_1 \right. \right].$$

By the definition of β_k in (4.1), the designated bidder stays in the auction as long as the above inequality holds. Hence, the strategy β is a best reply to the strategies $\psi_k(.)$ of non-ring bidders.

In an English auction with the same bidders and no collusion, the designated bidder uses the bidding strategy defined by (3.1) and (3.2). So, after $m+k-1$ bidders dropped out, he bids up to:

$$p_{m+k} = \mathbb{E}\left[V_m \left| X_i = Y_{n-1} = \ldots = Y_{m+k} = w_m;\ y_{m+k-1}, \ldots, y_1 \right. \right]. \tag{A1}$$

On the other hand, with collusion, the designated bidder's expected valuation when the price is p_{m+k} (after k non-ring bidders dropped out) is no lower than:

$$\mathbb{E}\left[V_m \left| Z_{n-m} = \ldots = Z_{k+1} = \psi_k^{-1}(p_{m+k});\ w_m, \ldots, w_1, z_k, \ldots, z_1 \right. \right]. \tag{A2}$$

Notice that, since w_m is the highest signal among ring bidders, even without collusion the $m-1$ bidders with signals $w_1, \ldots, w_{m-1}$ drop out of the auction before the designated bidder. It follows that expectations (A1) and (A2) are conditioned on the same signals $w_1, \ldots, w_m$ and $z_1, \ldots, z_k$. Moreover, $\psi_k^{-1}(p_{m+k}) \geq w_m$ since, after observing $m-1$ bidders quit at price zero, a non-ring bidder must have a signal at least as high as w_m to be willing to remain active up to the same price at which the designated bidder with signal w_m is willing to remain active. Therefore, (A2) is greater than (A1): with collusion, at price p_{m+k} the valuation of the designated bidder is greater than p_{m+k} and, hence, he does not drop out of the auction. ■

Proof of Lemma 4. The probability that a buyer with signal w_m wins an auction with n potential buyers and no collusion is:

$$\Pr\left[w_m > y_{n-1}\right] = \Pr\left[\alpha^i_{n-2}\left(w_m; y_1, \ldots, y_{n-2}\right) > \alpha^i_{n-2}\left(y_{n-1}; y_1, \ldots, y_{n-2}\right)\right].$$

The probability that the designated bidder wins the auction when the ring adopts mechanism (M) (i.e., the probability that he bids higher than the $n-m$ non-ring bidders) is:

$$\Pr\left[\beta_{n-m-1} > \psi_{n-m-1}\left(z_{n-m}\right)\right].$$

The latter probability is greater than the former because:

(*i*) $\psi_{n-m-1}\left(z_{n-m}\right) < \alpha^i_{n-2}\left(y_{n-1}; y_1, \ldots, y_{n-2}\right)$ by Lemma 2 and the fact that $z_{n-m} \leq y_{n-1}$;

(*ii*) $\beta_{n-m-1} > \alpha^i_{n-2}\left(w_m; y_1, \ldots, y_{n-2}\right)$ by Lemma 3. ■

Proof of Lemma 5. Follows from Lemma 2 and the fact that $m-1$ ring bidders drop out at price zero. ■

Proof of Proposition 3. We can focus on the expected profit of the designated bidder. The expected profit of a bidder with signal w_m in an auction without collusion is:

$$\mathbb{E}\left[\left(V_m - \alpha^i_{n-2}\left(Y_{n-1}; Y_1, \ldots, Y_{n-2}\right)\right) \cdot \mathbf{1}_{\{w_m > Y_{n-1}\}} \left| w_m\right.\right].$$

From Lemmas 4 and 5, it follows that this expression is lower than the expected ring profit, given by (6.1). ■

Proof of Lemma 6. Consider a generic equilibrium in linear and increasing bidding functions. Let the price at which the designated ring bidder drops out of the auction in equilibrium be:

$$h\left(x_1, x_2\right) = a + b\left(x_1 + x_2\right),$$

where a and b are two constants. We are going to use the fact that bidders' equilibrium bidding functions must be reciprocal best replies to determine the values of a and b.

If bidder 3 wins the auction at price p, then she expects the sum of the two ring bidders' signals $(x_1 + x_2)$ to be equal to $h^{-1}(p) = \frac{p-a}{b}$. In equilibrium bidder 3 bids up to the expected value of the prize conditional on winning. Therefore, she bids up to price p_3 such that:

$$p_3 = (1+\varepsilon)\, x_3 + \frac{p_3 - a}{b} \quad \Leftrightarrow \quad p_3 = \frac{b(1+\varepsilon)}{b-1} x_3 - \frac{a}{b-1}.$$

It then follows that, if the designated ring bidder wins at price p, he expects bidder 3's signal x_3 to be equal to $\frac{b-1}{b(1+\varepsilon)}\left[p+\frac{a}{b-1}\right]$. And in equilibrium the designated bidder bids up to the expected value of the prize conditional on winning. So he bids up to price p_1 such that:

$$p_1 = (1+\varepsilon)(x_1+x_2) + \frac{b-1}{b(1+\varepsilon)}\left[p_1 + \frac{a}{b-1}\right] \quad \Leftrightarrow \quad p_1 = \frac{b(1+\varepsilon)^2}{b\varepsilon+1}(x_1+x_2) + \frac{a}{b\varepsilon+1}.$$

In order for the function $h(.)$ to be an equilibrium bidding function, it must consistent with the above expression for p_1. Therefore, it must be that:

$$a = \frac{a}{b\varepsilon+1} \quad \text{and} \quad b = \frac{b(1+\varepsilon)^2}{b\varepsilon+1}.$$

The unique meaningful solution to these two equations is $b = 2+\varepsilon$ and $a = 0$ (the other solution being $b = 0$).

An identical argument holds for the bidding function of bidder 3. Finally, notice that the other ring bidder can do no better than abstain form the auction, since bidder 3 would not make any inference from his bidding behaviour. ■

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