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Colluding through Suppliers

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Abstract
In a dynamic game between N retailers and a large number of suppliers, I show that inefficient contracting emerges as a mechanism to implement collusion among retailers, building on the natural 'complementarity' between retail and wholesale prices. When efficient collusion is not sustainable, this complementarity allows retailers to rely on inefficient input supply, entailing double marginalization and negative franchise fees, to squeeze the wedge between collusive and deviation profits. I also study the role of communication on the equilibrium outcomes of games where retailers have the initiative. It turns out that communication is indeed fundamental to strengthen cartels' sustainability, although generating efficiency losses.

Keywords: Bertrand competition, double marginalization, collusion, competing hierarchies.

JEL classification: D21, D43, L42.

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Table of contents

1. Introduction

Games played through agents are pervasive in many interesting economic environments. A number of strategic interactions involve competing principals whose behavior is, explicitly or implicitly, influenced by the contractual relationships that they have previously established with independent agents. In recent years, a large and growing body of theoretical work on vertical structures has deeply influenced the way of thinking about competition in corporate finance, industrial organization, international trade and political economy.

In the IO literature manufacturer-retailer relationships have been widely studied examples of these instances. Existing agency models have underscored several important aspects of these games. For instance, by studying the link between pre-commitment effects and renegotiation (Caillaud et al., 1995, Fershtman et al., 1991, and Katz, 1991), the rationale behind alternative forms of vertical restraints (Blair and Lewis, 1994, Gal-Or, 1991a, Khun, 1997, Jullien and Rey, 2007, Martimort and Piccolo, 2007 and 2009, Rey and Stiglitz, 1995, and Rey and Tirole, 1986, among many others), or by emphasizing the anticompetitive nature of exclusive dealings (for instance, Bernheim and Whinston, 1986 and 1998, Gal-Or, 1991b, Martimort, 1996, and Rey et al., 2008).

But this body of work has mainly taken a static view, and has thus often overlooked the strategic aspects stemming from the intertemporal dimension of vertical contracting. For example, so far, the effects produced by different wholesale trading rules on the outcome of the repeated interaction between upstream and downstream firms have been poorly understood, few recent exceptions being Jullien and Rey (2007) and Nocke and White (2007). This theoretical gap raises a number of natural, yet unanswered issues, which are related to the old literature on the determinants of firms' boundaries, as well as to the more recent antitrust debate about the attitude that competition policy should take towards different and evolving forms of vertical arrangements. What is the link between collusion and vertical contracting in markets where the bargaining power is in the retailers' hands? How does the strategic design of wholesale contracts affect the dynamics of competition in these games? Can public contracting help downstream firms to enforce non-cooperative outcomes? In which markets this is likely to happen?

The present paper makes a step forward towards understanding these questions and, as I shall argue, it advances in several dimensions the growing literature studying collusion in dynamic games of repeated interaction between upstream and downstream firms. Two main objectives are at the core of the analysis. First, I wish to shed new light on the hidden determinants of vertical contracting, and on the role that a careful design of wholesale arrangements might play in softening competition in a dynamic framework where retailers have the initiative. In a nutshell, the main idea is that inefficient vertical contracting emerges as a mechanism to implement collusion among retailers, building on the natural 'complementarity' between retail and wholesale prices. When collusion between retailers is not sustainable with efficient wholesale deals, this complementarity makes it advantageous for retailers to rely on inefficient input supply in order to squeeze the wedge between collusive and deviation profits, whereby weakening the incentive to deviate from the implicit agreement. In addition, profitable, collusive market outcomes must be supported by wholesale contracts featuring negative franchise fees, that is, by payments made by manufacturers to retailers (e.g., listing fees and slotting allowances, see Rey et al., 2008).

My second main objective is to comment on the role played by communication between competing hierarchies on the set of collusive outcomes achievable in games where the initiative is in the retailers' hands. Communication is, indeed, fundamental to strengthen cartels' sustainability, although generating some efficiency losses. In my framework communication will be formalized by allowing retailers to publicly commit to wholesale contracts. More specifically, I study two simple communication regimes: one where retailers share information about their wholesale contracts (public contracting), and the other where bilateral negotiations between downstream and upstream firms are secret (private contracting). I show that collusion possibilities broaden under public contracting. Moreover, in these circumstances, the value of communication increases the less patient firms are and the larger the number of retailers in the downstream market is.

I consider an industry where, in each period, N retailers sell a homogenous good and compete by setting prices. The final good can be recovered from an intermediate input, which is supplied by $M \geq N$ competing upstream firms (suppliers). This interaction is repeated over an infinite horizon and, in each period, retailers dictate the wholesale trading rules (two-part tariffs) by making take-it or leave-it offers to suppliers. Therefore, in contrast to the pioneering papers on the subject (Jullien and Rey, 2007, and Nocke and White, 2007) which deal with collusion between upstream firms, I shall focus on collusion between retailers.

The analysis is initially conducted under the hypothesis that wholesale contracts are public. In this regime, downstream firms observe the wholesale contracts offered by rivals before taking their retail pricing decisions, whereby making it possible to tailor these decisions to the specific contingencies of the game contractual history. In this framework I show that, as long as wholesale contracts purposefully specify inefficient trading rules, retailers can make positive profits by charging prices higher than marginal costs (i.e., wholesale prices) even when the discount factor δ falls short of the critical value $(N-1)/N$.[1] While in the static game the unique symmetric equilibrium features no double marginalization (i.e., wholesale prices are equal to zero) and null franchise fees, retailers might prefer to pay positive wholesale prices for collusive purposes in the repeated game.

There are two relevant effects that the design of an inefficient wholesale contract generates on the self-enforceability condition needed to sustain collusion on the downstream market. First, excessively high wholesale prices introduce double marginalization, which stifles the difference between deviation and collusive profits. To understand this effect remember that when retailers face zero (or very low) marginal costs, by undercutting the monopoly price a deviant retailer grabs a spot gain close to the monopoly profit. This is no longer true when retailers are committed to pay large wholesale prices, undercutting would then secures lower profits to the deviant. In this perspective, my analysis underscores a trade-off between efficiency and collusion similar in spirit to the insights of Rotemberg and Saloner (1986).

Second, the optimal collusive strategy entails negative franchise fees which increase the continuation value of being in a cooperative phase. In fact, as long as downstream firms follow a penal code based on trigger strategies (i.e., infinite reversion to the repeated play of the static game), in the punishment phase retailers offer the efficient wholesale contract which requires no fixed fees. As a result, the gain that a deviant downstream firm can grab by undercutting its rivals must compensate for the stream of future profits generated by the per period franchise fees that upstream firms would pay in the cooperative phase.

Building on these insights, I characterize the optimal collusion strategy and show that the (efficient) monopoly profit is sustainable for large values of the discount factor, in this parameter region downstream firms charge the monopoly price, set a wholesale price equal to zero and share uniformly the final demand; for intermediate values of the discount factor, instead, collusion is still sustainable but it becomes inefficient, i.e., wholesale prices need to be positive and retail prices will generically not coincide with the monopoly level; obviously, for very impatient firms (discount factors close to zero) the unique equilibrium is perfect competition.

In the second part of the analysis I consider the case of private contracting. The objective is to show that limits on the retailers' communication possibilities might help tempering cartel formation incentives. I show that when retail pricing decisions cannot be contingent on rivals' wholesale contracts, preventing retailers from grabbing spot deviation gains becomes impossible below the critical discount factor $(N-1)/N$. The reason for this result is that,

[1] Below this threshold collusion would not arise in the standard repeated Bertrand game, that is, in the game where retailers do not need to rely on suppliers to produce the final good.

with private contracts, non-deviating members of the cartel cannot instantaneously react to deviations relying on a wholesale contract different than that specified by the implicit collusive agreement. This insight has a simple, but novel testable implication concerning the link between market concentration, firms' intertemporal preferences and the strategic role of communication. It turns out that the value of public contracting increases in very competitive environments or in circumstances where firms' discount rate is small.

Summarizing, the paper offers two novel insights to the literature on dynamic competing hierarchies. First, it emphasizes the coordination role that suppliers play in dynamic games where retailers have an incentive to behave cooperatively. The analysis robustly shows that there exists a mechanism which allows to sustain (inefficient) collusion even in the region of parameters where self-enforceability would not be met in the standard (repeated) Bertrand analysis. Second, on the policy ground, the paper provides a novel rationale for payments made by suppliers to retailers. Negative franchise fees are quite common in practice and are often treated as anticompetitive by antitrust legislations.[2] Yet, the economic rationale and the welfare impact of listing fees and more generally of slotting allowances remain two open research questions. While the existing literature has mainly focused on suppliers' incentive to offer such payment schemes, the evidence seems to show that the use of slotting allowances is correlated with the exercise of buyer power[3], a feature which is captured by my model.[4]

Few conclusive remarks are worthwhile about the modeling approach taken in the paper. First, the key result relies mainly on the hypothesis of Bertrand competition in the downstream market and public contracts. These assumptions can be motivated on the basis of the insights offered by the existing literature. In contrast to static models analyzing Cournot or differentiated Bertrand competition, in my framework the static game has a unique perfectly competitive equilibrium where, in spite of public contracts, retailers price at marginal costs and wholesale prices are set to zero. This feature shows quite clearly that the strategic value of public contracts cannot always be rationalized in static models, whereby motivating the dynamic analysis which is at the core of the paper. Second, one might argue that collusive outcomes could also be sustained by way of more complex vertical arrangements, e.g., 'three-part tariffs', resale price maintenance or, similarly, by wholesales contracts more explicit about the amount of inputs that each retailer commits to buy. Clearly, these mechanisms might improve firms' ability to sustain collusion; but, by their own nature, they would also

[2] The magnitude of such payments is considerable according to a study published by the FTC, 2003.

[3] See Rey et al. (2008) for a detailed discussion on this point.

[4] In a static framework Marx and Shaffer (2008) as well as Rey et al. (2008) show that slotting allowances might be anticompetitive as long as retailers have the initiative.

appear more suspicious to antitrust authorities relative to standard two-part tariffs (which are usually seen as pro-competitive). This is the reason why, following the previous literature, I will restrict my attention to the latter form of contracts.

The rest of the paper is organized in the following way: Section 2 below links my contribution to the existing literature. I introduce the model and develop the analysis with public contracting in Section 3. Section 4 characterizes optimal collusion with private contracting. I study the value of public contracting in Section 5. Section 6 concludes. All proves are provided in the Appendix.

2. Related literature

My analysis shares common features with several strands of the vertical contracting literature. First, given its dynamic perspective, my model is closely related to the literature studying the repeated interaction between upstream and downstream firms. Although through different mechanisms, my model shares with this growing literature the idea that public contracts help sustaining collusion. In a dynamic setting with long-lived suppliers and myopic retailers, Jullien and Rey (2007) show that vertical price fixing (RPM) helps upstream firms to sustain collusion. Retail prices that are centrally set by suppliers do not fully adjust to local variations on retail costs or demand. As a result, retail prices are more uniform under vertical price control and deviations from a tacit agreement between upstream firms are more easily detected. Nocke and White (2007) also study a model where upstream and downstream firms interact in a dynamic framework. They focus on the anticompetitive effects of vertical integration. My analysis, differs from the approach taken in these papers in one main respect: while I am interested in downstream collusion, they study the opposite case of upstream collusion.

Second, by emphasizing the role of payments made by manufacturers to retailers in enforcing downstream collusion, my analysis overlaps with the literature on 'buyer power'. Marx and Shaffer (2008) consider a vertical contracting model where downstream firms have the initiative and show that strong retailers can exclude competitors by offering "three-part tariffs" that include slotting allowances. In contrast to their model, where negative fees are a way of creating negative externalities (exclusion) between downstream firms, in my framework these instruments create positive externalities, they are in used as a tool to sustain cooperative behavior. Rey et al. (2008), instead, analyze the competitive effects of up-front payments in a contracting situation where rival retailers offer contracts to a single manufacturer. In contrast to Bernheim and Whinston (1986, 1998), they show that two-part tariffs do not suffice to implement the monopoly outcome in a static game, and argue that more complex arrangements, that combine slotting allowances and standard two-part tariffs, are necessary to internalize all the contractual externalities stemming from common agency. My model departs from Rey et al. (2008) in two main respects. First, I study a dynamic framework while they focus on a static game. Second, while I purposefully abstract from common agency issues, which would unnecessarily complicate the analysis, their results mainly rely on the externalities that these games feature and do not hold in a model with Bertrand competition in the downstream market, which is instead key to my analysis.

Finally, by linking collusion to specific forms of vertical contracting, my model shares some features with the strategic delegation literature. Stemming from the seminal contributions by Schelling (1956, 1960), over the years, many scholars have pushed forward the idea that credible delegation may have a strategic value in static games of competing agencies (Fershtman and Judd, 1986 and 1987, Sklivas, 1987, and Vickers, 1985). In somewhat general environments, Fershtman and Kalai (1997), Fershtman et al. (1991) and Katz (1991) show that one credible way to achieve commitments in two stage games is by letting delegates represent the players of a game. Fershtman and Kalai (1997) and Katz (1991) focus on unobservable contracts, and show that these can serve as pre-commitments as long as there does not exist a contract that solves the agency problem between principals and agents, and these players have non-congruent preferences. Fershtman et al. (1991), instead, take a public contracting perspective and argue that the players of a two stage game can use agents strategically to play on their behalf and the contract they sign with them. Their analysis is developed in a Cournot-type game, where every Pareto optimal outcome of the game can become the unique subgame perfect Nash equilibrium of the delegation game. On a similar note, Caillaud et el. (1995), show that public announcements of contracts may generate beneficial precommitment effects when competing principals deal with privately informed agents.

My analysis differs from all these papers in several respects. First, while they take a static view and usually postulate that agents take decisions on the behalf of principals, my focus is on repeated games where agents (suppliers) are passive. Second, while these papers do not provide predictions on the link between market structure, firms' time preferences and the value of public contracting, my analysis offers precise insights on this link. Finally, and perhaps more fundamentally, in my Bertrand framework delegation has no bite in the static game in spite of public contracts, whereby the need for focusing on intertemporal incentive constraint rather than on static ones.

3. The model

Players and environment: Consider a simple competitive environment where $N \geq 2$ independent and identical downstream firms (retailers), $R_1, R_2, .., R_N$, sell a homogenous good on a final (retail) market by setting prices. The market demand for the final good is denoted by $D(p)$, so that for any given vector of retail prices $\mathbf{p} = (p_1, .., p_N)$, each downstream firm R_i faces an individual demand given by:

$$D_i(\mathbf{p}) = \begin{cases} 0 & \text{if } p_i > p_j \text{ for some } j \neq i, \\ D(p_i) & \text{if } p_i < \min_{j \neq i} p_j, \\ \frac{D(p)}{n} & \text{if } n - 1 = \#\{j : p_j = p_i = p < p_{j'},\ j' \neq i,\ j' \neq j\}. \end{cases}$$

Retailers' production functions are linear and marginal costs are normalized to zero. Nevertheless, the final output must be recovered from an intermediate input which is supplied by a large number M of suppliers, $S_0, S_1, .., S_M$. As standard, I shall assume that the intermediate input is transformed into the final output according to a one-to-one technology. Moreover, as already said before, to make the point in the simplest possible way for my purposes, I assume that the upstream market is perfectly competitive, i.e., there are as many suppliers as retailers: $M \geq N$. For simplicity, production technologies are assumed to be linear and marginal costs are normalized to zero also in the upstream market.

Wholesale contracts: Since the input supply market is perfectly competitive, I assume that retailers have the initiative, meaning that they have full bargaining power in dictating the wholesale trading rules and make take-it or leave-it offers to suppliers. A wholesale contract between R_i and S_i is a two-part tariff, $C_i \equiv (T_i, w_i)$, specifying a wholesale price w_i for each unit of intermediate input ordered by R_i and a franchise fee T_i. Following Rey et al. (2008), I assume that the franchise fee can be contingent on actual trades, so that suppliers as well as retailers have the option of "opting out" from the vertical agreement as long as it yields negative profits to one of the contracting counterparts. This hypothesis implies that fixed fees can be waived in circumstances where input supply is not mutually profitable. Essentially, it avoids the free riding problem that would emerge when a retailer is undercut by its competitors.

Commitment rules and timing: I will consider two different regimes concerning contracts' observability:

- (*Public contracts*) In this regime wholesale offers are public: retailers as well as suppliers can observe the contracts issued by their competitors before taking their retail pricing decisions.

- (*Private contracts*) In this regime, wholesale contracts are instead secret: each retailer does not observe the wholesale contracts offered by its competitors.

Essentially, while with public contracts retailers can condition their retail pricing decisions on competitors' wholesale offers within each stage τ of the game, in case of private contracts this behavior is no longer feasible.

The existing literature on competing supply chains often assumes that wholesale contracts are public (Jullien and Rey, 2007, Nocke and White, 2007, and Rey and Stiglitz, 1995, among many others). This hypothesis implicitly relies on the widespread use of information sharing agreements and strategic alliances in retail markets. An important trend in product distribution is, in fact, the growth of information-intensive channels. These are usually characterized by channel partners who invest in bundles of sophisticated information technology like telecommunication and satellite linkages, bar coding and electronic scanning systems, database management systems etc. to not only disseminate information within a given organization but also among competing organizations (Stern et al. 1996).[5] These information sharing agreements may clearly facilitate communication about retail costs and thus on wholesale contracts.

I consider an infinitely repeated game with discrete time, $\tau = 0, .., +\infty$. The sequence of events within the stage game, thereafter denoted by $\mathcal{G}$, unfolds as follows:

T=1 (Contracting): Retailers offer contracts to suppliers on a pairwise basis.

T=2 (Acceptance): Suppliers simultaneously accept or refuse the received offers.

T=3 (Competition): Contractual rules become public information depending on the commitment regime and competition takes place: retailers set prices and the market clears, i.e., final demands materialize and input orders are placed. At this stage, opting out from a contract is possible as long as wholesale trades are not mutually profitable.

T=4 (Enforcement) Contractual obligations are finally executed.

[5] As noted by Niraj and Narasimhan (2004), major retailers such as Sainsbury and Marks & Spencers in U.K. as well as A&P grocery stores, Super Valu Stores and Von's supermarket in U.S. have made substantial investments in these technologies. Similarly, leading manufacturers such as Procter and Gamble have responded to the availability of greater information by developing tracking and information systems at the retail store level.

Each firm's objective is to maximize the discounted sum of its future profits, discounting future at the common rate $\delta \in (0,1)$.

Histories and information: After each stage τ, the public information h^τ depends on the contracts' observability regime. When wholesale contracts are public, at the end of stage τ all firms observe the same public history $h^\tau \equiv (\mathbf{p}^\tau, \mathbf{C}^\tau)$, which contains the sequence of prices offered up to that stage by all retailers $\mathbf{p}^\tau \equiv (\mathbf{p}_1^\tau, .., \mathbf{p}_N^\tau)$, with $\mathbf{p}_i^\tau \equiv (p_i^0, .., p_i^\tau)$, along with the sequence of contracts $\mathbf{C}^\tau \equiv (\mathbf{C}_1^\tau, ..., \mathbf{C}_N^\tau)$ offered by each retailer i up to τ, with $\mathbf{C}_i^\tau \equiv (C_i^0, .., C_i^\tau)$. Hence, in this regime, the game is one of perfect monitoring: All past actions become common knowledge at the end of each play. With private contracting, instead, at stage τ each retailer R_i only observes past retail prices but not contractual histories, that is, R_i's information set $h_i^\tau \equiv (\mathbf{p}^\tau, \mathbf{C}_i^\tau)$ contains the public history $\mathbf{p}^\tau$ (past retail pricing decisions) and its own contractual history $\mathbf{C}_i^\tau$.

Collusion: I look for symmetric and stationary pure strategy equilibria such that, whenever it is possible, retailers seek to collude. I assume that the implicit agreement between downstream firms (cartel) is organized by a benevolent third party whose objective is to implement the retail price and the contractual strategy that maximize the (downstream) industry profits subject to the relevant self-enforceability constraints and the suppliers' participation constraints. A collusive symmetric and stationary strategy, thereafter denoted by $\hat{\sigma}$, then requires all downstream firms to issue the contract $C^c \equiv (T^c, w^c)$ and set the retail price p^c in the collusive phase, and a contract $C^p \equiv (T^p, w^p)$ and a retail price p^p in the punishment phase.

I shall focus on punishment codes requiring infinite Nash reversion[6]: that is, following any deviation by one retailer, all its rivals will price at marginal costs and offer the competitive contract $C^* \equiv (0,0)$ for the rest of the game. There are two types of deviations that a retailer may envision: First, it may stick to the collusive contractual rule C^c and deviate only by undercutting its rivals. Following the notation introduced by Athey et al. (2004), I will refer to this behavior as to 'on-schedule deviation'. Second, a retailer may deviate 'off-schedule', that is, it may deviate by changing both its wholesale contract and retail price.

Clearly, off-schedule deviations can be punished in a different manner depending on the contracts' observability regime. With public contracting, this type of behavior is instantaneously detected and thus punished. Consistently with the hypothesis of Nash reversion trigger strategies, I shall assume that if in period τ retailer R_i offers a contract different

[6] As it will be clear in the next sections, using more complex penal codes can only reinforce my conclusions.

than C^c, its rivals will price at marginal costs from that period on, and in the subsequent stages they offer the competitive and efficient contract C^*. Formally, this amounts to assume that as long as $C_i^\tau \neq C^c$ for some i and τ, all other retailers follow the rule: $p_\tau^p = w^c$ and $p_{\tau'}^p = 0$ for $\tau' > \tau$. With private contracting this type of deviation can be detected and thus punished only with one period lag: the difference between on- and off-schedule deviations becomes immaterial. As I will argue, this feature is key for understanding the differences between public and private contracting in my framework.

Technical and simplifying assumptions: The analysis will be developed under the following unrestrictive assumptions:

A1 The demand $D(p)$ is a strictly decreasing and twice continuously differentiable function satisfying the standard Inada conditions: (i) $D(0) > 0$, and (ii) there exists an upper-bound $(\overline{p})$ on the retail price such that $D(p) = 0$ for all $p \geq \overline{p}$.

A2 The function $D(p)p$ is single peaked and thus features a unique internal maximum p^m, which is identified by the first-order necessary and sufficient condition:

$$D'(p^m)\, p^m + D(p^m) = 0.$$

Both these assumptions are standard and are imposed only for expositional purposes.

A3 Whenever indifferent between accepting a wholesale contract and opting out, suppliers prefer to accept the contract and secure input supply.

This hypothesis simplifies the analysis insofar as it allows to restrict attention to the class of equilibria with positive sales. The equilibrium concept that I shall use in solving the repeated game is Subgame Perfect Nash Equilibrium (SPNE) for the regime of public contracts, and Perfect Bayesian Equilibrium (PBE) for that with private contracts.

3.1 Equilibrium analysis: public contracting

In this section I provide the equilibrium characterization. For the sake of clarity I will first briefly analyze the static game and then move to the repeated game. The objective of the analysis is to unveil the strategic role that the design of inefficient wholesale contracts has on downstream collusion.

3.1.2 The static game

In the static game perfect competition between identical retailers leads to the following very intuitive result:

Lemma 1 *In the unique SPNE of the stage game retailers and suppliers make zero profits. All retailers set retail prices equal to marginal costs and offer the competitive and efficient contract* $C^* \equiv (0, 0)$*, that is, wholesale prices and franchise fees are both equal to zero.*

The intuition for this result is simple. First, a standard undercutting argument implies that competing retailers price at marginal costs in the unique SPNE of the static game. Second, since retailers have full bargaining power and offering contracts which entail positive wholesale prices limit their ability to undercut competitors, the equilibrium must feature wholesale contracts that require minimum franchise fees and wholesale prices, that is, $w^* = T^* = 0$. As already mentioned before, the static outcome will turn out to be very useful in describing the punishment code that retailers follow in the repeated game.

3.1.3 Repeated interaction

I now suppose that the stage game analyzed above is infinitely repeated. The objective is to identify the conditions under which the equilibrium features collusion, and to characterize the properties of the implicit agreement that supports cooperative outcomes in the downstream market.

Two important remarks are worthwhile before describing how collusion is formalized in my model. First, in order to simplify the equilibrium analysis, it is convenient to show that, in the framework under consideration, there is no loss of generality in restricting attention to equilibria featuring exclusive dealings, that is, to outcomes of the contracting game where each retailer deals only with one supplier. The next proposition makes this point clear:

Proposition 1 *In solving the repeated game, there is no loss of generality in restricting attention to equilibria with exclusive dealings, that is, where retailers and suppliers trade on a pairwise basis.*

This result can be easily explained. Since in the economy there are as many suppliers as retailers, that is, $M \geq N$, the input supply market is perfectly competitive. As a result, retailers can extract the whole surplus from their suppliers: in equilibrium these latter break-even. Moreover, since I have assumed that contractual rules are executed only after final prices have been announced and individual demands have materialized, in equilibrium this break-even condition holds no matter how downstream and upstream firms are matched. Suppliers are thus indifferent between dealing with one or multiple retailers.

Second, it is also important to observe that as long as downstream firms symmetrically offer the competitive and efficient contract C^*, the unique equilibrium of the continuation game features collusion supported by Nash reversion trigger strategies if and only if the discount factor δ is large enough:

Lemma 2 ***(Friedman, 1971)*** *Suppose that retailers are forced to offer the competitive and efficient contract C^* in each stage τ, then every retail price level between monopoly (p^m) and perfect competition (0) can be supported by Nash reversion trigger strategies if and only if $\delta \geq (N-1)/N$. Otherwise, for $\delta < (N-1)/N$, there exists a unique SPNE featuring perfect (efficient) competition: Retail prices are equal to marginal costs ($p^* = 0$).*

Although this lemma restates in a slightly more complex environment the textbook version of Folk Theorem for the Bertrand game with perfect monitoring, it will help to understand the novel insights that I provide for the more interesting case where retailers can choose to deal with suppliers according to inefficient trading rules. I shall argue that equilibrium outcomes featuring positive retail prices can emerge also in the region of parameters where the standard version of the Folk Theorem does not hold, that is, for $\delta < (N-1)/N$.

To gain insights about the mechanism which makes this new collusion opportunities viable, it is worthwhile discussing the incentive constraints that an implicit agreement between downstream firms must satisfy. As already discussed before, I will focus on a very simple and intuitive form of implicit collusive agreements: Retailers play symmetrically in each period and equally share the stakes of collusion.

Some additional notation will turn helpful at this point. Let H^τ be the set of possible public histories at stage τ, and denote by $\hat{\sigma} \equiv (\hat{\sigma}^\tau(h^\tau))_{\tau=1}^{+\infty}$, with $\hat{\sigma}^\tau : H^\tau \to \mathbb{R}^3$, the collusive strategy which specifies for each history h^τ at stage τ a retail price $\hat{p}^\tau$ and wholesale contract $\hat{C}^\tau$. Denote by $s(h^\tau) \in \{0,1\}$ the dicothomic state variable taking value 1 if at stage τ the game is in a cooperative phase, i.e., all retailers have obeyed to the implicit agreement up to τ, and 0 if a deviation has occurred at $\tau - 1$. Moreover, in the subset of histories such that a deviation has occurred at $\tau - 1$ (i.e., those for which the state variable $s(h^\tau)$ takes value 0) denote by $z(h^\tau)$ the dicothomic state variable taking value 1 if the deviation was on-schedule and 0 if, instead, it was off-schedule. A stationary and symmetric collusion strategy enforcing the non-competitive retail price p^c and the wholesale contract C^c on the cooperative phase can be thus formally described as follows:

$$\forall\ \tau \text{ and } h^\tau \in H^\tau:\ \hat{\sigma}^\tau\left(h^\tau\right) \equiv \begin{cases} \left(p^c, C^c\right) & \text{if } s\left(h^\tau\right) = 1, \\ \left(w^c, C^c\right) & \text{if } z\left(h^\tau\right) = 1, \\ \left(0, C^*\right) & \text{if } z\left(h^\tau\right) = 0. \end{cases}$$

Self-enforceability requires that all players must find it profitable to follow the rules dictated by $\hat{\sigma}$, i.e., downstream and upstream firms ought not have profitable deviations. I start by characterizing the set of incentive compatibility constraints that the pair (p^c, C^c) must meet. Obviously, suppliers must make non-negative profits when accepting C^c, as otherwise they would refuse to supply the intermediate input, that is:

$$\frac{1}{N} D\left(p^c\right) w^c + T^c \geq 0. \tag{1}$$

This participation constraint simply implies that, in the collusive phase, the sum of the franchise fees and each supplier's sales revenue is non-negative. As for downstream firms, instead, two types of deviations must be considered. First, each retailer must not find it worthwhile to issue the collusive wholesale contract C^c, and then set the retail price slightly below the collusive level p^c, whereby stealing the whole market. Given the strategy $\hat{\sigma}$, this type of behavior cannot be profitable as long as the intertemporal gain from collusion exceeds the spot gain that a retailer can make by secretly cutting its retail price:

$$\frac{\frac{1}{N} D\left(p^c\right)\left(p^c - w^c\right) - T^c}{1-\delta} \geq D\left(p^c\right)\left(p^c - w^c\right) - T^c. \tag{2}$$

Second, retailers must also not find it worthwhile to deviate 'off-schedule': They must not gain by offering both a contract and a retail price different than those specified by $\hat{\sigma}$. Once again, given the punishment code described above, this type of deviation is not profitable as long as the following self-enforceability constraint is met:

$$\frac{\frac{1}{N} D\left(p^c\right)\left(p^c - w^c\right) - T^c}{1-\delta} \geq \max_{(w_i, T_i)\in\mathbb{R}^2} \left\{D\left(w^c\right)\left(w^c - w_i\right) - T_i :\ D\left(w^c\right) w_i + T_i \geq 0\right\}, \tag{3}$$

where it is easy to verify that the maximal deviation profit (right-hand side in the above equation) is obtained when the efficient contract C^* is issued:

$$\max_{C_i \in \mathbb{R}^2} \{D(w^c)(w^c - w_i) - T_i : D(w^c) w_i + T_i \geq 0\} \equiv D(w^c) w^c.$$

The incentive constraint (3) thus rewrites as:

$$\frac{\frac{1}{N} D(p^c)(p^c - w^c) - T^c}{1 - \delta} \geq D(w^c) w^c. \tag{4}$$

In this case, the intertemporal profit that each retailer earns on the cooperative path must exceed the profit that it would make by cutting the retail price down to $w^c - \varepsilon$, i.e., slightly below the marginal cost of its competitors, and choosing the competitive and efficient wholesale contract C^*. The next definition formally introduces the notion of an optimal symmetric collusive strategy with public contracts.

Definition 2 *Suppose that wholesale contracts are public, then an optimal symmetric and stationary collusive strategy $\hat{\sigma}$ maximizes retailers' joint profits subject to the self-enforceability and participation constraints described above:*

$$\mathcal{P} : \begin{cases} \max_{(p,(w,T)) \in \mathbb{R}^3} D(p)(p - w) - NT, \\ s.t.\ (1),\ (2)\ and\ (4). \end{cases}$$

The best symmetric and stationary collusive strategy has to maximize the (downstream) industry profits provided that no player can profitably deviate. The cartel's payoff is defined as total industry profits $D(p)(p - w)$ net of the total franchise fees NT. Obviously, in the region of parameters where positive profits cannot be sustained, the optimal collusive strategy must require retail prices equal to marginal costs (perfect competition) and efficient wholesale contracting. The next lemma shows the simple and intuitive result that whenever the collusive strategy $\hat{\sigma}$ can support the (efficient) monopoly profit (each downstream firm gets $\pi_i^C = D(p^m) p^m / N$) the optimal wholesale contract has to be the efficient one:

Lemma 3 *As long as the collusive strategy $\hat{\sigma}$ requires retailers to charge a (strictly) positive retail price and to issue the efficient and competitive contract C^* in the cooperative phase, collusion is efficient and the monopoly price p^m can be sustained in equilibrium.*

To gain insight about this result it is useful to recall that when all retailers issue the competitive and efficient contract C^* in the cooperative phase, the self-enforceability constraints

(1) and (4) rewrite as:

$$\frac{\frac{1}{N}D\left(p^c\right)p^c}{1-\delta}\geq\max\left\{0,D\left(p^c\right)p^c\right\}.$$

Intuitively, under efficient wholesale contracting, downstream firms can share the monopoly profits as long as collusion is viable in the simple repeated Bertrand analysis where each retailer owns the upstream technology and can thus produce the intermediate input at no cost. As already discussed above, this result can hold only if retailers are patient enough (i.e., the discount factor exceeds the standard threshold $(N-1)/N$). Otherwise, when retailers are not sufficiently patient to sustain monopoly profits, the analysis becomes more complex. A natural question is then whether there exists a symmetric collusive strategy, still securing positive profits to downstream firms, which is supported by positive wholesale prices and negative franchise fees. Clearly, the answer depends on the interplay between the incentive constraints (1) and (4). Simple algebraic manipulations of these equations lead to the following more intuitive and compact formula for incentive compatibility:

$$\frac{D\left(p^c\right)\left(p^c-w^c\right)}{N}\geq\max\left\{\left(1-\delta\right)D\left(p^c\right)\left(p^c-w^c\right)+\delta T^c,\left(1-\delta\right)D\left(w^c\right)w^c+T^c\right\}.$$

There are two relevant effects that the design of an inefficient wholesale contract generates on the above self-enforceability condition. First, excessively high wholesale prices introduce double marginalization, which stifles the difference between deviation and collusive profits. To understand this effect remember that when retailers face zero (or very low) marginal costs, by undercutting the monopoly price a deviant retailer grabs a spot gain close to the monopoly profit. This is no longer true when retailers are committed to pay large wholesale prices, in such a case undercutting secures lower profits to the deviant.

Second, negative franchise fees increase the continuation value of being in a cooperative phase. In fact, as long as downstream firms follow a penal code based on trigger strategies, they offer the efficient wholesale contract in the punishment phase. As a result, the gain that a deviant firm can grab by undercutting its rivals must compensate for the stream of future profits generated by the per-period franchise fees that upstream firms would pay to downstream firms on the cooperative phase. Summarizing:

Lemma 4 *As long as an equilibrium features inefficient collusion, the contract C^c must require positive wholesale prices ($w^c>0$) and negative franchise fees ($T^c<0$).*

Building on this simple and intuitive result, the next theorem identifies the conditions that the optimal implicit collusive agreement between downstream firms must satisfy. Its objective is to identify a range of discount factors lying below the critical value $(N-1)/N$ where a collusive outcome can still emerge in equilibrium.

Theorem 3 *For $\delta \geq (N-1)/N$ collusion is efficient, retailers share the monopoly profit $\pi^m = D(p^m)\, p^m$ and offer the competitive and efficient contract C^*. Otherwise, there exists a lower bound $\underline{\delta}(N) > 0$, such that for $\underline{\delta}(N) < \delta < (N-1)/N$ collusion is inefficient but still viable: retail and wholesale prices are strictly positive, $\widetilde{p}(\delta, N) > 0$ and $\widetilde{w}(\delta, N) > 0$ respectively, and are chosen so as to solve the following system of equations:*

$$\frac{\widetilde{p}(.) - \widetilde{w}(.)}{\widetilde{p}(.)} = \frac{1}{N(1-\delta)} - \frac{\widetilde{w}(.)}{\widetilde{p}(.)\, N}, \tag{5}$$

$$\frac{\widetilde{p}(.) - \widetilde{w}(.)}{\widetilde{p}(.)} = \frac{D(\widetilde{w}(.))\, \widetilde{w}(.)}{D(\widetilde{p}(.))\, \widetilde{p}(.)} - \frac{\widetilde{w}(.)}{\widetilde{p}(.)\, N}. \tag{6}$$

The lower-bound $\underline{\delta}(N)$ is identified by the zero retail price condition $\widetilde{p}(\delta, N) = 0$.

This result rests on the following simple idea: If retailers are patient enough, collusion is efficient and self enforceable, exactly as in the infinitely repeated Bertrand game where retailers own the upstream technology and can thus produce the intermediate input at no costs. However, while for $\delta < (N-1)/N$ a model without suppliers leads to the conclusion that any implicit collusive agreement would be broken by the temptation of grabbing too high deviation gains, so that only the competitive outcome is compatible with the notion of SPNE. In contrast, with vertical relationships, this is no longer the case. In a nutshell, when downstream firms rely on input supply by upstream firms, a careful design of wholesale contracts can still make cooperation viable in this parameter region. The implicit cost of collusion, though, materializes in vertical contracts which feature double marginalization: although these deals are still fair in the sense that suppliers break even in the collusive phase, positive wholesale prices and negative franchise fees stifle the difference between deviation and collusion profits, so as to make the stakes of undercutting less attractive.

The system of equations (5)-(6) formalizes the simple idea that equilibrium retail prices must be not too high to make deviations profitable and that wholesale prices have to be positive in order to make negative franchise fees compatible with the suppliers' zero profit condition. The next corollary describes the optimal collusive strategy as a function of the discount factor δ.

Corollary 1 *The optimal collusive strategy specifies a retail price $p^c(\delta, N)$ and a wholesale* *price* $w^c(\delta, N)$ *having the following features:*

$$p^c(\delta, N) = \begin{cases} 0 & \textit{for } \delta \leq \underline{\delta}(N), \\ \widetilde{p}(\delta, N) & \textit{for } \delta \in (\underline{\delta}(N), (N-1)/N), \\ p^m & \textit{for } \delta \geq (N-1)/N. \end{cases}$$

and

$$w^c(\delta, N) = \begin{cases} 0 & \textit{for } \delta \leq \underline{\delta}(N), \\ \widetilde{w}(\delta, N) & \textit{for } \delta \in (\underline{\delta}(N), (N-1)/N), \\ 0 & \textit{for } \delta \geq (N-1)/N. \end{cases}$$

Figure 1 below provides a graphical illustration of the retail and wholesale prices implemented by the collusive strategy characterized above. Retailers price at marginal costs in the competitive region ($\delta < \underline{\delta}(N)$) where wholesale prices are equal to zero: in this region there is no way of preventing undercutting since firms are myopic. For intermediate values of the discount rate ($\delta \in (\underline{\delta}(N), (N-1)/N)$), instead, the optimal implicit agreement requires strictly positive retail and wholesale prices. As already discussed above, here double marginalization kicks in to temper deviation incentives. Finally, when retailers are very patient ($\delta > (N-1)/N$), the monopoly price is sustainable with the efficient contract, in this region there is no reason to introduce double marginalization to sustain positive profits.

[Insert Figure 1 about here]

To gain insight about the effect that firms' intertemporal preferences and the degree of concentration in the downstream market have on retailers' market power, it is interesting to study how the mark-up $m^c(.) = (p^c(.) - w^c(.))/w^c(.)$ changes with respect to the underlying parameters of the model (δ and N) in the most interesting region of parameters where collusion is viable but inefficient, that is, for $\underline{\delta}(N) < \delta < (N-1)/N$. Indeed, while in the case where collusion is efficient the mark-up is mainly shaped by the characteristics of demand through the first-order condition in **A2**, in the inefficient collusion region the mark-up will also depend on the characteristics of the supply side through the self-enforceability conditions.

Proposition 4 *In the region of parameters where* $\underline{\delta}(N) < \delta < (N-1)/N$*, the mark-up* $m^c(\delta, N)$ *is increasing in* δ *and decreasing in* N*.*

Industries with fewer and more patient firms feature higher price-cost margins. The economic intuition for this result is simple: As δ increases, retailers become more patient and weight less spot deviation gains. As a consequence, there is less need to distort upward wholesale prices to sustain positive price-cost margins on the downstream market. By the same token, when the stakes of collusion are shared by fewer downstream firms, higher price-cost margins can be sustained in equilibrium.

The linear example: I now construct a simple example putting Theorem 3 and Proposition 4 at work. Consider the linear demand function $D(p) = \max\{0, 1-p\}$, such that $p^m = 1/2$ and $\overline{p} = 1$. Moreover, assume that there are only two firms active on the downstream market, i.e., $N = 2$. It is easy to check that in this simple environment the solution of the system (5)-(6) yields:

$$\widetilde{p}(\delta, 2) = \frac{(1-\delta)(1-4\delta)}{1+8\delta^2-7\delta},$$

and

$$\widetilde{w}(\delta, 2) = \frac{(1-2\delta)(1-4\delta)}{1+8\delta^2-7\delta}.$$

where $\widetilde{p}(\delta, 2) = \widetilde{w}(\delta, 2) = 0$ for $\underline{\delta}(2) = 1/4$. So, in the range $\delta \in (0.25, 0.5)$ both $\widetilde{p}(\delta, 2)$ and $\widetilde{w}(\delta, 2)$ are positive and the mark-up is increasing with respect to δ:

$$m(\delta, 2) = \frac{\widetilde{p}(\delta, 2) - \widetilde{w}(\delta, 2)}{\widetilde{p}(\delta, 2)} = \frac{\delta}{1-\delta}.$$

Moreover, as shown in Figure 2 below, the retail price is also increasing in δ: more patient players can enforce implicit agreement sustaining larger retail prices. The wholesale price is instead concave in δ: the more patient retailers are, the lower is the marginal impact that higher wholesale prices have on the self-enforceability conditions.

[Insert Figure 2 about here]

Another interesting exercise to perform in this linear framework is to illustrate the impact of an increase in the number of active retailers on the optimal collusion strategy. This can be readily seen by solving the system (5)-(6) for $N = 3, 4...$ When $N = 3$, for instance, we have:

$$\widetilde{p}(\delta, 3) = \frac{2(1-\delta)(4-9\delta)}{8+27\delta^2-32\delta},$$

and

$$\widetilde{w}(\delta, 3) = \frac{(2-3\delta)(4-9\delta)}{8+27\delta^2-32\delta},$$

where $\widetilde{p}(\delta, 3) = \widetilde{w}(\delta, 3) = 0$ for $\underline{\delta}(3) = 4/9$.

It is immediate to show that the overall region of parameters where collusion can be sustained shrinks when moving from a downstream industry with 2 retailers to one with 3 competitors, that is, $\underline{\delta}(3) > \underline{\delta}(2)$, and that the mark-up diminishes with the number of firms:

$$m(\delta, 3) = \frac{\widetilde{p}(\delta, 3) - \widetilde{w}(\delta, 3)}{\widetilde{p}(\delta, 3)} = \frac{\delta}{2(1-\delta)} < m(\delta, 2).$$

Once again, the wholesale price in concave in δ and the retail price is increasing in δ. Moreover, Figure 3 below illustrates how the optimal retail price moves in response of entry into the downstream market.

[Insert Figure 3 about here]

As already mentioned before, the region where collusion can be sustained shrinks when the retail market becomes more competitive in the sense that the number N of competing outlets increases.

4. Private contracting

I now suppose that wholesale contracts are unobservable. As I will argue, the main difference with the case of public contracts is that, in this new framework, on-schedule deviations can no longer be distinguished by off-schedule ones. In a nutshell, when bilateral negotiations are private, wholesale contracts loose their strategic value: An off-schedule deviation can no longer be detected, and thus punished, within the same stage game where it occurred. Therefore, punishments can only take place with one period lag. The objective of this section is to show that this limit on retailers' communication about wholesale deals dramatically stifles the collusion possibility frontier, i.e., it increases the lowest discount factor above which positive profits can be sustained in equilibrium.

Before introducing the formal analysis, it is worthwhile recalling that also with private contracts the static game features a unique competitive and efficient equilibrium, and that in the dynamic analysis there is no loss of generality in looking for equilibria with exclusive dealings. Proposition 1 still applies in this setting because the upstream market is perfectly competitive and retailers have no incentive to leave rents to suppliers irrespective of whether wholesale contracts are observable or not.

Building on these results I shall, once again, focus on symmetric and stationary implicit agreements, whereby retailers play symmetrically *on* and *off* the cooperative phase. As before, I assume that the cartel's penal code follows Nash reversion trigger strategies. One important aspect to emphasize, though, is that with private contracts the public history only contains information about past retail prices, that is, $h^\tau \equiv (\mathbf{p}_1^\tau, .., \mathbf{p}_N^\tau)$. Let H^τ be the set of possible public histories at stage τ, and denote by $\hat{\sigma} \equiv (\hat{\sigma}^\tau(h^\tau))_{\tau=1}^{+\infty}$, with $\hat{\sigma}^\tau : H^\tau \to \mathbb{R}^3$, the collusive strategy which specifies for each public history h^τ at stage τ a retail price $\hat{p}^\tau$ and wholesale contract $\hat{C}^\tau$. Moreover, denote by $y(h^\tau) \in \{0,1\}$ the dicothomic state variable taking value 1 if at stage τ the game is in a cooperative phase, i.e., all retailers have obeyed to the implicit agreement strategy up to τ, and 0 if a deviation has occurred at $\tau - 1$. A stationary and symmetric collusion strategy enforcing the non-competitive retail price p^c and the wholesale contract C^c on the cooperative phase can be thus formally described as follows:

$$\forall\ \tau \text{ and } h^\tau \in H^\tau :\ \hat{\sigma}^\tau(h^\tau) \equiv \begin{cases} (p^c, C^c) & \text{if } y(h^\tau) = 1, \\ \\ (0, C^*) & \text{if } y(h^\tau) = 0. \end{cases}$$

Of course, the strategy $\hat{\sigma}$ must satisfy suppliers' participation participation constraint:

$$\frac{1}{N} D(p^c) w^c + T^c \geq 0.$$

The main difference with the previous analysis lies on the self-enforceability constraints. Given any pair (p^c, C^c) specifying a retail price and wholesale contract to be implement in the cooperative phase, with unobservable contracts self-enforceability requires that no downstream firm must find it profitable to deviate by undercutting its rivals with a retail price slightly below p^c and issuing the best wholesale contract given that its rivals stick to C^c. This immediately implies the following inequality:

$$\frac{\frac{1}{N} D(p^c)(p^c - w^c) - T^c}{1-\delta} \geq \max_{(w_i, T_i) \in \mathbb{R}^2} \{D(p^c)(p^c - w_i) - T_i :\ D(p^c) w_i + T_i \geq 0\}. \qquad (7)$$

There is one key discrepancy between this self enforceability constraint and the one described in equation (3) for the case of public contracts: when a downstream firm deviates from the implicit agreement and wholesale contracts are not observable, the best retail price the retailer can charge is $p^c - \varepsilon$, so that undercutting now secures the full monopoly profit. This is because competitors cannot react by lowering prices down to their marginal cost w^c. As I shall argue below, this inability to react instantaneously to deviations makes it harder to sustain collusion. Similarly to the public contracting case, though, the optimal deviation contract must be such that the supplier's participation constraint binds. As a result, using $T_i = -D(p^c)\, w_i$, the incentive compatibility constraint (7) rewrites as:

$$\frac{\frac{1}{N} D(p^c)(p^c - w^c) - T^c}{1-\delta} \geq D(p^c)\, p^c. \tag{8}$$

Essentially, since the input supply market is perfectly competitive, there is no reason for why a deviant retailer should leave some rents to its supplier when undercutting the rivals. This leads to the following definition:

Definition 5 *Suppose that wholesale contracts are private, then an optimal symmetric and stationary collusive strategy $\hat{\sigma}$ maximizes retailers' joint profits subject to the self-enforceability and participation constraints described above:*

$$\mathcal{P}' : \begin{cases} \max\limits_{(p,(w,T))\in\mathbb{R}^3} D(p)(p-w) - NT, \\ s.t. \text{ (1) and (8)}. \end{cases}$$

As before, the implicit collusive agreement has to maximize the downstream industry joint profits subject to the relevant participation and self-enforceability constraints. The next proposition then characterizes the optimal collusion behavior:

Proposition 6 *Suppose that wholesale contracts are private, then collusion can be sustained only in the region of parameters where $\delta \geq (N-1)/N$. In this range downstream firms are able to sustain monopoly profits (π^m) and issue the competitive and efficient wholesale contract. Otherwise, for $\delta < (N-1)/N$, the unique PBE of the repeated game features perfect competition and efficient wholesale contracts.*

This result shows that limits on retailers' communication ability dramatically weaken their collusion possibilities. The key insight is that when there is no way of making wholesale contracts public, deviation spot gains become so large to prevent any scope for cooperation below the critical discount factor $(N-1)/N$. While with public contracts the cartel's members can avoid this moral hazard problem by changing their retail price decisions within the same period in which a deviation has occurred, with private contracts this mechanism is no longer viable. Clearly, this difference opens the issue of studying the conditions linking the value of communication between downstream firms, their intertemporal preferences and the market structure, an issue that will be at the core of the next concluding section.

5. The strategic value of public contracts for repeated games

Building on the results characterized in the previous analysis, I now turn to study the strategic value of public contracts in dynamic games of repeated interaction between downstream and upstream firms. What is the link between the strategic value of public contracts, firms' discount rate and product market competition in retail markets?

My analysis offers a simple answer to this question. The results stated in Theorem 3 and Proposition 6 clearly imply that public contracting broadens the collusive possibility frontier as long as the downstream industry features a discount rate δ that falls short of the critical value $(N-1)/N$. In this region of parameters, in fact, the possibility of making wholesale contracts public leads to new collusive outcomes relying on inefficient input supply. This simple consideration leads to the following result:

Proposition 7 *The value of public contracting is higher the more myopic firms are and the more intense product market competition is.*

This result, together with the characterization analysis provided in Theorem 3, offer simple testable predictions on the link between the anticompetitive use of wholesale contracts on the one hand, the downstream market structure and retailers' time preferences on the other. Of course, besides the aforementioned static work on buyer power, there might be several other stories which could be consistent with inefficient wholesale deals and public contracting (see, for instance, the large body of work on double marginalization, Motta, 2000, Ch. 6). But, in this literature, where the initiative is on the suppliers' hands, franchise fees are typically positive and their welfare effects are positive insofar as two-part tariffs prevent double marginalization. My analysis, instead, unambiguously suggests that antitrust and competition policy authorities should regard as anticompetitive *per se* forms of communication between retailers, which make it possible to share information about wholesale contracts and are bundled with negative franchise fees.

6. Concluding remarks

This paper makes a step forward towards understanding some natural economic issues emerging in dynamic games of repeated interaction between upstream and downstream firms where these latter have the initiative. Two main objectives have been pursued throughout the analysis. First, the model throws new light on the hidden determinants of vertical contracting, and on the role that a careful design of wholesale arrangements might play in softening competition in a dynamic framework. The main idea developed in the paper is that inefficient vertical contracting emerges as a mechanism to implement collusion among retailers, building on the natural 'complementarity' between retail and wholesale prices. When collusion between retailers turns out not to be sustainable with efficient wholesale deals, this complementarity makes it advantageous for retailers to rely on inefficient input supply provision in order to squeeze the wedge between collusive and deviation profits, whereby weakening deviation incentives. In addition, profitable collusive market outcomes must be supported by wholesale contracts featuring negative franchise fees: a practice which is currently at the heart of an intense antitrust and competition policy debate.

The second main insight of the paper is about the role played by communication mechanisms between competing hierarchies on the set of collusive outcomes achievable in games where the initiative is in the retailers' hands. Communication turns out to be indeed fundamental to strengthen cartels' sustainability, although generating some efficiency losses. Moreover, its strategic value increases the less patient firms are and the larger the number of retailers in the downstream market is.

One last comment is worthwhile about the scope of my contribution. My results are fairly general and, although I have developed the formal arguments in a stylized IO example, my conclusions are of wider scope. They can apply basically within any competing hierarchies model involving downstream price competition, be it procurement contracting, executive compensations, patent licensing, insurance or credit relationships, to name only a few.

Appendix

Proof of Lemma 1: The proof of this result is based on a standard undercutting logic. First, showing that there exists an equilibrium where all retailers offer the competitive and efficient contract C^* and set the retail price equal to zero follows from the Bertrand logic. Second, uniqueness can be readily shown by contradiction: Assume that there exists an equilibrium where all retailers offer a contract with positive wholesale and retail prices, $w > 0$ and $p > 0$ respectively. Then, a standard undercutting argument implies that a retailer can profitably deviate by issuing the same contract as its rivals, but by setting its retail price at $p - \varepsilon$, with $\varepsilon > 0$ and small, whereby stealing the whole market. This concludes the argument. ■

Proof of Proposition 1: In order to show this result one needs to argue that every equilibrium outcome (i.e., an intertemporal profile of wholesale contracts $(\mathbf{C}^\tau)_{\tau=1}^{+\infty}$ and retail prices $(\mathbf{p}^\tau)_{\tau=1}^{+\infty}$) emerging with non-exclusive relationships, i.e., in the game where some suppliers deal with more than one retailer, is also an equilibrium outcome of the game with exclusive dealings, i.e., the case where contracting takes place on a pairwise basis. The proof mainly rests on the hypothesis that in the economy there are as many suppliers as retailers, that is, $M \geq N$. The argument is by construction: Consider an equilibrium of the game where some suppliers deal with more than one retailer and, in each period τ, every retailer R_i offers the contract $C_i^\tau(h^\tau)$ and sets the retail price $p_i^\tau(h^\tau)$ depending on the public history h^τ. First, since the bargaining power is on the retailers' hands, each supplier must break-even in equilibrium irrespective of the exclusivity clauses. Moreover, by definition of best replies, the equilibrium outcome emerging with non-exclusive dealings must be an equilibrium in a framework with exclusivity: each retailer offers the same profile of wholesale contracts $C_i^\tau(h^\tau)$ to its exclusive supplier, sets the same retail price $p_i^\tau(h^\tau)$ and gets the same per period utility. This immediately implies that moving from a non-exclusivity regime to one with exclusivity not only leaves unchanged the equilibrium price system, but it also does not affect the way the market pie is distributed among the players. ■

Proof of Lemma 2: Assume that all retailers issue the efficient contract C^*. A collusive equilibrium can then be sustained as long as the self-enforceability conditions (2) and (4) hold. Since I assumed that $w^c = 0$, it can be easily verified that these two inequalities are met if and only if $\delta \geq (N-1)/N$. Finally, as self-enforceability is not met when all retailers offer the efficient contract C^* and $\delta < (N-1)/N$, it is immediate to show that, in this region of parameters, the unique SPNE of the game where retailers perpetually offer the efficient and competitive contract C^* entails zero retail prices. ■

Proof of Lemma 3: In order to show this result one first needs to argue that as long as there exists a collusive equilibrium featuring (strictly) positive retail prices in the region of parameters where $\delta < (N-1)/N$, the wholesale contract cannot be the efficient one. The argument is by contradiction, suppose that there exists a collusive symmetric equilibrium where retailers charge strictly positive retail prices and issue the efficient contract C^*. Then, by the self enforceability constraint (2) it is immediate to verify that on-schedule deviations are always profitable, so that there exists no implicit agreement which sustains this price system. This fact, together with the suppliers' participation constraint, which in equilibrium must bind for reasons that I have already discussed in the text, imply that if a collusive equilibrium exists in this region of parameters, it must be either the case that wholesale prices are strictly positive (with associated negative franchise fees) or strictly negative (with associated positive franchise fees). Suppose that $w^c < 0$, and take $T^c = -(1/N)\, D(p^c)\, w^c$, then the self-enforceability condition (2) implies:

$$p^c \frac{1 - N(1-\delta)}{N(1-\delta)} + \frac{w^c (N-1)}{N} \geq 0,$$

but this cannot be true in the relevant range of parameters. In fact, for $p^c \geq 0$ and $w^c < 0$ the above condition requires $\delta > (N-1)/1$: a contradiction. Therefore, if a collusive equilibrium exists it must be the case that $w^c > 0$ and $T^c < 0$. ■

Proof of Theorem 3: To begin with, it is useful to rewrite the cartel's program as:

$$\mathcal{P}: \begin{cases} \max_{(p,(w,T))\in\mathbb{R}_+^2\times\mathbb{R}} D(p)(p-w) - NT \\ \text{s.t.} \\ \frac{1}{N} D(p)\, w + T \geq 0 \quad \text{(A.1)}, \\ D(p)(p-w) \geq N \max\left\{(1-\delta) D(p)(p-w) + \delta T, (1-\delta) D(w)\, w + T\right\} \quad \text{(A.2)}. \end{cases}$$

where equation (A.2) rewrites the self-enforceability constraints (2)-(4) in a more compact way. Clearly, the suppliers' participation constraint must be binding in the optimum, as otherwise retailers could profitably decrease the franchise fee (or the wholesale price) to make higher profits. Then, (A.1) yields $T = -(1/N)\, D(p)\, w$, substituting this term into (A.2) and into the objective of program $\mathcal{P}$ one has:

$$\mathcal{P}: \begin{cases} \max_{(p,w)\in\mathbb{R}_+^2} D(p)\, p \\ \text{s.t.} \\ D(p)\, p \geq N(1-\delta) \max\left\{D(p)\left(p - \frac{w(N-1)}{N}\right), D(w)\, w\right\}. \end{cases}$$

The cartel program then amounts to find a pair of retail and wholesale price that maximize joint revenues $D(p)\,p$ subject to the self enforceability constraints. Showing that for $\delta \geq (N-1)/N$ the monopoly price is self-enforceable and is supported by the efficient contract C^* is immediate and will thus be omitted. Consider then the more interesting case where $\delta < (N-1)/N$, I will develop the arguments of the proof in several steps.

Step 1: *As long as $p^c > 0$ it must be $w^c > 0$.*

This fact can be easily shown by contradiction, suppose $w = 0$, from the self-enforceability constraints one has:

$$D(p)\,p \geq N(1-\delta)\max\{D(p)\,p, 0\} = N(1-\delta)\,D(p)\,p,$$

which cannot hold for $\delta < (N-1)/N$.

Step 2: *As long as $w^c > 0$ one must have $w^c < p^m$.*

The argument is again by contradiction, suppose that $w^c \geq p^m$ and that $p^c \geq w^c$ (a condition that I will check later), then strict concavity of $D(x)\,x$, together with the fact that $\delta < (N-1)/N$ is equivalent to $1 < N(1-\delta)$, imply:

$$D(p^c)\,p^c \leq D(w^c)\,w^c < N(1-\delta)\,D(w^c)\,w^c,$$

which is clearly incompatible with the self-enforceability condition (4).

Step 3: *For each retail price $p^c > 0$, there exists a unique wholesale price $w(p^c)$ such that:*

$$w(p^c) = \arg\min_{w \geq 0} \max\left\{D(p)\left(p - \frac{w(N-1)}{N}\right), D(w)\,w\right\}.$$

The argument to show this fact is simple. First, observe that for $w^c = 0$

$$\max\left\{D(p^c)\left(p^c - \frac{w^c(N-1)}{N}\right), D(w^c)\,w^c\right\} = D(p^c)\,p^c > 0,$$

moreover, for $w^c = p^c$ one also has:

$$\max\left\{D(p^c)\left(p^c - \frac{w^c(N-1)}{N}\right), D(w^c)\,w^c\right\} = D(p^c)\,p^c > 0.$$

Now, observe that the function $D(p^c)\,(p^c - w^c(N-1)/N)$ is strictly decreasing in w^c and positive at $w^c = 0$ for any $p^c > 0$. Moreover, the function $D(w^c)\,w^c$ is strictly concave, it is equal to zero at $w^c = 0$ and features a unique maximum at p^m. Since $D(x)\,x$ is single peaked, it must then be the case that for any $p^c > 0$ there exists a unique $w(p^c)$ which equalizes $D(p^c)(p^c - w^c(N-1)/N)$ and $D(w^c)w^c$, and thus minimizes

$$\max\{D(p^c)(p^c - w^c(N-1)/N), D(w^c)w^c\}.$$

Hence, the result.

Step 4: *If there exists a positive solution* $(\widetilde{p}(.), \widetilde{w}(.)) \in \mathbb{R}^2_{++}$ *of* P*, it must satisfy the following system of equations:*

$$D(p)\left(p - \frac{w(N-1)}{N}\right) = D(w)w, \tag{A.3}$$

$$\frac{p}{N(1-\delta)} = p - \frac{w(N-1)}{N}. \tag{A.4}$$

The argument follows directly from the fact that for any retail price $p^c > 0$, in the optimum, the cartel will set $w^c = \hat{w}(p^c)$ in order to minimize the upper bound imposed by (A.2) on the strictly concave function $D(p)p$. It must then be true that the unique positive solution of $\mathcal{P}$ must solve the system (A.3)-(A.4). Observe that the above system of equations is equivalent to the system (5)-(6).

Now, I will check that any collusive strategy must specify a retail price larger than the wholesale price, that is, $p^c \geq w^c$. The proof of this fact follows immediately from (A.4): Indeed, it is easy to check that for $\delta < (N-1)/N$ it must be:

$$\frac{(N-1)(1-\delta)}{(1-\delta)N-1} > 1,$$

so that $p^c > w^c$.

Step 5: *For* δ *close to zero, the unique solution compatible with positive sales of program* P *must entail* $p^c = w^c = 0$.

To develop the argument let me first analyze the case where $\delta = 0$. In that case, it is immediate to verify that the solution of the system (A.3)-(A.4) yields $p^c = w^c$ and $p^c \in \Pi \equiv \{p \geq 0 : D(p)p = 0\}$. Given the assumptions made on the demand function, it is easy to verify that $\Pi \equiv \{0, \overline{p}\}$. Hence, the only price level compatible with positive sales must be that where $p^c = w^c = 0$ at $\delta = 0$.

I now show that in a neighborhood of $\delta = 0$ it is still the case that collusion is not sustainable at any price level, so that as long as sales are positive in equilibrium one must have $p^c = 0$ for δ close to 0. In order to prove this result one needs to show how the implicit function $p^c(\delta, N)$ solving the system (A.3)-(A.4) varies around the point $\delta = 0$. So, differentiating (A.3)-(A.4), one obtains:

$$\frac{\partial p^c(\delta, N)}{\partial \delta} = \frac{-p^c(\delta, N)\left(D'(w^c(\delta, N))\, w^c(\delta, N) + D(w^c(\delta, N))\right)}{(1-\delta)\,\Delta\left(p^c(\delta, N), w^c(\delta, N), \delta, N\right)}, \tag{A.3}$$

where $\Delta(.)$ is the determinant of the matrix of the first-order derivatives of the system (A.3)-(A.4),

$$\Delta(.) = \left(\frac{D'(p^c(\delta, N))\, p^c(\delta, N)}{(1-\delta) N} + D(p^c(\delta, N))\right)(1-\delta)(N-1) + \\ + \left(D'(w^c(\delta, N))\, w^c(\delta, N) + D(w^c(\delta, N))\right)(1 - N(1-\delta)).$$

Now, taking the limit for $\delta = 0$ and selecting the solution $\overline{p}$ such that $D(\overline{p}) = 0$ one gets:

$$\lim_{\delta \to 0} \left.\frac{\partial p^c(\delta, N)}{\partial \delta}\right|_{w^c(0,N) = p^c(0,N) = \overline{p}} = \frac{N D'(\overline{p})\, \overline{p}}{D'(\overline{p})(N-1)^2} > 0,$$

so that around $\delta = 0$ there will be no sales as long as one selects $p^c(0, N) = \overline{p}$. As a consequence, positive sales around $\delta = 0$ are compatible only with $p^c(0, N) = 0$, where by definition $D(0) > 0$.

Step 6: *There exists a lower-bound $\underline{\delta}(N) \leq (N-1)/N$ such that for all $\delta \leq \underline{\delta}$, the unique solution of P compatible with positive sales entails $w^c(\delta, N) = p^c(\delta, N) = 0$.*

This result can be easily shown by noticing that at $\delta = 0$ the solution of the system (A.3)-(A.4) entails $p^c(\delta, N) > \overline{p}$. Hence, for all δ close to 0 one must have $p^c(\delta, N) = 0$. Now, observe that for $\delta \to (N-1)/N$ the system (A.3)-(A.4) yields $w^c = 0$ and $D(p^c)\, p^c = 0$, implying once again $p^c \in \Pi$. Taking the solution with the highest price $p^c = \overline{p}$, substituting $\delta = (N-1)/N$ and $w^c = 0$ into (A.3) one has:

$$\Delta(.) = D'(\overline{p})\, \overline{p} \frac{N-1}{N},$$

and thus:

$$\left.\frac{\partial p^c(\delta, N)}{\partial \delta}\right|_{\delta \to \left(\frac{N-1}{N}\right)^-,\ p^c(.) = \overline{p}} = \frac{-D(0)\, N^2}{D'(\overline{p})(N-1)} > 0,$$

implying that $p^c(\delta, N) < \overline{p}$ for all δ close to $(N-1)/N$.

Observe that for any $\delta \in (0, (N-1)/N)$ the solution of the system (A.3)-(A.4) is continuos since the demand function $D(p)$ is continuos. From step 5 it then follows that there must exist a lower bound $\underline{\delta}(N) < (N-1)/N$ such that $p^c(\underline{\delta}(N), N) \equiv 0$ for all $\delta < \underline{\delta}(N)$; moreover, from step 6 it must be $0 < p^c(\delta, N) < \overline{p}$ for all $\underline{\delta}(N) < \delta < (N-1)/N$. Showing that when $p^c(\underline{\delta}(N), N) \equiv 0$ one also has $w^c(\underline{\delta}(N), N) \equiv 0$ is immediate from the system (A.3)-(A.4).

Step 7: Finally, the statement of the theorem can be shown by gathering the results demonstrated in steps 1-6. ■

Proof of Corollary 1: The proof of this result follows immediately from Theorem 3. In fact, the system (A.3)-(A.4) yields $w^c(\underline{\delta}(N), N) = 0$ and $w^c(\delta, N) = 0$ for $\delta = (N-1)/N$. ■

Proof of Proposition 4: This result follows immediately from differentiation of (A.2) with respect to δ and N, so it will be omitted. ■

Proof of Proposition 6: The proof of this result relies upon the characterization of the solution of program $\mathcal{P}'$. Indeed, the suppliers' break-even condition yields $T = -(1/N) D(p) w$, which leads to

$$\mathcal{P}' : \begin{cases} \max_{p \in \mathbb{R}} D(p) p, \\ \text{s.t.} \\ \frac{D(p)p}{1-\delta} \geq N D(p) p. \end{cases}$$

This program immediately imply that the monopoly profit can be sustained as long as $\delta \geq (N-1)/N$ and that for $\delta < (N-1)/N$ the unique PBE of the game with private contracting implies perfect competition. ■

Proof of Proposition 7: The proof of this result simply follows from the fact that with public contracting inefficient collusion emerges in the region of parameters where $\delta < (N-1)/N$. It is then immediate to show that the critical value $(N-1)/N$ is increasing in N. ■

References

[1] Athey, S., K. Bagwell and C. Sanchirico, (2004), "Collusion and Price Rigidities" *The Review of Economic Studies*: 71, 317-349.

[2] Bernheim, D., and M., Whinston, (1986), "Common Agency", Econometrica, 54: 923-42.

[3] Bernheim, D., and M. Whinston, (1998), "Exclusive Dealing", *Journal of Political Economy*, 106: 64-103.

[4] Blair, F., and T. Lewis, (1994), "Optimal Retail Contracts with Asymmetric Information and Moral Hazard", *Rand Journal of Economics,* 25: 284-296.

[5] Caillaud, B., B. Jullien, and P. Picard, (1995), "Competing Vertical Structures: Precommitment and Renegotiation", *Econometrica*, 63: 621-646.

[6] Federal Trade Commission, (2001), Report on the FTC Workshop on Slotting Allowances and Other Marketing Practices in the Grocery Industry.

[7] Fershtman, C., and E. Kalai, (1997), "Unobserved Delegation", *International Economic Review*, 38: 763-774.

[8] Fershtman, C., and K. Judd, (1987), "Equilibrium Incentives in Oligopoly Equilibrium Incentives in Oligopoly", *American Economic Review,* 77: 927-940.

[9] Fershtman, C., K. Judd and E. Kalai, (1991), "Observable Contracts: Strategic Delegation and Cooperation Observable Contracts: Strategic Delegation and Cooperation", *International Economic Review*, 32: 551-559.

[10] Friedman, J., (1971), "A Non-cooperative Equilibrium for Supergames", *Review of Economic Studies*, 38: 1-12.

[11] Gal-Or, E., (1991a), "Vertical Restraints with Incomplete Information", *The Journal of Industrial Economics,* 39: 503-516.

[12] Gal-Or, E., (1991b), "A Common Agency with Incomplete Information", *Rand Journal of Economics,* 22: 274-286.

[13] Jullien, B., and P. Rey, (2007), "Resale Price Maintenance and Collusion", *Rand Journal of Economics*, 38:

[14] Katz, M., (1991), "Game-Playing Agents: Unobservable Contracts as Precommitments", *Rand Journal of Economics*: 22: 307-328.

[15] Kuhn, K.U., (1997), "Nonlinear Pricing in Vertically Related Duopolies", *Rand Journal of Economics*, 28: 37-62.983-1001.

[16] Martimort, D., (1996), "Exclusive Dealing, Common Agency, and Multiprincipals Incentive Theory", *Rand Journal of Economics,* 27: 1-31.

[17] Martimort D., and S. Piccolo, (2007), "Resale Price Maintenance under Asymmetric Information", *International Journal of Industrial Organization*, 25: 315-339.

[18] Martimort, D., and S. Piccolo, (2009), "The Strategic Value of Quantity Forcing Contracts", forthcoming in the *American Economic Journal: Microeconomics*.

[19] Marx, L., and G. Shaffer, (2008), "Upfront Payments and Exclusion in Downstream Markets", *Rand Journal of Economics,* 38: 823 - 843.

[20] Motta, M., (2004), *Competition Policy: Theory and Practice,* Cambridge University Press.

[21] Niraj, R., and C. Narasimhan, (2004), "Vertical Information Sharing in Distribution Channels", mimeo.

[22] Nocke, V., and L. White, (2007), "Do Vertical Mergers Facilitate Upstream Collusion?", *American Economic Review,* 97: 1321-1339.

[23] Rey, P., J. Thal and T. Vergé, (2008), "Buyer Power and Intrabrand Coordination", mimeo.

[24] Rey, P., and J. Tirole, (1986), "The Logic of Vertical Restrains", *American Economic Review*, 76: 921-939.

[25] Rey, P., and J. Stiglitz., (1995), "The Role of Exclusive Territories in Producers' Competition", *Rand Journal of Economics*, 26: 431-451.

[26] Rotemberg, J., and G. Saloner, (1986), "A Supergame-Theoretic Model of Price Wars during Booms", *The American Economic Review*, 76: 390-407.

[27] Schelling, T., (1956), "An Essay on Bargaining", *American Economic Review*, 46: 281-306.

[28] Schelling, T., (1960), *The Strategy of Conflict,* New York: Oxford University Press.

[29] Sklivas, D., (1987), "The Strategic Choice of Managerial Incentives", *Rand Journal of Economics*, 18: 452-458.

[30] Vickers, J., (1985), "Delegation and the Theory of the Firm", E*conomic Journal*, 95: 138-147.

[31] Stern, L.W., A. El-Ansary and A. Coughlan, (1996), *Marketing Channels*. Upper Saddle River, NJ: Prentice Halt.

Figure 1: wholesale and retail equilibrium prices

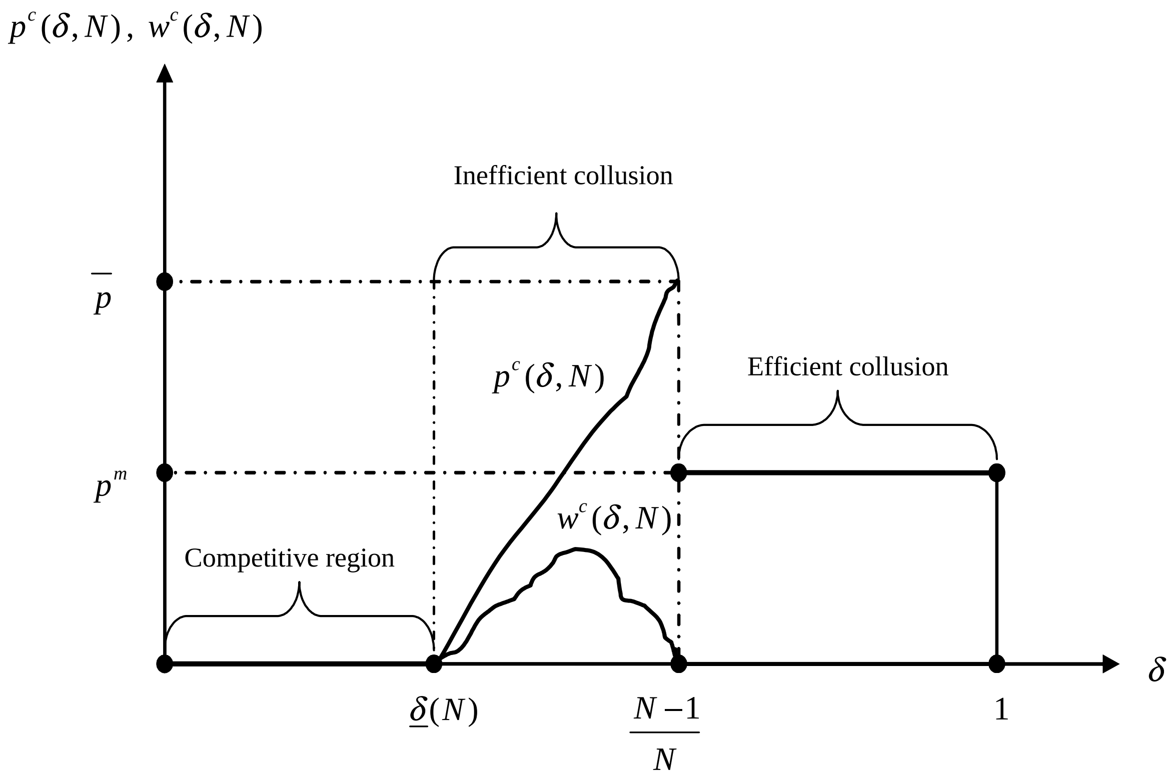

Figure 2: The linear example

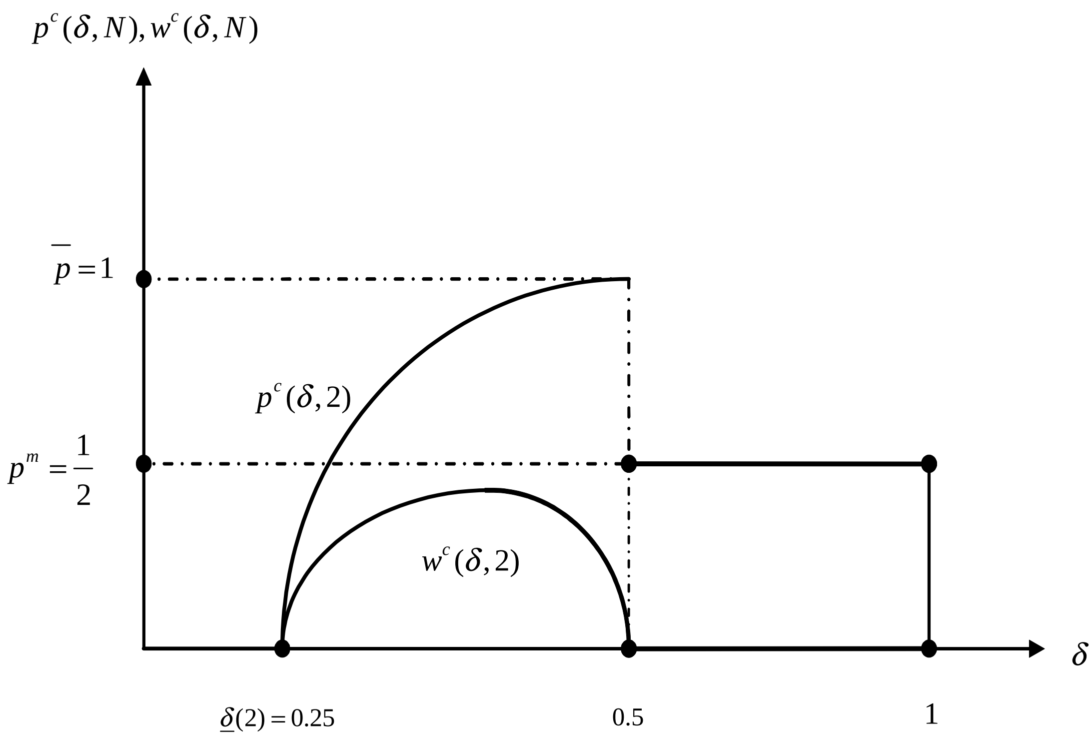

Figure 3: Market concentration