Growth and Social Security: The Role of Human Capital

Alexander Kemnitz and Berthold U. Wigger

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Abstract

This paper studies the growth and efficiency effects of pay-as-you-go financed social security when human capital is the engine of growth. Employing a variant of the Lucas (1988) model with overlapping generations, it is shown that a properly designed unfunded social security system leads to higher output growth than a fully funded one. Furthermore, the economy with unfunded social security is efficient while the other one is not. These results stand in sharp contrast to those that obtain in models where economic growth is driven by physical capital accumulation.

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* University of Mannheim

** University of Mannheim and CSEF, University of Salerno
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References
I. Introduction

Various authors have shown that unfunded social security stunts economic growth in Romer (1986)-type endogenous growth models (see, e.g. Saint-Paul, 1992, Wiedmer, 1996). The economic mechanism behind this result is straightforward. Unfunded social security discourages individuals to save privately for old age without compensating for this via forced public savings. As economic growth is positively related to the aggregate capital stock and, hence, total savings, an economy with unfunded social security exhibits slower growth. A corollary of this statement is that the introduction of a pay-as-you-go pension system cannot lead to a Pareto-improvement as the decline in growth rates harms future generations.

However, economic growth need not solely be due to physical capital accumulation as in Romer (1986). There is another strand of literature pioneered by Lucas (1988) that identifies human capital accumulation as the engine of growth. But once the effects of human capital are taken into account, the relation between social security and growth may appear in a completely different light. Employing a variant of the Lucas (1988) model with overlapping generations, we show that a properly designed unfunded social security system leads to higher output growth than a fully funded one. Moreover, the competitive economy with such an unfunded social security system is efficient whereas an economy without any or with a fully funded social security system is not.

The reason for the inefficiency of the latter is the following. In Lucas (1988), growth is driven by the ability of human capital per worker to increase without bound. But as in overlapping generations models individual human capital depreciates fully with retirement, any accumulation of human capital over time requires that succeeding generations inherit some part of the human capital stock of their ancestors. This implies a positive effect of actual investment in human capital on the productivity of future generations. However, this positive effect is disregarded by every single individual since it only has an negligible impact on the average human capital stock that is transferred to succeeding generations. In contrast, an unfunded social security program in which the size of the transfers to a particular old individual is properly tied to his human capital renders the competitive allocation efficient by providing socially optimal incentives to invest in human capital. We show that this can be accomplished by a pension formula that displays some stylized features of the german pension system. The higher human capital investment under this scheme translates into faster output growth.

The plan of the paper is as follows. Section 2 presents the economy without any or with a fully funded pension scheme and establishes its inefficiency. In section 3 an unfunded social security system is introduced and its impact on efficiency and growth is derived. Section 4 concludes.
2. Laissez-Faire and Efficient Allocations

The economy to be studied is a variant of the Diamond (1965) overlapping generations model where the engine of growth is of the Lucas (1988) type disregarding externalities in the production of aggregate output. Individuals live for two periods. A representative member of the generation born at time $t$ consumes $c_t^1$ units of a homogeneous good when young and $c_t^{2+1}$ units when old. Preferences are represented by $u(c_t^1, c_t^{2+1})$, where $u$ satisfies the standard concavity, monotonicity and Inada assumptions with respect to both arguments. The size of each generation is normalized to one.

The technology to produce the homogeneous output is represented by a neoclassical constant returns to scale production function $F(K_t, H_t)$, where $K_t$ denotes physical capital and $H_t$ labor in efficiency units at time $t$. The homogeneous output can either be consumed or stored as capital for production in the next period. Labor in efficiency units is determined by: $H_t = (1 - \lambda_t) \bar{h}_t$, where $1 - \lambda_t$ is the fraction of non-leisure devoted to labor market activities, and $\bar{h}_t$ is the average human capital stock at time $t$. A single worker’s human capital, $h_t$, depends on the time he devotes to studying, $\lambda_t$, and the average human capital of workers at time $t - 1$, $\bar{h}_{t-1}$. It evolves according to:

$$h_t = g(\lambda_t)\bar{h}_{t-1},$$

where studying time is essential, i.e. $g(0) = 0$, and raises the human capital stock in a weakly concave manner: $g(\cdot) > 0$, and $g'(\cdot) \leq 0$. Equation (1) establishes a proportional link between human capital accumulated at time $t$ and human capital accumulated in previous periods by individuals who are old or already nonexistent at time $t$. This relation is a prerequisite for this model to sustain endogenous long-run growth. Only then human capital per worker can increase over time without bound. For diminishing returns of human capital with respect to the accumulated stock, the ratio of human capital of two succeeding generations and, hence, growth must eventually drop to zero, irrespective of studying time.

The dependency of individual human capital on the average human capital of the parent generation is a usual assumption in the literature (see for example Azariadis and Drazen, 1990, and Galor and Tsiddon, 1994). It can be corroborated by the following two complementary findings. First, empirical studies suggest that a more educated workforce is more adaptive to technological changes, implying that the earnings possibilities

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1The externality discussed by Lucas (1988) is of an intragenerational type, as it works through the effects of the current human capital stock on current productivity. This should not be confused with the intergenerational externality we investigate here. In the Lucas (1988) model, this externality does not arise because households live forever. The additional consideration of the intragenerational externality would strengthen our results.

2In the infinite-horizon-variant of this equation, Lucas (1988) assumes the growth of human capital to be proportional in studying time.
of children are affected by how flexible the parent generation as a whole was.\textsuperscript{3} Second, human capital creates knowledge and is therefore often considered a determinant of technological progress itself. As some knowledge spills over to society due to its non-excludable and non-rival nature (Freeman and Polasky, 1992), it becomes available to succeeding generations, allowing them to build on their ancestors’ knowledge rather than having to reinvent the wheel or the steam engine each period.\textsuperscript{4}

After spending a fraction $\lambda_t$ of non-leisure time for accumulating human capital, a young individual is endowed with a human capital stock of $g(\lambda_t) \bar{h}_{t-1}$, which it supplies for the remaining part of period $t$, $(1 - \lambda_t)$, in the labor market. For every efficiency unit of labor supplied, the individual receives a wage of $w_t$ in terms of the homogeneous good.

Without loss of generality we assume that production decisions are made by old individuals. In every period they hire $H_t$ units of efficient labor for production using $K_t$, the physical capital they have saved when young and consume the amount of his accumulated physical capital and its rent. In such a framework which can either be interpreted as a laissez-faire economy where social security is absent or an economy with a fully funded pension system, the problem of an individual born at time $t$ is to maximize:

$$u(c^1_t, c^2_{t+1}),$$

subject to (1) and:

$$c^1_t = (1 - \lambda_t) h_t w_t - K_{t+1},$$

$$c^2_{t+1} = F(K_{t+1}, H_{t+1}) - w_{t+1} H_{t+1} + K_{t+1},$$

by choice of $c^1_t, c^2_{t+1}, \lambda_t, K_{t+1}$ and $H_{t+1}$. The first order conditions are given by:

$$u_1(c^1_t, c^2_{t+1}) [(1 - \lambda_t) g(\lambda_t) - g(\lambda_t)] \bar{h}_{t-1} w_t = 0,$$

$$u_2(c^1_t, c^2_{t+1}) [F_t(K_{t+1}, H_{t+1}) + 1] - u_1(c^1_t, c^2_{t+1}) = 0,$$

$$u_2(c^1_t, c^2_{t+1}) [F_t(K_{t+1}, H_{t+1}) - w_{t+1}] = 0,$$

where $u_i$ and $F_i$ are the partial derivatives of $u$ and $F$ with respect to the $i$-th argument. Notice that (6) implies that the marginal rate of substitution between consumption in the two periods equals the gross rate of return on physical capital, and that (7) implies that the real wage is equal to the marginal product of labor. Considering (5), the

\textsuperscript{3}For both an elaboration of this point and empirical evidence see Gakor and Tsiddion (1994).

\textsuperscript{4}Note that equation (1) does not imply that learning about existing technologies is costless. It only says that it is the easier to acquire a certain level of knowledge, be it new or old, the larger is the human capital stock of the parent generation.
fraction of non-leisure time a young individual devotes to human capital accumulation results from the equality of its marginal gains in production and human capital accumulation. As all members of a generation are identical, individual and average human capital coincide, i.e. \( \tilde{h}_t = h_t \). Considering this and substituting (7) into (5), the first order conditions for maximum utility may be written as:

\[
\begin{align*}
    & u_1(c^1_t, c^{2}_{t+1}) \left[ (1 - \lambda_t)g(\lambda_t) - g(\lambda_t) \right] h_{t-1} F_2(K_t, H_t) = 0, \quad (8) \\
    & u_2(c^1_t, c^{2}_{t+1}) \left[ F_1(K_{t+1}, H_{t+1}) + 1 \right] - u_1(c^1_t, c^{2}_{t+1}) = 0. \quad (9)
\end{align*}
\]

In this form the first-order conditions will be employed for later comparisons. Since (8) reduces to:

\[
(1 - \lambda_t)g'(\lambda_t) - g(\lambda_t) = 0, \quad (10)
\]

our assumptions on \( g(\cdot) \) imply that \( \lambda_t \) is uniquely determined and is constant for all \( t \). We will denote this level by \( \bar{\lambda} \). A competitive equilibrium of the laissez-faire economy is then implicitly defined by equation (9), \( \lambda_t = \bar{\lambda} \) and the labor market clearing condition \( H_t = (1 - \lambda_t) h_t \).

As a first result we will show that this equilibrium is Pareto-inefficient. For this purpose, we derive the set of efficient allocations by considering a social planner who aims at the maximization of the present value of a discounted stream of future utilities at time \( t = 0 \) and demonstrate that the laissez-faire equilibrium does not belong to this set.

The planner’s problem can then be formulated as:

\[
\max \sum_{t=0}^{\infty} \mu_t u(c^1_t, c^{2}_{t+1}), \quad (11)
\]

subject to:

\[
\begin{align*}
    c^1_t + c^2_t & = F(K_t, H_t) + K_t - K_{t+1}, \quad (12) \\
    H_t & = (1 - \lambda_t) h_t, \quad (13) \\
    h_t & = g(\lambda_t) h_{t-1}, \quad (14)
\end{align*}
\]

given \( K_0 \) and \( h_{-1} \), for some geometric sequence of the planner’s discount factors, \( \{\mu_t\}_{t=0}^{\infty} \), rendering the objective function finite. Under the assumptions of the model the following Euler equations are necessary and sufficient for an optimum allocation:

\[
\begin{align*}
    & \mu_{t+1} u_1(c^1_{t+1}, c^{2}_{t+2}) \left[(1 - \lambda_{t+1})g(\lambda_{t+1})g'(\lambda_t)h_{t-1} F_2(K_{t+1}, H_{t+1}) \right. \\
    & \left. + \mu_t u_1(c^1_t, c^{2}_{t+1}) \left[(1 - \lambda_{t})g'(\lambda_t) - g(\lambda_t) \right] h_{t-1} F_2(K_t, H_t) \right] = 0, \quad (15) \\
    & \mu_{t+1} u_1(c^1_{t+1}, c^{2}_{t+2}) \left[F_1(K_{t+1}, H_{t+1}) + 1 \right] - \mu_t u_1(c^1_t, c^{2}_{t+1}) = 0, \quad (16) \\
    & -\mu_{t+1} u_1(c^1_{t+1}, c^{2}_{t+2}) + \mu_t u_2(c^1_t, c^{2}_{t+1}) = 0. \quad (17)
\end{align*}
\]

Substituting (17) into (15) and (16) and rearranging terms, we get:

\[ u_1(c_t, c_{t+1}^2) [(1 - \lambda_t)g' (\lambda_t) - g(\lambda_t)] h_{t-1} F_2(K_t, H_t) + u_2(c_{t+1}^1, c_{t+1}^2) (1 - \lambda_{t+1}) g(\lambda_{t+1}) g' (\lambda_t) h_{t-1} F_2(K_{t+1}, H_{t+1}) = 0, \quad (18) \]

\[ u_2(c_t^1, c_{t+1}^2) [F_1(K_{t+1}, H_{t+1}) + 1] - u_1(c_t^1, c_{t+1}^2) = 0. \quad (19) \]

These conditions must be met by any efficient allocation, irrespective of the discount factors attached to different generations. It is convenient for the subsequent analysis to denote solutions to these equations by an asterisk.

A comparison of the efficiency conditions with those for the economy without any or with fully funded social security leads to:

**PROPOSITION 1:** The laissez-faire equilibrium is Pareto-inefficient. A Pareto-efficient allocation requires young individuals to devote more non-leisure time to human capital accumulation than they choose to do in the laissez-faire economy.

**PROOF:** This result is easily established by evaluating the above equations at the laissez-faire equilibrium values. Considering (10), the left-hand side of (18) reduces to the second summand for \( \lambda = \bar{\lambda} \). As this summand is positive and the right-hand side is zero, \( \bar{\lambda} \) fails to satisfy the efficiency conditions. A positive left-hand side of (18) at \( \lambda = \bar{\lambda} \) implies that the marginal gains of studying time exceed its marginal costs. Consequently, an efficient allocation requires individuals to spend more time for human capital accumulation than they do in the laissez-faire economy. Q.E.D.

The inefficiency of the laissez-faire economy is caused by the fact that the intergenerational transmission of human capital is not priced. Spending time for study enhances not only one's own human capital, but also that of succeeding generations. However, the single individual has no incentive to take this intergenerational spillover into account because it can only be controlled at the economy-wide level: newborn generations are endowed with the average of last period's human capital, on which a single individual only has a negligible impact.

Furthermore, another inefficiency arises if physical and human capital are complements in production \( (F_{12} > 0) \). In equilibrium, individuals choose their savings such that the gross return of physical capital equals the marginal rate of substitution of consumption between both periods. But for complementary inputs, the gross return of physical capital positively depends on the amount of effective labor employed. Consequently, an inefficiently low stock of human capital entails inefficient low accumulation of physical capital. Then, the reason for the inefficiency of the competitive market can also be identified as the lack of a mechanism which signals to young individuals the effect of their human capital investment decision on the rate of return on physical capital that they receive for their savings when old.
3. An Economy with Pay-as-you-go Pensions

In this section we present an unfunded social security system that is able to remedy the inefficiency of the laissez-faire competitive allocation. The financing of the scheme is of the pay-as-you-go type, where workers contribute \( \tau_t \) units of the homogeneous good which are distributed to pensioners according to a stylized form of the German pension formula, where pension payments are positively linked to former working income and on the time spent for human capital formation:\(^6\)

\[
P_t = P_t ( (1 - \lambda_{t-1}) h_{t-1} w_{t-1}, \lambda_{t-1} ).
\]

For the following analysis the effect of this pension formula on the individual allocation of time between studying and working will be decisive. To simplify the notation, we will therefore work with an abbreviated form:

\[
P_t = \theta_t (\lambda_{t-1}),
\]

where all other variables are suppressed as they cannot be individually influenced. Because of the positive dependence of benefits on both studying and working time, it is convenient to assume concavity of pension benefits with respect to \( \lambda_t \). Budget balancing of the pension scheme requires:

\[
\tau_t = \theta_t (\lambda_{t-1}). \quad (20)
\]

We will now show that for a properly chosen sequence of pension schemes the competitive equilibrium of this economy is Pareto-efficient and displays a higher rate of balanced output growth than the laissez-faire competitive equilibrium.

In the economy with the above pension scheme, individuals choose \( c_t^1, c_{t+1}^2, \lambda_t, K_{t+1} \) and \( H_{t+1} \) in order to maximize:

\[
u(c_t^1, c_{t+1}^2),
\]

subject to (1) and to the new consumption constraints:

\[
c_t^1 = (1 - \lambda_t) h_t w_t - \tau_t - K_{t+1}, \quad (22)
\]

\[
c_{t+1}^2 = F(K_{t+1}, H_{t+1}) - w_{t+1} H_{t+1} + \theta_{t+1} (\lambda_t) + K_{t+1}. \quad (23)
\]

\(^6\)Without delving too much into detail, the features of the German system in this context are as follows. Benefits depend on former annual earnings and on the number of years the individual has worked and hence contributed to the system. Furthermore, years spent in the educational system are at least partially treated as contribution years.
The first order conditions now obtain as:

\[ u_1(c^1_t, c^2_{t+1}) [(1 - \lambda_t)g' (\lambda_t) + g(\lambda_t)] \ h_{t-1} \ w_t + u_2(c^1_t, c^2_{t+1}) \theta_{t+1}'(\lambda_t) = 0, \quad (24) \]

\[ u_2(c^1_t, c^2_{t+1}) \ [F_1(K_{t+1}, H_{t+1}) + 1] - u_1(c^1_t, c^2_{t+1}) = 0, \quad (25) \]

\[ u_2(c^1_t, c^2_{t+1}) \ [F_2(K_{t+1}, H_{t+1}) - w_{t+1}] = 0. \quad (26) \]

A competitive equilibrium with unfunded social security then obeys (24)-(26), the labor market equilibrium condition \( H_t = (1 - \lambda_t) h_t \) and the budget balance restriction (20). From the above conditions, one can infer:

\[ u_1(c^1_t, c^2_{t+1}) \ [(1 - \lambda_t)g' (\lambda_t) - g(\lambda_t)] \ h_{t-1} F_2(K_t, H_t) \]

\[ + u_2(c^1_t, c^2_{t+1}) \theta_{t+1}'(\lambda_t) = 0, \quad (27) \]

\[ u_2(c^1_t, c^2_{t+1}) \ [F_1(K_{t+1}, H_{t+1}) + 1] - u_1(c^1_t, c^2_{t+1}) = 0. \quad (28) \]

A comparison of these equations with the conditions for an efficient allocation (18) and (19) immediately reveals that the solutions of both systems coincide as long as:

\[ \theta_{t+1}'(\lambda^*_t) = (1 - \lambda^*_{t+1}) g(\lambda^*_t) g'(\lambda^*_t) h_{t-1} F_2(K^*_{t+1}, H^*_{t+1}) \quad \text{for all } t. \quad (29) \]

where variables with an asterisk denote efficient levels. This establishes the following result:

**PROPOSITION 2**: A competitive equilibrium with an unfunded social security system where the pension formula satisfies (29) is Pareto-efficient.

The social security system presented here induces every generation to take the beneficial effect of their human capital investment on the following generation into account. This is accomplished by tying together pension benefits and studying time such that the marginal increase in pension benefits due to longer study equals its marginal return for the succeeding generation. In fact, a pension scheme in line with (29) plays a Pigouvian role by rewarding each generation with the marginal external benefit of its studying time on the following generation’s human capital.\(^7\)

There are infinitely many pension formulas consistent with (29). However, any optimal scheme requires some positive reward of studying time as such. Otherwise, if this reward worked solely indirectly through the dependency of benefits on former working income, individuals would continue to maximize current labor income by choosing

\(^7\)The inefficiency of savings as discussed in the previous section is only a consequence of inefficient human capital investment. It therefore vanishes under efficient studying and needs no separate correction.
\( \lambda \) as they do in the laissez-faire equilibrium. Therefore, an additional incentive to accumulate human capital by making allowances for studying time is needed.

Turning to the growth effects of social security, we investigate the balanced growth equilibrium, i.e. the equilibrium where physical capital and effective labor grow at a constant rate which also implies a constant studying time per period. In this equilibrium output and the stock of human capital grow at the same factor which amounts to \( g(\lambda) \) in the laissez-faire economy and to \( g(\lambda^S) \) in the economy with unfunded social security according to (29), where \( \lambda^S \) is implicitly determined by:

\[
F_1 \left[ (1 - \lambda^S)g'(\lambda^S) - \lambda^S \right] + (1 - \lambda^S)g(\lambda^S)g'(\lambda^S) = 0, \quad (30)
\]

with the equilibrium interest factor \( F_1 \) being a constant.\(^8\) Evaluating (30) for \( \lambda = \lambda \) leads to \( \lambda^S > \lambda \) and, hence, to the following result:

**Proposition 3:** In an economy with an unfunded social security system satisfying (29), balanced growth is higher than in the laissez-faire economy.

4. Conclusions

We have shown that the connection between social security, economic growth and efficiency may take on a different or even a reverse form than suggested by the recent literature if human instead of physical capital accumulation is the engine of growth. In our model a higher growth rate can be attained by employing a properly designed unfunded social security system which internalizes intergenerational spillovers of human capital formation. Furthermore, this internalization leads to an efficient allocation, while a laissez-faire economy generally fails to achieve efficiency.

Although we have carried out our analysis in a model that sustains long-run growth, our (in-)efficiency conclusion would also hold in models that are more in the neoclassical tradition provided that there is some human capital transmitted from generation to generation. Only if every generation were to "reinvent the wheel", the type of intergenerational externalities we investigate here would not exist.

A positive effect of unfunded social security on human capital investment has also been derived by Sinn (1998). However, his approach differs from ours in some important respects. First, Sinn's (1998) case for social security rests on an intrafamily moral hazard problem, namely that children, once they are educated, may refuse to compensate their parents for educational investments. This induces parents to invest too little in their

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\(^8\)The equality of output and human capital growth follows immediately from \( \frac{Y_{t+1}}{Y_t} = \frac{f(k_{t+1})}{f(k_t)H_t} = \frac{(1-\lambda)g(\lambda)}{(1-\lambda)g(\lambda)} = g(\lambda) \), since the production function is linear homogeneous, i.e.: \( Y_t = f(K_t/H_t)H_t = f(k_t)H_t \), with \( k_t = K_t/H_t \).
childrens' human capital. This effect can be mitigated by the pension scheme which gives parents a stake in their successors' working income.9 In our model, however, individuals underinvest in their own human capital because they neglect positive side effects of their study, unless they are compensated by a proper pension rule. Second, Sinn (1998) investigates a pension scheme where individual benefits are independent of the human capital investment in one's offspring. He shows that such a system may enhance welfare, but never leads to an efficient allocation. We, instead, derive an pension scheme that restores efficiency by conditioning individual pension benefits on the studying time chosen. And third, his model is static and is not concerned with the growth implications of social security as we are.

It is not our aim to advocate unfunded social security as such. Rather, we provide a counterexample to the existing literature in order to point to a possibly positive feature of real world pension schemes. The relevance of this feature depends, among other things, on the effect that human capital and its intergenerational transmission has on economic development relative to physical capital. Our paper demonstrates that as long as this is an unresolved issue, general statements regarding the impact of social security on economic efficiency and per capita income growth should be made with great caution.

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9There may be other reasons why present workers should be given a share in future wages. Merton (1983), for example, argues that a PAYG system allows individuals to diversify retirement income between physical and human capital in a world where the factor income shares are uncertain.
References


