Stock Market Development and Economic Growth:
A Matter of Information Dynamics

Salvatore Capasso

September 2006
**Abstract**

The aim of this paper is to provide further insights into the linkages between stock market development and economic growth. When it is not possible to distinguish between investment projects with different rates of return, market valuation of those projects is an “average value” reflecting the expected return across all investment opportunities. Consequently, as in a typical lemon’s market, higher return projects are penalised since they attract lower than fair prices. This informational cost, or dilution cost, depends on the degree of informational asymmetry in the market, as well as on the type of financial contract issued by the firm to finance those projects – typically, equity or debt. On this basis, we interpret the development of stock market as the result of a change in the level of informational costs which decrease with capital accumulation and induce firms to switch from debt financing to a less costly equity financing.

**Keywords:** Credit Markets, Economic Growth, Information Asymmetries, Stock Markets

**JEL Classification:** O16, O40, G10

---

* University of Napoli “Parthenope” and CSEF. Address for correspondence: Department of Economics, University of Naples “Parthenope”, Via Medina 40, 80133, Naples, Italy. E-mail: scapasso@unipathenope.it
Contents

1. Introduction

2. The Environment
   2.1 Households
   2.2 Capital Production
   2.3 Output Production

3. The Capital Market

4. Optimal Contracts

5. Capital Accumulation

6. Numerical Simulations

7. Final Considerations

Appendix: The Optimal Choice of Financial Contract

References
1. Introduction

In the wake of substantial empirical evidence, considerable research has been devoted to modelling and understanding the strong positive linkages between real and financial development. Much of this research has followed the so-called “functional” approach in the analysis of such linkages. For example, it is argued that financial markets and financial institutions can affect capital accumulation because they can affect the real allocation of resources between alternative technologies (Greenwood and Jovanovich, 1990; Saint-Paul, 1992; Bencivenga, Smith and Star, 1996; Blackburn and Hung, 1998) or because they can affect agents’ savings decisions by reducing liquidity costs and offering greater opportunities for diversifying risks (Bencivenga and Smith, 1991; Levine, 1991). The common feature of this literature has been the modification of the Arrow-Debreu framework through the introduction of information costs or other forms of transaction costs which impede the smooth functioning of financial markets.

While much has been learnt from the above research, many issues have still to be explained and the co-evolution of the real and financial sectors of an economy remains a fertile area for investigation. Undoubtedly, the development of financial markets is a complex process that is intimately connected to real economic activity. As such, the metamorphosis and transformation of the financial system cannot be fully understood unless this is interpreted as a truly endogenous process involving dynamic structural change which is linked to changes in the real economy. Without recognising this, it would be difficult to explain how financial institutions evolve and how new financial arrangements emerge. As Levine (1997) emphasises, there is need for further research into many aspects of financial development which have so far received relatively little attention. The emergence and expansion of stock markets as economies develop is one such aspect.

Empirical evidence shows clearly the existence of a strong positive correlation between stock market development and economic growth (Atje and Jovanovich, 1993, Demirgüç-Kunt and Levine, 1996a, b, Korajczyk, 1996, Levine and Zervos, 1996, 1998). The development of stock markets is associated not only with an increase in the number of firms listed in the market, but also with an increase in the capitalisation of firms. The process of stock market development is also non-linear, occurring abruptly and rapidly at first and continuing more gradually thereafter.

At the theoretical level, the study of stock markets and growth has been given new impetus with recent analyses of the design of optimal financial contracts under asymmetric information in dynamic general equilibrium models (Bernanke and Gertler, 1989; Bencivenga and Smith, 1993; Bose and Cothren, 1996, 1997). This new body of research enables one to understand the evolution of the financial system and to explain how alternative types of financial contract may emerge to solve problems of moral hazard and adverse selection. These problems, arising from asymmetric information between lenders and borrowers, imply a violation of the Modigliani-Miller theorem concerning the irrelevance of a firm’s capital structure. Essentially, firms in need of external finance face a cost minimisation problem which they must solve by issuing different forms of financial contracts under different circumstances. The crucial point is that this choice is affected by the level of capital accumulation.

This paper aims to provide further insights into the linkages between stock market development and economic growth within the context of a dynamic general equilibrium framework of informational asymmetries, endogenous contract choice and capital
accumulation (Boyd and Smith, 1998; Blackburn, Bose and Capasso, 2005). The analysis is based on a model of optimal capital structure, developed by Bolton and Freixas (2000), in which firms design optimal securities to finance risky investment projects. This model predicts that, when borrowers and lenders face ex ante informational asymmetries, the optimal securities are always of the form of debt and/or equity. Because of asymmetric information, both the issue of equity and the issue of debt involve “dilution costs” as the market “averages” the expected value of the unobservable project outcomes of firms. As in Myers and Majluf (1984), the “pecking order” condition is that, in the absence of other distortions, debt always dominates equity due to the assumption that debt involves lower dilution costs. Unlike equity, however, debt may also entail some bankruptcy costs (e.g. liquidation may imply missed opportunities of future production), which may cause the preferred mode of financing to switch to equity. In what follows, we modify this framework by allowing for a higher level of heterogeneity among firms. The degree of asymmetric information in the economy is determined endogenously by the fact that the incentive of low productivity firms to mimic high productivity firms changes over time with the level of capital accumulation. We show that, as capital accumulation takes place, a lower number of non-creditworthy firms enter the capital market and this reduces the degree of informational asymmetry which, in turn, has consequences for the structure of the financial system. In addition, the model also seems capable of accounting for certain specific aspects of stock market development that some observers have found puzzling. In particular it has been noted that “further development of stock markets may affect firms differently in economies where the markets already play a significant role than in those where they do not. If stock markets are already significant, further development leads to a substitution of equity financing for debt. However, in economies where stock markets are too small to have a significant role in the economy,…, development permits large firm to increase their leverage” (Demirgüç-Kunt, A. and Maksimovic V., 1996, p. 364). In other words, the effect of stock market development on the debt-equity ratio appears to depend on the level of development itself – the effect being positive at low levels of development but negative at high levels of development so that the debt-equity ratio displays non-monotonic behaviour.

The paper is organised as follows. In Section 2 we describe the economic environment in terms of the technologies, preferences and endowments of agents. In Section 3 we outline the structure of the credit market in which borrowing and lending take place. In Section 4 we study the optimal financial contract and identify the conditions under which one type of contract (debt or equity) dominates the other. Implications for growth and capital accumulation are discussed in Section 5. Section 6 presents some numerical simulations of the model which confirm the analytical results and illustrate the transitional dynamics. Section 7 offers some concluding remarks.

2. The Environment

Time is discrete and indexed by $t = 1, 2, \ldots \infty$. We consider an overlapping generations economy in which there is a constant population (normalised to 2) of two-period-lived, non-altruistic agents. Agents of each generation are divided at birth into two equal sized groups of unit mass comprising households (workers or lenders) and firms (entrepreneurs or borrowers). Firms produce capital (for which they require loans from households) when young, and output
when old. All agents are risk neutral and derive utility only from second period consumption. All markets are competitive.

2.1 Households

Each young household is endowed with one unit of labour which is supplied inelastically to an old producer of final output in return for the wage, $w_t$. This income can be stored for consumption in the second period, or lent out to a (young) producer of capital. The storage technology of households converts one unit of output at time $t$ into $\rho \geq 1$ units of output at time $t+1$. A loan to a capital producer is repaid in the subsequent period in terms of capital which the household rents out to final output producers.

2.2 Capital Production

Capital is produced from risky investment projects to which all entrepreneurs have access when young. The expected returns on these projects are different for different types of entrepreneur: there is a fraction, $n_1 \in (0,1)$, of type-1 entrepreneurs (skilled capital producers) whose expected return is high; a fraction, $n_2 \in (0,1)$, of type-2 entrepreneurs (semi-skilled capital producers) whose expected return is low; and a fraction, $n_3 = 1 - n_1 - n_2$, of type-3 entrepreneurs (unskilled capital producers) whose expected return is zero.\(^1\) The last group of firms also have access to a safe capital project which yields a certain rate of return.

In addition to the above, firms are heterogeneous within each group according to their efficiency in running a project. Efficiency is defined in terms of the minimum amount of resources needed to be invested at time $t$ in order to obtain a given amount of capital at time $t+1$. As regards the risky capital project, the efficiency of a firm is indexed by $\alpha$, which is uniformly distributed on $(0,1)$. The fixed cost of investment is given by $a(\alpha)$, where $a'(\alpha) > 0$, so that higher values of $\alpha$ are associated with lower levels of efficiency. As regards the safe capital project, the efficiency of a (type-3) firm is indexed by $\beta$, uniformly distributed on $(0,1)$ as well, and the fixed cost of investment is given by $b(\beta)$, where $b'(\beta) > 0$. We assume that the level of efficiency in running risky and safe projects are independent of each other. That is, a type-3 entrepreneur who is relatively efficient in operating the risky project is not necessarily efficient in running the safe project. This assumption captures the idea that individuals may have different skills in different activities.

The fundamental informational asymmetry in the model is that while the efficiency levels ($\alpha$ and $\beta$) of firms are public knowledge, the type of firm (type-1, type-2 or type-3) is private information. This informational asymmetry is the source of capital market imperfections which drive the results of the analysis.

The outcomes of risky projects for each group of firms is specified as follows: all type-1 firms produce $\kappa_1$ units of capital with probability $p$ and 0 units of capital with probability $1-p$; all type-2 firms produce $\kappa_2 < \kappa_1$ units of capital with probability $p$ and 0 units of capital with

\(^1\) Private information and heterogeneity among entrepreneurs is needed in order to generate informational costs in the credit market. A three-type framework allows to simplify matter by making the investment choice of one group of agents (type 3) independent of the type of contract (debt or equity) issued by the other groups (type 1 and type 2). At the same time, because of this three dimensional heterogeneity, equity and debt show different informational cost.
probability 1−p; and all type-3 firms produce 0 units of capital with certainty. Let \( r_{t+1} \) denote the price of capital at \( t+1 \) (which is constant in equilibrium). We assume that 
\[ r_{t+1} p k_1 > \rho a(\alpha) > r_{t+1} p k_2 \]
for all \( \alpha \). These restrictions imply that only type-1 firms obtain an expected return on the risky project which is sufficient to repay the minimum amount required by a lender for a loan of size \( a(\alpha) \).\(^2\) Essentially, this means that households would never knowingly lend to type-2 and type-3 firms since households can always earn \( \rho a(\alpha) \) amount of income from storage. The fact that such lending may take place is due to the existence of informational asymmetries.

A safe capital project (operated only by type-3 firms) is governed by a linear technology which yields \( q \) units of capital at time \( t+1 \) per unit of output invested at \( t \) after the fixed initial outlay \( b(\beta) \). For a loan size of \( w_t \), therefore, a type-3 firm produces \([w_t - b(\beta)]q\) units of capital from the safe project.\(^3\)

The foregoing description of capital production is summarised as follows:

<table>
<thead>
<tr>
<th>Risky Project</th>
<th>Safe Project</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Type-1</strong></td>
<td><strong>Type-2</strong></td>
</tr>
<tr>
<td>Initial outlay: ( a(\alpha) )</td>
<td>Initial outlay: ( a(\alpha) )</td>
</tr>
<tr>
<td>Capital production: ( k_1 ) with prob. ( p )</td>
<td>Capital production: ( k_2 ) with prob. ( p )</td>
</tr>
<tr>
<td>0 with prob. 1−p</td>
<td>0 with prob. 1−p</td>
</tr>
</tbody>
</table>

### 2.3 Output Production

Entrepreneurs produce final output in the second period of their lives. We assume that, at a minimum, running risky projects confers non-marketable skills that enable all firms to produce (and consume) a subsistence amount of output, \( \phi > 0 \), from home production. This assumption ensures that even type-2 and type-3 entrepreneurs (who always go bankrupt in the case of debt) may have the incentive to operate risky projects. Type-1 firms also have access to a production technology for combining their own skills with labour (supplied by young households of the next generation) and capital (acquired from projects undertaken previously by the same generation), to produce output in the market according to

\[
y_{t+1} = \Theta k_{t+1}^{\theta} (l_{t+1} K_{t+1})^{1-\theta} + \Phi h_{t+1}, \quad \Theta, \Phi > 0, \theta \in (0,1),
\]

where \( y_{t+1} \) denotes output, \( l_{t+1} \) denotes labour, \( k_{t+1} \) denotes capital, \( K_{t+1} \) denotes aggregate capital and \( h_{t+1} \) is a zero-one non-marketable skills variable. This production technology incorporates an externality effect associated with learning-by-doing, as in many types of endogenous growth model. Assuming that \( \Phi > \phi \), it is always optimal for a type-1 firm to use

\(^2\) Since \( r_{t+1}=r \) (a constant) in equilibrium, and since \( a'(\alpha)>0 \), then the assumption is satisfied by imposing the restriction \( rp k_1 > \rho a(1) \) and \( rp k_2 < \rho a(0) \).

\(^3\) This is just a simplifying assumption. We could enrich the framework by letting type 1 and type 2 agents having access to the safe projects and assuming that these are always dominated by risky investments.
this technology rather than home production. At the same time, however, we assume that only firms that are non-bankrupt are able to retain control over the market technology with a positive contribution of their own skills, \( h_{t+1} = 1 \). Those that go bankrupt cede control of this technology to lenders with zero contribution of manufacturing skills, \( h_{t+1} = 0 \). In other words, when a type-1 firm goes bankrupt it loses control of the output production technology and can supply its skills only in the home technology.

Let \( \kappa_{t+1} \) denotes the amount of capital produced by a type-1 firm. The firm is a net lender of capital if \( k_{t+1} < \kappa_{t+1} \) and a net borrower of capital if \( k_{t+1} > \kappa_{t+1} \). Accordingly, its profits are given by

\[
\pi_{t+1} = \Theta k_{t+1}^{\theta} (l_{t+1} \cdot K_{t+1})^{1-\theta} + \Phi h_{t+1} - w_{t+1} l_{t+1} - r_{t+1} (k_{t+1} - \kappa_{t+1}).
\]

Maximising (2) with respect to \( k_{t+1} \) and \( l_{t+1} \) gives \( \Theta k_{t+1}^{\theta-1} l_{t+1} K_{t+1}^{1-\theta} = r_{t+1} \), and \( (1-\Theta) d_{t+1} k_{t+1}^{\theta} l_{t+1} K_{t+1}^{1-\theta} = w_{t+1} \). Recall that the number of type-1 firms is \( n_1 \), and that households supply one unit of labour inelastically to these firms. In equilibrium, therefore, \( K_{t+1} = n_1 k_{t+1} \) and \( n_1 l_{t+1} = 1 \), so that the profit maximising conditions may be written as

\[
\Theta = r_{t+1} = r, \tag{3}
\]

\[
(1-\Theta) n_1 k_{t+1} = (1-\Theta) n_1 K_{t+1} = w_{t+1}. \tag{4}
\]

The latter implies \( w_{t+1} = W(K_{t+1}) \), where \( W(\cdot) > 0 \). Substituting (3) and (4) into (2) gives

\[
\pi_{t+1} = r \kappa_{t+1} + \Phi h_{t+1}. \tag{5}
\]

3. The Capital Market

As indicated earlier, young entrepreneurs require external finance from young households in order to run the capital production technology. Borrowing and lending take place in the credit market which operates in the following way. At the beginning of each period, a newly-born borrower approaches a newly-born lender with a request for a loan to finance a capital project. Following others (e.g. Bencivenga and Smith, 1993; Bose and Cothren 1996, 1997) we make the simplifying assumption that there is a one-to-one matching between borrowers and lenders. Each borrower proposes a financial contract which specifies the size of the loan in terms of output (i.e. the lender’s wage), and the repayment of the loan in terms of capital (i.e. the outcome of the project). The contract also states whether repayment is in the form of debt, denoted by \( d_{t+1} \) (a lump-sum repayment from the proceeds of the project), or equity, denoted by \( s_{t+1} \) (a payment that is proportional to the net profit from the project). It is possible to show that these are the only optimal forms of security that will be issued by the borrower. Under both arrangements, a lenders’ participation constraint must be satisfied: that is, the expected income from a loan must be at least equal to the income that could be obtained from storage.

---

4 Assuming that agents can transfer skills from one activity to another is common in the literature (see for example Jovanovic and Nyarko, 1996). Moreover, it has been shown that firms in transferring skills from some of their branches to others can improve the overall productivity (Blomstrom et. al, 1998).

5 See, for example, Bolton and Freixas (2000).
In the case of safe capital projects, where returns are non-stochastic and there are no informational asymmetries (only type-3 firms run these projects), the entrepreneur will be indifferent between debt and equity contracts. By running such a project with a loan size of \( w_t \), a type-3 firm of efficiency level \( \beta \) produces \( (w_t - b(\beta))q \) units of capital. Under a debt contract, the firm makes a payment of \( \bar{d}_{t+1} \) units of capital to the lender and rents out the remaining capital to final goods producers to receive \( r([w_t - b(\beta)]q - \bar{d}_{t+1}) \) units of output as income. In turn, the lender rents out the debt payment to final producers to receive \( r\bar{d}_{t+1} \) units of output as income. The lender’s participation constraint is therefore \( r\bar{d}_{t+1} = \rho w_t \), and bankruptcy will never occur provided that \( (w_t - b(\beta))q > \bar{d}_{t+1} \). Under an equity contract, the firm makes a payment of \( \bar{s}_{t+1}(w_t - b(\beta))q \) units of capital to the lender and rents out the remaining capital to final producers to receive \( r(1 - \bar{s}_{t+1})(w_t - b(\beta))q \) units of output as income. The lender rents out the equity payment to receive \( r\bar{s}_{t+1}[(w_t - b(\beta))q = \rho w_t \) units of output as income. The lender’s participation constraint is therefore \( r\bar{s}_{t+1}[(w_t - b(\beta))q = \rho w_t \). In both cases, the borrower and lender receive the same compensation – namely, \( r(w_t - b(\beta))q - \rho w_t \) and \( \rho w_t \), respectively – and the two contracts are completely equivalent.

This is not the case for risky projects, which display stochastic returns and involve asymmetric information about borrowers’ type. As a result, the financial contracts associated with these projects may involve bankruptcy as well as dilution costs. These costs are different for equity and debt contracts. While an equity contract does not require any minimum repayment to the lender, a debt contract entails a pre-determined re-payment. Therefore, the possibility of bankruptcy only arises in the case of debt but not in the case of equity. In contrast, since informational problems concern both the issue of equity and debt, both types of contract may involve dilution costs.

With probability \( 1 - p \) the risky project delivers zero units of capital, whatever the firm’s type. Under such circumstances, any positive debt repayment, \( d_{t+1} > 0 \), will always involve bankruptcy. Recall also that, with probability \( p \), the project delivers \( \kappa_t \) units of capital to a type-1 firm, \( \kappa_t < \kappa_t \) units of capital to a type-2 firm, and zero units of capital to a type-3 firm. We assume that \( \kappa_t > d_{t+1} > \kappa_t \), which means that only type-1 firms remain solvent when the project succeed; type-2 and type-3 firms are always bankrupt regardless of the project outcome.

Given the above, it follows that type-2 and type-3 firms must masquerade as type-1 firms if they are to receive loans to finance risky projects. The maximum expected loan repayment of a type-2 (type-3) firm is \( \rho \kappa_2 \) (0) units of capital which a household can rent out to earn a maximum expected income of \( \rho \kappa_2 \) (0) units of output. But the household can

---

6 Since \( \bar{d}_{t+1} = \rho w_t / r \), this condition may be written as \( r([w_t - b(\beta)]q - \rho w_t) > 0 \) or \( (rq - \rho)w_t - rqb(\beta) > 0 \), which requires that \( rq > \rho \). Given this, since \( w_t = W(K_t) \) (from (4)), with \( W'(\cdot) > 0 \), and \( b'(\cdot) > 0 \), the condition is satisfied by imposing the restriction \( (rq - \rho)w_t - rqb(1) > 0 \).

7 It can be shown that, for suitable parameter restrictions, the expression for \( d_{t+1} \) obtained under the assumption \( \kappa_t > d_{t+1} > \kappa_t \) is such that the assumption is always satisfied. The restrictions are \( r(z, p\kappa_t + (1 - z)\eta \rho \kappa_2) > \rho a(1) \) and \( \rho \kappa_2 [n_t + (1 - n)\eta t] < \rho a(\alpha), \) the latter of which is always satisfied by virtue of the earlier assumption that \( \rho \kappa_2 < \rho a(\alpha) \) for all \( \alpha \).
always earn $\rho a(\alpha)(> r p k_2 > 0)$ units of output from storing $a(\alpha)$ instead of lending it to firm. Consequently, no household will knowingly lend to a type-2 or type-3 firm that plans to operate a risky project.

It is evident that type-2 firms, for which the only investment opportunities are risky projects, will always have an incentive to mimic type-1 firms. Type-3 firms, however, may or may not act accordingly depending on whether the returns from risky projects are greater or less than the returns from running the safe project. A type-3 firm running the risky project produces zero units of capital with certainty, but it also acquires skills which can be used in home production. The payoff from risky projects (under either debt or equity contracts) is therefore $\phi$ units of home produced output. By contrast, a safe projects yields $r[(w_i - b(\beta))]q - \rho w_i$ units of output (under either debt or equity contracts) to a type-3 firm of efficiency level $\beta$. It follows, therefore, that a type-3 firm will not run a risky project (i.e. will not mimic a type-1 firm) if $r[(w_i - b(\beta))]q - \rho w_i > \phi$. Now, define a $\overline{\beta}_t \in (0,1)$ such that

$$r[(w_i - b(\overline{\beta}_t))]q - \rho w_i = \phi.$$  

In words, $\overline{\beta}_t$ defines the marginal type-3 firm which is indifferent between the risky and the safe project. Since $b'(\cdot) > 0$, then all firms with $\beta < \overline{\beta}_t$ undertake the safe project, while all firms with $\beta > \overline{\beta}_t$ undertake the risky project. The marginal firm can be considered as a function of the wage rate, $\overline{\beta}_t = B(w_i)$, such that (i) $\overline{\beta}_t = 0$ for $B(\cdot) \leq 0$, (ii) $\overline{\beta}_t = 1$ for $B(\cdot) \geq 1$, and (iii) $\overline{\beta}_t \in (0,1)$ with $B'(\cdot) > 0$ for $B(\cdot) \in (0,1)$. Moreover, since $\beta$ is uniformly distributed on $(0,1)$, then $\overline{\beta}_t$ is also understood to be the fraction of type-3 firms that do not run risky projects, with $1 - \overline{\beta}_t$ being the fraction of type-3 firms that do run risky projects.

4. Optimal Contracts

In what follows we determine the optimal financial contract for risky projects when lenders are faced with loan applications from borrowers of unknown type. In considering these applications, a household takes account of the fact that all type-2 firms have an incentive to mimic type-1 firms and that some type-3 firms may have the same incentive as well. It follows that the level of informational asymmetry – the probability that the household faces a type-2 or type-3 firm (instead of a type-1 firm) – is an endogenous variable since it depends on the incentive of type-3 firms to cheat, which depends, in turn, on the prevailing wage rate. Recall that the populations of type-1, type-2 and type-3 firms are $n_1$, $n_2$ and $1 - n_1 - n_2$, respectively. From above, the fraction of type-3 firms that mimic type-1 firms is $\overline{\beta}_t$. Accordingly, the probability that a firm applying for a loan is actually a type-1 firm is given by

$$z_t = \frac{n_1}{(1-n_1-n_2)(1-\overline{\beta}_t) + n_1 + n_2}.$$  

Thus, $z_t = Z(\overline{\beta}_t) \in [n_1, n_1/(n_1 + n_2)]$, where, $Z'(\cdot) > 0$. This probability provides a measure of the degree of asymmetric information. A decrease in the fraction of type-3 firms
masquerading as type-1 firms (i.e. an increase in $\beta$) leads to an increase in this probability. Higher values of $z_i$ are therefore associated with lower degrees of information asymmetry.

It is now possible to determine the expected market value of a risky project. This is given by a weighted average of the expected return on the risky project for each type of firm, where the weights correspond to the fraction of each type of firm in the population. That is, the expected value of a project is $z_i p \kappa_1 + (1 - z_i) \eta p \kappa_2$, where $\eta = n_2 / (n_2 + n_3) = n_2 / (1 - n_1)$ (i.e., the fraction of non-type-1 firms that are type-2). Given this, one can then turn to the determination of the optimal financial contract, either equity or debt. In doing this it is important to emphasise that, while both debt and equity involve dilution costs (due to the averaging of project outcomes across the market), only debt entails the possibility of bankruptcy. As such, the preferences for one type of contract over the other will depend on the relative magnitude of these costs. As in Bolton and Freixas (2000) we will assume that equity is associated with higher dilution costs, otherwise equity will always dominate. It follows that, in absence of bankruptcy costs debt will always dominate equity. It also follows that firms issue either debt or equity, but never a combination of the two.

**Characterisation of equity contract.**

When a type-1 firm issues equity to finance its project, a lender expects to receive an equity payment of $s_{t+1}[z_i p \kappa_1 + (1 - z_i) \eta p \kappa_2]$ units of capital which can be rented out to receive an expected income of $r_{t+1}[z_i p \kappa_1 + (1 - z_i) \eta p \kappa_2]$ units of output. Since the lender’s participation constraint is $r_{t+1}[z_i p \kappa_1 + (1 - z_i) \eta p \kappa_2] = \rho a(\alpha)$, it follows that a type-1 firm of efficiency level $\alpha$ needs to issue an amount of equity equal to

$$s_{t+1} = \frac{\rho a(\alpha)}{r[z_i p \kappa_1 + (1 - z_i) \eta p \kappa_2]}.$$  \(8\)

Thus, the amount of equity needed to be issued in order to undertake the risky project is a function of both the firm’s efficiency level and the degree of informational asymmetry in the economy. Formally, $s_{t+1} = S(z_i, \alpha)$, where $S_\alpha(\cdot) < 0$ and $S_{z_i}(\cdot) > 0$. \(^8\)

From (5), the expected net income of the firm is given as $V_{t+1}^E = r(1 - s_{t+1}) p \kappa_1 + \Phi$. Substitution of (8) yields

$$V_{t+1}^E = \left\{ \frac{r[z_i p \kappa_1 + (1 - z_i) \eta p \kappa_2] - \rho a(\alpha)}{z_i p \kappa_1 + (1 - z_i) \eta p \kappa_2} \right\} p \kappa_1 + \Phi.$$  \(9\)

Hence, $V_{t+1}^E = V^E(z_i, \alpha)$, where $V^E_\alpha(\cdot) > 0$ and $V^E_{z_i}(\cdot) < 0$.

---

\(^8\) Obviously, it is required that $s_{t+1} \in (0,1)$. This is satisfied under the same parameter restrictions needed to assure $\kappa_i > d_{t+1}$. 

8
Characterisation of debt contract.

When a type-1 firm issues debt to finance its project, the expected repayment to a lender is $z_t p_d + (1 - z_t) \eta p \kappa_z$ units of capital which can be rented out to obtain an expected income of $r[z_t p_d + (1 - z_t) \eta p \kappa_z]$ units of output. Since the lender’s participation constraint is $r[z_t p_d + (1 - z_t) \eta p \kappa_z] = \rho a(\alpha)$, then a type-1 firm of efficiency level $\alpha$ makes a payment equal to

$$d_{\alpha,t+1} = \frac{\rho a(\alpha) - r(1 - z_t) \eta p \kappa_z}{rz_t p}.$$  \hspace{1cm} (10)

As for the equity payment, the debt payment is a function of the firm’s efficiency level and the degree of asymmetric information: $d_{\alpha,t+1} = D(z_t, \alpha)$, where $D_1(\cdot) < 0$ and $D_\alpha(\cdot) > 0$. \hspace{1cm} (5)

From (5), the firm’s expected net income is deduced as $V_{t+1}^D = rp(\kappa_z - d_{t+1}) + p\Phi + (1 - p)\phi$. \hspace{1cm} (10)

Substitution of (10), gives:

$$V_{\alpha,t+1}^D = \frac{r[z_t p \kappa_z + (1 - z_t) \eta p \kappa_z] - \rho a(\alpha) + p\Phi + (1 - p)\phi}{z_t}.$$ \hspace{1cm} (11)

Thus, $V_{\alpha,t+1}^D = V^D(z_t, \alpha)$, where $V^D_\alpha(\cdot) > 0$ and $V^D_\alpha(\cdot) < 0$.

Debt Versus Equity

The optimal choice of contract for a type-1 firm is determined by a comparison of the firm’s expected payoff in the two cases. An equity contract is chosen if $V_{\alpha,t+1}^E - V_{\alpha,t+1}^D > 0$, while a debt contract is chosen if $V_{\alpha,t+1}^E - V_{\alpha,t+1}^D < 0$. Let $\bar{\alpha}_t$ denote the efficiency level of a firm such that $V_{\alpha,t+1}^E - V_{\alpha,t+1}^D = 0$. That is, from (9) and (11),

$$(1 - p)(\Phi - \phi) - \frac{\{r[z_t p \kappa_z + (1 - z_t) \eta p \kappa_z] - \rho a(\bar{\alpha}_t)\}(1 - z_t) \eta p \kappa_z}{z_t [z_t p \kappa_z + (1 - z_t) \eta p \kappa_z]} = 0.$$ \hspace{1cm} (12)

This $\bar{\alpha}_t$ defines the marginal type-1 firm which is indifferent between issuing debt and issuing equity. Since $a'(\alpha) > 0$, then all firms with $\alpha < \bar{\alpha}_t$ issue debt and all firms with $\alpha > \bar{\alpha}_t$ issue equity. The marginal firm is a function of the degree of informational asymmetry, $\bar{\alpha}_t = A(z_t)$, such that (i) $\bar{\alpha}_t = 1$ for $A(\cdot) \geq 1$, (ii) $\bar{\alpha}_t = 0$ for $A(\cdot) \leq 0$, and (iii) $\bar{\alpha}_t \in (0,1)$ for $A(\cdot) \in (0,1)$. Moreover, since $\alpha$ is uniformly distributed on $(0,1)$, then $\bar{\alpha}_t$ is understood to be the fraction of type-1 firms issuing debt, with $1 - \bar{\alpha}_t$ being the fraction of type-1 firms issuing equity. A detailed analysis of the condition under which one type of contract dominates the other is contained in the Appendix and summarised in Fig. 1.

---

9 Since $rp \kappa_z < \rho a(\alpha)$ by assumption, then $d_{t+1} > 0$.

10 Recall that, with probability $p$, the firm is successful, repays the loan and retains control over output production; with probability $1 - p$, the firm is unsuccessful, goes bankrupt and produces output at home.
For each $\alpha \in (0,1)$, the difference in payoffs under equity and debt, $\hat{V}(\cdot;\alpha) = V^E(\cdot;\alpha) - V^D(\cdot;\alpha)$ is a concave function in $z_t \in (0,1)$ (note that $\lim_{z_t \to 0} \hat{V}(z_t;\alpha) = +\infty$ and $\lim_{z_t \to 1} \hat{V}(z_t;\alpha) = (1-p)(\Phi - \phi) > 0$, $\forall \alpha$). The highest of these curves is the locus corresponding to $\alpha = 1$, the lowest is the locus corresponding to $\alpha = 0$ (recall $a'(\alpha) > 0$). Given an initial value of $z_t$, $z_0 \in [n_1, n_2/(n_1 + n_2)]$, the marginal type-1 firm is determined by (12), for which $\hat{V}(z_0;\alpha) = 0$. Since $\hat{V}_\alpha(z_0;\cdot) > 0$, each $\hat{V}(z_0;\alpha)$ curve, for $\alpha > \bar{\alpha}_0$, lies above the horizontal axis at $z_0$, implying that the type-1 firm of efficiency level $\alpha > \bar{\alpha}_0$ prefers equity to debt (see Fig. 1). Conversely, each $\hat{V}(z_0;\alpha)$ for $\alpha < \bar{\alpha}_0$, lies below the horizontal axis at $z_0$, so that the type-1 of efficiency level $\alpha < \bar{\alpha}_0$ prefers debt to equity. As $z_t$ increases the fraction of firms preferring one contract to the other, $\bar{\alpha}_t$, changes.
5. Capital Accumulation

In order to determine the path of capital accumulation in the economy, it is necessary to compute the amount of capital produced in each period by each type of firm. The total population of type-1 firms in the economy is \( n_1 \). All of these firms run the risky project and each of them produces \( \kappa_1 \) units of capital with probability \( p \). Therefore, the total amount of capital produced by type-1 firms is \( n_1 \kappa_1 \) with probability \( p \). The total population of type-2 firms in the economy is \( n_2 \). All of these firms run the risky project as well and each of them produces \( \kappa_2 \) units of capital with probability \( p \). Thus the total amount of capital produced by these firms is \( n_2 \kappa_2 \) with probability \( p \). The total number of type-3 firms is \( 1 - n_1 - n_2 \). A fraction of these, \( (1 - \bar{\beta}_t) \), runs the risky project and produce zero units of capital. The remaining fraction, \( \bar{\beta}_t \), runs the safe project and produce \( [(w_t - b(\beta))]q \) units of capital. The total amount of capital produced by these firms is therefore \( (1 - n_1 - n_2) \int_0^{\bar{\beta}} [(w_t - b(\beta))]q d\beta \).

By applying the law of large numbers it is possible to determine the aggregate capital accumulation path as

\[
K_{t+1} = n_1 p \kappa_1 + n_2 p \kappa_2 + (1 - n_1 - n_2) \int_0^{\bar{\beta}} [(w_t - b(\beta))]q d\beta \\
= n_1 p \kappa_1 + n_2 p \kappa_2 + (1 - n_1 - n_2)q \bar{\beta} w_t - (1 - n_1 - n_2)q \int_0^{\bar{\beta}} b(\beta) d\beta .
\]

Since \( w_t = W(K_t) \) from eq. (4), and \( \bar{\beta}_t = B(w_t) \) from eq. (6), it follows that \( K_{t+1} = f(K_t, \bar{\beta}_t) = F(K_t) \), where \( F'(\cdot) = (1 - n_1 - n_2)q W'(\cdot) \{\bar{\beta}_t + B'(\cdot)[w_t - b(\cdot)]\} > 0 \) and \( F(0) = n_1 p \kappa_1 + n_2 p \kappa_2 \).

The main factor in influencing the capital accumulation path (and long run equilibrium) is the incentive for type-3 firms to mimic type-1 firms and undertake risky project investment (as opposed to safe project investment). This incentive changes over time with capital accumulation itself. Specifically, as \( K_t \) increases, then so too does \( w_t \) and so too does \( \bar{\beta}_t \), implying a fall in the number of type-3 firms that are inclined to run the risky projects. The implications of this may be evaluated by examining the two extreme cases where \( \bar{\beta}_t \) takes on its corner values.

Suppose, first, that \( \bar{\beta}_t = 0 \). This is the case in which all type-3 firms mimic type-1 firms, so that the capital accumulation path would be \( f(K_t, 0) = n_1 p \kappa_1 + n_2 p \kappa_2 \). This path implies a constant level of capital at

\[
K^* = n_1 p \kappa_1 + n_2 p \kappa_2
\]

Suppose, alternatively, that \( \bar{\beta}_t = 1 \), which is the case in which no type-3 firm mimics type-1 firms. Then the capital accumulation path becomes

\[
f(K_t, 1) = n_1 p \kappa_1 + n_2 p \kappa_2 + (1 - n_1 - n_2)q(1 - \theta) \Theta K_t + (1 - n_1 - n_2)q \int_0^{\bar{\beta}} b(\beta) d\beta ,
\]

which implies either a steady state point at
\[ K^{**} = \frac{n_1 p k_1 + n_2 p k_2 - (1 - n_1 - n_2)q \int_0^1 b(\beta) d\beta}{1 - (1 - n_1 - n_2)q(1 - \Theta)} \]  \hspace{1cm} (15) 

if \( 1 > (1 - n_1 - n_2)q(1 - \Theta) \), or a long run growth rate of 

\[ g = (1 - n_1 - n_2)q(1 - \Theta) \]  \hspace{1cm} (16) 

if \( 1 < (1 - n_1 - n_2)q(1 - \Theta) \). These scenarios are depicted in Fig. 2. Given the positive correlation between the structure of the financial system and the level of real economic activity, it is possible to distinguish between two regimes: a low development regime with a relatively low level of economic activity and predominance of debt and high development regime with a relatively high level of economic activity and predominance of equity. Transition between these regimes depends on the initial level of capital and on the shape of the capital accumulation path. The case of multiple equilibria is represented in Fig. 2a. Depending on the initial level of capital, the economy can either be trapped in the low development regime with a low steady state level of capital \( k^* \) (if \( k_0 < k^c \)), or it can converge towards a high development regime (if \( k_0 > k^c \)), with high steady state level of capital, \( k^{**} \), or a positive long run growth rate (bold section of the capital accumulation path). The case of unique equilibrium is represented in Fig. 2b. In this case, the economy displays always transition between low development regime and high development regime (with a positive steady state level of capital, \( k^{**} \) or a positive long run growth rate).

By affecting the incentives of type-3 firms to mimic type-1, capital accumulation affects the degree of informational asymmetry and, with it the choice of financial contract. Given that \( K_t \) increases, then the resulting increase in \( \bar{y}r_1 \) leads to an increase in \( z_t \) and a decrease in \( \bar{\alpha}_1 \). Along the trajectory path, therefore, the amount of informational asymmetry falls and the number of firms issuing equity in preference to debt increases.

![Fig. 2a](image1)

![Fig. 2b](image2)
6. Numerical Simulations

The foregoing analytical results are confirmed by numerical simulations of the model. For these simulations, we assume that \(a(\alpha) = a_0 + a_1\) and \(b(\beta) = b_0 + b_1\) and experiment with parameter values around the following benchmark set: \(n_1=0.5; n_2=0.125; p =0.5; \kappa_1=2; \kappa_2=0.25; a_0 =0.025; a_1 =0.125; b_0 = 0.5; b_1 =0.125; Q = 20; \Phi=1; \varphi=0.9875; \Theta=1; \theta=0.5; \rho =1\). This benchmark set of values gives rise to transition between development regimes with positive long-run growth. Under such circumstances, the typical shapes of the trajectories of variables are as shown in Figure 2b. Thus, the proportion of type-3 firms that do not mimic type-1 firms \((\beta)\) increases over time while the proportion of type-1 firms that issue debt \((\alpha)\) decreases over time. These events are reflected in a surge of growth which eventually converges to a long-run stationary value. An example of multiple equilibria is provided by the case in which all parameters remain at their benchmark values except for \(Q = 14.75\). The critical level of capital in this case is \(k^c = 0.90\). Initial levels of capital below this value cause the economy to converge to the low steady state equilibrium, \(k^*\), while initial levels of capital above this value level lead the economy onto a path of perpetual growth.

Figure 2 depicts the case in which \(\alpha\) is monotonically decreasing over time as the economy moves from a low development regime to a high development regime. Interestingly, however, the model has the potential to generate non-monotonic behaviour in \(\alpha\), which may increase at first and then decrease subsequently. This is indicated by the simulation results which show that the relationship between \(\alpha\) and \(z_t\) is an inverted u-shape function. If the model is able to produce such behaviour in \(\alpha\), then it will be able to explain the apparent puzzle of why stock market development appears to be associated with an increase in the debt-equity ratio at low levels of its development and decrease in this ratio at high levels of its development.

7. Final Considerations

The main objective of this paper has been to provide an account of the role played by economic development in the evolution of financial markets. The empirical evidence shows clearly that stock market activity is closely related to real activity, with firms having a greater preference towards issuing equity (rather than debt) as capital accumulation proceeds. In other words, the optimal capital structure of firms depends fundamentally on the level of economic development.

In order to understand the link between financial markets and growth, it is necessary to depart from the fiction of a perfectly functioning representative agent paradigm and to move towards a framework based on market imperfections where the Modigliani-Miller theorem fails to hold. The recent literature that has taken on this challenge has provided significant new insights and raised important issues for further consideration. The model presented in this chapter has the distinguishing feature that the degree of informational asymmetry is not exogenous, but rather changes over time with changes in the incentives of low productivity firms to masquerade as high productivity firms. The higher is the level of development, the lower is the proportion of firms that have such incentives, and the lower is the degree of asymmetric information. This leads to a higher value attached by the market to risky investment projects and a greater number of firms that prefer to issue equity rather than debt.
Appendix: The Optimal Choice of Financial Contract

From (9),
\[ V^E(0, \alpha) = \left[ \frac{r\eta p\kappa_2 - \rho a(\alpha)}{\eta p\kappa_2} \right] p\kappa_1 + \Phi \quad \text{and} \quad V^E(1, \alpha) = rp\kappa_1 - \rho a(\alpha) + \Phi > 0. \]

In addition,
\[ V^E_z(z, \alpha) > 0, \quad V^E_{zz}(z, \alpha) < 0 \quad \text{for} \quad z > (\zeta) - \frac{\eta p\kappa_2}{p(\kappa_1 - \eta\kappa_2)}; \quad \text{and} \quad V^E_{z\alpha}(z, \alpha) > 0. \]

From (11),
\[ \lim_{z \to 0^+} V^D(z, \alpha) = -\infty \quad \text{and} \quad V^D(1, \alpha) = rp\kappa_1 - \rho a(\alpha) + p\Phi + (1 - p)p > 0. \]

In addition,
\[ V^D_z(z, \alpha) > 0, \quad V^D_{zz}(z, \alpha) < 0 \quad \text{for} \quad z > (\zeta) 0 \quad \text{and} \quad V^D_{z\alpha}(z, \alpha) < 0. \]

The above results imply that, for a given \( \alpha \), the payoffs under equity and debt are both increasing in \( z \in (0, 1) \), and that the payoff under equity dominates the payoff under debt for \( z \to 1^- \) and \( z \to 0^+ \).

Now, define \( \hat{\nu}(z, \alpha) = V^E(z, \alpha) - V^D(z, \alpha) \). By using (9) and (11)
\[ \hat{\nu}(z, \alpha) = (1 - p)(\Phi - \phi) - \frac{\left\{ r[z, p\kappa_1 + (1 - z)\eta p\kappa_2] - \rho a(\alpha)\right\}(1 - z)\eta p\kappa_2}{z[z, p\kappa_1 + (1 - z)\eta p\kappa_2]} \] (A1)

with
\[ \lim_{z \to 0^+} \hat{\nu}(z, \alpha) = +\infty \quad \text{and} \quad \hat{\nu}(1, \alpha) = (1 - p)(\Phi - \phi) > 0 \] (A2)

Figure 3 gives a diagrammatic representation of \( \hat{\nu}(z, \alpha) \), \( V^E(z, \alpha) \) and \( V^D(z, \alpha) \) for a given \( \alpha \in (0, 1) \).

The expression for \( \hat{\nu}(z, \alpha) \) determines a set of u-shaped concave functions in \( z \in (0, 1) \), each for a given value of \( \alpha \in (0, 1) \). Since, \( \hat{\nu}_z(z, \alpha) > 0 \), the highest of these curves is the one associated with \( \alpha = 1 \), the lowest is the one associated with \( \alpha = 0 \) (see Fig. 1). If all of these curves were lying above the \( z \)-axis, then no type-1 firm would ever issue debt (in this case \( \hat{\nu}(z, \alpha) > 0 \ \forall \alpha \in (0, 1) \)). It follows that the presence of debt in the economy requires that, for some values of \( z \in (0, 1) \), a fraction, or all of the \( \hat{\nu}(z, \alpha) \) curves should intersect the \( z \)-axis and, hence, \( \hat{\nu}(z, \alpha) < 0 \) for some \( \alpha \). If this is the case, it is possible to determine an initial value \( z_0 \in [n_1, n_1/(n_1 + n_2)] \) for which there is a type-1 firm of efficiency level \( \alpha_0 \), indifferent between debt and equity (i.e. \( \hat{\nu}(z_0; \alpha_0) = 0 \)). All type-1 firms with \( \alpha > \alpha_0 \) issue equity (since \( \hat{\nu}(z_0; \alpha > \alpha_0) > 0 \)) and all type-1 firms with \( \alpha < \alpha_0 \) issue debt (since \( \hat{\nu}(z_0; \alpha < \alpha_0) < 0 \)).
Formally, the above requires that, at least for some initial value of \(z_t\) and for some \(\alpha\), the (12) is holding. Setting \(\hat{V}(\cdot, \alpha) = 0\) gives the following quadratic equation in \(z_t\):

\[
C(z_t; \alpha) = c_0 z_t^2 + c_1 z_t + c_2
\]  

(A3)

with

\[
c_0 = (\kappa_1 - \eta \kappa_2)[(1 - p)(\Phi - \phi) + r\eta p \kappa_2] > 0
\]

\[
c_1 = (1 - p)(\Phi - \phi) - r\eta p \kappa_2(\kappa_1 - \eta \kappa_2) - \eta \kappa_2[\rho a(\alpha) - r\eta p \kappa_2]
\]

\[
c_2 = \eta \kappa_2[\rho a(\alpha) - r\eta p \kappa_2]
\]

The existence of an \(\alpha_0\) requires that the quadratic equation has at least one real root. The roots of the quadratic equation, \(z_{1t}\) and \(z_{2t}\), are real if \(c_1^2 - 4c_0c_2 > 0\). Given this, then both roots must lie between 0 and 1.

Given the shape of the payoff functions, for each \(z_t\) there is only one \(\hat{V}(\cdot, \alpha)\) intersecting the horizontal axis at \(z_t\). As \(z_t\) increases the value of \(\alpha_t\) (and, therefore, the specific \(\hat{V}(\cdot, \alpha)\)) changes, and so does the corresponding fraction of type-1 firms issuing debt or equity. Condition for \(\alpha_t\) to decrease monotonically with \(z_t\) is that given an initial value of \(z_0 \in [n_1, n_1/(n_1 + n_2)]\), the difference in payoffs must be such that \(\partial \hat{V}(\cdot)/\partial z > 0\) \(\forall \alpha \in (0,1)\) and
\( \forall z_i \in [n_i, n_i/(n_1 + n_2)] \). It is tedious but straightforward to show that this condition is equivalent to:

\[
\frac{r\eta p\kappa_2 [2z_i (\eta p\kappa_1 - \eta p\kappa_2) + \eta p\kappa_2]}{z_i (\eta p\kappa_1 - \eta p\kappa_2) (2 - z_i) + \eta p\kappa_2} > \rho(\alpha). 
\]
References


