Access Price Cap Mechanisms and Industry Structure with Information Acquisition

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Abstract
This paper considers a network industry characterized by an upstream natural monopoly with cost uncertainty, regulated through an access price mechanism, and an unregulated downstream market with Cournot competition. The upstream monopolist can acquire information on the upstream cost while the information acquisition is prohibitively costly for the regulator and downstream firms. The information acquisition is also unobservable. I investigate how the presence of costly and socially valuable information on the upstream cost affects the desirability of allowing the upstream monopolist to produce in the downstream market (integration) rather than excluding it (separation). I show that when the upstream monopolist is regulated only through an access price cap, the information acquisition problem provides an argument in favour of vertical integration. However, when the regulator also obliges the upstream monopolist to share her access profits with consumers, a bias emerges in favour of separation via the impact of the access-profit sharing plan on the upstream monopolist's incentives to transmit information to her rival in the downstream market.

Keywords: Access Price Cap Mechanisms, Integration, Separation, Information Acquisition

JEL Classification: D82; D83; L5

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References
1 Introduction

Over the past twenty years, the regulated network industries in Europe and the United States have been affected by very important regulatory reforms, concerning both their structural organization and the degree of price regulation. Indeed, during the 1980s and the 1990s, the telecommunications and electricity industries were targeted for the structural separation of natural monopolistic segments (essential facilities) from potentially competitive businesses. Moreover, they underwent a partial deregulation process aimed at favouring service-based competition. However, the persistence of essential facilities and the continuing dominance of the incumbents seem to suggest that total deregulation is unrealistic at this stage, and that policy makers should instead concern themselves with the industrial structure of the regulated network industries and the type of price regulation mechanisms used.

In network industries characterized by an upstream naturally monopolistic sector and a downstream unregulated sector, the upstream monopolist has often been excluded from the downstream market based on the assumption that such exclusion would favour downstream competition. The economic literature has shown that vertical integration gives the integrated firm a greater incentive to degrade the quality of its input in order to harm downstream competitors (Armstrong and Sappington, 2007) or, under asymmetric information, to exaggerate its cost in order to convince the regulator to set a higher access price (Vickers, 1995). On the other hand, vertical integration can lead efficiency gains due to economies of scope and lower production costs in the downstream market compared to separation when the access price is higher than the marginal cost of the essential input.

This literature is mainly concerned with optimal access regulatory mechanisms which, however, are prohibitively costly to implement in practice, given their high information requirements. Instead, a widely used form of regulation, although inefficient, is the price cap mechanism, which only sets a price ceiling allowing price discretion below it. Access price caps, as distinct from retail price caps, were introduced in the telecommunication industries in the USA in 1991 and in the UK in 1997, with the dual aim of preventing exclusionary practices and of providing incentives for efficient access charges (Vogelsang, 2003).

In network industries characterized by rapid technological change and universal service obligations, the use of an access price cap mechanism for pricing network elements is likely to be more appropriate than other regulatory rules, such as the "Long Run Average Incremental Cost", recommended by the Euro-

\footnote{In recent debates (see Cave, 2006), the term "separation" in network industries has been used to refer to a variety of different options, ranging from accounting separation (i.e. the weakest form) to structural separation (i.e. separation of ownership). An intermediate form which emerged from the regulation of the telecommunications industry (Ofcom, 2005) is the operational or functional separation between the non- replicable access network and the rest of the incumbent’s activities. The underlying logic of this approach is that accounting separation can eliminate price discrimination on the part of the incumbent, but not other forms of non-price discrimination and that structural separation is too costly a remedy. Those distinctions are irrelevant for the aim of this paper which considers only non-replicable network activities.}
pean Commission (CEC, 1997; CEC, 1998) and the "Total Element Long Run Incremental Cost", introduced by the Telecommunication Act in the USA in 1996. Under those rules, the prices of the unbundled network elements are based on the incremental costs of providing the specific service, where these costs are estimated to be those of a hypothetically efficient network built using the best available technology, rather than the technology incorporated in the incumbent's network.

One of the main weaknesses of the above approach is that, in a dynamic industry characterized by rapid technological change, it impedes the incumbent from recovering past investments. On the other hand, the suggested remedy – namely periodic price revisions which take into account the depreciation of capital equipment due to technological obsolescence – assumes that the regulator can predict the direction in which technology and demand change. However, since this prediction is likely to be inaccurate, the risk of forecasting errors could fall on the regulated firm, inducing her not to produce the essential input (see Littlechild, 2003).

On the contrary, the use of an access price cap mechanism, which sets the maximum price (cap) the incumbent is allowed to charge for each network element based on the cost corresponding to the existing technology, could avoid the risk of a service interruption and induce the upstream monopolist to make innovative investments.

This paper focuses on a particular issue which may have a different effect on the performance of the access price cap mechanism, depending on the specific industrial structure of network industries. In network industries characterized by highly uncertain technological conditions, the evaluation of technological shocks may be very costly, not only for the regulator but for the firm itself. An accurate estimate of cost conditions is valuable to the regulator when it improves the performance of the access price cap mechanism; in this case, a welfare loss may arise if the firm's incentives to estimate cost conditions turn out to be inadequate.

In what follows, I will explore those issues in two different settings. First, I will investigate how the problem of acquiring socially valuable information on the upstream cost affects the welfare comparison between integration and separation, when the only tool available to the regulator is the access price cap. I will prove that when the upstream monopolist is allowed to produce in the downstream market, the access price cap mechanism gives her a greater incentive to acquire socially valuable information on the uncertain upstream cost compared to the exclusion scenario, and that this favours integration.

Second, I will consider the possibility that the regulator may force the upstream monopolist to share some access profits with consumers. I will show that the transfer of a high fraction of access profits to consumers brings about the additional effect, under integration with respect to separation, of inducing the upstream monopolist to transmit information to her rival, thus creating a bias in favour of separation.

A part from being related to the literature on vertical integration, this paper is also related to the literature on access price regulation (see, for example,
Armstrong, 2002, Laffont and Tirole, 1996, for a survey on access price regulation in the telecommunications industry; Riezmann, 2000, for a theoretical and empirical analysis of the strategic pricing effects of access price cap regulation in the electricity industry) as well as to the literature on information acquisition, which has investigated the impact of information acquisition on the performance of regulatory mechanisms (see, for example, Cremer et al., 1998, for an analysis of optimal regulation and, in particular, Iossa and Stroffolini, 2002, for an analysis of price cap regulation).

The model presented in this paper is built on Iossa and Stroffolini (2007), who also investigate the effects of information acquisition on the welfare desirability of vertical integration, but with two important differences compared to the present analysis: in their paper, the information acquisition concerns an uncertain demand in the downstream market and the access price is regulated through an optimal mechanism.

I will examine a network industry characterized by an upstream natural monopoly with cost uncertainty, regulated through an access price cap mechanism, and an unregulated downstream market with Cournot competition, producing a homogeneous good. The cost of producing the essential input is random and, since any cost-reducing activity is assumed away, the actual cost reflects exogenous technological changes. I will assume that only the upstream monopolist has the necessary know-how to acquire information on the upstream cost, while the information acquisition is prohibitively costly for the regulator and the downstream firms. The regulator is concerned only about consumer welfare and the acquisition of information on the upstream cost is valuable to the regulator to the extent that it can affect the firm’s output choice complying with the access price cap constraint. The regulator knows the cost of acquiring information but he cannot observe the process of information acquisition which prevents him from simply instructing the firm to acquire information. Within this context, I will compare two industrial structures: integration, where the upstream monopolist is integrated with a downstream firm, and separation, where the upstream firm does not operate in the downstream market. In both cases only two firms produce in the downstream market.

In the first part of the paper, I will describe the results obtained when the upstream monopolist is regulated through a standard access price cap mechanism. First, I will analyze the incentives to acquire information on the upstream cost, under integration and separation, when the access price cap is designed for the case of asymmetric information. I will show that integration gives the upstream monopolist a greater incentive to acquire socially valuable information compared to separation. This can be explained as follows. Under separation, information on the upstream cost is valuable to the upstream monopolist only to the extent that it allows her to charge the monopoly access price whenever it lies below the access price cap. This implies that the gains from acquiring information depend on the probability that favourable technological conditions make the monopoly access price lower than the access price cap. On the contrary, when the upstream monopolist is allowed to produce in the downstream market, as under integration, she can use the information on the upstream cost
to adjust her output in the downstream market accordingly, whatever the actual cost is.

Moreover, I will show that the very fact that the information remains private makes it profitable for the upstream monopolist, under integration, to charge the access price cap even when it lies above the monopoly access price. This is because, with an ignorant rival setting her output on the basis of the expected value of the upstream cost, the access profits obtainable from selling the essential input to the rival more than compensate the gain in downstream profits obtainable from excluding her from the market.

When the cost of acquiring information is greater than the value of information to the upstream monopolist, and the access price cap is the only regulatory instrument, higher incentives to acquire information can only be provided by increasing the overall level of the price cap in order to increase the firm’s profits.

I will show that an increase in the access price cap raises the upstream monopolist’s value of information under separation but not under integration. This is due to the fact that the integrated upstream monopolist always charges the access price cap, both when she acquires information and when she does not; therefore, the value of information is independent of the level of the access price cap. On the contrary, under separation, the higher the access price cap, the greater the probability that the informed upstream monopolist will charge the monopoly access price and, therefore, the greater the value of information.

It follows that, when the cost of acquiring information is sufficiently high, the integrated upstream monopolist chooses to remain ignorant and sets its output on the basis of the expected value of the upstream cost; hence the welfare gains arising from the variability of industry output are lost. Instead, under separation, a trade-off occurs. On the one hand, an increase in the level of the access price cap induces the upstream monopolist to acquire information, but at the cost of reducing the industry output on the range where the access price cap is binding. On the other hand, if the level of the price cap is not modified, the firm chooses to remain ignorant and charges the access price cap; therefore, any welfare gains arising from the potential reduction of the access price are lost. Hence, the lower those welfare gains and the higher the cost of acquiring information, the less likely it is that inducing information acquisition will be optimal under separation.

The above results suggest that the problem of information acquisition is more likely to increase the welfare desirability of integration with respect to separation, whenever inducing information acquisition under separation is unlikely to be optimal.

Indeed, I will show that there is an entire range of information acquisition costs where, under integration, the access price cap mechanism designed for the case of asymmetric information naturally gives the monopolist an incentive to acquire information, whereas, under separation, the regulator has to increase the level of the cap to induce information acquisition. Therefore, over this range, the information acquisition problem generates a welfare loss under separation but not under integration.

However, when the cost of acquiring information increases, a welfare loss
occurs under integration as well, due to the lack of socially valuable information. As a consequence, there might be a range of information acquisition costs where the access price cap mechanism induces information acquisition under separation but not under integration. I will argue that this could occur in those network industries where, under asymmetric information, the access price cap mechanism is not able to affect the profit-maximizing behaviour of an integrated upstream monopolist whatever the actual value of the cost. In these cases a regulator, with the access price cap as the only regulatory instrument, would not allow the upstream monopolist to produce in the downstream market, i.e., integration would never occur.

The conclusion is that in network industries where the access price cap mechanism, under asymmetric information, is effective in regulating an integrated upstream monopolist, the information acquisition problem is likely to favour integration.

The above analysis has implicitly assumed that it is unprofitable for the integrated upstream monopolist to transmit the privately acquired information to her rival in the downstream market. This assumption is justified by the results obtained in the second part of the paper, where the access price cap mechanism is modified with the introduction of an access-profit sharing plan that obliges the upstream monopolist to share a constant fraction of the access profits with consumers.

Earnings-sharing plans are usually advocated since they allow consumers to share some of the gains from production with the firm, either through lump sum transfers or through price reductions. However, regardless of the manner in which earnings are shared, the requirement to share earnings with consumers reduces the regulated firm’s incentives to minimize operating costs (Mayer and Vickers, 1996; Sappington, 2002).

Having assumed away any cost-reducing activity, the fundamental trade-off associated with earnings-sharing plans does not play any role in the context of this paper. In our model there is a new trade-off, which is due to the fact that only the upstream monopolist can privately acquire information on the upstream cost. Within this setting, the transfer of a high fraction of access profits to consumers, under integration, induces the upstream monopolist to transmit information to her rival in the downstream market, thereby reducing any welfare gains generated by the adoption of an access-profit sharing plan.

This result can be explained as follows. When the rival becomes informed on the upstream cost, she adjusts her output accordingly. This adjustment has two opposite effects on the upstream monopolist’s expected profits. On the one hand, it increases the variability in the upstream monopolist’s equilibrium output, which, in turn, raises the expected profits obtained in the downstream market. This result is in line with those obtained by the literature on information sharing, according to which firms competing in an unregulated Cournot market with homogeneous good find it profitable to symmetrically share information about their own costs (see Fried, 1984; Shapiro, 1986; Raith, 1996)². On

²Fried (1984) explores the firm’s incentives to produce and disclose information on the
the other hand, the rival’s output variability caused by the information transmission reduces the expected access profits the monopolist obtains from selling the essential input. This result strictly depends on the access price cap breaking the link between access price and cost, which makes the access revenues linear in the cost parameter. As a consequence, the information transmission does not affect the expected access revenues, while it increases the upstream monopolist’s expected costs of supplying the access.

It follows that the greater the fraction of access profits shared with consumers, the more the information transmission is likely to increase the upstream monopolist’s expected profits and thereby her value of information.

I will show that, with a regulator only concerned with consumer welfare, the information transmission causes a welfare loss even when it induces the acquisition of socially valuable information. This is mainly due to the fact that the information transmission reduces the expected access profits and therefore brings about lower expected transfers to consumers compared to the case where the rival remains ignorant. This negative welfare effect is greater than the positive effect arising from the adjustment of industry output to the actual cost, which occurs when the transmission of information induces the acquisition of information.

Besides, when this information acquisition effect does not occur, the transmission of information to the rival increases the variability in each firm’s equilibrium output, thereby causing a reduction in the variability in the industry equilibrium output, which further contributes to reduce the expected welfare. Since the only way the regulator can make the information transmission less profitable for the upstream monopolist is to reduce the fraction of access profits transferred to consumers, the adoption of an access-profit sharing plan generates a bias in favour of separation where, instead, no transmission effect occurs and all access profits are transferred to consumers.

The rest of the paper is organized as follows. In Section 2 I outline the basic model where the upstream monopolist is regulated only through an access price cap (section 2.1); then I derive the equilibrium outputs under Cournot competition and analyze the choice of the access price charged by the upstream monopolist, under integration (section 2.2) and separation (section 2.3); further I analyze the regulator’s choice of the access price cap (section 2.4). In Section 3 I investigate the performance of the access price mechanism in the presence of the problem of information acquisition under integration (section 3.1) and separation (section 3.2); then I analyze the impact of the information acquisition problem on the welfare comparison between integration and separation (section 3.3). In Section 4, after modifying the basic model with the introduction of an access-profit sharing plan (section 4.1), I investigate the information transmission effect arising from the adoption of these plans under integration (section 4.2); then I determine the optimal fraction of access profits transferred to consumers under integration and separation (section 4.3); finally, I investi-

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diopolist’s cost functions; Shapiro (1986) analyzes the profit and welfare effects of cost sharing in standard oligopoly models; Raith (1996) proposes a general model.
gate how the adoption of an access-profit sharing plan affects the desirability of integration with respect to separation (section 4.4). Section 5 concludes the paper. All proofs missing from the main paper are in the Appendix.

2 Access price cap mechanisms

2.1 The basic model

In this paper, a very simple analytical framework is used where a regulated upstream monopolist sells network access to downstream firm (s) in a retail market characterized as an unregulated duopoly with Cournot competition and homogeneous product.

Two industrial structures are considered: integration (I) and separation (S); I indicates a situation where the upstream monopolist is allowed to produce, through a subsidiary, also in the downstream market while under S it is excluded. The number of firms in the downstream market is fixed and equal to two in both industrial structures; I will assume that only one firm, a part the subsidiary of the incumbent, knows the technology required to produce the output. Thus, the difference between the two industrial structures is solely that under S the downstream firm which was the subsidiary of the upstream monopolist in the final market becomes independent. This makes it possible to highlight the effects of information issues on the comparison between I and S and has no qualitative impact on the results.

The marginal cost of producing the essential input is \( c = \beta \), where \( \beta \) is a parameter of adverse selection, with \( \beta \in [\beta, \overline{\beta}] \); it has density function \( f(\beta) \) and distribution function \( F(\beta) \) which are common knowledge; \( \beta_0 \) and \( \sigma^2 \) denote the mean value and the variance of the distribution of \( \beta \), respectively. The realizations of \( \beta \) can be interpreted as the result of exogenous technological changes and any cost-reducing investment is assumed away. The upstream monopolist (she) can observe at some cost \( K \geq 0 \) the true realization of \( \beta \), while the information acquisition is prohibitively costly for the regulator (he) and downstream firms. I will assume that the regulator knows the value of \( K \) but cannot observe the information acquisition process; he observes only the access price.

The upstream market is regulated through an access price cap mechanism with downward flexibility which sets an upper bound on the price that the upstream monopolist can charge for the essential input sold to the downstream firms. Hereafter, let \textbf{ACI} denote the access price cap regulatory mechanism under I and \textbf{ACS} the regulatory mechanism under S.

The downstream market is characterized by a linear inverse demand function: \( P(Q) = d - Q \). For the sake of simplicity, it is assumed that the technology used to produce the downstream output, which is the same under both industrial structures, only requires the essential facility.\(^3\) Therefore, the cost of producing

\(^3\)I will assume away efficiency gains arising from integration and assume that the fixed cost of entering the downstream market is equal for each firm and normalized to zero. These
the final good is the marginal cost of producing the essential input for the upstream monopolist, since the access price paid by her subsidiary is just an internal transfer, while for the downstream firm the cost of producing the final good is the regulated access price.

Consider now the payoff of the firms, net of the information-acquisition cost. Under $I$ the profit function of the upstream monopolist is

$$\Pi^M_I = (d - Q_I - \beta)q^M + (a_I - \beta)q^R$$  \hspace{1cm} (1)

where $Q_I = q^M + q^R$, and $q^M$ and $q^R$ denote the quantity produced by the upstream monopolist and the rival firm in the downstream market, respectively; $a_I$ denotes the access price paid by the rival. The profit function of the rival is

$$\Pi^R_I = (d - Q_I - a_I)q^R$$  \hspace{1cm} (2)

Under $S$, the profit function of the upstream monopolist is given by

$$\Pi^M_S = (a_S - \beta)Q_S$$  \hspace{1cm} (3)

where $Q_S = 2q_S$ and $q_S$ denotes the quantity produced by a downstream firm; $a_S$ denotes the access price paid by the downstream firms and the downstream firms profit is

$$\Pi^R_S = (d - Q_S - a_S)q_S$$  \hspace{1cm} (4)

The regulator maximizes, both under $I$ and under $S$, the expected net consumer surplus; denoting by $S(Q_h)$ the gross consumer surplus with $S' = P_h$ and $S'' < 0$, the regulator’s objective function is given by\footnote{The exclusion of any concern about firm’s profits on the part of the regulator can be justified by the fact that in industries characterized by cost uncertainty a price cap mechanism usually allows the firm to make high profits.}

$$\int \left[ S(Q_h) - (d - Q_h)Q_h \right] f(\beta) d\beta = \frac{1}{2} \int \frac{Q_h^2 f(\beta) d\beta}{2} \quad h = I, S$$  \hspace{1cm} (5)

The timing of the game is the following. 1) Nature chooses $\beta$; 2) the regulator sets the access price cap under ACI and ACS; 3) the upstream monopolist decides whether to acquire information on the cost parameter $\beta$ by investing $K$, and, if she does, observes $\beta$; 4) the upstream monopolist decides whether to accept the regulatory mechanism and, if she does, chooses the access price; 5) firms in the downstream market simultaneously choose their quantities and the access prices are paid.

The game will be solved, both under $I$ and under $S$, through backward induction, i.e., first by finding the equilibrium at the last stage of the game and then at the previous stages. The analysis begins with the computation of the downstream equilibrium outputs resulting from Cournot competition as a
function of the access price charged by the upstream monopolist; then, by using these results, I will derive the access price charged by the upstream monopolist. Finally, the access price cap solving the regulator’s problem will be determined.

2.2 Integration

Let \( a_I(\beta) \epsilon \{ a_I^m(\beta), \overline{A}_I \} \), with \( a_I(\beta) \leq \overline{A}_I \), denote the access price charged by the upstream monopolist under information acquisition, where \( a_I^m(\beta) \) is the monopoly access price and \( \overline{A}_I \) is the access price cap. It is worth noticing that if the upstream monopolist chooses the monopoly access price, her rival in the downstream market can deduce the true realization of \( \beta \) from the access price. It follows that the upstream monopolist, in choosing the monopoly access price, has to take into account the equilibrium outputs which would arise from Cournot competition in the downstream market if her rival were informed about \( \beta \). The maximization of (1) w.r.t. \( q^M \) and of (2) w.r.t. \( q^R \) yield the equilibrium variables in the downstream market as a function of \( \beta \) and \( a_I \)

\[
q^M(\beta, a_I) = \frac{d - 2a_I + \beta}{3} ; \quad q^R(\beta, a_I) = \frac{d - 2a_I + \beta}{3} \quad ; \quad Q_I(\beta, a_I) = \frac{2d - a_I - \beta}{3} ;
\]

Substituting for (6) in (1) and maximizing it w.r.t. \( a_I \) yields the monopoly access price

\[
a_I^m(\beta) = \frac{d + \beta}{2} \quad (7)
\]

Let \( \beta^m(\overline{A}_I) \) denote the value of \( \beta \in [\overline{\beta}, \overline{\beta}] \) such that: \( \beta^m(\overline{A}_I) = a_I^m(\overline{A}_I) \). Since \( a_I^m(\beta) \) increases in \( \beta \), it follows that

\[
a_I^m(\beta) \leq \overline{A}_I \text{ for all } \beta \leq \beta^m(\overline{A}_I) ;
\]

\[
a_I^m(\beta) > \overline{A}_I \text{ for all } \beta > \beta^m(\overline{A}_I) ;
\]

Therefore, under ACI, the upstream monopolist has to charge the access price cap for \( \beta \epsilon [\beta^m(\overline{A}_I), \overline{\beta}] \), while she is allowed to charge both the monopoly access price and the access price cap for \( \beta \epsilon [\overline{\beta}, \beta^m(\overline{A}_I)] \). The upstream monopolist’s choice is determined by comparing the firm’s profits obtainable from charging \( a_I^m(\beta) \) and \( \overline{A}_I \), respectively, for \( \beta \epsilon [\overline{\beta}, \beta^m(\overline{A}_I)] \).

By charging the monopoly access price, the upstream monopolist excludes her rival from the downstream market; so she will gain monopolistic profits in the downstream market, but lose the access profits that she would obtain by selling at \( \overline{A}_I \) the essential input to the rival. In this case, the upstream monopolist’s profit function is given by (1) with the equilibrium outputs defined in (6) for \( a_I = a_I^m(\beta) \).

Now consider the case where the upstream monopolist charges \( \overline{A}_I \). Since the access price cap breaks the link between access price and cost, the upstream monopolist’s rival cannot deduce the true realization of \( \beta \) from the access price. Hence, it will choose its output \( q^R \) so as to maximize its expected profits \( E\Pi^R \),

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with \( \Pi^\beta \) given by (2) where the expectation is taken over \( \beta \), while the informed upstream monopolist will set her output \( q^M \) so as to maximize (1), yielding the following equilibrium outputs:

\[
q^M(\beta, \beta_0, \bar{A}_I) = \frac{d + \bar{A}_I}{3} - \frac{\beta}{2} + \frac{\beta_0}{6}, \quad Q_I(\beta, \beta_0, \bar{A}_I) = \frac{2d - \bar{A}_I}{3} - \frac{\beta}{2} - \frac{\beta_0}{6}
\]

The fact that at \( \bar{A}_I \) the upstream monopolist’s rival sets its output on the basis of \( \beta_0 \) makes the industrial structure in the downstream market dependent on the value of the demand parameter \( d \) and on the shape of the distribution function \( F(\beta) \).

Indeed, by using the definition of \( \beta^m(\bar{A}_I) \), the rival’s output can be written as \( q^R(\beta_0, \bar{A}_I) = \frac{\beta_0 - \beta^m(\bar{A}_I)}{4} \). Therefore, \( q^R(\beta_0, \bar{A}_I) = 0 \) for demand and distribution functions giving \( \beta_0 \leq \beta^m(\bar{A}_I) \) and the upstream monopolist will gain monopolistic downstream profits as in the case in which she charges \( q^T(\beta) \). Instead, \( q^R(\beta_0, \bar{A}_I) > 0 \) for demand and distribution functions giving \( \beta_0 > \beta^m(\bar{A}_I) \) and the upstream monopolist will obtain Cournot profits in the downstream market and access profits from selling the essential input to the rival; in this case the upstream monopolist’s profit function is given by (1) with the equilibrium outputs defined in (8).

The comparison between the profits obtainable by the upstream monopolist from charging the monopoly access price and those from charging the access price cap for \( \beta \in \left[\beta, \beta^m(\bar{A}_I)\right] \), leads to the following Lemma.

**Lemma 1** Under \( ACI \), the upstream monopolist will charge the access price cap for all \( \beta \in \left[\beta, \beta^m(\bar{A}_I)\right] \).

According to Lemma 1 the upstream monopolist chooses to charge the access price cap also when favourable technological conditions make the monopoly access price lower than the access price cap.

The rationale for Lemma 1 can be understood as follows. On the one hand, for demand and distribution functions yielding \( \beta_0 \leq \beta^m(\bar{A}_I) \), her rival does not produce at \( \bar{A}_I \) and therefore the upstream monopolist is indifferent about charging the access price cap or the monopoly access price. On the other hand, for demand and distribution functions yielding \( \beta_0 > \beta^m(\bar{A}_I) \), the upstream monopolist, by charging \( \bar{A}_I \), allows her rival to produce in the downstream market. Therefore, she gains access profits from selling the essential input to her rival, but loses downstream profits with respect to the case in which she charges the monopoly access price. Since for \( \beta \in \left[\beta, \beta^m(\bar{A}_I)\right] \) the gain in access profits is greater than the loss in downstream profits, the upstream monopolist will find it profitable to charge the access price cap whatever the actual value of the cost.

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5 This is obtained by substituting for \( 2\bar{A}_I = d + \beta^m(\bar{A}_I) \) in \( q^R(\beta) \).
It is worth noticing that this result strictly depends on the fact that the ignorant rival firm sets its output on the basis of $\beta_0$ and this is greater than the output that an informed rival would produce on the basis of the true realization of $\beta$ for $\beta e \left[ \beta, \beta^m(\overline{A}_I) \right].$ In the rest of the paper I will restrict the exposition to the case of $\beta_0 > \beta^m(\overline{A}_I)$; this is not a serious limitation because, as will be shown in the appendix, the results of the paper also hold in the case of $\beta_0 \leq \beta^m(\overline{A}_I).$

When the upstream monopolist does not acquire information, she will choose $q^M$ so as to maximize $E\Pi^M_I$ with $\Pi^M_I$ defined by (1) and her rival chooses $q^R$ so as to maximize $E\Pi^R_I$, with $\Pi^R_I$ given by (2), where the expectations are taken with respect to $\beta$. The equilibrium outputs under no information acquisition are given by

\[
q^M(\beta_0, a^N_I) = \frac{d + a^N_I - 2\beta_0}{3}; \quad q^R(\beta_0, a^N_I) = \frac{d - 2a^N_I + \beta_0}{3}; \quad (9)
\]

\[
Q_I(\beta_0, a^N_I) = \frac{2d - a^N_I - \beta_0}{3}
\]

where $a^N_I = \min \{ \overline{A}_I, a^m_I(\beta_0) \}$ with $a^m_I(\beta_0)$ indicating the monopoly access price maximizing the upstream monopolist’s expected profits. Since $\beta^m(\overline{A}_I) < \beta_0$ by assumption, $a^m_I(\beta_0) > \overline{A}_I$ and thereby an ignorant upstream monopolist will charge $a^N_I = \overline{A}_I$. The above analysis has shown that the information on $\beta$ allows the upstream monopolist to adjust her output to the actual value of the cost, while, when ignorant, she sets her output on the expected value of the cost. Therefore, the information acquisition is socially valuable, under integration, because it increases the variability in the industry equilibrium output which, in turn, with linear demand, increases the expected net consumer surplus.

\footnote{For the same reason the industry output, as well as the expected consumer surplus, is greater when the upstream monopolist charges the access price cap rather than the monopoly access price for $\beta e \left[ \beta, \beta^m(\overline{A}_I) \right]$.}

\footnote{There is also an economic rationale for excluding the case of $\beta_0 \leq \beta^m(\overline{A}_I)$. Since in this case the upstream monopolist’s rival would not produce at the access price cap, the integrated industry would become an unregulated monopoly. As a consequence, a regulator with the access price cap as the only instrument of control would not allow the upstream monopolist to produce in the downstream market, i.e. integration would never occur under asymmetric information. This makes the case of $\beta_0 \leq \beta^m(\overline{A}_I)$ irrelevant for the purpose of this paper.}

\footnote{The welfare effects of information acquisition can be intuitively understood by realizing that the expected net consumer surplus $\frac{1}{2}E[Q(.)^2]$ is equal to

\[
\frac{1}{2}E[Q(.)]^2 + \frac{1}{2}\text{var}[Q(.)]
\]

Due to linear demand, the acquisition of information does not affect the expected equilibrium output, while it increases its variance.}
2.3 Separation

This paragraph will derive Cournot equilibrium outputs and the access price charged by the upstream monopolist under separation. The downstream firms simultaneously choose their output by maximizing their profit function (4) w.r.t. $q^S$ yielding:

$$q_S(a_S(\beta)) = \frac{d - a_S(\beta)}{3}; Q_S(a_S(\beta)) = \frac{2d - 2a_S(\beta)}{3};$$ (10)

where $a_S(\beta) = \min \{a_w^S(\beta), \overline{A}_S\}$, is the access price charged by an informed upstream monopolist, with $a_w^S(\beta)$ denoting the monopoly access price and $\overline{A}_S$ the access price cap. It is easy to demonstrate that the monopoly access price under $S$ is equal to that under $I$, defined in (7). Therefore, there is a value of $\beta, \beta^m(\overline{A}_S) \in (\underline{\beta}, \overline{\beta})$ defined by $\beta^m(\overline{A}_S) = a_w^S\overline{A}_S(\overline{A}_S)$, such that

$$a_S(\beta) = a_w^S(\beta) \text{ for all } \beta \leq \beta^m(\overline{A}_S)$$

$$a_S(\beta) = \overline{A}_S \text{ for all } \beta > \beta^m(\overline{A}_S)$$

which substituted in (10) yield

$$Q_S(\beta) = \frac{d - \beta}{3} \text{ for all } \beta \leq \beta^m(\overline{A}_S)$$ (11)

$$Q_S(\overline{A}_S) = \frac{2d - 2\overline{A}_S}{3} \text{ for all } \beta > \beta^m(\overline{A}_S)$$ (12)

Notice that the information on $\beta$ acquired by the upstream monopolist can affect the downstream output only through the monopoly access price charged to downstream firms. As a consequence, for $\beta \in (\beta^m(\overline{A}_S), \overline{\beta})$ where the access price cap is binding, the industry equilibrium output becomes insensitive to $\beta$.

If the upstream monopolist does not acquire information on $\beta$, she sets the access price $a_w^S = \min \{A_S, a_w^I(\beta_0)\}$ with $a_w^I(\beta_0)$ indicating the monopoly access price maximizing the upstream monopolist’s expected profits. In what follows, as under $I$, I will restrict the exposition to the case where the demand and distribution functions are such that $\beta^m(\overline{A}_S) < \beta_0$; this is not a limitation because, as shown in the appendix, all the results still hold when $\beta^m(\overline{A}_S) \geq \beta_0$.

Therefore, the access price set by an ignorant upstream monopolist is $a_w^S = \overline{A}_S$ leading to the equilibrium output $Q_S(\overline{A}_S)$ for all $\beta \in [\underline{\beta}, \overline{\beta}]$.

The above analysis has shown that the information acquisition allows the upstream monopolist to charge an access price which is lower than the access price cap on the range $\beta \in [\underline{\beta}, \beta^m(\cdot)]$ leading to a greater industry output level. Therefore, the information acquisition is valuable to the regulator under separation because it makes it possible to realize the welfare gains arising from the downward flexibility of the access price cap. These welfare gains are greater, the greater the probability that favourable technological conditions make the monopoly access price lower than the price cap.
2.4 The regulator’s problem

In the first stage the regulator sets the access price cap \( \bar{A}_h \), \( h = I, S \), taking into account the Cournot equilibrium output, \( Q_h(a_h(\beta)) \), the access price \( a_h(\beta) \in \{ a^M_h(\beta), \bar{A}_h \} \), charged by the upstream monopolist under information acquisition and the access price \( \bar{A}_h \) charged under ignorance.

Since the information acquisition on \( \beta \) is valuable to the regulator both under integration and under separation, the regulator’s problem is

\[
\max_{\bar{A}_h} \frac{1}{2} \int_{\bar{\beta}}^{\bar{\beta}} Q_h(a_h(\beta))^2 f(\beta) d\beta \quad h = I, S \tag{P1}
\]

\[
\text{s.t.} \quad \Pi^M_h(\beta, a_h(\beta)) - K \geq 0 \quad \text{for all } \beta \in [\bar{\beta}, \bar{\beta}] \tag{IR-IA}
\]

\[
\Pi^M_h(\beta, a_h(\beta)) \geq 0 \tag{IR}
\]

\[
\Pi^M_h(\beta, a_h(\beta)) - K \geq E\Pi^M_{hN}(\beta, \bar{A}_h) \tag{IC-IA}
\]

with \( E\Pi^M_h(\beta, a_h(\beta)) \) and \( E\Pi^M_{hN}(\beta, \bar{A}_h) \) denoting the expected profits of an informed and an ignorant upstream monopolist, respectively.

Constraint (\( IR - IA \)) ensures that the upstream monopolist anticipates non-negative expected profits when she invests \( K \) in information acquisition. Constraint (\( IR \)) guarantees that the upstream monopolist finds it profitable to produce after having observed \( \beta \). Constraint (\( IC - IA \)) ensures that the firm prefers to incur cost \( K \) to become informed about the realization of \( \beta \) rather than remaining uninformed.

In a standard adverse selection setting where the upstream monopolist privately observes the true realization of \( \beta \) at no cost, the access price cap, denoted by \( \bar{a}_h \), solves (P1) disregarding constraints (\( IR - IA \)) and (\( IC - IA \)). Since the profit function of the upstream monopolist is decreasing in \( \beta : \frac{\partial \Pi^M_h}{\partial \beta} = -Q_h(.) < 0 \), \( \bar{a}_h \) solves \( \Pi^M_h(\bar{\beta}, \bar{a}_h) = 0 \) with \( \bar{a}_I \leq \bar{a}_S \).

In the following section I will analyze the incentives of the upstream monopolist to acquire information when the access price cap is \( \bar{a}_h \); then I will investigate if the access price cap can be modified in order that constraints (\( IR - IA \)) and (\( IC - IA \)) be satisfied when the cost of acquiring the value of information is greater than the value of information for the upstream monopolist.

\(^9\)The participation constraints of downstream firms are always satisfied in equilibrium

\(^10\)It is easy to show, from eqs (1) and (3), that \( \bar{a}_I = \bar{\beta} \) and \( \bar{a}_S \leq \bar{\beta} \), because the upstream losses arising from setting an access price cap \( \bar{a}_I < \bar{\beta} \) can be compensated by the profits obtainable by the integrated upstream monopolist in the imperfectly competitive downstream market.
3 ACI and ACS under costly information acquisition

3.1 Incentives to acquire information under ACI

Initially, I will analyze the incentives of the upstream monopolist to acquire information under ACI at the access price cap $\pi_1$ designed by the regulator for the case in which the upstream monopolist privately observes $\beta$ at no cost. Lemma 1 has shown that the informed upstream monopolist chooses to charge the access price cap whatever the actual value of the cost. As a consequence, the information acquisition does not affect the choice of the access price which is $\pi_1$ both if the firm acquires information and if she does not.

Let $\Pi^M_1(\beta, \beta_0, \pi_1)$ denote the upstream monopolist’s expected profits under information acquisition and $\Pi^{M,N}_1(\beta, \beta_0, \pi_1)$ those under ignorance, where $\Pi^M_1(\beta, \beta_0, \pi_1)$ and $\Pi^{M,N}_1(\beta, \beta_0, \pi_1)$ are obtained by substituting for (8) and (9), respectively, in (1) for $\lambda_1 = a^n = \pi_1$. Then the following Proposition is obtained.

**Proposition 2** i) Under ACI, at the access price cap $\pi_1$, there is a value of $K$, denoted by $K_1$, where

$$K_1 = \frac{\beta_x^2}{\tau}$$

solves $\Pi^M_1(\beta, \beta_0, \pi_1) - \Pi^{M,N}_1(\beta, \beta_0, \pi_1) = K_1$, such that for all $K \leq K_1$, the upstream monopolist acquires information, while for all $K > K_1$, the upstream monopolist remains ignorant.

ii) $\partial [\Pi^M_1(\beta, \beta_0, \pi_1) - \Pi^{M,N}_1(\beta, \beta_0, \pi_1)]/\partial \pi_1 = 0$, i.e., the upstream monopolist’s information value does not depend on the access price cap.

The intuition behind Proposition 2 is as follows. Under ACI the value of information for the upstream monopolist is given by the profitability of adjusting her output to the actual value of $\beta$ for all $\beta \in [\beta, \bar{\beta}]$. The greater the variance of the cost parameter, the greater the increase in the profit arising from the output’s adjustments and so the greater the gain from acquiring information.

Besides, since the upstream monopolist charges $\pi_1$ regardless whether she is informed or not, an increase in the access price cap raises the firm’s expected profits to the same extent both when she acquires information and when she does not. Therefore, an increase in the access price cap has no effect on the upstream monopolist’s information value. As a consequence, a regulator, with the access cap as the only instrument of control, is unable to induce the upstream monopolist to acquire information for $K > K_1$, as stated in the following Lemma.

**Lemma 3** Under integration, the access price cap mechanism cannot induce information gathering on the part of the upstream monopolist for all $K > K_1$.

A straightforward consequence of Proposition 2 and Lemma 3 is stated in the following corollary which highlights the welfare effects of the information acquisition problem under ACI..
Corollary 4 Under ACI, costly information acquisition does not affect welfare for all $K \leq K_I$; it generates a welfare loss for all $K > K_I$ due to the lack of socially valuable information and this welfare loss is greater, the greater the variance of the cost parameter.

The intuition behind corollary 4 lies in the fact that information acquisition is socially valuable because it makes the industry output sensitive to $\beta$. When $K \leq K_I$, the access price cap designed for the case of asymmetric information naturally provides the upstream monopolist with incentives to acquire information and so the presence of costly information acquisition does not affect welfare.

However, when $K > K_I$, the upstream monopolist remains ignorant and sets her output on the expected value of $\beta$; as a consequence, the welfare gains arising from the industry output variability are lost. Besides, the greater the cost uncertainty, the greater the welfare gains arising from the adjustment of the output to the actual value of the cost and thereby more is lost from the lack of information acquisition.

3.2 Incentives to acquire information under ACS

This paragraph will evaluate the effects of the information acquisition problem on the performance of the access price cap mechanism under separation. First, I will analyze the incentives of the upstream monopolist to acquire information on $\beta$ under ACS at the access price cap $\pi_S$ designed by the regulator for the case of asymmetric information.

The expected profits of the upstream monopolist when she acquires information are

$$E\Pi^M_S(\beta, \pi_S) = \int_0^\beta \Pi^M_S(\beta) f(\beta) d\beta - \int_0^\pi\Pi^M_S(\beta, \pi_S) f(\beta) d\beta$$

where $\Pi^M_S(\beta)$ and $\Pi^M_S(\beta, \pi_S)$ are obtained by substituting for (11) and (12), respectively, in (3) for $\bar{A}_1 = \pi_S$.

The expected profits of an ignorant upstream monopolist who charges $\pi^N_S = \pi_S$, are

$$E\Pi^{MN}_S(\beta, \pi_S) = \int_0^\pi \Pi^M_S(\beta, \pi_S) f(\beta) d\beta$$

Then, by expressing the upstream monopolist’s information value as follows (this expression is derived in Appendix)

$$E\Pi^M_S(\beta, \pi_S) - E\Pi^{MN}_S(\beta, \pi_S) = \int_0^\beta [Q_S(\beta) - Q_S(\pi_S)] F(\beta) d\beta$$

the following proposition is obtained.
Proposition 5 Under ACS at the access price cap \( \pi_S \), there is a value of \( K \), denoted by \( K_S(\pi_S) \), where \( K_S(\pi_S) > 0 \) solves \( E\Pi^M_0(\beta, \pi_S) - E\Pi^M_N(\beta, \pi_S) = K_S(\pi_S) \), such that: for all \( K \leq K_S(\pi_S) \) the upstream monopolist acquires information, while for all \( K > K_S(\pi_S) \), the upstream monopolist remains ignorant.

The intuition behind Proposition 5 is straightforward. Under ACS the information on \( \beta \) has a value for the upstream monopolist since it allows the firm to charge the monopoly access price whenever it lies below the access price cap, i.e. for \( \beta \in [\hat{\beta}, \beta^m(\pi_S)] \). Over this range, the firm’s information value is proportional to the difference in output levels, for each \( \beta \), when the firm acquires information and when she remains ignorant. Indeed, the greater the difference in output levels, the greater the increase in the sensitivity of the profit to \( \beta \) arising from information acquisition and so the greater the gain from acquiring information.

From Proposition 5 it follows that for \( K > K_S(\pi_S) \) the level of the access price cap needs to be modified in order to satisfy the constraints \((IR - IA)\) and \((IC - IA)\) in problem \((P1)\). Under ACS the access price cap turns out to be an instrument for the regulator to induce information gathering on the part of the upstream monopolist. The same result is obtained in Iossa and Stroppolini (2002) for the case of a price cap mechanism.

Denoting by \( \pi_S(K) \) the access price cap satisfying the constraints \((IR - IA)\) and \((IC - IA)\) in problem \((P1)\), the following proposition is obtained.\(^{11}\)

Proposition 6 Under separation for \( K > K_S(\pi_S) \), information acquisition can be induced by raising the access price cap, \( \pi_S(K) > \pi_S \) with \( \frac{\partial \pi_S(K)}{\partial K} > 0 \).

The intuition behind Proposition 6 lies in the fact that the higher the access cap, the higher the range of \( \beta \) where, by acquiring information, the upstream monopolist can charge the monopoly access price. This increases the expected profits obtainable from acquiring information leaving unaffected those obtainable under ignorance.

Let \( W_S(K) \) denote the maximum value function of the expected net consumer surplus in problem \((P1)\) under ACS. A straightforward consequence of Propositions 5 and 6 is stated in the following corollary.\(^{12}\)

Corollary 7 \( dW_S(K)/dK \) is a function of \( K \): it is equal to 0 for all \( K \leq K_S(\pi_S) \) and it is strictly negative for all \( K > K_S(\pi_S) \).

For \( K \leq K_S(\pi_S) \) the access price cap mechanism designed for the case of asymmetric information naturally provides the upstream monopolist with the incentives to acquire information and so the information acquisition problem does not affect welfare. When \( K \) rises, the access price cap needs to be increased to induce information acquisition; the greater \( K \), the greater the increase required in the access price cap and the lower the expected net consumer surplus.

\(^{11}\)Proposition 6 is similar to Proposition 2 in Iossa and Stroppolini (2002).

\(^{12}\)The statement of Corollary 7 is found in Corollary 1 in Iossa and Stroppolini (2002).
On the other hand, if the access price cap is not modified for \( K > K_S(\bar{\pi}_S) \),
the upstream monopolist will prefer to remain ignorant and charge \( \bar{\pi}_S \) whatever
the realizations of \( \beta \). Denoting by \( W^N_S \) the net expected consumer surplus under
no information acquisition, the following Lemma is obtained.

**Lemma 8** Under ACS there is a value of \( K \), denoted by \( K^N_S \) where \( K^N_S > K_S(\bar{\pi}_S) \), solves \( W_S(K^N_S) - W^N_S = 0 \), such that for all \( K \leq K^N_S \) it is optimal
to induce information acquisition, while for all \( K > K^N_S \) it is not optimal to
induce it.

The economic intuition behind Lemma 8 can be explained as follows. On the
one hand, inducing information acquisition makes it possible to realize the
welfare gains arising from the downward flexibility of the access price cap
mechanism. On the other hand, inducing information acquisition is welfare costly
because it requires an increase in the access price cap which reduces the output
on the upper range of \( \beta, \beta \epsilon (\beta^m(:,\bar{\beta}]) \).

It is easy to show that whether or not it is optimal to induce information
acquisition depends on the value of \( K \), on the value of the demand parameter
d and on the shape of the distribution function \( F(\beta) \). Indeed, the greater the
value of \( K \), the greater the increase required in the access price cap and so the
greater the welfare cost of inducing information acquisition.

Moreover, for high values of the demand parameter \( d \), the range \( (\beta, \beta^m(\bar{\pi}_S]) \)
is more likely to be smaller than the range \( (\beta^m(:,\bar{\beta}] \).\(^{13}\) Besides, for distribution
functions of \( \beta \) sufficiently skewed to the right, the probability that \( \beta \) falls in the
region \( (\beta, \beta^m(\bar{\pi}_S]) \), rather than in the region \( (\beta^m(\bar{\pi}_S), \bar{\beta}] \), is lower. Both these
effects reduce the probability that an informed upstream monopolist charges an
access price which is lower than the access price cap; this, in turn, reduces the
welfare gains arising from information acquisition.

The above analysis suggests that in sectors where the social value of the
service offered is high and where the distribution function of the upstream cost
realizations is sufficiently skewed to the right, the less likely it is that inducing
information acquisition will be optimal.

### 3.3 Welfare comparison between integration and separation

In the light of the above analysis, this paragraph will study how the information
acquisition problem affects the welfare comparison between integration and
separation under an access price cap mechanism. The following proposition compares the incentives to acquire information under ACI and ACS at the access prices cap \( \bar{\pi}_I \leq \bar{\pi}_S \), designed for the case in which the upstream monopolist privately observes \( \beta \) at no cost.

**Proposition 9** \( K_I > K_S(\bar{\pi}_S) \), i.e., the incentives to acquire information under
ACI are greater than under ACS.

\(^{13}\)This arises from \( \beta^m(\cdot) = 2\bar{\pi}_I - d \).
The rationale for Proposition 9 can be understood by noticing (from (13) and (22)) that the gain from information acquisition is proportional to how sensitive the industry output is to the cost and this sensitivity is greater under integration than under separation for each value of the cost. Indeed, under $I$, the information on $\beta$ is used by the upstream monopolist to adjust her output accordingly, whatever the value of $\beta \epsilon [\bar{\beta}, \beta]$. Instead, when the upstream monopolist is excluded from the downstream market, the industry output can adjust to $\beta$ only through the monopoly access price charged to downstream firms, i.e. only for $\beta \epsilon [\bar{\beta}, \beta^m(.)]$ where the monopoly access price lies below the access price cap. Moreover, for $\beta \epsilon [\bar{\beta}, \beta^m(.)]$ the sensitivity of the industry output under $S$ is lower than under $I$. This is because, while under separation the output of both downstream firms adjust to $\beta$, under integration only the upstream monopolist’s output adjusts to $\beta$, as the information is private.

The above analysis leads to the following Proposition.

**Proposition 10** Costly information acquisition does not affect the welfare comparison between integration and separation for $K \leq K_S(\bar{\pi}_S)$; it generates a bias against separation for $Ke(K_S(\bar{\pi}_S), K_I)$ and this bias is non-decreasing in $K$ for $Ke(K_S(\bar{\pi}_S), K_I)$ and it is increasing in $K$ for $Ke(K_S(\bar{\pi}_S), \min \{K_I, K_S^N\})$.

The rationale for Proposition 10 can be understood as follows. First, since the information has a value for the upstream monopolist both under ACI and under ACS, there is no need to modify the level of the access cap to induce information acquisition for all $K \leq K_S(\bar{\pi}_S)$. However, as the information is more valuable to the upstream monopolist under ACI than under ACS, inducing information acquisition introduces an inefficiency under separation and not under integration when $Ke(K_S(\bar{\pi}_S), K_I)$ holds. This inefficiency increases in $K$ for all $Ke(K_S(\bar{\pi}_S), K_S^N)$ where it is optimal to induce information acquisition under separation.

Moreover, from Corollary 4, it has been shown that for $K > K_I$ a welfare loss arises under integration as well, due to the lack of socially valuable information acquisition. Therefore, if $K_S^N \leq K_I$ the information acquisition problem favours integration for all $K \leq K_S^N$, while if $K_S^N > K_I$ no clear cut results can be obtained for $Ke(K_I, K_S^N)$.

It follows that the information acquisition problem is more likely to increase the welfare desirability of integration, the lower the value of $K_S^N$, i.e. whenever inducing information acquisition under separation is unlikely to be optimal. As the discussion following Lemma 8 has highlighted, this is more likely to occur in network industries characterized by high social values of output and sufficiently right-skewed distribution functions of the upstream cost. The first condition refers to industries where the losses arising from the interruptability of the service are socially relevant, as in the case of the universal service obligation. The second condition refers to cases where the probability of cost-reducing technological changes is greater when the cost is high than when the cost is low. Intuitively, this is likely to characterize sectors with very complex technology,
exhibiting a form of decreasing return to scale, where the greater the number of realized technological improvements, the lower the probability of realizing other improvements.

I argue that the other cases, namely the network industries characterized by low social values of output and left-skewed distribution functions, are not worth considering. This can be explained as follows. Consider low values of the demand parameter and left-skewed distribution functions giving $\beta_0 \leq \beta^m(\bar{\pi}_I)$. In this case, as shown in section 2, the rival of the upstream monopolist in the downstream market would not produce at the access price cap. Therefore, a regulator, with the access price cap as the only instrument, would not be able to limit the profit-maximizing behaviour of an integrated upstream monopolist whatever the actual value of the cost, namely, an integrated industry would become an unregulated monopoly. In these network industries the only way the regulator could affect the upstream monopolist’s strategy would be to exclude the firm from the downstream market, i.e., integration would never occur under asymmetric information.

The following Corollary summarizes the above results.

**Corollary 11** In network industries where the access price cap mechanism, under asymmetric information, is effective in regulating an integrated upstream monopolist, the presence of costly and socially valuable information acquisition is more likely to increase the welfare desirability of integration.

Finally, the following Lemma evaluates the two industrial structures when $K > \max \{K_I, K_S}\}$, where there is no information acquisition on $\beta$ both under integration and separation.

**Lemma 12** Integration is welfare preferable to separation when no information acquisition occurs.

The economic intuition of Lemma 12 lies in the fact that under ignorance the downstream output is not sensitive to $\beta$ and the only difference between downstream outputs under integration and separation is due to the expected production costs. Now the production cost is greater under separation than under integration for two reasons. First, because the access price cap, which is the access price charged under ignorance, is strictly higher under $S$ than under $I$, being $\bar{\pi}_S > \bar{\pi}_I$ for all $K > K_S(\bar{\pi}_S)$ and $\bar{\pi}_I \leq \bar{\pi}_S$. This makes the production cost of downstream firms under $S$ greater than the cost of the upstream monopolist’s rival under $I$. Second, because the ignorant upstream monopolist sets its output on $\beta_0$ which is lower than $\bar{\pi}_I$. Both these effects lead to a greater output when the ignorant upstream monopolist is allowed to produce in the downstream market, as under integration, than when she is excluded, as under separation.

4 Access profit-sharing plans

In this section the access price cap mechanism is modified with the introduction of an access profit-sharing plan which obliges the upstream monopolist to share
a constant fraction of access profits with consumers. The following analysis tackles a problem which arises, under integration, when only the upstream monopolist is able to acquire information regarding the upstream cost. This issue concerns the upstream monopolist’s decision on whether to transmit the privately acquired information to her rival in the downstream market. I will investigate whether the introduction of an access profit-sharing plan affects the upstream monopolist’s incentives to transmit information and, in this way, the welfare desirability of integration with respect to separation.

4.1 The model

To take into account the upstream monopolist’s decision on the transmission of information, the timing of the game is modified as follows.

1) Nature chooses \( \beta \); 2) the regulator sets the access price cap under \( I \) and under \( S \); 3) The upstream monopolist decides whether to acquire information on \( \beta \) by investing \( K \) and, under \( I \), whether to transmit it to the rival; the upstream monopolist has to precommit to a decision transmission rule, i.e., to reveal all the information or keep it private before observing the true realization of \( \beta \). If the upstream monopolist spends \( K \), she observes \( \beta \); 4) the upstream monopolist decides whether to accept the regulatory mechanism and, if she does, chooses the access price and, under \( I \), reveals the information to her rival if she precommitted to do so. The information revealed is assumed to be verifiable by her rival so the revelation is truthful; 5) firms in the downstream market simultaneously choose their quantities on the basis of the information available as a consequence of the decisions taken in stage 3 and the access prices are paid.

I will now derive the upstream monopolist’s profit functions corresponding to the access price cap mechanisms with access profit sharing, under \( I \) (denoted hereafter by \( \text{ACIps} \)) and under \( S \) (denoted by \( \text{ACSpS} \)).

Denoting by \( \gamma^h, h = I, S \) the fraction of the access profits related to consumers, with \( \gamma^h \in [0, 1] \), the upstream monopolist’s profit function under \( \text{ACIps} \) is given by

\[
\Pi^M_I(\gamma^I) = (d - Q_I - \beta)q^M + (1 - \gamma^I)(a_I - \beta)q^R
\]

(14)

and under \( \text{ACSpS} \)

\[
\Pi^M_S(\gamma^S) = (1 - \gamma^S)(a_S - \beta)Q_S
\]

In the following analysis I will make two assumptions to isolate those welfare effects arising from the adoption of an access profit-sharing plan related to

\[\text{Notice that profit-sharing can also be realized through a reduction of the cap by an amount equal to the percentage of profits transferred to consumers. Since this form of access profit-sharing has the same effect on the upstream monopolist’s behaviour as the transfers of profits to consumers, the two forms are equivalent for the purpose of this analysis.}^{14}\]

\[\text{I will assume that the upstream monopolist under integration cannot strategically manipulate her profits by reporting access profits as downstream profits. This assumption does not affect the results qualitatively and helps to focus on the effect of an access profit-sharing plan on the welfare desirability of integration, via its impact on the upstream monopolist’s incentives to transmit information.}^{15}\]
information transmission and acquisition issues. First, I will assume that the access price cap \(\bar{\pi}_I\), designed under ACI for the case of asymmetric information, is such that non-negative access profits occur whatever the value of \(\beta\in [\beta, \bar{\beta}]\).\(^{16}\)

This assumption implies that the adoption of an access profit-sharing plan does not affect the value of the access price cap whatever the fraction \(\gamma^h\) of access profits transferred to consumers, both under \(I\) and under \(S\).\(^{17}\) Second, I will assume that the demand functions and the distribution functions of \(\beta\) are such that, for \(\gamma = 0\), the monopoly access price at \(\bar{\beta}\) is at least equal to the access price cap, \(a^n_h(\beta) \geq \bar{\pi}_h\), \(h = I, S\), i.e., the access price cap is always binding for all \(\beta\in [\beta, \bar{\beta}]\).\(^{18}\) These assumptions imply that the access price charged by the upstream monopolist, both under ACIps and ACIips, is the access price cap \(\bar{\pi}_h\) for all \(\beta\in [\beta, \bar{\beta}]\) and \(\gamma^h \in [0, 1]\).\(^{19}\)

4.2 Information transmission effect under integration

This paragraph will analyze whether the introduction of an access profit-sharing plan in the access price cap mechanism under integration affects the upstream monopolist’s incentives to reveal the true realization of \(\beta\) to her rival in the downstream market.

When the upstream monopolist does not transmit information to her rival, Cournot competition under ACIps gives the same equilibrium outputs as under ACI; therefore, the upstream monopolist’s profit function under no information transmission, denoted by \(\Pi^A_h(\beta, \beta_0, \pi_I, \gamma^I)\), is obtained by substituting for the

\(^{16}\)This is equivalent to assuming \(\pi_I = \bar{\beta}\).

\(^{17}\)In note 11 it was stated that, under ACI, \(\pi_I < \bar{\beta}\) satisfies the upstream monopolist’s participation constraint when the downstream profits compensate the access losses. In these cases the adoption of an access profit-sharing plan would lead to the sharing of access losses with consumers for high values of \(\beta\) creating two opposite effects on welfare. On the one hand, the sharing of losses would lead to a reduction in consumer income. On the other hand, the sharing of losses would increase the upstream monopolist’s profits, thereby causing a reduction of the access price cap required to satisfy the firm’s participation constraint. The following analysis will ignore these opposite welfare effects in order to isolate those related to information acquisition and transmission issues.

\(^{18}\)When the access price cap is binding under separation the information is not valuable to the regulator as the output is not sensitive to \(\beta\). However, this assumption is completely irrelevant for assessing the welfare effect of the introduction of an access profit-sharing plan under S, which is given only by the increase in consumer income. Instead, under integration, if \(a^n_h(\beta) < \bar{\pi}_h\) for \(\beta \leq \beta_m(\pi_I)\), the transfer of a fraction of access profits to consumers could make it profitable for the upstream monopolist to charge \(a^n_h(\beta)\) for \(\beta \leq \beta_m(\pi_I)\) instead of \(\pi_I\), so that Lemma 1 no longer holds. In this case ACIps would lead to a lower level of industry output for \(\beta \leq \beta_m(\pi_I)\) with respect to ACI reducing the positive welfare effect due to the transfer of access profits to consumers. I do not consider this welfare effect being irrelevant for the purpose of this analysis.

\(^{19}\)The monopoly access price under ACI, \(a^n_h(\beta, \gamma^I)\), is defined by

\[
\frac{2}{\gamma^I} q^M + (1 - \gamma^I) \frac{\partial}{\partial q^M} a^n_h(\beta - \gamma^I) q^R = 0
\]

which is obtained by the maximization of eq (14) with respect to \(a_I\). Implicitly differentiating the above eq. with respect to \(a^n_h\) and \(\gamma^I\), gives

\[
\frac{d\gamma^I}{a^n_h} > 0.
\]

Therefore \(a^n_h(\beta) \geq \pi_I\) implies \(a^n_h(\beta, \gamma^I) > \pi_I\) for all \(\gamma^h \in [0, 1]\).

The case of separation is easily proved following the same reasoning.
equilibrium outputs defined in (8), in (14), for \( \bar{\lambda}_t = \pi_t \). Instead, when the upstream monopolist chooses to transmit to her rival the acquired information on \( \beta \), the latter will adjust its output accordingly; the upstream monopolist’s profit function under information transmission, denoted by \( \Pi_I^M(\beta, \pi_I, \gamma^I) \), is obtained by substituting for the equilibrium outputs defined in (6), in eq (14), for \( a_t = \pi_I \).

Taking the expectations of \( \Pi_I^M(\beta, \beta_0, \pi_I, \gamma^I) \) and of \( \Pi_I^M(\beta, \pi_I, \gamma^I) \), the following Lemma is obtained.

**Lemma 13** Under ACIps \( \frac{\partial}{\partial \gamma^I} [\Pi_I^M(\beta, \pi_I, \gamma^I) - \Pi_I^M(\beta, \beta_0, \pi_I, \gamma^I)] / \partial \gamma^I > 0 \) and there is a value of \( \gamma^I \), denoted by \( \gamma^I_0 \), where \( \gamma^I_0 > 0 \) solves \( \Pi_I^M(\beta, \beta_0, \pi_I, \gamma^I_0) = \Pi_I^M(\beta, \beta_0, \pi_I, \gamma^I_0) \), such that: for all \( \gamma^I \leq \gamma^I_0 \) the upstream monopolist does not find it profitable to transmit to her rival the information on \( \beta \), while for all \( \gamma^I > \gamma^I_0 \) she finds it profitable to do so.

The economic intuition of Lemma 13 lies in the fact that the information transmission has two opposite effects on the upstream monopolist’s expected profits. On the one hand, it increases the expected downstream profits obtained by the upstream monopolist from selling her output in the downstream market. On the other hand, the information transmission reduces the expected access profits obtained from selling the essential input to her rival at the access price cap \( \pi_I \).

The key to understanding intuitively the effects on downstream expected profits is to realize that, whether or not the information is transmitted to the rival, the downstream expected profits of the upstream monopolist are equal

\[
E [q_M(\cdot)^2] = E [q_M(\cdot)]^2 + \text{var} [q_M(\cdot)]
\]

Due to linear demand, the information transmission does not affect the expected equilibrium quantity of the upstream monopolist, while it increases the variance of the equilibrium output. Indeed, if the rival firm learns that the cost of the upstream monopolist is higher (lower) than expected, it will produce more (less) in equilibrium. These strategic adjustments to the true realization of \( \beta \) increase the variability in the equilibrium output of the upstream monopolist which is beneficial to the firm.

On the contrary, the rival’s output variability caused by the information transmission reduces the expected access profits. This strictly depends on the access price cap breaking the link between access price and cost which makes the access revenues linear in \( \beta \). As a consequence, the information transmission does not affect the expected access revenues, while it raises the upstream monopolist’s expected costs of supplying the access.

The above analysis implies that the greater the fraction of access profits transferred to consumers (i.e., the greater \( \gamma^I \)), the more likely it is that the information transmission increases the upstream monopolist’s expected profits and that the upstream monopolist chooses to transmit information.

From Lemma 13 it follows that under ACI—where no fraction of access profits is shared with consumers \( (\gamma^I = 0) \)—the upstream monopolist will never
reveal to her rival the value of $\beta$. This justifies the implicit assumption made in the previous part of this paper where any transmission effect has been assumed away.

A straightforward consequence of Lemma 13 is stated in the following corollary where $EE^M_N(\beta, \beta_0, \bar{\pi}_I, \gamma')$ denotes the expected profits of an ignorant upstream monopolist under ACIps.

**Corollary 14** The incentives of the upstream monopolist to acquire information on $\beta$ under ACIps are no lower than under ACI, the upstream monopolist acquiring information for all $K \leq K_I(\bar{\pi}_I, \gamma')$, where i) $K_I(\bar{\pi}_I, \gamma') = K_I(\bar{\pi}_I)$ solves $EE^M_N(\beta, \beta_0, \bar{\pi}_I, \gamma') = K_I(\bar{\pi}_I)$ for all $\gamma' \in [0, \gamma'_0]$ and ii) $K_I(\bar{\pi}_I, \gamma') > K_I(\bar{\pi}_I)$ solves $EE^M_N(\beta, \beta_0, \bar{\pi}_I, \gamma') = K_I(\bar{\pi}_I, \gamma')$ for all $\gamma' \in (\gamma'_0, 1]$.

The economic intuition of Corollary 14 is as follows.

When the rival is ignorant, the access profits are the same whether or not the upstream monopolist is informed. In this case, the upstream monopolist’s value of information does not depend on the access profits and so it does not alter whatever the fraction of access profits transferred to consumers. Since from Lemma 13, when $\gamma' \leq \gamma'_0$, the transfer of access profits to consumers does not induce the upstream monopolist to transmit information to her rival, the firm’s value of information under ACI is equal to that under ACIps for all $\gamma' \in [0, \gamma'_0]$.

Instead, when $\gamma' > \gamma'_0$, the information transmission increases the upstream monopolist’s expected profits which, in turn, raises the firm’s gain from acquiring information as opposed to the case where the rival remains ignorant. It follows that the upstream monopolist’s value of information under ACIps is greater than under ACI, for all $\gamma' \in (\gamma'_0, 1]$.

### 4.3 Optimal access profit-sharing plans

This paragraph will derive the optimal fraction of access profits that should be transferred to consumers under separation and under integration, denoted by $\gamma^h$, $h = I, S$.

Under an access price cap mechanism with access profit-sharing, the expected welfare is given by the sum of the expected net consumer surplus and the fraction $\gamma^h$ of the expected access profits rebated to consumers.\(^{20}\)

The expected welfare under ACIps, denoted by $W^{ps}_S(\gamma, \bar{\pi}_S)$, is

$$W^{ps}_S(\gamma^S, \bar{\pi}_S) = \frac{Q_S(\bar{\pi}_S)^2}{2} + \gamma^S(\bar{\pi}_S - \beta_0)Q_S(\bar{\pi}_S)$$  \hspace{1cm} (15)

with $Q_S(\bar{\pi}_S)$ defined in (12). Since the expected welfare under ACIps increases in $\gamma^S$, the optimal fraction of access profits transferred to consumers, i.e. the value of $\gamma^S$ maximizing (15), is $\gamma^{S^*} = 1$.

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\(^{20}\)Because of linear demand, there is no income effect due to the increase in consumer income.
Under integration, however, the determination of the optimal fraction of access profits transferred to consumers needs to take into account not only the effect of the access profit-sharing on the consumer income, as under separation, but also its effects on the upstream monopolist’s decisions to acquire and transmit information which alter the equilibrium outputs. Indeed, in the light of Lemma 13 and Corollary (14), when \( \gamma^I \in [0, \gamma_{\text{opt}}^I] \) of access profits is transferred to consumers, for \( K \leq K(\pi_I) \), the upstream monopolist acquires information but does not find it profitable to transmit it to her rival, while, for \( K(\pi_I), K(\pi_I, \gamma^I) \), she prefers to remain ignorant. Instead, when \( \gamma^I \in (\gamma_{\text{opt}}^I, 1] \) of access profits is transferred to consumers, the information transmission is profitable to the upstream monopolist for all \( K \leq K(\pi_I, \gamma^I) \).

For \( K > K_I(\pi_I, \gamma^I) \) the upstream monopolist will never acquire information and thereby there is no information transmission. In the light of this, the expected welfare under ACFps, denoted by \( W^{ps}_I(K, \gamma^I, \pi_I) \), can be expressed as follows:

\[
W^{ps}_I(K, \gamma^I, \pi_I) = E \left[ F(\beta, \beta_0, K, \gamma^I, \pi_I) \right] \frac{E \left[ (\pi_I - \beta) f(\beta, \beta_0, K, \gamma^I, \pi_I) \right]}{2} + \gamma^I \left[ f(\beta, \beta_0, K, \gamma^I, \pi_I) \right] \]

(16)

where the functions \( F(\beta, \beta_0, K, \gamma^I, \pi_I) \) and \( f(\beta, \beta_0, K, \gamma^I, \pi_I) \) are so defined:

\[
F(\beta, \beta_0, K, \gamma^I, \pi_I) = Q_I(\beta, \beta_0, \pi_I) \text{ for } K \leq K_I(\pi_I) \]
\[
= Q_I(\beta, \pi_I) \text{ for } K > K_I(\pi_I, \gamma^I) \]

(17)

\[
Q_I(\beta, \pi_I) = \left\{ \begin{array}{ll}
Q_I(\beta_0, \pi_I) & \text{for } K \leq K_I(\pi_I, \gamma^I), \gamma^I \in [0, \gamma_{\text{opt}}^I] \\
Q_I(\beta_0, \pi_I) & \text{for } K > K_I(\pi_I, \gamma^I), \gamma^I \in (\gamma_{\text{opt}}^I, 1]
\end{array} \right.
\]

\[
f(\beta, \beta_0, K, \gamma^I, \pi_I) = q^R(\beta_0, \pi_I) \left\{ \begin{array}{ll}
q^R(\beta, \pi_I) & \text{for } K \leq K_I(\pi_I, \gamma^I), \gamma^I \in [0, \gamma_{\text{opt}}^I] \\
q^R(\beta, \pi_I) & \text{for } K > K_I(\pi_I, \gamma^I), \gamma^I \in (\gamma_{\text{opt}}^I, 1]
\end{array} \right.
\]

(18)

with \( Q_I(\beta, \beta_0, \pi_I) \) and \( q^R(\beta_0, \pi_I) \) defined in (8) for \( \pi_I = \pi_I; Q_I(\beta_0, \pi_I) \) and \( q^R(\beta, \pi_I) \) defined in (9) for \( \pi_I = \pi_I; Q_I(\beta, \pi_I) \) and \( q^R(\beta, \pi_I) \) in (6) for \( \alpha_I = \pi_I \).

The determination of the optimal fraction of access profits transferred to consumers, denoted by \( \gamma^I_* \), is carried out in two stages for \( K \leq K_I(\pi_I, \gamma^I) \).

First, maximizing \( W^{ps}_I(K, \gamma^I, \pi_I) \) w.r.t. \( \gamma^I \), separately for \( \gamma^I \in [0, \gamma_{\text{opt}}^I] \) — where there is no information transmission — and for \( \gamma^I \in (\gamma_{\text{opt}}^I, 1] \) — where the information transmission occurs — yields \( \gamma^I = \gamma_{\text{opt}}^I \) and \( \gamma^I = 1 \), respectively. This holds both for \( K \leq K_I(\pi_I) \) and for \( K(\pi_I, K(\pi_I, 1]) \).

Then, the value of \( \gamma^I_* \) is determined by solving

\[
\max_{\gamma^I \in [0, 1]} W^{ps}_I(K, \gamma^I, \pi_I)
\]

(18)

separately on the three ranges of \( K : K \leq K_I(\pi_I), K(\pi_I), K(\pi_I, 1]) \) and \( K > K(\pi_I, 1) \) where different effects are generated by the adoption of an access
profit-sharing plan according to the values of $\gamma^I$. This can be explained as follows.

When $\gamma^I = 1$, the regulatory mechanism induces the upstream monopolist to transmit information to her rival and this affects the expected welfare in two ways. On the one hand, the information transmission increases the variability in the rival’s equilibrium output and thereby reduces the expected access profits; this, in turn, reduces the expected consumer transfers with respect to the case in which the rival is ignorant. On the other hand, the information transmission affects the variability in the industry equilibrium output in the opposite way according to the values of $K$.

For $K \leq K(\bar{\pi}_I)$, where there is no information acquisition effect, the information transmission increases the variability in each firm’s equilibrium output, thereby causing a reduction in the variability of the industry equilibrium output. Indeed, as previously explained, when the rival is informed, its equilibrium output increases (decreases) if the upstream cost is higher (lower) than expected and this, in turn, reduces (increases) the upstream monopolist’s output compared to the case in which the rival is ignorant. Since the direct effect of the information transmission on the rival’s output is stronger than its counter-effect on the upstream monopolist’s output, the information transmission reduces the sensitivity of the industry equilibrium output to $\beta$. This effect further contributes to decrease the expected welfare.

Instead, for $K \in (K(\bar{\pi}_I), K(\bar{\pi}_I, 1)]$, the information transmission makes it profitable for the upstream monopolist to acquire information on $\beta$. The adjustments of the firms’ outputs to the true realization of $\beta$ raise the variability in the industry equilibrium output with respect to the case where no information acquisition and transmission occur, thereby increasing the expected net consumer surplus. However, this positive welfare effect is lower than the negative effect of information transmission on the expected consumer transfers.

It follows that the information transmission generates a welfare loss for all $K \leq K(\bar{\pi}_I, 1)$, even when it induces socially valuable information acquisition.

When a fraction of access profits equal to $\gamma^I_0$ is transferred to consumers, the welfare loss due to information transmission is eliminated, but the consumer transfers are lower with respect to the case of $\gamma^I = 1$, for the same level of expected access profits. Since the welfare loss due to information transmission is an increasing function of the variance of the cost parameter, the greater $\sigma^2$, the more likely it is that the optimal fraction of access profits transferred to consumers is equal to $\gamma^I_0$. Besides, the range of values of $\sigma^2$ where $\gamma^I = 1$ is

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21By using (6) and (8), the equilibrium outputs when the information on $\beta$ is transmitted to the rival can be written as

$q^M(\beta, \bar{\pi}_I) = q^M(\beta, \bar{\pi}_0, \bar{\pi}_I) + \frac{\beta_0 - \beta}{\beta - \beta_0}$

$q^R(\beta, \bar{\pi}_I) = q^R(\beta_0, \bar{\pi}_I) + \frac{\beta - \beta_0}{\beta_0 - \beta}$

where $q^M(\beta, \beta_0, \bar{\pi}_I)$ and $q^R(\beta_0, \bar{\pi}_I)$ are the equilibrium outputs under no information transmission. The above expressions clearly show that the direct effect of information transmission on the rival’s output—measured by $\frac{1}{\sigma}$—is greater than its counter-effect on the upstream monopolist’s output—measured in the absolute value by $\frac{1}{\sigma}$.
greater when \( K \epsilon (K(\bar{\pi}_I), K(\bar{\pi}_I, 1)) \) than when \( K \leq K(\bar{\pi}_I) \), because of the welfare gain due to the information acquisition effect generated by the information transmission.

For \( K > K(\bar{\pi}_I, 1) \), the upstream monopolist never acquires information and so there is no information transmission effect. In this case, the welfare maximization requires, as under separation, that all access profits be transferred to consumers, i.e. \( \gamma^I = 1 \).

The above analysis is summarized in the following Lemma where

\[
\Delta W_{I}^{ps}(K, \sigma^2) = W_{I}^{ps}(K, 1, \bar{\pi}_I) - W_{I}^{ps}(K, \gamma^I, \bar{\pi}_I)
\]
denotes the difference between the expected welfare functions under ACIps for \( \gamma^I = 1 \) and \( \gamma^I = \gamma^I_0 \), respectively.

**Lemma 15** Under ACIps there are two values of \( \sigma^2 \), denoted by \( \sigma_1^2 \) and \( \sigma_2^2 \), with \( \sigma_2^2 > \sigma_1^2 > 0 \), where \( \sigma_1^2 \) solves \( \Delta W_{I}^{ps}(K, \sigma_1^2) = 0 \) for all \( K \leq K(\bar{\pi}_I) \) and \( \sigma_2^2 \) solves \( \Delta W_{I}^{ps}(K, \sigma_2^2) = 0 \) for all \( K \epsilon (K(\bar{\pi}_I), K(\bar{\pi}_I, 1)) \), such that

\[
\begin{align*}
\gamma^I = & \; 1 \left\{ \begin{array}{ll}
\text{for all } K \leq K(\bar{\pi}_I) & \text{if } \sigma^2 \leq \sigma_1^2 \\
\text{for all } K > K(\bar{\pi}_I, 1) & \text{if } \sigma^2 \leq \sigma_2^2 
\end{array} \right. \\
\gamma^I = & \; \gamma^I_0 \left\{ \begin{array}{ll}
\text{for all } K \leq K(\bar{\pi}_I) & \text{if } \sigma^2 > \sigma_1^2 \\
\text{for all } K \epsilon (K(\bar{\pi}_I), K(\bar{\pi}_I, 1)) & \text{if } \sigma^2 > \sigma_2^2 
\end{array} \right. 
\end{align*}
\]

Eqs (19) can be explained as follows. For low values of \( \sigma^2 \), \( \sigma^2 \leq \sigma_1^2 \), the welfare loss due to information transmission, when \( \gamma^I = 1 \), is lower than the welfare loss due to the reduction in consumer transfers, when \( \gamma^I = \gamma^I_0 \) and so \( \gamma^I = 1 \) for all \( K \leq K(\bar{\pi}_I, 1) \). For \( \sigma^2 \in [\sigma_1^2, \sigma_2^2] \) the welfare loss due to information transmission increases so that \( \gamma^I = \gamma^I_0 \) for all \( K \leq K(\bar{\pi}_I) \), while \( \gamma^I = 1 \) for all \( K \epsilon (K(\bar{\pi}_I), K(\bar{\pi}_I, 1)) \) where the information transmission induces information acquisition. For very high cost uncertainty, \( \sigma^2 > \sigma_2^2 \), the information transmission is so welfare costly that \( \gamma^I = \gamma^I_0 \) for all \( K \epsilon (K(\bar{\pi}_I), K(\bar{\pi}_I, 1)) \), where for \( K \epsilon (K(\bar{\pi}_I), K(\bar{\pi}_I, 1)) \) the upstream monopolist prefers to remain ignorant.

### 4.4 Integration versus separation

The above analysis has shown that the adoption of an access profit-sharing plan may generate the additional effect, under integration with respect to separation, of inducing the upstream monopolist to transmit information to her rival regarding \( \beta \). This reduces the welfare gains arising from the adoption of a profit-sharing plan under integration as opposed to under separation for two reasons. On the one hand, the information transmission increases the variability in the rival’s equilibrium output and thereby has a negative effect on the expected welfare even when it induces socially valuable information acquisition. On the other hand, the only way the regulator can make the information transmission...
less profitable for the upstream monopolist is to reduce the fraction of access profits transferred to consumers as opposed to the case of separation.

This is summarized in the following proposition where

$$\Delta_{S}^{ps} = W_{S}^{ps}(\gamma^{S*}, \pi_{S}) - W_{S}^{ps}(0, \pi_{S})$$

and

$$\Delta_{I}^{ps}(K, \sigma^{2}) = W_{I}^{ps}(K, \gamma^{I*}, \pi_{I}) - W_{I}^{ps}(K, 0, \pi_{I})$$

with $\gamma^{S*} = 1$ and $\gamma^{I*}$ defined in Lemma 15, denote the welfare gains generated by the adoption of an access profit-sharing plan under $I$ and under $S$, respectively.

**Proposition 16** $\Delta_{S}^{ps} - \Delta_{I}^{ps}(K, \sigma^{2}) > 0$ for all $K$ with $\partial \Delta_{I}^{ps}(K, \sigma^{2})/\partial \sigma^{2} \leq 0$, i.e., the introduction of access profit-sharing into an access price cap mechanism generates a welfare bias in favour of separation for all $K$ and this bias does not decrease in $\sigma^{2}$.

The economic intuition of Proposition 16 is as follows. First, consider the range of $K$, $K \leq K(\pi_{I}, 1)$ where the upstream monopolist may acquire information under integration. In the light of Lemma 15, the transfer of all access profits to consumers, $\gamma^{I*} = \gamma^{S*} = 1$, gives a lower welfare gain under integration than under separation on account of the negative welfare effect of information transmission occurring under integration. Since the greater the variance of the cost parameter, the greater the welfare loss due to information transmission, the welfare bias versus separation, generated by the access profit sharing plan, increases in $\sigma^{2}$ for all $K \leq K(\pi_{I}, 1)$.

On the other hand, if a fraction of access profits equal to $\gamma^{I}_{0}$ is transferred to consumers under integration, the negative welfare effect of information transmission disappears, but the increase in consumer income under $I$ is lower than under $S$.

Only when the upstream monopolist never acquires information—as when $K > K(\pi_{I}, 1)$—the transfer of all access profits to consumers under integration could generate the same welfare effect as under separation if the access profits were equal. The statement of Proposition 16 stems from the fact that the level of access profits are always greater under separation than under integration.

## 5 Conclusions

In this paper I have investigated the desirability of allowing an upstream monopolist, regulated through an access price cap mechanism, to produce in the downstream market (integration) as opposed to excluding her (separation), in the presence of costly and socially valuable information on the upstream cost. I have shown that, when the upstream monopolist is regulated only through an access price cap, the information acquisition problem provides an argument in favour of vertical integration. This is more likely to occur in network industries
characterized by universal service obligation and complex technologies exhibiting some form of decreasing returns of scale. However, when the regulator also obliges the upstream monopolist to share her access profits with consumers, a bias emerges in favour of separation via the impact of the access-profit sharing plan on the upstream monopolist’s incentive to transmit information to her rival in the downstream market. This bias is more pronounced the higher the cost uncertainty characterizing the network industry.

The analysis carried out in this paper suggests that, in industries characterized by a highly dynamic technology, the desirability of integration is more likely to decrease the greater the redistributive concerns on the part of the regulator. Indeed, allowing the upstream monopolist to produce in the downstream market increases the likelihood of realizing the welfare gains from information acquisition, albeit at the cost of reducing income transfers from the firm to consumers compared with the case of separation.

A straightforward extension of this paper is to investigate the welfare desirability of integration when the acquisition of information concerns an uncertain demand in the downstream market.

Another line of research is to evaluate the performance of other forms of partial regulation when there is a problem of acquiring socially valuable information on uncertain demand and/or cost functions. One form of partial regulation is the global price cap (Laffont and Tirole, 1996) which regulates the upstream monopolist in both markets, but introduces flexibility in the price structure by applying a single price-cap index to both the end-user services and the essential input provided to downstream competitors. The supporters of this mechanism have shown that it can lead to efficient prices when the regulator can forecast the true quantities corresponding to the prices that will be set by the firm, and uses these quantities to weight the corresponding price in the cap. However, in the context of an unstable and unknown demand function more realistic weights are used, such as the past quantities produced by the firms. An interesting issue to investigate is how the upstream monopolist’s incentives to acquire information on uncertain demand and/or cost are affected by the type of variables chosen as weights in the global cap and by the tightness of the cap.

6 Appendix

Proof of Lemma 1. The profit function of the upstream monopolist who charges the monopoly access price is

$$\Pi_I^M(\beta) = \frac{(d - \beta)^2}{4}$$  \hspace{1cm} (20)

and when she charges \(\overline{A}_I\) is

$$\Pi_I^M(\beta, \beta_0, \overline{A}_I) = \left[ \frac{d + \overline{A}_I}{3} - \frac{\beta}{2} - \frac{\beta_0}{6} \right]^2 + (\overline{A}_I - \beta) \left( \frac{d - 2\overline{A}_I + \beta_0}{3} \right)$$  \hspace{1cm} (21a)
Taking the difference between (21a) and (20) and substituting for \( d = 2\bar{A}_I - \beta^m \), easy calculations yield

\[
\Pi^M_I(\beta, \beta_0, \bar{A}_I) - \Pi^M_I(\beta) = -\frac{5}{36} \beta^m(\bar{A}_I)^2 + \frac{\beta_0}{6} + \frac{\beta_0 \beta^m}{9} \left( \bar{A}_I - \beta(\beta - \beta^m(\bar{A}_I)) \right)
\]

Since \( \frac{\partial}{\partial \beta} [\Pi^M_I(\beta, \beta_0, \bar{A}_I) - \Pi^M_I(\beta)] < 0 \) and \( \Pi^M_I(\beta^m(\bar{A}_I), \beta_0, \bar{A}_I) - \Pi^M_I(\beta^m(\bar{A}_I)) = \frac{(\beta_0 - \beta^m(\bar{A}_I))^2}{\beta^m(\bar{A}_I)} > 0 \), it follows that \( \Pi^M_I(\beta, \beta_0, \bar{A}_I) - \Pi^M_I(\beta) > 0 \) for all \( \beta \in [\beta, \beta^m(\bar{A}_I)]. \) ■

Proof of Proposition 2 i) First show that the constraint \( (IC - IA) \) is satisfied. Since \( E\Pi^M_I(\beta, \beta_0, \bar{A}_I) = \Pi^M_I(\beta_0, \bar{A}_I) \), by using Taylor’s expansion

\[
E\Pi^M_I(\beta, \beta_0, \bar{A}_I) - \Pi^M_I(\beta_0, \bar{A}_I) = -\frac{1}{2} \frac{\partial Q_I(\beta, \beta_0, \bar{A}_I)}{\partial \beta} \sigma^2 = \frac{\sigma^2}{4}
\] (22)

Second, since \( \Pi^M_I(\beta_0, \bar{A}_I) \geq \Pi^M_I(\beta, \bar{A}_I) \), the constraint \( (IR - IA) \) is satisfied as \( (IR) \) and \( (IC - IA) \) hold.

Using the same procedure it is easy to show that the same result is obtained in the case of \( \beta_0 < \beta^m(\bar{A}_I) \) where the upstream monopolist’s profit function under information acquisition is \( \Pi^M_I(\beta) \) defined in (20) and \( \alpha^N = \alpha^N(\beta_0) \).

ii) Easy calculations give \( \frac{\partial E\Pi^M_I(\beta, \beta_0, \bar{A}_I)}{\partial \sigma} = \frac{\partial E\Pi^M_I(\beta, \beta_0, \bar{A}_I)}{\partial \bar{A}_I} = \frac{5d - 2\sigma^2}{9} \). ■

Proof of Corollary 4. Substituting for (8) in (5) gives the expected net consumer surplus under information acquisition

\[
\frac{1}{2} \int_{\beta}^{\bar{A}_I} Q_I(\beta, \beta_0, \bar{A}_I)^2 f(\beta) d\beta
\] (23)

while the expected consumer surplus under ignorance is

\[
\frac{Q_I(\beta_0, \bar{A}_I)^2}{2}
\] (24)

with \( Q_I(\beta_0, \bar{A}_I) \) defined in (9) at \( \alpha^N = \bar{A}_I \).

Taking the difference between eqs (23) and (24) and using Taylor’s expansion, gets

\[
\frac{1}{2} E \left[ \frac{2d - \bar{A}_I - \beta}{3} + \frac{\beta_0}{6} \right]^2 - \frac{1}{2} \left[ \frac{2d - \bar{A}_I - \beta_0}{3} \right]^2 = \frac{\sigma^2}{8}
\]

■

Proof of Proposition 5. First show that the constraint \( (IC - IA) \) is satisfied. Notice that \( \frac{\partial m^M_I(\beta)}{\partial \beta} = -Q_S(\beta) \) for all \( \beta \leq \beta^m(\bar{A}_S) \) and \( \frac{\partial m^M_I(\beta, \bar{A}_S)}{\partial \beta} = -Q_S(\bar{A}_S) \) for all \( \beta > \beta^m(\bar{A}_S) \). Integrating \( \frac{\partial m^M_I(\beta, \bar{A}_S)}{\partial \beta} \) yields

\[
\Pi^M_S(\beta^m(\bar{A}_S)) = \int_{\beta^m(\bar{A}_S)}^{\bar{A}_S} Q_S(\bar{A}_S) d\beta
\] (25)
since $\Pi^M_S(\bar{\pi}, \pi_S) = 0$ at $\pi_S = \bar{\pi}$

Integrating $\frac{\partial \Pi^M_S(\beta)}{\partial \beta}$ yields

$$\Pi^M_S(\beta) = \Pi^M_S(\beta^m(\pi_S)) + \int_{\beta}^{\beta^m(\pi_S)} Q_S(s)ds$$

and substituting for (25) gives

$$\Pi^M_S(\beta) = \int_{\beta}^{\beta^m(\pi_S)} Q_S(s)ds + \int_{\beta^m(\pi_S)}^{\bar{\beta}} Q_S(\bar{\pi}_S)d\beta$$

for all $\beta \leq \beta^m(\bar{\pi}_S)$ (26)

Applying the same procedure $\Pi^M_S(\beta, \pi_S)$ is obtained

$$\Pi^M_S(\beta, \pi_S) = \int_{\beta}^{\bar{\beta}} Q_S(\pi_S)ds$$

for all $\beta > \beta^m(\pi_S)$ (27)

Taking the expectation of (26) and (27) yields

$$E \Pi^M_S(\beta, \pi_S) = \int_{\beta}^{\beta^m(\bar{\pi}_S) \beta^m(\pi_S)} Q_S(s)ds F(\beta) +$$

$$\int_{\beta}^{\beta^m(\pi_S)} Q_S(\pi_S)d\beta F(\beta) +$$

$$\int_{\beta^m(\pi_S)}^{\bar{\beta}} Q_S(\pi_S)ds F(\beta)$$

Applying the same procedure

$$E \Pi^{MN}_S(\beta, \pi_S) = \int_{\beta}^{\bar{\beta}} Q_S(\pi_S)ds F(\beta)$$

Taking the difference between (28) and (29) and integrating by parts gives

$$E \Pi^M_S(\beta, \pi_S) - E \Pi^{MN}_S(\beta, \pi_S) = \int_{\beta}^{\beta^m(\pi_S)} [Q_S(\beta) - Q_S(\pi_S)] F(\beta)d\beta$$

which, by using (11) and (12) and substituting for $d = 2\pi_S - \beta^m$, becomes

$$E \Pi^M_S(\beta, \pi_S) - E \Pi^{MN}_S(\beta, \pi_S) = \int_{\beta}^{\beta^m(\pi_S)} \frac{\beta^m(.) - \beta}{3} F(\beta)d\beta > 0$$

(30)

Second, since $E \Pi^{MN}_S(\beta, \pi_S) > \Pi^M_S(\bar{\pi}, \pi_S)$, the constraint $(IR - IA)$ is satisfied as $(IR)$ and $(IC - IA)$ hold. Similar results are obtained when $a^N_S = a^N_S(\beta_0)$, since $\Pi^M_S(\beta, a_S(\beta)) > \Pi^M_S(\beta, a^N_S(\beta_0))$ for all $\beta \neq \beta_0$. ■

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**Proof of Proposition 6.** Let’s consider (30) evaluated at \( \bar{\pi}_S \), Differentiating it with respect to \( \bar{\pi}_S \) gives

\[
\frac{d}{d\pi_S} \left[ \Pi_S^M(\beta, \bar{\pi}_S) - \Pi_S^M(\beta, \bar{\pi}_S) \right] = \frac{2}{3} \int_{\beta}^\infty F(\beta) d\beta
\]  

(31)

By implicitly differentiating the constraint \((IC - IA)\), the result follows

\[
\frac{d\bar{\pi}_S(K)}{dK} = \frac{1}{\beta^m(\bar{\pi}_S)} > 0
\]  

(32)

\[\Box\]

**Proof of Corollary 7.** Notice that

\[
W_S(K) = \frac{1}{2} \int_{\beta}^\infty Q_S(\beta)^2 dF(\beta) + \frac{1}{2} \int_{\beta^m(\bar{\pi}_S)}^\infty Q_S(\bar{\pi}_S)^2 dF(\beta)
\]  

(33)

where \( \bar{\pi}_S = \pi_S \) and \( \beta^m(\bar{\pi}_S) = \beta^m(\bar{\pi}_S) \) for \( K < K(\pi_S) \); \( \bar{\pi}_S = \pi_S(K) \) and \( \beta^m(\bar{\pi}_S) = \beta^m(\pi_S(K)) \) for \( K > K(\pi_S) \). Therefore, \( dW_S(K)/dK = 0 \) for \( K < K(\pi_S) \) and \( dW_S(K)/dK = (dW_S(\beta^m(\pi_S), \pi_S)/d\pi_S)(d\pi_S(K)/dK) \) for \( K > K(\pi_S) \) where

\[
\frac{dW_S(\beta^m(\pi_S), \pi_S)}{d\pi_S} = \frac{-2}{3} Q_S(\pi_S)(1 - F(\beta^m(\pi_S))) < 0
\]

and \( d\pi_S(K)/dK \) is given in (32) \( \Box \)

**Proof of Lemma 8.** Let

\[
W_S^N = \frac{Q_S(\pi_S)^2}{2}
\]  

(34)

with \( Q_S(\pi_S) \) defined in (12) for \( \bar{\pi}_S = \pi_S \). Taking the difference between (33) and (34) leads

\[
W_S(K) - W_S^N = \int_{\beta}^\infty \left( \frac{Q_S(\beta)^2}{2} - \frac{Q_S(\pi_S)^2}{2} \right) dF(\beta) + \int_{\beta^m(\bar{\pi}_S)}^\infty \left( \frac{Q_S(\bar{\pi}_S)^2}{2} - \frac{Q_S(\pi_S)^2}{2} \right) dF(\beta)
\]

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where $\overline{\beta}_S = \overline{\pi}_S$ and
\[
W_S(K) - W_S^N = \int_\beta \left( \frac{d - \overline{\beta}}{9} \right) F(\beta) d\beta > 0 \quad \text{with} \quad d (W_S(K) - W_S^N)/dK = 0
\]
for all $K \leq K(\overline{\pi}_S)$; $\overline{\beta}_S = \overline{\pi}_S(K)$ and $d (W_S(K) - W_S^N)/dK < 0$ for $K > K(\overline{\pi}_S)$.

**Proof of Proposition 9.** The proof is made for the case $\overline{\pi}_I = \overline{\pi}_S$ and $\beta^m(\overline{\pi}_I) = \beta^m(\overline{\pi}_S)$. It holds also for $\overline{\pi}_I \leq \overline{\pi}_S$, being $dK_I/d\overline{\mu}_I = 0$. By using (30), it is easy to show that sufficient condition for $K_I > K_S(\overline{\pi}_S)$ is
\[
\beta^m(\overline{\pi}_I) = \int_\beta \left[ \Pi_I^M(\beta, \beta_0, \overline{\pi}_I) - \Pi_I^{MN}(\beta, \beta_0, \overline{\pi}_I) \right] dF(\beta) - \int_\beta \frac{\beta^m(\beta) - \beta}{3} F(\beta) d\beta > 0
\]

Integrating by part the first term yields
\[
\int_\beta \left[ \Pi_I^M(\beta^m, \beta_0, \overline{\pi}_I) - \Pi_I^{MN}(\beta^m, \beta_0, \overline{\pi}_I) \right] dF(\beta) = Q_I(\beta, \beta_0, \overline{\pi}_I) - Q_I(\beta_0, \overline{\pi}_I) F(\beta) d\beta.
\]

with $Q_I(\beta, \beta_0, \overline{\pi}_I)$ and $Q_I(\beta_0, \overline{\pi}_I)$ defined in (8 ) and (9), respectively, for $\overline{\pi}_I = a^N_I = \overline{\pi}_I$. Since the first term is positive, it is easy to show that the inequality in expression (35) is satisfied if
\[
\int_\beta \frac{3\beta_0 - \beta - 2\beta^m(\beta)}{6} F(\beta) d\beta > 0
\]
which is true being $\beta^m(\beta) \leq \beta_0$.

The result a fortiori holds for $\beta^m(\beta) \geq \beta_0$ where $a^N_h = a^m_h(\beta_0)$, $h=I, S$. Indeed, $K_I$ does not change, while $K_S(\overline{\pi}_S)$ decreases, being $\Pi_S^{MN}(\beta_0, a^m_S(\beta_0)) > \Pi_S^{MN}(\beta_0, \overline{\pi}_S)$ for $\overline{\pi}_S \neq a^m_S(\beta_0)$.

**Proof of Proposition 10.** From Corollary (7) and Lemma (8) $dW_S(K)/dK = 0$ for all $K \leq K_S(\overline{\pi}_S)$ and $dW_S(K)/dK < 0$ for all $K e (K_S(\overline{\pi}_S), K_S^N$ ). In the light of Corollary (4) and Proposition (9), the result follows.

**Proof of Lemma (12).** Taking the difference between (24) and (34) yields
\[
\frac{(2d_1 - \overline{\pi}_I - \beta_0)^2}{18} - \frac{(2d_2 - 2\overline{\pi}_S)^2}{18} > 0
\]
since $\overline{\pi}_I + \beta_0 < 2\overline{\mu}_S$. Similar result is obtained in the case of $\beta_0 < \beta^m(\beta)$ where $a^N_h = a^m_h(\beta_0)$, $h=I, S$. 

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Proof of Lemma 13. Substituting for (8) in (14) for \( \bar{A}_I = a_I = \overline{\pi}_I \), yields

\[
\Pi^M_I(\beta, \beta_0, \overline{\pi}_I, \gamma^I) = \left[ \frac{d + \overline{\pi}_I}{3} - \frac{\beta}{2} - \frac{\beta_0}{6} \right]^2 + (1 - \gamma^I) (\overline{\pi}_I - \beta) \left( \frac{d - 2\overline{\pi}_I + \beta_0}{3} \right)
\]

Then substituting for (6) in (14) for \( a_I = \overline{\pi}_I \), yields

\[
\Pi^M_I(\beta, \overline{\pi}_I, \gamma^I) = \left[ \frac{d + \overline{\pi}_I - 2\beta}{3} \right]^2 + (1 - \gamma^I) (\overline{\pi}_I - \beta) \left( \frac{d - 2\overline{\pi}_I + \beta}{3} \right)
\]

Taking the expectation of (37) and subtracting the expectation of (36), yields

\[
E \Pi^M_I(\beta, \overline{\pi}_I, \gamma^I) - E \Pi^M_I(\beta, \beta_0, \overline{\pi}_I, \gamma^I) = \frac{7}{36} \sigma^2 - \frac{(1 - \gamma^I)}{3} \sigma^2 = \left( - \frac{5}{36} + \frac{1}{3} \gamma^I \right) \sigma^2
\]

where i) \( \partial \left[ E \Pi^M_I(\beta, \overline{\pi}_I, \gamma^I) - E \Pi^M_I(\beta, \beta_0, \overline{\pi}_I, \gamma^I) \right]/\partial \gamma^I = -\frac{1}{3} \sigma^2 \) and

ii) \( E \Pi^M_I(\beta, \overline{\pi}_I, \gamma^I_0) - E \Pi^M_I(\beta, \beta_0, \overline{\pi}_I, \gamma^I_0) = 0 \) at \( \gamma^I_0 = 0, 42 \)

Proof of Corollary 14. i) Since

\[
E \Pi^M_I(\beta, \beta_0, \overline{\pi}_I, \gamma^I) = E \Pi^M_I(\beta, \beta_0, \overline{\pi}_I) - \gamma^I (\overline{\pi}_I - \beta_0) q^R(\beta_0, \overline{\pi}_I)
\]

and

\[
E \Pi^{MN}_I(\beta, \beta_0, \overline{\pi}_I, \gamma^I) = E \Pi^{MN}_I(\beta, \beta_0, \overline{\pi}_I) - \gamma^I (\overline{\pi}_I - \beta_0) q^R(\beta_0, \overline{\pi}_I)
\]

from proposition (2) it follows

\[
E \Pi^M_I(\beta, \beta_0, \overline{\pi}_I, \gamma^I) - E \Pi^{MN}_I(\beta, \beta_0, \overline{\pi}_I, \gamma^I) = \frac{\sigma^2}{4} \text{ for } \gamma^I \in [0, \gamma^I_0]
\]

ii) By adding (38) and (39) it is obtained

\[
E \Pi^M_I(\beta, \overline{\pi}_I, \gamma^I) - E \Pi^{MN}_I(\beta, \beta_0, \overline{\pi}_I, \gamma^I) = \frac{\sigma^2}{9} + \gamma^I \frac{\sigma^2}{3} > \frac{\sigma^2}{4} \text{ for } \gamma^I \in [\gamma^I_0, 1]
\]

Proof of Lemma 15. By using the definitions in (16) and (17) and denoting \( A(\beta_0) = (\overline{\pi}_I - \beta_0) q^R(\beta_0, \overline{\pi}_I) \), it is easy to show that

\[
\Delta W^P_I(K, \sigma^2) = \begin{cases} 
-0.47\sigma^2 + (1 - \gamma^I_0) A(\beta_0) & \text{for } K \leq K(\overline{\pi}_I) \\
-0.347\sigma^2 + (1 - \gamma^I_0) A(\beta_0) & \text{for } K \epsilon (K(\overline{\pi}_I), K(\overline{\pi}_I, 1)] \\
1 - \gamma^I_0 A(\beta_0) & \text{for } K > K(\overline{\pi}_I, 1) 
\end{cases}
\]

(40)
which leads to

\[
\Delta W_{I}^{ps}(K, \sigma^2) = \begin{cases} 
0 & \text{at } \sigma_1^2 > 0 \text{ for } K \leq K(\pi_I) \\
\text{for } K \epsilon (K(\pi_I), K(\pi_I, 1)) & \text{at } \sigma_2^2 > \sigma_1^2 \text{ for } K > K(\pi_I, 1) 
\end{cases}
\]

with \( \sigma_1^2 = \frac{1-\gamma_0}{\alpha_0}(\pi_I - \beta_0)q^R(\beta_0, \pi_I) \).

Since from (40) \( \Delta W_{I}^{ps}(K, \sigma^2) > 0 \), \( \partial \Delta W_{I}^{ps}(K, \sigma^2)/\partial \sigma^2 < 0 \), for all \( K \leq K(\pi_I, 1) \) and \( \partial \Delta W_{I}^{ps}(K, \sigma^2)/\partial \sigma^2 = 0 \) for all \( K > K(\pi_I, 1) \), the maximum value function of the expected welfare in problem (18) is

\[
W_{I}^{ps}(K, \pi_I) \begin{cases} 
\text{for } K \leq K(\pi_I) & \text{if } \sigma^2 \leq \sigma_1^2 \\
\text{for } K \epsilon (K(\pi_I), K(\pi_I, 1)) & \text{if } \sigma^2 \leq \sigma_2^2 \\
\text{for all } K > K(\pi_I, 1)
\end{cases}
\]

(41)

\[
W_{I}^{ps}(K, \gamma_0, \pi_I) \begin{cases} 
\text{for } K \leq K(\pi_I) & \text{if } \sigma^2 > \sigma_1^2 \\
\text{for } K \epsilon (K(\pi_I), K(\pi_I, 1)) & \text{if } \sigma^2 > \sigma_2^2 \\
\text{for all } K > K(\pi_I, 1)
\end{cases}
\]

\[\blacksquare\]

**Proof of Proposition 16.** Under \( S \), from (15)

\[\Delta S^{ps} = (\pi_S - \beta_0)Q_S(\pi_S) \text{ for all } K\]

Under \( I \), from eqs (41), using (16) and (17), it is easy to show that

\[
\Delta I^{ps}(K, \sigma^2) = \begin{cases} 
-0.47\sigma^2 + A(\beta_0) & \text{for } K \leq K(\pi_I), \sigma^2 \leq \sigma_1^2 \\
-0.347\sigma^2 + A(\beta_0) & \text{for } K \epsilon (K(\pi_I), K(\pi_I, 1)), \sigma^2 \leq \sigma_2^2 \\
\gamma_0^2A(\beta_0) & \text{for } K \leq K(\pi_I), \sigma^2 > \sigma_1^2 \\
A(\beta_0) & \text{for } K \epsilon (K(\pi_I), K(\pi_I, 1)), \sigma^2 > \sigma_2^2 \\
\end{cases}
\]

where \( \partial \Delta I^{ps}(K, \sigma^2)/\partial \sigma^2 \leq 0 \) for all \( K \leq K(\pi_I, 1) \) and \( d\Delta I^{ps}(K, \sigma^2)/d\sigma^2 = 0 \) for all \( K > K(\pi_I, 1) \).

The result of Proposition (16) stems from \( q^R(\beta_0, \pi_I) < Q_S(\pi_S) \) at \( \pi_I = \pi_S \). This ensures that \( \Delta S^{ps} - \Delta I^{ps}(K, \sigma^2) > 0 \) for all \( K > K(\pi_I, 1) \) and a fortiori \( \Delta S^{ps} - \Delta I^{ps}(K, \sigma^2) > 0 \) for all \( K \) and \( \sigma^2 \). \[\blacksquare\]

**References**


